Statistical approach for the evaluation of an effective conductivity in a complex solid

Léa Penazzi, Jens Biele, Stéphane Blanco, Cyril Caliot, Anne Castelan, C. Coustet, Sébastien Dutour, Mouna El-Hafi, Vincent Forest, Richard A Fournier

IMT Mines Albi-Carmaux
École Mines-Télécom

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L. Penazzi[1,4], J. Biele[2], S. Blanco[3], C. Caliot[4], A. Castelan[5], C. Coustet[6], S. Dutour[3], M. El Hafi[1], V. Forest[6], R. Fournier[3]

(1) RAPSODEE
(2) DLR
(3) Laplace
(4) PROMES
(5) icam
(6) |Méso|Star> Understanding Complex Systems

Methodology

Statistical method of Monte Carlo to solve combined conductive-radiative heat transfer in the solid phase of a porous medium.

Model

- **Solid spheres** at the stationnary state

$$\nabla^2 T(\boldsymbol{x}) = 0 \qquad \text{(solide)} \quad (1)$$

- **Vacuum** at the stationnary state

$$\boldsymbol{\omega}.\nabla I = 0 \qquad \text{(void)} \quad (2)$$

- **Solid/Void boundary condition** : conduction/radiation

$$-\lambda \nabla T.\boldsymbol{n} = \epsilon\sigma T^4 - \epsilon \int_{\partial\mathcal{H}(\boldsymbol{n})} \boldsymbol{\omega}'.\boldsymbol{n} I(\boldsymbol{y}, -\boldsymbol{\omega}')d\boldsymbol{\omega}' \quad (3)$$

Statistical formulation

- **Solid spheres** (with $\boldsymbol{v_i} = (-1)^i \boldsymbol{e}_{\lfloor \frac{i+1}{2} \rfloor}$)

$$T(\boldsymbol{x}) = \frac{1}{6}\sum_{i=1}^{6} T(\boldsymbol{x} + \delta \boldsymbol{v_i}) \quad (4)$$

Discretization of space on an orthonormal basis $(\boldsymbol{x}, \boldsymbol{e_1}, \boldsymbol{e_2}, \boldsymbol{e_3})$

- **Vacuum**

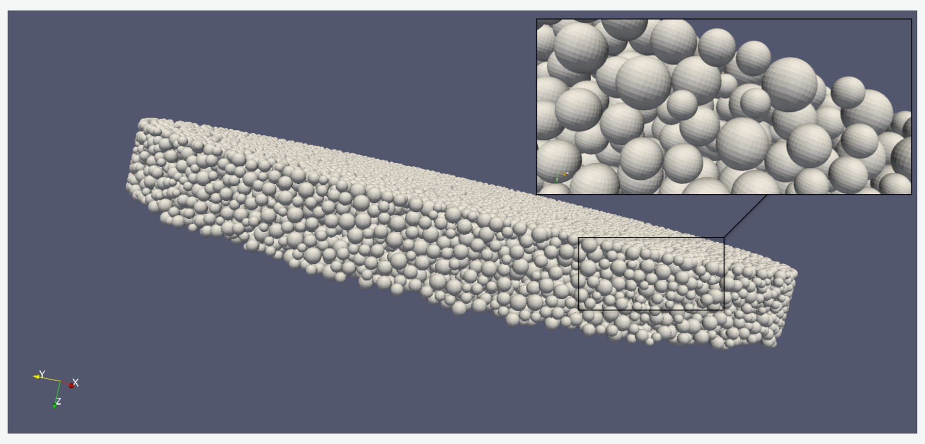
$$T_R = \mathbb{E}(A_{R,bot}\, T_{min} + A_{R,C}\, T(\boldsymbol{Y}_R) + A_{R,top}\, (\frac{\varphi}{\pi\ h} + T_R(\boldsymbol{Y}_R, \boldsymbol{\Omega}_R)) \quad (5)$$

- **Linearized solid/void boundary condition**

$$T(\boldsymbol{y}) = p_S\, T(\boldsymbol{y} - \boldsymbol{\delta_b}) + p_R\, T_R(\boldsymbol{y}) \quad (6)$$

