

Content-Distribution Strategies in Markets With Locked-in
Customers

Online Supplementary Material

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I Price discrimination

In markets with repeat purchases, firms often are able to identify whether or not a consumer purchased from a rival in the past. This allow them to poach their competitors' current customers by offering special inducements to switch. This form of price discrimination called behavior-based price discrimination, is widespread in many markets.¹ Besides, the development of business intelligence tools,² the length of the contracts³ and even the switching processes⁴ can increase the opportunities for the firms to infer the price-sensitivity of their own customers. Thus, as well as being able to know whether or not a consumer purchased from a rival in the past, some firms can also evaluate the likelihood that a current customer is willing to switch.

We now assume that price discrimination is permitted, and that distributors can observe consumers' past purchases, and correctly classify their own loyal customers and switchers. Thus, each distributor can categorize customers into three segments: old "sticky" customers, old price-sensitive customers, and rival's current customers. However, only two customized prices are relevant: the price offered to loyal customers and the price offered to others (i.e., old price-sensitive customers and rival's current customers). We denote by p_l^c and p_s^c the prices offered by the larger and smaller distributors to their loyal customers and q_l^c and q_s^c the prices offered by the larger and smaller distributors to the others, where $c \in \{m, l, s\}$ is the content-distribution strategy. Note that since loyal customers face prohibitively high switching costs which prevent switching, only price-sensitive customers benefit from the offers q_l^c and q_s^c . Without loss of generality, we assume that

¹Fudenberg and Villas-Boas (2007) provide overviews of the literature on customer recognition and behavior-based price discrimination.

²Business intelligence tools provide novel opportunities to collect and analyze vast amounts of information. Customer information (e.g. income, age, gender, purchasing history) can be used to predict future customer behaviors, and to categorize customers into segments according to their willingness to pay for new services or their likelihood to switch.

³Contracts with service providers (such as broadband or mobile ISP) usually include a penalty in the form of an early termination fee. Since these early termination fees are often based on the amount of time left in the contract, customers with long term engagement face higher switching costs and so are less likely to switch suppliers.

⁴Consumers sometimes are obliged to undergo a validation process with their existing provider in order to complete a switch. Consequently, before switching takes place, service providers are informed of the customer intentions and may be able to price discriminate accordingly. For instance, in many countries, consumers wanting to switch but to retain their telephone number are obliged to contact their existing mobile telephony provider to request a porting authorization code.

price-sensitive customers buy from *exclusive* distributor if distributors offer similar net utilities.

The trade-off between price high to extract surplus from locked-in customers and a discounted price to win price-sensitive customers no longer exists if distributors can price discriminate. Hence, there is a Nash equilibrium in pure strategies in Stage II. If C affiliates with one distributor exclusively, say distributor i with $i \neq j \in \{l, s\}$, the lowest price that distributor j can offer to price-sensitive customers is the marginal cost, which is equal to zero. To prevent price-sensitive customers from being tempted by its rival's offer, distributor i needs to offer a price that generates a similar net utility. It is straightforward to show that in equilibrium $q_i^i = \tau$ and $q_j^i = 0$ with $i \neq j \in \{l, s\}$. If C multi-homes, distributors compete *à la Bertrand* in the segment of the market of price-sensitive customers. The equilibrium prices are such that $q_l^m = q_s^m = 0$. Finally, since they face prohibitively high switching costs, distributors extract the full amount of surplus from their locked-in customers: $p_i^c = v + \tau$ if $c \in \{m, i\}$ and $p_i^j = v$ with $i \neq j \in \{l, s\}$. Let $\tilde{\pi}_i^c$ be the equilibrium profit of distributor i in Stage II given that distributors can price discriminate and that the content-distribution strategy is c , with $i \in \{l, s\}$ and $c \in \{m, l, s\}$. We can show that $\tilde{\pi}_l^m = (v + \tau) K_l$ and $\tilde{\pi}_s^m = (v + \tau) K_s$ if C multi-homes; $\tilde{\pi}_i^i = (v + \tau) K_i + (1 - K_i - K_j) \tau$ and $\tilde{\pi}_j^i = v K_j$ if C affiliates exclusively with the larger distributor $i \neq j \in \{l, s\}$. As we know from Section 4.2, the content provider chooses the form of distribution that maximizes Stage-II industry profits. We use these payoffs to show the following proposition.

Proposition 6 *There exists a unique $\bar{\alpha}^{pd} = \frac{1}{2-\sigma}$ such that the content provider affiliates exclusively with the larger distributor if $\alpha < \bar{\alpha}^{pd}$, and multi-homes otherwise.*

Proposition 6 shows that the content provider multi-homes if the mass of locked-in customers is sufficiently high and affiliates with the larger distributor exclusively otherwise. Interestingly enough, and in contrast to our baseline model, the content provider never affiliates exclusively with the smaller distributor. This is due to the fact that the intensity of competition does not depend anymore on the initial market split.⁵ Hence, the content provider can no longer soften

⁵The independence between equilibrium prices and market shares is a common result in the literature on switching costs whenever firms can price discriminate and products are *ex-ante* homogeneous (e.g., Chen, 1997; Taylor, 2003). Gabrielsen (2014) shows that this result is not robust to introducing *ex-ante* product differentiation.

price competition in the distribution market by affiliating exclusively with the smaller rather than the larger distributor. An exclusive affiliation with the larger rather than the smaller distributor is therefore more profitable since it guarantees that a larger number of consumers have access to the content.

When they both have content access, distributors fully exploit their base of locked-in customers, but make zero profits on the segment of price-sensitive customers. Their profit is then similar to the minimum value that they are certain to obtain when price discrimination is not permitted. Therefore, price discrimination reduces industry profits when the content provider multi-homes. Under exclusivity, price discrimination reduces industry profits only if content quality is not too high. However, in this case, price discrimination hurts industry profitability more strongly when the content provider multi-homes. Overall, $\bar{\alpha}^{pd} \geq \bar{\alpha}$ and price discrimination promotes exclusivity. Finally, relation between the likelihood of exclusivity and the asymmetry in market shares is now reversed such that greater asymmetry in market shares makes exclusivity more likely to occur. This is because an exclusive affiliation with the larger distributor no longer strengthens price competition as σ rises.

Here, we assumed that distributors can perfectly classify their own loyal customers and switchers. With a less restrictive assumption that distributors classify customers with a less-than-perfect probability as in Chen et al. (2001), an exclusive distribution with the smaller distributor may still exist. More specifically, the content provider's incentive to sign an exclusive deal with a smaller distributor would probably decrease as distributors' ability to recognize customers more accurately (and to price discriminate accordingly) improves.

II Bidding Game

We here study a “bidding game”, similar to that in Hagiu and Lee (2011), in which distributors compete for content access by offering menus of prices contingent on whether content is sold on an exclusive or non-exclusive basis. Let m_i be the menu of prices offered by distributor $i \in$

$\{l, s\}$ to the content provider: $m_i \in \{\{T_i^i, T_i^m\} / T_i^i \in \mathbb{R}, T_i^m \in \mathbb{R}\}$, with T_i^i and T_i^m the lump-sum transfers for exclusive and non-exclusive distribution, respectively. The content provider receives T_l^l if it affiliates exclusively with the larger distributor, T_s^s if it affiliates exclusively with the smaller distributor and $T_s^m + T_l^m$ if it multi-homes. Whenever the content provider is indifferent between two forms of distribution, we assume that it chooses the content-distribution strategy that maximizes expected industry profits.

By offering a sufficiently low payment for non-exclusive distribution each distributor can unilaterally rule out multi-homing. The distributor with the larger relative gain from exclusive content can then guarantee itself these exclusive content rights. To do so, it offers a low payment for non-exclusive distribution (e.g., $-\infty$) and a payment for exclusivity that makes its rival indifferent between exclusively carrying the content and being the *excluded* distributor. An exclusive equilibrium therefore always exists, and involves a transfer $T_i^i = \pi_j^j - \pi_j^i$ from distributor i to C if $\pi_i^i + \pi_j^i \geq \pi_i^j + \pi_j^j$.

However, both distributors could potentially obtain higher profits under multi-homing. They should increase their non-exclusive fee to induce the content provider to opt for a non-exclusive distribution. Formally, assume that $T_i^i \geq T_j^j$, the content provider multi-homes if

$$T_i^m + T_j^m \geq T_i^i \quad (1)$$

It follows that multi-homing can be achieved at lower cost if both distributors reduce their exclusive fee to a level below which at least one of them has an incentive to deviate in order to gain exclusivity. None of the distributors has an incentive to gain the exclusivity, neither to be the *excluded* distributor only if

$$\pi_i^m - T_i^m \geq \pi_i^i - T_j^j \text{ and } \pi_j^m - T_j^m \geq \pi_j^j - T_i^i \quad (2)$$

$$\pi_i^m - T_i^m \geq \pi_i^j \text{ and } \pi_j^m - T_j^m \geq \pi_j^i \quad (3)$$

Finally, condition (1) must be binding, otherwise distributors could lower their non-exclusive fee and still induce the content provider to multi-home. Conditions (1)-(2)-(3) can hold only if multi-homing maximizes expected industry profits. We can state the following lemma.

Lemma 6 *An equilibrium in which C affiliates exclusively with one of the distributors always exists, and involves a transfer $T_i^i = \pi_j^j - \pi_j^i$ from distributor i to C if $\pi_i^i + \pi_j^i \geq \pi_i^j + \pi_j^j$. An equilibrium in which C multi-homes exists if and only if*

$$\pi_l^m + \pi_s^m \geq \max \{ \pi_l^l + \pi_s^l, \pi_l^s + \pi_s^s \}. \quad (4)$$

Proof. See Appendix C ■

Since each of them can unilaterally rule out multi-homing, distributors face a coordination problem and both exclusive and non-exclusive equilibria exist when multi-homing maximizes expected industry profits. We select among multiple equilibria via the Pareto-undominated solution (for the distributors). When multi-homing maximizes expected industry profits, the unique Pareto-undominated equilibrium is such that C multi-homes with transfers $T_i^m = \pi_j^j - \pi_j^m$ and $T_j^m = \pi_i^i - \pi_i^m$. Given that distributors coordinate on the Pareto-undominated equilibrium, the industry structure that emerges from Stage I is still that which maximizes expected industry profits in Stage II. This coordination among the distributors avoids any “inefficiency” at the contracting stage.⁶

Coordination & Multiple equilibria. In the following, we show that under some plausible assumptions, no exclusive equilibrium exists when multi-homing maximizes expected industry profits. Assume that $\pi_i^m + \pi_j^m > \pi_i^i + \pi_j^i \geq \pi_i^j + \pi_j^j$ and $T^e = \pi_j^j - \pi_j^i$, with $i \neq j \in \{l, s\}$.

(i) Any exclusive equilibrium is such that $T_i^i = T_j^j = T^e$. Distributor i earns a total expected

⁶Even though distributors fail to coordinate, multi-homing will occur when condition (4) is satisfied if the distributor that receives the exclusive content can resell it to its rival. The renegotiation stage being equivalent to the situation in which one of the distributors is vertically integrated with the content provider, potential coordination failure will be resolved in this latter renegotiation stage. Indeed, when a vertically-integrated distributor chooses whether or not to supply its content to its rival, the industry structure that emerges is that which maximizes expected industry profits (see, Section VI).

profit $\pi_i^i - T^e$ and distributor j a total expected profit π_j^i . In addition, distributor i can guarantee itself the exclusive content rights, by offering a sufficiently low payment for non-exclusive distribution. Distributors can therefore guarantee themselves a minimum total expected profit, which is $\pi_i^i - T^e$ for distributor i and π_j^i for distributor j .

- (ii) When distributor j chooses a non-exclusive fee $T_j^m \geq \bar{\bar{T}}_j^m \equiv \pi_j^m - \pi_j^i$, it obtains a total expected profit lower than or equal to π_j^i if multi-homing occurs. We then assume that distributor j always sets a non-exclusive fee $T_j^m < \bar{\bar{T}}_j^m$.

Using (i) and (ii), we restrict our attention to potential exclusive equilibria that are such that $m_i \in \{\{T^e, T_i^m\} / T_i^m \in \mathbb{R}\} \equiv \bar{M}_i$ and $m_j \in \{\{T^e, T_j^m\} / T_j^m < \bar{\bar{T}}_j^m\}$. Let $\bar{T}_i^m = T^e - \bar{\bar{T}}_j^m$, $\bar{\bar{T}}_i^m = \pi_i^m - (\pi_i^i - T^e)$ and $\mathbb{M} = \{\bar{M}_i^1, \bar{M}_i^2, \bar{M}_i^3\}$ forms a partition of \bar{M}_i , with $\bar{M}_i^1 = \{\{T^e, T_i^m\} / T_i^m \leq \bar{T}_i^m\}$, $\bar{M}_i^2 = \{\{T^e, T_i^m\} / T_i^m \in (\bar{T}_i^m, \bar{\bar{T}}_i^m)\}$ and $\bar{M}_i^3 = \{\{T^e, T_i^m\} / T_i^m \geq \bar{\bar{T}}_i^m\}$.

- When $m_i \in \bar{M}_i^1$: since $T_j^m < \bar{\bar{T}}_j^m$, distributor i rules out multi-homing. Indeed, $T^e > T_i^m + T_j^m$ and C prefers to affiliate exclusively with distributor i . Distributor i earns a total expected profit $\pi_i^i - T^e$.
- When $m_i \in \bar{M}_i^2$: exclusivity with distributor i occurs and distributor i earns a total expected profit $\pi_i^i - T^e$ if $T^e > T_i^m + T_j^m$. Otherwise, multi-homing occurs and distributor i earns a total expected profit strictly higher than $\pi_i^i - T^e$.
- When $m_i \in \bar{M}_i^3$: exclusivity with distributor i occurs and distributor i earns a total expected profit $\pi_i^i - T^e$ if $T^e > T_i^m + T_j^m$. Otherwise, multi-homing occurs and distributor i earns a total expected profit which is less than or equal to $\pi_i^i - T^e$.

It follows that distributor i is either strictly better off or at least as good by choosing a strategy $m_i \in \bar{M}_i^2$ rather than $m_i \in \bar{M}_i^1 \cup \bar{M}_i^3$. When $m_i \in \bar{M}_i^2$, distributor j 's best response is to set $T_j^m = T^e - T_i^m$ (which is strictly lower than $\bar{\bar{T}}_j^m$) and thus the content provider multi-homes.

To summarize, any exclusive equilibrium is characterized by $T_i^i = T_j^j = T^e$. Given that $T_i^m \in$

$(\bar{T}_i^m, \bar{T}_i^m)$ when distributor i chooses an exclusive fee equal to T^e , no exclusive equilibrium exists when multi-homing maximizes expected industry profits.

The value of a large customer base. Even if the content-distribution outcome is similar to the one obtained in the baseline model, the transfer fees are now different. Distributor i will pay $T_i^i = \pi_j^j - \pi_j^i$ in exchange for exclusive content and $T_i^m = \pi_j^j - \pi_j^m$ for non-exclusive content, with $i \neq j \in \{l, s\}$. We define the value of having a large customer base as $\Delta^c = \Pi_i^c - \Pi_s^c$, with Π_i^c the equilibrium expected total profit of distributor $i \in \{l, s\}$ and $c \in \{m, l, s\}$ the content-distribution strategy. Since the equilibrium expected total profits depend upon the content distribution strategy, we distinguish three situations according to whether C multi-homes, C is exclusive to the larger distributor or C is exclusive to the smaller distributor.

- Non-exclusive distribution: there are monetary transfers $T_l^m = \pi_s^s - \pi_s^m$ and $T_s^m = \pi_l^l - \pi_l^m$ from the larger and smaller distributors to the content provider. The equilibrium expected total profits are $\Pi_l^m = \pi_l^m - (\pi_s^s - \pi_s^m)$ and $\Pi_s^m = \pi_s^m - (\pi_l^l - \pi_l^m)$. The value of a large customer base is $\Delta^m = \pi_l^l - \pi_s^s$.
- Exclusive distribution with the larger distributor: there is a monetary transfer $T_l^l = \pi_s^s - \pi_s^l$ from the larger distributor to the content provider. The equilibrium expected total profits are $\Pi_l^l = \pi_l^l - (\pi_s^s - \pi_s^l)$ and $\Pi_s^l = \pi_s^l$. The value of a large customer base is $\Delta^l = \pi_l^l - \pi_s^s$.
- Exclusive distribution with the smaller distributor: there is a monetary transfer $T_s^s = \pi_l^l - \pi_l^s$ from the smaller distributor to the content provider. The equilibrium expected total profits are $\Pi_l^s = \pi_l^s$ and $\Pi_s^s = \pi_s^s - (\pi_l^l - \pi_l^s)$. The value of a large customer base is $\Delta^s = \pi_l^l - \pi_s^s$.

