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Semi-analytical IGA-based computation of wave dispersion in fluid-coupled anisotropic poroelastic plates

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Abstract

Understanding dispersion relations and wave mode shapes is vital in nondestructive control of dynamic behaviors of poroelastic composites. In the framework of semi-analytical finite element (SAFE) method, this paper presents two numerical approaches so-called semi-analytical isogeometric Galerkin (SAIGA-G) and semi-analytical isogeometric collocation (SAIGA-C) for computing dispersion of guided-waves in anisotropic poroelastic plates immersed in acoustic fluids. Biot's theory was used for describing the dynamic behavior of anisotropic poroelastic material. Assuming the structures is homogeneous along its axial direction, the Non-Uniform Rational B-splines (NURBS) was successful employed in procedures using isogeometric Galerkin and collocation methods. The numeral studies showed that the SAIGA-G method using high continuity NURBS basis allowed to significantly improve the accuracy as well as the convergence rate of the wave dispersion solutions in compared with the conventional SAFE method, which used Lagrange basis functions. Otherwise, the SAIGA-C method was shown to have similar performance in terms of accuracy to the SAFE method.

Keywords: Guided-waves, anisotropic poroelasticity, wave dispersion, isogeometric analysis, NURBS basis, IGA collocation

1. Introduction

A poroelastic material consists of two phases, being the elastic solid phase and the fluid phase filling the pore spaces. Many materials encountered in civil, mechanical, geophysical and biomedical engineering can be considered as poroelastic media such as composites, rock, bone, etc. This work is concerned by studying the guided-wave propagation behavior in poroelastic media, which has a broad range of applications, for (non-exhaustive) example, the design of sound absorbing

29 materials [1, 2, 3] or the development of non-destructive testing (NDT) methods [4, 5, 6]. The
30 wave propagation problem in poroelastic materials has been studied many investigations. When
31 the dominant wavelengths of interests are sufficiently large with respect to the characteristic length
32 of microscopic scale, Biot's and mixture theories were mostly employed [7], for which the governed
33 dynamic equations are based on the displacement field of both solid and fluid phases.

34 Many analytical and numerical methods were developed for studying the propagation of guided-
35 waves in poroelastic media. For example, Green's function of a homogeneous poroelastic halfspace
36 may be found in [8, 9]. Transmission and reflection phenomena from poroelastic plate were carefully
37 studied in many works [10, 11]. To study multi-layered poroelastic media, the dynamic stiffness
38 method has been used for deriving the solutions in both frequency and time domains dynamic
39 stiffness method [12, 13]. When the material properties are not piecewise-constant as in layered
40 media but continuously varied, the asymptotic formulation using Peano's series has been developed
41 [14, 15]. Although the analytical methods are generally fast, it has been shown that numerical
42 instabilities may occur due to positive exponential terms for the cases of large layer thickness
43 (in comparing with wavelengths in the medium), requiring some particular techniques [16, 17].
44 Moreover, analytical methods don't allow to consider the structures with complex geometries.
45 Therefore, aside from developed analytical techniques, the so-called semi-analytical finite element
46 method (SAFE) has been widely used. The core idea of SAFE method is to apply the Fourier
47 transform along the direction in which the structure is assumed to be have infinite extent, and
48 use FE discretization in its cross-section's plan. In many studies on visco-elastic waveguides,
49 SAFE method has been shown to be very efficient for determining the primitive characteristics of
50 dispersive guided waves (such as phase velocity, group velocities and attenuation), even when the
51 coupling with surrounding fluids were considered [18, 19]. Using SAFE allows to easy handle the
52 waveguides with functionally-graded properties [20, 21] or with arbitrary section's geometry [22, 23].
53 SAFE method was also successfully used for computing solutions of transient waves propagating in a
54 anisotropic viscoelastic[24], poroelastic plates [25], multi-layered periodic piezoelectric composited
55 plates [26] or thermo-acousto-elastic plates [27].

56 The accuracy of FE solution of wave propagation problems may be improved by employing
57 smooth and higher-order basis functions. It has been shown that using higher-order FE methods
58 (spectral element or p -FE) are efficient for the simulation of wave propagation in poroelastic layered

59 structures [28, 29, 30]. In some studies of guide-waves in elastic structures, spectral element
60 methods (SEM) were also used. In [31], SAFE-SEM coupled with PML has been applied for
61 modeling the leaky wave propagation in the embedded waveguides with cylindrical and square
62 cross-section. Their results of convergence analysis show that the using high order SEM reduces
63 the N_{dof} to achieve a given precision at the high frequency regime. Kalkowski et al. [32] recently
64 used a SAFE-SEM formulation based on Gauss-Lobatto-Jacobi (GLJ) to examine buried and
65 immersed fluid filled pipes. Although attractive features, the computational cost of SAFE for the
66 estimation of high-frequency modes was still expensive because some extensive h - or p - refinements
67 would be required. Moreover, when considering waveguides coupled with fluids, the mode shapes
68 computed by SAFE may have some significant discontinuities at interfaces due to numerical errors
69 of derivative approximation [33].

70 Isogeometric analysis (IGA) [34, 35], which uses the NURBS for the approximation of solutions
71 fields, turned out to be very interesting technique as it exhibits increased accuracy and robustness
72 in comparison to standard FE methods in the context of wave propagation simulation [36, 37]. In
73 a recent work [33], NURBS basis functions was introduced within the SAFE framework and was
74 shown to significantly increase the accuracy of dispersion curves of elastic guided waves in com-
75 parison to the standard SAFE method. Moreover, by employing this isogeometric-based approach
76 (denoted by semi-analytical igeometric analysis - SAIGA), the continuity in mode shapes due the
77 numerical errors at fluid-solid interfaces was much better described, even when simple geometry
78 (plate-like) was considered. Using NURBS has also been shown to better describe the complex
79 section's geometry and curvilinear interface between the solid and the fluid [38].

80 Although the NURBS-based discretization has been proposed for studying the guided-wave
81 propagation in elastic media in numerous works [37, 39], its benefits when considering poroelastic
82 media have not been investigated. For modeling poroelastic materials saturated by a fluid, the
83 system of equations is more complex as it also includes the generalized Darcy's law, of which the
84 resolution has usually numerical difficulties. Especially when the porosity is important and the
85 problem is studied at high frequencies, the attenuation induced from interaction between the fluid
86 and solid phases become much more significant and the robustness of numerical procedure for find-
87 ing complex eigenvalues needs to be carefully investigated. In the literature, it has been shown that
88 using IGA leads to a superior accuracy over FEA [40] for modeling static behavior of poroelastic

89 media,. In the dynamic context, Morganti et al. [41] presented an isogeometric collocation method
90 for studying an 1D poroelastic wave propagation problem. The IGA collocation method [42] has
91 been developed to combine the accuracy and the smoothness advantages of the IGA method with
92 the computational efficiency of the collocation method. Since there are no volume integrals in the
93 IGA collocation, the collocation methods is considerably cheaper from a computational point of
94 view. Moreover, its implementation is simple, since one only need to evaluate the shape functions
95 and the right-hand side data at chosen collocation points. The boundary conditions are imposed as
96 additional constraints in the linear system, which is typically non-symmetric even for self-adjoint
97 problem but more sparse as in the Galerkin method.

98 This paper aims to investigate the potential of isogeometric finite element methods for com-
99 puting the dispersion of guided-waves in poroelastic media. We will focus on the studying the
100 effectiveness of NURBS-based shape functions for the approximation of poroelastic equations in
101 the context of guided waves analysis. To do so, we develop two semi-analytical procedures for solv-
102 ing the dispersion relation equations of guided waves in an anisotropic poroelastic plate immersed
103 in acoustic fluids. The first one employs the isogeometric Galerkin method, which is based on
104 the NURBS discretization of the weak formulation derived in the frequency-wavenumber domain
105 to derive the eigenvalue (dispersion) system of equations. The second one uses the collocation
106 isogeometric method, which based on an NURBS approximation of the strong form of the consid-
107 ered problem to derive the eigenvalue (dispersion) system of equations. To the best of authors'
108 knowledge, the numerical performance of IGA-based methods for computing the dispersion curves
109 of guided waves in poroelastic plates has not been studied in the literature.

110 The paper is organized as follows. Section 2 presents the equations (in strong and weak forms)
111 describing wave propagation problem in coupled fluid-poroelastic system. According to the con-
112 ventional semi-finite element method's framework, Section 3 presents the equations transformed
113 into frequency-wavenumber domain. In Section 4, after introducing the main concepts of IGA, two
114 methods of numerical resolution will be presented: (1) the semi-analytical isogeometric Galerkin
115 (SAIGA-G) method, and (2) the semi-analytical isogeometric collocation (SAIGA-C) method. Sec-
116 tion 5 subsequently carries out the validation and convergence analysis of the proposed methods
117 through several examples. The paper will end with conclusive discussions in Section 6.

118 In the following, the symbol $\partial_i(\star)$ ($i = 1, 2$) stands for the partial derivative of (\star) with respect

119 to x_i ; the symbol ‘ \cdot ’ for the scalar product and the symbol ‘ \cdot ’ between tensors of any order for their
 120 double contraction; \mathbf{a}^* stands for the conjugate transpose (or Hermitian transpose) of a matrix \mathbf{a} .
 121 Einstein summation convention is not used in this manuscript.

122 2. Problem statement

123 In this section, we will first describe the geometry of the fluid-solid coupled waveguide, then
 124 the equations (in both strong and weak forms) for modeling the wave propagation phenomena in
 125 the considered system.

126 2.1. Geometry description

127 Let us consider an anisotropic poroelastic layer (Ω^b) with infinite extents along \mathbf{e}_1 -axis. The
 128 thickness of the layer is assumed to be constant and is denoted by h . The surfaces of this layer
 129 may be stress-free or loaded by two surrounding fluid halfspaces. The domains occupied by the
 130 poroelastic layer and the fluids are denoted by $\Omega^b = \{(x_1, x_2) \in [-\infty, \infty] \times [0, h]\}$, $\Omega_1^f = \{(x_1, x_2) \in$
 131 $(-\infty, \infty) \times (-\infty, 0]\}$, $\Omega_2^f = \{(x_1, x_2) \in (-\infty, \infty) \times [h, +\infty)\}$, respectively. The interfaces between
 132 Ω and the fluid domains Ω_j^f are denoted by Γ_j^{bf} ($j = \{1, 2\}$), respectively. As the thickness is
 133 constant, the outward directed, normal vectors of Ω^b at Γ_1^{bf} and Γ_2^{bf} are given by $\mathbf{n}_1^b = \{0, -1\}^T$
 and $\mathbf{n}_2^b = \{0, 1\}^T$, respectively.

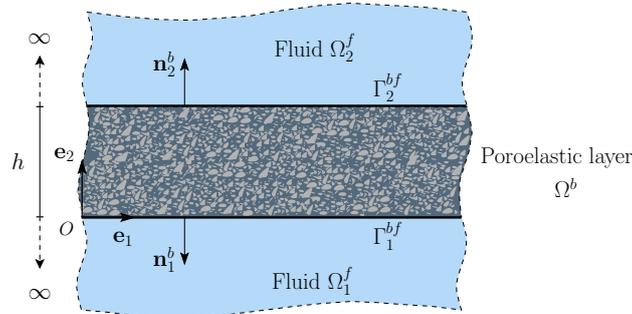


Figure 1: Schematic of a two-dimensional poroelastic waveguide coupled with fluid half-spaces

134

135 2.2. Governing equations

136 2.2.1. Wave equations in the fluids

The fluids occupying the domains Ω_j^f ($j = \{1, 2\}$) are modeled by an acoustic fluid whose mass density and bulk modulus at rest are denoted by K_j and ρ_j , respectively. The linearized wave and

Euler equations in fluids are expressed as follows:

$$\rho_j \ddot{p}_j - K_j \nabla^2 p_j = 0, \quad (1)$$

$$\rho_j \dot{\mathbf{v}}_j + \nabla p_j = \mathbf{0}, \quad (2)$$

137 where p_j and $\mathbf{v}_j(\mathbf{x}, t)$ denote the acoustic pressure and velocity of fluids in Ω_j^f ; $\nabla^2(\star)$ is the Laplace
138 operator. The wave celerity in Ω_j^f can be defined by $c_j = \sqrt{K_j/\rho_j}$.