Test Case

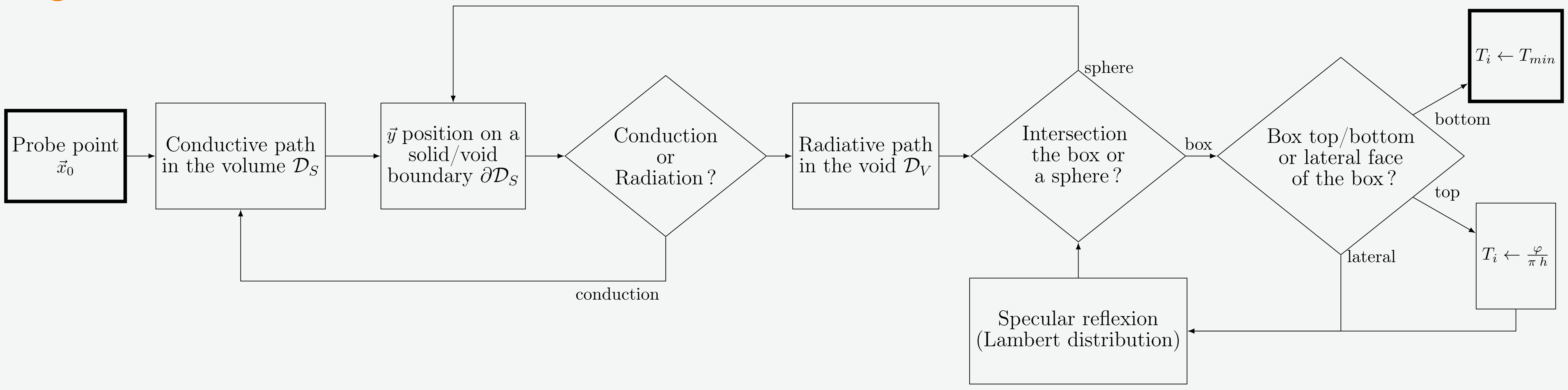
- **3D slab-shape** Assembly of solid spheres interpenatrating slightly in the void



Assumptions :

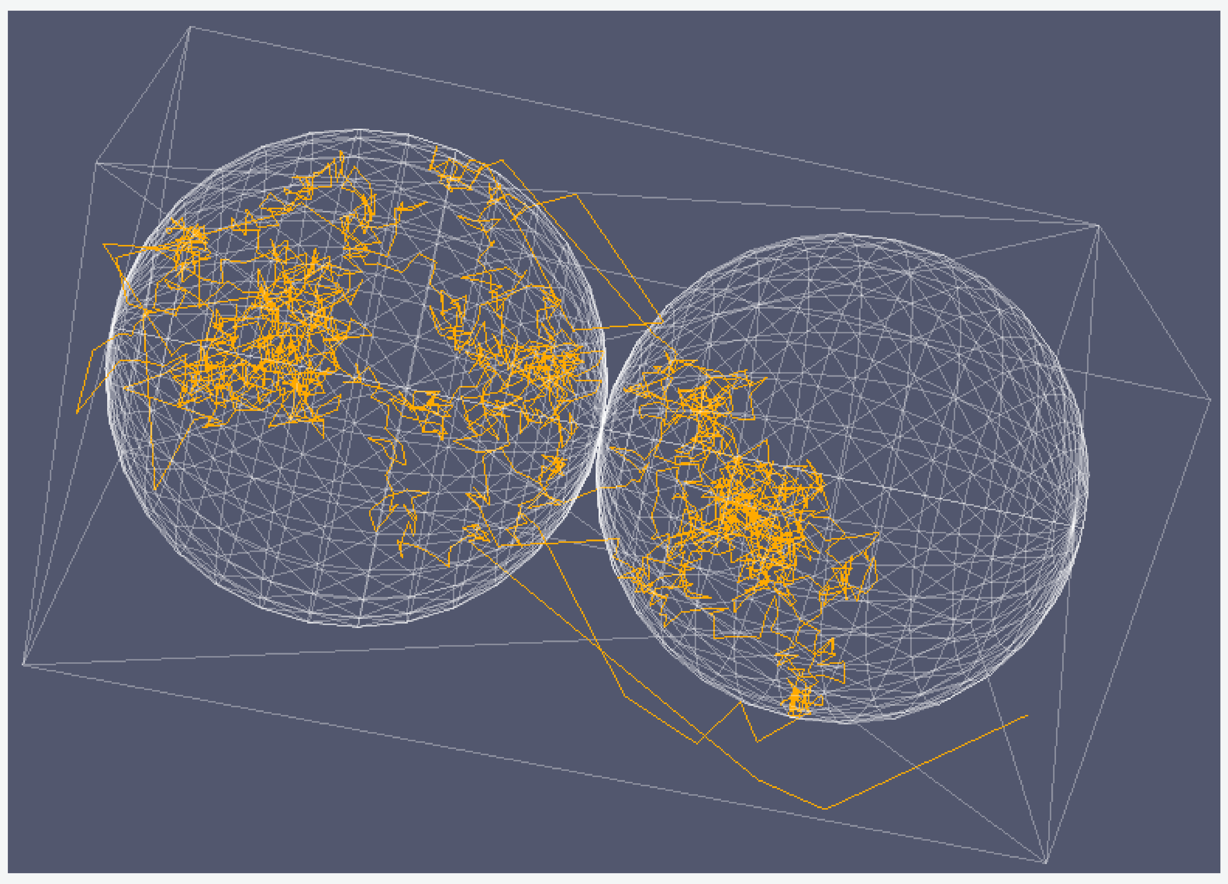
— Grey opaque spheres ϵ
— Radiative heat flux φ
— Radiative temperature T_{min}