We see that $\Delta^c = \pi_l^l - \pi_s^s$ for $c \in \{m, l, s\}$. Thus, the value of a large customer base is equal to the Stage II expected profit of a *large exclusive* distributor minus the Stage II expected profit of a *small exclusive* distributor. The key point here is that the value of a large customer base does not depend on the content-distribution strategy and $\Delta^c \equiv \Delta$ for $c \in \{m, l, s\}$. We can decompose

the value of a large customer base as follows

$$\Delta = \Delta^0 + \Sigma(\tau), \quad (5)$$

with $\Delta^0 = vK_l \frac{K_l - K_s}{1 - K_s}$ the value of a large customer base if there were no content provider, and

$$\Sigma(\tau) = \begin{cases} (2K_l - 1)\tau & \text{if } \tau < \hat{\tau} \\ \frac{K_s(1 - K_s) - K_l(1 - K_l)}{1 - K_l}v + (K_l - K_s)\tau & \text{if } \tau \geq \hat{\tau} \end{cases}.$$

Since Δ^0 is positive, a large customer base would always have a positive value if there were no content provider. The second term, Σ , expresses the bargaining power surplus provided by a large customer base. We will refer to Σ as the *bargaining* value of a large customer base. The fee paid by a distributor to access to the content (exclusively or not) increases with the extent to which its rival grants the exclusivity. Indeed, $T_i^i = \pi_j^j - \pi_j^i$ and $T_i^m = \pi_j^j - \pi_j^m$ with $i \neq j \in \{l, s\}$. The bargaining power of a distributor will therefore be greater if its rival does not highly value the exclusivity. Since it can offer the content to a larger base of loyal customers, it seems natural to expect that the larger distributor values the exclusivity more than the smaller distributor. Hence, the larger distributor should have greater bargaining power, and the *bargaining* value of a large customer base should be positive. However, the initial advantage of having a larger customer base progressively disappears when the proportion of locked-in customers decreases. Besides, an *exclusive* distributor will face more intense competition if it has a large rather than a small customer base. Thus, the larger distributor does not necessarily value the exclusivity more than its smaller rival. This leads to the following proposition.

Proposition 7 *There exists a threshold $\tau_2 = \frac{(1-\alpha)v}{1-\alpha\sigma}$ such that the bargaining value of a large customer base is negative if $\tau < \tau_2$ and $\alpha\sigma < 0.5$, and the bargaining value of a large customer base is positive otherwise.*

Proof. See Appendix D ■

Proposition 7 shows that the *bargaining* value of a large customer base is always positive when the larger distributor is certain to serve more than half of the market ($\alpha\sigma \geq 0.5$). Otherwise, the *bargaining* value of a large customer base is positive if $\tau \geq \tau_2$ and negative if $\tau < \tau_2$. When $\Sigma < 0$, competition over content access reduces the value of having a large customer base (compared to the case where there is no content provider). As long as expression (5) remains positive, distributors benefit from a large customer base. However, if Σ is negative and more than offsets Δ^0 (which is always positive), distributors are worse off having the largest customer base. This leads to the following proposition.⁷

Proposition 8 *There exist values τ_0 , τ_1 and $\tilde{K}_l < 0.5$ such that the value of a large customer base is negative if $\tau \in (\tau_0, \tau_1)$ and $\alpha\sigma < \tilde{K}_l$, and the value of a large customer base is positive otherwise.*

Proof. See Appendix E ■

Proposition 8 shows that when the base of captive customers of the larger distributor is lower than \tilde{K}_l , the value of a large customer base is negative for $\tau \in (\tau_0, \tau_1)$. Otherwise, the value of a large customer base is positive. To illustrate Propositions 7 and 8, we propose a numerical example with the following parameter values, $v = 1$ and $\sigma = 0.7$. In Figure 1, we determine the signs of the *overall* and the *bargaining* values of a large customer base as a function of the mass of locked-in customers α (on the horizontal axis) and the level of content quality τ (on the vertical axis).

⁷The threshold values displayed in Propositions 7 and 8 have the following characteristics $0 < \tau_0 < \hat{\tau} < \tau_1 < \tau_2$.

Figure 1: The *overall* and the *bargaining* values of a large customer base

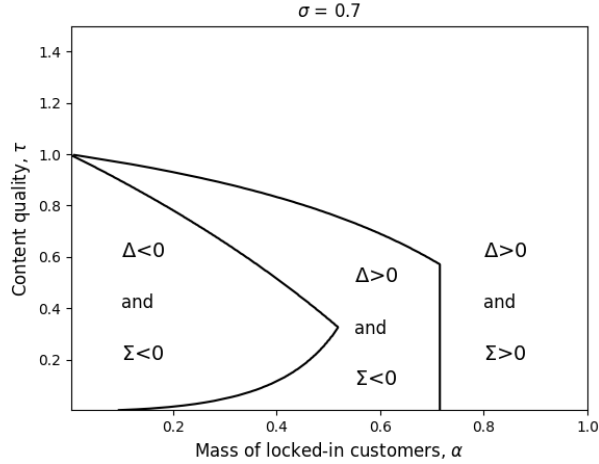


Figure 1 shows that competition over content access can reduce the value of having a large customer base (compared to the case where there is no content provider). When the captive-customer base of the larger distributor is not too large and the content is of low-quality, this negative effect can be strong enough for the largest customer-base distributor to be worse off.

The commonly held belief that firms benefit from a large customer base in markets with switching costs does not necessarily hold. Klemperer (1987a) and Schmidt (2010) have already pointed out several reasons to explain why firms can be made worse off having a larger customer base. Our paper introduces a new motive that may reduce the value of a large customer base: when distributors compete to acquire content exclusivity, a large customer base may worsen their bargaining power.

III Discriminatory fixed-fee policy

We here assume that the content provider sells its content by means of a discriminatory fixed-fee policy. The content provider announces two fees (T_l, T_s) , and distributor i obtains the content rights if it is willing to pay the fee T_i . Contrary to the baseline model, the contracts do not specify whether the rights are granted exclusively or not. We assume that distributors accept an offer

when they are indifferent between accepting or refusing it.⁸

Suppose that the content provider announces two fees (T_l, T_s) such that $T_l \leq \pi_l^m - \pi_l^s$ and $T_s \leq \pi_s^m - \pi_s^l$. Since $\pi_l^l - \pi_l^0 \geq \pi_l^m - \pi_l^s$, the larger distributor purchases the content regardless of the choice made by its rival.⁹ The best-reply choice of the smaller distributor is then to purchase the content. It follows that the unique equilibrium is such that both distributors purchase the content. Otherwise, when $T_l > \pi_l^m - \pi_l^s$ or $T_s > \pi_s^m - \pi_s^l$, no equilibrium exists in which both distributors buy the content rights since at least one of them will be better off rejecting the offer given that its rival purchases the content. The highest profit the content provider can obtain when both distributors purchase content access is $\pi_l^m + \pi_s^m - (\pi_l^s + \pi_s^l)$. As those who fail to pay the fee run the risk of being the *excluded* distributor, this revenue is similar to the one obtained in the baseline model under multi-homing.

Given that its rival does not purchase the content, distributor $k \in \{l, s\}$ is willing to pay up to $\pi_k^k - \pi_k^0$ for access to the content. When $\pi_i^i - \pi_i^0 > \pi_j^j - \pi_j^0$, the content provider obviously obtains the highest profit if distributor i rather than distributor j purchases the content, with $i \neq j \in \{l, s\}$. The content provider can rule out the possibility that distributor j purchases the content by setting $T_j > \max\{\pi_j^j - \pi_j^0, \pi_j^m - \pi_j^i\}$. In that case, distributor i knows that if it rejects the offer, the content rights will not be allocated to any distributor and therefore purchases the content if $T_i \leq \pi_i^i - \pi_i^0$. Thus, the content provider's revenue is $\max\{\pi_l^l - \pi_l^0, \pi_s^s - \pi_s^0\}$ for exclusive distribution. Note that $\pi_s^0 \geq \pi_s^l$ and $\pi_l^0 = \pi_l^s$. The cost of rejecting an offer at the exclusive equilibrium point is therefore similar to that in the baseline model for the larger distributor. However, this selling format reduces the bargaining power of the content provider when it wants to affiliate exclusively with the smaller distributor. Indeed, the cost of a deviation from an exclusive equilibrium for the smaller distributor is now lower.

To summarize, the content provider's revenue is lower to that obtained in the baseline model

⁸Clearly, the content provider could lower the relevant fee by some $\varepsilon > 0$ so that distributors will strictly prefer to accept the offer.

⁹The larger distributor is better off purchasing the content rights as long as $\pi_l^m - T_l \geq \pi_l^s$ if the smaller distributor has content access, and as long as $\pi_l^l - T_l \geq \pi_l^0$ if the smaller distributor does not have content access. Since $\pi_l^l - \pi_l^0 \geq \pi_l^m - \pi_l^s$, the larger distributor is better off purchasing the content regardless of the choice made by its rival if $T_l \leq \pi_l^m - \pi_l^s$.

only when content rights are granted exclusively to the smaller distributor. Otherwise, the content provider's revenue remains unchanged. This leads to the following proposition.

Proposition 9 *An exclusive affiliation with the smaller distributor is less likely to occur than in the baseline model.*

Last, the equilibrium expected total profits of the smaller distributor are $\Pi_s^s = \pi_s^0$ when it has exclusivity and $\Pi_s^c = \pi_s^l$ otherwise. The equilibrium expected total profits of the larger distributor are similar regardless of the content-distribution choice, and $\Pi_l^c = \pi_l^0 = \pi_l^s$ for $c \in \{m, l, s\}$. The value of a large customer base, Δ^c , is therefore contingent on the content-distribution choice: $\Delta^c = \pi_l^s - \pi_s^l$ if $c \in \{m, l\}$ and $\Delta^s = \pi_l^s - \pi_s^0$, with $0 \leq \Delta^s \leq \Delta^m = \Delta^l$. When the content provider is exclusive to the smaller distributor, being the smaller distributor is less harmful than in the baseline model. Notwithstanding, since $\Delta^c \geq 0$ for $c \in \{m, l, s\}$, distributors are still better off having a larger customer base, regardless of the content-distribution choice.

IV Advertising revenue collected by the content provider

The literature on content distribution shows that whether content is exclusive or multi-homes depends crucially on whether the content provider collects revenue directly from the final consumers (see, Hagiu and Lee, 2011; Weeds, 2012; Stennek, 2014). So far, to put the emphasis on our main result that multi-homing can occur even if the content provider's revenue stems only from the lump-sum fee(s) collected from the distributors, we have assumed that the content provider could not collect any revenue directly from the final consumers. In this section we relax this assumption: the content provider now receives advertising revenue depending on exposure to end users. The expected profits of the content provider are now composed of two parts: the fixed fee(s) from the distributor(s) in Stage I and the advertising revenue in Stage II from the consumers with access to its content. For simplicity, per-user advertising revenue, θ , is exogenous and independent of content quality. Let σ_i^c be the equilibrium expected market share of distributor i (at the end of Stage II) and π_{cp}^c the content provider's equilibrium expected profit in Stage II: $\pi_{cp}^m = \theta$, $\pi_{cp}^i = \sigma_i^i \times \theta$

and $\pi_{cp}^m > \pi_{cp}^l \geq \pi_{cp}^s$, with $i \in \{l, s\}$ and $c \in \{m, l, s\}$.

Since the distributors' profits in Stage II are left unchanged, the per-user advertising revenue does not affect the lump-sum fees that the content provider can extract from the distributors. However, the content provider's profit is now $\pi_l^m + \pi_s^m - (\pi_l^s + \pi_s^l) + \pi_{cp}^m$ if it multi-homes and $\pi_i^i - \pi_i^j + \pi_{cp}^i$ if it joins exclusively distributor i , with $i \neq j \in \{l, s\}$. Multi-homing is more profitable for the content provider if

$$\pi_l^m + \pi_s^m + \pi_{cp}^m \geq \max \{ \pi_l^l + \pi_s^l + \pi_{cp}^l, \pi_l^s + \pi_s^s + \pi_{cp}^s \};$$

while an exclusive affiliation with distributor i is more profitable if

$$\pi_l^i + \pi_s^i + \pi_{cp}^i \geq \max \{ \pi_l^m + \pi_s^m + \pi_{cp}^m, \pi_l^j + \pi_s^j + \pi_{cp}^j \}$$

with $i \neq j \in \{l, s\}$. The allocation of content across distributors in Stage I is still that which maximizes expected industry profits in Stage II, which are now the sum of the expected revenues collected by the distributors and the content provider in Stage II. Given that $\pi_{cp}^m > \pi_{cp}^l \geq \pi_{cp}^s$, we can state the following proposition.

Proposition 10 *When θ rises,*

- *multi-homing is more likely to prevail over exclusivity.*
- *an exclusive affiliation with the larger rather than the smaller distributor is more likely to occur.*

When it chooses its content-distribution, the content provider must now take account of the relative loss in terms of market exposure to some end users. An exclusive affiliation with distributor i rather than with both distributors results in a decline in advertising revenue of $\sigma_j^i \theta$. Exclusive distribution thus produces an opportunity cost for the content provider and becomes less likely as θ rises. Similarly, by exclusively joining distributor i rather than distributor j , the content

provider's exposure changes by $\sigma_i^i - \sigma_j^j$. Since $\sigma_i^l \geq \sigma_s^s$, an exclusive affiliation with the larger distributor rather than with the smaller distributor is more likely to occur as θ increases.

V Advertising revenue collected by the distributors

In this section, distributors finance themselves through subscription fees and advertising. For instance, even with considerable heterogeneity across pay-TV networks, most distributors earn a combination of affiliation and advertising revenues.¹⁰ We here assume that distributors earn a subscription fee and an advertising revenue, θ , from each of their customers. Following the reasoning in the proof of Lemma 1, we derive the equilibrium pricing in Stage II taking into account that distributors also collect a per-user advertising revenue (see Appendix F). The price competition in Stage II is slightly different from that in the baseline model. On the one hand, the distributors' incentives to price-cut their rival to attract price-sensitive customers are greater since those customers bring advertising revenue to the distributors. On the other hand, by focusing on their locked-in customers, distributors can guarantee themselves higher profits than in the baseline model. Indeed, when it sets the monopoly price (i.e. that with net utility of 0), distributor i obtains a profit $(u_i^c + \theta) K_i$ with probability one, while in the baseline model it could guarantee itself a profit $u_i^c K_i$, with $u_i^c = v + \tau$ if $c \in \{m, i\}$ and $u_i^j = v$. Since advertising revenue is partly passed on to consumers in the form of lower prices, distributors compete more fiercely as θ rises.¹¹ This tougher competition is however more than offset by the gain in advertising revenue, and the expected industry profits in Stage II increase with θ . Let $\pi_i^{c,\theta}$ be the Stage II expected profit of distributor i when the content distribution is $c \in \{m, l, s\}$ and the per-user advertising revenue is θ . We can show that $\frac{d\pi_i^{m,\theta} + \pi_s^{m,\theta}}{d\theta} = \frac{d\pi_i^{s,\theta} + \pi_s^{s,\theta}}{d\theta} = \frac{d\pi_i^{l,\theta} + \pi_s^{l,\theta}}{d\theta} > 0$ if $\tau < \widehat{\tau}^\theta$ and $\frac{d\pi_i^{m,\theta} + \pi_s^{m,\theta}}{d\theta} = \frac{d\pi_i^{s,\theta} + \pi_s^{s,\theta}}{d\theta} \geq \frac{d\pi_i^{l,\theta} + \pi_s^{l,\theta}}{d\theta} > 0$, otherwise. Expected industry profits are affected differently by θ only if the content-distribution strategy alters the identity of the distributor that has the

¹⁰Crawford (2015) reports the relative importance of affiliation and advertising revenues for the 25 largest U.S. Pay-TV networks in 2012: the share of advertising revenue varies from 17% to 76% and is of 43% on average.

¹¹Note that it can be profitable for distributors to subsidize customers with revenue from the advertising market. Hence, prices can potentially be below marginal cost (which is here zero).

lowest incentive to compete for price-sensitive customers. The larger distributor is always the less inclined to offer a high net utility to attract price-sensitive customers, unless it has exclusive high-quality content (higher than $\widehat{\tau}^\theta$). Expected industry profits increase with θ , but to a lower extent when the content is of high-quality and C is exclusive to the larger distributor. As we know from Section 4.2, the content provider chooses the form of distribution that maximizes Stage-II expected industry profits. We can state the following proposition.

