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► To cite this version:

Noureddine Chabbi. Schematisation and general relativity: 1) space-time and gravitation 2) Tensors and special relativity 3) the dynamics of expansion. 2022. <hal-03515360>

HAL Id: hal-03515360

<https://hal.science/hal-03515360v1>

Preprint submitted on 6 Jan 2022

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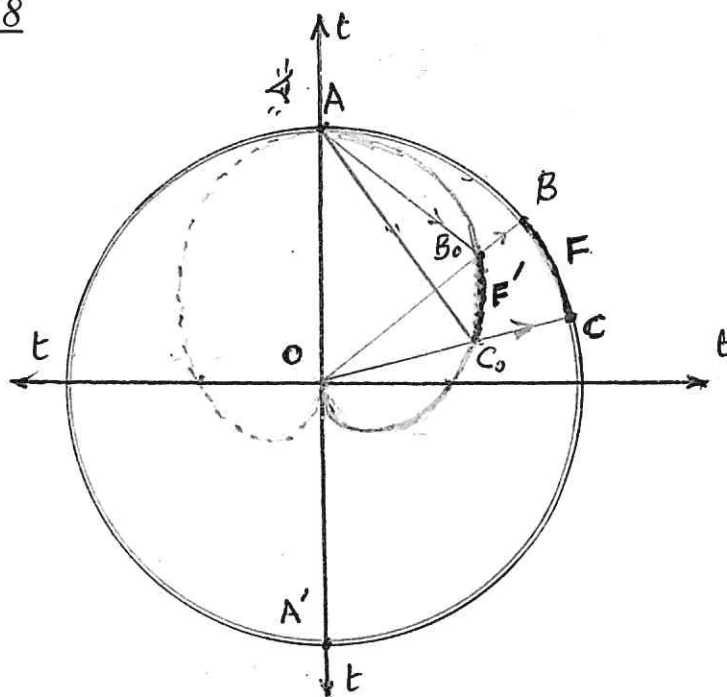
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Schematisation of General Relativity

- I) Space-time and gravitation*
- II) Tensors and Special Relativity*
- III) The Dynamics of Expansion*

Noureddine Chabbi

Tours ' (France) : November 2018



Schematisation of General Relativity

Space-time and gravitation PART I

Contents :

This work begins with a brief presentation of the transition from a Newtonian theory whose methods and approaches are based on rigid and rigorous physical laws and on a conception of a static Universe, to a scientific theory based on probability and refutation.

Newton's absoluteness and determinism no longer apply to the character based on the refutation of the achievements of scientific research.

the theory of general relativity provides the best explanation of the Universe. Thanks to this theory, the Universe has become an object of physical science; its essential feature is its simplicity and accessibility to all.

The theory of general relativity, whose main tools are the tensors, remains until further notice, the most efficient to explain the Universe and the nonrigidity of space-time. Its most important aspect is the disappearance of the concept of gravitational force, which is replaced by the concept of the curvature of space-time.

The first part is a treatise on already known ideas and scientific achievements relating to the Universe, matter and gravitation.

The second part is the most important, it is a treatise made up of diagrams and figures that support a new vision, totally different from all the classical visions. It is an attempt to popularise general relativity diagrams. In order to visualize the cosmological horizon, I do not place the earth or the solar system in the middle of the Universe, but I place them on the sphere Universe at any point (no point of the sphere has its own particularity). The proposed diagram represents the sphere (or hypersphere) space-time or simply the Universe; and for the analysis, this swelling sphere is cut by a plane passing through its centre, we obtain a disc whose edge is a large circle, this is then the place of our study in order to discern the various parts of the sphere.

of the whole sphere. A point, instead of the observer, is also taken arbitrarily on this large circle, as I have already noticed.

The points of the great circle are the singularities of space-time, they represent matter: stars, galaxies, etc... which are visible only through their images on a cardioid and all the points of the sphere are visible only on a "heart" inside this sphere (which will be called the space-time sphere). The constant speed of light means that the points of the sphere belong to the present and those of the heart, i.e. the images of these points belong to the past. The future is outside the sphere, it is not yet visible. This proposed schematisation of general relativity not only allows for the popularisation of the theory, but also tries to support and reinforce the credibility of a number of points of view and suggestions that are not always in line with current ideas and convictions.

These suggestions focus on the following points :

*The quasi-sphericity of the Universe

*The extension (the swelling) of the Universe at constant speed and not at accelerated speed and it is then demonstrated that this speed $v = C/2$

*Shifts in the real positions of galaxies and light sources in the Universe in relation to their apparent positions: the moment we look at a star in the sky, that star is no longer where we see it, it has moved away and what we see is its image when it was there or when we saw it in the past.

*The construction of a "space-time sphere" diagram, a quasi-sphere whose farthest point is located at a distance of $21.98 \cdot 10^9$ light years and whose extent is $43.96 \cdot 10^9$ light years.

*The construction of a "path of light" curve $d(t)$ used for the analysis and a core (inside the space-time) set of image points of all the points of the space-time sphere. This "heart" represents the visible Universe, image of the actual hyper-spherical non-visible Universe.

*On the difference between apparent distance and real non-apparent distance, on the shift of galaxies and on their escape velocities.

*On a new schematization of the cosmological horizon and on the crumpled Universe wave.

*The Doppler-Fizeau effect which supports and confirms the proposed schemes of the "general relativity theory".

Introduction:

The methods of the scientific process:

1) Scientific methods and uncertainties:

In the universe of knowledge everything is uncertain:

In his book "The Logic of Scientific Discovery", Karl R. Popper compares scientific theories to nets intended to capture what we call "the world", to make it rational, to explain it and to master it, and we strive to tighten the mesh more and more.

There is no absolute knowledge and scientific theories are not verifiable but only reinforced and corroborated. In science, there is no absolute truth, no perfect theory.

A theory, as I wrote above, has a life. It reigns for a while, then disappears leaving traces. It never happens that old experiments give new results. It only happens that new experiments decide against an old theory. The old theory, even if it is ousted, often retains its validity as a kind of borderline case of the new theory. It is still applicable at least to a high degree of approximation to the cases where it was successfully applied before (K.R. Popper).

For the Newtonians, a physical phenomenon must be explained by another well-determined physical cause, but the new theorists pose problems on the basis of probability; and for some, probability is a concept subjective, reflecting the scientist's ignorance of nature. The probability of a phenomenon expresses not its actual existence, but the probability of its occurrence, given the information available. We do not have enough information to be certain, so probability is linked to ignorance. But it is recognised that probability is the only way to reason about billions of particles at once. If the theory talks about probability, then it gives imperfect information about reality. And what do we know about information? Does what we know accurately reflect the reality of nature?

And is the world described by physical theories real?

We can conceive of mathematical infinity, but what is physical infinity?

Nels Bohr, one of the giants of the precursors of quantum physics, said: "There is no quantum world, there is only an abstract quantum description. It is wrong to think that the task of physics is to know what nature is. Physics describes the world in terms of what we know.

In physics, a proposition is exact or true if it is deduced from axioms in the generally accepted way. It is therefore necessary to know whether the accepted axioms are true or not. Can a system be shown to be consistent on the basis of axioms and postulates alone, i.e. on the basis of propositions accepted without demonstrations; and to impose axioms and postulates outside the system is to say that the system is incomplete in itself. This shows that there are limits to rational reasoning, and that rational reasoning alone cannot take us all the way. A coherent and non-contradictory system inevitably contains undecidable propositions, i.e. statements which can be told by logical reasoning alone whether they are true or false. Without considering the Universe as spatially homogeneous, isotropic and symmetrical on a large scale, one cannot reason. Without admitting the existence of three possible geometries for space-time, namely a Euclidean, a spherical and a hyperbolic geometry, we will never be able to solve the questions posed.

Moreover, we only know the world and quantum mechanics through the information we possess, but who says that the information we possess is complete and integral?

We apply to the Universe methods of calculation legitimated on systems included in the Universe;

is there any other way, other than the traditional methods of physics?

