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# Reciprocity relations in 3D vector radiative transport applied to Diffuse Optical Tomography

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**Abstract:** We describe a reciprocity relation for polarized radiative transport between arbitrarily positioned sources and detectors separated by a scattering medium. Applications to polarized Diffuse Optical Tomography are shown which allow for efficient computation of the sensitivity kernel.

**OCIS codes:** (110.6960) Tomography; (290.5855) Scattering, polarization; (110.5405) Polarimetric imaging.

## 1. Introduction

In the typical setup for Optical Tomography (OT), a light source and detector are placed exterior to a medium, and are used to locate an inhomogeneity within. In the present case, we consider a uni-directional light source incident on the surface of a slab type medium at location  $\mathbf{r}_a$ , with direction  $\hat{\mathbf{s}}_a$ , and a detector at a location  $\mathbf{r}_b$ , accepting light only in the direction  $\hat{\mathbf{s}}_b$  (see Fig. 1a). When a purely absorbing perturbation is present at some location within the medium, the signal at the detector will be different from the homogeneous case. The measured difference, or ‘sensitivity’ to the perturbation, is considered as the data function of the problem (or ‘sensitivity function’). An Inversion method is then sought which takes the data function and attempts to recover the location of the perturbation, which requires a suitable forward model. However, this problem is severely ill-posed, and furthermore, it is typically only approached by considering the radiance as a scalar quantity, for which the diffusion equation is often used as the forward model. Here however, we present a formalism for the forward modelling of the data function, using the vector Radiative Transport Equation (v-RTE). It is obvious that a forward model which accounts for the unavoidable effects of polarization is preferred over one which does not, and it is hoped that the added degrees of freedom available in various linearly independent polarization states will reduce the ill-posedness of the inverse problem. In addition, we have invoked a well known reciprocity relation of the single scattering phase matrix of the v-RTE, to derive a reciprocity relation for the Green’s function describing transport between two spatially separated points in a three-dimensional medium [1]. This reciprocity relation, along with additional symmetries present in the problem, allows for a significant increase in efficiency when computing the kernel of the sensitivity function (data function) via Monte Carlo methods.

## 2. The sensitivity function for polarized optical tomography

Here we present a model using the v-RTE, with which we look to describe the forward problem of OT for arbitrary combinations of incident polarization states, and polarization filters upon detection. Within the validity of the first Born approximation, it can be shown [1] that the resulting sensitivity, or data function,  $\Phi$ , for a given source-detector pair, and absorbing perturbation,  $\delta\mu_a(\mathbf{r})$ , can be expressed as:

$$\Phi(\mathbf{r}_b, \hat{\mathbf{s}}_b; \mathbf{r}_a, \hat{\mathbf{s}}_a) = \int [\mathbf{S}_{out} \cdot \mathbf{K}(\mathbf{r}_b, \hat{\mathbf{s}}_b, \mathbf{r}_a, \hat{\mathbf{s}}_a; \mathbf{r}) \mathbf{S}_{in}] \delta\mu_a(\mathbf{r}) d^3r, \quad (1)$$

where

$$\mathbf{K}(\mathbf{r}_b, \hat{\mathbf{s}}_b, \mathbf{r}_a, \hat{\mathbf{s}}_a; \mathbf{r}) = \int \mathbf{G}(\mathbf{r}_b, \hat{\mathbf{s}}_b; \mathbf{r}, \hat{\mathbf{s}}) \mathbf{G}(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}_a, \hat{\mathbf{s}}_a) d^2s, \quad (2)$$

is the Kernel of the sensitivity function. Here,  $\mathbf{S}_{in}$  is the incident state of polarization, and  $\mathbf{S}_{out}$ , is the state of polarization for which the polarizing filter at the detector is totally transmissive. Note that the Kernel,  $\mathbf{K}$ , is a  $4 \times 4$  matrix, while the

inner product of  $KS_{in}$ , with the detecting filter state,  $S_{out}$ , which results in the data function, is itself a scalar. This scalar quantity physically represents the sensitivity of the measured intensity passing through the polarizing filter at the detector (if no polarizing filter is present,  $S_{out}$  is a unit unpolarized Stokes vector). Each of the Green's functions appearing in the expression are  $4 \times 4$  matrices, and are solutions of the completely homogeneous problem (where  $\delta\mu_a(\mathbf{r}) = 0$ ). In our case, these Green's functions are computed via Monte Carlo methods. Note however that for each location and direction within the medium, a new Green's function  $G(\mathbf{r}_b, \hat{\mathbf{s}}_b; \mathbf{r}, \hat{\mathbf{s}})$  must be calculated, describing the response of an effective source (at  $\mathbf{r}, \hat{\mathbf{s}}$ ) measured at the detector (again see Fig. 1a). This seriously limits the efficiency of such an approach, even if only considering one particular source-detector pair. The solution we present, involves the exploitation of reciprocity relations involving the single scattering phase matrix,  $Z$ , of the v-RTE, and by consequence, the Green's functions themselves.