Proposition 11 *An exclusive affiliation with the larger distributor is less likely to occur when θ rises.*

Proof. See Appendix F ■

Regardless of the content-distribution strategy, the advertising revenue collected by the industry is always equal to θ . However, advertising revenue increases price competition differently according to the form of distribution. The competition-enhancing effect is stronger when the larger distributor has exclusivity and the content is of high-quality. Hence, a rise in θ makes an exclusive affiliation with the larger distributor less likely.

The trade-off facing the distributors, which involves rent extraction from locked-in customers *versus* a discounted price to win price-sensitive customers, is affected in the same way by an increase in θ as by an increase in the value of “regular” content, v .¹² Both terms appear together in the profit expressions, and a rise in θ is therefore equivalent to an increase in v (see (20)-(21) in Appendix F). In the baseline model, the allocation of content across distributors is determined by the relative value of the content $\frac{\tau}{v}$. The mechanisms underlying the result in Proposition 11 is therefore similar to that presented in the baseline model when we discuss the effect of a variation in content quality. A rise in v (which is equivalent to a decrease in τ) makes an exclusive affiliation with the larger distributor less likely, which is consistent with the result of Proposition 11.

¹²Assume that distributor j offers a net utility equal to w . The best reply for distributor i is either to fully exploit its locked-in customers or to offer a net utility slightly higher than w . Distributor i receives a profit $(u_i^c + \theta) K_i$ if it focuses on its locked-in customers and a profit slightly lower than $(1 - K_j)(u_i^c + \theta - w)$ otherwise, with $u_i^i = u_i^m = v + \tau > u_i^j = v$. It is profitable for distributor i to undercut its rival if $(u_i^c + \theta)(1 - K_i - K_j) - w(1 - K_j) > 0$. An increase in v or in θ affects in the same way the distributor i 's incentive to undercut its rival.

VI Vertical Integration

This section investigates the incentives for a vertically-integrated operator to supply its content to a downstream rival. We assume that the content provider is integrated with distributor i , with $i \in \{l, s\}$. The vertically-integrated (*VI*) firm can deny access to its content, sell its content non-exclusively or give up its content rights. In the latter case, its rival becomes the *exclusive* distributor. If it supplies its content to the rival distributor, the *VI* firm requires a lump-sum payment $T^m = \pi_j^m - \pi_j^i$ for non-exclusive distribution and $T^e = \pi_j^j - \pi_j^i$ for waiving the rights related to the content. In both cases, distributor j is indifferent between accepting the offer (and paying the associated fee) and being the *excluded* distributor, and therefore accepts the offer. The profit of the *VI* firm is equal to $\pi_i^m + T^m$ if it signs a non-exclusive contract, $\pi_i^j + T^e$ if it gives its content rights up to its downstream rival, and π_i^i otherwise. The *VI* firm prefers non-exclusive distribution if $\pi_i^m + \pi_j^m \geq \max\{\pi_i^i + \pi_j^i, \pi_i^j + \pi_j^j\}$ and to fully transfer its content rights to its downstream rival if $\pi_i^j + \pi_j^j > \max\{\pi_i^m + \pi_j^m, \pi_i^i + \pi_j^i\}$. Otherwise, the *VI* firm prefers to withhold its content from the rival distributor. We can state the following proposition.

Proposition 12 *When the content provider is vertically integrated with one of the distributors, the industry structure that emerges from Stage I is that which maximizes expected industry profits in Stage II.*

The allocation of content across distributors is therefore unaffected by the vertical structure of the industry. Note that if the *VI* firm cannot give up its content rights, the content will be less likely to be exclusive to one distributor than in our baseline model: non-exclusive distribution will occur as soon as $\pi_i^m + \pi_j^m \geq \pi_i^i + \pi_j^i$.

VII Multi-homing consumers

We have up to now assumed that consumers purchase from one distributor only. In the following, we extend our model by allowing consumers to purchase from both distributors in order to see

how the content-distribution choice is affected when consumers can multi-home. Without loss of generality, consumers multi-home if they are indifferent between purchasing from one distributor and from both distributors. We furthermore assume that the market is split evenly, meaning that distributors are symmetric with respect to their initial market share. There are two distributors, A and B , and we denote by $K = 0.5\alpha$ each distributor's locked-in customer base.

Apart from the value created by the content, a consumer receives similar utility from purchasing access to one or both distributors. Joint consumption can therefore provide an incremental surplus only when the content provider is exclusive. In this case, the locked-in customers of the *excluded* distributor multi-home if the *exclusive* distributor offers a net utility of v (or more); while the other customers never purchase from both distributors.¹³

The content provider multi-homes. Allowing consumers to multi-home does not change the results of the baseline model, and $\pi_A^m = \pi_B^m = (v + \tau)K$.

The content provider is exclusive. By setting a net utility equal to v , the *exclusive* distributor can serve the entire demand with probability one and obtain a profit equal to τ . Thereafter, we consider three possibilities according to the quality of the content: (a) $\tau \leq \tau^*$, (b) $\tau^* < \tau < \tau^+$ and (c) $\tau \geq \tau^+$, with $\tau^* = Kv/(1 - K)$ and $\tau^+ = (1 - K)v/K$. Without loss of generality, we assume that C is exclusive to distributor A.

(a) $\tau \leq \tau^*$. Since $(v + \tau)K \geq \tau$, the *exclusive* distributor is better off fully exploiting its locked-in customers rather serving the entire demand. Both distributors are still confronted with a trade-off between fully exploiting their locked-in customers and a discounted price to win price-sensitive customers. In that case, allowing consumers to multi-home will not

¹³Assume that distributor i has the exclusivity. The net utility of the locked-in customers of distributor i is reduced by $p_j^i > 0$ if they also purchase from distributor j . Thus, they never purchase from both distributors. For the other customers, the incremental utility from multi-purchasing is strictly negative if $p_j^i > \tau$. In this case, none of the customers multi-homes, and price-sensitive customers purchase from the distributor offering the highest net utility. When $p_j^i \leq \tau$, price-sensitive customers only purchase from distributor i , and the locked-in customers of distributor j purchase from both distributors (and gain an incremental surplus of $\tau - p_j^i$). Note that $p_j^i \leq \tau$ is equivalent to $w_i^j \geq v$: the locked-in customers of the *excluded* distributor multi-home if the *exclusive* distributor offers a net utility of v (or more).

change the results of the baseline model and $\pi_A^A = \tau(1 - K) + Kv$ and $\pi_B^A = vK$.

(b) $\tau^* < \tau < \tau^+$. Since $(v + \tau)K < \tau$, the *exclusive* distributor is better off serving the entire demand rather than fully exploiting its locked-in customers. When $\bar{w}_A^A = v$, the *excluded* distributor cannot attract the price-sensitive customers and is better off quoting the monopoly price. However, when $w_A^B = 0$, the *exclusive* distributor prefers to slightly undercut its rival (i.e., w_A^A slightly higher than 0); which in turn incites the *excluded* distributor to also compete for price-sensitive customers. There is thus no pure-strategy equilibrium in prices, and we return to the situation described in our benchmark framework with one exception: the *exclusive* distributor is now able to guarantee itself a higher profit. Suppose that a mixed strategy involves distributor i offering net utility no higher than w with probability $F_i^A(w)$ over the interval with lower bound 0, and that distributor A has no mass point at 0. By choosing a net utility of 0, distributor B can guarantee itself a profit vK . Let \bar{w}_B^A be the net utility that will guarantee distributor B a similar profit when it succeeds in gaining the price-sensitive customers: \bar{w}_B^A is such that $(v - \bar{w}_B^A)(1 - K) = vK$. Any $w > \bar{w}_B^A = \frac{(1-2K)v}{1-K}$ is a dominated strategy for distributor B . By setting net utility equal to v , distributor A can guarantee itself a profit of τ . Let \bar{w}_A^A be the net utility that guarantees distributor A a similar profit when it serves the entire demand apart from the locked-in customers of the *excluded* distributor: \bar{w}_A^A is such that $(v + \tau - \bar{w}_A^A)(1 - K) = \tau$, and so $\bar{w}_A^A = \frac{v(1-K) - K\tau}{1-K}$. Any w such that $\{w | w > \bar{w}_A^A, w \neq v\}$ is a dominated strategy for distributor A .

- If $\tau \in (\tau^*, v]$, $\bar{w}_B^A \leq \bar{w}_A^A$. Any $w > \bar{w}_B^A$ is a dominated strategy for distributor A . In equilibrium, distributors must be indifferent between all of the values in their support, and thus $\pi_A^A = \tau(1 - K) + Kv$ and $\pi_B^A = vK$. In this case, the mixing probabilities and expected profits are similar to those in our baseline model.
- If $\tau \in (v, \tau^+)$, $\bar{w}_B^A > \bar{w}_A^A$. If distributor A quotes \bar{w}_A^A with positive probability, then by choosing $\bar{w}_A^A + \varepsilon$, where ε is infinitely small, distributor B increases its profit. There is a profitable deviation. Now assume that distributor A places a mass point at v and

otherwise chooses a net utility in the interval $(0, \bar{w}_A^A]$. Any $w \geq \bar{w}_A^A$ is a dominated strategy for distributor B .¹⁴ In equilibrium, distributors must be indifferent between all of the values in their support, and thus $\pi_A^A = \tau$ and $\pi_B^A = vK$. This leads to the following proposition.

Proposition 13 *When $\tau \in (v, \tau^+)$ and the content provider is exclusive to distributor A , there is a mixed-strategy Nash equilibrium in which distributor A sets net utilities in the domain $w_A^A \in (0, \bar{w}_A^A] \cup \{v\}$. The mixing probability of distributor A is*

$$F_A^A(w) = \begin{cases} \frac{wK}{(v-w)(1-2K)} & \text{if } w \in (0, \bar{w}_A^A] \\ 1 & \text{if } w = v \end{cases},$$

with a mass point at $w = v$ of probability $\frac{(1-K)(\tau-v)}{(1-2K)\tau}$. Distributor B sets net utilities in the domain $w_B^A \in [0, \bar{w}_A^A)$. The mixing probability of distributor B is $F_B^A(w) = \frac{\tau-K(v+\tau-w)}{(1-2K)(v+\tau-w)}$ with a mass point at $w = 0$ of probability $\frac{\tau(1-K)-Kv}{(v+\tau)(1-2K)}$.

When $\tau \in (v, \tau^+)$, the *exclusive* distributor occasionally offers high net utility in order to serve the entire demand. In that case, the *excluded* distributor is unable to attract the price-sensitive customers, this in turn reduces its incentive to compete fiercely. Compared to the baseline model, the *excluded* distributor offers more frequently low net utility, while the demand served by the *exclusive* distributor is likely to be greater. When the content provider is exclusive, expected industry profits are therefore higher than those in the baseline model.

(c) $\tau \geq \tau^+$. Since $(v + \tau)K < \tau$, the *exclusive* distributor is better-off serving the entire demand rather than fully exploiting its locked-in customers. When $\bar{w}_A^A = v$, the *excluded* distributor cannot attract the price-sensitive customers and is better off quoting the monopoly

¹⁴Neither distributor has a mass point at w^* , such that $0 < w^* < \bar{w}_A^A$. By way of contradiction, assume that distributor i quotes w^* with positive probability; then by choosing $w^* + \varepsilon$, where ε is infinitely small, distributor j increases its profit. There is a profitable deviation. This is a contradiction. Distributor B is always better off by not quoting \bar{w}_A^A but coming arbitrarily close to it, while distributor A is always better off by not quoting 0 but coming arbitrarily close to it.

price. Given that $w_B^A = 0$, the *exclusive* distributor has no incentive to set a net utility slightly higher than zero. There exists a unique Nash equilibrium in pure strategies in which the net utilities are $w_B^A = 0$, $w_A^A = v$ and profits are $\pi_B^A = Kv$, $\pi_A^A = \tau$. When the content provider chooses exclusive distribution, expected industry profits are higher than those in the baseline model.

Overall, when the content provider chooses exclusive distribution, expected industry profits are either higher or similar to those in the baseline model. Since none of the consumers multi-homes, the equilibrium pricing strategies and profits are similar to those in the baseline model when the content provider multi-homes. We can state the following proposition

Proposition 14 *Consumer multi-purchase promotes exclusivity.*

VIII Small switching costs

In our baseline model, some consumers are fully locked-in while others can switch supplier at no cost. We now consider an alternative approach where consumers have positive (but not prohibitively high) switching costs, such that some consumers may pay to switch in equilibrium. We furthermore assume that the market is split evenly, meaning that distributors are symmetric with respect to their initial market share. We denote by A and B the two distributors. Distributors still offer homogeneous products but the switching costs are now continuously distributed over a range including zero. This framework is similar to that used in Chen (1997). Each consumer faces a switching cost of s , which is a draw from a random variable uniformly-distributed over $[0, \bar{s}]$ across the population of consumers. We assume that the average switching cost \tilde{s} is above $\frac{\tau}{6}$, so that exclusion never occurs.

Switching occurs only if one distributor offers higher net utility than its competitor. For instance, assume that distributor i offers higher net utility than distributor j . The “old” customers of distributor j with switching cost $s < w_i^c - w_j^c \equiv \nabla_{i,j}^c$ will switch to distributor i , with $i \neq j \in$

$\{A, B\}$ and $c \in \{m, A, B\}$ the content-distribution strategy. In this case, the demand served by distributor i is $\frac{1}{2} + \frac{1}{2} \int_0^{\nabla_{i,j}^c} \frac{1}{s} ds$ and that served by distributor j is $\frac{1}{2} \int_{\nabla_{i,j}^c}^{\tilde{s}} \frac{1}{s} ds$. In equilibrium, switching occurs from the *excluded* to the *exclusive* distributor, and the number of consumers who pay to switch falls with average switching costs. There exists a unique Nash equilibrium in pure strategies in Stage II. Equilibrium prices and profits, which depend on the content-distribution choice and average switching costs, are as follows:

- $p_i^i = v + \tau$, $p_i^j = v$, $p_i^m = v + \tau$ and $\pi_i^i = \frac{v+\tau}{2}$, $\pi_i^j = \frac{v}{2}$, $\pi_i^m = \frac{v+\tau}{2}$ if $\tilde{s} \geq \frac{v+\tau}{2}$;
- $p_i^i = \frac{v+\tau+2\tilde{s}}{2}$, $p_i^j = v$, $p_i^m = 2\tilde{s}$ and $\pi_i^i = \frac{(v+\tau+2\tilde{s})^2}{16\tilde{s}}$, $\pi_i^j = \frac{(6\tilde{s}-v-\tau)v}{8\tilde{s}}$, $\pi_i^m = \tilde{s}$ if $\frac{3v+\tau}{6} < \tilde{s} < \frac{v+\tau}{2}$;
- $p_i^i = 2\tilde{s} + \frac{\tau}{3}$, $p_i^j = 2\tilde{s} - \frac{\tau}{3}$, $p_i^m = 2\tilde{s}$ and $\pi_i^i = \frac{(6\tilde{s}+\tau)^2}{36\tilde{s}}$, $\pi_i^j = \frac{(6\tilde{s}-\tau)^2}{36\tilde{s}}$, $\pi_i^m = \tilde{s}$ if $\frac{\tau}{6} \leq \tilde{s} \leq \frac{3v+\tau}{6}$.

with $i \neq j \in \{A, B\}$. We use these payoffs to show the following proposition.

Proposition 15 *There is a unique value $\tilde{s}_0 \in \left(\frac{3v+\tau}{6}, \frac{v+\tau}{2}\right)$ such that the content provider multi-homes if $\tilde{s} \geq \tilde{s}_0$. Otherwise, it affiliates exclusively with one of the distributors.*

Proof. See Appendix G ■

Consistent with the baseline model, Proposition 15 shows that less downstream competition – measured as a higher average switching cost – makes multi-homing more likely. The trade-off between the two distribution modes from the perspective of industry profits is roughly similar to that highlighted in the benchmark model. Exclusivity leads to vertical differentiation, which favors distributor rent extraction, while multi-homing increases the average consumer willingness to pay for distributor services. When \tilde{s} exceeds $\frac{v+\tau}{2}$ both distributors act as monopolists and multi-homing maximizes industry profits. Otherwise, the economic value created by content is entirely captured by consumers when C multi-homes. Somewhat surprisingly, exclusivity does not necessarily occur when \tilde{s} is below $\frac{v+\tau}{2}$. The intuition here is as follows. For intermediate values of $\tilde{s} \in \left(\frac{3v+\tau}{6}, \frac{v+\tau}{2}\right)$, the *excluded* distributor fully exploits its old customers and does not respond to a variation in \tilde{s} . The *exclusive* distributor does not benefit from a less-competitive environment as \tilde{s} rises. As a result, the beneficial competition-softening effect from higher \tilde{s} can be lower under exclusivity

than under multi-homing. Proposition 15 shows that multi-homing can arise in equilibrium even if both distributors do not act exactly as monopolists in their market segment. Finally, exclusivity always occurs when neither distributor has an incentive to fully exploit old customers (i.e., when $\tilde{s} \leq \frac{3v+\tau}{6}$).