The observer is part of the system he or she claims to study and cannot extract himself or herself to see things from above; moreover, scientific truths depend on our senses and perceptions. We must also remember that the work on the Universe is done on a single case and there will be no possible repetitions to verify. Niels Bohr would go so far as to assert that reality is not a matter for physics. It is theoretical physics that allows the existence of black holes and still nothing proves that they really exist; to recognise them directly, one must detect the gravity fields that surround them. A black hole was finally photographed for the first time, that of the centre of the giant galaxy M 87 a 1st true image, on 10 / 04 /2019 at 15h30mn at 55 million light years by Event Horizon Telescope. Being limited, we ask questions without being able to give answers and often the questions are more important than the answers.

Richard Feynman, (1918-1988); American physicist from Newark and winner of the 1965 Nobel Prize for his thesis: "The Interactions between the Photon and the Electric Field" noted: "In physics there may well be ideas that we like and that may be wrong and those that we find strange and that may be right". It should be noted that even in human strategies, the best conceived enterprises are often foiled by the unpredictable course of history.

In conclusion, nothing is known or determined with absolute precision. In all fields, uncertainty is present and nature is buffered by stochastic and random processes.

Science often proceeds by trial and error, with innumerable errors in its wake, even among the greatest, such as Einstein, who was firmly convinced and certain that the Universe is fixed and that it has always existed and will continue to do so for all eternity, whereas it was enough for him to focus his attention and thoughts on his own equations to see that these can only explain the extension of the Universe; this was the error of my life, he said. He persisted in altering the equations of his own theory in order to preserve his idea of a static cosmos.

A search gathers clues, makes traces speak and interprets phenomena. Everything seems to be based on deduction and induction, but also on imagination, symmetry, parallelism and equivalence.

All the difficulties lie in the interpretations of the observations and information received, which push the theorists to use their imagination.

For example, one has to interpret the intense flashes of s-rays and x-rays that sometimes flood our entire solar system (late 1990s): this could be a collision of two neutron stars located a few light years from our sun. To draw conclusions based on serious plausibility calculations, we must wait to observe the phenomenon several times. Sometimes it is the great chance and from unforeseeable and unforeseen situations that science advances; it is the case of the fossil radiation which was discovered in May 1965, by the two American physicists Robert Wilson and Arno Penzias. The singularity of this fossil radiation is a miracle.

An example of a successful prediction:

In 1948, George Gamow, a Russian-American cosmologist, predicted that if the Universe is cooling due to expansion, and if a flashback to a fiery era is justified, then there must be a trace of that era today in the form of millimetre radiation, at a time when the expansion theory had just entered the scene. This radiation reveals that there was a dense, hot state that was in the form of a fluid without structure. And as mentioned above in 1965 Wilson and Penzias have this millimetre radiation for which there is no source in the universe.

Another example of a successful prediction: Louis de Broglie had stated that the association between wave fields and particles is not only confirmed for photons, but also for all physical particles. By analogy with the photon field, there is also an electron field, a neutron field, etc... These fields are associated with their respective particles in the same way as the electrostatic field is associated with photons. However, one should note the incompatibility of Einstein's theory of gravitation and the quantum mechanics described during the 1920s. The theory fails to explain microscopic gravitational phenomena and the vicinity of the starting point of the big bang.

Another theory is called "string theory":

This theory tried to integrate the requirements of general relativity and quantum theory. It postulates that the particles mediating the gravitational force emanate from the vibrations of tiny strings in a particularly twisted space-time. The exchange of these particles is what attracts two bodies to each other. While providing a framework for describing the types of forces, this theory does not describe any of them. So far, no experiments have been conducted to prove the existence of strings, so this is an idea that will not find its way into the world of science.

Another theory, that of the multiverse:

Parallel universes would be in additional dimensions of space, without contact or also with intersections of each other. If this is the case, then these universes are reduced to one since they are in the same extra space. Moreover, the ultra-precise adjustment of universal constants repeated indefinitely would no longer make sense. In short, these are speculations of the imagination that have no relation to anything but science fiction.

However, a scientific theory can be plausible, probable, very probable, or almost certain. The theory of the expansion of the universe is almost certain. But a scientific theory should not also be questioned at the slightest difficulty. We often bet on a theory, but it is never definitive, we can always revise the choice.

2) Chance and chaos :

For fear of confusing science with belief, some people do not hesitate to bring in chance as an integral part of the universe (particularly in biology). Chance cannot be a reasonable justification against nature and causes cannot be attributed to chance and unpredictable events. The lack of understanding of nature will last as long as human thought lasts and we are not supposed to understand nature completely. The science we develop and we ourselves are part of this world, so we cannot explain the whole universe and its origins.

In science, there is no absolute conviction about nature, but there are no explanations based on chance either. To opt for the interference of chance in the universe is not only a lack of knowledge, but also a lack of humility; it is as if the scientist refuses to say: "I don't know". The language of science is itself limited and our way of seeing the world and our capacity for analysis and conception are limited; they can therefore hinder and constitute obstacles to scientific endeavours. By means of fine-tuning, we can definitively rule out the idea of the intervention of chance in the Universe; the cosmological constants are determined with hallucinating precision, revealed by the telescopes in orbit of the earth (Hubble, Planck and others), which leave no room for thinking about chance. To eliminate chance is to accept that this high precision can only hide behind it a superior reflection; and it seems that we do not like this situation.

In their races, usually only one of the millions of sperm will succeed in penetrating the egg. Each one has only one chance in 10 million to be the favourite. Is it chance that controls or is there ONE already predetermined and well chosen in advance to penetrate the egg.

Are the incredible complexities and unheard of combinations of events that result in life on Earth, beauty, harmony, the emergence of intelligence, consciousness and the unity of the Universe all the result of chance?

No scientist can dispute the very precise tuning of the physical constants and initial conditions of the Universe. Is this incredible and unexampled tuning due to chance?

Or is it the result of necessity?

Astrophysicist Trinh Xuan Thuan mentions this: The astounding precision of the initial density setting of our Universe is comparable to a probability that is equivalent to an archer hitting from the earth, a 1 cm² target located at a distance of 14 billion light years by shooting only once and without knowing where the target is located.

It should be noted that everything is timed to ultimately generate life on earth. Our Universe is set on just the right frequency which has nothing to do with chance; blind nature or coincidence does not create an intelligent being and it is clear that in this Universe there is no room for coincidence and chance.

To give chance a major importance when it explains nothing is to mask a profound ignorance and ignorance is always linked to a lack of humility.

For some, anything related to spirituality is pejorative and contains a depreciatory nuance, whereas by his nature, man is also spirituality. The Universe happens to have exactly the properties required to generate an intelligent being, capable of consciousness and also of spirituality.

What is wrong if the scientist testifies in favour of monotheism and joins spirituality?

Why is it that the scientist is so stubbornly stubborn that he must remain outside our spiritual interior?

Why not accept a complementarity between science and spirituality, of course, without altering scientific research by the nonsense of beliefs?

The great French biologist, Louis Pasteur, (a deist) made the distinction: "God never comes into the laboratory with me, he always waits for me outside the door". Another line of reasoning that reinforces my beliefs taken from someone who also dismisses randomness: until further notice, we know that all motion in the Universe occurs through the interaction of the 4 forces that are :

-The electromagnetic force

-The strong nuclear force (which maintains the elements constituting the nucleus of the atom

-The weak nuclear force (between protons and electrons

-Gravitation.

These forces have surprisingly different values from each other. The famous molecular biochemist Michael Denton describes this extraordinary balance between these 4 forces as follows:

If, for example, gravitation were one billionth of a billionth of a time greater, then the universe would be smaller and its lifetime much shorter, an average star would have a mass one billionth of a time smaller than that of the sun and its lifetime would be one year at most. On the other hand, if gravity had been one billionth of a billionth of a time less powerful, no galaxies would have formed. And if, for example, the strong nuclear force had been a little weaker by a billionth of a billionth of a time, the only stable element would have been Hydrogen, and no other atoms would have existed. (ditto for the other two forces).

Is it by chance that the initial conditions of the creation of the Universe were established with such finesse that surpasses all measurements?

