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Optimal Pricing Approach Based on Expected Utility Maximization with Partial Information

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Abstract. Real-time pricing is considered as a promising strategy to flatten the power consumption provided with perfect knowledge of consumers' demand level. However, the gathering of full information of demand levels might be cumbersome or even impossible for the provider in practical scenarios. In this paper, instead of assuming the perfectly known demand levels, we investigate the problem where the provider has the sole knowledge of the probabilistic distribution of the demand levels. Furthermore, a penalty term caused by the prediction error of the consumption prediction is introduced due to the incomplete information. By solving the stochastic optimization problem, the optimal consumption prediction and optimal price to maximize the expected social welfare is derived analytically. Numerical results show that the degradation on the social welfare brought by the partial information can be less than 1% when the price and consumption prediction are well designed.

Keywords: Real-time pricing · Demand response · Consumption prediction

1 Introduction

An essential goal of smart grids is to create reliable communications between many components. the exchange and control of information can provide more effective generation and transmission of electricity, resulting in flattening the power consumption. To reduce the power consumption during the peak time, demand side management (DSM) has been proposed and shown to be a promising strategy in certain scenarios [1, 2].

Pricing-based demand response (DR) is one of the most widely used DSM methods, where the electricity price designed by the provider is related to the overall demand (or aggregate load) of the served consumers. In a smart grid

system, the energy provider can send the tariff information to the energy consumption controller (ECC) unit located at the consumer's devices (e.g., the smart meters), and thus the consumers can schedule their activities to low price periods. To shift the power consumption from the rush hours, the dynamic pricing is in accordance with the overall demand levels. Several different pricing have been proposed recently. For instance, the time of use pricing has on-peak tariff and off-peak tariff [3], the day-ahead pricing predicts the following day's consumption and propose a tariff according to its consumption prediction [4]. Due to the large deployment of smart meters, the real-time pricing can be implemented by communicating the demand of current period and feeding back the real-time tariff to the consumers. Under the ideal communication environment, the real-time price can be designed to maximize the social welfare or the benefit of the provider [5,6].

However, due to the limitation of available resources in the communication channels, the perfect observation of the demand levels can be prohibitive. In this paper, we consider the case where only partial information can be acquired by the provider. More precisely, the provide has the sole knowledge of the demand level statistics instead of knowing its instantaneous realizations. Moreover, the information shortage about the demand levels in provider's side leads to the uncertainty of total power consumption of the system. As a consequence, the procurement (or generation) of electricity in advance will be affected and thus a penalty term has been introduced in this paper to model this impact. Knowing the statistics of demand and the penalty term to the provider's cost function, the stochastic optimization problem has been studied here. The rest of the paper is organized as follows. The system model is introduced in Sect. 2. The problem is formulated and The optimal price and optimal load prediction are derived in Sect. 3 and Sect. 4 respectively. The paper ends by numerical results.

2 System Model

In this paper, we consider a smart power system consisting of an unique energy provider and several consumers. It is assumed that there is an ECC unit embedded in each consumer's smart meter. The role of the ECC is to control the power consumption such that the consumer's utility can be maximized.

Let $\mathcal{N} = \{1, \dots, N\}$ denote the set of all the consumers. For each consumer $i \in \mathcal{N}$, denote x_i^k as the power consumption of consumer i at time slot k . In fact, considering the problems during several time slots $\mathcal{K} = \{1, \dots, K\}$, the solutions can be found separately in each time-slot. Without loss of generality, we consider our problem for one given time-slot and remove the index k for all the definitions. For each consumer i . the available power consumption interval I_i is defined as

$$I_i = [m_i, M_i] \quad (1)$$

and thus $m_i \leq x_i \leq M_i$.

2.1 Utility Function of Consumers

The energy demand of each consumer depends on several parameters, e.g., the climate condition, tariff, the variation of the energy demand over different time of the day. For all consumers, we denote the utility function as $U(x, \omega)$, where x represents the power consumption and ω is a parameter representing the satisfaction level (based on climate, time and so on). Here we choose the utility function proposed in [5] defined as:

$$U(x, \omega) = \begin{cases} \omega x - \frac{\alpha}{2} x^2, & \text{if } 0 \leq x \leq \frac{\omega}{\alpha} \\ \frac{\omega^2}{2\alpha}, & \text{if } x > \frac{\omega}{\alpha}. \end{cases} \quad (2)$$