IX Heterogeneity in the value of “regular” content

In the baseline model, distributors offer services of similar quality (excluding content). We here consider distributors with different values of “regular” content. Assume that distributor i offers a quality of service v_i (excluding content), with $v_i \neq v_j$ and $i \neq j \in \{l, s\}$. We can state the following proposition.

Proposition 16 *An exclusive affiliation with distributor i is more likely to occur as v_i rises.*

Proof. See Appendix H ■

We here provide the intuition behind Proposition 16. An increase in v_i has a greater effect on expected industry profits when distributor i can monetize this quality surplus over more customers. Distributor i is more likely to attract price-sensitive customers if it has exclusivity (compared to the other types of distribution). In this sense, exclusive content-access strengthens distributor i 's ability to reach more customers from whom it can extract the added value created by higher v_i . In addition, the degree of vertical differentiation is more likely to rise with v_i if the content provider is exclusive to distributor i rather than another type of distribution.¹⁵ Overall, when the content provider is exclusive to distributor i , an increase in v_i strengthens distributor i 's ability to reach more customers and can promote vertical differentiation (compared to the other types of distribution). Therefore, higher v_i has a greater effect on expected industry profits when

¹⁵We define the degree of vertical differentiation as $z^c = |u_i^c - u_j^c|$, with $c \in \{m, i, j\}$ and $u_k^k = u_k^m = v_k + \tau$, $u_k^q = v_k$ with $k \neq q \in \{i, j\}$. Given the content-distribution choice $c \in \{m, i, j\}$, the degree of vertical differentiation increases with v_i if $dz^c/dv_i > 0$ and decreases with v_i if $dz^c/dv_i < 0$. $dz^i/dv_i = dz^m/dv_i = dz^j/dv_i = 1$ if $v_i \geq v_j + \tau$; $dz^i/dv_i = dz^m/dv_i = 1$ and $dz^j/dv_i = -1$ if $v_j + \tau \geq v_i \geq v_j$; $dz^i/dv_i = 1$ and $dz^j/dv_i = dz^m/dv_i = -1$ if $v_j \geq v_i \geq v_j - \tau$; and $dz^i/dv_i = dz^m/dv_i = dz^j/dv_i = -1$ if $v_j - \tau \geq v_i$. It follows that $dz^i/dv_i \geq dz^m/dv_i$ and $dz^i/dv_i \geq dz^j/dv_i$.

the content provider chooses exclusive affiliation with distributor i rather than another type of distribution.

X Independent captive-customer bases

In the baseline model, a distributor increases its base of locked-in customers either if its initial market share or if the mass of locked-in customers rises. In both cases, the base of locked-in customers of its rival is also altered. However, in reality, a distributor could increase its base of locked-in customers without affecting that of its rival. Let $K_l > 0$ and $K_s > 0$ be the base of locked-in customers of distributors l and s , with $K_l + K_s < 1$. We assume that $K_l \geq K_s$ and that the bases of loyal customers are not interdependent. This extension can encompass two different scenarios.

- *Heterogeneity in lock-in efficiency.* Distributors can differ in their efficiency in retaining customers (i.e., $\alpha_i \neq \alpha_j$), such that $K_l = \alpha_l \sigma$ and $K_s = \alpha_s (1 - \sigma)$. Greater efficiency in retaining customers allows a distributor to broaden its own customer base without affecting that of its rival.
- *New uncommitted consumers.* There is a unit mass of consumers, but with an inflow of new uncommitted consumers, μ , such that $\mu = 1 - (\sigma_i + \sigma_j)$ and $\sigma_i + \sigma_j \leq 1$. In this case, $K_l = \alpha \sigma_l$ and $K_s = \alpha \sigma_s$. An increase in the initial market share of one of the distributors no longer reduces the base of locked-in customers of its rival.

We can prove the following proposition.

Proposition 17 (i) *If $K_l < \frac{1}{3}$ or if $K_l \in \left(\frac{1}{3}, \frac{3-\sqrt{5}}{2}\right)$ and $K_s < K_s^-$, the content provider affiliates exclusively with distributor l if $\tau > \tau^{k_1}$ and with distributor s otherwise. If $K_l \geq \frac{1}{3}$ and $K_s \in [K_s^-, K_s^+)$, the content provider affiliates exclusively with distributor l if $\tau > \tau^{k_2}$ and multi-homes otherwise. If $K_l \geq \frac{1}{3}$ and $K_s \geq K_s^+$, the content provider multi-homes.*

(ii) *Multi-homing is more likely to occur as K_l rises. Under exclusivity, the content provider is more likely to affiliate exclusively with distributor l as K_l rises.*

Proof. See Appendix I ■

The threshold values in Proposition 17 are such that $K_s^- = \frac{K_l^2 - 3K_l + 1}{1 - 2K_l}$, $K_s^+ = \frac{2 - \sqrt{K_l} \sqrt{4 - 3K_l} - K_l}{2}$, $K_s^- = K_s^+$ if $K_l = \frac{1}{3}$, $\lim_{K_l \rightarrow 1} K_s^+ = 0$, $\tau^{k_1} = \tau^{k_2}$ if $K_s = K_s^-$, $\tau^{k_1} = \frac{(2 - K_l - K_s)v}{K_l - K_s} \left[\frac{K_l}{1 - K_s} - \frac{K_s}{1 - K_l} \right]$ and $\tau^{k_2} = \frac{(2 - K_l - K_s)(1 - K_s)v}{(1 - K_s)^2 - (2 - K_l - K_s)K_l} \left[\frac{K_l}{1 - K_s} - \frac{K_s}{1 - K_l} \right]$. Item (i) in Proposition 17 characterizes the equilibrium content-distribution strategies as a function of the distributors' loyal customer bases and content quality. The main findings of the baseline model are qualitatively unchanged. Multi-homing emerges as the equilibrium outcome when K_l and K_s are high, while the content provider prefers exclusivity when there are few locked-in customers. For intermediate values of the distributors' loyal customer bases, the content provider sells high-quality content to the *dominant* distributor and multi-homes otherwise.

An increase in K_l lowers the number of price-sensitive customers, without reducing the base of locked-in customers of the smaller distributor. Both distributors have therefore less incentive to compete for price-sensitive customers when K_l rises. It follows that the larger distributor can more easily supply the content on a larger scale, without facing tougher competition, as K_l rises. As a result, when there are few locked-in customers, the larger distributor is more likely to obtain exclusive rights to content if its base of locked-in customers rises. Compared to the baseline model, the comparative statics analysis is therefore slightly different. In the baseline model, greater asymmetry in market shares broadens the base of the larger distributor's locked-in customers at the expense of the smaller distributor, but does not affect the number of price-sensitive customers. The smaller distributor is therefore willing to offer higher net utility to attract price-sensitive customers as σ rises. As a result, greater asymmetry in market shares strengthens competition when the larger distributor has exclusive high-quality content, and therefore encourages exclusivity with the smaller rather than the larger distributor.

Finally, a rise in K_l reduces the need to resort to multi-homing to guarantee broad content diffusion. However, an increase in K_l softens competition to a broader extent if the content provider

multi-homes rather than affiliating exclusively with the larger distributor. This explains why a rise in K_l promotes multi-homing over exclusivity.

XI Initial Period - competition for market share

Our benchmark framework describes a mature market in which distributors have a base of locked-in customers and compete for content access. In this section, we study the emerging market phase in which distributors are symmetric without any installed customer base. The content provider is assumed to enter after the distributors have built their customer base and therefore no competition for content exclusivity occurs in this initial period.¹⁶ This timing may reflect the delay between the time when new distribution technologies arrive on the market and the time that complementary content are developed. This delay can be due to technical uncertainty or to technical constraints which reduce the incentives for third-party firms to develop complementary content services for new distribution technologies. For instance, during the early stages of mobile telecommunication market development, networks were designed mainly for voice and text transmission. No content such as live sporting event broadcasting, film streaming services or interactive mobile applications was available when the mobile telecoms providers started to build their customer base. When deployment of mobile broadband networks started to widespread, giving mobile telecoms providers the opportunity to offer value-added content, the mobile telecommunication market had almost reached maturity.

For ease of presentation, we refer to the initial period as the first-period (or Period 1) and the mature market phase as the second-period (or Period 2). At the start of Period 1, we assume that distributors anticipate the future entry of a content provider and its content-distribution strategy. They set their price simultaneously to maximize their expected total discounted profit (the sum of first-period and second-period profits) using a discount factor (for second-period profits) $\delta \in [0, 1]$. Consumers are assumed to be naive, meaning that they only care about their current utility.¹⁷

¹⁶Contrary to Weeds (2016), exclusivity can not be used as a tool to gain (unattached) customers during the initial market phase.

¹⁷The effects of consumers being forward-looking on the market outcome are well-understood throughout the

Following Padilla (1992), we also assume that some consumers are informed about the price offers of the two distributors while others are only informed about one of these offers in Period 1.¹⁸ While fully informed consumers buy from the lowest-priced distributor, partially informed consumers buy from the distributor about which they are informed (as long as the price does not exceed v). We note $\phi \in (0, 1)$ the mass of fully informed consumers and $(1 - \phi)/2$ the mass of consumers only informed of distributor i 's offer in Period 1.¹⁹

When both distributors quote similar first-period prices, we assume that they split the market equally (i.e., each fully informed consumer buys from either distributor with equal probability). The number of consumers buying from distributor i in Period 1 can only take one of three values: $(1 - \phi)/2$ if distributor i fails to offer the lower price, $1/2$ if both distributors offer the same price, and $(1 + \phi)/2$ if distributor i succeeds in offering the lower price. Let $\bar{c} \in \{m, s, l\}$ be the equilibrium distribution strategy in Period 2 if $\sigma = (1 + \phi)/2$ (i.e., distributors set different prices in Period 1) and $\bar{c} \in \{m, x\}$ be the equilibrium distribution strategy in Period 2 if $\sigma = 1/2$ (i.e., distributors set similar prices in Period 1), where x stands for exclusivity. Since the content-distribution strategy can be affected by the customer base asymmetry, \bar{c} and \bar{c} are not necessarily similar.

We will assume that the contracting game in Period 2 is either similar to that in the baseline model or to that in the bidding game.

switching costs literature (e.g., Klemperer, 1987b; Villas-Boas, 2006). The main result is that forward-looking customers are less price-sensitive because they anticipate that they will be locked-in after choosing a firm. Relaxing this naivety assumption would add complexity to our model and would mean that consumers are highly sophisticated. In particular, consumers would have to predict the entry of a content provider and its content-distribution strategy.

¹⁸Assuming that some customers are partially informed in Period 1 and all customers are fully informed in Period 2 may reflect the idea that in emerging markets, information is not perfectly distributed. Some customers may not be aware of the full set of products available. As information is disseminated, the number of fully informed customers increases over time. A similar analytical structure could also reflect the idea that the decision of some customers would be limited to a single distributor in the early phases of market development. Then, the variety of choices available would increase over time. For instance, in the mobile telecommunication market, many customers subscribed to the historical incumbent simply because they did not have any other choice due to limited network availability.

¹⁹If $\phi = 0$, each distributor enjoys local monopoly power in Period 1 in half of the market. It exists a unique Nash equilibrium in pure strategies in Period 1, whereby both distributors use their market power to extract the entire consumer surplus by charging a price v . Unfortunately, the overall game does not have any Nash equilibrium if $\phi = 1$. According to Padilla (1992), two ways of dealing with this problem consist of either searching for correlated equilibria or finding the equilibrium with an endogenous sharing rule.

- In the baseline model (see Section 4.3): $\Pi_s^c = \pi_s^l$ and $\Pi_l^c = \pi_l^s$ with $c \in \{m, s, l\}$.
- In the bidding game (see Section II): $\Pi_s^m = \pi_s^m - (\pi_l^l - \pi_l^m)$ and $\Pi_l^m = \pi_l^m - (\pi_s^s - \pi_s^m)$;
 $\Pi_s^s = \pi_s^s - (\pi_l^l - \pi_l^s)$ and $\Pi_l^s = \pi_l^s$; $\Pi_s^l = \pi_s^l$ and $\Pi_l^l = \pi_l^l - (\pi_s^s - \pi_s^l)$.

In both cases, the value of a large customer base in Period 2, Δ , does not depend on the content-distribution strategy. However, in the baseline model $\Delta = \pi_l^s - \pi_s^l > 0$, while in the bidding game, $\Delta = \pi_l^l - \pi_s^s$ may either be positive or negative.

Remark. When the market is split evenly, distributors are indifferent between having the exclusivity (and paying for it) and being the *excluded* distributor ($\Pi_i^i = \Pi_j^i$ with $i \neq j \in \{l, s\}$ if $\sigma = 0.5$). Let Π^x be the equilibrium expected profit in period 2 of the *excluded* and the *exclusive* distributors if the content provider sells its rights on an exclusive basis and $\sigma = 0.5$. We denote by Π^m the equilibrium expected profit in period 2 of both distributors if the content provider multi-homes and $\sigma = 0.5$. Π^x can be computed by substituting σ by $1/2$ in Π_l^l , and Π^m can be computed by substituting σ by $1/2$ in Π_l^m .

Given that $p_{i,1} \leq v$, the distributor i 's expected total discounted profit is given by

$$\Pi_i^T(p_{i,1}) = \begin{cases} \frac{1-\phi}{2}p_{i,1} + \delta\Pi_s^{\bar{c}} & \text{if } p_{i,1} > p_{j,1} \\ \frac{1}{2}p_{i,1} + \delta\Pi_l^{\bar{c}} & \text{if } p_{i,1} = p_{j,1} \\ \frac{1+\phi}{2}p_{i,1} + \delta\Pi_l^{\bar{c}} & \text{if } p_{i,1} < p_{j,1} \end{cases},$$

with $i \neq j$, $\bar{c} \in \{m, s, l\}$, $\bar{c} \in \{m, x\}$ and where $\Pi_s^{\bar{c}}$ and $\Pi_l^{\bar{c}}$ are the equilibrium expected profit in Period 2 of the smaller and larger distributors given that σ is equal to $(1 + \phi)/2$. The presence of a positive fraction of fully informed consumers (who buy from the cheaper distributor) creates tension between the distributors' incentives to price low, in order to attract these consumers, and price high, in order to extract rents from captive (partially informed) customers. Therefore, no equilibrium in pure strategies exists. However, there is an equilibrium in mixed strategies. Given the symmetric structure of the full game, it seems natural to look for a symmetric equilibrium.

Assume that both distributors use the same pricing strategy defined by an identical distribution function, G . Since no symmetric equilibrium can be characterized by pricing strategies involving mass points (we provide a formal proof of this statement in Appendix J), there is a zero probability of a tie. The expected total discounted profit of distributor i is such that

$$\Pi_i^T(p) = G(p) \underbrace{\left(\left(\frac{1-\phi}{2} \right) p + \delta \Pi_s^c \right)}_{\tilde{\Pi}^-(p)} + (1-G(p)) \underbrace{\left(\left(\frac{1+\phi}{2} \right) p + \delta \Pi_t^c \right)}_{\tilde{\Pi}^+(p)},$$

where $\tilde{\Pi}^-(p)$ is the expected total discounted profit of a distributor charging p when it fails to offer the lower first-period price, and $\tilde{\Pi}^+(p)$ is the expected total discounted profit of a distributor charging p when it succeeds in offering the lower first-period price. Focusing on customers that are solely informed of its offer, each distributor can guarantee itself a profit of $\tilde{\Pi}^-(v)$. Distributors must be indifferent between all prices in their support, hence $\Pi_i^T(p) = \tilde{\Pi}^-(v)$. This results in

$$G(p) = 1 - \frac{\tilde{\Pi}^-(v) - \tilde{\Pi}^-(p)}{\tilde{\Pi}^+(p) - \tilde{\Pi}^-(p)} = 1 - \frac{(1-\phi)(v-p)}{2\delta\bar{\Delta} + 2\phi p}. \quad (6)$$

$\bar{\Delta}$ in expression (6) is the value of a large customer base given that $\sigma = (1+\phi)/2$. $\bar{\Delta}$ can be computed by replacing σ by $(1+\phi)/2$ in Δ . We can easily check that G satisfies the conditions of a cumulative distribution function. We can state the following lemma.