The fundamental constants of the Universe have been established with an incredible acuity and subtlety that leaves nothing to chance.

It is also true that man has no central place in this Universe, neither his planet, nor his sun, nor even his galaxy which does not even represent a point in this vast Universe, had any particular importance; but it seems suddenly that this Universe has exactly the properties required for this conscious man to appear!

Whoever reads me may say that I have a message to pass on! hey...well yes! and why not!...recognizing the only absolute thing: the absolute truth, that is to say GOD, the only one, is essential; the rest is secondary. It is necessary to recognize that everything is relative and nothing is absolute as proposed by Einstein's theory...except this absolute intelligence which created the Universe. the idea of the absolute, how we got it, is the proof that the absolute exists.

For the great French poet Victor Hugo, "science without conscience is the ruin of the soul"; in fact, in my opinion, a man who thinks, aims and misses the point, is a man who does not see clearly I remember the liturgical chant of a great Algerian Sufi (assayah assoufi), who was close to the divine all his life: 'I adore you, O you who hear the sound of the ant's footsteps in the dark night, you the

attentive calculator who has left nothing to chance'.

The theory of general relativity was attacked by the Marxists in the first half of the 20th century as a bourgeois ideology (Lenin's famous pamphlet against the theory of general relativity) and the theory was only recognised by the USSR late in the Second World War, thanks to its truthfulness and thanks to ($E=mc^2$) which was a real eye-opener.

In any case, the absolute time of the materialists, where they believed that the Universe has no beginning and no end and that it is absolute and independent of space, is over, just as communism is over.

II TRANSITION: From Newtonian theory to General Relativity and gravitation:

1) From Newton to Einstein

a) Newton:

1687 is the year of the publication of Isaac Newton's "Principia", which marked the beginning of a scientific revolution. From this nascent school, a synthesis between empirical and rational deductive methods was delivered. According to this Newtonian school, everything is based on physical laws. The planets, for example, obey laws and follow well-defined and determined trajectories. As soon as we know the initial conditions of the motion of a material point, we can at any moment determine its positions and speeds with precision and nothing is left to chance. The motion of bodies is determined by differential equations and the trajectories are specified in an exclusive and univocal manner.

Kant, who belongs to this school, asserted that through science, everything will be determined, explained and revealed with rigour and without any uncertainty. In particular, this classical determinism treats the universe as a static and infinite space, parallel to an independent time without beginning and without end. The existence of a disjointed, absolute and infinite space and time, obeying Euclidean geometry, was considered to be an absolutely flat universe.

In this context, any body, in a normal state without any influence, is either at rest (immobile) or in uniform rectilinear motion (Newton's 1st law).

In addition, anything that causes a change in momentum is called a force: The fundamental relation of dynamics is $f = m \dots$ (Newton's 2nd law) with m : the inertial mass of the moving material point and \dots its acceleration.

Newton also accepted that all bodies exert an attraction or gravitational force on each other $f = (k.m.m').u / d^2$, (Newton's 3rd law) with m and m' the two masses of the two material points, separated by a distance d and u is a unit vector designating the direction of the force.

The greater the distance d , the weaker the force of attraction and all bodies in the cosmos influence each other. Newton explains the state of equilibrium of a body in the following way: the force exerted on a material point (or body) would be the same in all directions. The resultant at this material point would be zero (an indeterminate compensation of the form infinity - infinity). Moreover, the cosmos has no beginning and no end and has no initial state. The initial state is always pushed further into the infinite past time.

The Newtonian theory, like all scientific theories, has tried to paint a coherent picture of the universe, but like all scientific theories, it has a life of its own. It runs into several problems of time and space and its authenticity can only be verified for speeds small compared to the speed of light. Despite its lack of breath, and its inability to fully explain nature, it remains the basis for almost all practical and technology-related calculations. For calculating forces and trajectories of shells, rockets and aeroplanes, for topographic measurements of land, for determining angles, for building constructions and bridges etc...

There is no longer any absolute that holds, the Newtonian absolute has not stood up to reality. Neither time nor space are absolute and Kant's strict determinism can no longer stand

.b) Einstein's General Relativity:

which superseded the Newtonian theory to better explain nature. This new theory therefore implies the abandonment of the concept of absolute time and absolute space. The time measured by a clock

depends on the speed of its movement. Time and space are not separate, but they are one thing: space-time. This space-time is the fabric of the universe, as we will see later. There is no direct action at a distance, even for a magnet, which according to Faraday always produces something real in the space around it, which we call a magnetic field. This field in turn acts on the piece of iron and the effects of gravitation are conceived in a similar way.

A star creates and generates a curvature in its vicinity, it is a gravitational field that acts on the objects causing the falling movement (example: the earth).

The gravitational field, in contrast to the electric and magnetic fields, has one fundamental property: bodies move under the sole influence of the gravitational field with the same constant acceleration.

Two bodies of different composition fall in exactly the same way in a gravitational field. No shield can isolate the gravitational field. For this theory, published in 1916, all matter deforms the space-time frame and the gravitational force is confused with this space-time deformation, which imposes a modification of its trajectory on the body that passes through it; planet or grain of sand, all follow the same trajectory in space-time. A surface or space of any dimension is curved, if the rules of geometry differ from those of Euclid.

It should be noted that the new theory should also lead to the same results as Newton's gravitation in the same domain where it is verified. Equivalence is therefore possible between Einstein and Newton.

The principle of equivalence: An accelerated system and a system subject to a gravitational field are equivalent, i.e. The laws of mechanics are the same in a system subjected to gravity and in an accelerated system.

Einstein goes even further by adopting the following principle: All the laws of physics, not just gravity, are locally identical in all reference systems, regardless of their relative motion or acceleration. If the dynamic effects of gravitational forces do not depend on the nature and characteristics of the bodies subjected to them, it is because these effects depend only on the nature and characteristics of the space-time in which the bodies move under the sole influence of the gravitational field and undergo the same constant acceleration. If gravitation disappears in the geometry of space-time, it is because it possesses an extraordinary property, as we have already noticed: all bodies move in exactly the same way under its influence, a 1/2 kg wooden ball and a 1 kg piece of lead, dropped from the same height, arrive at the ground at the same time (neglecting air resistance, of course). Gravitation behaves like pure acceleration, so equivalence is possible. It is therefore a relativistic theory where the equations of physics and in particular that of gravitation have the same form in all reference systems, i.e. also covariant equations.

General relativity does not impose any constraints on the global properties of space-time (it says nothing about the limits of space-time); but it does allow the representation of a space that is both finite and unbounded, escaping any 'edge' paradox. It was therefore a spherical and static Universe (the static side was later abandoned). It should be noted that in the case of the sphericity of the Universe, finiteness is acceptable, contrary to the two other cases, namely the Euclidean and the hyperbolic. The blackness of the night is a proof on which rests the finiteness of cosmic time and the constancy of the speed of light; and who says finiteness of time says also finiteness of cosmic space.

As in Newton's time, scientific theory has made since the beginning of the 20th century with Einstein, Karl Popper, Niels Bohr, Louis de Broglie, Heisenberg, and others a 2nd major turn. For Einstein, space exists because of the masses it contains, these masses give it a certain curvature. Where there is mass or energy, there is local deformation of space. It is therefore a plastic space. Time is not independent either, it deforms with matter; in fact, time and space are one and the same

thing: space-time.

According to Einstein and the theory of general relativity, space-time does not precede matter, but is born at the same time as it. There is not a space and a time in which we place masses, but it is a whole constituted by space-time and its singular points which are the masses and the energy. The latter give it its metric, i.e. its form. This determines the movements of bodies, for example, the earth, which is not subject to any apparent force, performs a movement that appears to be rectilinear and uniform, but in a curved space, such a curvature is caused by the sun, which makes the earth's movement elliptical (geodesic). The sun itself undergoes the same thing around the centre of the galaxy (the Milky Way). The curvature on the scale of the sun is probably caused by an opaque star (black hole) located at the centre of the galaxy.

According to the theory of general relativity, time passes more slowly the greater the gravity field, i.e. the closer we get to matter or energy, the slower time passes.

