



HAL
open science

Joint estimation of balanced motions and internal tides from future wide-swath altimetry

Florian Le Guillou, Noé Lahaye, Clement Ubelmann, Sammy Metref,
Emmanuel Cosme, Aurelien L.S. Ponte, Julien Lesommer, Eric Blayo, Arthur
Vidard

► **To cite this version:**

Florian Le Guillou, Noé Lahaye, Clement Ubelmann, Sammy Metref, Emmanuel Cosme, et al.. Joint estimation of balanced motions and internal tides from future wide-swath altimetry. 2021. hal-03517332v1

HAL Id: hal-03517332

<https://hal.science/hal-03517332v1>

Preprint submitted on 29 Nov 2021 (v1), last revised 8 Jan 2022 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

1 **Joint estimation of balanced motions and internal tides from**
 2 **future wide-swath altimetry**

3 **Florian Le Guillou¹, Noël Lahaye², Clément Ubelmann³, Sammy Metref¹, Emmanuel**
 4 **Cosme¹, Aurélien Ponte⁴, Julien Le Sommer¹, Eric Blayo⁵, Arthur Vidard⁵**

5 ¹Univ. Grenoble Alpes, CNRS, IRD, Grenoble INP, IGE; Grenoble, France

6 ²Inria & IRMAR, campus universitaire de Beaulieu, Rennes, France

7 ³Ocean Next, Grenoble, France

8 ⁴Univ. Brest, CNRS, IRD, Ifremer, Laboratoire d’Océanographie Physique et Spatiale (LOPS), IUEM, Brest, France

9 ⁵Univ. Grenoble Alpes, CNRS, INRIA, Grenoble INP, LJK; Grenoble, France

10 **Key Points:**

- 11 • We present a dynamical method that estimates and separates balanced motions (BM)
 12 and internal tides (IT) from sea surface height data.
 13 • The method uses iteratively two data assimilation techniques, each specific to the es-
 14 timation of one component (BM or IT).
 15 • Although idealized, this study shows encouraging results for the disentanglement of BM
 16 and IT signature on future SWOT data.

Abstract

The future Surface Water and Ocean Topography (SWOT) mission will soon provide Sea Surface Height (SSH) measurements resolving scales of a few tens of kilometers. Over a large fraction of the globe, the SSH signal at these scales is essentially a superposition of a component due to balanced motions (BM) and another component due to internal tides (IT). Several oceanographic applications require the separation of these components and their mapping on regular grids. For that purpose, the paper introduces an alternating minimization algorithm that iteratively implements two data assimilation techniques, each specific to the mapping of one component: a quasi-geostrophic model with Back-and-Forth Nudging for BM, and a linear shallow-water model with 4-Dimensional Variational (4DVar) assimilation for IT. The algorithm is tested with Observation System Simulation Experiments (OSSE) where the truth is provided by a primitive-equation ocean model in an idealized configuration simulating a turbulent jet and a mode-one IT. The algorithm reconstructs almost 80% of the variance of BM and IT, the remaining 20% being mostly due to dynamics that cannot be described by the simple models used. Importantly, in addition to the reconstruction of stationary IT, the amplitude and phase of nonstationary IT are reconstructed. Although idealized, this study represents a step forward towards the disentanglement of BM and IT signals from real SWOT data.

Plain Language Summary

The future Surface Water and Ocean Topography (SWOT) mission will soon provide Sea Surface Height (SSH) images with pixels of 2 km, informing about ocean motions at scales of a few tens of kilometers. At these scales, SSH variations are essentially due to the superposition of slow, balanced motions primarily driven by Earth rotation, and fast, propagating motions due to internal waves mainly generated by interactions between bathymetry and tidal water displacements. Several oceanographic applications require the separation of these two SSH components and their mapping on regular grids. This paper presents an original method to achieve this separation, based on data assimilation approaches and simple dynamical models. Experiments with synthetic SWOT observations, simulated from an ocean circulation model with detailed physics, show the efficiency of the method.

1 Introduction

For more than 25 years, altimetry has allowed the study of near-global sea surface height (SSH) at scales longer than 150 km and drastically transformed our understanding of mesoscale processes in the oceans. In particular, altimetry has given access to the upper-ocean circulation dynamics through geostrophy and revealed that most kinetic energy in the world ocean is contained in mesoscale eddies with wavelengths between 100 km and 300 km (Wunsch & Ferrari, 2004). But the scales resolved by nadir (along-track) altimetry are limited by the 100-300 km spacing between satellite ground tracks. Even by merging several nadir measurements (Taburet et al., 2019), the space time resolution of the resulting 2D SSH maps does not allow to conveniently characterize and study small mesoscale (<150km) motions (Ballarotta et al., 2019; Amores et al., 2018).

With wide-swath radar interferometry, the future SWOT mission (Morrow et al., 2019) will provide SSH observations over a 120km-wide swath with a near-global coverage, resolving spatial scales down to 15-30km depending on the sea state (Fu & Uebelmann, 2014). Recent studies, based on numerical models, have highlighted the impacts of short-mesoscale and submesoscale processes on ocean dynamics. Submesoscale motions have been found to trigger a large part of vertical motions (McWilliams, 2016; Klein & Lapeyre, 2009) that drive the exchanges of heat, carbon and nutrients between the ocean surface and subsurface. Small mesoscale processes also provide kinetic energy to larger scales through an inverse cascade (Ajayi et al., 2019; Capet et al., 2016), hence impacting the mesoscale dynamics. The SWOT mission rep-

65 represents a unique opportunity to validate our present-day understanding of submesoscale mo-
66 tions and their role in the oceanic processes in a broad sense.

67 At the scales measured by SWOT, some SSH variations due to internal tide (IT) may
68 become comparable to those due to the short mesoscale balanced motions (BM; Qiu et al., 2018).
69 IT, producing high frequency sea level fluctuations at scales around 200 km and below, are
70 internal gravity waves generated when the barotropic tidal flow encounters variations of topog-
71 ography (Garrett & Kunze, 2007). Although the generation processes of IT are well under-
72 stood, the dissipation processes (that greatly influence the ocean's energy budget) remain barely
73 known. Recent studies emphasize that IT and BM interact in a complex way (Kelly & Ler-
74 musiaux, 2016), with BM scattering and refracting IT (Dunphy et al., 2017). These complex
75 interactions are thought to strongly modify local mixing but need to be validated with obser-
76 vations. SWOT capacity to observe even a part of IT SSH signal will help to better understand
77 these interactions and the processes related to ocean mixing and dissipation.

78 The accurate characterization of both IT and BM from SWOT, in the small mesoscale
79 spectrum where their signatures overlap, will require scientific and technological developments
80 for the processing of observations. The first challenge concerns the design of gridded prod-
81 ucts. For BM, the main objective is to increase the space-time resolution of the present-day
82 SSH maps using SWOT. The mismatch between the (high) spatial and (low) temporal sam-
83 pling of SWOT fosters the development of innovative inversion techniques, which typically in-
84 clude dynamical constraints (Ubelmann et al., 2015). For IT, the main objective is to map the
85 incoherent part of the signal, i.e. the part due to waves which characteristics have been altered
86 by interactions with BM and stratification variations. This is further developed in the next para-
87 graph. The second challenge consists in the separation of IT and BM components from SWOT
88 to make the differentiated gridded products possible. Both components are driven by differ-
89 ent dynamics, and separation is needed to properly estimate quantities associated to each (e.g.
90 surface currents for tracer advection, or energy dissipation due to waves). Separation is also
91 made difficult by geographical and seasonal variations in the spatial scales and the relative strength
92 of IT and BM (Qiu et al., 2018).

93 Disentangling the contribution of BM and IT on SSH is still an unsolved challenge. Due
94 to its partial sampling in space and time, altimetry does not capture the fast SSH pulsation due
95 to IT. Stationary IT (which are phase-locked to the astronomical forcing) can be mapped by
96 harmonic analysis of long time series (>10 years) of conventional nadir altimetry data (Ray
97 & Zaron, 2015). IT become non stationary when interacting with BM (Dunphy et al., 2017;
98 Ponte & Klein, 2015) and/or being modulated by the stratification (Ray & Zaron, 2011). The
99 phase and amplitude modulations of the IT field can evolve on time scales between 5 and 20
100 days, what make their predictability nearly impossible with conventional satellite altimetry (Nash
101 et al., 2012; Haren et al., 2004). Recent studies (Zaron, 2017; Nelson et al., 2019) suggest that
102 nonstationarity represents half of the total IT variance on average. SWOT will considerably
103 increase the SSH measurement density and open the way to predicting nonstationary IT. In the
104 context of SWOT development, the BM-IT separation problem has been addressed with var-
105 ious approaches. Torres et al. (2019) make use of a spatial scale threshold above which BM
106 dominate and below which IT dominate. This spectral technique is less effective when the com-
107 mon spatial scale interval extends too much, what is particularly the case in winter due to the
108 emergence of small vortices (< 50 km) associated with mixed layer instabilities (Ajayi et al.,
109 2020). Ponte et al. (2017) and Gonzalez-Haro et al. (2019) explore multi-sensor approaches
110 with altimetry and sea surface temperature observations, motivated by the fact that BM and
111 IT have distinct footprints on both fields.

112 In this paper, we propose to simultaneously and dynamically map and separate SSH com-
113 ponents of BM and IT from 2D altimetric observations, distributed in time, as SWOT data will
114 be. The proposed approach, illustrated on Figure 1, implements an alternating minimization
115 algorithm that iteratively calls two mapping techniques, each one specific to the estimation of
116 a component (BM or IT). At each iteration, the observations used to map one component is
117 made of the difference between the full observation and the previous estimation of the other

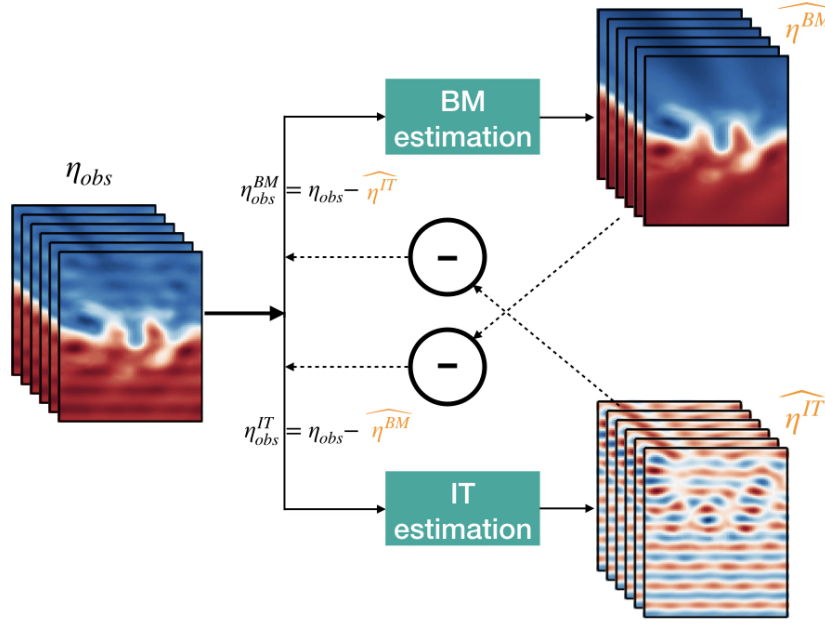


Figure 1. The joint estimation algorithm alternates BM estimations and IT estimations with observations recursively corrected from the other component. The algorithm is formalized and described in Section 4.

118 component. Both mapping techniques are based on data assimilation. BM are reconstructed
 119 with a Quasi-Geostrophic (QG) model assimilating observations with a Back-and-Forth Nudg-
 120 ing (BFN) technique (Auroux & Blum, 2008). The algorithm is called BFN-QG hereafter. IT
 121 are reconstructed with a Shallow-Water model (SW) assimilating observations with a 4DVar
 122 technique (DIMET & TALAGRAN, 1986). The algorithm is called 4DVar-SW hereafter. The
 123 joint estimation algorithm is tested with observing system simulation experiments (OSSE). The
 124 true state of the ocean (illustrated on Figure 2) is provided by a primitive-equation regional
 125 model, ROMS, in an idealized configuration that simulates the propagation of a mode-one IT
 126 through a turbulent mesoscale zonal jet (Ponte et al., 2017; Dunphy et al., 2017).

127 The paper is structured as follows: Sections 2 and 3 present the data assimilation algo-
 128 rithms reconstructing BM and IT, respectively; Section 4 formalizes and describes the gen-
 129 eral joint estimation approach that implements both assimilation algorithms; The experimen-
 130 tal set-up is presented in Section 5 and the results are discussed in Section 6; Conclusions and
 131 perspectives are drawn in section 7.

132 **2 Mapping the balanced motions: BFN-QG algorithm.**

133 The mapping technique for BM is presented in detail in Guillou et al. (2021). It is based
 134 on a quasigeostrophic (QG) model and a data assimilation method called the Back and Forth
 135 Nudging (Auroux & Blum, 2008), and referred to as BFN-QG. The dynamical model is a 1.5-
 136 layer QG model (Ubelmann et al., 2015). Forcing and mixing terms are omitted, resulting in
 137 the simplified QG equations:

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0 \quad (1)$$

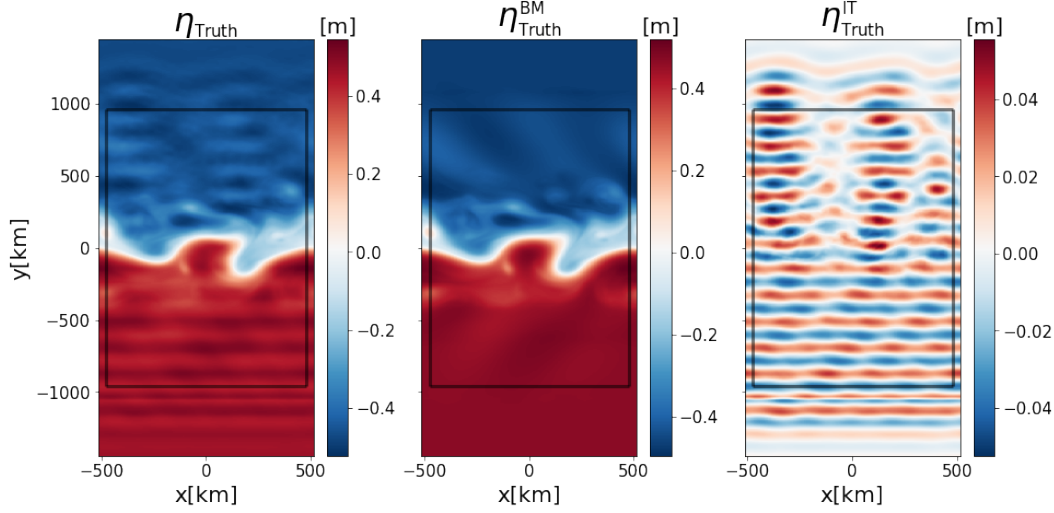


Figure 2. SSH snapshot of the simulated true state of the ocean (left), its BM component (middle) and its IT component (right). The black square represents the study domain in which the reconstructions are performed.