Lemma 7 *There is a symmetric mixed-strategy Nash equilibrium in period 1 in which:*

- (i) *each distributor sets its price according to a common distribution function $G(p)$, as given in expression (6), with support $[p^{\min}, v]$ and*

$$p^{\min} = \frac{(1-\phi)v - 2\delta\bar{\Delta}}{1+\phi};$$

- (ii) *each distributor earns equilibrium expected total discounted profit*

$$\left(\frac{1-\phi}{2} \right) v + \delta \Pi_s^c.$$

Recall that the value of having a large customer base does not depend on the content-distribution strategy, but is affected by the quality of the content. The first-period pricing strategies are therefore affected by the future entry of a content provider but not by its distribution strategy. However, the equilibrium expected total discounted profits depend on the market structure that emerges in the second period.

First-period competition and switching costs. It is usually acknowledged that, when consumers are myopic, first-period prices are lower than they would be in a market without switching costs (e.g., Padilla, 1992; Villas-Boas, 2015). This result stems from the fact that a large customer base is valuable in the future. Hence, firms have strong incentives to build a large customer base and so compete fiercely in the early market stage. We have shown that the value of a large customer base may either be positive or negative when distributors compete for content access in the mature market phase. This depends, among other things, on the balance of bargaining power between the parties. Distributors are always better off with the largest customer base when the content provider has the entire bargaining power in Period 2. In that case, switching costs strengthens first-period price competition. However, a large customer base could be detrimental to distributors when they offer exclusivity-contingent contracts to the content provider in the mature market phase. It follows that first-period prices are not necessarily expected to be lower than they would be in a market without switching costs if distributors bid strategically for content access in Period 2.

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Appendix

A Lemma 1

In our baseline model, distributors offer services of similar quality (excluding content). We relax this assumption in Section IX. The proof is therefore built considering less restrictive assumptions than our baseline model and holds true for $v_l \neq v_s$. Let v_i be the standalone utility generated from access to distributor i (excluding content). In the baseline model, $v_l = v_s = v$, $K_l = \sigma\alpha$, $K_s = (1 - \sigma)\alpha$. Let \underline{w}_i^c and \bar{w}_i^c be the lower and upper bounds of the support of F_i^c where c is the content-distribution strategy. The existence of a mixed-strategy Nash equilibrium is proved by construction. Suppose that $\underline{w}_i^c = \underline{w}_j^c = \underline{w}^c$ and distributor j has no mass point at \underline{w}^c .

- Since $F_j^c(\underline{w}^c) = 0$, the expected profit of distributor i decreases with \underline{w}^c . By choosing $\underline{w}^c - \varepsilon$, with ε infinitely small, distributor i increases its expected profit and so $\underline{w}^c = 0$.
- The expected profit of distributor i decreases with w if $\bar{w}_j^c < w \leq \bar{w}_i^c$. Thus, there is a profitable deviation for distributor i . A similar argument holds for distributor j if $\bar{w}_i^c < w \leq \bar{w}_j^c$, which leads to $\bar{w}_i^c = \bar{w}_j^c = \bar{w}^c$.
- Next, we prove that neither distributor has a mass point w^* , such that $\underline{w}^c < w^* \leq \bar{w}^c$. By way of contradiction, assume that distributor i quotes w^* with positive probability; then by choosing $w^* + \varepsilon$, where ε is infinitely small, distributor j increases its profit. There is a profitable deviation. A contradiction. A similar argument holds if distributor j quotes w^* with positive probability.
- By choosing \underline{w}^c , distributor i can guarantee itself a profit $u_i^c K_i$. Let \hat{w}_i^c be the net utility that will guarantee for distributor i a similar profit when it succeeds in gaining also the price-sensitive customers: \hat{w}_i^c is such that $(u_i^c - \hat{w}_i^c)(1 - K_j) = u_i^c K_i$. Any $w > \hat{w}_i^c$ is a dominated strategy for distributor i , which leads to $\bar{w}^c \leq \hat{w}_i^c = \frac{(1-K_j-K_i)u_i^c}{1-K_j}$.
- In equilibrium, distributor i must be indifferent between all values in its support. It thus follows that $\bar{w}^c = \hat{w}_i^c$ and $\pi_i^c = u_i^c K_i$. By choosing \bar{w}^c , distributor j can guarantee itself a

profit $(u_j^c - \bar{w}^c)(1 - K_i) = (u_i^c K_i - (u_i^c - u_j^c)(1 - K_j)) \times \frac{1 - K_i}{1 - K_j}$, which will be its expected profit π_j^c since it must be indifferent between all values in its support.

- In equilibrium, the following condition must be satisfied

$$(u_k^c - w)(1 - K_q) F_q^c(w) + (u_k^c - w) K_k (1 - F_q^c(w)) = \pi_k^c \quad (7)$$

for $k \neq q \in \{i, j\}$. From condition (18), it follows that $F_j^c(w) = \frac{w K_i}{(1 - K_i - K_j)(u_i^c - w)}$ and $F_i^c(w) = \frac{w K_j (1 - K_j) + (1 - K_i - K_j)((1 - K_j)u_j^c - (1 - K_i)u_i^c)}{(1 - K_j)(1 - K_i - K_j)(u_j^c - w)}$. We can check that F_i^c and F_j^c are strictly increasing and continuous over the interval with lower bound 0 and upper bound \bar{w}^c , $F_j^c(\bar{w}^c) = F_i^c(\bar{w}^c) = 1$ and $F_j^c(0) = 0$.

- It remains to check that $F_i^c(0) \geq 0$. This is satisfied if

$$(1 - K_i) u_i^c \leq (1 - K_j) u_j^c. \quad (8)$$

If distributor i has a mass point at 0 (i.e. $F_i^c(0) > 0$), distributor j is always better off not quoting 0 but coming arbitrarily close to it. In this case, distributor j quotes net utilities that belong to the half open interval $(0, \bar{w}^c]$.

Let us define $\hat{\tau}^l = \frac{(1 - K_s)v_s - (1 - K_l)v_l}{1 - K_l}$, $\hat{\tau}^s = \frac{(1 - K_l)v_l - (1 - K_s)v_s}{1 - K_s}$ and $\hat{\tau}^m = \frac{(1 - K_l)v_l - (1 - K_s)v_s}{K_l - K_s}$. When we substitute i by l and j by s in the proof hereinabove (where l and s stand for larger and smaller distributors), condition (19) holds if (i) $c = m$ and $\tau \geq \hat{\tau}^m$, (ii) $c = s$ and $\tau \geq \hat{\tau}^s$ or (iii) $c = l$ and $\tau < \hat{\tau}^l$. By substituting the subscripts i by s and j by l , condition (19) holds if (iv) $c = m$ and $\tau < \hat{\tau}^m$, (v) $c = s$ and $\tau < \hat{\tau}^s$ or (vi) $c = l$ and $\tau \geq \hat{\tau}^l$. Note that in the benchmark model: $v_l = v_s$ and so $\hat{\tau}^m$ and $\hat{\tau}^s$ are lower than 0 and thus situations (iv)-(v) never occur, and $\hat{\tau}^l$ corresponds to $\hat{\tau}$.

If C multi-homes. The larger distributor quotes a net utility no higher than w with probability $F_l^m(w)$ with support $(0, \bar{w}^m]$ if $\tau < \hat{\tau}^m$ and $[0, \bar{w}^m]$ if $\tau \geq \hat{\tau}^m$; the smaller distributor quotes a net utility no higher than w with probability $F_s^l(w)$ with support $[0, \bar{w}^m]$ if $\tau < \hat{\tau}^m$ and $(0, \bar{w}^m]$ if $\tau \geq \hat{\tau}^m$, where

$$F_l^m(w) = \frac{wK_s}{(1-K_s-K_l)(v_s+\tau-w)}, \quad F_s^m(w) = \frac{wK_l(1-K_l)+(1-K_s-K_l)((1-K_l)(v_l+\tau)-(1-K_s)(v_s+\tau))}{(1-K_l)(1-K_s-K_l)(v_l+\tau-w)}$$

$$\text{and } \bar{w}^m = \frac{(1-K_l-K_s)(v_s+\tau)}{1-K_l} \text{ if } \tau < \hat{\tau}^m,$$

$$F_l^m(w) = \frac{wK_s(1-K_s)+(1-K_l-K_s)((1-K_s)(v_s+\tau)-(1-K_l)(v_l+\tau))}{(1-K_s)(1-K_l-K_s)(v_s+\tau-w)}, \quad F_s^m(w) = \frac{wK_l}{(1-K_l-K_s)(v_l+\tau-w)}$$

$$\text{and } \bar{w}^m = \frac{(1-K_s-K_l)(v_l+\tau)}{1-K_s} \text{ if } \tau \geq \hat{\tau}^m.$$

The equilibrium expected profits in Stage II are given by

$$\pi_l^m = ((v_s + \tau) K_s - (v_s - v_l) (1 - K_l)) \frac{1-K_s}{1-K_l} \text{ and } \pi_s^m = (v_s + \tau) K_s \text{ if } \tau < \hat{\tau}^m$$

$$\pi_l^m = (v_l + \tau) K_l \text{ and } \pi_s^m = ((v_l + \tau) K_l - (v_l - v_s) (1 - K_s)) \times \frac{1-K_l}{1-K_s} \text{ if } \tau \geq \hat{\tau}^m.$$

If C affiliates exclusively with the smaller distributor. The larger distributor quotes a net utility no higher than w with probability $F_l^s(w)$ with support $(0, \bar{w}^s]$ if $\tau < \hat{\tau}^s$ and $[0, \bar{w}^s]$ if $\tau \geq \hat{\tau}^s$; the smaller distributor quotes a net utility no higher than w with probability $F_s^l(w)$ with support $[0, \bar{w}^s]$ if $\tau < \hat{\tau}^s$ and $(0, \bar{w}^s]$ if $\tau \geq \hat{\tau}^s$, where

$$F_l^s(w) = \frac{wK_s}{(1-K_s-K_l)(v_s+\tau-w)}, \quad F_s^s(w) = \frac{wK_l(1-K_l)+(1-K_s-K_l)((1-K_l)v_l-(1-K_s)(v_s+\tau))}{(1-K_l)(1-K_s-K_l)(v_l-w)} \text{ and } \bar{w}^s = \frac{(1-K_l-K_s)(v_s+\tau)}{1-K_l}$$

$$\text{if } \tau < \hat{\tau}^s,$$

$$F_l^s(w) = \frac{wK_s(1-K_s)+(1-K_l-K_s)((1-K_s)(v_s+\tau)-(1-K_l)v_l)}{(1-K_s)(1-K_l-K_s)(v_s+\tau-w)}, \quad F_s^s(w) = \frac{wK_l}{(1-K_l-K_s)(v_l-w)} \text{ and } \bar{w}^s = \frac{(1-K_s-K_l)v_l}{1-K_s}$$

$$\text{if } \tau \geq \hat{\tau}^s.$$

The equilibrium expected profits in Stage II are given by

$$\pi_l^s = ((v_s + \tau) K_s - (v_s + \tau - v_l) (1 - K_l)) \frac{1-K_s}{1-K_l} \text{ and } \pi_s^s = (v_s + \tau) K_s \text{ if } \tau < \hat{\tau}^s$$

$$\pi_l^s = v_l K_l \text{ and } \pi_s^s = (v_l K_l - (v_l - v_s - \tau) (1 - K_s)) \times \frac{1-K_l}{1-K_s} \text{ if } \tau \geq \hat{\tau}^s.$$

If C affiliates exclusively with the larger distributor. The larger distributor quotes a net utility no higher than w with probability $F_l^l(w)$ with support $[0, \bar{w}^l]$ if $\tau < \hat{\tau}^l$ and $(0, \bar{w}^l]$ if $\tau \geq \hat{\tau}^l$; the smaller distributor quotes a net utility no higher than w with probability $F_s^l(w)$ with support $(0, \bar{w}^l]$ if $\tau < \hat{\tau}^l$ and $[0, \bar{w}^l]$ if $\tau \geq \hat{\tau}^l$, where

$$F_l^l(w) = \frac{wK_s(1-K_s)+(1-K_l-K_s)((1-K_s)v_s-(1-K_l)(v_l+\tau))}{(1-K_s)(1-K_l-K_s)(v_s-w)}, F_s^l(w) = \frac{wK_l}{(1-K_l-K_s)(v_l+\tau-w)} \text{ and } \bar{w}^l = \frac{(1-K_s-K_l)(v_l+\tau)}{1-K_s},$$

if $\tau < \hat{\tau}^l$,

$$F_l^l(w) = \frac{wK_s}{(1-K_s-K_l)(v_s-w)}, F_s^l(w) = \frac{wK_l(1-K_l)+(1-K_s-K_l)((1-K_l)(v_l+\tau)-(1-K_s)v_s)}{(1-K_l)(1-K_s-K_l)(v_l+\tau-w)} \text{ and } \bar{w}^l = \frac{(1-K_l-K_s)v_s}{1-K_l} \text{ if } \tau \geq \hat{\tau}^l.$$

The equilibrium expected profits in Stage II are given by

$$\pi_l^l = (v_l + \tau) K_l \text{ and } \pi_s^l = ((v_l + \tau) K_l - (v_l + \tau - v_s) (1 - K_s)) \times \frac{1-K_l}{1-K_s} \text{ if } \tau < \hat{\tau}^l$$

$$\pi_l^l = (v_s K_s - (v_s - v_l - \tau) (1 - K_l)) \frac{1-K_s}{1-K_l} \text{ and } \pi_s^l = v_s K_s \text{ if } \tau \geq \hat{\tau}^l.$$

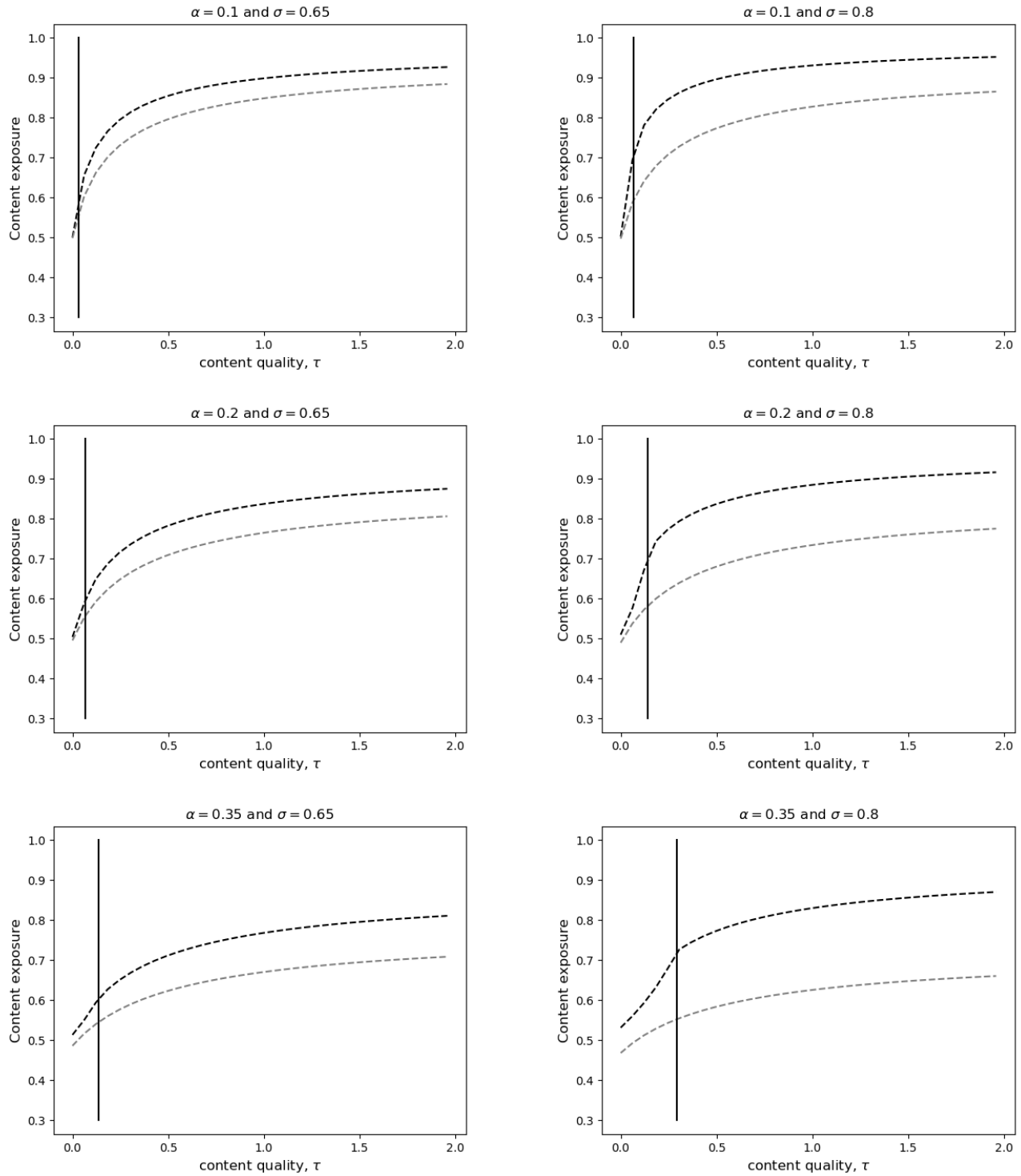
This completes the proof of Lemma 1.