C.) General relativity describes two kinds of deformation of space-time:

The first: concerns the distortions due to tidal effects: The gravitational force varies inversely proportional to the square of the distance between the earth and the moon according to the

Newtonian formula: $f = k.m.m' / d^2$ locally valid formula. This force must be slightly greater on the front part of the earth, close to the moon, than on the rear part. The effect is tiny, but real, and to neglect it would be to overlook an essential aspect of gravitation. Gravitation therefore not only pulls the earth towards the moon, but also deforms it and transforms it into an ellipsoid: this is the phenomenon of tides. General relativity attributes these effects to the curvature of space-time (diagram: 1)

The second: concerns the contraction of volumes: the earth is filled with matter, which exerts an attractive force on everything on its surface: the water in the oceans, us and living beings. This force, which is directed towards the centre, contracts the earth and the reduction in its volume is proportional to the quantity of matter contained. This effect can be added to that of the tide.

The presence of matter therefore creates a local depreciation around itself.

A star like the sun causes a depreciation

A white dwarf causes a deep crater

A neutron star causes a precipice

A black hole causes a bottomless pit

It should be remembered that time passes more slowly the closer you are to a source of gravity.

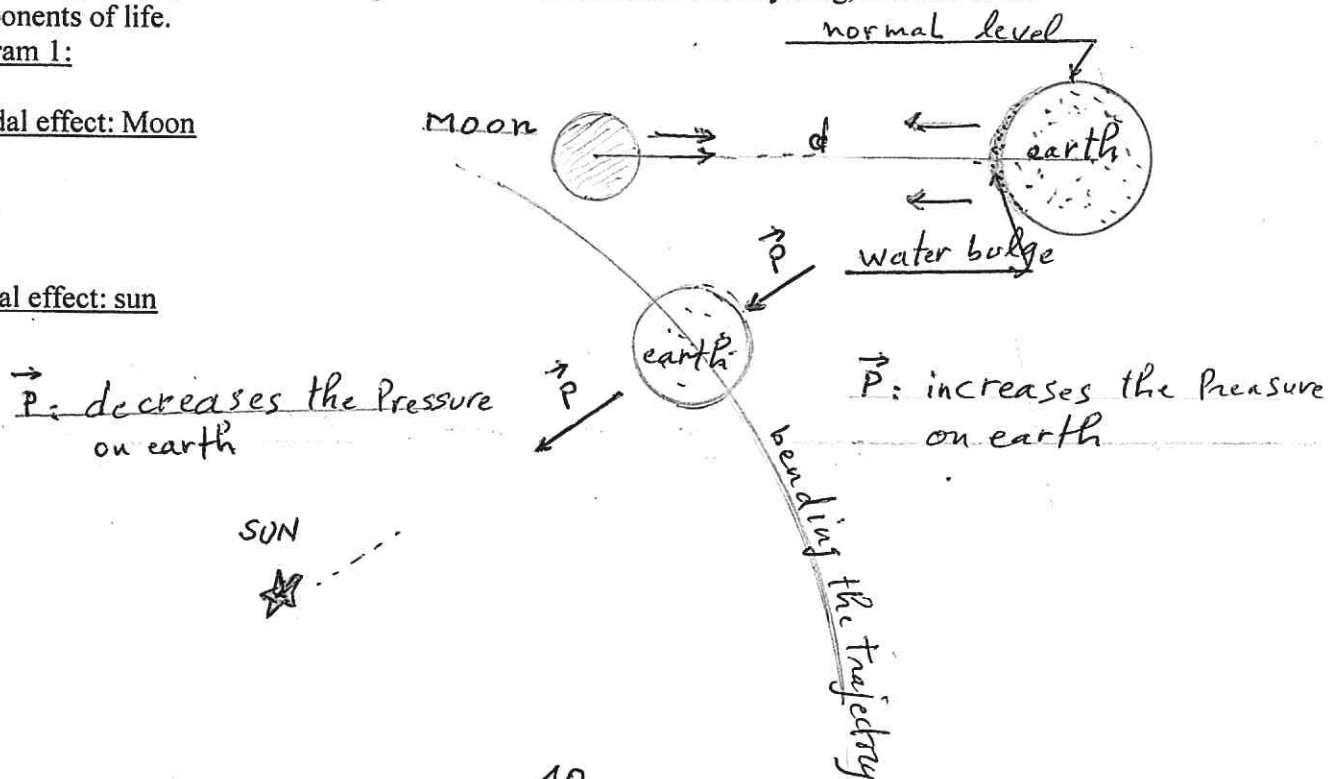
Time stops completely and space folds completely in a black hole (does space-time go into nothingness or what?).

Gravity is in us, we feel it, the day is light and the night is heavy; we sleep better at night than during the day and we work better during the day than during the night. On earth, gravitation acts on all living beings, animals and vegetation and it intervenes in everything, it is one of the components of life.

Diagram 1:

1- Tidal effect: Moon

2- Tidal effect: sun



2) The theory of general relativity and the Universe

General relativity is the ideal theoretical tool to describe the Universe. This theory describes the interaction between matter and space-time. Matter curves space-time and this curvature causes matter to move.

With the developments of Friedman and Le maitre from 1922 onwards, the relativistic equations require the Universe to be dynamic, dynamic meaning expanding or contracting, because the static model proposed at that time by Einstein is unstable (the mistake of my life he admitted).

Edwin Hubble discovered in 1925 that the Universe is vast and extends beyond our galaxy. There are billions of billions of galaxies other than ours (the Milky Way). In 1929, Hubble presented in an article, a relationship associating the red shift of the spectrum of galaxies to their distance from the solar system. The red shift of the lines of distant galaxies is the consequence of the expansion of the Universe.

Today, almost the entire scientific community believes that the Universe is expanding. The commonly accepted model is the 'hot big bang' model, described by the theory of general relativity.

Three assumptions underpin the model in question:

The first: is that the Universe is spatially homogeneous and isotropic on a large scale, since it is considered that there is no preferred direction at the time of the huge first shot of creation. This postulate will allow to constrain the geometric structure of space-time.

The second is that the theory of general relativity is the correct theory of gravitation. It is this theory that will give the dynamic aspect to the model.

There are no particular regions, no preferred directions. Wherever you look, you don't see anything special. Nor are we in a particular place compared to others. The Universe is isotropic and the major consequence of this isotropy is that it is homogeneous on a cosmological scale. In short, the Universe is considered to be homogeneous and isotropic on a large scale, and the distribution of matter (the galaxies) is also considered to be symmetrical and homogeneous. These postulates and this uniform distribution on a large scale are intended and are even estimated by theorists, because without basic hypotheses, we cannot advance in our reasoning. On the cosmological scale, the curvature of space-time is necessarily the same everywhere. It is also accepted that there are only three possible geometries for a four-dimensional space-time, three for space and one for time: open hyperbolic, closed spherical and infinite flat.

we define a quantity, the ratio between the density of the Universe and a critical density, this critical density can be the density of the apparent matter in the Universe.

So let $\Omega = (\text{density of the Universe})/(\text{critical density})$

(these geometries were established by Friedmann)

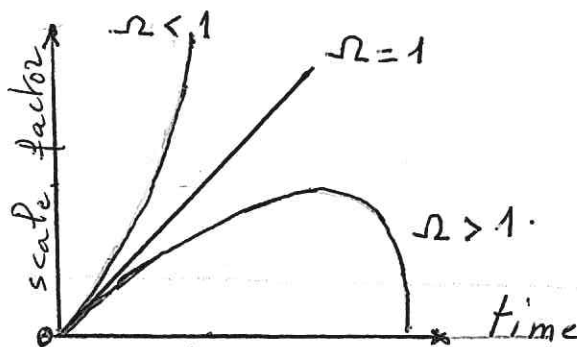
The critical density is taken to be 6 protons/m³ or 3 hydrogen atoms/m³.

If $\Omega < 1$ then the curvature is negative and we are in an open or hyperbolic universe whose volume is also infinite.

