
Responses and actions taken to the comments of manuscript:
“Wedge indentation of elastoplastic solids – from single
indentation to interaction between indenters”

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We thank the reviewers for their careful reading and valuable comments on the manuscript. We have cautiously considered all of them and modified the new version of the manuscript accordingly as summarised in the actions taken below together with the following responses and the attached revised version.

Report of Reviewer #1

1. **Reviewer’s comment:** Open data and FE scripts should be provided. Methods are appropriate and results appear to be interesting for the computational contact mechanics community

Authors’ response: We agree with the reviewer regarding the availability of the data and scripts for a journal such as JTCAM. This should let the interested reader perform his own analysis of the data or create his own with the FEM scripts of the authors. This is crucial for reproducibility purposes. However, due to the size of the data generated by a FEM simulation, only the processed results of interest shall be provided (e.g., hardness). A FEM model could be generated by the scripts using any material and geometrical parameters, including those used by the authors, to produce a full data set on the FEM mesh if necessary.

Action taken: Typical ABAQUS input files and associated scripts of FEM single and multiple indentions will be provided to an accessible repository. Simulation results will be stored in a HDF file format and take the shape of a Python nested dictionary. A ”readme.txt” file will help the reader to understand how is the data sorted and can be processed and how to use the scripts.

Report of Reviewer #2

1. **Reviewer's comment:** When the authors state "At this point the sealing is stopped but not ended", it is not very clear what does it mean.

Authors' response: In perfectly plastic theoretical framework (Hill [1950a], Salikhyanov [2019]), the sealing would stop at this point, except if the substrate is not semi-infinite. This means that a gap will remain between the two surfaces in contact, that cannot be further closed by increasing the vertical force. In the present article, the FEM simulations show a slow down of sealing (closing of the gap) but full closing of the groove eventually occurs. We agree that the sentence could be confusing.

Action taken: In the revised manuscript, the sentence has been changed to "At this point the sealing is temporarily stopped and the gap is not entirely closed".

2. **Reviewer's comment:** When describing their FE model, the authors write: "The mesh is especially refined near the contact zone, but is also sufficiently wide to approximate a semi-infinite solid." I would invite the author to quantitatively justify this rather vague statement by concrete numbers and, probably, even a size-convergence results. Notably, for a particular (logarithmic) force-displacement relation in 2D problems, it would be interesting to specify what was the criterion used by the authors to demonstrate the convergence. Moreover, to justify accuracy of the obtained finite element results a mesh-convergence should be demonstrated. At some point (p.10) the authors admit "The finite element mesh might be too coarse to catch this [deviation of hardness for small indentation depths] regime properly as well" but they do not fix the problem. May be a non-homogeneous mesh could be used to obtain accurate results over the entire interval of considered loads.

Authors' response: A mesh size convergence study was carried out for single and multiple indentation. The physical quantity considered for this investigation was the mean contact pressure (or hardness). It is computed over the entire simulation since the strain and strain field are supposed to be geometrically similar. For the sake of accuracy, we performed some single indentations with increasing and decreasing numbers of elements (i.e., mesh density under the contact). Only a weak mesh dependency on the hardness is observed, even for the coarsest mesh. The evolution of the calculated hardness as a function of the number of elements in the contact area is presented on Fig. 16 of the revised manuscript for single indentation for various materials. With a rather weak difference, especially when compared to possible errors coming from approximate models of hardness, a comparatively coarse mesh was finally chosen for the study in order to minimize the computational time. Similar observations can be made in multiple indentation (Fig. 17 to 19 in the revised manuscript).

Action taken: New paragraphs have been added in Sec. 3.1 and 3.2 to discuss the effect of mesh convergence on the presented results. Appendixes A and B have been added as well, to present in more details the convergence studies performed for single and multiple wedges indentation.

3. **Reviewer's comment:** Having the same indentation angle β for the case of indentation with a single-wedge and a periodic array of wedges would allow the authors to compare the results before the plastic cores start to interact in the latter problem... and also to comment on the difference between isolated indenter and equally spaced indenters, and even to comment on the mesh convergence. However, the authors dealt with different indenter angles and different mesh densities which does not help in such an analysis. This choice was not justified, so I would invite the authors to comment on it in the paper.

Authors' response: Most of the single indentation studies deal with a $\beta = 20^\circ$ angle. It is the usual tip angle for indentation experiments since it is related to Berkovich and Vickers tips. Fig. 7 of the submitted paper extends the points made on 20° angles by comparing both models (REM and ECC) for 5° , 10° and 15° angles. A good agreement between the

models and the simulations is found, indicating that the effect of the contact angle is well captured by the models. For double indentation (renamed multiple indentation), angles of 5° to 20° were studied but only 5° and 10° tip angles were presented in the paper. This choice was mainly motivated by the fact that in sealing applications, the angle of the grooves are expected to be relatively small.