139 2.2.2. Dynamic equations in the poroelastic domain

The anisotropic poroelasticity theory is used to describe the behavior of porous domain (Ω^b). The poroelastic equations employed here are based upon Biot's original works [43, 44, 45] as well as the recent developments in anisotropic constitutive equations [46, 47, 48]. In Biot's theory the porous media is assumed to consist of a solid skeleton and a connected pore network saturated by fluid (with mass density ρ_f). The sealed pores are considered as part of the solid. It also assumes that the size of the pores is small compared to the characteristic wavelength, and applicability of continuum mechanics to macroscopic two-phase medium. At a point \mathbf{x} and at time t , the vectors of displacement of the solid skeleton and of the interstitial fluid are denoted by $\mathbf{u}^s(\mathbf{x}, t)$ and $\mathbf{u}^f(\mathbf{x}, t)$, respectively; the relative displacement between the fluid and the solid frame weighted by the porosity ϕ is denoted by $\mathbf{w} = \phi(\mathbf{u}^f - \mathbf{u}^s)$. For the purpose of convenience, the Voigt's notation was used for representing the stress, strain and Biot's effective tensors under the vectorial forms as : $\mathbf{s}(\mathbf{x}, t) = \{\sigma_{11}, \sigma_{22}, \sigma_{12}\}^T$, $\mathbf{e}(\mathbf{x}, t) = \{\varepsilon_{11}, \varepsilon_{22}, 2\varepsilon_{12}\}^T$, $\boldsymbol{\alpha}(\mathbf{x}, t) = \{\alpha_{11}, \alpha_{22}, \alpha_{12}\}^T$, where σ_{ij} , ε_{ij} and α_{ij} are the components of the corresponding tensors. By using Voigt's notation, the Biot's constitutive equations for the anisotropic linear poroelastic material [49] read:

$$\mathbf{s} = \mathbf{C}\mathbf{e} - \boldsymbol{\alpha}p, \quad (3)$$

$$p = -M(\mathbf{m}^T \mathbb{L}\mathbf{w} + \boldsymbol{\alpha}^T \mathbb{L}\mathbf{u}^s), \quad (4)$$

140 where p is the interstitial pore pressure; $\mathbf{C}_{3 \times 3}$ is the drained elastic tensor; the scalar M is the
141 Biot's modulus; $\mathbf{m} = (1, 1, 0)^T$; and the operator \mathbb{L} is defined by: $\mathbb{L} = \mathbf{L}_1 \partial_1 + \mathbf{L}_2 \partial_2$, in which
142 \mathbf{L}_1 and \mathbf{L}_2 are 3×2 matrices whose the elements are zeros except the following ones: $L_1(1, 1) =$
143 $L_1(3, 2) = L_2(2, 2) = L_2(3, 1) = 1$.

The wave propagation in the poroelastic medium is described by a system of two coupled equations representing the momentum equation and the dynamic Darcy law [50]. Neglecting the

body forces (other than inertia), this system reads:

$$\rho \ddot{\mathbf{u}}^s + \rho_f \ddot{\mathbf{w}} - \mathbb{L}^T \mathbf{s} = 0 , \quad (5)$$

$$\rho_f \ddot{\mathbf{u}}^s + \tilde{\mathbf{a}} * \ddot{\mathbf{w}} + \mathbb{L}^T \mathbf{m}p = 0 , \quad (6)$$

where $\rho = \phi \rho_f + (1 - \phi) \rho_s$ is the mixture density (or apparent density); ρ_s and ρ_f are mass densities of the solid (matrix) material and the saturating fluid, respectively and ϕ is the porosity. The viscodynamic operator $\tilde{\mathbf{a}}$ is a frequency-dependent symmetric second-order tensor which depend on the permeability and tortuosity of the medium. For materials with orthorhombic symmetry, it has only non-zero diagonal components. In this work, we used the Johnson-Koplik-Dashen (JKD) model [51] in which the diagonal components of $\tilde{\mathbf{a}}$ are given by :

$$a_{jj}(\omega) = \frac{\rho_f}{\phi} \left(a_j^\infty + \frac{i\phi\eta F_j(\omega)}{\omega\rho\kappa_j} \right) , \quad (7)$$

144 where, η denotes the dynamic viscosity of the pore fluid, a_j^∞ and κ_j are the tortuosity and the
145 low-frequency limit permeability of the skeleton in the direction \mathbf{e}_j , respectively.

By noting that $\mathbf{e} = \mathbb{L}\mathbf{u}^s$ and by substituting Eq. (4) into Eq. (3), the vectors \mathbf{s} and $\mathbf{m}p$ in Eqs. (5-6) may be expressed in terms of \mathbf{u}^s and \mathbf{w} as follows:

$$\mathbf{s} = \mathbf{C}_u \mathbb{L}\mathbf{u}^s + \mathbf{C}_\alpha \mathbb{L}\mathbf{w} , \quad (8)$$

$$\mathbf{m}p = -(\mathbf{C}_M \mathbb{L}\mathbf{w} + \mathbf{C}_\alpha^T \mathbb{L}\mathbf{u}^s) , \quad (9)$$

146 where $\mathbf{C}_u = \mathbf{C} + M\boldsymbol{\alpha}\boldsymbol{\alpha}^T$, $\mathbf{C}_\alpha = M\boldsymbol{\alpha}\mathbf{m}^T$, $\mathbf{C}_M = M\mathbf{m}\mathbf{m}^T$. The tensor \mathbf{C}_u is known as the undrained
147 elasticity tensor, which may be considered as the rigidity of an equivalent elastic medium in which
148 the relative movement between the interstitial fluid and solid skeleton is null (*i.e.* when $\mathbf{w} = \mathbf{0}$).
149 One may also notice that while \mathbf{C}_u and \mathbf{C}_M are symmetric, \mathbf{C}_α of an anisotropic medium is not;
150 it could be symmetric only if the considered poroelastic medium is isotropic (*i.e.* when $\alpha_{11} = \alpha_{22}$
151 and $\alpha_{12} = 0$).

152 2.2.3. Boundary conditions

At interfaces between the porous media and the fluid domains. At both interfaces Γ_1^{bf} and Γ_2^{bf} , open pore condition was assumed, leading to a continuity condition of the normal velocity between the poroelastic medium and the surrounding fluids [50] :

$$\dot{\mathbf{w}} \cdot \mathbf{n}_j^b = (\mathbf{v}_j - \dot{\mathbf{u}}^s) \cdot \mathbf{n}_j^b , \quad \forall \mathbf{x} \in \Gamma_j^{bf} \quad (j = \{1, 2\}). \quad (10)$$

In view of the Euler equation (Eq. (2)), this interface condition may be rewritten as:

$$\left(\frac{1}{\rho_j}\nabla p_j + \ddot{\mathbf{w}} + \ddot{\mathbf{u}}^s\right) \cdot \mathbf{n}_j^b = 0, \quad \forall \mathbf{x} \in \Gamma_j^{bf} \quad (j = \{1, 2\}). \quad (11)$$

In addition, the conditions of stress continuity at the porous-fluid interfaces [50] impose that :

$$\left. \begin{array}{l} \mathbf{t} = -p_j \mathbf{n}_j^b \\ p = p_j \end{array} \right\} \quad \forall \mathbf{x} \in \Gamma_j^{bf} \quad (j = \{1, 2\}), \quad (12)$$

153 where \mathbf{n}_j^b is the outward normal unit vector at the interfaces from Ω^b toward Ω_j^f and \mathbf{t} is the
 154 traction vector. It is worth to note that the outward unit vector of the solid domain is related to
 155 unit vector of the fluid domain by $\mathbf{n}_j^b = -\mathbf{n}_j^f$ and $\mathbf{t} = \mathbf{L}_2^T \mathbf{s} = \{\sigma_{12}, \sigma_{22}\}^T$.

Exterior boundaries. The radiation condition of fluid pressure in Ω_j^f ($j = \{1, 2\}$) may be formally written by :

$$p_j \rightarrow 0 \quad \text{when } |\mathbf{x}| \rightarrow \pm\infty, \quad (13)$$

156 2.3. Weak formulations

Weak formulation in the fluid domains. The pressure field in the fluid occupying the domain Ω_j^f is described by Eq. (1). The weak form of this equation by taking into account the boundary conditions Eqs. (13)-(11) reads

$$\int_{\Omega_j^f} \delta p_j^* \rho_j \ddot{p}_j d\Omega_j^f + \int_{\Omega_j^f} \nabla \delta p_j^* K_j \nabla p_j d\Omega_j^f - \int_{\Gamma_j^{bf}} \delta p_j^* \rho_j K_j (\ddot{\mathbf{u}}^s + \ddot{\mathbf{w}}) \cdot \mathbf{n}_j^f d\Gamma_j^{bf} = 0, \quad \forall \delta p_j \in \mathcal{C}^{ad}, \quad (14)$$

Weak formulation in the poroelastic domain. Upon integrating Eqs. (5) and Eqs. (6) against the test function $\delta \mathbf{u}$ and $\delta \mathbf{w}$, respectively and considering the interface conditions Eqs. (13)-(11), the weak formulation of the boundary valued problem in the poroelastic layer Ω^b may be derived as:

$$\begin{aligned} & \int_{\Omega^b} \delta \mathbf{u}^* \rho \ddot{\mathbf{u}}^s dv + \int_{\Omega^b} \delta \mathbf{u}^* \rho_f \ddot{\mathbf{w}} dv + \int_{\Omega^b} \mathbb{L}^T \delta \mathbf{u}^* (\mathbf{C}_u \mathbb{L} \mathbf{u}^s + \mathbf{C}_\alpha \mathbb{L} \mathbf{w}) dv + \int_{\Gamma_j^{bf}} \delta \mathbf{u}^* (p_j \mathbf{n}^b) ds = \mathbf{0}, \\ & \int_{\Omega^b} \delta \mathbf{w}^* (\tilde{\mathbf{a}} \ddot{\mathbf{w}}) dv + \int_{\Omega^b} \delta \mathbf{w}^* \rho_f \ddot{\mathbf{u}}^s dv + \int_{\Omega^b} \mathbb{L}^T \delta \mathbf{w}^* (\mathbf{C}_M \mathbb{L} \mathbf{w} + \mathbf{C}_\alpha^T \mathbb{L} \mathbf{u}^s) dv + \int_{\Gamma_j^{bf}} \delta \mathbf{w}^* (p_j \mathbf{n}^b) ds = \mathbf{0}, \\ & \forall (\delta \mathbf{u}, \delta \mathbf{w}) \in \mathcal{C}^{ad}. \end{aligned} \quad (15)$$

157 3. Equations in the frequency-wavenumber domain

For studying the behavior of guided wave propagating along the longitudinal direction (\mathbf{e}_1) in the medium, we look for harmonic solutions for which a field φ ($\varphi \in \{u_1, u_2, w_1, w_2, p_1, p_2\}$) in Ω may be expressed under following form:

$$\varphi(x_1, x_2, t) = \tilde{\varphi}(x_2)e^{i(k_1x_1 - \omega t)}, \quad (16)$$

158 where $i^2 = -1$; k_1 is the wavenumber in the \mathbf{e}_1 -direction. Note that in the frequency-wavenumber
 159 $(\omega - k_1)$ domain, the derivatives with respect to t and to x_1 can be replaced by: $(\star) \rightarrow -i\omega(\star)$ and
 160 $\partial_1(\star) \rightarrow ik_1(\star)$, respectively. Hence, the considered two-dimensional problem may be simplified
 161 into a system of one-dimensional (1D) PDEs with respect only to x_2 . Moreover, in this study,
 162 as the semi-infinite fluid domains were modeled by using the well-known Perfect Matched Layer
 163 (PML) technique (see [Appendix A](#)), following finite 1D domains could be introduced $\bar{\Omega}^b = [0, h]$,
 164 $\bar{\Omega}^{f1} = [-h_1^f, 0]$ and $\bar{\Omega}^{f2} = [h, h + h_2^f]$ for modeling the poroelastic and fluid regions.

165 3.1. Strong form

By replacing Eq. (16) into Eqs. (1,5 & 6), the performing some algebraic manipulations, the later equations may be transformed into the $(\omega - k_1)$ domain as follows:

$$-\omega^2 \rho_j \tilde{p}_j + k_1^2 K_j \tilde{p}_j - K_j \partial_2^2 \tilde{p}_j = 0, \quad \forall x_2 \in \bar{\Omega}_j^f, \quad j = \{1, 2\}, \quad (17)$$

$$-\omega^2 \mathbf{B}_\rho \mathbf{v} + k_1^2 \mathbf{B}_0 \mathbf{v} - ik_1 (\mathbf{B}_1 + \mathbf{B}_1^T) \partial_2 \mathbf{v} - \mathbf{B}_2 \partial_2^2 \mathbf{v} = \mathbf{0}, \quad \forall x_2 \in \bar{\Omega}^b. \quad (18)$$

where $\mathbf{v} = (\tilde{\mathbf{u}}^s, \tilde{\mathbf{w}})^T$ and:

$$\mathbf{B}_\rho = \begin{bmatrix} \rho \mathbf{I}_2 & \rho_f \mathbf{I}_2 \\ \rho_f \mathbf{I}_2 & \tilde{\mathbf{a}} \end{bmatrix}, \quad \mathbf{B}_0 = \begin{bmatrix} \mathbf{A}_u^{11} & \mathbf{A}_\alpha^{11} \\ \mathbf{A}_\alpha^{11} & \mathbf{A}_M^{11} \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} \mathbf{A}_u^{12} & \mathbf{A}_\alpha^{12} \\ \mathbf{A}_\alpha^{12} & \mathbf{A}_M^{12} \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} \mathbf{A}_u^{22} & \mathbf{A}_\alpha^{22} \\ \mathbf{A}_\alpha^{22} & \mathbf{A}_M^{22} \end{bmatrix} \quad (19)$$

166 in which \mathbf{I}_2 denotes the 2-by-2 identity matrix; the matrices \mathbf{A}_u^{ab} , \mathbf{A}_α^{ab} and \mathbf{A}_M^{ab} with $a, b = \{1, 2\}$
 167 are defined by: $\mathbf{A}_u^{ab} = \mathbf{L}_a^T \mathbf{C}_u \mathbf{L}_b$, $\mathbf{A}_\alpha^{ab} = \mathbf{L}_a^T \mathbf{C}_\alpha \mathbf{L}_b$, $\mathbf{A}_M^{ab} = \mathbf{L}_a^T \mathbf{C}_M \mathbf{L}_b$.