If $\Omega > 1$ then the curvature is positive and we are in a closed spherical Universe whose volume is also infinite.

If $\Omega = 1$ then there is no curvature and we are in a flat Universe, also with an infinite volume, devoid of large-scale curvature.

diagram2 :



It should be noted that the sum of the densities of dark matter and dark energy evaluated by the observers, gives a value very close to the critical density which allows ... to be very close to 1 positively (... 1). This allows us to say that we are in an almost flat or quasi-flat Universe, a conclusion that pleases theorists very much.

The third: Geometrically, general relativity assumes that there are no absolute motions, but there are motions relative to selected and supposedly immobile reference points.

The FLRW metric:

This is a mathematical formulation of : Friedman, Lemaitre, Robertson and Walker.

A metric: (definition) : Let E be a set E , provided with a distance d ; (E, d) is said to be a metric space, the distance d is related to a matrix (or tensor), this is related to the density of matter contained in E .

It is a concept from Riemann's tensor theory; it is derived from general considerations about the symmetry properties of space-time: more details in the chapter 'modelling space-time' When we pass this formulation, which is the FLRW metric, into the field equations of general relativity, we realise that the curvature of space-time varies with time.

What is the Universe in extension like?

Is it flat, closed or open?

In the FLRW metric, the curvature of spacetime is related to the density of matter. Matter, via gravitation, always tries to slow down the extension unlike the cosmological constant which plays the role of accelerating the extension (an energy which counterbalances gravitation, which we will see next).

Poisson equation :

If we pose $Df =$ the variation of the gravitational potential

and $r =$ the mass density

we can write $Df = r$

This is the Poisson equation which indicates that the more mass there is, the more the gravitational potential deepens and the more intense the acceleration of the phenomenon becomes. The similar equation between mass distribution and space-time geometry is written as follows

Curvature = Matter

This is Einstein's equation which summarises gravitation and the deformation of space-time. This is also another way of writing the fundamental equation of dynamics $f = m \cdot Y$ according to the Newtonian theory in flat space-time.

If we put $M =$ the quantity related to the mass distribution and $E(g) =$ a metric tensor function g ; there are parallels between $E(g)$ and Df and between M and r

M cannot depend only on the mass distribution since there is also energy (e) and momentum which can be transformed into mass (m), since we can write $e = mC^2$

Here we see that matter is a form of energy. So mass is not the only one responsible for the curvature of the geometry, but all forms of energy contribute to it: thermal energy, kinetic energy, radiation, pressure, potential energy stored in the gravitational field Df .

The equation: matter = curvature is the source of gravitation and geometry. The energy of a gravitational field is itself a source of gravitational field and the curvature of geometry is itself a source of curvature.

The energy-impulse tensor is represented by the quantity M and gravitation is therefore not considered as a force, but is the manifestation of the structure of space-time, made curved by the matter and energy present within it. Knowing the geometry of a surface or space means knowing how lengths, angles, areas and volumes vary on that surface or space. These quantities are then calculated from the metric tensor (see below: modelling space-time).

3) Gravitation, creative and destructive :

a) The four types of forces :

1) The weak nuclear interaction: this is a weak force that binds the electrons to the nucleus of the atom, i.e. to the protons; a radioactive force that acts within the nucleus.

2) The strong nuclear interaction: this is a colour force that binds the protons in the nucleus of the atom. It binds the (+) and (+). It glues the elementary particles together in the nucleus. It is domesticated in radiological treatment centres and in nuclear laboratories and centres.

3) The electromagnetic force: It is caused by the electric current. It ensures the solidity of matter on our scale by binding electrons to nuclei.

4) The gravitational force: It is based on a particular charge: the mass. There is no neutralization of its effects. No shield can stop it. It reigns on an astronomical scale without sharing. It is creative and destructive, and it even affects our own existence and our lives, as well as all living things (animals, insects and plants, as I noted earlier) It ensures the link of the whole cosmos: if we move a stone of 1 kg, we are sure to influence the whole Universe and to modify something in the balance of the cosmos; we must mention this even if it is in a tiny and negligible way. Gravitation is the weakest of the four forces, 10^{40} times weaker than the electromagnetic force.

b) Gravitation and the cosmos:

Since the beginning of the cosmos, gravitation has allowed gigantic tears and cracks in the clouds of gas and hydrogen and then generated condensation of islands of gas and hydrogen of dimensions amounting to billions of light years one. Within these islands, it causes further cracking and condensation to give rise to clusters of galaxies, and it is always the case that it imposes the

grouping of clusters. Gravitation then causes the galaxy to contract. When matter is drawn towards the centre, a bathor effect develops; the rotation tends to accentuate with the contraction, its diameter shrinks over time and within it, stars are born, evolve and die. Gravitation accumulates and piles up matter in galaxies, and it is always gravitation that holds them together. It agitates the nebulae in the galaxies, accumulates and concentrates the gases until they heat up to ignite the stars, and it is still the gravitational force that builds the solar systems like ours.

But its activity does not stop there, it will pull down and demolish what it has built. Once all the energy of a star is consumed, it will crash into itself and become a very dense body of degenerate matter in a small volume. Stars then turn into white dwarfs, neutron stars and sometimes a black hole (this has been observed indirectly through gravitational fields).

In our Milky Way galaxy, there seem to be billions of white dwarfs, neutron stars, black holes and other unknowns. The black holes in the vortex can join the main one in the centre to make it even heavier and larger. These degenerate stars, mentioned above, may constitute the so-called 'invisible dark matter' in our galaxy. It is even claimed that this dark matter is more present than the visible matter, constituting according to the estimates of specialists more than 90% of the total mass density of the Universe.

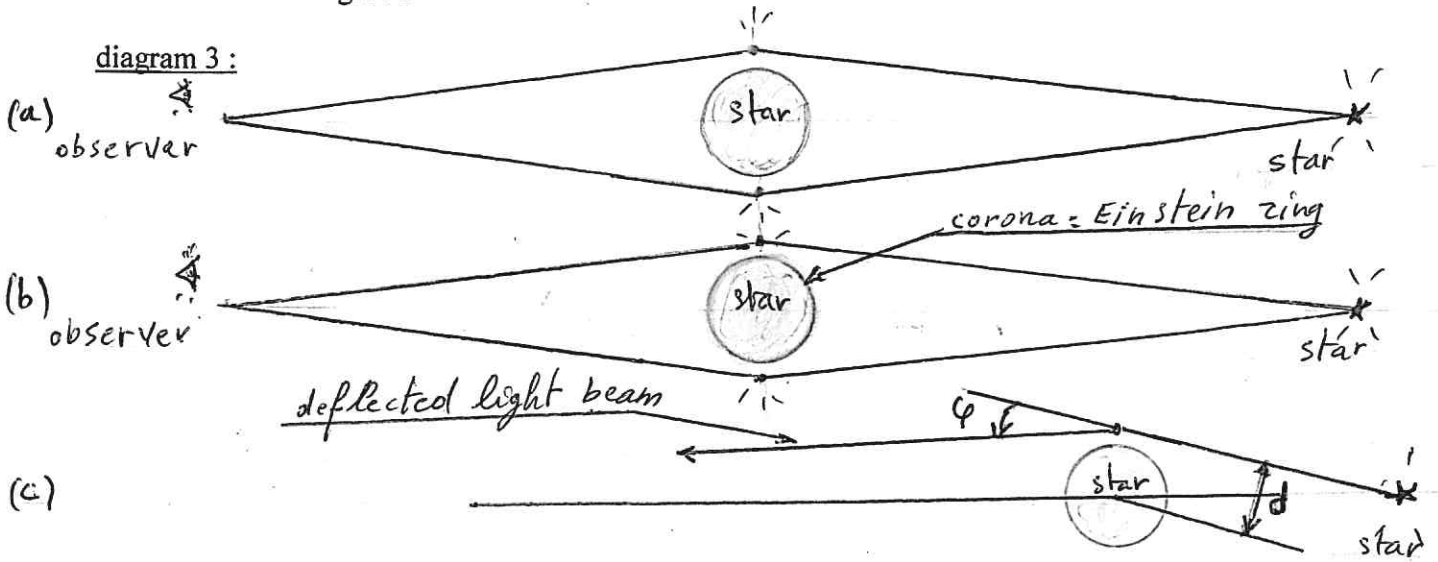
For a star, as long as the nuclear reactions persist, they keep the gravitation in check and tend to contract it; the star itself remains inflated thanks to this energy and continues to oppose the gravitation and maintain the balance. But once the nuclear energy is exhausted, it cools down, deflates and shrinks under the action of gravitation. Often, the decay enriches the Universe with stellar material and new cracks and contractions start again.