### 3. Reciprocity of the Green's functions

If we have a macroscopically isotropic and non-chiral medium, for which the single scattering phase matrix satisfies the reciprocity relation [2, 3]:

$$Z(-\hat{\mathbf{s}}', -\hat{\mathbf{s}}) = PZ^T(\hat{\mathbf{s}}, \hat{\mathbf{s}}')P, \quad (3)$$

where  $P = \text{diag}[1, 1, -1, 1]$ , then we have recently show that there also exists the following reciprocity relation involving the Green's functions [1]:

$$G(\mathbf{r}_{in}, -\hat{\mathbf{s}}_{in}; \mathbf{r}_{out}, -\hat{\mathbf{s}}_{out}) = PG^T(\mathbf{r}_{out}, \hat{\mathbf{s}}_{out}; \mathbf{r}_{in}, \hat{\mathbf{s}}_{in})P. \quad (4)$$

With this relation, it is then possible to simplify the computation of the sensitivity kernel  $K$ . For instance, if we take  $\mathbf{r}_{in} = \mathbf{r}$ ,  $\hat{\mathbf{s}}_{in} = \hat{\mathbf{s}}$  and  $\mathbf{r}_{out} = \mathbf{r}_b$ ,  $\hat{\mathbf{s}}_{out} = \hat{\mathbf{s}}_b$ , then one of the Green's functions in Eq. 2, can be re-written as:

$$G(\mathbf{r}_b, \hat{\mathbf{s}}_b; \mathbf{r}, \hat{\mathbf{s}}) = PG^T(\mathbf{r}, -\hat{\mathbf{s}}; \mathbf{r}_b, -\hat{\mathbf{s}}_b)P. \quad (5)$$

This relation is represented in Fig. 1(a) as a reversal of the path which leaves the buried source at  $r$ , and arrives at the detector location,  $\mathbf{r}_b$ . If we now exploit the translational symmetry present in the slab geometry, we can show that the kernel in Eq. 2, reduces to:

$$K(\mathbf{r}_b, \hat{\mathbf{s}}_b, \mathbf{r}_a, \hat{\mathbf{s}}_a; \mathbf{r}) = \int PG^T(\mathbf{r} + \mathbf{r}_{ab}, -\hat{\mathbf{s}}; \mathbf{r}_a, -\hat{\mathbf{s}}_b)PG(\mathbf{r}, \hat{\mathbf{s}}; \mathbf{r}_a, \hat{\mathbf{s}}_a)d^2s, \quad (6)$$

where  $\mathbf{r}_{ab} = \mathbf{r}_a - \mathbf{r}_b$ . Importantly, this kernel now only requires the calculation of two Green's functions, each representing the response to a source at location  $\mathbf{r}_a$ , and with the directions  $\hat{\mathbf{s}}_a$ , and  $-\hat{\mathbf{s}}_b$ , respectively. Furthermore, in the special case where  $-\hat{\mathbf{s}}_b = \hat{\mathbf{s}}_a$ , as shown in the Fig. 1(a), then in fact we need only to calculate one Green's function. Calculating the kernel for a given pair of source-detector directions, no longer requires the computation of an entirely new Green's function for each internal point  $\mathbf{r}$ , rather, only the evaluation of the same single (or two) Green's function(s) at various locations.

### 4. Computing the sensitivity function

To calculate the sensitivity kernel at all locations within a medium, we need to compute at most two Green's functions for each pair of source-detector directions ( $\hat{\mathbf{s}}_a, \hat{\mathbf{s}}_b$ ). We compute these functions numerically via Monte Carlo simulations, which requires keeping track not only of the voxels visited by a photon (and its polarization state arriving at the voxel), but also of its incoming direction. A single run of a the MC code produces the vector specific intensity  $I(\mathbf{r}, \hat{\mathbf{s}})$ , for a given input polarization of the source,  $S_{in}$ . Storage of this data is achieved by expanding each of the four components of the vector specific intensity in spherical functions  $Y_{lm}(\hat{\mathbf{s}})$ , *e.g.*