138 where J is the Jacobian operator and the streamfunction ψ is proportional to SSH η :

$$\psi = \frac{g}{f}\eta \quad (2)$$

139 with g the gravity constant, and f the Coriolis parameter. The PV is linked to the streamfunction
140 by the elliptical equation:

$$q = \nabla^2\psi - \frac{1}{L_R^2}\psi \quad (3)$$

141 where ∇^2 is the Laplace operator and L_R is the first baroclinic Rossby radius of deformation.
142 Propagating SSH from time t_i to time t_{i+1} is a succession of elementary steps: (i) compute
143 q_i from η_i with equations 2 and 3, (ii) propagate q_i with equation 1 to obtain q_{i+1} , (iii) invert
144 equation 3 to retrieve ψ_{i+1} , then η_{i+1} .

145 The BFN technique is based on the Nudging method (Anthes, 1974), which consists of
146 adding an extra term to the model prognostic equation (equation 1) to pull the model variable
147 towards the observations. This term is proportional to the difference between the observations
148 and the model variable. Here, the model variable is PV q and the observations are 2D SSH
149 η_{obs} . Thanks to equations 2 and 3, a corresponding PV observation q_{obs} can be derived from
150 η_{obs} and used to nudge Eq. 1 as:

$$\frac{\partial q}{\partial t} + J(\psi, q) - K(q_{\text{obs}} - q) = 0 \quad (4)$$

151 where K is a tunable coefficient controlling the strength of nudging. As explained in (Guillou
152 et al., 2021), K must exhibit smooth variations in time to avoid the emergence of numerical
153 instabilities and enhance the constraint of the observations on the model dynamics. A Gaus-
154 sian kernel is used for K :

$$K[t] = \sum_{t_{\text{obs}}} K_0 e^{-\left(\frac{t-t_{\text{obs}}}{\tau}\right)^2} \quad (5)$$

155 where K_0 and τ are the nudging parameters and t_{obs} are the observation times.

156 Nudging is also performed backward in time, by adding a feedback term with a sign op-
 157 posite to that of the forward nudging. Note that the backward integration of the model is possi-
 158 ble because the model equation 1 is reversible in time.

159 The BFN is the combination of the forward nudging and the backward nudging over a
 160 sliding time window of length T_{BFN} . Over a specific time window $[t_i, t_i + T_{\text{BFN}}]$, the algorithm
 161 works as follows. The model state at time t_i is prescribed from the previous time window. The
 162 forward nudging is first computed from t_i to $t_i + T_{\text{BFN}}$. The final state at $t_i + T_{\text{BFN}}$ is then used
 163 as initial condition to run the backward nudging back to t_i . The result initializes the forward
 164 nudging, and so on until convergence. In a few iterations, the model converges towards a tra-
 165 jectory that both fits the observations and complies with the QG dynamics. The process can
 166 then be carried out in the next time window.

167 3 Mapping the internal tide signal: 4Dvar-SW algorithm.

168 3.1 Dynamical model

169 The dynamical model used to simulate the surface propagation of IT motions is a lin-
 170 ear shallow water (SW) model. This model represents the first baroclinic mode dynamics as-
 171 sumed to capture the largest part of the IT signal (Ray & Zaron, 2016). The equations are (Gill,
 172 1982):

$$\frac{\partial u}{\partial t} - fv = -g \frac{\partial \eta}{\partial x}, \quad (6a)$$

$$\frac{\partial v}{\partial t} + fu = -g \frac{\partial \eta}{\partial y}, \quad (6b)$$

$$\frac{\partial \eta}{\partial t} = -H_e \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right), \quad (6c)$$

173 where (u, v) are the velocity components, η is the SSH, f is the Coriolis frequency, and H_e
 174 the equivalent depth that determines the baroclinic deformation radius (through the relation
 175 $R_d = \sqrt{gH_e}/f$). Equations 6 are discretized on a C-grid and a leap-frog time-stepping scheme
 176 (Sadourny, 1975). The domain is a square, land-free domain with a flat bottom and open bound-
 177 aries. IT enter through the boundaries and are not generated within the domain.

178 After entering the domain, IT can be made incoherent by time variations of the equiv-
 179 alent depth, since H_e modulates the phase speed of the waves, given by $u_{\text{phase}} = \sqrt{\frac{f^2}{k^2} + (gH_e)^2}$
 180 where k is the wavenumber. Space variations of H_e (along with the spatial variation of f) changes
 181 the wave propagation direction and its wavelength (Rainville & Pinkel, 2006).

182 Boundary conditions are designed to introduce and let out waves. They are of the ra-
 183 diative type (Blayo & Debreu, 2005). The amplitudes and directions of incoming waves are
 184 prescribed with a Flather condition (Flather, 1987) as:

$$v^n \pm \sqrt{\frac{g}{H_e}} \eta = v_{\text{ext}}^n \pm \sqrt{\frac{g}{H_e}} \eta_{\text{ext}} \quad (7)$$

185 where v^n is the normal component of the wave velocity at the boundary. Parameters v_{ext}^n and
 186 η_{ext} must be prescribed. The signs in the above equation vary with the boundary (– for south-
 187 ern and western boundaries, + for the others).

188 Incoming waves are superpositions of progressive monochromatic waves with frequency
 189 ω and absolute wavenumber k . The frequency is a tidal frequency (details are given in Sec-
 190 tion 5) and k is deduced from ω using the dispersion relation:

$$\omega^2 = gH_e k^2 + f^2 \tag{8}$$

191 The superposing, monochromatic waves differ by their amplitudes and their propagation
 192 directions. Denoting v_θ^n and η_θ the amplitudes of velocity and SSH of the wave entering the
 193 domain with an angle θ with the inward boundary normal direction, and denoting κ_θ the unit
 194 vector of the propagation direction, the expressions of parameters v_{ext}^n and η_{ext} at boundary
 195 grid point $\mathbf{r} = (x, y)$ and time t are:

$$\eta_{ext}[t, x, y] = \sum_{\theta} \Re \left(\underline{\eta_\theta}[t, x, y] e^{i(\omega t - k \kappa_\theta \cdot \mathbf{r})} \right), \tag{9a}$$

$$v_{ext}^n[t, x, y] = \sum_{\theta} \Re \left(\underline{v_\theta^n}[t, x, y] e^{i(\omega t - k \kappa_\theta \cdot \mathbf{r})} \right) \tag{9b}$$

196 where i is the imaginary unit and $\underline{\cdot}$ is the complex notation.

197 Parameters v_θ^n and η_θ are linked together by the polarisation relations (obtained from Equa-
 198 tions 6). Thus, prescribing boundary conditions with Eq. 7 implies setting only η_θ at each bound-
 199 ary and for each angle θ . A limited number of angles are predefined. The prescription of the
 200 boundary conditions involves $2N_\theta N_b N_t$ values, with N_θ the number of prescribed wave direc-
 201 tions, N_b the number of grid points at the boundary, and N_t the number of model time steps.
 202 In the following, we denote all these external values to be prescribed by η_{ext} .

203 3.2 4DVar cost function

204 The forward problem described in Section 3.1 is here formulated shortly as:

$$(u, v, \eta)_{t,x,y} = M(\phi) \tag{10}$$

205 where $\phi = (H_e, \eta_{ext})_{t,x,y}$ are the input parameters, M is the non linear operator solving the prog-
 206 nostic equations (6) associated with the OBCs (7) over a fixed time interval $[t_0, t_1]$. In Eq. 10,
 207 we neglect the initial condition in the function arguments. Given the high speed of the waves
 208 crossing the domain, we consider that the initial condition is quickly "forgotten" by the model,
 209 and the sea state simulated over a long enough time window essentially depends on bound-
 210 ary conditions.

211 The associated inverse problem consists in seeking the set of parameters $\phi^{opt} = (H_e^{opt}, \eta_{ext}^{opt})$
 212 that produces a model state trajectory that best fits the observations η_{obs} in the time window
 213 $[t_0, t_1]$. We formulate this problem as the minimization of the 4DVar cost function:

$$J(\phi) = (\phi - \phi^b)^T B^{-1} (\phi - \phi^b) + (\eta_{obs} - H \cdot M(\phi))^T R^{-1} (\eta_{obs} - H \cdot M(\phi)) \tag{11}$$

214 where $\phi^b = (H_e^b, \eta_{ext}^b)$ is the background control vector, H is the linear operator that projects
 215 the model state in the observation space, B and R denote the background and observation er-
 216 ror covariance matrices, respectively. In this work, these matrices are set diagonal.

217 The minimum of J is found using a descent method that requires the calculation of the
 218 gradient ∇J with respect to ϕ :

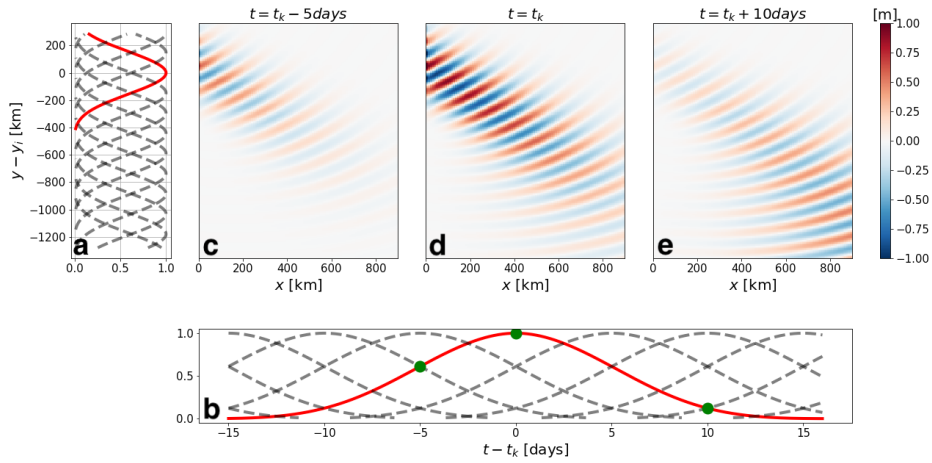


Figure 3. Illustration of the reduced-order basis for η_{ext} at the western border for $\theta = -\pi/4$: (a) the spatial decomposition with $\sigma = 150\text{km}$ (in red the spatial gaussian centered around y_i); (b) the temporal decomposition with $\tau = 5\text{days}$ (in red the temporal gaussian centered around t_k); (c,d,e) SSH snapshots produced by one element $G^{(k,i)}$ of the basis around the space-time coordinate (y_i, t_k) at three distinct times (represented by the green circles in (b)).

$$\nabla J(\phi) = 2B^{-1}(\phi - \phi^b) - 2M^* \cdot H^T \cdot R^{-1}(\eta_{\text{obs}} - H \cdot M(\phi)) \quad (12)$$

219 This gradient is computed with an adjoint method that makes use of the adjoint model M^* .
 220 Since the model M is non linear, an incremental 4Dvar algorithm (Courtier et al., 1994) is im-
 221 plemented to find the optimal set of parameters ϕ^{opt} that cancels ∇J .

222 3.3 Reduced-order 4Dvar

223 Order reduction is a standard practice in geophysical data assimilation or inversion, to
 224 overcome the issues of ill-posedness and numerical complexity (Blayo et al., 1998; Robert et
 225 al., 2005, e.g., for 4DVar). Control variables are projected into a low-dimensional basis of vec-
 226 tors (EOFs - Empirical Orthogonal Functions - are a usual choice, relevant to many inverse
 227 problems) and the cost function minimization is performed in this low-dimensional space to
 228 make it possible or faster.

229 In this work, we propose to express the control vector (i.e. the model parameters ϕ) in
 230 a reduced basis G made of space-time gaussian functions. This choice has been motivated by
 231 the fact that we know H_e and η_{ext} should vary "weakly" in space and time. Depth H_e inher-
 232 its its time variation scales from the balanced flow, and the tidal forcing is quasi-stationary in
 233 comparison with the generated internal tides. Mathematically, we call $G^{(k,i)}$ one basis func-
 234 tion centered at the point (t_k, r_i) (t_k is a timestamp and r_i is a spatial coordinate), expressed
 235 as:

$$G^{(k,i)}(t, r) = e^{-\frac{(t-t_k)^2}{2\tau^2}} e^{-\frac{\|r-r_i\|^2}{2\sigma^2}} \quad (13)$$

236 where τ (respectively σ) is the time (respectively space) characteristic scale. For H_e , r and
 237 r_i both refer to 2D coordinates (x, y) while for η_{ext} they refer to a 1D coordinate (along the

238 domain boundary). Thus, $G^{(k,i)}$ is a 3D function for H_e and a 2D function for η_{ext} . As an il-
 239 lustration, we represent in Figure 3 the space-time distribution of the basis functions for η_{ext}
 240 at the western boundary and the wavy SSH produced by one element of the basis. Two suc-
 241 cessive basis functions are separated by $t_{k+1} - t_k = \tau$ in time and $\|r_{i+1} - r_i\| = \sigma$ in space.

242 In this basis, ϕ writes as:

$$\phi = G \cdot w = \sum_k \sum_i w_{ki} G^{(k,i)} \quad (14)$$

243 where $w = (w_{ik})$ are the coordinates of ϕ in the basis G , and the new control variables of the
 244 4DVar. The cost function J and its gradient ∇J are then obtained by replacing H by $H \cdot G$ in
 245 equations 11 and 12.

246 For the order reduction to be efficient, the characteristic scales τ and σ of the gaussian
 247 basis functions has to be carefully chosen. They should be short enough to explain a maxi-
 248 mum of the variability of H_e and η_{ext} and long enough to reduce significantly the dimension
 249 of the control space. In the experiments presented in Section 5, the dimension of the control
 250 space is reduced by a factor of 10^4 .