Since the beginning of time and at all times, stars, galaxies and clusters of galaxies are born and die under the effects of gravitation.

c) The effects of gravitation on light:

The Universe is full of light distortions due to the curvatures of space-time. From its source, light follows successive curvatures due to masses and energy and its trajectory will be constantly

deviated. In 1916, Einstein predicted the existence of strange mirages in the cosmos, which was later verified: see figure 3



(a) and (b): a star interposed between us and a light source can act as a gravitational lens. Not only is the light deflected by the curvature caused by the interposed star, but it is also amplified. We can observe a very distant galaxy amplified by the intermediary of a closer one when there is an alignment. This forms a corona or ring: the 'Einstein ring'.

(c): The gravitational field of a star causes light to be deflected by an angle μ

Light deflection :

Applying the method related to the equations of the Newtonian theory, the angle of deflection is:

$$\phi_n = (2G.M)/RC^2 = 0,875''$$

with M = the mass of the star (the sun); R is its radius; C is the speed of light and G is the gravitational constant.

General relativity had predicted a deviation twice as large as that of Newton's equations:

$$\phi_e = (4.G.M/RC^2) = 1.75''; \text{ see diagram 3; (c)}$$

Through the deviations of light by the phenomenon of gravitational lensing, it has been proven that the mass of a cluster of galaxies far exceeds the mass of the galaxies in the cluster. This betrays the presence of a large amount of invisible matter or dark matter. This may be degenerate matter dust, neutron stars, massive compact objects composed of neutrons, etc.

What is the nature of this dark matter?

The mass orbiting galaxies is probably several times greater than the mass of the stars that make them up. This very heavy dark matter does not seem to be sufficient to stop the expansion of the Universe. At the relatively close level, the gravitational lensing phenomenon can also be used to study the mass distribution of our galaxy. Indeed, studies of other spiral galaxies similar to ours suggest that about 90% of these galaxies are made up largely of dark matter.

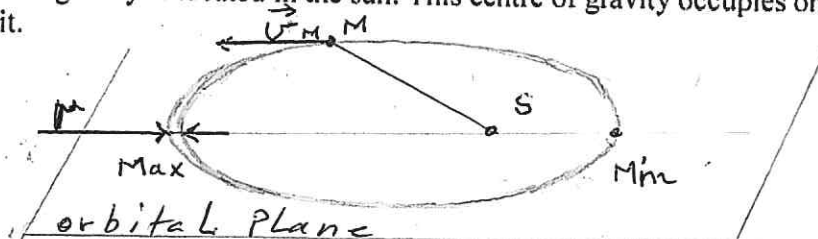
In principle, the rotational velocities of stars decrease as one moves away from the centre of a galaxy, but we often find that the opposite is true; we then conclude that the mass of the galaxy is not reduced to what is apparent and visible and we therefore arrive at the surprising conclusion: most of the mass of the Universe is invisible. To detect this matter, we could do as Einstein predicted, by the microlensing effect, which can be summarily explained as follows:

If a massive lens passes in front of a star, the star will be amplified, the duration of which depends on the mass of the massive dark object in front of it; but the small number of observed events does not yet resolve the problem of the nature of most of the dark matter. Through gravitational optics, we may be able to study the distributions of this matter, but to date we are not certain of the results and the knowledge obtained.

4) The truth of the general relativity theory:

The case of mercury: as mentioned above, there is a non-conformity between Newtonian mechanics and the theory of general relativity. According to Newton, an isolated planet revolving around a sun (like ours), describes an ellipse around the sun or more exactly around the common centre of gravity. Often this centre of gravity is located in the sun. This centre of gravity occupies one of the foci of the elliptical orbit.

Figure 4



The SM radius varies between a maximum and a minimum, but it oscillates with respect to the orbital plane in such a way that, when M makes a complete turn, it does not return to the same point. The angle described during such a period, from one perihelion to the next perihelion by the line joining S and M would be $\neq 360^\circ$ and the orbit would not be closed

According to the theory of general relativity, which deviates slightly from Newton's theory, there is a small deviation from the Kepler-Newton law of orbital motion with respect to the angle μ

described by the SM ray between one perihelion and the next. It differs from the full angle of revolution, i.e. 2π

$$\mu = \frac{24 \cdot \pi^3 a^2}{(T^2 C^2 (1 - e^2))}$$

with :

$a = 1/2$ (major axis)

$C =$ speed of light

$T =$ duration of a revolution

$e =$ eccentricity

The major axis of the ellipse rotates around the sun in the orbital direction of the planet. This rotation must reach 43 arc seconds per century for the planet Mercury; for the other planets it is negligible.

5) Analogy: Gravitation and Thermodynamics :

A striking analogy between gravitation and thermodynamics, it seems that there is a link between Einstein's gravitational equation: Matter = Curvature; and the fundamental equation of

thermodynamics: $n.R.T = P.V$ with:

n : number of moles

R : constant of perfect gases

T : temperature

P : pressure

V : volume

The quantity $P.V$ corresponds to the deformation of space under the pressure P and $n.R.T$

corresponds to energy or matter.

Perhaps there is a link between thermodynamics and gravitation? asked Ted Jacobson of the University of Maryland in 1995.

The change in the volume of gas under pressure leads to a change in the location of the molecules; they become closer to each other and this is similar to what happens to matter during the extortion of space-time: small bodies fall on top of large ones.

6) The limits of the theory of general relativity :

The theory of general relativity, which has so far succeeded in making the Universe an object of physical science, remains unable to resolve several aspects of the Universe :

_It fails to solve the problem of the edge paradox; it says nothing about the finiteness of space, i.e. whether it is finite or infinite.

_It ignores everything about the world of fluctuations; extrapolating the equations of the theory to the original point leads to absurdities, i.e. it cannot explain the world of the infinitely small. Its domain does not encompass the immutable and flat quantum space and is limited beyond the Planck wall to plus infinity. The theory of general relativity remains incompatible with the theory of quantum mechanics to make a single unitary theory.

7) The language of physics :

It is also essential to remember that to approach nature, physics has used mathematical language, it does not encounter any hindrance to advance because "mathematical tools" have always been ahead of it and the way is paved for it ,(for example, Riemann theory and differential geometry were ahead of general relativity and electromagnetism).

Physics therefore uses mathematics which is regulated by an unambiguous language. Each term used in a text has an extremely precise meaning, because mathematical language is different from

the usual language, which is in essence plural, where shifts in meaning are permanent. In short, in physics as in mathematics, the word must keep the same meaning in its different uses.

Physics gives all credit to mathematics, physics is sufficient in itself.

II) SPACE-TIME MODELLING :

1) The concept of linear variety:

The mathematical approach to general relativity is based on the concept of variety. Space-time appears to us to be regular, continuous and connected, without holes or gaps and constituting a separate space; therefore mathematically it can be modelled by a smooth 4-dimensional differential variety: a 4-dimensional pseudo-Riemannian variety.

The objective of this choice is to reflect the physical properties of space-time as well as possible. Each point M (see following diagram) of the variety is considered to belong to a neighbourhood with a coordinate system (map) as a local representation of the space-time around the observer represented by this point M . The neighbourhood of M looks locally like a 4-dimensional Euclidean space.

The map with its coordinate system describes locally the space around the point M of the variety and the region looks like a flat space as we have noticed (more details will be given in the next chapter on special relativity)

A map describes this region around the observer at M and the whole space-time, i.e. the Universe, requires a whole atlas: several maps grouped and compatible. The main tools used in the geometry of gravitation are tensor fields, defined on a pseudo-Riemannian variety representing space-time.

