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# Evaluation of the temporal auto-correlation function sensitivity to Brownian motion in the radiative transport regime

Ugo Tricoli<sup>1,\*</sup>, Callum M. Macdonald<sup>1</sup>, Turgut Durduran<sup>2</sup>, Anabela Da Silva<sup>1</sup>, Vadim A. Markel<sup>1</sup>

<sup>1</sup>Aix Marseille Univ, CNRS, Centrale Marseille, Institut Fresnel, F-13013 Marseille, France

<sup>2</sup>ICFO-Institut de Ciències Fotòniques, The Barcelona Institute of Science and Technology, 08860 Castelldefels (Barcelona), Spain. Institució Catalana de Recerca i Estudis Avançats (ICREA), 08015 Barcelona, Spain.

\*ugo.tricoli@fresnel.fr

**Abstract:** We apply first order perturbation theory and reciprocity to the scalar radiative transport equation for the temporal field auto-correlation function to study its sensitivity to changes in the Brownian motion of the constituent scattering particles.

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## 1. Introduction

Numerous methods consider the temporal auto-correlation function in order to study the dynamical properties of a medium, e.g. diffuse wave spectroscopy. Our interest is directed to methods based on measurements of the electric field temporal auto-correlation function,  $G_1(\mathbf{r}, \tau) = \langle \mathbf{E}(\mathbf{r}, \mathbf{t}) \mathbf{E}^*(\mathbf{r}, \mathbf{t} + \tau) \rangle$ , [1]. In this work, we want to quantify the sensitivity of the field auto-correlation function to a change into the diffusion constant of the constituent particles undergoing random relative Brownian motion. Thus, we apply first order perturbation theory on the Brownian motion diffusion constant  $D_B$  in order to calculate the effect of the perturbation through the correlation transport equation (CTE). We show an analytical expression for the field auto-correlation sensitivity function and we evaluate its kernel through Monte Carlo simulations. Since the kernel preserves the translational and reciprocity symmetries, we can apply the method introduced in [2] in order to evaluate the sensitivity kernel. It is interesting to note that the presence of correlations introduces additional absorption w.r.t. the uncorrelated case, since not only photons can be absorbed but also their correlations. Consequently, for normal illumination and detection, the field auto-correlation is sensitive to a smaller region of the sample. This behavior is interesting for diffuse correlation tomography and speckle contrast optical tomography in order to increase the resolution of the optical reconstruction of blood flow [3].

## 2. Perturbation theory on the Brownian motion of the particles in the scattering medium

We consider a multiply-scattering medium with its constituent particles undergoing Brownian motion. Inside the medium, the electric field temporal autocorrelation function  $G_1(\mathbf{r}, \hat{\mathbf{s}}, \tau)$  obeys the scalar CTE [1]

$$(\hat{\mathbf{s}} \cdot \nabla + \mu_t) G_1(\mathbf{r}, \hat{\mathbf{s}}, \tau) = \mu_s \int g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) f(\hat{\mathbf{s}}, \hat{\mathbf{s}}') G_1(\mathbf{r}, \hat{\mathbf{s}}', \tau) d^2 s' + \mathcal{S}(\mathbf{r}, \hat{\mathbf{s}}), \quad (1)$$

where  $f$  is the phase function,  $\mathcal{S}$  is the source term,  $\mu_s$  and  $\mu_t$  are the absorption and the extinction coefficients. We define the dynamic absorption coefficient as  $k_c = 2\mu_s D_B k_0^2 \tau$  and the normalized temporal field correlation function for single scattering as

$$g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) = \exp[-1/3k_0^2 \langle \Delta r^2(\tau) \rangle (1 - \hat{\mathbf{s}} \cdot \hat{\mathbf{s}}')]. \quad (2)$$

The mean-square displacement of the scattering particles undergoing Brownian motion is  $\langle \Delta r^2(\tau) \rangle = 6D_B \tau$ , with  $\tau$  being the correlation time. If  $\tau \ll (2D_B k_0^2)^{-1}$

$$g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) = 1 - 2D_B k_0^2 \tau + 2D_B k_0^2 \tau (\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}') \quad (3)$$

as proposed in [1]. If we now perturb the Brownian motion diffusion coefficient so that  $D_B' = D_B + \delta_B$  then

$$g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) = 1 - 2D_B k_0^2 \tau + 2D_B k_0^2 \tau (\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}') - 2\delta_B k_0^2 \tau + 2\delta_B k_0^2 \tau (\hat{\mathbf{s}} \cdot \hat{\mathbf{s}}'). \quad (4)$$