251 In addition to reducing the size of the control vector, the reduced order basis justifies
 252 the choice of a diagonal matrix for B , assuming that the basis element are independent from
 253 each other.

254 4 Joint estimation approach

255 As stated literally in the introduction, our goal is to map and separate BM and IT based
 256 on SWOT observations that contains both components, and our approach is based on data as-
 257 simulation. Let us represent by η^{BM} the SSH field related to BM, η^{IT} the SSH field related to
 258 IT, and η_{obs} the observation informing about $\eta^{BM} + \eta^{IT}$. The reconstruction problem is formu-
 259 lated as finding:

$$\arg \min_{\eta^{BM}, \eta^{IT}} \|\eta^{BM} + \eta^{IT} - \eta_{obs}\|^2 \quad (15)$$

260 where the norm typically considered here is the L_2 norm based on the space-time integral. The
 261 resolution of the minimization problem is addressed using a classical alternating minimiza-
 262 tion procedure (Tseng, 1990). The use of an alternating minimization algorithm is based on
 263 the strong assumption of the decoupling between the dynamics driving BM and IT. In real-
 264 ity, BM impact the propagation of IT. Here, the BM effect on IT propagation is introduced through
 265 the equivalent depth H_e only. Iterations are performed, alternating the two minimization sub-
 266 problems:

$$\widehat{\eta}_k^{BM} = \arg \min_{\eta^{BM}} \|\eta^{BM} + \eta_{k-1}^{IT} - \eta_{obs}\|^2, \quad (16a)$$

$$\widehat{\eta}_k^{IT} = \arg \min_{\eta^{IT}} \|\widehat{\eta}_k^{BM} + \eta^{IT} - \eta_{obs}\|^2 \quad (16b)$$

267 In these equations, k is an iteration index. $\widehat{\eta}_k^{BM}$ and $\widehat{\eta}_k^{IT}$ are BM and IT estimations, respec-
 268 tively, at iteration k .

269 Both minimization subproblems in Eq. 16a and 16b can be solved with methods stan-
 270 dard in geophysical data assimilation (4DVar, 3DVar, Kalman filters, etc). In what follows, Eq.
 271 16a is solved with BFN-QG (see section 2) assimilating the corrected observation $\eta_{obs} - \eta_{k-1}^{IT}$.

272 Equation 16b is solved with 4DVar-SW (see section 3) assimilating the corrected observation
 273 $\eta_{\text{obs}} - \eta_k^{\text{BM}}$. The joint estimation pseudo-code is presented in Algorithm 1 and illustrated in
 274 Figure 1.

Algorithm 1: The joint estimation algorithm. Variables $\widehat{\eta}^{\text{BM}}$ and $\widehat{\eta}^{\text{IT}}$ are BM and IT estimations, respectively. K is a convergence criterion.

Result: $\widehat{\eta}^{\text{BM}}$ and $\widehat{\eta}^{\text{IT}}$
 Initialize $\eta_{\text{obs}}^{\text{BM}} = \eta_{\text{obs}}$;
 $K = \text{True}$;
while K **do**
 1. Compute $\widehat{\eta}^{\text{BM}}$ from $\eta_{\text{obs}}^{\text{BM}}$ solving 16a ;
 2. Do $\eta_{\text{obs}}^{\text{IT}} = \eta_{\text{obs}} - \widehat{\eta}^{\text{BM}}$;
 3. Compute $\widehat{\eta}^{\text{IT}}$ from $\eta_{\text{obs}}^{\text{IT}}$ solving 16b ;
 4. Do $\eta_{\text{obs}}^{\text{BM}} = \eta_{\text{obs}} - \widehat{\eta}^{\text{IT}}$;
 5. Update K
end

275 5 Experiment setup

276 The joint estimation algorithm is tested with OSSEs (Observing System Simulation Ex-
 277 periments): a simulation provides reference surface data containing both BM and IT motions;
 278 SSH observations are extracted from this simulation and are assimilated; the estimated fields
 279 are then compared to the reference fields.

280 5.1 Reference simulation

281 The test flow consists of a turbulent zonal jet and a mode-1 internal tide propagating sig-
 282 nal described in Ponte et al. (2017) and Dunphy et al. (2017). This simulation is carried out
 283 with the Coastal and Regional Ocean Community (CROCO, <https://www.croco-ocean.org>) model.
 284 The horizontal resolution is 4 km and the domain size is 1024 km x 2880 km. The mode-one
 285 internal tide is generated by a zonally uniform tide excitation in the southern part of the do-
 286 main. The time period of the tide is $T = 12$ hours. The plane wave produced is propagating
 287 northward and is encountering the meridional jet located at the center of the domain (Figure
 288 2).

289 The reference SSH, called η_{truth} , is decomposed into BM and IT (Dunphy et al., 2017).
 290 The reference BM field $\eta_{\text{truth}}^{\text{BM}}$ is obtained by low pass filtering η_{truth} :

$$\eta_{\text{truth}}^{\text{BM}}(t) = \frac{1}{W} \int_{t-2T}^{t+2T} \eta_{\text{truth}}(t) e^{-\left(\frac{t_i-t}{T}\right)^2} dt_i$$

291 where $W = \int_{t-2T}^{t+2T} e^{-\left(\frac{t_i-t}{T}\right)^2} dt_i$. The reference IT field $\eta_{\text{truth}}^{\text{IT}}$, is obtained with an harmonic fit-
 292 ting:

$$\eta_{\text{truth}}^{\text{IT}}(t) = a_c(t) \cos\left(\frac{2\pi t}{T}\right) + a_s(t) \sin\left(\frac{2\pi t}{T}\right)$$

293 where:

$$a_c(t) = \frac{2}{W} \int_{t-2T}^{t+2T} \eta_{\text{truth}}(t_i) \cos\left(\frac{2\pi t_i}{T}\right) e^{-\left(\frac{t_i-t}{T}\right)^2} dt_i$$

$$a_s(t) = \frac{2}{W} \int_{t-2T}^{t+2T} \eta_{\text{truth}}(t_i) \sin\left(\frac{2\pi t_i}{T}\right) e^{-\left(\frac{t_i-t}{T}\right)^2} dt_i$$

294 Figure 2 shows snapshots of the total field, BM, and IT flows. The IT surface signature
 295 on SSH is not exceeding 5 cm, while BM's is around 50 cm. The IT wavelength is about 150
 296 km. South of the jet, the wave is rectilinear and aligned with the southern boundary, which
 297 reflects its stationarity. North of the jet, the wave is dispersed due to its interaction with tur-
 298 bulent BM, producing nonstationarity in the wave field (Dunphy et al., 2017).

299 5.2 Observational scenarii

300 Observations are drawn from the true SSH (η_{truth}), containing both BM and IT compo-
 301 nents. They are free of error, complete in space, and regularly distributed in time. The obser-
 302 vation frequency is 75 h in the reference experiments discussed in Section 6.1.

303 In order to help evaluation of the mapping performance of the joint estimation exper-
 304 iments (hereafter denoted as *joint*), two additional scenarii are considered. The first one, de-
 305 noted *idealized*, refers to the case where both components can be observed separately, and each
 306 observation is processed with the appropriate assimilation system. This means that BFN-QG
 307 (4Dvar-SW) is fed with observations extracted from $\eta_{\text{truth}}^{\text{BM}}$ ($\eta_{\text{truth}}^{\text{IT}}$, respectively) fields. This ideal
 308 scenario provides upper bounds for the mapping performance of the *joint* experiments. The
 309 second scenario, hereafter denoted as *independent*, implements each assimilation system in-
 310 dependently (without iteration) with observations of the full signal η_{obs} . This scenario provides
 311 a baseline to be outperformed by the *joint* experiments.

312 This study also investigates the impact of the observational temporal sampling on the
 313 mapping performances. For that purpose, a sensitivity experiment is performed and discussed
 314 in section 6.2.

315 5.3 Configuration of assimilation

316 Assimilation is performed in an inner domain of the reference simulation, illustrated by
 317 the black squares in Figure 2, to avoid the effects of boundary conditions and forcing in the
 318 reference simulation. The horizontal resolution is degraded to 16 km to reduce computational
 319 complexity, and the experiments are run over 4 months.

320 The BFN-QG (4Dvar-SW) parameters are listed in Table 1 (Table 2, respectively). For
 321 the *joint* scenario, the background values H_e^b and η_{ext}^b are updated at each iteration by the op-
 322 timal values found at the previous iteration. We consider two values for the observation er-
 323 ror covariance matrix R : 0.01 m for the *joint* and *idealized* scenarii, 0.1 m for the *independ-*
 324 *ent* scenario. The value for the *independent* scenario is higher to reflect representativeness
 325 errors and allow convergence of the 4Dvar-SW even with the presence of BM in the obser-
 326 vations.

327 5.4 Diagnostics

328 For measuring the performance of the reconstruction, we use the time varying Root Mean
 329 Score Error (*RMSE*) and its associated score S , defined as follows:

Parameter	Description	Value
L_R	First Rossby radius of deformation	30 km
$K_0 dt$	(adimensionalised) Nudging coefficient	0.1
τ	Nudging time scale	2 days
T_{BFN}	BFN sliding time window length	15 days

Table 1. BFN-QG parameters

Parameter	Description	Value
θ	Entry angles of incoming waves	$-\pi/2, 0, \pi/2$
σ	Gaussian space scale	150 km
τ	Gaussian time scale	20 days
H_e^b	Background value for H_e	0.9 m
η_{ext}^b	Background value for η_{ext}	0 m
B_{H_e}	Background error for H_e	0.2 m
$B_{\eta_{ext}}$	Background error for η_{ext}	0.001 m
R	observational error	0.01 m or 0.1 m

Table 2. 4Dvar-SW parameters. As the covariance matrix (B and R) are diagonal, only standard deviations are given.

$$RMSE^{BM}(t) = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\widehat{\eta}^{BM}(t, i) - \eta_{truth}^{BM}(t, i) \right)^2}, \quad (17a)$$

$$S^{BM} = 1 - \frac{\overline{RMSE^{BM}}}{RMS\left(\eta_{truth}^{BM} - \eta_{truth}^{BM}\right)}, \quad (17b)$$

$$RMSE^{IT}(t) = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\widehat{\eta}^{IT}(t, i) - \eta_{truth}^{IT}(t, i) \right)^2}, \quad (17c)$$

$$S^{IT} = 1 - \frac{\overline{RMSE^{IT}}}{RMS\left(\eta_{truth}^{IT}\right)} \quad (17d)$$

330 where N is the number of pixel in the study domain, \overline{X} is the time-average of the variable X
 331 and RMS is the root mean square function. A score of 1 indicates a perfect reconstruction in
 332 terms of $RMSE$, while a score of 0 indicates that the time-averaged $RMSE$ is as large as the
 333 RMS of the reference field. Note that for BM, the score is defined relative to the time anomaly
 334 of the reference field.

335 6 Results

336 6.1 Reconstruction performances

337 Figure 4 illustrates the performances of the joint estimation algorithm over the iterations
 338 for a temporal sampling of 75 h (6.25 IT periods). The joint estimation algorithm converges
 339 after 10 iterations. The scores obtained with the *idealized* and *independent* scenarii are also
 340 shown. As the joint estimation is started by a BM reconstruction, the first iteration of the joint
 341 algorithm corresponds to the *independent* scenario for BM.

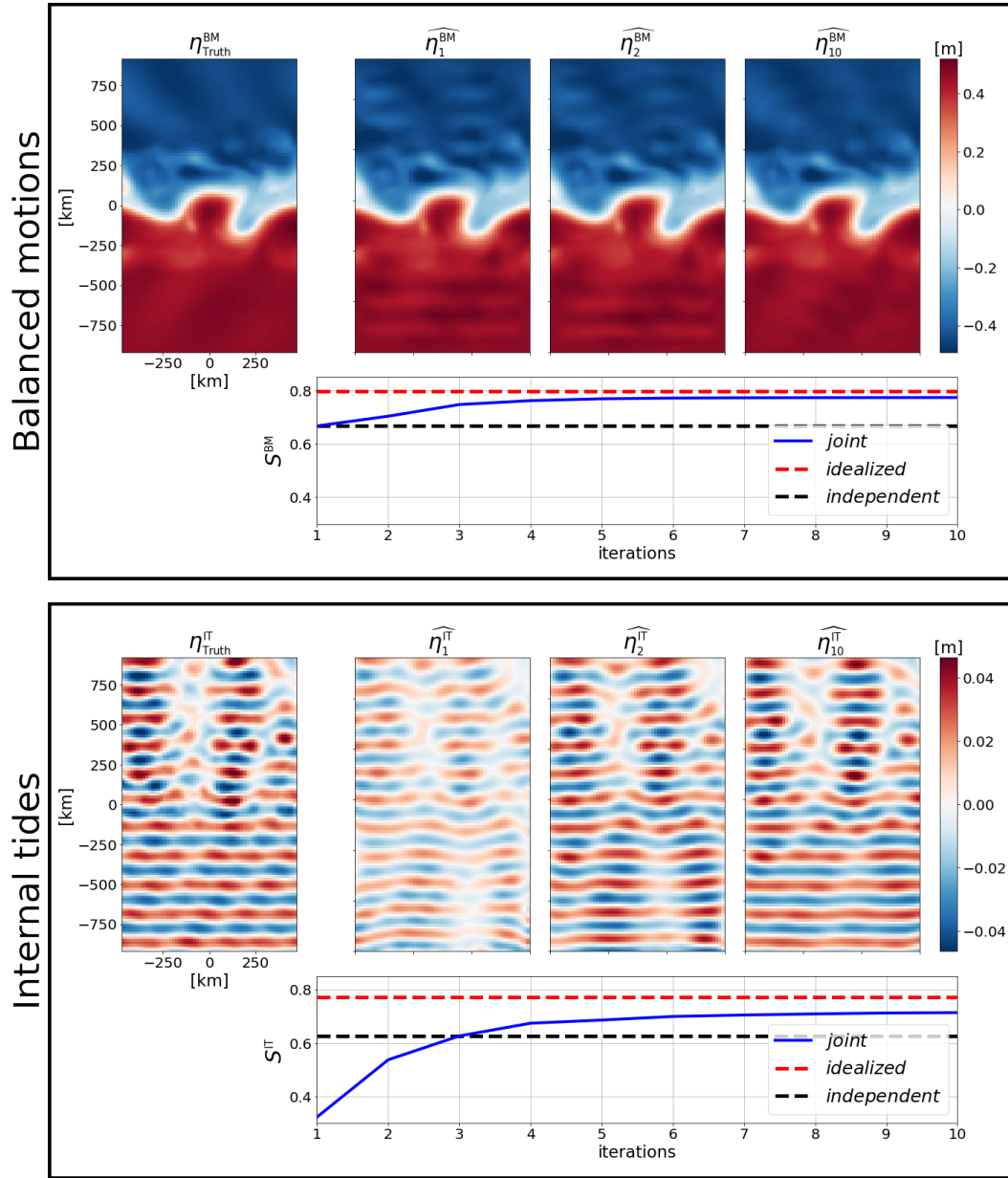


Figure 4. Performances of the estimation of BM (top) and IT (bottom). For each class of motion, snapshots at the first, the second, and the tenth iteration are shown. The evolution of the scores for the *joint* (solid blue curve), *idealized* (dash red curve) and *independent* (dash black curve) scenarii are plotted in function of the iterations of the joint estimation algorithm.