The boundary conditions (Eqs. 11,13) for the fluid domains $\bar{\Omega}^{fj}$ ($j = \{1, 2\}$) may be now expressed as:

$$\partial_2 \tilde{p}_j = \omega^2 \rho_j (\tilde{u}_2^s + \tilde{w}_2), \quad \text{at } x_2 = x_2^{\Gamma_j^b}, \quad (20)$$

$$\tilde{p}_1 = \tilde{p}_2 = 0 \quad \text{at } x_2 = x_2^{\Gamma_j^\infty} \quad (21)$$

where $x_2^{\Gamma_1^b} = 0$, $x_2^{\Gamma_2^b} = h$, $x_2^{\Gamma_j^\infty} = -h_1^f$, $x_2^{\Gamma_2^\infty} = h + h_2^f$ are the vertical coordinates of fluid domains' boundaries, respectively. Similarly, the stress continuity conditions (Eq. 12) transformed into $(\omega - k_1)$ domain read:

$$\left. \begin{aligned} ik_1 (\mathbf{A}_u^{21} \tilde{\mathbf{u}} + \mathbf{A}_u^{22} \tilde{\mathbf{w}}) + (\mathbf{A}_\alpha^{21} \partial_2 \tilde{\mathbf{u}} + \mathbf{A}_\alpha^{22} \partial_2 \tilde{\mathbf{w}}) &= (0, -\tilde{p}_j)^T \\ - ik_1 M (\alpha_{11} \tilde{u}_1 + \alpha_{12} \tilde{u}_2 + \tilde{w}_1) - M (\alpha_{12} \partial_2 \tilde{u}_1 + \alpha_{22} \partial_2 \tilde{u}_2 + \partial_2 \tilde{w}_2) &= \tilde{p} \end{aligned} \right\} \text{ at } x_2 = x_2^{\Gamma_j^b} \quad (22)$$

168 3.2. Weak form

In the frequency-wavenumber domain, the weak formulations of the fluids ($\bar{\Omega}^{f_j}$, $j = \{1, 2\}$), and of the poroelastic layer ($\bar{\Omega}^b$) may be derived by replacing Eq. (16) into Eqs. (14,15):

$$\begin{aligned} -\omega^2 \int_{\bar{\Omega}_j^f} \delta \tilde{p}_j^* \rho_j \tilde{p}_j \gamma_2 dx_2 + k_1^2 \int_{\bar{\Omega}_j^f} \delta \tilde{p}_j^* K_j \tilde{p}_j \gamma_2 dx_2 + \int_{\bar{\Omega}_j^f} \partial_2 \delta \tilde{p}_j^* K_j \partial_2 \tilde{p}_j (1/\gamma_2) dx_2 \\ - \omega^2 \delta \tilde{p}_j^* \rho_j K_j \left(\tilde{u}_2^s(x_2^{\Gamma_j}) + \tilde{w}_2(x_2^{\Gamma_j}) \right) = 0, \end{aligned} \quad (23)$$

$$\begin{aligned} -\omega^2 \int_{\bar{\Omega}^b} \delta \mathbf{v} \cdot \mathbf{B}_\rho \mathbf{v} dx_2 + k_1^2 \int_{\bar{\Omega}^b} \delta \mathbf{v} \cdot \mathbf{B}_0 \mathbf{v} dx_2 + ik_1 \int_{\bar{\Omega}^b} (\partial_2 \delta \mathbf{v} \cdot \mathbf{B}_1 \mathbf{v} - \delta \mathbf{v} \cdot \mathbf{B}_2 \partial_2 \mathbf{v}) dx_2 \\ + \int_{\bar{\Omega}^b} \partial_2 \delta \mathbf{v} \cdot \mathbf{B}_2 \partial_2 \mathbf{v} dx_2 + (\delta \tilde{u}_2(0) + \delta \tilde{w}_2(0)) \tilde{p}_1(0) + (\delta \tilde{u}_2(h) + \delta \tilde{w}_2(h)) \tilde{p}_2(h) = 0, \end{aligned} \quad (24)$$

169 in which $\gamma_2(x_2)$ is a continuous function which was introduced in the PMLs of the fluid domains
170 (see Appendix A).

171 4. Numerical resolutions using NURBS-based approximations

172 In the framework of the conventional the semi-analytical finite element (SAFE) method, the
173 Lagrange polynomials may be used for the approximation of the Eqs. (23)-(24). To do so, it is
174 well known that a sufficient number of grid points per wavelength has to be adjusted, to be able
175 to capture the oscillation of the solution. Thus, the quality of numerical solutions depends on the
176 wave-number k_1 . It has also been shown that for large wave-number problems, using high-order
177 interpolation polynomials would be better to reduce the numerical errors than using lower-order
178 ones with refined element sizes. In this section, we propose to use the high-order interpolation
179 based on NURBS for improve computational efficiency on determining the wave dispersion in
180 high-frequency. The idea is based on the NURBS-based isogeometric analysis allowing the use
181 of globally C^k -continuous basis functions, with $k \leq p - 1$, p being the polynomial degree. In
182 this section, we firstly briefly recall the concept of isogeometric analysis with the main focus on

183 the B-spline and NURBS basis functions. Next, we develop two numerical approaches based
 184 on isogeometric Galerkin and collocation methods to compute the dispersion of guided-waves in
 185 anisotropic poroelastic plates immersed in fluids.

186 4.1. B-spline and NURBS basis functions

In the framework of standard SAFE method, Lagrange polynomials usually served as interpo-
 lation functions for the discretization of weak formulations presented in Sec. 3.2. In this paper, we
 adopt isogeometric-based approaches where the NURBS basis functions are used for discretizing
 the problems (in strong or weak forms) presented in Sec. 3. Basically, B-spline basis functions of
 order q are determined in a parameter domain $\hat{\Omega} \subset \mathbb{R}$ using a sequence of non-decreasing set of
 coordinates called *knot vector* defined as $\Xi = \{\xi_1, \xi_2, \xi_3, \dots, \xi_{n+q+1}\}$, where $\xi_i \in \mathbb{R}$ ($i = 1, 2, \dots, n$) is
 the i^{th} knot and n is the number of basis functions used to construct the B-spline curve. For a given
 knot vector, the corresponding set of B-spline basis functions $B_{i,q}$ are defined by the well-known
 Cox-de Boor recursion formula as:

$$q = 0 : \quad B_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i < \xi < \xi_{i+1}, \\ 0 & \text{otherwise,} \end{cases} \quad (25a)$$

$$q > 0 : \quad B_{i,q}(\xi) = \frac{\xi - \xi_i}{\xi_{i+q} - \xi_i} B_{i,q-1}(\xi) + \frac{\xi_{i+q+1} - \xi}{\xi_{i+q+1} - \xi_{i+1}} B_{i+1,q-1}(\xi). \quad (25b)$$

187 Note that the quotient $0/0$ is assumed to be zero. Open knot vectors, where the first and the last
 188 knot each have a multiplicity of $q + 1$, are standard in CAD NURBS basis functions are built from
 189 the B-spline functions by assigning a weigh w_i to every B-spline function $B_{i,q}(\xi)$:

$$R_{i,q}(\xi) = \frac{B_{i,q}(\xi)w_i}{\sum_{j=1}^n B_{j,q}(\xi)w_j}. \quad (26)$$

190 The NURBS basis functions have some advantage such as higher continuity across the element
 191 boundaries, partition of unity, variation diminishing, linear independence and compact support.
 192 Such a complete set of basis functions can be employed in any finite finite element or collocation
 193 framework. According to the isogeometric concept in which the same basis functions are used
 194 for the approximation of solution fields and for geometry representation, the approximation of a
 195 complex-valued function $\varphi(x)$, denoted by $\hat{\varphi}^h$, is given by :

$$\varphi(x) \approx \hat{\varphi}^h = \sum_{i=1}^n R_{i,q}(\xi)\Phi_i, \quad (27)$$

196 where the complex-valued coefficients Φ_i are the corresponding control variables (values at the
 197 control points). Using the inversion of geometrical mapping $x_2(\xi)$ the function \hat{v}^h over the physical
 198 domain Ω can be define such that $v^h = \hat{v}^h \circ x^{-1}$.

199 4.2. Semi-analytical isogeometric Galerkin method (SAIGA-G)

For simplify the presentation, we assume that each domain $\bar{\Omega}_1^f, \bar{\Omega}_2^f$ and $\bar{\Omega}^b$ is discretized using only one patch with n_1, n_b, n_2 being the number of basis functions in the patches of $\bar{\Omega}^{f1}, \bar{\Omega}^b, \bar{\Omega}^{f2}$, respectively. By using the Galerkin finite element method, same approximations are applied for both functions $\tilde{\mathbf{u}}^h$ and $\delta\tilde{\mathbf{u}}^h$ (as well as for $\tilde{\mathbf{w}}^h$ and $\delta\tilde{\mathbf{w}}^h$; \tilde{p}_α^h and $\delta\tilde{p}_\alpha^h$) on each patch:

$$\tilde{\mathbf{u}}^h = \mathbf{R}^u \mathbf{U}, \quad \delta\tilde{\mathbf{u}}^h = \mathbf{R}^u \delta\mathbf{U}, \quad (28a)$$

$$\tilde{\mathbf{w}}^h = \mathbf{R}^w \mathbf{W}, \quad \delta\tilde{\mathbf{w}}^h = \mathbf{R}^w \delta\mathbf{W}, \quad (28b)$$

$$\tilde{p}_j^h = \mathbf{R}_j^p \mathbf{P}_j, \quad \delta\tilde{p}_j^h = \mathbf{R}_j^p \delta\mathbf{P}_j, \quad (28c)$$

where $\mathbf{R}_{2 \times 2n_b}^u, \mathbf{R}_{2 \times 2n_b}^w$ and $(\mathbf{R}_j^p)_{1 \times n_j}$ are the interpolation matrix containing the NURBS basis functions (Eq. (26)); $\mathbf{U}_{2n_b \times 1}, \delta\mathbf{U}_{2n_b \times 1}, \mathbf{W}_{2n_b \times 1}$ and $\delta\mathbf{W}_{2n_b \times 1}$ are the vectors of control displacements; $(\mathbf{P}_j)_{n_j \times 1}$ and $(\delta\mathbf{P}_j)_{n_j \times 1}$ are the pressure vectors of control pressures. In this work, we used the same NURBS basis functions for approximation of the solution fields in Eqs. (28a-28c). By substituting the approximations (Eqs. 28a-28c) into the weak formulations (Eqs. 23-24), one obtains a eigenvalue problem:

$$(-\omega^2 \mathbf{M} + \mathbf{K}_0 + ik_1 \mathbf{K}_1 + k_1^2 \mathbf{K}_2) \mathbf{V} = \mathbf{0}, \quad (29)$$

where $\mathbf{V} = (\mathbf{P}_1, \mathbf{U}, \mathbf{W}, \mathbf{P}_2)^T$ contains the global eigenvectors the fluid pressures ($\mathbf{P}_1, \mathbf{P}_2$) in fluids and of displacements (\mathbf{U}, \mathbf{W}) in the proelastic domain, respectively; the global matrices \mathbf{M} and $\mathbf{K}_0, \mathbf{K}_1, \mathbf{K}_2$ are are square matrices of order $(n_1 + 4n_b + n_2)$ and may be determined from the

assembling of corresponding elementary matrices in poroelastic and fluid domains:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}^{p_1} & \mathbf{M}^{p_1 u} & \mathbf{M}^{p_1 w} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^u & \mathbf{M}^{uw} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{wu} & \mathbf{M}^w & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{p_2 u} & \mathbf{M}^{p_2 w} & \mathbf{M}^{p_2} \end{bmatrix}, \mathbf{K}_0 = \begin{bmatrix} \mathbf{K}_0^{p_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{K}_0^{up_1} & \mathbf{K}_0^u & \mathbf{K}_0^{uw} & \mathbf{K}_0^{up_2} \\ \mathbf{K}_0^{wp_1} & \mathbf{K}_0^{wu} & \mathbf{K}_0^w & \mathbf{K}_0^{wp_2} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_0^{p_2} \end{bmatrix}, \quad (30a)$$

$$\mathbf{K}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_1^u & \mathbf{K}_1^{uw} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_1^{wu} & \mathbf{K}_1^w & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} \mathbf{K}_2^{p_1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2^u & \mathbf{K}_2^{uw} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}_2^{wu} & \mathbf{K}_2^w & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{K}_2^{p_2} \end{bmatrix}, \quad (30b)$$