2.2 The Cost Function of the Provider

Denoting the total power consumption L as $L = \sum_{i=1}^N x_i$, the cost function $C_1(L)$ for the provider, representing the cost of providing L units of energy, is chosen as [5]:

$$C_1(L) = aL^2 + bL + c \quad (3)$$

Different from most of the existing works, we also consider the cost caused by the prediction error of the power consumption. Note that it is assumed that the provider has imperfect knowledge of ω_i . The imperfect information can be induced by privacy issue, or the bad quality of communication channels. Indeed, in the worst case (all consumer want to keep its private information), it is assumed that the provider knows solely the probability density function (p.d.f.) of the ω_i , which is possible to be acquired by knowing the past realizations (values) of the ω_i . As a consequence, the provider needs to predict the total consumption by \tilde{L} and thus brings a penalty term depending on $L - \tilde{L}$. For instance, the provider purchased \tilde{L} units of energy in advance from the energy generator. If $\tilde{L} > L$, the provider will sell the superfluous energy to the Transmission System Operator (TSO) with a *lower* price. If $\tilde{L} < L$, the provider needs to purchase more energy from the TSO with a *higher* price to satisfy the energy need by all the consumers. Define the penalty term as $C_2(L - \tilde{L})$, with $C_2(\cdot)$ fulfilling the following properties:

- 1) The penalty function is non-negative.

$$C_2(x) \geq 0 \quad (4)$$

- 2) The penalty function is non-decreasing when $L - \tilde{L} > 0$ and non-increasing otherwise.

$$\frac{\partial C_2(x)}{\partial x} \geq 0 \text{ if } x > 0 \quad (5)$$

$$\frac{\partial C_2(x)}{\partial x} \leq 0 \text{ if } x < 0 \quad (6)$$

In this paper, we choose the absolute value function to describe the penalty term as follows:

$$C_2(L - \tilde{L}) = d|L - \tilde{L}| \quad (7)$$

Hence, the total cost of the provider can be expressed as:

$$C(L, \tilde{L}) = C_1(L) + C_2(L - \tilde{L}) \quad (8)$$

3 Problem Formulation

3.1 Demand Side Response of Consumers

For user $i \in \{1, \dots, N\}$, consuming x_i kW electricity with a tariff P_i dollars per kWh needs to pay $P_i x_i$ dollars to the provider, its utility can be expressed as:

$$W_i(x_i, \omega_i) = U(x_i, \omega_i) - P_i x_i \quad (9)$$

To maximize its own utility, the optimal power consumption can be written as:

$$x_i^* = \begin{cases} m_i, & \text{if } \frac{\omega_i - P_i}{\alpha} \leq m_i \\ \frac{\omega_i - P_i}{\alpha}, & \text{if } m_i \leq \frac{\omega_i - P_i}{\alpha} \leq M_i \\ M_i, & \text{if } \frac{\omega_i - P_i}{\alpha} \geq M_i \end{cases} \quad (10)$$

For the sake of clarify, we assume the power consumption interval is sufficient large so that the condition $m_i \leq \frac{\omega_i - P_i}{\alpha} \leq M_i$ is always met. Hence, the optimal consumption can be simplified to

$$x_i^* = \frac{\omega_i - P_i}{\alpha} \quad (11)$$

3.2 Expected Social Welfare Maximization Problem

When the communication channel between consumers and the provider is assumed to be perfect (lossless information can be exchanged), the social welfare can be optimized by using the algorithm proposed in [5]. However, for the scenario with no available communication channels unreliable communication channels, the maximization problem becomes totally different. In our case, it is assumed that the provider has the sole knowledge of the p.d.f. of all the ω_i . Hence, due to the lack of information, it is difficult to optimize the instantaneous social welfare. Consequently, we propose to maximize the expected social welfare as follows:

$$\mathbb{E}_{x_i, \omega_i} \left[\sum_{i \in \mathcal{N}} U(x_i, \omega_i) - C(L, \tilde{L}) \right] \quad (12)$$

Note that the power consumption x_i has been determined by (11) in the consumer's side. Plug (11) into (12), the expected social welfare can be rewritten as:

$$\begin{aligned} & \overline{W}(P_1, \dots, P_N, \tilde{L}) \\ &= \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U(x_i^*, \omega_i) - C(L^*, \tilde{L}) \right] \\ &= \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P_i}{\alpha}, \omega_i\right) - C(L^*, \tilde{L}) \right] \end{aligned} \quad (13)$$

where $L^* = \sum_{i \in \mathcal{N}} x_i^*$. To maximize the expected social welfare defined by (13), the provider can optimize the tariff P_i and the prediction of total power consumption \tilde{L} as follows:

$$\max_{P_1, \dots, P_N, \tilde{L}} \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P_i}{\alpha}, \omega_i\right) - C(L^*, \tilde{L}) \right] \quad (14)$$

Additionally, from the following proposition, it can be seen that the expected welfare can be optimized by using a common tariff P for all consumers rather than a different tariff for each consumer.