B Content exposure under exclusivity

Distributor i attracts price-sensitive customers with probability $\eta_i^c = \int_0^{\bar{w}^c} F_j^c(w) f_i^c(w) dw$, with $f_i^c(w) = dF_i^c(w)/dw$, $i \neq j \in \{l, s\}$ and $c \in \{m, l, s\}$. The expected market share of distributor i is $\bar{\eta}_i^c = (1 - K_i - K_j) \times \eta_i^c + K_i$. Let $\vartheta = \bar{\eta}_l^l - \bar{\eta}_s^s$ be the difference in content exposure, according to whether the content provider affiliates exclusively with the larger or the smaller distributor. When there is no content provider, $\vartheta = \frac{K_l(K_l-K_s)}{1-K_s} \equiv \vartheta^0 \geq 0$ and the larger distributor is expected to serve a greater share of the market than its rival. On the other extreme, when τ is really high, a *large exclusive* distributor and a *small exclusive* distributor will attract price-sensitive customers with roughly similar probabilities (close to 1), and ϑ is thus close to $K_l - K_s$ (with $K_l - K_s > \vartheta^0$). However, determining analytically the relationship between ϑ and τ is far from being trivial - in particular, because this relationship can be non-monotonic. We therefore propose a numerical example to illustrate the difference in content exposure, according to whether the upstream provider affiliates exclusively with the larger or the smaller distributor. We consider two

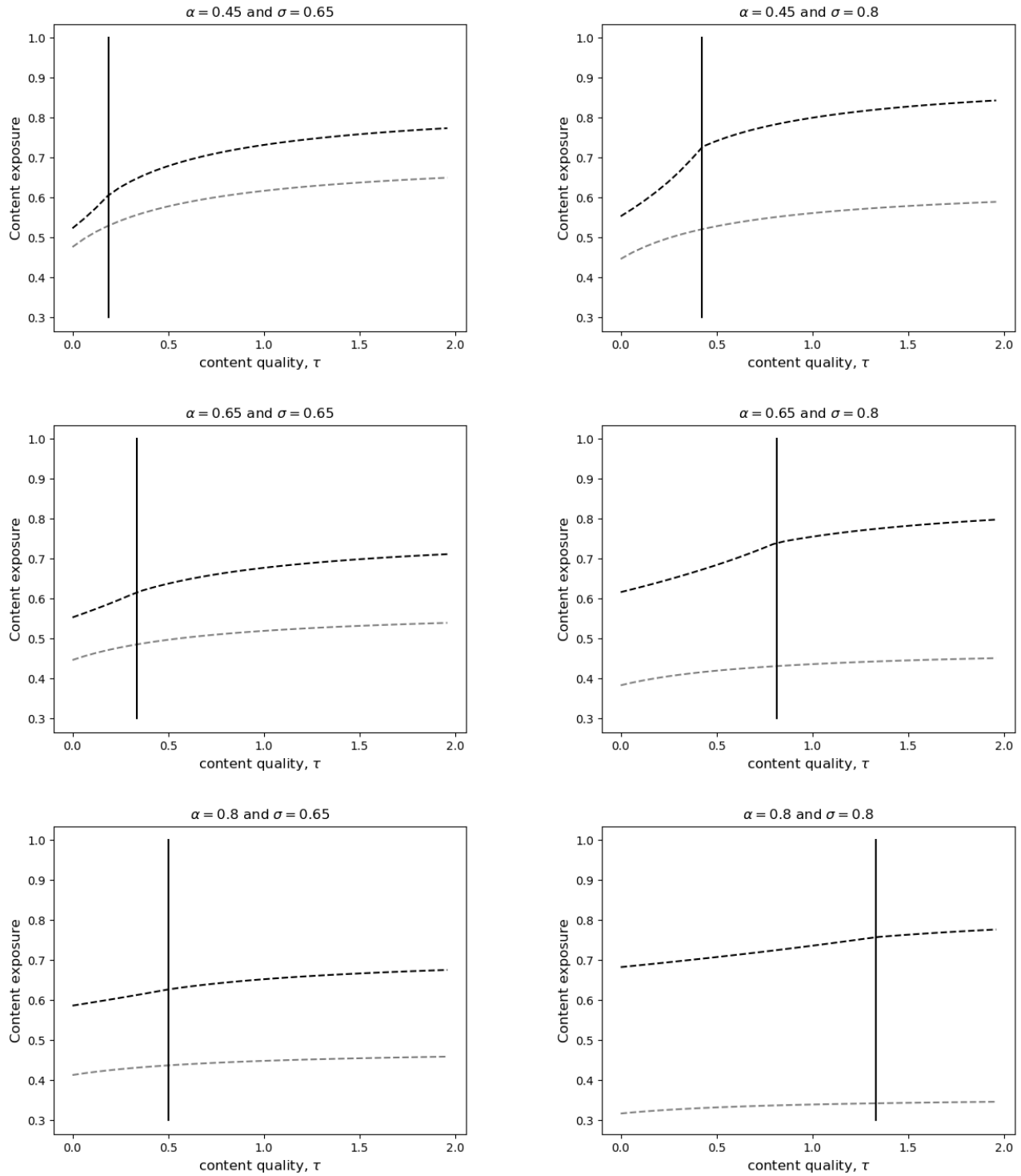
different levels of the initial market split, $\sigma \in \{0.65, 0.80\}$, and six different levels for the mass of locked-in customers, $\alpha \in \{0.1, 0.2, 0.35, 0.45, 0.65, 0.8\}$, with $v = 1$. Figure 2 and Figure 3 depict the expected market shares of a *large exclusive* distributor and of a *small exclusive* distributor as a function of the level of content quality τ (on the horizontal axis). The black-dashed line represents the expected market share of a *large exclusive* distributor, and the grey-dashed line represents the expected market share of a *small exclusive* distributor. The black-solid line represents $\hat{\tau}$. These examples highlight that content exposure is greater if the upstream provider is exclusive to the larger rather than the smaller distributor. Moreover, the loss of content exposure that comes with an exclusive affiliation with the smaller rather than the larger distributor tends to be lower when content is of low-quality.

Figure 2: Expected market share of the *exclusive* distributor



Notes: The black-dashed line represents the expected market share of a *large exclusive* distributor, and the grey-dashed line represents the expected market share of a *small exclusive* distributor. The black-solid line represents $\hat{\tau}$.

Figure 3: Expected market share of the *exclusive* distributor



Notes: The black-dashed line represents the expected market share of a *large exclusive* distributor, and the grey-dashed line represents the expected market share of a *small exclusive* distributor. The black-solid line represents $\hat{\tau}$.

C Lemma 6

This proof is based on Hagiu and Lee (2011) and is comprised of the following two lemmas.

Lemma A2. *An exclusive equilibrium in which C affiliates exclusively with distributor i always exists if $\pi_i^i + \pi_j^i \geq \pi_i^j + \pi_j^j$ for $i \neq j \in \{l, s\}$.*

Proof. Let $T_i^i = \pi_j^j - \pi_j^i$, $T_j^j = \pi_j^j - \pi_j^i$ and $T_i^m = T_j^m = -\infty$. The following necessary and sufficient conditions for equilibrium are satisfied:

- Content provider incentive compatibility. C does not want to switch to be exclusive to distributor j or to multi-home.
- Distributor incentive compatibility. Distributor j does not want to slightly increase transfer to gain the exclusivity. Distributor i does not want to lower transfer and loses the exclusivity.

This completes the proof of Lemma A2.

Lemma A3. *An equilibrium in which C multi-homes can exist only if*

$$\pi_i^m + \pi_j^m \geq \pi_i^i + \pi_j^i \text{ for } i \neq j \in \{l, s\}. \quad (9)$$

Proof. Assume that $T_i^i \geq T_j^j \geq 0$ for $i \neq j \in \{l, s\}$.

The following conditions are necessary and sufficient for equilibrium:

- Content provider incentive compatibility. C does not want to switch to be exclusive to distributor i

$$T_i^m + T_j^m \geq T_i^i. \quad (10)$$

- Distributor incentive compatibility (i). Neither distributor wants to deviate to gain the exclusivity, which can be done by setting $T_i^m = -\infty$ and $T_i^i = T_j^j$ for distributor i and by

setting $T_j^m = -\infty$ and $T_j^j = T_i^i$ for distributor j

$$\pi_i^m - T_i^m \geq \pi_i^i - T_j^j, \quad (11)$$

$$\pi_j^m - T_j^m \geq \pi_j^j - T_i^i. \quad (12)$$

- Distributor incentive compatibility (ii). Neither distributor wants to deviate and have no content

$$\pi_i^m - T_i^m \geq \pi_i^j, \quad (13)$$

$$\pi_j^m - T_j^m \geq \pi_j^i. \quad (14)$$

- Distributor incentive compatibility (iii). Neither distributor can lower its non-exclusive fee and still induce content provider to multi-home, which is equivalent to requiring that condition (10) is binding.

From now on we consider condition (10) as binding. Using conditions (11) and (12), we show that $T_j^j \geq \pi_i^i + \pi_j^j - (\pi_i^m + \pi_j^m)$ and therefore $T_j^j \geq \pi_i^i + \pi_j^j - (\pi_i^m + \pi_j^m) > 0$. Conditions (11) and (14) imply (15) and conditions (12) and (13) imply (16)

$$\pi_i^i - \pi_i^m + T_i^i - T_j^j \leq T_j^m \leq \pi_j^m - \pi_j^j, \quad (15)$$

$$\pi_j^j - \pi_j^m \leq T_i^m \leq \pi_i^m - \pi_i^i. \quad (16)$$

The profit of the content provider is higher than

$$\pi_i^i + \pi_j^j - (\pi_i^m + \pi_j^m) + T_i^i - T_j^j \quad (17)$$

Expression (17) is positive. Given that $T_i^i \geq T_j^j$, condition (9) is necessary for (15) and (16) to hold. This completes the proof of Lemma A3. Now assume that a non-exclusive equilibrium exists. The unique Pareto-undominated equilibrium for the distributors involves $T_i^i = T_j^j = T_i^m + T_j^m$,

$$T_i^m = \pi_j^j - \pi_j^m \text{ and } T_j^m = \pi_i^i - \pi_i^m.$$

D Proposition 7

For $\tau < \hat{\tau}$, $\Sigma(\tau) < 0$ if $\alpha\sigma < 0.5$. For $\tau \geq \hat{\tau}$, $\Sigma(\tau) < 0$ only if $\alpha\sigma < 0.5$ and $\tau \in [\hat{\tau}, \tau_2)$ with $\tau_2 = \frac{(1-\alpha)v}{1-\alpha\sigma}$. Therefore $\Sigma(\tau) < 0$ if $\alpha\sigma < 0.5$ and $\tau < \tau_2$, $\Sigma(\tau) \geq 0$ otherwise. This leads to Proposition 7.

E Proposition 8

Δ is continuous in τ , $\Delta^{\tau=0} > 0$, $\lim_{\tau \rightarrow \infty} \Delta > 0$ and

$$\frac{d\Delta}{d\tau} = \begin{cases} 2\sigma\alpha - 1 = \Upsilon^{inf} & \text{if } \tau < \hat{\tau} \\ \alpha(2\sigma - 1) = \Upsilon^{sup} & \text{if } \tau > \hat{\tau} \end{cases},$$

with $\Upsilon^{sup} > 0$. Let $\tilde{K}_l = \frac{2\sigma+1-\sqrt{8\sigma^2-4\sigma+1}}{2(1+\sigma)}$. For $\alpha\sigma \geq 0.5$, Υ^{inf} is positive and so $\Delta \geq 0$. For $\alpha\sigma < 0.5$, $\Upsilon^{inf} < 0$ and so the minimum value of Δ is reached at $\tau = \hat{\tau}$. If $\alpha\sigma \in [\tilde{K}_l, 0.5)$, $\Delta^{\tau=\hat{\tau}} \geq 0$ and so $\Delta \geq 0$. If $\alpha\sigma < \tilde{K}_l$, $\Delta^{\tau=\hat{\tau}} < 0$ and so there exist values τ_0 and τ_1 , with $0 < \tau_0 < \hat{\tau} < \tau_1$, such that $\Delta < 0$ if $\tau \in (\tau_0, \tau_1)$ and $\Delta \geq 0$ otherwise. Therefore $\Delta < 0$ only if $\alpha\sigma < \tilde{K}_l$ and $\tau \in (\tau_0, \tau_1)$. This completes the proof of Proposition 8.

F Equilibrium pricing when the distributors collect advertising revenue & Proposition 11

Equilibrium pricing when the distributors collect advertising revenue. Note that we do not restrict distributors to charge prices above marginal cost and therefore distributor $i \in \{l, s\}$ can potentially offer a net utility higher than u_i^c . Indeed, it can be profitable for distributors to subsidize customers with revenues from the advertising market. The existence of a mixed-strategy Nash equilibrium is proved by construction. Suppose that $\underline{w}_i^c = \underline{w}_j^c = \underline{w}^c$ and distributor j has no mass point at \underline{w}^c .

- Since $F_j^c(\underline{w}^c) = 0$, the expected profit of distributor i decreases with \underline{w}^c . By choosing $\underline{w}^c - \varepsilon$, with ε infinitely small, distributor i increases its expected profit and so $\underline{w}^c = 0$.
- The expected profit of distributor i decreases with w if $\bar{w}_j^c < w \leq \bar{w}_i^c$. Thus, there is a profitable deviation for distributor i . A similar argument holds for distributor j if $\bar{w}_i^c < w \leq \bar{w}_j^c$, which leads to $\bar{w}_i^c = \bar{w}_j^c = \bar{w}^c$.
- Next, we prove that neither distributor has a mass point w^* , such that $\underline{w}^c < w^* \leq \bar{w}^c$. By way of contradiction, assume that distributor i quotes w^* with positive probability; then by choosing $w^* + \varepsilon$, where ε is infinitely small, distributor j increases its profit. There is a profitable deviation. A contradiction. A similar argument holds if distributor j quotes w^* with positive probability.
- By choosing \underline{w}^c , distributor i can guarantee itself a profit $(u_i^c + \theta) K_i$. Let \hat{w}_i^c be the net utility that will guarantee for distributor i a similar profit when it succeeds in gaining also the price-sensitive customers: \hat{w}_i^c is such that $(u_i^c + \theta - \hat{w}_i^c)(1 - K_j) = (u_i^c + \theta) K_i$. Any $w > \hat{w}_i^c$ is a dominated strategy for distributor i , which leads to $\bar{w}^c \leq \hat{w}_i^c = \frac{(1-K_j-K_i)(u_i^c+\theta)}{1-K_j}$.
- In equilibrium, distributor i must be indifferent between all values in its support. It thus follows that $\bar{w}^c = \hat{w}_i^c$ and $\pi_i^{c,\theta} = (u_i^c + \theta) K_i$. By choosing \bar{w}^c , distributor j can guarantee itself a profit $(u_j^c + \theta - \bar{w}^c)(1 - K_i) = ((u_i^c + \theta) K_i - (u_i^c - u_j^c)(1 - K_j)) \times \frac{1-K_i}{1-K_j}$, which will be its expected profit $\pi_j^{c,\theta}$ since it must be indifferent between all values in its support.
- In equilibrium, the following condition must be satisfied

$$(u_k^c + \theta - w)(1 - K_q) F_q^c(w) + (u_k^c + \theta - w) K_k (1 - F_q^c(w)) = \pi_k^c \quad (18)$$

for $k \neq q \in \{i, j\}$. From condition (18), it follows that $F_j^c(w) = \frac{wK_i}{(1-K_i-K_j)(u_i^c+\theta-w)}$ and $F_i^c(w) = \frac{wK_j(1-K_j)+(1-K_i-K_j)((1-K_j)(u_j^c+\theta)-(1-K_i)(u_i^c+\theta))}{(1-K_j)(1-K_i-K_j)(u_j^c+\theta-w)}$. We can check that F_i^c and F_j^c are strictly increasing and continuous over the interval with lower bound 0 and upper bound \bar{w}^c , $F_j^c(\bar{w}^c) = F_i^c(\bar{w}^c) = 1$ and $F_j^c(0) = 0$.