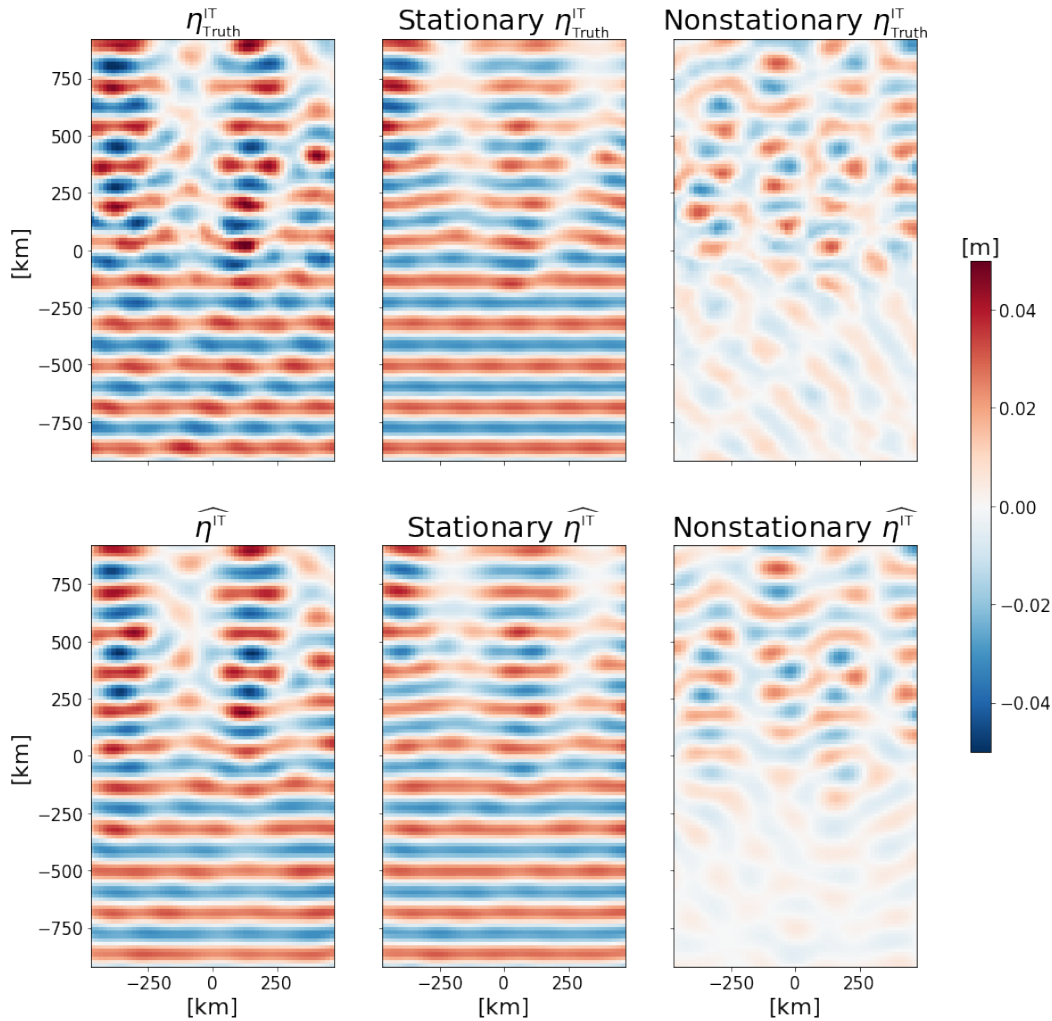


Figure 5. SSH snapshots of the IT field from the reference (top) and the reconstruction by the joint estimation algorithm (bottom). The full IT signal (left) is decomposed in the stationary (middle) and nonstationary (right) components.

342 Without the alternating minimization, the estimation scores for both BM and IT fall between
 343 60% (worst case, *independent* scenario) and 80% (best case, *idealized* scenario). The
 344 80% bound is likely due to the observation time sampling and to physical processes unresolved
 345 by the QG and SW models. These processes include the presence of higher baroclinic modes
 346 and ageostrophic dynamics for the QG model, higher tidal harmonics and neglected terms in
 347 the SW model. The lower 60% bound reflects the presence of the component that the system
 348 can not (or hardly) reconstruct (IT for the QG model, BM for the SW model). This compo-
 349 nent appears as noise in the observations.

350 The joint estimation algorithm is able to disentangle BM and IT components of SSH.
 351 Throughout iterations, both components are progressively separated. This is particularly vis-
 352 ible in the $\widehat{\eta}^{\text{BM}}$ snapshots, where IT signature is clearly visible at the first iteration, and grad-
 353 ually filtered out throughout iterations. We attribute the strength of the joint estimation to the
 354 fact that each data assimilation sub-system (BFN-QG and 4DVar-SW) filters out the compo-
 355 nit it is not supposed to reconstruct. This is reflected in the relatively high (closer to 1 than
 356 0) scores reached in the *independent* scenarii.

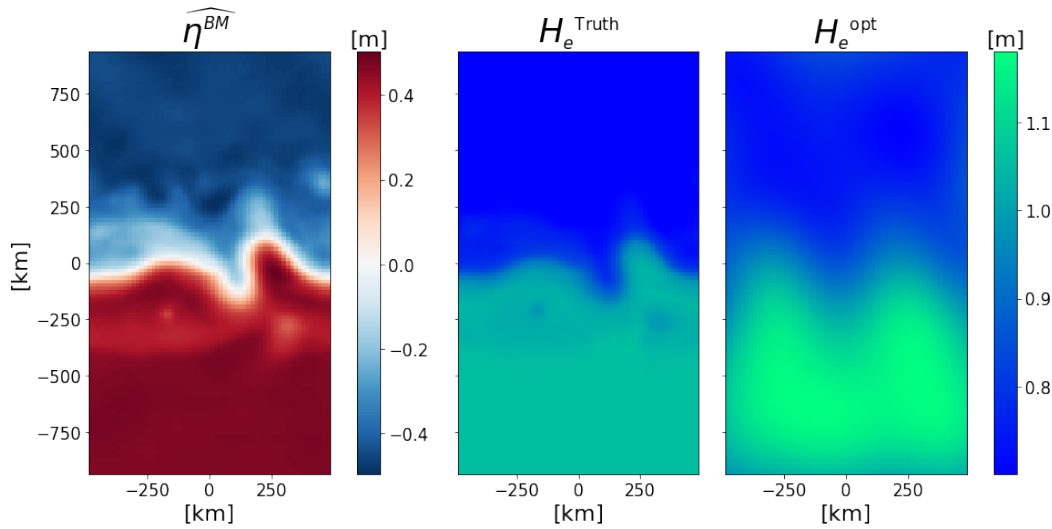


Figure 6. Left: BM SSH reconstructed by BFN-QG in the joint estimation algorithm. Middle: reference H_e computed from the 3D fields of the reference simulation. Right: H_e estimated by 4Dvar-SW in the joint estimation algorithm.

357 The 4Dvar-SW technique can reconstruct a large part of the nonstationary part of IT, as
 358 illustrated on Figure 5. 87 % of the stationary and 44% of the nonstationary IT variances are
 359 recovered on average. For the nonstationary IT fields, 4Dvar-SW captures 54% of the variance
 360 in the northern part of the domain, but only 28% in the southern part. This poor performance
 361 is attributed to the weakness of the nonstationary signal in the southern part (Figure 5, right
 362 column). Besides, some physical processes, such as IT reflection on the turbulent jet, may not
 363 be captured by 4Dvar-SW.

364 The spatial patterns of SSH $\widehat{\eta}^{BM}$ estimated by BFN-QG and the optimal equivalent depth
 365 H_e^{opt} obtained with 4Dvar-SW share similarities together and with the reference equivalent depth,
 366 as illustrated on Figure 6. A natural improvement of the method would be to use $\widehat{\eta}^{BM}$ to es-
 367 timate a background value H_e^b in the 4Dvar-SW to improve the IT reconstruction. This would
 368 require a climatology of the stratification to convert SSH to H_e . Reciprocally, 4Dvar-SW could
 369 help refine the estimation of H_e and stratification.

370 6.2 Sensitivity to the observation temporal sampling

371 In this section we investigate the impact of the observation temporal sampling on the es-
 372 timation of IT and BM. 13 time intervals Δt_{obs} are tested, from 69 h to 75 h by increments
 373 of 30 min. The IT period being $T=12$ h, the 6 h-interval length corresponds to $T/2$ and the
 374 central 72 h step corresponds to a multiple of T . For each Δt_{obs} , we run an *idealized*, an *in-*
 375 *dependent* and an *joint* experiments. The results are shown on Figure 7.

376 The IT estimation is strongly degraded near $\Delta t_{obs}=72$ h, what affects the reconstruction
 377 of BM in the joint estimation. At $\Delta t_{obs}=72$ h, IT estimation scores are even negative in the *joint*
 378 and *independent* scenarii. This particular time interval presents a singularity and leads to an
 379 aliasing issue, since it observes only one phase of the wave field and does not detect the wave
 380 propagation. The same occurs with multiples of $T/2$ (e.g. 66 h and 78 h, not shown here), when
 381 only two opposite phases of the wave field are observed. Since the IT component is not fil-
 382 tered out by 4Dvar-SW, the estimation of BM by BFN-QG in the *joint* scenario is similar to
 383 the *independent* one.

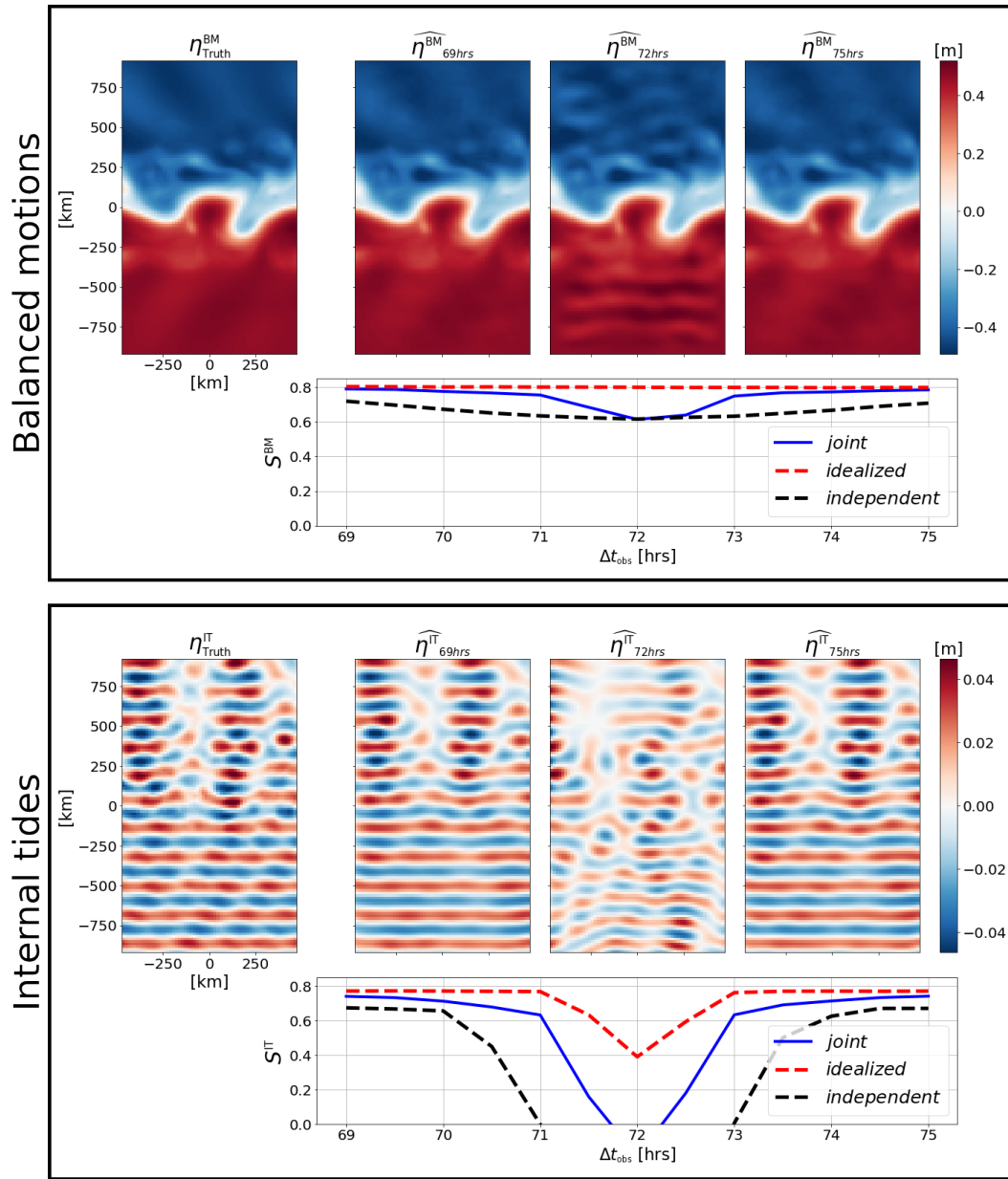


Figure 7. Performances of the joint estimation algorithm in function of the observation temporal sampling.