In these matrices, the sub-matrices related to the poroelastic solution fields (\mathbf{U}, \mathbf{W}) and fluid pressure fields ($\mathbf{P}_1, \mathbf{P}_2$) may be distinguished. For the poroelastic layer, the matrices related to the \mathbf{U} are defined as:

$$\mathbf{M}^u = \int_{\bar{\Omega}^b} (\mathbf{R}^u)^T \rho \mathbf{R}^u dx_2, \quad \mathbf{M}^{uw} = \int_{\bar{\Omega}^b} (\mathbf{R}^u)^T \rho_f \mathbf{R}^w dx_2, \quad (31a)$$

$$\mathbf{K}_2^u = \int_{\bar{\Omega}^b} (\mathbf{R}^u)^T \mathbf{A}_u^{11} \mathbf{R}^u dx_2, \quad \mathbf{K}_2^{uw} = \int_{\bar{\Omega}^b} (\mathbf{R}^u)^T \mathbf{A}_\alpha^{11} \mathbf{R}^w dx_2, \quad (31b)$$

$$\mathbf{K}_1^u = \int_{\bar{\Omega}^b} 2 \{ (\partial_2 \mathbf{R}^u)^T \mathbf{A}_u^{21} \mathbf{R}^u \}_a dx_2, \quad \mathbf{K}_1^{uw} = \int_{\bar{\Omega}^b} 2 \{ \partial_2 (\mathbf{R}^u)^T \mathbf{A}_\alpha^{21} \mathbf{R}^w \}_a dx_2, \quad (31c)$$

$$\mathbf{K}_0^u = \int_{\bar{\Omega}^b} \partial_2 (\mathbf{R}^u)^T \mathbf{A}_u^{22} \partial_2 \mathbf{R}^u dx_2, \quad \mathbf{K}_0^{uw} = \int_{\bar{\Omega}^b} \partial_2 (\mathbf{R}^u)^T \mathbf{A}_\alpha^{22} \partial_2 \mathbf{R}^w dx_2, \quad (31d)$$

$$\left(\mathbf{K}_0^{up_1} \right)_{\ell k} = \delta_{\ell 2} \delta_{kn_1}, \quad \left(\mathbf{K}_0^{up_2} \right)_{\ell k} = \delta_{\ell(2n_b)} \delta_{k1}, \quad (31e)$$

where $\{\cdot\}_a$ denotes the anti-symmetric part of a matrix; δ_{jk} denotes the Kronecker's delta; the upper indexes uw or up represent the couplings between \mathbf{U} & \mathbf{W} or \mathbf{U} & \mathbf{P}_j , respectively. Similarly, the matrices related to the \mathbf{W} are given by :

$$\mathbf{M}^w = \int_{\bar{\Omega}^b} (\mathbf{R}^w)^T \tilde{\mathbf{a}} \mathbf{R}^w dx_2, \quad \mathbf{M}^{wu} = \int_{\bar{\Omega}^b} (\mathbf{R}^w)^T \rho_f \mathbf{R}^u dx_2, \quad (32a)$$

$$\mathbf{K}_2^w = \int_{\bar{\Omega}^b} (\mathbf{R}^w)^T \mathbf{A}_M^{11} \mathbf{R}^w dx_2, \quad \mathbf{K}_2^{wu} = \int_{\bar{\Omega}^b} (\mathbf{R}^w)^T \mathbf{A}_\alpha^{11} \mathbf{R}^u dx_2, \quad (32b)$$

$$\mathbf{K}_1^w = \int_{\bar{\Omega}^b} 2 \{ (\partial_2 \mathbf{R}^w)^T \mathbf{A}_M^{21} \mathbf{R}^w \}_a dx_2, \quad \mathbf{K}_1^{wu} = \int_{\bar{\Omega}^b} 2 \{ (\partial_2 \mathbf{R}^w)^T \mathbf{A}_\alpha^{21} \mathbf{R}^u \}_a dx_2, \quad (32c)$$

$$\mathbf{K}_0^w = \int_{\bar{\Omega}^b} (\partial_2 \mathbf{R}^w)^T \mathbf{A}_M^{22} \partial_2 \mathbf{R}^w dx_2, \quad \mathbf{K}_0^{wu} = \int_{\bar{\Omega}^b} (\partial_2 \mathbf{R}^w)^T \mathbf{A}_\alpha^{22} \partial_2 \mathbf{R}^u dx_2, \quad (32d)$$

$$\left(\mathbf{K}_0^{wp_1} \right)_{\ell k} = \delta_{\ell(2n_b+2)} \delta_{kn_1}, \quad \left(\mathbf{K}_0^{wp_2} \right)_{\ell k} = \delta_{\ell(4n_b)} \delta_{k1}, \quad (32e)$$

where the upper indexes wu or wp represent the couplings between \mathbf{W} & \mathbf{U} or \mathbf{W} & \mathbf{P}_j , respectively. The matrices related to the \mathbf{P}_1 and \mathbf{P}_2 are defined as:

$$\mathbf{M}^{pj} = \int_{\bar{\Omega}} \rho_j \gamma_{2j} (\mathbf{R}_j^p)^T \mathbf{R}_j^p dx_2, \quad (33a)$$

$$\mathbf{K}_0^{pj} = \int_{\bar{\Omega}} (K_j / \gamma_{2j}) (\partial_2 \mathbf{R}_j^p)^T \partial_2 \mathbf{R}_j^p dx_2, \quad \mathbf{K}_2^{pj} = \int_{\bar{\Omega}} K_j \gamma_{2j} (\mathbf{R}^p)^T \mathbf{R}^p dx_2, \quad (33b)$$

$$\left(\mathbf{M}^{p_1 u} \right)_{\ell k} = \rho_1 K_1 \delta_{\ell(n_1)} \delta_{k2}, \quad \left(\mathbf{M}^{p_1 w} \right)_{\ell k} = \rho_1 K_1 \delta_{\ell(n_1)} \delta_{k(2n_b+2)}, \quad (33c)$$

$$\left(\mathbf{M}^{p_2 u} \right)_{\ell k} = \rho_2 K_2 \delta_{\ell 1} \delta_{k(2n_b)}, \quad \left(\mathbf{M}^{p_2 w} \right)_{\ell k} = \rho_2 K_2 \delta_{\ell 1} \delta_{k(4n_b)} \quad (33d)$$

Further more due to the fact that $\mathbf{A}_{\alpha\beta} = \mathbf{A}_{\beta\alpha}^T$, it can be shown that that the sub-matrices of the \mathbf{M} , \mathbf{K}_0 , \mathbf{K}_2 are symmetric while the sub-matrices of \mathbf{K}_1 is anti-symmetric. Note that the in the case where there is no fluid half-spaces the fluid contribution and the corresponding coupling matrices are zeros matrices. All these sub-matrices are computed by using Gauss–Legendre quadrature formula with $r = q + 1$ quadrature nodes per element along each parametric direction which has been shown to be efficient [52].

4.3. Semi-analytical isogeometric collocation (SAIGA-C)

To construct the 1D isogeometric collocation method, we follow [42] by firstly choosing three sets of collocation points $\tau_i^{f_1}$ ($i = 1, \dots, n_1^{\text{col}}$), τ_j^b ($j = 1, \dots, n_b^{\text{col}}$) and $\tau_k^{f_2}$ ($k = 1, \dots, n_2^{\text{col}}$) for $\bar{\Omega}^{f_1}$, $\bar{\Omega}^{f_2}$ and $\bar{\Omega}^b$, respectively. Then we seek the approximations of the fields \tilde{p}_1 , \tilde{p}_2 , $\tilde{\mathbf{u}}$, $\tilde{\mathbf{w}}$ at the collocation points by using the associated NURBS basis functions. In this study, the employed collocation points (τ_i^φ) of a field φ ($\varphi = \{u_1, u_2, w_1, w_2, p_1, p_2\}$) are located at the images of the so-called ‘‘Greville abscissa’’. In general, using a given open knot vector $\Xi = \{\xi_1 = 0, \xi_2, \xi_3, \dots, \xi_{n+q+1} = 1\}$, in which the first and the last knots have multicity $q + 1$, the Greville abscissae points $\bar{\xi}_i$ ($i = 1, 2, \dots, n$) in a parametric space are calculated by

$$\bar{\xi}_i = \frac{\xi_{i+1} + \xi_{i+2} + \dots + \xi_{i+q}}{q}. \quad (34)$$

By building matrices which contain the k -th derivatives ($k = 0, 1, 2$) of the NURBS shape functions of the field φ at all collocation points (τ_i^φ), the k -th derivatives of φ may be approximated by

(Eq. 26)

$$\begin{pmatrix} \partial_2^{(k)} \varphi(\tau_1^\varphi) \\ \partial_2^{(k)} \varphi(\tau_2^\varphi) \\ \vdots \\ \partial_2^{(k)} \varphi(\tau_{n_\varphi^{\text{col}}}^\varphi) \end{pmatrix} \approx \begin{bmatrix} \partial_2^{(k)} R_1^\varphi(\tau_1^\varphi) & \partial_2^{(k)} R_2^\varphi(\tau_1^\varphi) & \cdots & \partial_2^{(k)} R_{n_\varphi^{\text{col}}}^\varphi(\tau_1^\varphi) \\ \partial_2^{(k)} R_1^\varphi(\tau_2^\varphi) & \partial_2^{(k)} R_2^\varphi(\tau_2^\varphi) & \cdots & \partial_2^{(k)} R_{n_\varphi^{\text{col}}}^\varphi(\tau_2^\varphi) \\ \cdots & \cdots & \cdots & \cdots \\ \partial_2^{(k)} R_1^\varphi(\tau_{n_\varphi^{\text{col}}}^\varphi) & \partial_2^{(k)} R_2^\varphi(\tau_{n_\varphi^{\text{col}}}^\varphi) & \cdots & \cdots \partial_2^{(k)} R_{n_\varphi^{\text{col}}}^\varphi(\tau_{n_\varphi^{\text{col}}}^\varphi) \end{bmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_{n_\varphi^{\text{col}}} \end{pmatrix}, \quad (35)$$

215 where Φ_i denotes the solution of $\varphi(\tau_i^\varphi)$ at the control points.

In the fluid domain $\bar{\Omega}_j^f$ ($j = 1, 2$), giving a set of collocation points τ_i^{fj} ($i = 1, \dots, n_j^{\text{col}}$), one may *a priori* derive from (Eq. (17)) a system of (n_j^{col}) equations at every collocations points:

$$-\omega^2 \rho_j \tilde{p}_j(\tau_i^{fj}) + k_1^2 K_j \tilde{p}_j(\tau_i^{fj}) - K_j \partial_2^2 \tilde{p}_j(\tau_i^{fj}) = 0. \quad (36)$$

However, as this system should satisfy the boundary conditions (20), two equations at the first and last collocation points should be replaced, for instance in the lower fluid domain $\bar{\Omega}_1^f$, by:

$$\tilde{p}_1(\tau_1^{f1}) = 0, \quad (37)$$

$$\partial_2 \tilde{p}_1(\tau_{n_1^{\text{col}}}^{f1}) + \rho_1 \omega^2 \left(\tilde{u}_2(\tau_1^b) + \tilde{w}_2(\tau_1^b) \right) = 0. \quad (38)$$

216 Similar conditions may be set for the upper fluid domain $\bar{\Omega}_2^f$.

Considering the poroelastic domain $\bar{\Omega}^b$, the approximate solutions $\tilde{\mathbf{u}}$ and $\tilde{\mathbf{w}}$ are evaluated at the collocation points τ_i^b ($i = 1, \dots, n_b^{\text{col}}$) should satisfy (18):

$$-\omega^2 \mathbf{B}_\rho \mathbf{v}(\tau_i^b) + k_1^2 \mathbf{B}_0 \mathbf{v}(\tau_i^b) - ik_1 (\mathbf{B}_1 + \mathbf{B}_1^T) \partial_2 \mathbf{v}(\tau_i^b) - \mathbf{B}_2 \partial_2^2 \mathbf{v}(\tau_i^b) = \mathbf{0}, \quad (39)$$

At the Γ_1^{bf} (interface between Ω_1^f and Ω^b), the stress continuity conditions (22) may be expressed in matrix form as follows:

$$ik_1 \begin{bmatrix} \mathbf{A}_u^{21} & \mathbf{A}_u^{22} \\ [M\alpha_{11}, M\alpha_{12}] & [1, 0] \end{bmatrix} \mathbf{v}(\tau_1^b) + \begin{bmatrix} \mathbf{A}_\alpha^{21} & \mathbf{A}_\alpha^{22} \\ [M\alpha_{12}, M\alpha_{22}] & [0, 1] \end{bmatrix} \partial_2 \mathbf{v}(\tau_1^b) = - \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tilde{p}_1(\tau_{n_1^{\text{col}}}^{f1}) \quad (40)$$

217 Again, a similar condition may be set at the interface Γ_2^{bf} .

By applying the approximations (35) to the Eqs. (36-40), then performing some algebraic manipulations for assembling the matrices, we can establishing a eigenvalue system containing

$n_1^{\text{col}} + 4 \times n_b^{\text{col}} + n_2^{\text{col}}$ equations which has the same form as the one derived by SAIGA-G procedure (Eq. (29)):

$$(\mathbf{K}_0 + ik_1\mathbf{K}_1 + k_1^2\mathbf{K}_2 - \omega^2\mathbf{M})\mathbf{V} = \mathbf{0}, \quad (41)$$

218 4.4. Resolution of dispersion equations

The system of characteristic equations (29) and (41) are an eigenvalue problem which are used to determine the relationship between the pulsation ω and the wavenumber k_1 . By noting that all global matrices \mathbf{K}_0 , \mathbf{K}_1 , and \mathbf{K}_2 do not depend on k_1 , Eqs. (29) and (41) are a quadratics eigenvalue problem with respect to k_1 and could be solved by reformulating them under following linearized eigenvalue problem:

$$\left(\begin{bmatrix} \mathbf{0} & -\omega^2\mathbf{M} + \mathbf{K}_0 \\ -\omega^2\mathbf{M} + \mathbf{K}_0 & i\mathbf{K}_1 \end{bmatrix} - k_1 \begin{bmatrix} -\omega^2\mathbf{M} + \mathbf{K}_0 & \mathbf{0} \\ \mathbf{0} & -\mathbf{K}_2 \end{bmatrix} \right) \begin{pmatrix} \mathbf{V} \\ k_1\mathbf{V} \end{pmatrix} = \mathbf{0}. \quad (42)$$

For each value of the angular frequency ω , solving Eq. (42) allows us to determine the eigenvalues k_1 and their associated eigenvectors (also called by wave structures), $\mathbf{V}(\omega, k_1)$ of guided modes. The frequency-dependent phase velocity ($C_{\text{ph}}(\omega)$) and the attenuation ($\gamma(\omega)$) of a guided mode are given by:

$$C_{\text{ph}} = \frac{\omega}{\text{Re}(k_1)} [\text{m.s}^{-1}], \quad \gamma = \text{Im}(k_1) [\text{Np.m}^{-1}], \quad (43)$$

219 where $\text{Re}()$ and $\text{Im}()$ denote the real and imaginary parts of a complex function.