Proposition 1. *For any given tariff $\tilde{P}_1, \dots, \tilde{P}_N$, the following inequality always hold:*

$$\overline{W}(\tilde{P}_1, \dots, \tilde{P}_N, \tilde{L}) \leq \overline{W}(P_1^c, \dots, P_N^c, \tilde{L}) \quad (15)$$

where $P_i^c = \frac{1}{N} \sum_{j=1}^N \tilde{P}_j$ for every $i \in \mathcal{N}$.

Proof. It can be calculated that

$$\begin{aligned} & \overline{W}(\tilde{P}_1, \dots, \tilde{P}_N, \tilde{L}) - \overline{W}(P_1^c, \dots, P_N^c, \tilde{L}) \\ &= \frac{1}{2\alpha} \left(\sum_{i=1}^N (P_i^c)^2 - \sum_{i=1}^N \tilde{P}_i^2 \right) \end{aligned} \quad (16)$$

Note that $\sum_{i=1}^N \tilde{P}_i = \sum_{i=1}^N P_i^c$. Furthermore, it can be found that

$$(P_1^c, \dots, P_N^c) = \arg \min_{\sum_{i=1}^N P_i = C} \sum_{i=1}^N P_i^2 \quad (17)$$

As a consequence, it can be seen that

$$\sum_{i=1}^N (P_i^c)^2 - \sum_{i=1}^N \tilde{P}_i^2 \leq 0. \quad (18)$$

Our claim is proved.

According to Prop. 3.1, the optimization problems (14) can be further simplified to the following problem:

$$\max_{P, \tilde{L}} \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P}{\alpha}, \omega_i\right) - C(L^*, \tilde{L}) \right] \quad (19)$$

The provider will find the optimal price and the optimal power consumption prediction to maximize its expected social welfare. The approach to derive them will be presented in the subsequent section.

4 Optimal Consumption Prediction and Price

In this section, we proposed one approach to derive the optimal power consumption prediction and the optimal price. One can easily observe that these two variables are correlated. Without loss of optimality loss, the optimization problem (19) can be solved in two steps. Firstly, we focus on find the optimal consumption prediction for a given price P , i.e.,

$$\tilde{L}^*(P) \in \arg \max_{\tilde{L}} \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P}{\alpha}, \omega_i\right) - C(L^*, \tilde{L}) \right] \quad (20)$$

When the $\tilde{L}^*(P)$ has been derived, the optimal price can be obtained by solving the following problem:

$$P^* \in \arg \max_P \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P}{\alpha}, \omega_i\right) - C(L^*, \tilde{L}^*(P)) \right] \quad (21)$$

4.1 Optimal Prediction of the Total Power Consumption

We notice that only the term $C_2(L^* - \tilde{L})$ is related to the prediction \tilde{L} , thus the optimization problem (22) is equivalent to the following problem:

$$\tilde{L}^*(P) \in \arg \max_{\tilde{L}} \mathbb{E}_{\omega_i} \left[\left| \sum_{i=1}^N \frac{\omega_i - P}{\alpha} - \tilde{L} \right| \right] \quad (22)$$

Proposition 2. *For a given price P , the optimal prediction of the total power consumption can be written as*

$$\tilde{L}^*(P) = \text{MED}(\omega) - \frac{NP}{\alpha} \quad (23)$$

where $\text{MED}(\cdot)$ represents the median of the variable and $\omega = \sum_{i=1}^N \frac{\omega_i}{\alpha}$.

Proof. The proof is omitted because of the lack of space.

According to the Prop. 4.1, the optimal prediction decreases when the price rises. This can be explained by the fact that the rise of price will bring a degradation of the power consumption, and further the prediction of total power consumption will decline.

4.2 Optimal Price

Knowing the optimal prediction for a given price, the second step can be done by solving the optimization problem (21). Interestingly, according to (23), the problem can be further simplified. We notice that the term $C(L^*, \tilde{L}^*(P))$ can be rewritten as:

$$\begin{aligned} & C(L^*, \tilde{L}^*(P)) \\ &= C_1(L^*) + d \left| \sum_{i=1}^N \frac{\omega_i - P}{\alpha} - \tilde{L}^*(P) \right| \\ &= C_1(L^*) + d \left| \sum_{i=1}^N \frac{\omega_i}{\alpha} - \text{MED}(\omega) \right| \end{aligned} \quad (24)$$

Since $d \left| \sum_{i=1}^N \frac{\omega_i}{\alpha} - \text{MED}(\omega) \right|$ is independent of P , the optimization problem (21) can be further simplified as:

$$P^* \in \arg \max_P \mathbb{E}_{\omega_i} \left[\sum_{i \in \mathcal{N}} U\left(\frac{\omega_i - P}{\alpha}, \omega_i\right) - C_1(L^*) \right] \quad (25)$$

Therefore, when the prediction has been optimized, the pricing problem is independent of the optimal prediction \tilde{L}^* .