384 Using the joint estimation algorithm mitigates the poor IT reconstruction with time in-
385 tervals near 72 h. The IT estimation scores with $\Delta t_{\text{obs}} = 71$ h or 73 h are negative in the *in-*
386 *dependent* experiments, but exceed 0.6 in the *joint* experiments. This suggested robustness of
387 the joint estimation algorithm is encouraging for the processing of the future SWOT data.

388 7 Conclusions

389 In this study, we have presented an alternating minimization algorithm to simultaneously
390 and dynamically map and separate Internal Tide (IT) and Balanced Motions (BM) signals from
391 2D altimetric observations. Two data assimilation techniques are used in an iterative process
392 to estimate both signals: BM are reconstructed by the BFN-QG algorithm while IT are recon-
393 structed by the 4Dvar-SW algorithm. At any given iteration, the assimilated observations used
394 to map one signal are made of the difference between the full observation (containing both sig-
395 nals) and the previous estimation of the other signal. The use of the 4Dvar-SW algorithm for
396 IT reconstruction allows to map (i.e., estimate in magnitude and phase) both stationary and
397 nonstationary IT signals which is not obvious with other conventional separation methods such
398 as harmonic analyses.

399 The joint estimation of BM and IT has been tested in an idealized context, with a true
400 state of the ocean and artificial observations extracted from a numerical simulation that prop-
401 agates a mode-one internal tide through a turbulent quasigeostrophic jet. This numerical ex-
402 periment aims at illustrating the feasibility of this approach in a simplified context and should
403 constitute a step closer to the processing of future SWOT data.

404 The results show that the joint estimation allows to simultaneously separate and estimate
405 78% and 75% of the BM and IT variance, respectively. The remaining variance is interpreted
406 as being due to non resolved dynamics by the QG and SW models.

407 A sensitivity study of the observational temporal sampling has also been performed and
408 shows that an unfortunate temporal sampling of the observations can be detrimental to the joint
409 estimation performances. In particular, the performances of the IT reconstruction are consid-
410 erably reduced when the time step between two consecutive observations is a multiple of half
411 the IT time period $T/2$. Fortunately, it turns out that this situation is singular for non sun-synchronous
412 orbits, meaning that a slight shift from a multiple of $T/2$ in the observation time sampling en-
413 sures good performances again.

414 Although the results of the observational temporal sampling sensitivity study are encour-
415 aging, future work should investigate the method applied to a realistic SWOT sampling. One
416 of the major next steps will be to assess the potential of the method with spatially sparse ob-
417 servations. An interesting avenue to improve the method in that context could be to use ad-
418 ditional information, such as conventional nadir data or SWOT nadir data, to ensure the joint
419 estimation method convergence hence improving the reconstructions.

420 Another crucial step towards operational SSH mapping will be to experiment on a more
421 realistic ocean simulation, with multiple tidal components and harmonics in constant interac-
422 tion. To allow the 4Dvar-SW to efficiently extract IT signal from realistic SSH data, some tech-
423 nical improvement may be needed. Source terms, that generate IT signal inside the study do-
424 main, may have to be implemented and controlled by the assimilation to map IT signals around
425 generation sites (located at ocean ridge and sea mounts for instance). One can also think of
426 improving the resolved dynamics by including higher baroclinic modes. Another improvement
427 would be to add interaction terms between BM and IT in the shallow water equations. For that
428 last remark, one strategy would be to estimate a background term of H_e from the reconstruc-
429 tion of BM.

430 **Acknowledgments**

431 This research was funded by ANR (project number ANR-17- CE01-0009-01) and CNES through
 432 the SWOT Science Team program. The numerical codes and the data used in this study can
 433 be accessed here: <https://github.com/leguillf/MASSH>

434 **References**

435 Ajayi, A., Le Sommer, J., Chassignet, E., Molines, J.-M., Xu, X., Albert, A., & Dewar, W.
 436 (2019). Diagnosing cross-scale kinetic energy exchanges from two submesoscale
 437 permitting ocean models. *Earth and Space Science Open Archive*, 26. Retrieved
 438 from <https://www.essoar.org/doi/abs/10.1002/essoar.10501077.1> doi:
 439 10.1002/essoar.10501077.1

440 Ajayi, A., Sommer, J. L., Chassignet, E., Molines, J.-M., Xu, X., Albert, A., & Cosme,
 441 E. (2020). Spatial and temporal variability of the north atlantic eddy field
 442 from two kilometric-resolution ocean models. *Journal of Geophysical Re-*
 443 *search: Oceans*, 125(5), e2019JC015827. Retrieved 2020-06-15, from [https://](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JC015827)
 444 agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JC015827
 445 (_eprint: <https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/2019JC015827>)
 446 doi: 10.1029/2019JC015827

447 Amores, A., Jordão, G., Arsouze, T., & Le Sommer, J. (2018). Up to what extent can we
 448 characterize ocean eddies using present-day gridded altimetric products? *Journal*
 449 *of Geophysical Research: Oceans*, 123(10), 7220-7236. Retrieved from [https://](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JC014140)
 450 agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JC014140 doi:
 451 <https://doi.org/10.1029/2018JC014140>

452 Anthes, R. (1974). Data assimilation and initialization of hurricane prediction models. *J. At-*
 453 *mos. Sci.*, 31, 701-719.

454 Auroux, D., & Blum, J. (2008). A nudging-based data assimilation method: the back and
 455 forth nudging (BFN) algorithm. *Nonlinear Process Geophys.*, 15(2), 305-319. (Place:
 456 Gottingen Publisher: Copernicus Gesellschaft Mbh WOS:000255511000007) doi: 10
 457 .5194/npg-15-305-2008

458 Ballarotta, M., Ubelmann, C., Pujol, M.-I., Taburet, G., Fournier, F., Legeais, J.-F., . . . Picot,
 459 N. (2019). On the resolutions of ocean altimetry maps. *Ocean Sci.*, 15(4), 1091-1109.
 460 (Place: Gottingen Publisher: Copernicus Gesellschaft Mbh WOS:000481992900001)
 461 doi: 10.5194/os-15-1091-2019

462 Blayo, E., Blum, J., & Verron, J. (1998). Assimilation variationnelle de données en
 463 océanographie et réduction de la dimension de l'espace de contrôle (variational
 464 data assimilation in oceanography and reduction of the control-space dimension).

465 Blayo, E., & Debreu, L. (2005). Revisiting open boundary conditions from the point of view
 466 of characteristic variables. *Ocean Modelling*. Retrieved from [https://hal.inria](https://hal.inria.fr/inria-00134856)
 467 [.fr/inria-00134856](https://hal.inria.fr/inria-00134856)

468 Capet, X., Roulet, G., Klein, P., & Maze, G. (2016, 11). Intensification of Upper-Ocean
 469 Submesoscale Turbulence through Charney Baroclinic Instability. *Journal of Physi-*
 470 *cal Oceanography*, 46(11), 3365-3384. Retrieved from [https://doi.org/10.1175/](https://doi.org/10.1175/JPO-D-16-0050.1)
 471 [JPO-D-16-0050.1](https://doi.org/10.1175/JPO-D-16-0050.1) doi: 10.1175/JPO-D-16-0050.1

472 Courtier, P., Thépaut, J., & Hollingsworth, A. (1994). A strategy for operational implementa-
 473 tion of 4D-Var, using an incremental approach. *Quarterly Journal of the Royal Me-*
 474 *teorological Society*, 120, 1367-1387.

475 DIMET, F.-X. L., & TALAGRAND, O. (1986). Variational algorithms for analysis
 476 and assimilation of meteorological observations: theoretical aspects. *Tellus A*,
 477 38A(2), 97-110. Retrieved from [https://onlinelibrary.wiley.com/doi/](https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1600-0870.1986.tb00459.x)
 478 [abs/10.1111/j.1600-0870.1986.tb00459.x](https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1600-0870.1986.tb00459.x) doi: [https://doi.org/10.1111/](https://doi.org/10.1111/j.1600-0870.1986.tb00459.x)
 479 [j.1600-0870.1986.tb00459.x](https://doi.org/10.1111/j.1600-0870.1986.tb00459.x)

480 Dunphy, M., Ponte, A., Klein, P., & Gentil, S. (2017, 01). Low-mode internal tide propaga-
 481 tion in a turbulent eddy field. *Journal of Physical Oceanography*, 47. doi: 10.1175/
 482 JPO-D-16-0099.1

483 Flather, R. (1987). A tidal model of the northeast pacific. *Atmosphere-Ocean*, 25(1), 22–45.
 484 Retrieved from <https://doi.org/10.1080/07055900.1987.9649262> (Publisher:
 485 Taylor & Francis) doi: 10.1080/07055900.1987.9649262

486 Fu, L.-L., & Ubelmann, C. (2014). On the transition from profile altimeter to swath altimeter
 487 for observing global ocean surface topography. *Journal of Atmospheric and Oceanic*
 488 *Technology*, 31(2), 560 - 568. Retrieved from [https://journals.ametsoc.org/
 489 view/journals/atot/31/2/jtech-d-13-00109_1.xml](https://journals.ametsoc.org/view/journals/atot/31/2/jtech-d-13-00109_1.xml) doi: 10.1175/JTECH-D
 490 -13-00109.1

491 Garrett, C., & Kunze, E. (2007). Internal tide generation in the deep ocean. *Annual Review*
 492 *of Fluid Mechanics*, 39(1), 57-87. Retrieved from [https://doi.org/10.1146/
 493 annurev.fluid.39.050905.110227](https://doi.org/10.1146/annurev.fluid.39.050905.110227) doi: 10.1146/annurev.fluid.39.050905
 494 .110227

495 Gill, A. E. (1982). Atmosphere-ocean dynamics. *Int. Geophys. Ser.*, 30, 662p.

496 Gonzalez-Haro, C., Isern-Fontanet, J., Tandeo, P., & Autret, E. (2019, May). High-resolution
 497 currents from the synergy between Infrared SST and altimetry. *LPS 2019 : Living*
 498 *Planet Symposium*.

499 Guillou, F. L., Metref, S., Cosme, E., Ubelmann, C., Ballarotta, M., Sommer, J. L., &
 500 Verron, J. (2021). Mapping altimetry in the forthcoming swot era by back-and-
 501 forth nudging a one-layer quasigeostrophic model. *Journal of Atmospheric and*
 502 *Oceanic Technology*, 38(4), 697 - 710. Retrieved from [https://journals
 503 .ametsoc.org/view/journals/atot/38/4/JTECH-D-20-0104.1.xml](https://journals.ametsoc.org/view/journals/atot/38/4/JTECH-D-20-0104.1.xml) doi:
 504 10.1175/JTECH-D-20-0104.1

505 Haren, H. v., Laurent, L. S., & Marshall, D. (2004). Small and mesoscale processes
 506 and their impact on the large scale: an introduction. , 51(25), 2883–2887. Re-
 507 trieved from [https://www.sciencedirect.com/science/article/pii/
 508 S0967064504002061](https://www.sciencedirect.com/science/article/pii/S0967064504002061) doi: <https://doi.org/10.1016/j.dsr2.2004.09.010>

509 Kelly, S. M., & Lermusiaux, P. F. J. (2016). Internal-tide interactions with the gulf
 510 stream and middle atlantic bight shelfbreak front. *Journal of Geophysical Research:*
 511 *Oceans*, 121(8), 6271-6294. Retrieved from [https://agupubs.onlinelibrary
 512 .wiley.com/doi/abs/10.1002/2016JC011639](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2016JC011639) doi: [https://doi.org/10.1002/
 513 2016JC011639](https://doi.org/10.1002/2016JC011639)

514 Klein, P., & Lapeyre, G. (2009, 01). The oceanic vertical pump induced by mesoscale and
 515 submesoscale turbulence. *Annual review of marine science*, 1, 351-75. doi: 10.1146/
 516 annurev.marine.010908.163704

517 McWilliams, J. C. (2016). Submesoscale currents in the ocean. *Proceedings of the Royal*
 518 *Society A: Mathematical, Physical and Engineering Sciences*, 472(2189), 20160117.
 519 Retrieved from [https://royalsocietypublishing.org/doi/abs/10.1098/
 520 rspa.2016.0117](https://royalsocietypublishing.org/doi/abs/10.1098/rspa.2016.0117) doi: 10.1098/rspa.2016.0117

521 Morrow, R., Fu, L.-L., Arduin, F., Benkiran, M., Chapron, B., Cosme, E., . . . Zaron, E. D.
 522 (2019). Global observations of fine-scale ocean surface topography with the sur-
 523 face water and ocean topography (swot) mission. *Frontiers in Marine Science*,
 524 6, 232. Retrieved from [https://www.frontiersin.org/article/10.3389/
 525 fmars.2019.00232](https://www.frontiersin.org/article/10.3389/fmars.2019.00232) doi: 10.3389/fmars.2019.00232

526 Nash, J., Kelly, S., Shroyer, E., Moum, J., & Duda, T. (2012, 11). The unpredictable nature
 527 of internal tides on continental shelves. *Journal of Physical Oceanography*, 42, 1981-
 528 2000. doi: 10.1175/JPO-D-12-028.1

529 Nelson, A. D., Arbic, B. K., Zaron, E. D., Savage, A. C., Richman, J. G., Buijsman, M. C.,
 530 & Shriver, J. F. (2019). Toward realistic nonstationarity of semidiurnal baroclinic
 531 tides in a hydrodynamic model. *Journal of Geophysical Research: Oceans*, 124(9),
 532 6632-6642. Retrieved from [https://agupubs.onlinelibrary.wiley.com/doi/
 533 abs/10.1029/2018JC014737](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2018JC014737) doi: <https://doi.org/10.1029/2018JC014737>

534 Ponte, A. L., & Klein, P. (2015, March). Incoherent signature of internal tides on sea level in
 535 idealized numerical simulations. *Geophysical Research Letters*, 42(5), 1520-1526. Re-
 536 trieved from <https://hal.archives-ouvertes.fr/hal-01149664> doi: 10.1002/
 537 2014GL062583