Action taken: Error maps for indentation angles 5° , 10° and 15° were redrawn in the modified version of the paper. Some sentences have been rewritten or added in Sec. 4.1 for a better clarity.

4. **Reviewer's comment:** Comparing analytic models for material hardness with finite element simulations provide the reader with really valuable results. However, the lack of mesh convergence study does not allow us yet to use it as a reference result. Additionally, the grid discretization for the parameteric study was not mentioned in the manuscript.

Authors' response: A convergence study has been performed for both single and multiple indentation (see the comment #2 above). The sampling of the material parameters was not mentioned in the first version to make the paper easier to read. Though, we agree with reviewer that this lacks in the paper so it has been added to the revised version. A total of 450 (n , σ_y) couples are studied, with a relatively fine regular sampling.

Action taken: The grid discretization employed for the parametric study of single indentation has been explicitly specified in Sec. 3.1 of the revised manuscript.

5. **Reviewer's comment:** It would be reasonable to explain in detail how the authors compute the hardness ((i) either through computing the contact pressure or (ii) by dividing the reaction by the contact width). Because of a rather coarse discretization, errors could be considerable. Notably, it seems to me that sliding average filter used by the authors could be not very accurate. It will all depend on the way how the authors measure the contact width. I would invite the authors to comment on it in more details to make their results reproducible by the community.

Authors' response: The reviewer is right. The way the contact pressure is computed is of primary importance in contact mechanics problems. However in the case of single sharp indentation, we can take advantage of the principle of geometric similarity that leads to a constant contact pressure during the whole indentation process. Its value is then computed by dividing the vertical force of the tip by the projected contact area, averaged on all simulation steps. This method should decrease the mesh dependency, hence permitting a relatively coarser mesh. Fig. 8 (blue curve) of the revised manuscript shows the evolution of the computed hardness during the penetration. A weak dependence is observed due to geometrical similarity, and the average value is kept.

Sliding average filters are used only for multiple wedges indentation. In this case, geometric similarity does not hold anymore and the current (or apparent) hardness should be considered. We are thus bound to calculate an instantaneous hardness that becomes mesh-dependent. The coarsest acceptable mesh is used (to keep the computational time reasonable) so apparent hardness obviously presents some minor artefacts due to mesh dependency (in the determination of the contact length for example). The sliding average filters are thus necessary to smooth the plots and focus on the general behaviour of multiple indentation. Nevertheless, the effect of this filter on the results is presented in the mesh convergence study.

Action taken: Fig. 18 and 19 have been added in appendix B of the revised paper. To prevent all misunderstanding, some explanations were added for filters, mesh dependency and calculation method of single indentation hardness and multiple wedges indentation apparent hardness in the same appendix. A related paragraph in Sec. 3.2 has also been added.

6. **Reviewer’s comment:** It is important for the reader’s understanding to show hardness results of FEA and analytical models (at least for one of them) whereas Fig. 5 and 6 show only the relative error. Notably, the reader cannot make his/her own judgement on the statement ”Finally, its heterogeneity is a problem in itself while the relative error of the REM model can be compared to a constant offset.”

Authors’ response: We agree with the reviewer’s comment. Such data are lacking for the understanding of what have been done. An illustration of the performance of the two hardness models as a function of tip angle for typical material will be shown in the revised manuscript. The other data will be made available as supplementary materials.

Action taken: In the revised manuscript, Fig. 5 has been integrated, and a related comment was written in Sec. 4.1. We also wish to notify that Fig. 7 was corrected, the contact angle used in the hardness formulas was incorrectly implemented. This is corrected in the revised manuscript and has no impact on the conclusions previously drawn.

7. **Reviewer’s comment:** I wonder why in Fig. 10(a) and 11(a) the nominal hardness \mathcal{H}^* does not saturate at 1 for $L^* \rightarrow 0$.

Authors’ response: The reviewer is right, as the dimensionless hardness should saturate at 1 for low contact length. The reason it occurs is a numerical issue related to the mesh-dependency of hardness at the beginning of the simulation. It is therefore more difficult to catch the expected plateau-like evolution of single indentation. Fig. 18 somehow highlights this mesh effect. The parametric study was based on a coarser mesh for saving computational time. The aforementioned sliding average filters were used to soften the mesh effect but it might make the results a bit misleading in this zone. We choose to present the results of the parametric studies and the coarse mesh with the understanding we have thanks to more refined simulation. Another reason of this observation is that high strain hardening coefficient make a quicker interaction response, decreasing the duration of the first stage (single indentation) and effectively restraining it to the least detailed part of the simulation. Finally, we are less interested in the beginning of sealing ($L^* \leq 0.2$) so this inaccuracy is considered acceptable in the context of the present study, further considering that this regime possesses a well known solution from single indentation theory. The mesh convergence study discussed in a previous comment shows that this effect is attenuated as the mesh is refined, as expected.