Proposition 3. *The optimal price to maximize the expected social welfare can be derived by solving (25) and written as:*

$$P^* = \frac{2a\alpha\mathbb{E}[\omega] + \alpha b}{\alpha + 2aN} \quad (26)$$

Proof. The proof is omitted because of the lack of space.

Knowing the optimal price P^* , the optimum prediction of the total power consumption can be calculated. Plug (26) into (22), the optimal consumption prediction can be written as:

$$\tilde{L}^*(P^*) = \text{MED}(\omega) - \frac{2Na\alpha\mathbb{E}[\omega] + N\alpha b}{\alpha^2 + 2Na\alpha} \quad (27)$$

5 Simulation Results

In this section, numerical results are shown to evaluate the performance of our approach. We consider a system with an unique provider and $N = 10$ consumers. For the sake of simplicity, we assume each ω_i is identically independent distributed (i.i.d.) and remains fixed during one time-slot. Each ω_i is uniformly distributed over the interval $[\omega_{\min}, \omega_{\max}]$. The scenario with asymmetric ω_i can be treated in the same way. The parameter d to define the penalty term is set to

be 0.1. For other parameters, they are chosen same as [5], i.e., $\alpha = 0.5$, $a = 0.01$, $b = 0$, and $c = 0$.

Firstly, we assess the performance degradation by using our approach compared with the algorithm proposed in [5]. The reason for the degradation is twofold: the imperfect knowledge of ω_i in our scenario leads to the sub-optimal power consumption solution, and also the deviation between the real power consumption and predicted power consumption brings the penalty to the provider. Assume $\omega_{\min} = 2.5 - \sigma$ and $\omega_{\max} = 2.5 + \sigma$, Fig. 1 represents the expected social welfare against σ . It can be observed that the performance degradation induced by the imperfect knowledge is quite small. Even with largest σ , the optimality loss is close to 1%. Furthermore, the performance degradation rises when σ increases. Indeed, larger σ leads to higher variance of ω_i . When ω_i changes faster, knowing the real value of ω_i becomes more important. As a consequence, when the provider has limited resource to communicate with the consumers, it is better to communicate with the consumer which has higher variance of its ω_i .

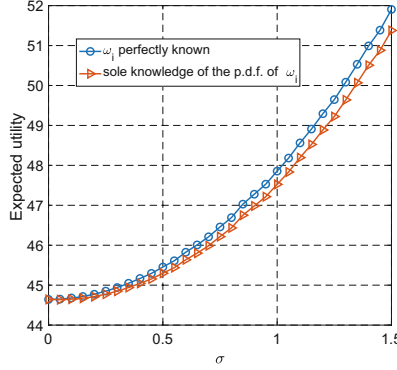


Fig. 1. Even have the sole knowledge of the p.d.f. of ω_i , the relative optimality loss is less than 1% under typical scenarios.

In Fig. 2, the optimal price against the expectation of ω_i is shown. Assume $\omega_{\min} = \mathbb{E}[\omega_i] - 1.5$ and $\omega_{\max} = \mathbb{E}[\omega_i] + 1.5$. The optimal price is derived by computing the expected utility defined in (25) with exhaustive search. From this figure, it can be seen that the optimal price is linear to $\mathbb{E}[\omega_i]$, which verified our analytical result shown in Prop. 4.2. When ω_i increases, the consumer prefers to respond with a higher power consumption to maximize its individual welfare. Therefore, the provider needs to increase the price to avoid the high load for the system. Therefore, it is better to have a rise in price in the evening since consumers are more demanding and have a drop in price after midnight since consumers are much less demanding at that time.

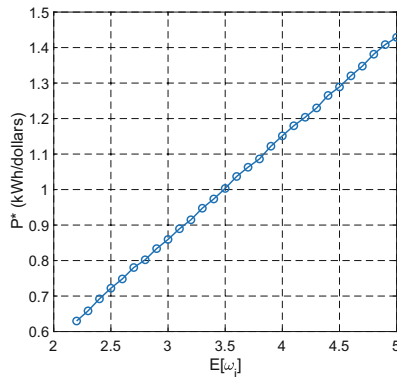


Fig. 2. When consumers are more probably to have higher power consumption, the price of the electricity designed by the provider will rise.

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