- 538 Ponte, A. L., Klein, P., Dunphy, M., & Le Gentil, S. (2017). Low-mode internal tides and
 539 balanced dynamics disentanglement in altimetric observations: Synergy with surface
 540 density observations. *Journal of Geophysical Research: Oceans*, *122*(3), 2143-
 541 2155. Retrieved from [https://agupubs.onlinelibrary.wiley.com/doi/abs/](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2016JC012214)
 542 [10.1002/2016JC012214](https://doi.org/10.1002/2016JC012214) doi: 10.1002/2016JC012214
- 543 Qiu, B., Chen, S., Klein, P., Wang, J., Torres, H., Fu, L.-L., & Menemenlis, D. (2018, 01).
 544 Seasonality in transition scale from balanced to unbalanced motions in the world
 545 ocean. *Journal of Physical Oceanography*, *48*. doi: 10.1175/JPO-D-17-0169.1
- 546 Rainville, L., & Pinkel, R. (2006). Propagation of low-mode internal waves through the
 547 ocean. , *36*(6), 1220 – 1236. Retrieved from [https://journals.ametsoc.org/](https://journals.ametsoc.org/view/journals/phoc/36/6/jpo2889.1.xml)
 548 [view/journals/phoc/36/6/jpo2889.1.xml](https://journals/phoc/36/6/jpo2889.1.xml) (Place: Boston MA, USA Publisher:
 549 American Meteorological Society) doi: 10.1175/JPO2889.1
- 550 Ray, R. D., & Zaron, E. D. (2011). Non-stationary internal tides observed with satellite al-
 551 timetry. *Geophysical Research Letters*, *38*(17). Retrieved from [https://agupubs](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011GL048617)
 552 [.onlinelibrary.wiley.com/doi/abs/10.1029/2011GL048617](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2011GL048617) doi: [https://doi](https://doi.org/10.1029/2011GL048617)
 553 [.org/10.1029/2011GL048617](https://doi.org/10.1029/2011GL048617)
- 554 Ray, R. D., & Zaron, E. D. (2015, 12). M2 Internal Tides and Their Observed Wavenum-
 555 ber Spectra from Satellite Altimetry*. *Journal of Physical Oceanography*, *46*(1), 3-22.
 556 Retrieved from <https://doi.org/10.1175/JPO-D-15-0065.1> doi: 10.1175/JPO
 557 -D-15-0065.1
- 558 Ray, R. D., & Zaron, E. D. (2016). M2 internal tides and their observed wavenumber spectra
 559 from satellite altimetry. , *46*(1), 3 – 22. Retrieved from [https://journals.ametsoc](https://journals.ametsoc.org/view/journals/phoc/46/1/jpo-d-15-0065.1.xml)
 560 [.org/view/journals/phoc/46/1/jpo-d-15-0065.1.xml](https://journals/phoc/46/1/jpo-d-15-0065.1.xml) (Place: Boston MA,
 561 USA Publisher: American Meteorological Society) doi: 10.1175/JPO-D-15-0065.1
- 562 Robert, C., Durbiano, S., Blayo, E., Verron, J., Blum, J., & Le Dimet, F.-X. (2005, Aug). A
 563 reduced-order strategy for 4d-var data assimilation. *Journal of Marine Systems*, *57*(1-
 564 2), 70–82. Retrieved from [http://dx.doi.org/10.1016/j.jmarsys.2005.04](http://dx.doi.org/10.1016/j.jmarsys.2005.04.003)
 565 [.003](http://dx.doi.org/10.1016/j.jmarsys.2005.04.003) doi: 10.1016/j.jmarsys.2005.04.003
- 566 Sadourny, R. (1975). The dynamics of finite-difference models of the shallow-water equa-
 567 tions. , *32*(4), 680 – 689. Retrieved from [https://journals.ametsoc.org/view/](https://journals.ametsoc.org/view/journals/atsc/32/4/1520-0469_1975_032_0680_tdofofm_2_0_co_2.xml)
 568 journals/atsc/32/4/1520-0469_1975_032_0680_tdofofm_2_0_co_2.xml
 569 (Place: Boston MA, USA Publisher: American Meteorological Society) doi:
 570 10.1175/1520-0469(1975)032<0680:TDOFDM>2.0.CO;2
- 571 Taburet, G., Sanchez-Roman, A., Ballarotta, M., Pujol, M.-I., Legeais, J.-F., Fournier,
 572 F., . . . Dibarboue, G. (2019). DUACS DT2018: 25 years of reprocessed sea
 573 level altimetry products. *Ocean Science*, *15*(5), 1207–1224. Retrieved from
 574 <https://www.ocean-sci.net/15/1207/2019/> (Publisher: Copernicus GmbH)
 575 doi: <https://doi.org/10.5194/os-15-1207-2019>
- 576 Torres, H. S., Klein, P., Siegelman, L., Qiu, B., Chen, S., Ubelmann, C., . . . Fu, L.-
 577 L. (2019). Diagnosing ocean-wave-turbulence interactions from space. *Geo-*
 578 *physical Research Letters*, *46*(15), 8933-8942. Retrieved from [https://](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019GL083675)
 579 agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019GL083675 doi:
 580 10.1029/2019GL083675
- 581 Tseng, P. (1990). Further applications of a splitting algorithm to decomposition in variational
 582 inequalities and convex programming. , *48*(1), 249–263. Retrieved from [https://doi](https://doi.org/10.1007/BF01582258)
 583 [.org/10.1007/BF01582258](https://doi.org/10.1007/BF01582258) doi: 10.1007/BF01582258
- 584 Ubelmann, C., Klein, P., & Fu, L.-L. (2015). Dynamic interpolation of sea surface height
 585 and potential applications for future high-resolution altimetry mapping. *J. Atmos.*
 586 *Ocean. Technol.*, *32*(1), 177–184. (Place: Boston Publisher: American Meteorological Soc
 587 WOS:000348221100011) doi: 10.1175/JTECH-D-14-00152.1
- 588 Wunsch, C., & Ferrari, R. (2004). VERTICAL MIXING, ENERGY, AND THE GEN-
 589 ERAL CIRCULATION OF THE OCEANS. , *36*(1), 281–314. Retrieved 2021-03-29,
 590 from <http://arjournals.annualreviews.org/doi/abs/10.1146%2Fannurev>
 591 [.fluid.36.050802.122121](http://arjournals.annualreviews.org/doi/abs/10.1146%2Fannurev) doi: 10.1146/annurev.fluid.36.050802.122121
- 592 Zaron, E. D. (2017). Mapping the nonstationary internal tide with satellite altimetry. *Jour-*

593 *nal of Geophysical Research: Oceans*, 122(1), 539-554. Retrieved from [https://](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2016JC012487)
594 agupubs.onlinelibrary.wiley.com/doi/abs/10.1002/2016JC012487 doi:
595 10.1002/2016JC012487

Figure 1.

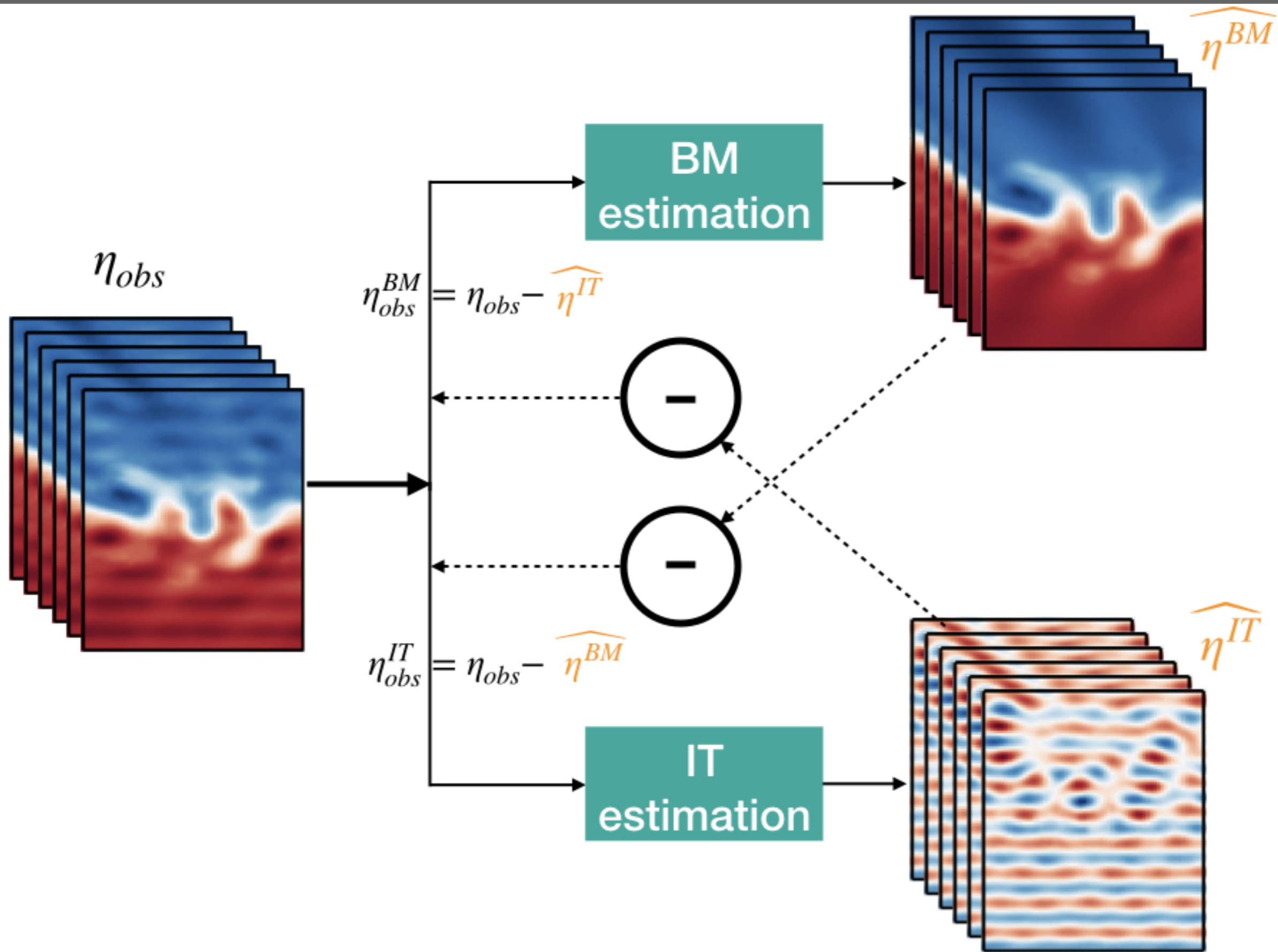


Figure 2.