$$I(\mathbf{r}, \hat{\mathbf{s}}) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l i_{lm}(\mathbf{r})Y_{lm}(\hat{\mathbf{s}}), \quad (7)$$

and similarly for the  $Q$ ,  $U$ , and  $V$  Stokes components. Here  $l_{\max}$  is the truncation order and the functions  $i_{lm}(\mathbf{r})$ ,  $q_{lm}(\mathbf{r})$ ,  $u_{lm}(\mathbf{r})$ ,  $v_{lm}(\mathbf{r})$  are to be computed numerically. It can be shown that, in a stochastic MC process and for each voxel containing the point  $\mathbf{r}$ , the coefficients are given by:

$$i_{lm}(\mathbf{r}) \xrightarrow{N \rightarrow \infty} \sum_j I_j Y_{lm}(\hat{\mathbf{s}}_j), \quad (8)$$

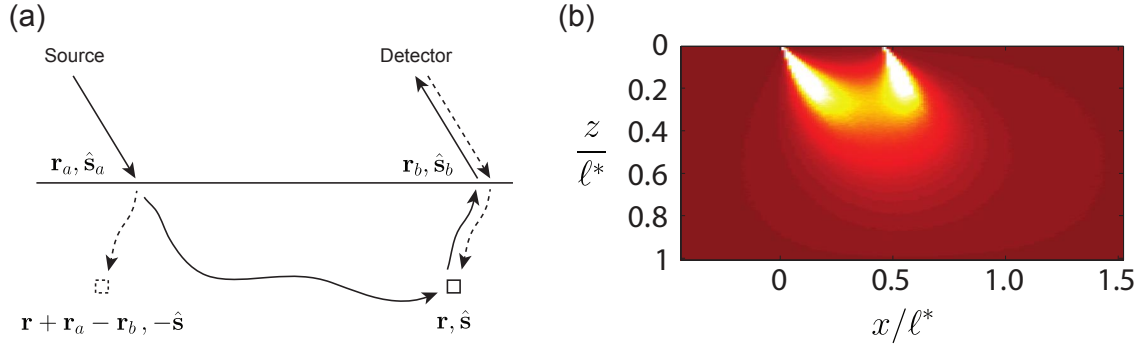


Fig. 1. (a) The basic geometry relevant to reciprocity, and the optical tomography problem, along with the co-ordinates discussed. (b) An example of a calculated sensitivity kernel matrix element ( $K_{11}$ ), computed via Monte Carlo.

where  $N$  is the total number of photons used in the MC simulation,  $\hat{s}_j$  are the incoming direction of the photons entering a given voxel, and  $I_j$  are the respective first components of the Stokes vector of the incoming ray. The same applies to the remaining three coefficient sets,  $q_{lm}$ ,  $u_{lm}$  and  $v_{lm}$ , describing the other three Stokes components. So, for each voxel and each incident state of polarization, we will compute and store in memory  $4(l_{\max} + 1)^2$  coefficients to represent the angular dependence of the specific intensity. Constructing a  $4 \times 4$  Green's function, requires carrying out four separate runs of the simulation, each with a linearly independent polarization state, and combining the coefficients from each in a linear fashion (outlined in [4]). Once the two Green's functions are obtained, one for  $\hat{s}_a$ , and one for  $-\hat{s}_b$  (or just one if  $\hat{s}_a = -\hat{s}_b$ ), the sensitivity kernel  $K$  can be calculated from Eq. (6) analytically. Here it is required that we use the symmetry relation  $Y_{lm}(-\hat{s}) = (-1)^l Y_{lm}(\hat{s})$  and also, the orthogonality of the spherical functions. In Figure 1(b), we show an example of a kernel element,  $K_{11}$ , computed for a 2D slice of a 3D slab medium consisting of spherical scattering particles of size parameter  $X = 7.15$ , and refractive index contrast of 1.04. To stress the efficiency of the computation presented here, creating the same image *without* exploiting reciprocity would take a minimum of 200 times the number of simulations run here, given that there are  $200 \times 200$  voxels in just this 2D slice (assuming translational symmetry was still fully exploited).

## 5. Conclusions

In summary, we have presented an approach to compute the DOT sensitivity kernel for polarized light. Significant reductions in computational requirements were obtained by utilizing a reciprocity relation for the vRTE. Further, with the ability to accurately treat the optical tomography problem with polarized sources and filters, the perturbation of polarized transport in general can now be better understood, and additional applications involving the analysis of polarized light can be assessed, such as polarization gating imaging.

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