Action taken: Figs. 17 and 18 are added to appendix B, providing further information for the readers regarding the mesh dependence of our results. It is shown that the total error on the hardness–contact length curve due to the mesh size is less than 3% for the mesh used.

8. **Reviewer’s comment:** The problem that I see in Fig. 10(a)-11(a), is that, apparently (not sufficient data is provided to let me judge), the indentation hardness of the simulation is sensitive to the penetration depth whereas the considered models for a single indentation do not have this dependence.

Authors’ response: The principle of geometric similarity is well known in single indentation Hill [1950a]. As noted by Salikhyanov [2019], multiple wedges indentation triggers the interaction of slip lines fields of indenter tips. This interaction denied the necessary hypothesis of geometric similarity so that multiple indentation is effectively dependent of the penetration depth or, more accurately, dependent of any observable used to described the evolution of the indentation. As can be noticed in appendix B, the dependency is also observed for the most refined mesh that we consider converged, hence it is not a mesh effect.

In this article we choose the contact ratio (L^*) to follow the evolution of the indentation. Generally, the penetration depth that is mentioned by the reviewer is used instead (slip lines of Hill, Hertz contact...). In our case, the material is elasto-plastic with strain hardening so the studied indentation is accompanied by an elastic deformation of the bulk which

is difficult to extract without arbitrary assumptions on this behaviour. The penetration depth (displacement of indenter relative to the undeformed non-contact surface) is hence badly defined. Indeed, the displacement of the indenter is polluted by the bulk elastic displacement and the non contact surface present some displacement due to both bulk elastic displacement and plastic flow between the tips. For these reasons we choose not to study the dependence in penetration depth but in contact ratio which is far more reliable in our case.

Action taken: The principle of geometric similarity is described as well as its negation in multiple wedges indentation in Sec. 3.2 of the article. The appendix B of the revised manuscript confirms that the observed dependence does not come from mesh.

9. **Reviewer’s comment:** In both models for a single wedge indentation, the main geometrical characteristics is the indenter’s angle β , its importance was also highlighted by the authors on page 9, sec. 4.2.1. Nevertheless, the intensive parameteric study carried out by the authors focused exclusively on material parameters n and σ_y . As demonstrated in periodic wedge indentation, the nominal hardness is very weakly dependent on the indenter’s angle. I wonder if the angle’s effect is present in a single indenter simulation and if yes, how can it be characterized.

Authors’ response: The indentation angle has indeed a great effect in single indentation. It is often highlighted in literature with hardness formula generally involving the indenter’s angle and some material mechanical properties Hill [1950a], Johnson [1987]. That is why we were somehow surprised by the weak dependence observed in our case of periodic indentation. We think that it can be partly explained by the fact that symmetric indentation is performed (angles are the same in both sides) and that the normalization by \mathcal{H}_{mat} takes into account the majority of the effect of the contact angle on the response. Some further studies were made with asymmetric angles and it makes the angle one of the most influential parameter in the contact shape. Though, these results are not presented as they fall out of the scope of the present article.

Action taken: Fig. 5 which was added, presents the dependency of the hardness for different angles.

10. **Reviewer’s comment:** “Double wedge indentation” in reality is not a double-wedge but rather an infinite array of wedge indentations because of the symmetric (nominally periodic) boundary conditions. Therefore this term should be adjusted.

Authors’ response: It is totally true, and this double wedge represents the “unit cell”. This term is inherited from the first simulations where such conditions were not applied yet. In the literature, we found no occurrence of such experiments or problem with two or more indenters with the exception of Meguid et al. [1977] with two neighbouring flat punches. The authors proposed the term “co-indentation” that we personally find misleading.

Action taken: The term was modified to “multiple indentation” or “multiple wedges indentation” in the revised version of the article.

11. **Reviewer’s comment:** The authors write “Once a given contact ratio of $L^* \approx 0.75$ is reached, the hardness starts increasing exponentially.” Within such a small interval it is impossible to distinguish between exponential or, for example, power-law increase. Therefore, I would recommend the authors to adjust this sentence or justify an exponential increase.

Authors’ response: We wrote “exponentially” with a weak common sense of “quickly increasing” but the confusion with a mathematical exponential increase is indeed possible and not justified.

Action taken: The term was changed with various words for “quick increase” without scientific connotation.

12. **Reviewer’s comment:** The authors state: “The height of the bulk B has very little influence contrary to what was expected from the theoretical investigation of Hill (1950).” Normally, the slip line theory used by Rodney Hill is valid for rigid perfectly plastic materials only, so no hardening was taken into account. I would invite the authors to comment on it in a more elaborated way and notably to make a connection between a single indenter and Hill’s results in terms of substrate height B.