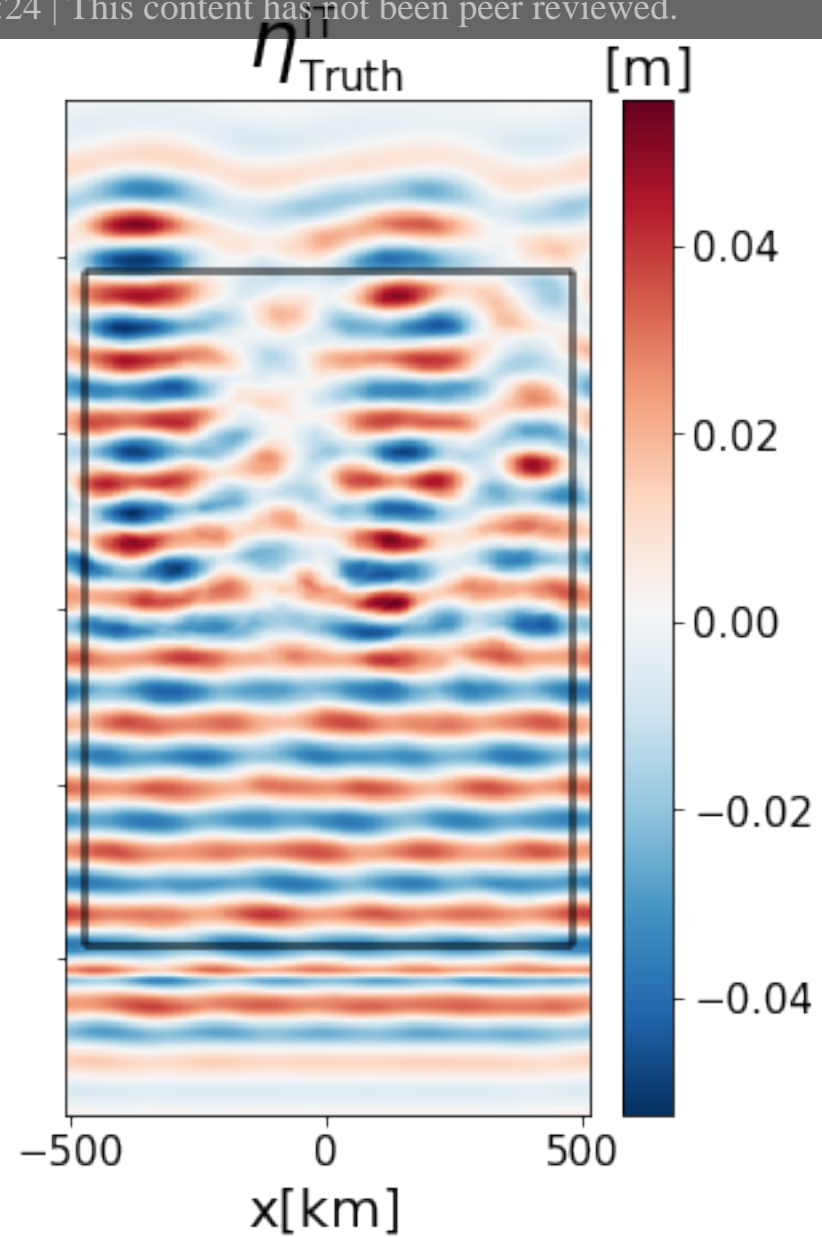
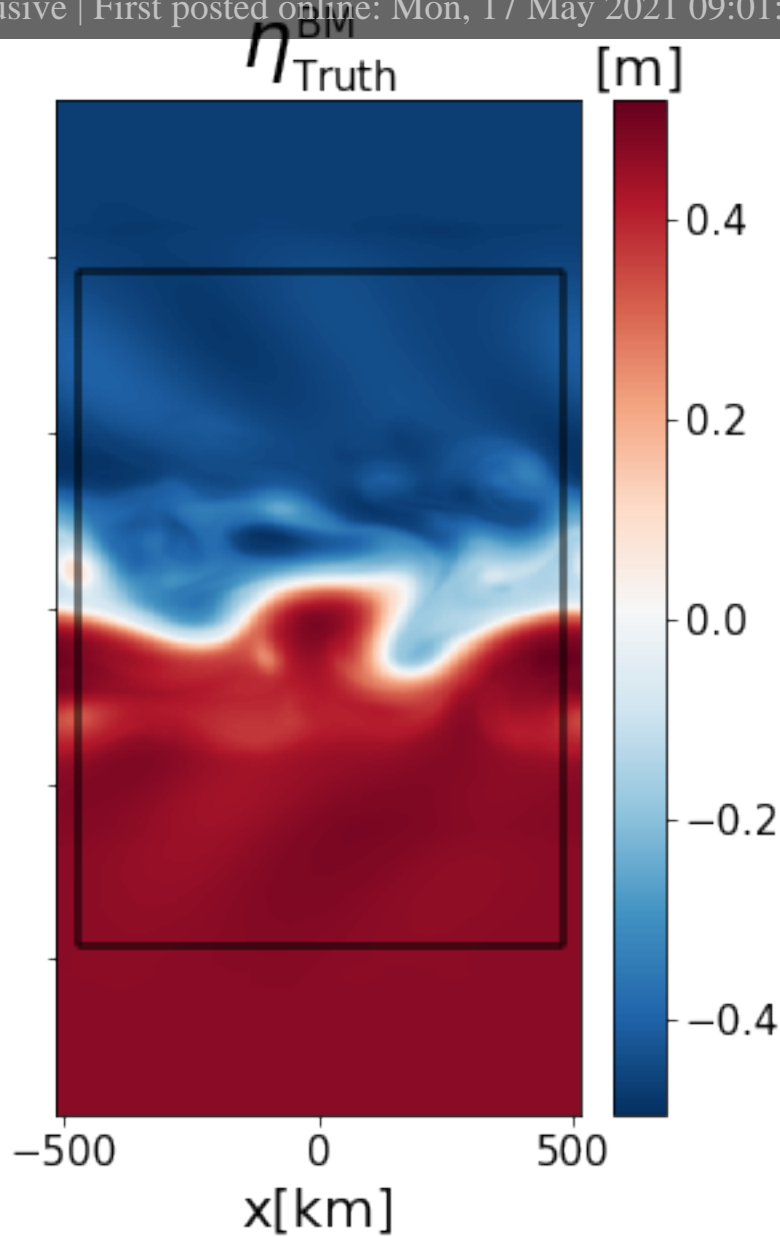
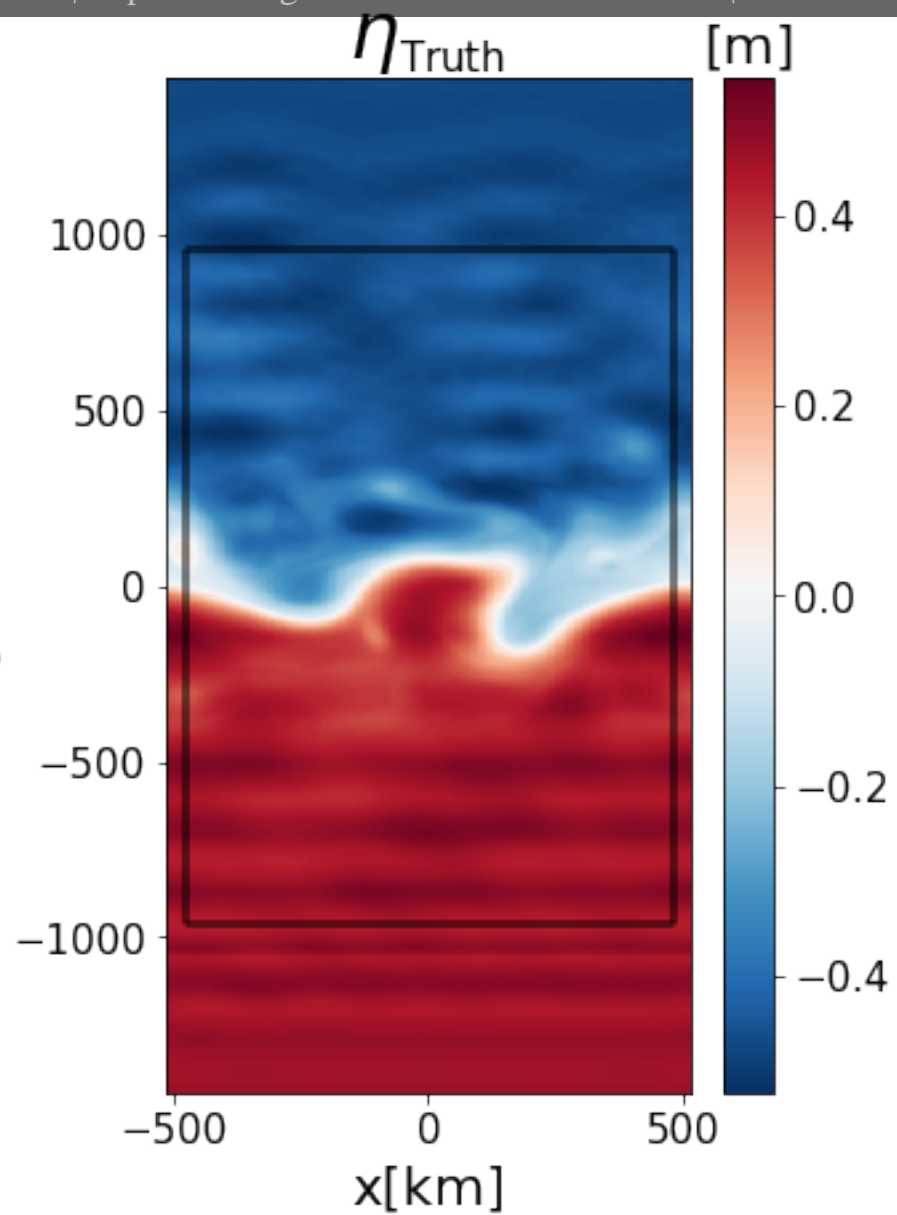


Figure 3.

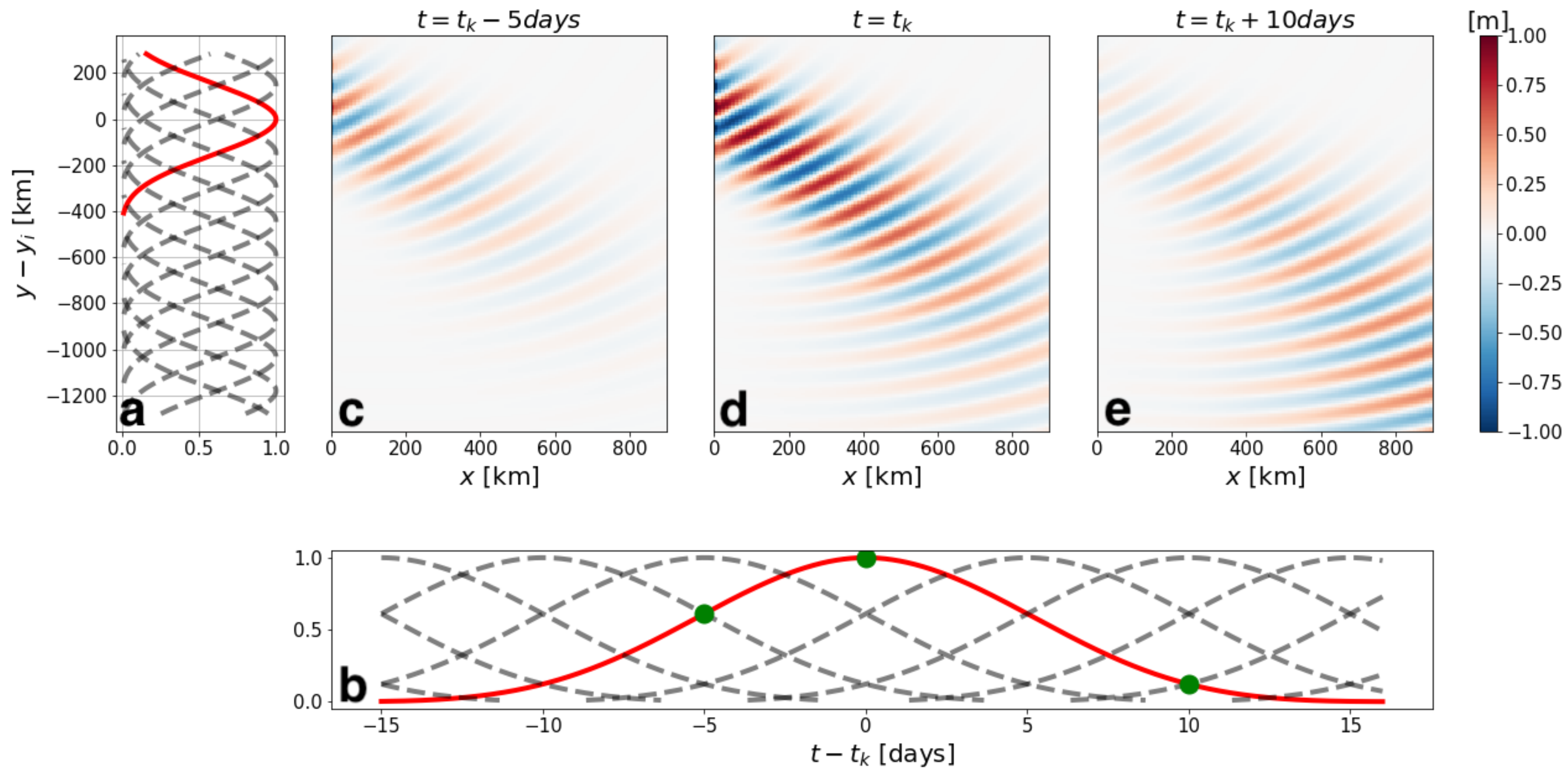
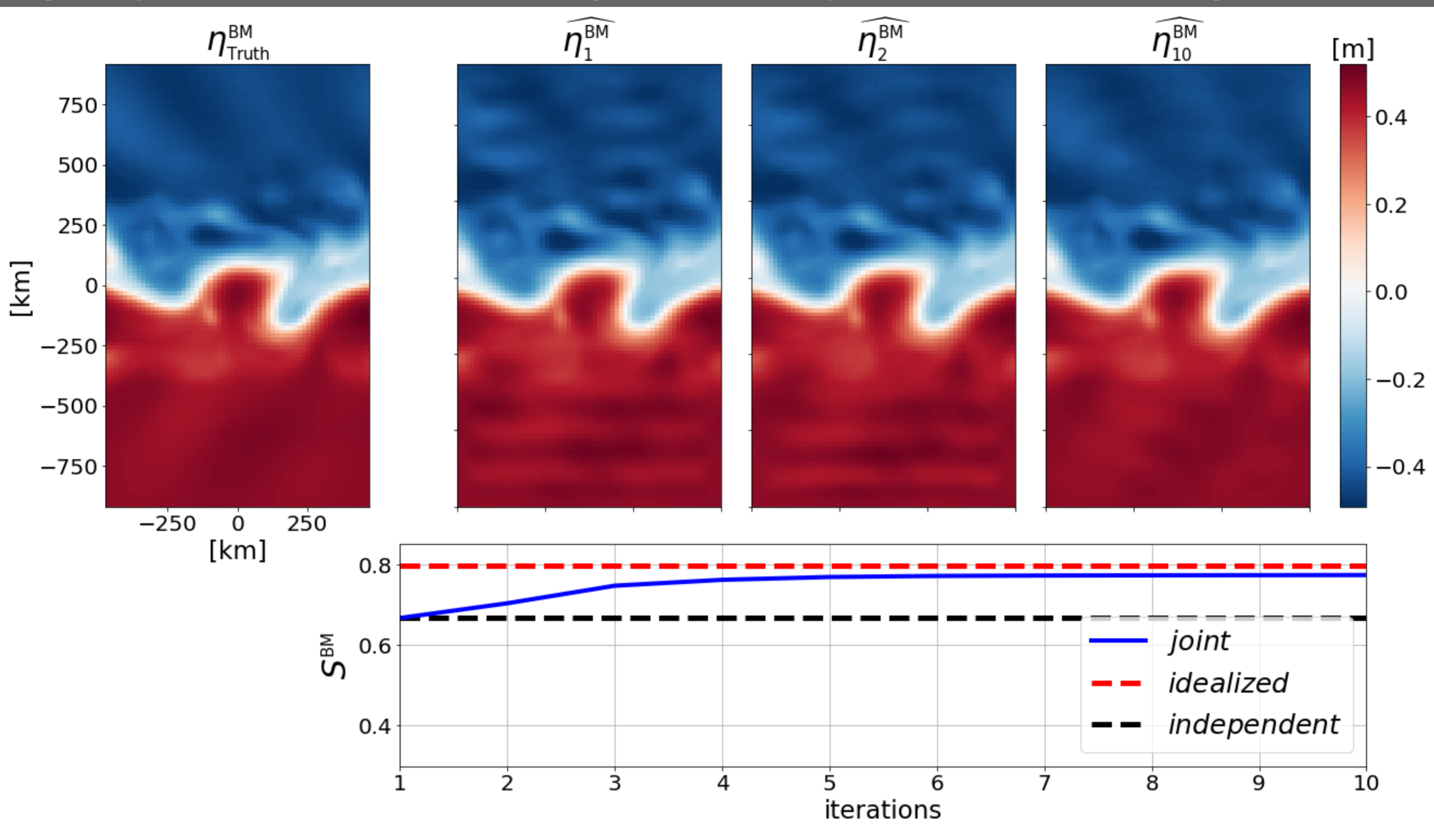


Figure 4.