- It remains to check that $F_i^c(0) \geq 0$. This is satisfied if

$$(1 - K_i)(u_i^c + \theta) \leq (1 - K_j)(u_j^c + \theta). \quad (19)$$

If distributor i has a mass point at 0 (i.e. $F_i^c(0) > 0$), distributor j is always better off not quoting 0 but coming arbitrarily close to it. In this case, distributor j quotes net utilities that belong to the half open interval $(0, \bar{w}^c]$. When we substitute i by l and j by s in the proof hereinabove (where l and s stand for larger and smaller distributors), condition (19) holds if (i) $c = m$, (ii) $c = s$ or (iii) $c = l$ and $\tau < \hat{\tau}^\theta = \frac{(K_l - K_s)(v + \theta)}{1 - K_l}$. By substituting the subscripts i by s and j by l , condition (19) holds if (vi) $c = l$ and $\tau \geq \hat{\tau}^\theta$. Given the content-distribution choice $c \in \{m, l, s\}$, the equilibrium expected profits in Stage II are:

$$\pi_l^{c,\theta} = \begin{cases} (v + \theta + \tau) K_l & \text{if } \tau < \hat{\tau}^\theta \text{ and } c = l \\ ((v + \theta) K_s + \tau(1 - K_l)) \times \frac{1 - K_s}{1 - K_l} & \text{if } \tau \geq \hat{\tau}^\theta \text{ and } c = l \\ (v + \theta) K_l & \text{if } c = s \\ (v + \theta + \tau) K_l & \text{if } c = m \end{cases} \quad (20)$$

$$\pi_s^{c,\theta} = \begin{cases} ((v + \theta + \tau) K_l - \tau(1 - K_s)) \times \frac{1 - K_l}{1 - K_s} & \text{if } \tau < \hat{\tau}^\theta \text{ and } c = l \\ (v + \theta) K_s & \text{if } \tau \geq \hat{\tau}^\theta \text{ and } c = l \\ ((v + \theta) K_l + \tau(1 - K_s)) \times \frac{1 - K_l}{1 - K_s} & \text{if } c = s \\ ((v + \theta + \tau) K_l) \times \frac{1 - K_l}{1 - K_s} & \text{if } c = m \end{cases} \quad (21)$$

Proof of Proposition 11. As we know from Section 4.2, the content provider chooses the form of distribution that maximizes Stage-II expected industry profits. Besides, the three forms of distribution can arise in equilibrium when there is no advertising revenue (i.e. $\theta = 0$).

$$\pi_l^{c,\theta} + \pi_s^{c,\theta} = \begin{cases} (v + \theta) K_l + [(v + \theta) K_l + \tau(1 - K_s)] \times \frac{1 - K_l}{1 - K_s} & \text{if } c = s \\ (v + \theta + \tau) K_l + [(v + \theta + \tau) K_l] \times \frac{1 - K_l}{1 - K_s} & \text{if } c = m \end{cases}.$$

$\pi_l^{m,\theta} + \pi_s^{m,\theta}$ is higher than $\pi_l^{s,\theta} + \pi_s^{s,\theta}$ if $K_l(1 - K_s) - (1 - K_l)(1 - K_l - K_s) \geq 0$. Therefore, θ does not affect the relative preference of the content distributor between an exclusive affiliation with the smaller distributor and multi-homing.

- When $\tau < \hat{\tau}^\theta$: $\pi_l^{l,\theta} + \pi_s^{l,\theta} = (v + \theta + \tau)K_l + [(v + \theta + \tau)K_l - \tau(1 - K_s)] \times \frac{1 - K_l}{1 - K_s}$. Since $\pi_l^{l,\theta} + \pi_s^{l,\theta} - (\pi_l^{m,\theta} + \pi_s^{m,\theta}) = -\tau(1 - K_l) \leq 0$, the content provider never affiliates exclusively with the larger distributor.
- When $\tau \geq \hat{\tau}^\theta$: $\pi_l^{l,\theta} + \pi_s^{l,\theta} = (v + \theta)K_s + [(v + \theta)K_s + \tau(1 - K_l)] \times \frac{1 - K_s}{1 - K_l}$. $\pi_l^{l,\theta} + \pi_s^{l,\theta}$ is higher than $\pi_l^{s,\theta} + \pi_s^{s,\theta}$ if $\tau \geq \tau^{x,\theta} = (v + \theta) \left[1 + \frac{K_l \frac{1 - K_l}{1 - K_s} - K_s \frac{1 - K_s}{1 - K_l}}{K_l - K_s} \right]$. Since $\tau^{x,\theta}$ increases in θ , an exclusive affiliation with the smaller rather than the larger distributor is more likely to occur when θ rises. $\pi_l^{l,\theta} + \pi_s^{l,\theta}$ is higher than $\pi_l^{m,\theta} + \pi_s^{m,\theta}$ if $\tau \times A_1 \geq (v + \theta) \times A_2$ with $A_1 = \frac{(1 - K_l - K_s)(1 - K_s) - K_l(1 - K_l)}{1 - K_s}$ and $A_2 = (2 - K_l - K_s) \left(\frac{K_l}{1 - K_s} - \frac{K_s}{1 - K_l} \right) \geq 0$. When $A_1 \leq 0$, expected industry profits are always higher when C multi-homes rather than affiliates exclusively with the larger distributor (this corresponds to the case where α is high). Otherwise, C is more likely to multi-home rather than affiliating exclusively with the larger distributor when θ rises.

G Proposition 15

Let $\Lambda^{m,i} = \pi_i^m + \pi_j^m - (\pi_i^i + \pi_j^i)$. $\Lambda^{m,i}$ is continuous. $\Lambda^{m,i} < 0$ if $\frac{\tau}{6} \leq \tilde{s} \leq \frac{3v + \tau}{6}$ and $\Lambda^{m,i} > 0$ if $\tilde{s} \geq \frac{v + \tau}{2}$. $\Lambda^{m,i}$ increases over the interval $(\frac{3v + \tau}{6}, \frac{v + \tau}{2})$. Therefore, there is a unique value $\tilde{s}_0 \in (\frac{3v + \tau}{6}, \frac{v + \tau}{2})$ such that $\Lambda^{m,i}$ is positive if $\tilde{s} \geq \tilde{s}_0$ and $\Lambda^{m,i}$ is negative otherwise.

H Proposition 16

Let $S_v^{i,c} = \{\tau | \Lambda_{v_i=v}^{i,c} \geq 0\}$ with $\Lambda^{i,c} = \pi_i^i + \pi_j^i - (\pi_i^c + \pi_j^c)$, $c \in \{j, m\}$ and $i \neq j \in \{l, s\}$.²⁰ We define the following conditions

$$\forall v_0 \leq v_1, S_{v_0}^{l,c} \subset S_{v_1}^{l,c} \text{ with } v_1 \leq \frac{1-K_s}{1-K_l} v_s \quad (22)$$

$$\forall v_0 \leq v_1, S_{v_0}^{s,c} \subset S_{v_1}^{s,c} \text{ with } v_0 \geq \frac{1-K_l}{1-K_s} v_l \quad (23)$$

$$\forall v_0 \leq v_1, S_{v_0}^{l,c} \subset S_{v_1}^{l,c} \text{ with } v_0 > \frac{1-K_s}{1-K_l} v_s \quad (24)$$

$$\forall v_0 \leq v_1, S_{v_0}^{s,c} \subset S_{v_1}^{s,c} \text{ with } v_1 < \frac{1-K_l}{1-K_s} v_l \quad (25)$$

with $c \in \{l, s, m\}$. $S_{v_0}^{i,j} \subset S_{v_1}^{i,j}$ and $S_{v_0}^{i,m} \subset S_{v_1}^{i,m}$ mean that the likelihood of an exclusive distribution with distributor i is either not affected or more likely to occur when v_i rises from v_0 to v_1 . Note that $S_{v_0}^{i,c} \subset S_{v_1}^{i,c}$ if $S_{v_0}^{i,c} = S_{v_1}^{i,c}$.

(i) $v_l \leq \frac{1-K_s}{1-K_l} v_s$.

- *Exclusivity with the smaller distributor vs Exclusivity with the larger distributor.*

– $\tau < \hat{\tau}^l$. $\Lambda^{l,s} = q_a \tau$, with $q_a = \frac{-K_l^2 - 3K_l K_s + 4K_l + 2K_s - 2}{1-K_s}$. Conditions (22)-(23) hold for $c = s$ and $c = l$, respectively.

– $\tau \geq \hat{\tau}^l$. $\Lambda^{l,s} = q_b \left(v_l - \frac{1-K_s}{1-K_l} v_s \right) + (K_l - K_s) \tau$ and $\Lambda_{\tau=\hat{\tau}^l}^{l,s} = -q_a \left(v_l - \frac{1-K_s}{1-K_l} v_s \right)$ with q_a defined above and $q_b = \frac{K_l^2 + 2K_l K_s - 3K_l + K_s^2 - 3K_s + 2}{1-K_s} > 0$. $q_a \geq 0$ only if $\alpha \geq \alpha_v^a = \frac{\sigma + 1 - \sqrt{5\sigma^2 - 4\sigma + 1}}{\sigma(3-2\sigma)}$. If $q_a \geq 0$, $\Lambda^{l,s} \geq 0 \forall v_l \leq \frac{1-K_s}{1-K_l} v_s$. If $q_a < 0$, there exists a unique $\bar{\tau}_0$ such that $\Lambda^{l,s}$ is positive if $\tau \geq \bar{\tau}_0$ and negative otherwise.

* $d\Lambda^{l,s}/dv_l > 0$ and so $\bar{\tau}_0$ decreases with v_l ,

* $d\Lambda^{l,s}/dv_s < 0$ and so $\bar{\tau}_0$ increases with v_s .

²⁰Appendix A provides the analytical expressions of π_i^c , as well as the threshold value $\hat{\tau}^c$.

Conditions (22)-(23) hold for $c = s$ and $c = l$, respectively.

- *Multi-homing vs Exclusivity with the larger distributor.*

- $\tau < \hat{\tau}^l$. $\Lambda^{l,m} = \tau(K_l - 1) < 0$. Condition (22) holds for $c = m$.
- $\tau \geq \hat{\tau}^l$. $\Lambda^{l,m} = q_c \left(v_l - \frac{1-K_s}{1-K_l} v_s \right) + q_d \tau$ and $\Lambda_{\tau=\hat{\tau}^l}^{l,m} = (1 - K_l) v_l - (1 - K_s) v_s < 0$, with $q_c = \frac{K_l^2 + 2K_l K_s - 3K_l + K_s^2 - 3K_s + 2}{1 - K_s} > 0$ and $q_d = \frac{K_l^2 + K_l K_s - 2K_l + K_s^2 - 2K_s + 1}{1 - K_s}$. $q_d \geq 0$ only if $\alpha \leq \alpha_v^d = \frac{1 - \sqrt{\sigma(1-\sigma)}}{1 - \sigma + \sigma^2}$. If $q_d \leq 0$, $\Lambda^{l,m} < 0 \forall v_l \leq \frac{1-K_s}{1-K_l} v_s$. If $q_d > 0$, there exists a unique $\bar{\tau}_1$ such that $\Lambda^{l,m}$ is positive if $\tau \geq \bar{\tau}_1$ and negative otherwise. Since $d\Lambda^{l,m}/dv_l > 0$, $\bar{\tau}_1$ decreases with v_l . Condition (22) holds for $c = m$.

- *Multi-homing vs Exclusivity with the smaller distributor.*

- $\forall \tau$. $\Lambda^{s,m} = \tau q_e$ with $q_e = \frac{K_l^2 + 2K_l K_s - 3K_l - K_s + 1}{1 - K_s}$. Condition (23) holds for $c = m$.

(ii) $v_l > \frac{1-K_s}{1-K_l} v_s$

- *Exclusivity with the smaller distributor vs Exclusivity with the larger distributor.*

- $\tau < \hat{\tau}^s$. $\Lambda^{l,s} = \tau \times q_f$, with $q_f = \frac{3K_l K_s - 2K_l + K_s^2 - 4K_s + 2}{1 - K_l} > 0$. Conditions (24)-(25) hold for $c = s$ and $c = l$, respectively.
- $\tau \geq \hat{\tau}^s$. $\Lambda^{l,s} = q_b \left(v_l - \frac{1-K_s}{1-K_l} v_s \right) + (K_l - K_s) \tau$, with $q_b > 0$ defined above. Conditions (24)-(25) hold for $c = s$ and $c = l$, respectively.

- *Multi-homing vs Exclusivity with the larger distributor.*

- $\tau < \hat{\tau}^m$. $\Lambda^{l,m} = q_g \tau \frac{K_l - K_s}{1 - K_l}$ with $q_g = \frac{2K_l K_s - K_l + K_s^2 - 3K_s + 1}{K_l - K_s}$. Condition (24) holds for $c = m$.
- $\tau \geq \hat{\tau}^m$. $\Lambda^{l,m} = q_c \left(v_l - \frac{1-K_s}{1-K_l} v_s \right) + q_d \tau$ and $\Lambda_{\tau=\hat{\tau}^m}^{l,m} = q_g \left(v_l - \frac{1-K_s}{1-K_l} v_s \right)$ with $q_c > 0$, q_d and q_g defined above. $q_g \geq 0$ only if $\alpha \leq \alpha_v^g = \frac{3 - 2\sigma - \sqrt{5 - 12\sigma + 8\sigma^2}}{2(1 - \sigma^2)}$, with $\alpha_v^g > \alpha_v^d$. If

$q_g < 0$ then q_d is negative, and $\Lambda^{l,m} < 0 \forall v_l > \frac{1-K_s}{1-K_l}v_s$. If $q_g \geq 0$ and $q_d \geq 0$, $\Lambda^{l,m} > 0 \forall v_l > \frac{1-K_s}{1-K_l}v_s$. If $q_g \geq 0$ and $q_d < 0$, there exists a unique $\bar{\tau}_2$ such that $\Lambda^{l,m} \geq 0$ if $\tau \leq \bar{\tau}_2$ and $\Lambda^{l,m} < 0$ otherwise. Since $d\Lambda^{l,m}/dv_l > 0$, $\bar{\tau}_2$ increases with v_l . Condition (24) holds for $c = m$.

- *Multi-homing vs Exclusivity with the smaller distributor.*

– $\tau < \hat{\tau}^s$. $\Lambda^{s,m} = -\tau(1 - K_s) < 0$. Condition (25) holds for $c = m$.

– $\hat{\tau}^s \leq \tau < \hat{\tau}^m$. $\Lambda^{s,m} = -q_c \left(v_l - \frac{1-K_s}{1-K_l}v_s \right) + q_k\tau$, $\Lambda_{\tau=\hat{\tau}^s}^{s,m} = v_s(1 - K_s) - v_l(1 - K_l) < 0$ and $\Lambda_{\tau=\hat{\tau}^m}^{s,m} = q_e \frac{1-K_l}{K_l-K_s} \left(v_l - \frac{1-K_s}{1-K_l}v_s \right)$ with $q_k = \frac{K_l^2 + K_l K_s - 2K_l + K_s^2 - 2K_s + 1}{1-K_l}$, q_e and $q_c > 0$ defined above. $q_k \geq 0$ only if $\alpha \leq \alpha_v^k \equiv \alpha_v^d$ and $q_e \geq 0$ only if $\alpha \leq \alpha_v^e = \frac{2\sigma - \sqrt{8\sigma^2 - 4\sigma + 1}}{2\sigma(2-\sigma)}$ with $\alpha_v^d > \alpha_v^e$ and α_v^d defined above. If $q_e < 0$, $\Lambda^{s,m} < 0 \forall v_s < \frac{1-K_l}{1-K_s}v_l$. If $q_e \geq 0$ then $q_k \geq 0$, there exists a unique $\bar{\tau}_3$ such that $\Lambda^{s,m} \leq 0$ if $\tau \leq \bar{\tau}_3$ and $\Lambda^{s,m} > 0$ otherwise. Since $d\Lambda^{s,m}/dv_s > 0$, $\bar{\tau}_3$ decreases with v_s . Condition (25) holds for $c = m$.

– $\tau \geq \hat{\tau}^m$. $\Lambda^{s,m} = \tau q_e$ with q_e defined above. Condition (25) holds for $c = m$.