Rewriting for simplicity  $g_1' = g_1^s + \delta$  and substituting into the CTE equation gives

$$(\hat{\mathbf{s}} \cdot \nabla + \mu_t) G_1(\mathbf{r}, \hat{\mathbf{s}}, \tau) = \mu_s \int g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) f(\hat{\mathbf{s}}, \hat{\mathbf{s}}') G_1(\mathbf{r}, \hat{\mathbf{s}}', \tau) d^2 s' + \mu_s \int \delta(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) f(\hat{\mathbf{s}}, \hat{\mathbf{s}}') G_1(\mathbf{r}, \hat{\mathbf{s}}', \tau) d^2 s' + \mathcal{S}(\mathbf{r}, \hat{\mathbf{s}}). \quad (5)$$

The field auto-correlation function will be also modified by the presence of the perturbation, thus

$$G_1(\mathbf{r}, \hat{\mathbf{s}}, \tau) = G_1^{0B}(\mathbf{r}, \hat{\mathbf{s}}, \tau) + G_1^{sc}(\mathbf{r}, \hat{\mathbf{s}}, \tau) \quad (6)$$

where  $G_1^{0B}$  is the solution of the CTE for a homogeneous Brownian motion (considering as CTE kernel to be sampled with rejection the product  $g_1^s \cdot f$ ). If we now substitute this into Eq. (5) and we keep only the first order  $\delta$ -term

$$(\hat{\mathbf{s}} \cdot \nabla + \mu_t) G_1^{sc}(\mathbf{r}, \hat{\mathbf{s}}, \tau) = \mu_s \int g_1^s(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) f(\hat{\mathbf{s}}, \hat{\mathbf{s}}') G_1^{sc}(\mathbf{r}, \hat{\mathbf{s}}', \tau) d^2 s' + \mu_s \int \delta(\hat{\mathbf{s}}, \hat{\mathbf{s}}', \tau) f(\hat{\mathbf{s}}, \hat{\mathbf{s}}') G_1^{0B}(\mathbf{r}, \hat{\mathbf{s}}', \tau) d^2 s' \quad (7)$$

The solution of Eq. (7) is

$$G_1^{sc}(\mathbf{r}, \hat{\mathbf{s}}, \tau) = \mu_s^3 \int M_B^0(\mathbf{r}, \mathbf{r}'; \hat{\mathbf{s}}, \hat{\mathbf{s}}'; \tau) d^3 r' d^2 s' \int \delta(\hat{\mathbf{s}}', \hat{\mathbf{s}}'', \tau) f(\hat{\mathbf{s}}', \hat{\mathbf{s}}'') d^2 s'' \int M_B^0(\mathbf{r}', \mathbf{r}''; \hat{\mathbf{s}}'', \hat{\mathbf{s}}'''; \tau) \mathcal{S}(\mathbf{r}'', \hat{\mathbf{s}}''') d^3 r'' d^2 s'''. \quad (8)$$

where  $M_B^0(\mathbf{r}, \mathbf{r}'; \hat{\mathbf{s}}, \hat{\mathbf{s}}'; \tau)$  is the Green's function for photons in a medium undergoing uniform Brownian motion. This can be simplified considering a point source at  $\mathbf{r}_A$  with a unique incident direction  $\hat{\mathbf{s}}_A$  so that  $\mathcal{S}(\mathbf{r}'', \hat{\mathbf{s}}''') = \delta(\hat{\mathbf{s}}''', \hat{\mathbf{s}}_A) \cdot \delta(\mathbf{r}'', \mathbf{r}_A)$ , thus

$$G_1^{sc}(\mathbf{r}_B, \hat{\mathbf{s}}_B, \mathbf{r}_A, \hat{\mathbf{s}}_A, \tau) = \mu_s^2 \int k_c(\mathbf{r}') \int M_B^0(\mathbf{r}_B, \mathbf{r}'; \hat{\mathbf{s}}_B, \hat{\mathbf{s}}'; \tau) \int \delta(\hat{\mathbf{s}}', \hat{\mathbf{s}}'', \tau) f(\hat{\mathbf{s}}', \hat{\mathbf{s}}'') M_B^0(\mathbf{r}', \mathbf{r}_A; \hat{\mathbf{s}}'', \hat{\mathbf{s}}_A; \tau) d^2 s' d^2 s'' d^3 r'. \quad (9)$$

where  $\mathbf{r}_B$  is the position of the detector. This is the sensitivity of the field auto-correlation function to a change into the diffusion constant of an amount  $\delta_B$  of the Brownian-like moving particles.

In speckle contrast optical tomography (SCOT), the reconstruction of the distribution of the mean-square displacement  $\langle \Delta r^2(\tau) \rangle$  into the sample volume, is recovered from measurements at the surface of the speckle contrast  $k^2$ , [3]. Thus, Eq. (9) can be used to calculate the speckle-contrast remembering that

$$k^2(\mathbf{r}, \hat{\mathbf{s}}, \tau) = \frac{2\beta}{T} \int_0^T (1 - \tau/T) \frac{G_1(\mathbf{r}, \hat{\mathbf{s}}, \tau)}{I(\mathbf{r}, \hat{\mathbf{s}}, 0)} \quad (10)$$

with  $I$  being the specific intensity,  $T$  the camera exposure time and  $\beta$  is an experimental parameter depending on the number of speckles recorded.