Balanced motions



Internal tides

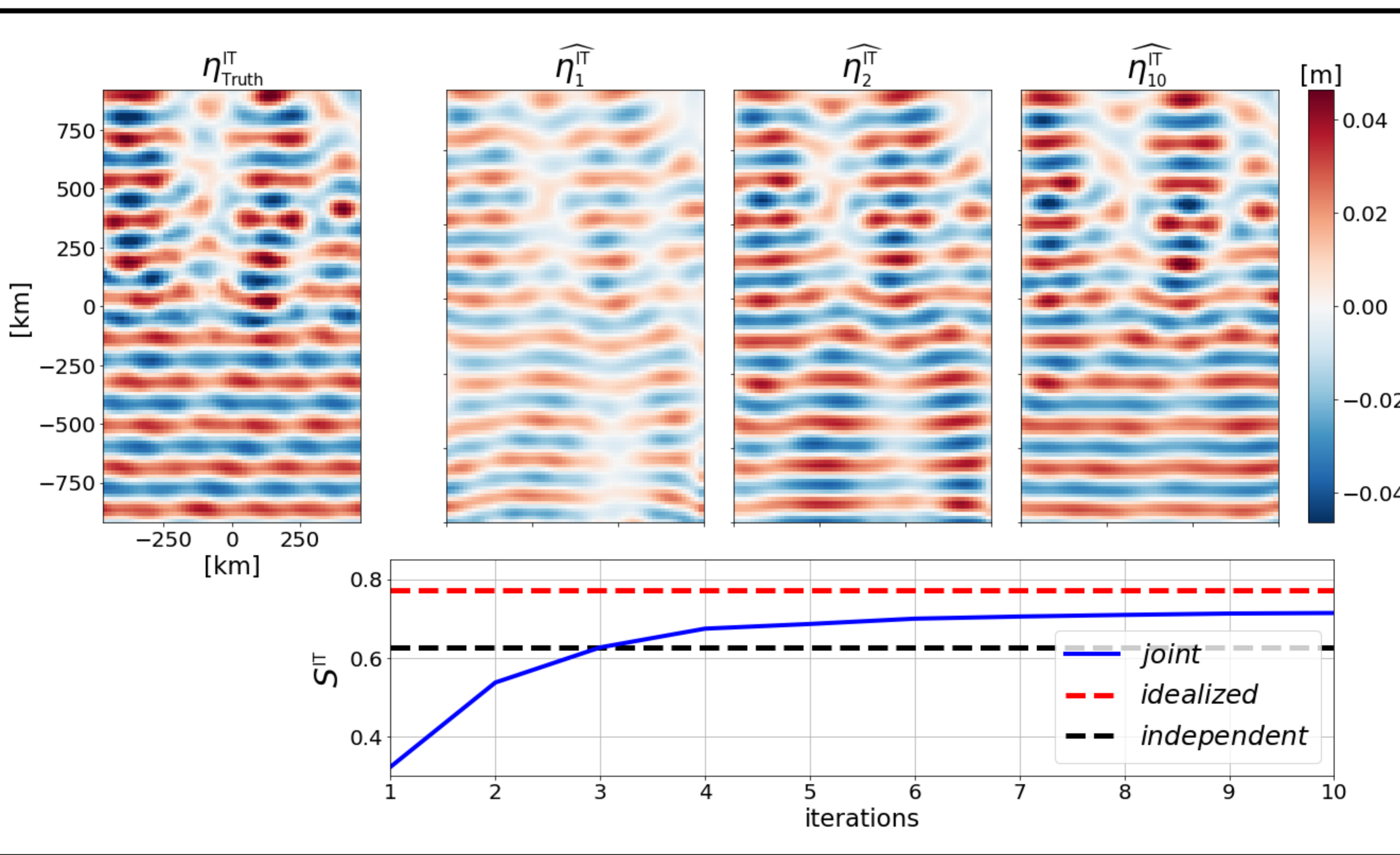


Figure 5.

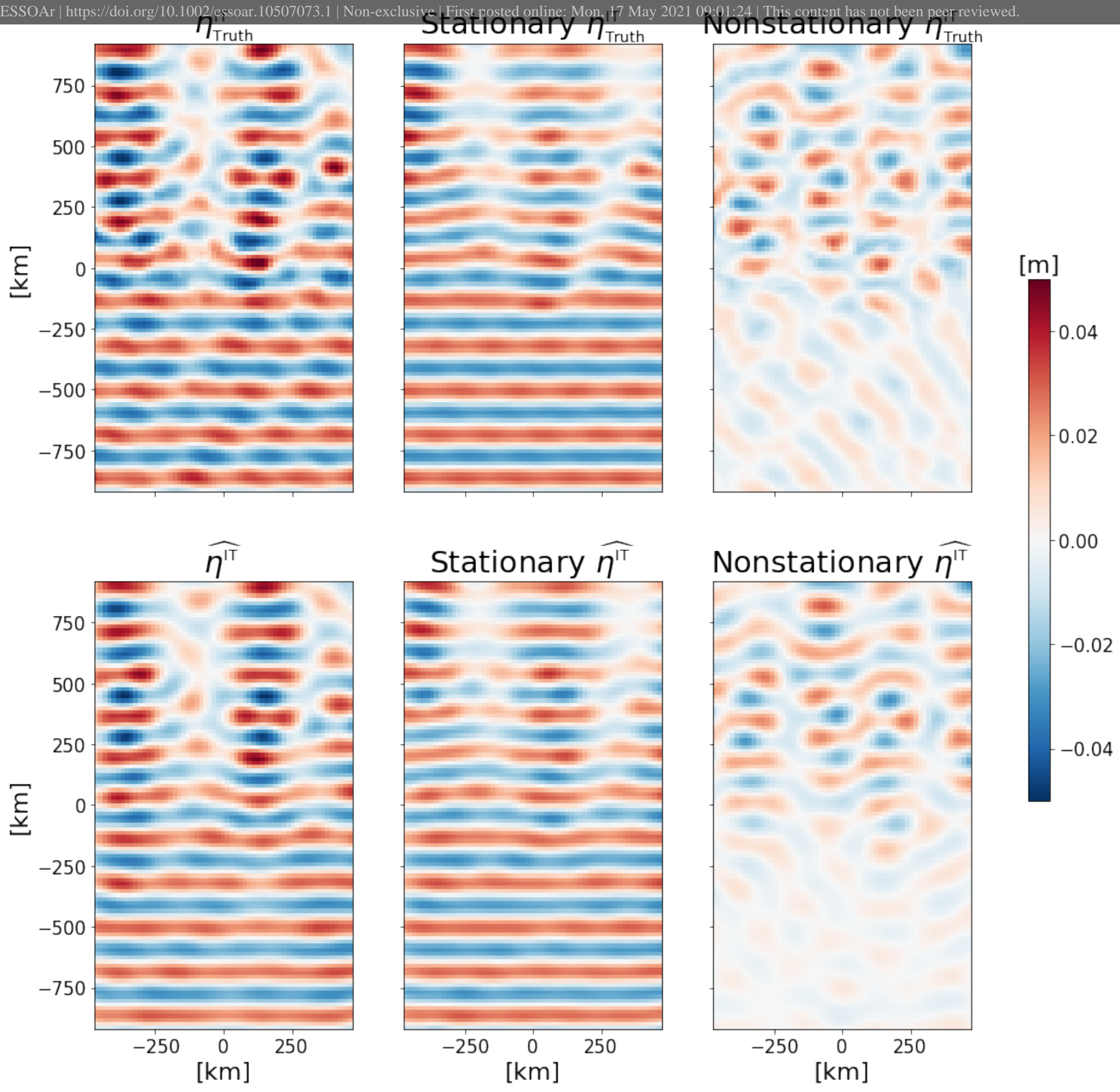


Figure 6.

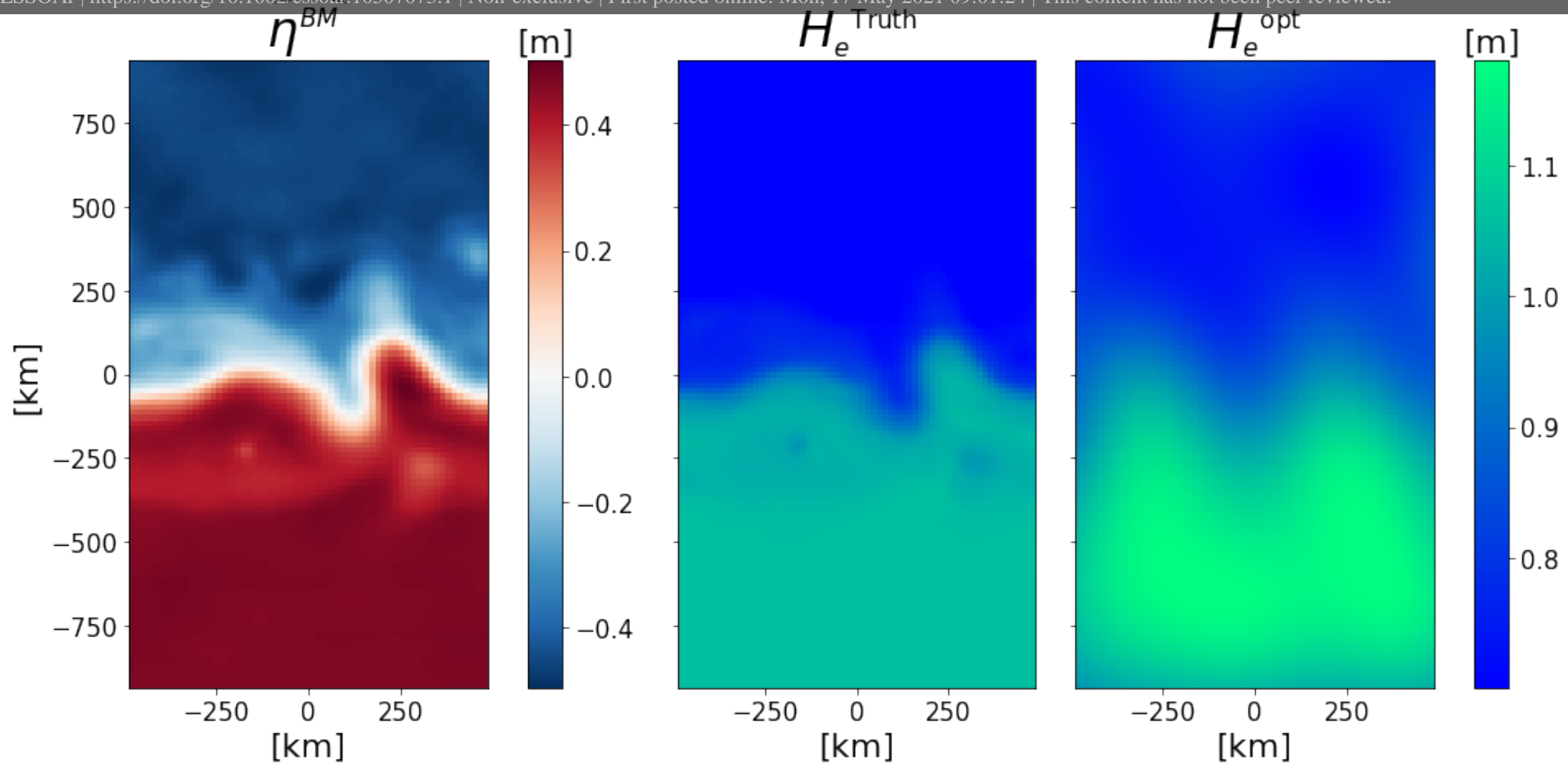
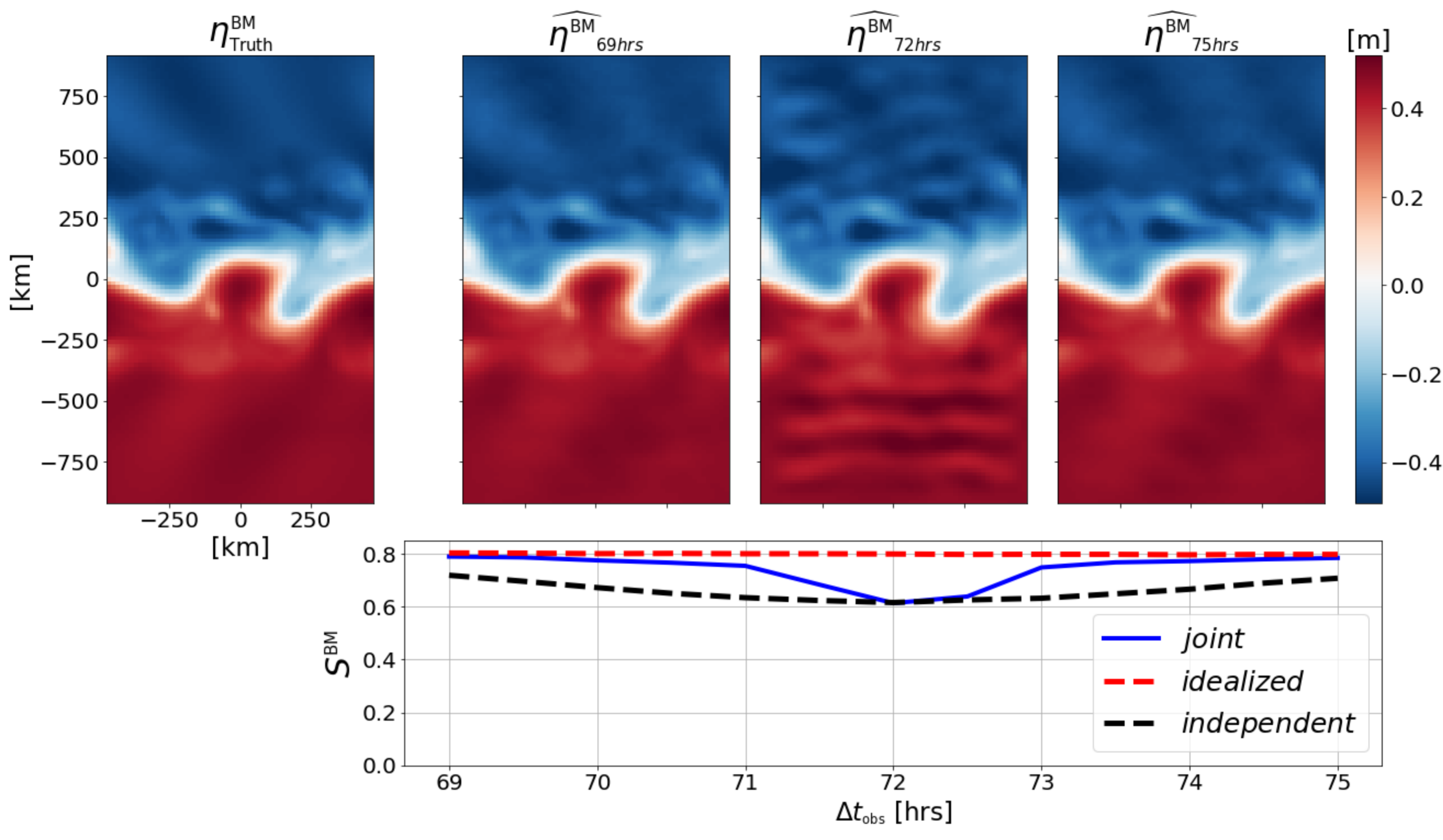


Figure 7.

Balanced motions



Internal tides