Authors’ response: Influence of bulk’s height is raised in perfect plasticity by Hill [1950b] and in perfect elasticity Johnson [1987], Sneddon [1995]. Hence, as we are using an elasto-plastic material, we assumed that this parameter should also affect the results. The hypothesis of semi-infinite (elastic) half-space is moreover often done in contact mechanics Greenwood and Williamson [1966], Johnson [1987], Salikhyanov [2019] letting the substrate’s height being of critical importance. With respect to these conclusions, we included this height in the parametric study but without specific conclusions on its influence. We should however qualify this lack of influence by the context of our study. The liner of industrial seals is generally less than 1 mm thick so the studied range of thickness is not very wide. Especially, contact length and bulk’s height are always comparable in our study so the answer of our contact problem does not depend on bulk height subject to such variations but a slight difference in behaviour may be observed compared to a taller bulk height (10 times more).

Action taken: References to elastic theory with finite half-space were added. The parallel we made with perfectly elastic or plastic theories is further explained in Sec. 4.2.1 and the difference with our material elasto-plastic strain hardening material more clearly stated.

13. **Reviewer’s comment:** To complement the discussion around Fig. 13 about pile-up and sink-in behavior with respect to the hardening exponent n , it would be interesting if the authors share their numerical results showing at least one of them. Otherwise, it is unclear if this well known behavior is recovered for a single indentation or not. The authors’ statement about higher loading: ”Within this regime there is no more indentation pile-up and the materials flow up homogeneously to fill up the remaining free volume” makes me guess that the pile-up is formed in the initial stage, but it remains unclear whether sink-in is formed for strong hardening or not.

Authors’ response: For $n = 0.3$ (consequentially $n = 0.4$), a sink-in is observed in single indentation as shown in Fig. 14. Referring to multiple wedge indentation sink-in or pile-up is more difficult. Indeed, as explained in comment #8, the contact depth over penetration depth ratio h_c/h is badly defined for multi indentation. Moreover, some additional factors can be involved in this behaviour with for example the plastic flow of the material confined by the groove formed by two neighbouring indenters. It makes a comparison with classical comprehension of the pile-up/sink-in as presented in Bolshakov and Pharr [1998], Cheng and Cheng [2004] difficult as more parameters are at stake. For these reasons, we choose to not include the study of h_c/h ratio in the article and rather analysed the advance of the closing thanks to the contact area ratio $L^* = L_c/W$.

Action taken: The part about pile-up and sink-in was rewritten in Sec. 4.2.2., clearly assessing the observed results in single indentation and potential modifications by multiple indentations. Single and multiple indentations geometries are shown. It highlights that bearing contact model seems to be precise enough to describe the contact geometry. The link is also made with previous figures (10, 11, 12) showing that the bearing contact model is not so far from the finite element model in terms of global opened area.

14. **Reviewer’s comment:** My final comment concerns the possible presence of a lubricant in the pocket made by neighbouring wedges. Entrapment of such a lubricant could strongly affect the apparent hardness and closing kinetics both in sealing and metal forming applications. It would be relevant, especially in view of sealing applications, to comment on this phenomenon, see [A,B,C].

Authors' response: We thank the reviewer for this useful comment that opens new perspectives on our work. An incompressible entrapped liquid should totally modify the result with a faster passage to a global response of the bulk to an apparent flat punch stimulation and no further closing of the groove without a liquid interface squeezing out. The seals of interest in the present context are generally put up on flanges in dry conditions so that the presence of lubricant of any sort at the interface has to be prevented. This is the reason why we did not consider that this phenomenon deserved to be mentioned at first, though it is of practical importance in some applications. Obviously, this is more complicated to introduce in the numerical modelling.

Action taken: The following comment about the modification of our observed behaviour in case of coupling with a lubricant at the interface, referring to studies in this domain Azushima and Kudo [1995], Bech et al. [1999], Shvarts and Yastrebov [2018], was added at the end of Sec. 4.2.2 in the revised manuscript.

“Our seals are put up on dry flanges and tighten immediately so the study holds for a dry medium. However the presence of lubricant is possible in similar applications. Submitted to pressure such fluid will at first partly flow out of the contact but could be entrapped in cavities according to surrounding surface deformation. In this case, which is not studied here, the plastic locking step is supposed to transform in a plastic-fluid locking step with the incompressible fluid preventing further deformation of the seal surface. As for the start of this locking, its study is not adapted to a 2D simulation because the start is triggered by the fluid entrapment. A 3D simulation of the contact would hence be necessary to describe such behaviour.”

Finally, the following references were added to the manuscript.

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