220 5. Numerical examples

221 In this section, we will investigate the performance of the proposed NURBS-based methods (*i.e.*
222 SAIGA-C and SAIGA-G methods) for computing the dispersion of poroelastic guided-waves in
223 free and fluid coupled anisotropic poroelastic layers. For all examples, the evaluation of the phase
224 velocities, the attenuations as well the mode shapes have been performed by using three approaches
225 SAIGA-C, SAIGA-G and conventional SAFE. As the closed form of analytical solutions are not
226 available, the SAFE's solutions obtained by very fine meshes, which are assumed to be the converged
227 solutions, are served as the reference ones for estimating the numerical errors.

228 5.1. Case of free-boundaries homogeneous poroelastic layers

229 5.1.1. Reference solutions

230 We consider a 4-mm-thick saturated anisotropic poroelastic plate, which represents a typical
 231 cortical bone sample [25]. The porous bone material is assumed to be saturated by water of which
 232 the bulk modulus is $K_f = 2.5$ GPa and the density is $\rho_f = 1000$ kg.m⁻³. The bone’s matrix has a
 233 density $\rho_s = 1772$ kg.m⁻³ and its components of the elastic tensor (using the Voigt’s notations) are
 234 given by : $c_{11}^m = 28.7$ GPa, $c_{22}^m = 23.6$ GPa, $c_{12}^m = 9.9$ GPa, $c_{66}^m = 7.25$ GPa and $c_{16}^m = c_{26}^m = 0$ GPa.
 235 By using the procedure presented in Appendix B, one can estimate the components of the drained
 236 elastic tensor \mathbf{C} . For example, the components of \mathbf{C} are given in Tab. 5.1.1 for the porosity of
 237 $\phi = 0.1$ and $\phi = 0.5$. In this numerical study, we assume that the permeability and the tortuosity
 238 in the considered medium are isotropic. The diagonal terms of the tortuosity tensor is given by:
 $a_{11} = a_{22} = 1$.

Table 1: Parameters of the poroelastic medium for different values of the porosity.

ϕ	c_{11}	c_{12}	c_{22}	c_{66}	$c_{16} = c_{26}$	M	$\kappa_{11} = \kappa_{22}$
-	(GPa)	(GPa)	(GPa)	(GPa)	(GPa)	(GPa)	(m ²)
0.1	24.84	7.27	17.5	5.49	0	18.32	3.33×10^{-13}
0.5	12.76	2.33	5.71	1.87	0	4.19	1.67×10^{-12}

239
 240 We first present in Figs. 2(a,b) the results of the phase velocity and attenuation computed by
 241 using the conventional SAFE formulation with a very fine mesh ($N_{\text{dof}} = 404$ and $q = 2$), which
 242 would be sufficient to be considered as a reference solutions. Due to the presence of fluid-filled
 243 pores, there exist some guided-wave modes with high attenuation. In Fig. 2(a,b), the dispersion
 244 curves are displayed in two different colors for separated the “low attenuation” (group 1) and
 245 “high attenuation” (group 2) modes, which have attenuation ($\gamma = \text{Im}(k_1)$) greater or smaller than
 246 a value $\bar{\gamma} = 100$ m⁻¹, respectively. The phase velocity curves of low-attenuation modes (Fig. 2a)
 247 are similar to the well-known Lamb-type modes in elastic plates, which are usually designated
 248 by the modes A or the modes S in depending to their antisymmetric/symmetric natures. For
 249 these modes, the relative fluid-solid displacement (\mathbf{w}), which is the main cause of viscous effect in
 250 poroelastic materials, are much weaker than the displacements of the solid (\mathbf{u}). As an illustration,

251 Fig. 3(a) depicts the shape modes of u_2 and w_2 of the mode A_1 at the frequency $f = 1$ MHz, in
 252 which w_2 is found much smaller than u_2 . Moreover, for this anti-symmetric mode, one may check
 253 that $u_2(0) = u_2(h)$ and $w_2(0) = w_2(h)$ with one wavelength over the plate thickness. Figs. 3(b,c)
 254 present the in-depth variation of u_2 and w_2 of two high attenuated modes, which correspond to
 255 two points B and C at the same frequency ($f = 1$ MHz) as marked in Fig. 2. One may observe
 256 that the orders of these modes are higher with more wavelengths over the thickness. Moreover, the
 257 fluid-solid relative displacement becomes more significant in comparing to the solid displacement
 258 u_2 , which induce more important attenuation as expected.

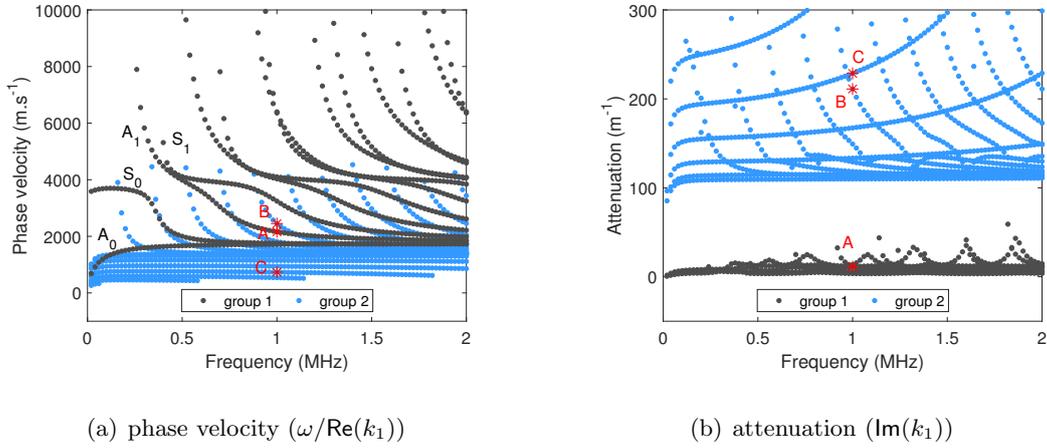


Figure 2: Dispersion curves (a) phase velocity versus frequency and (b) attenuation versus frequency of a free poroelastic bone layer; groups 1 and 2 represent low and high attenuated waves, respectively; a filtering criteria $\bar{\gamma} = 100 \text{ m}^{-1}$

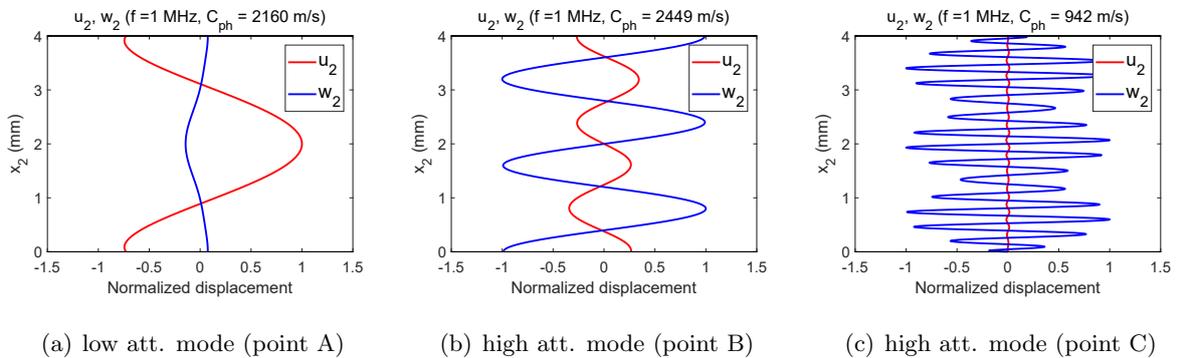


Figure 3: (Color online) (u_2, w_2) shape modes corresponding to the points (A),(B),(C) marked in Fig. 2

259 The wave dispersion in poroelastic plates depends on its porosity [53]. In Fig. 4, the phase
260 velocity (C_{ph}) and attenuation (γ) of the first symmetric mode S_0 are shown to be strongly modified
261 while the porosity changes. To visualize the effect of porosity on the wave dispersion, we present
262 in Fig. 4(a) the variation of real and imaginary parts of the wavenumber (k_1) *versus* frequency in
263 a 3D graph. Different to the case of elastic plates in which the real wavenumber solutions exist,
264 the wavenumbers of guided-waves poroelastic plates are complex. When $\phi = 0.1$, $\text{Im}(k_1)$ is small
265 even at high frequency and the k_1 -curve is found closely to the plane $\omega - \text{Re}(k_1)$. The peaks of the
266 attenuation curves may be observed at frequencies where higher modes are cut-on. When $\phi = 0.5$,
267 the imaginary part $\text{Im}(k_1)$ was found to be much higher because the viscous effect due to fluid-solid
268 relative movement becomes more significant, especially at high frequency. In high frequency range
269 ([1.5-2.0] MHz), the phase velocity of S_0 -mode tends to the Rayleigh wave's velocity (let us denote
270 it by c_R), which depend mainly to the porosity and to components of drained elasticity tensor. As
271 it has been shown in [54], the ratio c_R^2/c_{66} is monotonically increasing function with respect to the
272 porosity. However, the shear modulus c_{66} (estimated by using the model presented in Appendix
273 B) was found to be a monotonically decreasing function with respect to the porosity (see Table 1).
274 Hence, in contrast with the attenuation which depends on the permeability and typically increases
275 with higher porosity, varying the porosity may lead to insignificant change of the S_0 -mode's phase
276 velocity at high frequencies.

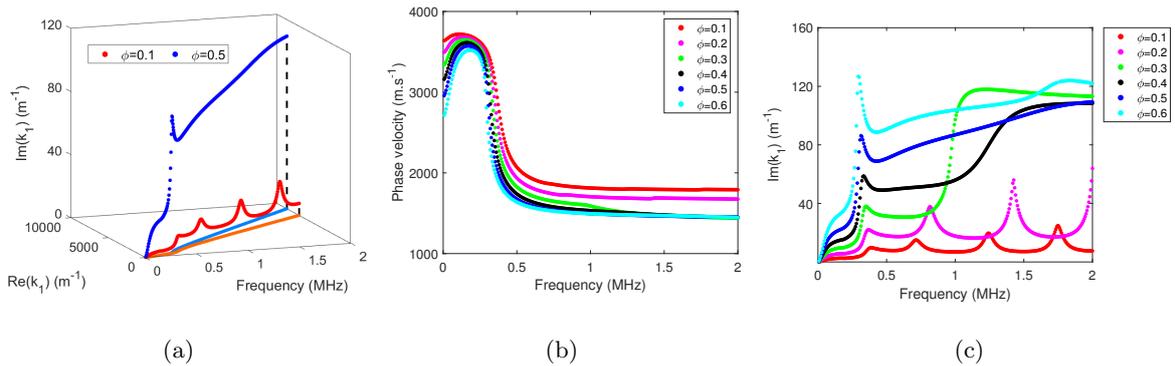


Figure 4: (Color online) Dependence of S_0 -mode dispersion on porosity: (a) wavenumber ($\text{Re}(k_1)$ and $\text{Im}(k_1)$); (b) phase velocity; (c) attenuation

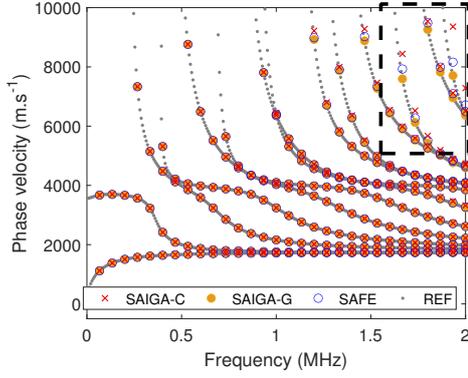
277 It worth noticing that, when considering an elastic plate at very low frequency range, the phase
278 velocity of the S_0 mode was found to be constant (non dispersive), being approximately equal to

279 the compressional wave velocity. This phenomenon may be explained by the fact that, at very low
 280 frequency, as the wavelength of the compressional wave is much longer than the plate's thickness,
 281 the S_0 mode may be seen as a plane compressional wave propagating along axial direction of
 282 the plate. In this study where the poroelastic plates were studied, the phase velocity of S_0 mode
 283 at low frequency limit (Fig. 4) was found not be constant but very dispersive, especially when
 284 the porosity was high. In fact, it might be guessed that the dispersive feature of the S_0 -branch
 285 at low frequency range lies to the dynamic behavior of poroelastic materials. In a poroelastic
 286 medium, there exists not only one but two compressional waves, which are known as the fast and
 287 slow P-waves. While the wavelength of the fast P-wave is quasi-constant and comparable to the
 288 compressional wave in the elastic medium, the wavelength of slow P-wave is typically very small at
 289 the low-frequency limit. The presence of the slow P-wave, which is very dispersive in low-frequency,
 290 would be the reason for which the S_0 mode is dispersive at low-frequency range. We may also
 291 observe that when the porosity tends to zero, the S_0 -mode branch tends to a constant function at
 292 low frequency, which is similar to elastic plate cases.