Conditions (22) and (24) are satisfied when $c \in \{s, m\}$ and conditions (23) and (25) are satisfied when $c \in \{l, m\}$. Since $\Lambda^{i,c}$ is a piecewise continuous function with respect to τ , conditions (22)-(25) are always satisfied. This completes the proof of Proposition 16.

I Proposition 17

Let $\hat{\tau} = \frac{K_l - K_s}{1 - K_l}v$, $\Lambda^{i,c} = \pi_l^i + \pi_s^i - (\pi_l^c + \pi_s^c)$, with $c \in \{j, m\}$ and $i \neq j \in \{l, s\}$.²¹ We study the sign of $\Lambda^{i,c}$ for $(K_l, K_s) \in M^{K_l, K_s} = \{(K_l, K_s) / 0 < K_s \leq K_l, K_l + K_s < 1\}$. Recall that C multi-homes if $\Lambda^{l,m} \leq 0$ and $\Lambda^{s,m} \leq 0$, C affiliates exclusively with distributor i if $\Lambda^{i,m} > 0$ and $\Lambda^{i,j} \geq 0$, with $i \neq j \in \{l, s\}$. We define M_i with $i \in \{1, 2, 3\}$ as follows:

$$M_1^{K_l, K_s} = \{(K_l, K_s) / 0 < K_s \leq K_l \leq K_l^-\} \cup \{(K_l, K_s) / 0 < K_s \leq K_l, K_l \in (K_l^-, K_l^+), K_s < K_s^-\}$$

²¹Appendix A provides the analytical expressions of π_i^c .

$$M_2^{K_l, K_s} = \{(K_l, K_s) / 0 < K_s \leq K_l, K_l \in (K_l^-, K_l^+), K_s^- \leq K_s < K_s^+\} \\ \cup \{(K_l, K_s) / 0 < K_s \leq K_l, K_l + K_s < 1, K_l^+ \leq K_l, K_s < K_s^+\}$$

$$M_3^{K_l, K_s} = \{(K_l, K_s) / 0 < K_s \leq K_l, K_l + K_s < 1, K_l^- \leq K_l, K_s \geq K_s^+\}$$

with $K_l^- = \frac{1}{3}$, $K_l^+ = \frac{3-\sqrt{5}}{2}$, $K_s^- = \frac{K_l^2-3K_l+1}{1-2K_l}$, $K_s^+ = \frac{2-\sqrt{K_l}\sqrt{4-3K_l}-K_l}{2}$. $\mathbb{M} = \{M_1, M_2, M_3\}$ forms a partition of M . In the following proof, the parameter k_r is a function solely of K_l and K_s , $\forall r \in \{a, b, c, d, e, f, g\}$. To define whether the sign of k_r changes or not, we run numerical simulations with incremental increases of 0.01.

- *Exclusivity with the smaller distributor vs Multi-homing.* $\Lambda^{s,m} = k_a \tau$. $\forall (K_l, K_s) \in M_1$, $k_a > 0$. Otherwise $k_a \leq 0$. Exclusivity with distributor s never occurs if $(K_l, K_s) \in M_2 \cup M_3$. Multi-homing never occurs if $(K_l, K_s) \in M_1$.

It follows that when $(K_l, K_s) \in M_2 \cup M_3$, C either multi-homes or affiliates exclusively with distributor l . When $(K_l, K_s) \in M_1$, C affiliates either with distributor s or with distributor l .

- *Exclusivity with the larger distributor vs Multi-homing.*
 - Assume that $(K_l, K_s) \in M_3^{K_l, K_s}$.
 - * $\tau < \hat{\tau}$. $\Lambda^{l,m} = \tau(1 - K_l) < 0$. Multi-homing occurs.
 - * $\tau \geq \hat{\tau}$. $\Lambda^{l,m} = k_b v + k_c \tau$ with $k_b < 0$ and $k_c < 0$. Multi-homing occurs.
 - Assume that $(K_l, K_s) \in M_2^{K_l, K_s}$.
 - * $\tau < \hat{\tau}$. $\Lambda^{l,m} = \tau(1 - K_l) < 0$. Multi-homing occurs.
 - * $\tau \geq \hat{\tau}$. $\Lambda^{l,m} = k_d v + k_e \tau$ with $k_d < 0$ and $k_e > 0$. $\Lambda^{l,m}$ increases with τ and $\Lambda^{l,m} < 0$ if $\tau = \hat{\tau}$. Therefore, there exists a unique $\tau^{k_2} = \frac{(2-K_l-K_s)(1-K_s)v}{(1-K_s)^2-(2-K_l-K_s)K_l} \left[\frac{K_l}{1-K_s} - \frac{K_s}{1-K_l} \right]$

such that C multi-homes if $\tau \leq \tau^{k_2}$ and affiliates exclusively with distributor l otherwise. In addition, τ^{k_2} increases with K_l .

- *Exclusivity with the larger distributor vs Exclusivity with the smaller distributor.*

– Assume that $(K_l, K_s) \in M_1^{K_l, K_s}$.

* $\tau < \hat{\tau}$. $\Lambda^{l,s} = \tau \frac{-K_l^2 - 3K_l K_s + 4K_l + 2K_s - 2}{1 - K_s} < 0$. C affiliates exclusively with distributor s .

* $\tau \geq \hat{\tau}$. $\Lambda^{l,s} = k_f v + k_g \tau$ with $k_f < 0$ and $k_g > 0$. $\Lambda^{l,s}$ increases with τ and $\Lambda^{l,s} < 0$ if $\tau = \hat{\tau}$. Therefore, there exists a unique $\tau^{k_1} = \frac{(2 - K_l - K_s)v}{K_l - K_s} \left[\frac{K_l}{1 - K_s} - \frac{K_s}{1 - K_l} \right]$ such that C affiliates exclusively with distributor s if $\tau \leq \tau^{k_1}$ and affiliates exclusively with distributor l otherwise. In addition, τ^{k_1} decreases with K_l .

To sum up:

- when $(K_l, K_s) \in M_1^{K_l, K_s}$, C affiliates exclusively with distributor s if $\tau \leq \tau^{k_1}$ and affiliates exclusively with distributor l otherwise
- when $(K_l, K_s) \in M_2^{K_l, K_s}$, C multi-homes if $\tau \leq \tau^{k_2}$ and affiliates exclusively with distributor l otherwise
- when $(K_l, K_s) \in M_3^{K_l, K_s}$, C multi-homes

$\forall K^- \leq K^+$, $M_1^{K^+, K_s} \subset M_1^{K^-, K_s}$ and $M_3^{K^-, K_s} \subset M_3^{K^+, K_s}$. In addition, τ^{k_1} decreases with K_l if $(K_l, K_s) \in M_1^{K_l, K_s}$ and τ^{k_2} increases with K_l if $(K_l, K_s) \in M_2^{K_l, K_s}$ (with $\tau^{k_1} = \tau^{k_2}$ if $K_s = K_s^-$).

Therefore, multi-homing is more likely to occur as K_l rises. Under exclusivity, the content provider is more likely to affiliate exclusively with distributor l as K_l rises. This completes the proof of Proposition 17.

J Period 1: there are no mass points in the symmetric equilibrium strategies

Lemma A1 *There are no mass points in the symmetric equilibrium strategies.*

Proof. In any symmetric equilibrium both distributors use the same pricing strategy defined by an identical distribution function over a coincident support. Since the number of mass points has to be countable, there exists $\epsilon > 0$ such that $p - \epsilon$ is named with zero probability. The deviant distributor will increase its profit for $\epsilon > 0$ sufficiently small and no mass points in the symmetric equilibrium exists if $\frac{1+\phi}{2}p + \delta\Pi_l^{\bar{c}} > \frac{p}{2} + \delta\Pi_l^{\bar{c}}$ which leads to the following condition

$$\Psi^{\bar{c},\bar{c}}(p, \delta) = \frac{\phi}{2}p + \delta(\Pi_l^{\bar{c}} - \Pi_l^{\bar{c}}) > 0 \quad (26)$$

where \bar{c} is the equilibrium distribution strategy if the market split is symmetric and \bar{c} the equilibrium distribution strategy if the market split is asymmetric in the start of period 2.

As we know from Section 4.2, the content provider chooses the form of distribution that maximizes Stage-II expected industry profits. Let $\Lambda_\sigma^{c_i, c_j} = \pi_l^{c_i} + \pi_s^{c_i} - (\pi_l^{c_j} + \pi_s^{c_j})$ with $\sigma = \frac{1+\phi}{2}$ if $p_{i,1} \neq p_{j,1}$ and $\sigma = 0.5$ if $p_{i,1} = p_{j,1}$.

- The content provider chooses the form of distribution \bar{c} when $\sigma = \frac{1+\phi}{2}$ if $\Lambda_{\frac{1+\phi}{2}}^{\bar{c}, c_2} \geq 0$ and $\Lambda_{\frac{1+\phi}{2}}^{\bar{c}, c_3} \geq 0$, with $\bar{c} \neq c_2 \neq c_3 \in \{m, l, s\}$.
- The content provider chooses the form of distribution \bar{c} when $\sigma = 0.5$ if $\Lambda_{0.5}^{\bar{c}, c_1} \geq 0$ with $\bar{c} \neq c_1 \in \{m, x\}$.

$\Lambda_\sigma^{c_i, c_j}$ can be rewritten as follows $\Lambda_\sigma^{c_i, c_j} = v \times g^{c_i, c_j}(\frac{\tau}{v}, \sigma)$ where $g^{c_i, c_j}(\frac{\tau}{v}, \sigma) = g_1^{c_i, c_j}(\sigma) + g_2^{c_i, c_j}(\sigma) \times \frac{\tau}{v}$, $g_1^{c_i, c_j}$ and $g_2^{c_i, c_j}$ are functions solely of α and σ . Let $\Omega_g^{\bar{c}, \bar{c}}$ be the set of $y \geq 0$ such that $g^{\bar{c}, c_2}(y, \frac{1+\phi}{2}) > 0$ and $g^{\bar{c}, c_3}(y, \frac{1+\phi}{2}) > 0$, with $\bar{c} \neq c_2 \neq c_3 \in \{m, l, s\}$, and $g^{\bar{c}, c_1}(y, \frac{1}{2}) > 0$ with

$\bar{c} \neq c_1 \in \{m, x\}$,

$$\Omega_g^{\bar{c}, \bar{c}} = \left\{ y \geq 0 \mid \bar{c} \neq c_2 \neq c_3 \in \{m, l, s\}, \bar{c} \neq c_1 \in \{m, x\}, \right. \\ \left. g^{\bar{c}, c_2} \left(y, \frac{1+\phi}{2} \right) > 0, g^{\bar{c}, c_3} \left(y, \frac{1+\phi}{2} \right) > 0, g^{\bar{c}, c_1} \left(y, \frac{1}{2} \right) > 0 \right\}.$$

To prove that no mass points in the symmetric equilibrium exists, we do not need to show that condition (26) holds for every pair (\bar{c}, \bar{c}) with $\bar{c} \in \{m, l, s\}$ and $\bar{c} \in \{m, x\}$. We only have to consider the pair (\bar{c}, \bar{c}) which is such that C chooses the form of distribution \bar{c} if $\sigma = \frac{1+\phi}{2}$ and \bar{c} if $\sigma = 0.5$. Therefore, the pair (\bar{c}, \bar{c}) can be disregarded from the analysis if $\Omega_g^{\bar{c}, \bar{c}} = \emptyset$.

Lemma A1_a *There is no mass point at $p = v$ in symmetric equilibrium strategies.*

Proof. A sufficient condition for Lemma A1_a to hold is given by

$$\Psi^{\bar{c}, \bar{c}}(v, 1) > 0 \quad \forall \bar{c} \in \{m, l, s\}, \bar{c} \in \{m, x\} \text{ such that } \Omega_g^{\bar{c}, \bar{c}} \neq \emptyset. \quad (27)$$

$\Psi^{\bar{c}, \bar{c}}(v, 1) \equiv v \times f^{\bar{c}, \bar{c}}\left(\frac{\tau}{v}\right)$ with $f^{\bar{c}, \bar{c}}\left(\frac{\tau}{v}\right) = f_1^{\bar{c}, \bar{c}} + f_2^{\bar{c}, \bar{c}} \times \frac{\tau}{v}$, $f_1^{\bar{c}, \bar{c}}$ and $f_2^{\bar{c}, \bar{c}}$ are functions solely of α and ϕ . Let $\Omega_f^{\bar{c}, \bar{c}}$ be the set of $y \geq 0$ such that $f^{\bar{c}, \bar{c}}(y) < 0$, i.e., $\Omega_f^{\bar{c}, \bar{c}} = \{y \geq 0 \mid f^{\bar{c}, \bar{c}}(y) < 0\}$. Condition (27) can be rewritten as follows

$$\Omega_f^{\bar{c}, \bar{c}} \equiv \emptyset \quad \forall \bar{c} \in \{m, l, s\}, \bar{c} \in \{m, x\} \text{ such that } \Omega_g^{\bar{c}, \bar{c}} \neq \emptyset. \quad (28)$$

We can not analytically prove that (28) holds. Since $\Omega_g^{\bar{c}, \bar{c}}$ and $\Omega_f^{\bar{c}, \bar{c}}$ are functions solely of α and ϕ , we resort to numerical simulations for $\alpha \in (0, 1)$, $\phi \in (0, 1)$ with incremental increases of 0.01 and show that (28) holds.²² This complete the proof of Lemma A1_a.

Lemma A1_b *There are no mass points at $p < p_l \equiv (1 - \phi)v + 2\delta(\Pi_s^{\bar{c}} - \Pi^{\bar{c}})$ in symmetric equilibrium strategies.*

Proof. Since there is no mass point at $p = v$ in symmetric equilibrium strategies (Lemma A1_a),

²²The computer program is available from the author upon request.

each distributor can guarantee itself a profit of $\frac{1-\phi}{2}v + \delta\Pi_s^{\bar{c}}$. Assume that there exists a mass point at p . The deviant distributor will increase its profit by setting v instead of p if $\frac{1-\phi}{2}v + \delta\Pi_s^{\bar{c}} > \frac{p}{2} + \delta\Pi_s^{\bar{c}}$. This yields $p < p_l$.

Lemma A1_c *There are no mass points at $p \geq p_l$ in symmetric equilibrium strategies.*

Proof. Since $\Psi^{\bar{c},\bar{c}}$ increases in p , a sufficient condition for Lemma A1_c to hold is given by

$$\Psi^{\bar{c},\bar{c}}(p_l, \delta) > 0 \quad \forall \bar{c} \in \{m, l, s\}, \bar{c} \in \{m, x\} \text{ such that } \Omega_g^{\bar{c},\bar{c}} \neq \emptyset.$$

with $\Psi^{\bar{c},\bar{c}}(p_l, \delta) = \frac{\phi(1-\phi)}{2}v + \delta(\Pi_l^{\bar{c}} + \phi\Pi_s^{\bar{c}} - (1+\phi)\Pi_s^{\bar{c}})$. $\Psi^{\bar{c},\bar{c}}(p_l, \delta) \equiv v \times h^{\bar{c},\bar{c}}(\frac{\tau}{v})$ with $h^{\bar{c},\bar{c}}(\frac{\tau}{v}) = h_1^{\bar{c},\bar{c}} + h_2^{\bar{c},\bar{c}} \times \frac{\tau}{v}$, $h_1^{\bar{c},\bar{c}}$ and $h_2^{\bar{c},\bar{c}}$ are functions solely of α , ϕ and δ . Let $\Omega_h^{\bar{c},\bar{c}}$ be the set of $y \geq 0$ such that $h^{\bar{c},\bar{c}}(y) < 0$, i.e., $\Omega_h^{\bar{c},\bar{c}} = \{y \geq 0 \mid h^{\bar{c},\bar{c}}(y) < 0\}$. Lemma A1_c holds if

$$\Omega_h^{\bar{c},\bar{c}} \equiv \emptyset \quad \forall \bar{c} \in \{m, l, s\}, \bar{c} \in \{m, x\} \text{ such that } \Omega_g^{\bar{c},\bar{c}} \neq \emptyset. \quad (29)$$

We can not analytically prove that (29) holds. Since $\Omega_g^{\bar{c},\bar{c}}$ and $\Omega_h^{\bar{c},\bar{c}}$ are functions solely of α , ϕ and δ , we resort to numerical simulations for $\alpha \in (0, 1)$, $\phi \in (0, 1)$ and $\delta \in [0, 1]$ with incremental increases of 0.01 and show that (29) holds.²³ This complete the proof of Lemma A1_c.

Hence, there are no mass points in the symmetric equilibrium strategies. This completes the proof of Lemma A1.

²³The computer program is available from the author upon request.