2) covariance principle :

The laws of physics must be expressed identically in all reference systems, inertial or not. The laws are therefore covariant by diffeomorphism (differentiable bijection whose reciprocal is also differentiable).

3) Tensor in general relativity:

(details of tensors in part 2: special relativity). In a 3 dimensional Euclidean space, we can define a metric of the form:

$$dl^2 = dx^2 + dy^2 + dz^2, \text{ relative to a matrix: } K = \text{diag}(1,1,1)$$

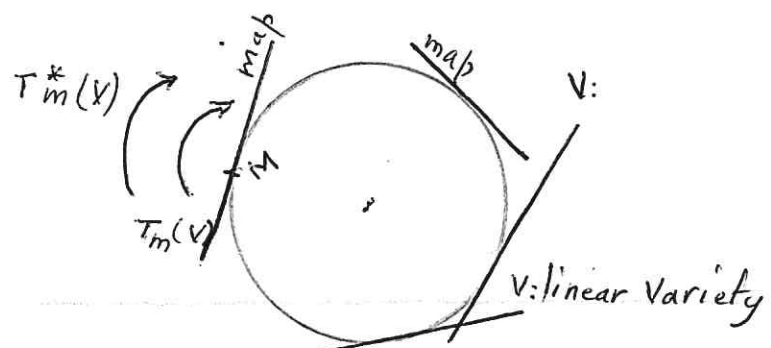
Similarly, in space-time, a metric can be defined as a measure of the infinitesimal distance between two infinitely close points (or events). We then evaluate the space-time interval between any two points by an appropriate integration. This metric generalises the Euclidean metric in 3-dimensional space mentioned above. In space-time, each product of the metric is assigned a coefficient which participates in the determination of the properties of the curvature of space-time:

$$dl^2 = \alpha dx^2 + \beta dy^2 + \gamma dz^2 + \delta dt^2$$

These coefficients ($\alpha, \beta, \gamma, \delta$) affecting the products also determine a matrix called the curve tensor and the curvature of space-time is shaped from this tensor (Riemann tensor).

To describe physical phenomena in space-time, there is, as we have noticed, no preferred reference frame; the most suitable mathematical tool is the tensor (more details in the chapter: special relativity). It is a linear operator: linear or multilinear function between 2 vector spaces: At a point $M \in V$ (V is the linear variety), one can construct a tangent space $T_m(V)$ and a contingent space $T^*m(V)$ to this linear variety V .

Diagram: 5



the contravariant vectors $\in T_m(V)$

Covectors or dual vectors: covariant vectors

... $T^*m(V)$; these are two vector spaces used to construct tensors

Tensor fields on a variety are applications :

$$\begin{array}{ccc} V & \longrightarrow & E(T) \\ T: M & \longrightarrow & T(M) \end{array}$$

With V ; the linear variety and $E(T)$ the tensor space, which associates a tensor $T(M)$ to each point M of the variety V .

Metric tensor :

The differential variety is provided with a Lorentzian metric defined by a metric tensor g which is thus a special case of the pseudo-Riemannian variety V .

The components $g(x)$ of the metric tensor vary continuously in space-time.

The metric tensor is the central object in general relativity, it describes the local geometry of space-time and is also considered as a representation of the gravitational potential.

Tensor and space-time:

We consider a 4 dimensional space: (x_1, x_2, x_3, x_4) whose last component characterises space-time and it is therefore possible to measure distances since it has a 16-component metric.

How to determine the nature of the deformation of space-time?

If we pose :

$$G(\alpha, \beta, \gamma, \delta) = \begin{pmatrix} \alpha_{11} & \beta_{12} & \gamma_{13} & \delta_{14} \\ \alpha_{21} & \beta_{22} & \gamma_{23} & \delta_{24} \\ \alpha_{31} & \beta_{32} & \gamma_{33} & \delta_{34} \\ \alpha_{41} & \beta_{42} & \gamma_{43} & \delta_{44} \end{pmatrix}$$

So what are the equations that give the 16 values $\alpha, \beta, \gamma, \delta$? What is the nature of the deformation induced in space-time by the presence of matter and energy?

Albert Einstein went even further by exploring with the mathematicians Walker and Meyer the idea that space-time was no longer subject to curvature, but also to torsion.

4) Minkowski's contribution:

Minkowski's greatest discovery was that, in order to emphasise the relationship with the 3-dimensional space continuum of Euclid's geometry, he substituted the imaginary quantity C for the ordinary time coordinate t , (C is the speed of light and i is such that $i^2 = -1$). The time coordinate thus plays exactly the same role as the 3 space coordinates. The 4 coordinates then correspond to the 3 space coordinates of Euclid's geometry and it is clear that i , an imaginary number, is the unit vector of time.

5) Formalisation of space-time :

Two examples in a Euclidean space:

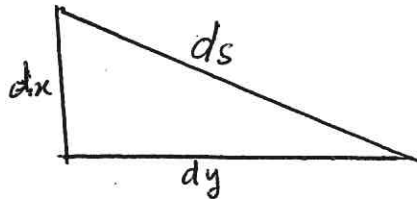
Example 1 :

a) In a Euclidean plane :

The Pythagorean theorem allows us to express the distance between two points;

Diagram 6

in Cartesian coordinates



This theorem can be written in a general form:

$$ds^2 = g_{ij}.dx_i.dy_j$$

and if g_{ij} takes the values: $g_{11} = g_{22} = 1$ and $g_{21} = g_{12} = 0$

i.e. $g = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then $ds^2 = dx^2 + dy^2$

b) Euclidean space in polar coordinates:

$(x, y) = (r, \theta)$ and $g = \begin{pmatrix} 1 & 0 \\ 0 & r^2 \end{pmatrix}$ and $ds^2 = dr^2 + r^2 d\theta$

c) Euclidean space in spherical coordinates :

$(x, y, z) = (r, \theta, \phi)$ and $g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$

and $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$

d) Euclidean space in cylindrical coordinates:

$(x, y, z) = (r, \theta, z)$ and $g = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$

Example 2 :

a) Space-time: Minkowski flat metric: it is the pseudo-Euclidean metric defined in the vicinity of point M in an empty space where the energy tensor is zero, so no source of gravitation, in a flat space-time (special relativity) in the vicinity of point M .

We have $(x, y, z, t) = (x, y, z, -Ct)$

so :

$$g = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

this matrix is associated with the metric :

$$ds^2 = dx^2 + dy^2 + dz^2 - C^2 dt^2$$

remembering that C, is the speed of light

The last component (-1) provides as we have noticed the properties of space-time. This gives us by considering dx, dy, and dz the components of the distance l (in space) :

$$s^2 = l^2 - C^2 t^2, \text{ the so-called Poincaré formula or the invariance formula, } s \text{ is the invariant interval.}$$

In ordinary space, when the distance between two points is zero, then the two points coincide, this is not the case in space-time. The interval between two points (or two events) can be zero without the two points coinciding (see diagrams in Part II, diagram n°12).

b) Space-time, Schwarz-Schild metric :

This is a particular solution of general relativity; here space is curved and the Schwarz-Schild metric describes the deformation of space-time in vacuum around a spherical mass, a star for

example.

The coordinates $(x, y, z, k) = (Ct, r, \theta, \phi)$ and

$$ds^2 = (-1 - \frac{2GM}{rC^2})C^2 dt^2 + (1 - \frac{2GM}{rC^2})^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

This metric is associated with the matrix:

$$g = \begin{pmatrix} (-1 - \frac{2GM}{rC^2}) & 0 & 0 & 0 \\ 0 & (1 - \frac{2GM}{rC^2})^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

With

G : the cosmological constant

C : the speed of light

M : the mass of the star

6) Einstein's equation:

a short presentation:

As mentioned, each point M of the linear variety is associated with a tensor which describes the local geometry around the point M. The tensor field on this linear variety is an application T :

$$M \xrightarrow{T} T(M)$$

This differential variety is endowed with a spatial metric ds² defined by the positive definite quadratic form: g (tensor)

$$ds^2 = g \cdot dx \cdot dy$$

By derivation of the tensor g we obtain the Christoffel symbols, then from these symbols we determine the Ricci tensor, which characterises the curvature of space-time. It can be considered as the Laplacian of the Riemannian metric tensor in the case of Riemannian varieties.