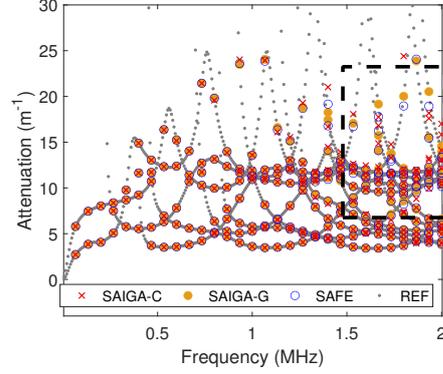
293 5.1.2. Validation of SAIGA-G and SAIGA-C methods

294 The results obtained by the proposed SAIGA-G and SAIGA-C methods were validated by
 295 comparing with the reference solution computed by conventional SAFE method. In this example,
 296 both SAIGA-G and SAIGA-C modeling are based on a single patch in which the NURBS basis
 297 functions with order q over an uniform knot vector were used. The number of elements (spans)
 298 in the patch is then given by $N_{\text{el}} = n_{\text{P}} - q = N_{\text{dof}}/n_{\text{dof}} - q$. Consequently, the total number
 299 of degrees of freedom (N_{dof}) is: $N_{\text{dof}}^{\text{SAIGA-G}} = N_{\text{dof}}^{\text{SAIGA-C}} = n_{\text{P}} \times n_{\text{dof}}$, where n_{P} is the number of
 300 control points (or nodes) and $n_{\text{dof}} = 4$ which is the number of unknowns (u_1, u_2, w_1, w_2) at each
 301 control point. On the other hand, by using the conventional SAFE method with $N_{\text{el}}^{\text{SAFE}}$ elements
 302 with q^{th} -order Lagrange interpolation function, the total number of degree of freedom is given by:
 303 $N_{\text{dof}}^{\text{SAFE}} = (q \times N_{\text{el}}^{\text{SAFE}} + 1) \times n_{\text{dof}}$.

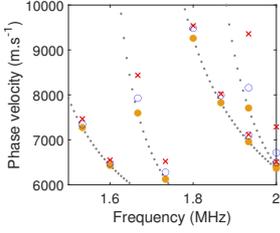
304 Dispersion curves of the phase velocity and attenuation *versus* the frequency of first 14 modes
 305 in the range $f = 0\text{--}2$ MHz were computed by using SAIGA-G, SAIGA-C and SAFE methods.
 306 Then these solutions were compared with the reference solution ($q = 2$, $N_{\text{dof}} = 404$) as shown
 307 in Figs. 5(a,b). In this example, all methods employed quartic-order ($q = 4$) basis functions and
 308 have the same $N_{\text{dof}} = 68$: the SAIGA-G method used 13 elements, the SAIGA-C method used 17



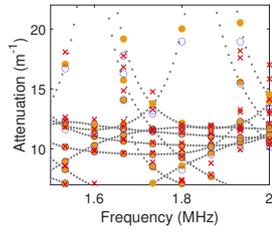
(a) C_{ph} using $q = 4$



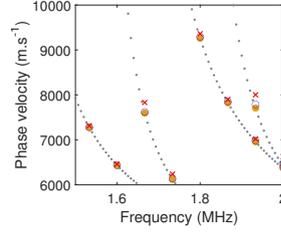
(b) γ using $q = 4$



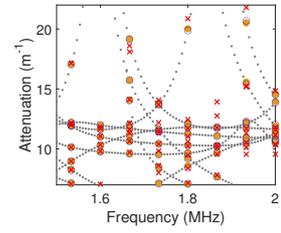
(c) C_{ph} with $q = 4$



(d) γ using $q = 4$



(e) C_{ph} using $q = 8$



(f) γ using $q = 8$

Figure 5: (Color online) Case of a free poroelastic layer ($\phi = 0.1$): validation of SAIGA-C, SAIGA-G and SAFE solutions computed with $q = 4$, $N_{\text{dof}} = 68$; sub-figures (c) and (d) are zoomed from the dashed-line windows in sub-figures (a) and (b), respectively.

309 collocation points, and the SAFE method used 4 elements. At low frequencies, all of three methods
 310 provide accurate estimations of C_{ph} as well as of γ . However, at high frequencies (1.5-2 MHz), the
 311 SAIGA-G solution has better precision than the ones obtained by SAIGA-C and SAFE methods,
 312 especially the modes presented in the dashed-line windows plotted in Fig. 5(a,b), which respectively
 313 are zoomed in Figs. 5(c,d) for a clearer comparison. Moreover, it may be observed in Figs. 5(c,d)
 314 that SAFE solutions didn't capture exactly the reference ones as the SAIGA-G solutions did, but
 315 they are better than SAIGA-C ones. In Figs. 5(e,f), we present the solutions computed by using
 316 $q = 8$ and by arranging the number of elements/spans ($N_{\text{el}}^{\text{SAIGA-C}} = N_{\text{el}}^{\text{SAIGA-G}} = 9$ and $N_{\text{el}}^{\text{SAFE}} = 2$)
 317 so that we still have $N_{\text{dof}} = 68$. It may be seen that using $q = 8$ improves the precision of SAFE
 318 solution. The precision the SAIG-C solution was even more significantly improved, although the
 319 attenuation of some modes were still not exactly predicted. This comparison affirms the fact that

320 the collocation based approach generally requires a higher order of NURBS basis functions than
 321 the Galerkin based approach.

322 Figs. 6(a,b) affirm the validation of C_{ph} solutions computed using SAIGA-G, SAIGA-C and
 323 SAFE methods for higher porosity cases ($\phi = 0.5$ and $\phi = 0.6$). Similar to the case ($\phi = 0.1$),
 324 the results of higher modes at higher frequency range (1.5-2 MHz) were shown to be less accurate.
 325 Moreover, the numerical errors seem to be more important when considering plates with higher
 326 porosity, in which the pore fluid movement becomes more significant. We may observe that the
 327 branch of mode S_0 , which was continuous in the cases ($\phi = 0.1, 0.5$), turned out to be discontinuous
 328 due to excessive values of attenuation when considering a higher porosity $\phi = 0.6$. One may observe
 329 that $f \approx 0.3$ MHz corresponds to the cut-on frequency of S_1 branch at which the high valued peak
 330 was found on attenuation curve (see Fig. 4(c))

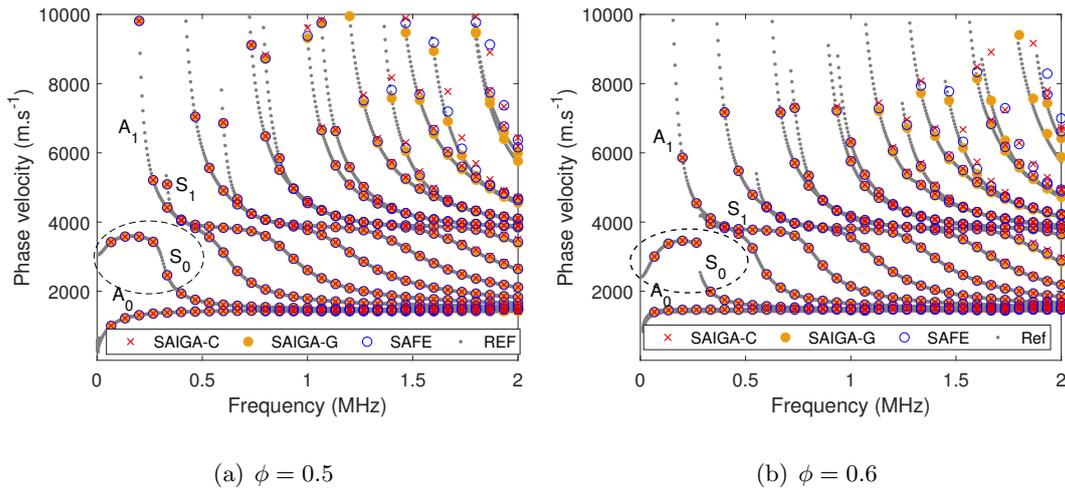


Figure 6: (Color online) Phase velocity curves of poroelastic bone layer: comparison between the SAIGA-C ($q = 4$ and $N_{dof} = 68$), SAIGA-G ($q = 4$ and $N_{dof} = 68$) and SAFE ($q = 4$ and $N_{dof} = 68$) and REF solution ($q = 2$ and $N_{dof} = 404$)

331 5.1.3. Convergence study

We aim to study the convergence rate of the proposed methods for evaluating the phase velocity in poroelastic plates. In particular, we are interested in investigating the accuracy of the numerical results when considering a specific mode over a frequency range. To do so, we introduce a function

$\text{err}(m)$ which estimates the relative errors of the numerical solution for a mode m as

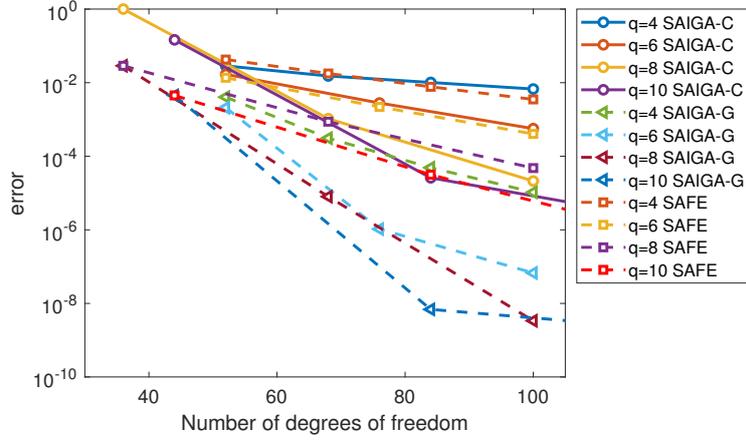
$$\text{err}(m) = \sqrt{\frac{1}{N_f} \sum_{i=1}^{N_f} \left(\frac{C_{\text{ph}}^{\text{num}}(f_i, m) - C_{\text{ph}}^{\text{REF}}(f_i, m)}{C_{\text{ph}}^{\text{REF}}(f_i, m)} \right)^2}, \quad (44)$$

332 where N^f is the number of the frequency values used for the computation in the considered fre-
 333 quency range, $C_{\text{ph}}^{\text{num}}(f_i, m)$ and $C_{\text{ph}}^{\text{REF}}(f_i, m)$ denote the phase velocities of the mode m evaluated
 334 at the frequency (f_i) by using three numerical approaches, and its reference values, respectively.

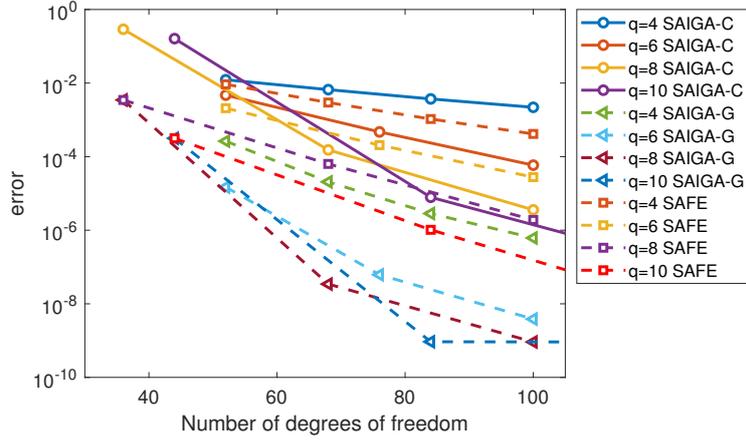
335 The convergence analysis were performed for the mode S_0 in two cases of porosity $\phi = 0.1$ and
 336 $\phi = 0.5$. Figs. 7(a,b) depict the variation of the errors of the phase velocity of S_0 -mode over the
 337 frequency range $f = 0 - 2$ MHz *versus* the number of DOF. Different orders q of NURBS and La-
 338 grange basis functions were investigated. Overall, the error analysis presented in Figs. 7(a,b) shows
 339 that all methods (SAFE, SAIGA-C, SAIGA-G) implied the h - and p -convergences as expected.