Algorithm

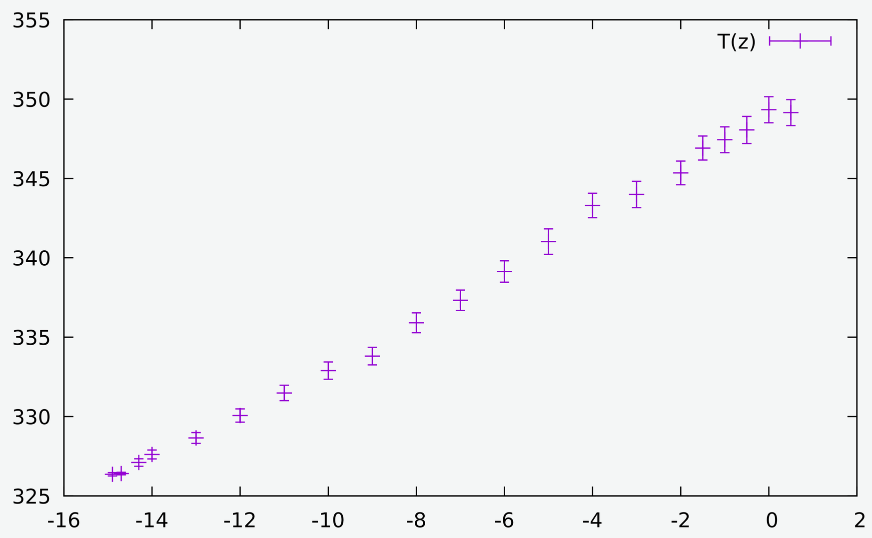


Conducto-radiative paths

- **Representation of the paths** from the probe point $\boldsymbol{x}_0$



Approach to obtain λ_{eff}



- **Temperature profile** $T_m(z)$ of the bed of spheres :

$$T_m(z) = \frac{1}{n}\sum_{i=0}^{n} T_i(\boldsymbol{x}) \quad (7)$$

- **Continuity of flux** at the solid/void boundary :

$$\lambda_{eff} = \frac{\varphi}{\frac{dT_m}{dz}} \quad (8)$$

NB : λ_{eff} computation is not complete yet, the aim is hence to present the method.

Conclusion

- A single Monte Carlo algorithm to solve combined conductive-radiative problem in a complex solid.
- Realizations of the algorithm can be interpreted as a succession of conductive-radiative paths.
- **Perspectives :**
  — Validation of the algorithm
  — Computation of the temperature as a function of the effective conductivity (Symbolic Monte Carlo)

Key References

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Contact

lea.penazzi
@mines-albi.fr

Notations (order of appearance) : $\boldsymbol{x}$ position in the solid ; $\boldsymbol{y}$ position at the solid/void boundary ; $\boldsymbol{\omega}$ unit vector indicating the radiation direction ; I spectrally integrated specific intensity ; λ solid thermal conductivity ; $\boldsymbol{n}$ lunit vector normal to the surface at $\boldsymbol{y}$; ϵ emissivity ; σ Stefan-Boltzmann constant ; ϕ radiative net-flux density imposed at the surface of the bed of spheres.

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