### 3. Results

We consider imaging of a scattering medium in the shape of a slab. The scattering and absorption coefficients ( $\mu_s = 500\text{cm}^{-1}$  and  $\mu_a = 0.03\text{cm}^{-1}$ ), and the scattering phase function (for Mie particles with asymmetry parameter  $g = 0.95$  and size parameter  $x = 15$ ) are constant inside the medium but the Brownian motion diffusion coefficient  $D_B$  can vary. Propagation of the field temporal auto-correlation is assumed to obey the scalar CTE. The source is a continuous-wave, collimated laser beam incident at some location  $\mathbf{r}_A$  on the surface of the slab and in the direction of the unit vector  $\hat{\mathbf{s}}_A$ . The detector measures the intensity of light exiting on the medium at a different point  $\mathbf{r}_B$  and in the direction  $\hat{\mathbf{s}}_B$ . The presence of motion heterogeneities inside the slab will result in a detectable variation of the measured field correlation function. In fact, we can define the sensitivity function as the difference between the registered correlation function exiting

a medium undergoing a homogeneous Brownian motion (reference) and a media with Brownian motion fluctuations. Within the accuracy of the first Born approximation, the equation that relates the fluctuations of the Brownian motion diffusion coefficient  $D_B$  to the sensitivity function is expressed through Eq. (9). To evaluate numerically the kernel of Eq. (9) we have to expand all the terms in spherical harmonics and to reduce their product through orthogonality. Then, the reciprocity as in [2] can be applied.

The results for the field auto-correlation sensitivity, i.e. the kernel of Eq. (9) are presented in Figure 1. The left column is the scalar uncorrelated case while the right column is the result considering temporal auto-correlations.

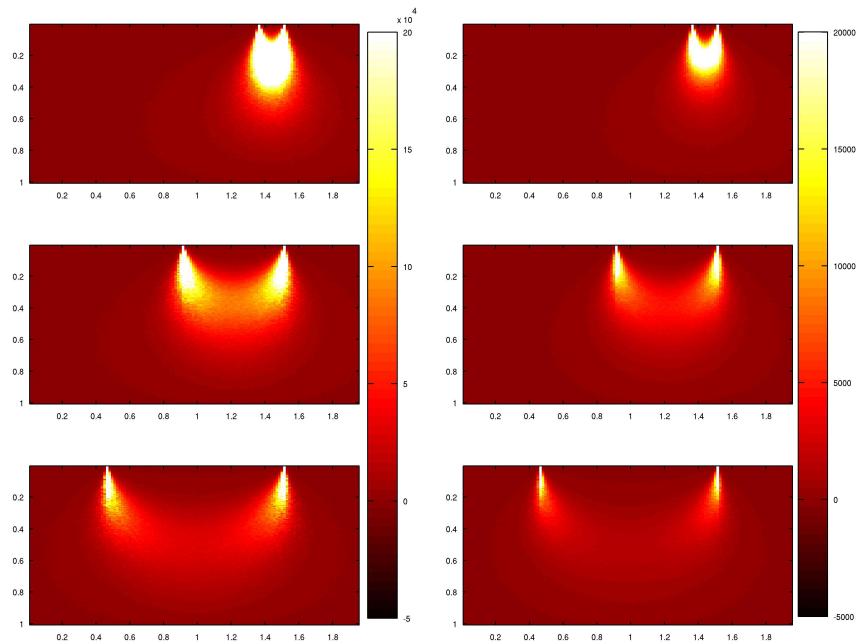


Fig. 1. Sensitivity kernel computed via Monte Carlo simulations for scalar uncorrelated specific intensity (left column) and for the field temporal auto-correlation (right column). From top to bottom the source-detector separations are  $0.225\ell^*$ ,  $0.45\ell^*$  and  $1.05\ell^*$ .

#### 4. Conclusions

We calculated the kernel of the field auto-correlation sensitivity function. We compared the vertical distribution of the sensitivity function for the uncorrelated scalar specific intensity with the vertical distribution of field temporal auto-correlation sensitivity function. We note that the banana-shape of the distribution is preserved in the presence of field correlations. In addition, the auto-correlated signal is weaker than the uncorrelated one for all source-detector distances. This is due to the extra absorption of correlations related to the term  $1/3\mu'_s k_0^2 \langle \Delta r^2(\tau) \rangle$ . Moreover, for short distances, the signal arriving at the detector comes from a smaller region. This can be useful for diffuse correlation tomography and speckle contrast optical tomography in order to increase the resolution.

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