340 While using the SAFE method with $q = 4$, we found that the numerical errors (in the both cases
 341 $\phi = 0.1$ and $\phi = 0.5$) are not efficiently reduced by applying mesh refinement. Using higher-orders
 342 ($q = 6, 8, 10$) allows to improve significantly the accuracy solutions. Moreover, the convergence rate
 343 (*i.e* the slope of the error curves) are shown to be increases when using higher-order Lagrangian
 344 shape function. A similar comment would be made regarding the errors of SAIGA-C solutions. It
 345 would also notice that the numerical errors of SAFE are slightly different than the one of SAIGA-C
 346 and this difference seems to be more significant when the porosity is higher ($\phi = 0.5$). However, the
 347 convergence rate of SAIGA-C, which is slower than of SAFE when using *low-order* basis functions
 348 ($q = 4, 6$), may be faster than when using *high-order* basis functions ($q = 8, 10$).

349 For both cases $\phi = 0.1$ and $\phi = 0.5$, it may be found that the SAIGA-G errors using a q -order
 350 NURBS basis function is much smaller comparing with to the errors obtained from SAIGA-C or
 351 SAFE methods which used the same order basis function. For example, to achieve a solution having
 352 numerical error about 10^{-3} , while using $q = 6$ SAIGA-C and SAFE require solving a system of
 353 about $N_{\text{dof}}=100$, using $q = 6$ SAIGA-G only requires a system with only $N_{\text{dof}}=56$. Overall, the
 354 convergence rates *versus* N_{dof} using SAIGA-G method were shown to be much faster than using
 355 SAIGA-C or SAFE methods. Moreover, the numerical errors weren't significantly reduced by
 356 increasing q from 8 to 10. Hence, using $p = 8$ would be a reasonable choice for this problem in
 357 practice.



(a) $\phi=0.1$



(b) $\phi=0.5$

Figure 7: (Color online) Error convergence of S_0 -mode computed for $\phi = 0.1$ and $\phi = 0.5$ by using SAFE, SAIGA-C and SAIGA-G methods

358 *5.2. Case of a poroelastic plate immersed in fluids*

359 Let us consider the case of a poroelastic bone plate, which has been studied in the previous
 360 section, but now is coupled with two half-spaces of water loaded on both sides as shown in Fig. 1.
 361 The acoustic properties of water are given by $\rho_1 = \rho_2 = 1000 \text{ kg}\cdot\text{m}^{-3}$ and $c_1 = c_2 = 1500 \text{ m}\cdot\text{s}^{-1}$.
 362 The infinite fluid domains Ω_1^f and Ω_2^f are modeled as finite-thickness layers with $h_1^f = h_2^f = 4 \text{ mm}$.
 363 To avoid the non-physical reflection from the upper and lower boundaries, we introduce 2 mm-
 364 thickness PMLs at the top $\bar{\Omega}_1^f$ and at the down of $\bar{\Omega}_2^f$, respectively. It worth noticing that the PMLs

365 have been shown to be efficient for the SAFE-based simulation of leaky guided waves in several
 366 previous works [55, 18]. In this study, the range of frequencies of interest is from 0 to 2 MHz, then
 367 PML parameters h^{pml} (PML thickness) and $\gamma_2(x_2)$ (PML function) (see Eq. A.3) were chosen by:
 368 $\hat{\gamma}_2 = 3 + 12i$ and $h^{\text{pml}} = 2.5$ mm. In addition, free pressure boundary conditions ($p_j = 0$) were
 imposed at the exterior boundaries of two fluid domains.

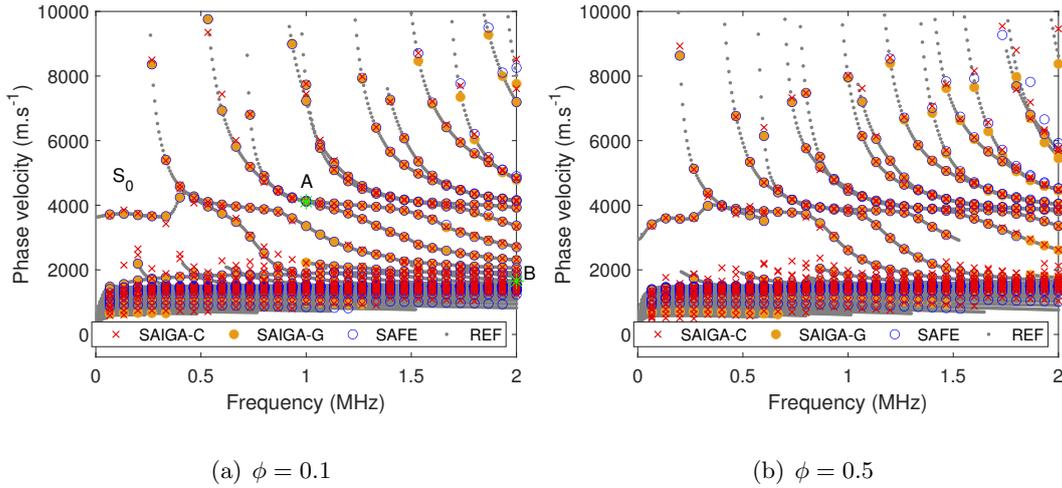


Figure 8: (Color online) Dispersion curves of immersed poroelastic bone layer: comparison between the SAIGA-C ($q = 4$ and $N_{\text{dof}} = 118$), SAIGA-G ($q = 4$ and $N_{\text{dof}} = 118$) and SAFE ($q = 4$ and $N_{\text{dof}} = 118$) and REF solution ($q = 2$ and $N_{\text{dof}} = 566$).

369
 370 Fig. 8 aims to compare the dispersion curves of leaky guided waves obtained using SAIGA-
 371 G, SAIGA-C and the conventional SAFE method with respect to the reference solution (REF)
 372 obtained by using SAFE with a very fine mesh. All of three solutions were computed by using
 373 $q = 4$ (NURBS or Lagrange functions), and all associated systems have $N_{\text{dof}} = 118$, in which
 374 the numbers of DOF in the poroelastic layer and each fluid layer are $N_{\text{dof}}^s = 68$ and $N_{\text{dof}}^{hs} = 25$,
 375 respectively. Two cases of porosity ($\phi = 0.1$ and $\phi = 0.5$) were considered as shown in Fig. 8(a)
 376 and Fig. 8(b), respectively. For both porosities, the SAIGA-G solutions were shown to have
 377 better agreement with the REF solutions. In the case $\phi = 0.1$, the numerical evaluation of C_{ph}
 378 using SAIGA-C or SAFE were failed for some modes (e.g on the S_0 branches). Note that similar
 379 issues were also reported in Hayashi *et al* [55] when studying an elastic plate immersed in fluids.
 380 Interestingly, it was found that using SAIGA-G method (with the same shape function order $q = 4$),

381 allows to capture perfectly all the points along the dispersion curve. These results exhibits the
 382 effectiveness of using NUBRS-based shape functions in this context. In the case $\phi = 0.1$, SAIGA-C
 383 and SAFE have a significant errors at the higher modes and frequencies whereas SAIGA-G is in
 384 very good agreement with the REF solution. The good approximation of SAIGA-G bring to a
 385 better presentation of discontinuous modes.

386 The attenuation of the S_0 mode is presented in Fig 9 for the porosity of $\phi = 0.1$ and $\phi = 0.5$.
 387 In the case $\phi = 0.1$, it may be seen that the presence of exterior fluids completely changes the
 388 attenuation of the considered mode. In this case, as the porosity is small, the attenuation caused by
 389 Darcy's infiltration effect in the considered poroelastic material is weak. Hence the main contribution
 390 to the attenuation to the S_0 mode is due to the leaky phenomena, in which the some energies of
 391 guided-waves are leaked into the surrounded fluid domains. When the porosity is more important
 392 ($\phi = 0.5$), the Darcy's infiltration effect in the porous material becomes much more important.
 393 Consequently, the contribution of leaky effect to the attenuation of guided waves is not dominant,
 394 even it is shown to be very significant, especially at high frequency range.

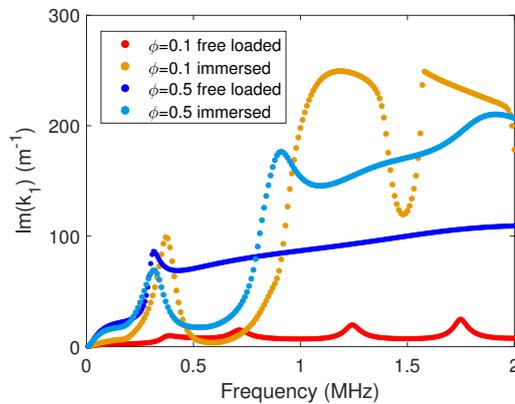


Figure 9: (Color online) Attenuation curves of S_0 mode for $\phi = 0.1$ and $\phi = 0.5$ for free loaded and immersed poroelastic plate

395 In order to examine the continuity in the shape modes, we first consider a mode in the dispersion
 396 curve (marked as point A in Fig. 8) which has $C_{ph}=4047 \text{ m.s}^{-1}$ at $f=1 \text{ MHz}$. Figures 10(a,b)
 397 present the shape modes of the displacement and pressure fields of this mode computed by three
 398 proposed methods. Note that the displacements in the fluid domains were derived from the gradient
 399 of pressure field (see Eq. 2). In the poroelastic layer, the stresses and pressure were derived the

400 solutions of \mathbf{U} and \mathbf{W} by using Eqs. (3) and (9), respectively. At the fluid-solid interface, the
 401 expected conditions of continuity of u_2 and p was perfectly verified for the SAIGA-G's solutions.
 402 However, some small discontinuities were found when using SAFE or SAIGA-C methods. As it
 403 could be expected, the SAIGA and SAFE solutions of displacement (u_2) and pressure (p_j) fields
 404 are progressively attenuated in the PML domains. It has also been checked that the obtained
 405 numerical solutions are not influenced by the distance between the PML and the poroelastic-fluid
 406 interface (data not shown). Thus, the use of PMLs as shown in Appendix A is an efficient way to
 407 consider the halfspace fluid domains in the considered study.

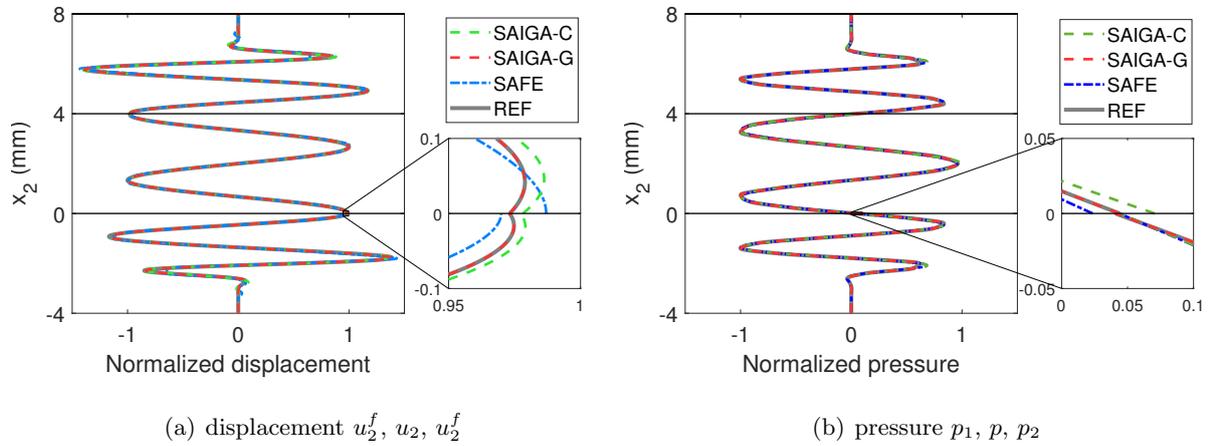


Figure 10: (Color online) Continuity of the mode shapes at $f=1$ MHz , $C_{ph}=4047$ m.s⁻¹ (point A in Fig. 8(a)) of immersed poroelastic cortical bone plate $\phi = 0.1$: comparison between the SAIGA-C ($q = 4$ and $N_{\text{dof}} = 118$), SAIGA-G ($q = 4$ and $N_{\text{dof}} = 118$),SAFE ($q = 4$ and $N_{\text{dof}} = 118$) and REF solution ($q = 2$ and $N_{\text{dof}} = 566$).

408 We next consider a mode at high frequency ($f = 2$ MHz) which corresponds to the point B in
 409 Fig. 8(a)). Figs 11(a,b) depicts images of the displacement and pressure fields of this mode in the
 410 ($\mathbf{e}_1, \mathbf{e}_2$) plane. The comparison of mode shapes computed by using SAFE, SAIGA-C and SAIGA-G
 411 methods are presented in Figs. 12(a,b,c,d). For this mode , using SAGA-G method ($q = 4$ and
 412 $N_{\text{dof}} = 118$) may produce perfectly the reference solutions of u_2, w_2, p and σ_{22} , which are expected
 413 to be continuous over all domains, even at the interfaces. However, at this high frequency, the
 414 considered mode shapes computed by SAFE and SAIGA-C methods, which were based on the
 415 same q and N_{dof} , may be found to be very erroneous, not only at the solid-fluid interfaces, but also
 416 over all domains. Hence, using the SAIGA-G method would be more appropriate for computing

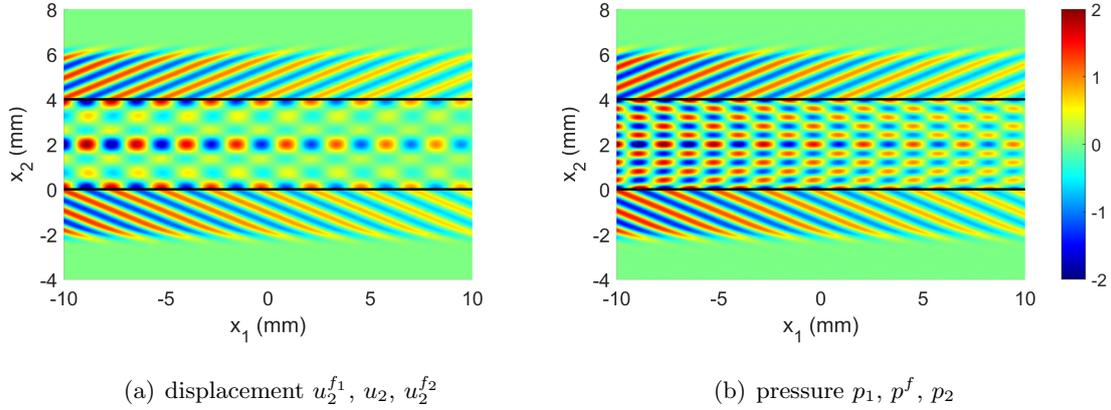


Figure 11: (Color online) Leaky wave propagation in an immersed poroelastic plate $\phi = 0.1$: symmetric mode shapes at $f=2$ MHz , $C_{ph}=2218$ m.s⁻¹ (point B in Fig. 8(a)) (a) displacement and (b) pressure fields

417 the mode shapes when higher frequency needs to be considered.

418 6. Conclusion

419 This presented work attempted to enlighten the potential of two SAIGA-based methods for
 420 studying the guided-waves in two-dimensional poroelastic plates with or without surrounding fluids.
 421 Two semi-analytical approaches, based on isogeometric Galerkin or collocation analysis (denoted
 422 by SAIGA-G and SAIGA-C, respectively) were developed. Due to the presence of the interstitial
 423 fluid phase, guided waves in the poroelastic plates are strongly attenuated, especially when con-
 424 sidering the cases of higher porosity and/or at high frequency. From a computational point of view,
 425 determination of guided waves' characteristics in poroelastic plates lead to quadratic eigenvalue
 426 problems of which all eigenvalues are complex. By comparing with the conventional SAFE method
 427 in which Lagrangian interpolation functions are used, the present study showed that using NURBS-
 428 based interpolation function may yield much more precise numerical solutions of complex-valued
 429 wavenumbers as well as mode shapes of the guided-wave modes. Moreover, the PML technique may
 430 be introduced without difficulties for efficiently simulating the infinite surrounding fluid domains
 431 in proposed formulations. It was shown that using SAIGA-C method doesn't give significant ad-
 432 vantages in terms of accuracy in comparison with the conventional SAFE method. However, the
 433 implementation of SAIGA-C method, which is based on the discretization of strong form equations,
 434 is straightforward and easier than the ones of SAFE or SAIGA-G methods, which are based on

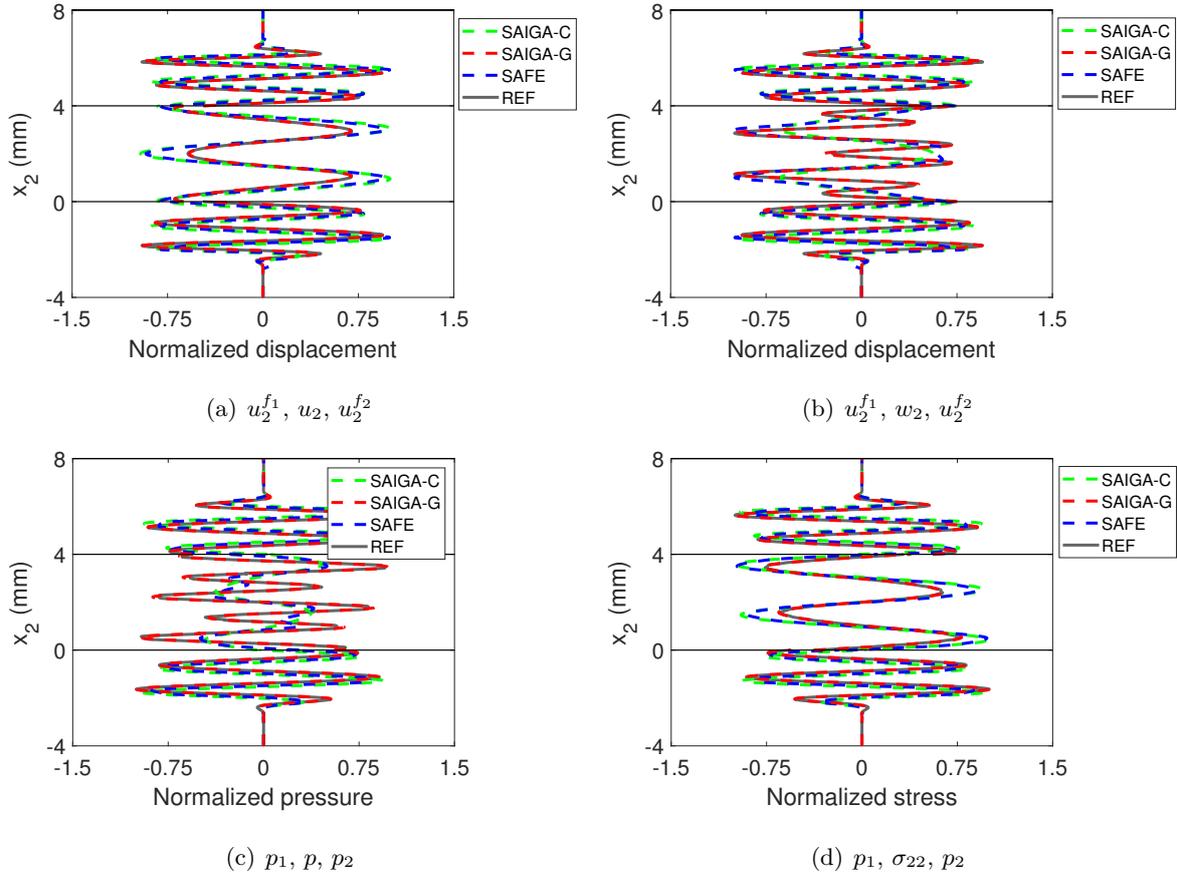


Figure 12: (Color online) Mode shapes at $f=2$ MHz , $C_{ph}=2218$ m.s $^{-1}$ (point B in Fig. 8(a)) for an immersed poroelastic cortical bone plate $\phi = 0.1$: comparison between the SAFE ($q = 4$ and $N_{\text{dof}} = 118$), SAIGA ($q = 4$ and $N_{\text{dof}} = 118$) and REF solution ($q = 2$ and $N_{\text{dof}} = 566$).

435 the discretization of weak form equations [56]. It worth noticing that, by considering the 2D prob-
 436 lems, the advantages of isogeometric methods were achieved thank to the high continuity feature
 437 of NURBS functions. For 3D problems, the SAIGA methods would be expected to be even more
 438 efficient, because they allow to describe more precisely complex geometries [38].

439 Appendix A. Perfectly Matched Layer (PML)

440 Finite element modeling of open waveguides requires special techniques to take into account
 441 the radiation condition of infinite surrounding media. In [57], a so-called 2.5D FEM-BEM method
 442 in which the SAFE method coupled with the boundary element method (BEM) has been proposed
 443 to simulate open waveguides. This method leads to a nonlinear eigenvalue problem that requires

444 expensive computational effort. Another technique represents the infinite medium by an absorbing
 445 layer (AL) [22, 58]. In this layer, a physical viscosity, which artificially increases with the distance
 446 from the central axis for the waveguide's core, is introduced. In practice, sufficient large layers
 447 is required in order to avoid artificial reflections by the absorbing layer, leading to a large set of
 448 eigenvalue equations and increases the computational cost. Recently, the Perfectly Matched Layer
 449 (PML) is proposed to model the infinite surrounding medium (solid or fluid) in the context of
 450 SAFE [18, 59, 60].

From mathematical point of view, the PML can be considered as a result of a mapping into complex coordinate, where the solutions of wave equations decay exponentially (as the new coordinate is complex)[61]. Therefore, the infinite medium can be truncated into a finite domain. The new stretched coordinates $\tilde{x}_1(x_1)$ and $\tilde{x}_2(x_2)$ in the waveguide are defined as

$$\tilde{x}_1(x_1) = x_1, \quad \tilde{x}_2(x_2) = \int_0^{x_2} \gamma_2(x_2) dx_2, \quad (\text{A.1})$$

where $\gamma_2(x_2)$ are non-zero, continuous, complex-valued coordinate stretching functions, also called PML functions, which satisfy:

$$\gamma_2(x_2) = 1 \text{ for } |x_2| \leq d_{x_2} \text{ and } \text{Im}\{\gamma_2(x_2)\} > 0 \text{ for } |x_2| > d_{x_2} \quad (\text{A.2})$$

where (d_{x_2}) is the position of the interfaces. We denote by h^{pml} the PML thicknesses in the e_2 directions. On the exterior boundary of the PML, the boundary condition can be arbitrarily chosen (Dirichlet or Neumann type). The absorption efficiency of leaky waves in the PML strongly depends on the choice of the PML function (γ_2), the position of the interface (d_{x_2}) and the thickness (h^{pml}). There are a number of variants to choose for the γ_2 function. For this study, a continuous parabolic function for both the real and imaginary parts of the PML function is therefore used which has demonstrated good performance in problems in the frequency domain [59, 62]:

$$\gamma_2(x_2) = \begin{cases} 1 & \text{if } |x_2| \leq d_{x_2}. \\ 1 + \hat{\gamma}_2 \left(\frac{|x_2| - d_{x_2}}{h^{\text{pml}}} \right)^2 & \text{if } |x_2| > d_{x_2}. \end{cases} \quad (\text{A.3})$$

451 Appendix B. Determination of poroelastic parameters

To describe the behavior of the poroelastic bone plate, the drained elasticity tensor \mathbb{C} as well as Biot's effective coefficients $\boldsymbol{\alpha}$ and \mathbf{M} used in (3) and (4) should be provided. For this study,

these parameters are derived from the characteristics of the interstitial fluid and solid skeleton phases by using a continuum micro-mechanics model proposed by Hellmich et al. [63]. According to this model, the micro-pores at the micro-structural scale are regarded as cylindrical pores with a circular cross section. In drained condition, the constitutive behavior of the material inside the pores does not possess stiffness. Hence, the estimated drained micro-structural stiffness of the bone whose solid bone matrix's elasticity tensor is \mathbb{C}^m reads:

$$\mathbb{C} = (1 - \phi)\mathbb{C}^m : \left\{ (1 - \phi)\mathbb{I} + \phi[\mathbb{I} - \mathbb{P}^{cyl} : \mathbb{C}^m]^{-1} \right\}^{-1}, \quad (\text{B.1})$$

where \mathbb{I} denotes the fourth-order identity tensor; \mathbb{P}^{cyl} the fourth-order tensor is the Hill's tensor for materials with periodical cylindrical inclusions, which may be derived in closed form [63]. The tensor $\boldsymbol{\alpha}$ the constant M can be then evaluated by [46, 48]:

$$\boldsymbol{\alpha} = \mathbf{I} - \mathbb{C} : (\mathbb{S}^m : \mathbf{I}), \quad -\frac{1}{M} = C - \boldsymbol{\alpha} : \mathbb{S} : \boldsymbol{\alpha}, \quad (\text{B.2})$$

where $\mathbb{S} = (\mathbb{C})^{-1}$ and $\mathbb{S}_m = (\mathbb{C}_m)^{-1}$ are respectively the drained and solid material compliance tensors, \mathbf{I} designates the second-order tensor identity and the scalar C denotes the effective compressibility of porous matrix material, which is given by

$$C = \frac{1}{K} - \frac{1}{K_m} + \phi \left(\frac{1}{K_f} - \frac{1}{K_m} \right), \quad (\text{B.3})$$

452 where $K = (\mathbf{I} : \mathbb{S} : \mathbf{I})^{-1}$, K_f and $K_m = (\mathbf{I} : \mathbb{S}_m : \mathbf{I})^{-1}$ are the bulk moduli of the drained porous
 453 matrix and of the interstitial fluid and of the poroelastic material, respectively.

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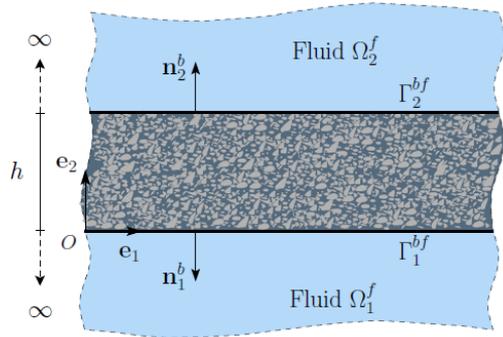
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Poroelastic plate immersed in fluids



Guided-wave analysis using semi-analytical isogeometric methods

- Galerkin method (SAGA-G)
- Collocation method (SAIGA-C)