The Einstein equation is written as follows: $R_{ij} \cdot (1/2) \cdot g^{ij} \cdot R + A \cdot g_{ij} = K \cdot T_{ij}$, with

R_{ij} : the Ricci tensor

g_{ij} : the metric signature tensor : (+, +, +, -)

R : the scalar curvature

A : the cosmological constant introduced by Einstein to allow static solutions to the cosmological model (the mistake of my life he said). Einstein finally admitted after Hubble's confirmations of the extension of the Universe that A=0.

T_{ij} : energy-impulse tensor: it represents the distribution of matter and energy in space-time.

K : Einstein's constant $K = (8\pi G)/C^4$

G : the gravitational constant $G = 6.6742 \cdot 10^{-11} \text{kg}^{-1} \text{s}^{-2}$

C : the speed of light $C = 299792458 \text{km} \cdot \text{s}^{-1}$

Einstein's gravitational field equations exploit the link between the curvature of space-time and the distribution of mass and energy. The curvature is constructed from the Riemann tensor and the mass and energy distribution is represented by another tensor: the momentum tensor; this is written as follows: Einstein tensor = impulse energy tensor.

These equations generalise the fish equation, which in Newtonian theory relates the gravitational potential to the density of matter.

Using Einstein's tensor, and eliminating his famous constant A (his abominable error as he said), while working in geometric units or $C = G = 1$

the formula is then reduced to writing like this: $G_{ij} = 8\pi T_{ij}$;

the first term represents the mass and energy content

and the second term represents the curvature of space-time or more simply:

$$\text{MATTER} = \text{CURVATURE}$$

Formula that shows how the curvature of space-time is related to the mass and energy content.

In summary, it can be said that: tensors, equations and geodesics form the CORE of the formulation of the theory of general relativity.

7) The mathematical framework of general relativity :

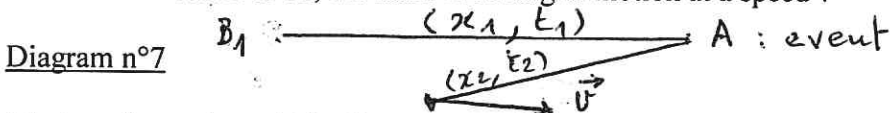
The identification between gravitation and geometry required a mathematical framework, that of a 4-dimensional space-time, with a non-Euclidean metric: the mathematics of Gauss and Riemann in the 19th century were decisive in the elaboration of the mathematical framework. It can be seen that mathematics has always preceded research in physics.

The metric used generalises the Pythagorean theorem, using coefficients that depend on the nature of the space-time in which we work. These coefficients, also called metric potentials, generalise the Newtonian potential and define the properties of the curvatures of space-time. They also determine the "tensors" of curvature which are matrices whose components are functions of these potentials. These tensors explain the links between the distribution of matter and energy and the curvature induced in space-time. These tensors generalise the Poisson equation that links gravitation to mass density in Newtonian theory.

8) The magic factor: $\sqrt{1 - v^2/C^2}$

a) The Poincaré system :

An event A (a point A), at a distance x_1 and a time t_1 from observer B_1 and at a distance x_2 and a time t_2 from observer B_2 , the latter B_2 being in motion at a speed v



The transformation which allows to pass from the measurements (x_1, t_1) , obtained by the first observer B_1 , to the measurements (x_2, t_2) obtained by the second observer B_2 is given by the Poincaré system.

$$(1) \begin{cases} x_2 = k \cdot (x_1 + \epsilon \cdot t_1) \\ t_2 = k \cdot (t_1 + \epsilon x_1) \end{cases}$$

$$\text{with } \epsilon = v/C \quad \text{and} \quad k = \frac{1}{\sqrt{1 - v^2/C^2}}$$

A trigonometric hyperbola can be used instead of a trigonometric circle to convert the system (1) into a rotation of space-time

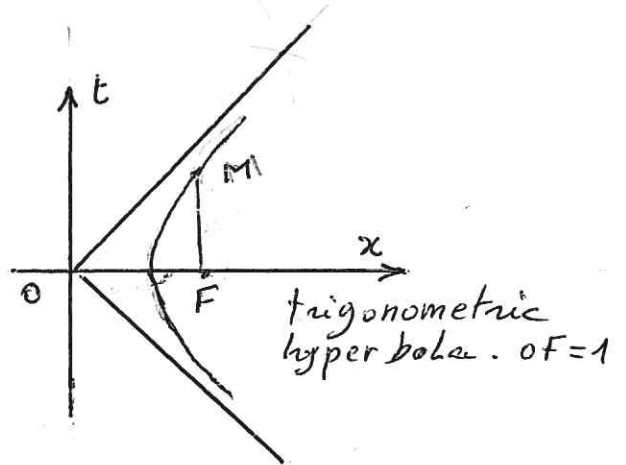
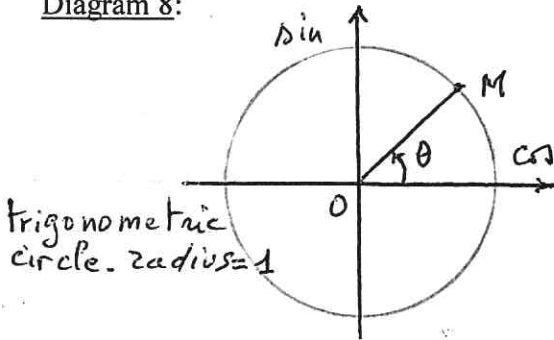
whose angle is given by the formulas: $\text{ch}^2 \theta = k$ and $\text{sh}^2 \theta = \epsilon \cdot k$

which satisfy the relation : $\text{ch}^2 \theta - \text{sh}^2 \theta = 1$

We thus obtain the system :

$$2) \begin{cases} x_2 = \text{ch} \theta x_1 + \text{sh} \theta t_1 \\ t_2 = \text{sh} \theta x_1 + \text{ch} \theta t_1 \end{cases}$$

Diagram 8:



b) The Lorentz transformation :

The world we live in is a 4-dimensional space-time continuum. The theory of general relativity has made the conception of the 4-dimensional world normal, because time is no longer independent as shown by the equations of the Lorentz transformation system

Let $M(x, y, z, t)$ and $M'(x', y', z', t')$

$$(1) \begin{cases} t' = (t - v/C^2) \cdot x / \sqrt{1 - v^2/C^2} \\ x' = (x - vt) / \sqrt{1 - v^2/C^2} \\ y' = y \\ z' = z \end{cases}$$

But if we take as basis the law of classical mechanics with absolute time, we obtain the Galilean transformation :

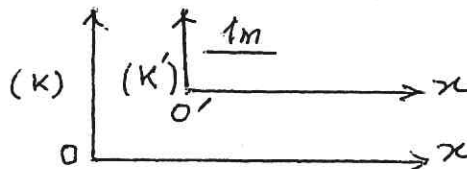
$$(2) \begin{cases} x' = x - v \cdot t \\ y' = y \\ z' = z \\ t' = t \end{cases}$$

c) graduated ruler and clock :

It is shown that any body moving at speed v undergoes a shortening in the direction of its movement, proportional to the factor $\sqrt{1 - v^2/C^2}$

Space-time considered as a graduated ruler plus a clock, the two are one thing. The length of a ruler of 1 metre which moves at speed v in the direction of its length with respect to the system (K) is equal to $\sqrt{1 - v^2/C^2}$

diagram 9



In motion the ruler is shorter than it is at rest and if $v = C$ (almost impossible) then $l = 0$

It is very difficult to define distances in cosmology, because these distances vary over time, due to the expansion of the Universe (expansion at a certain $\ln(K')$, a clock at point o such that $x'=0$, $t'_0=0$, $t'_1=1$, i.e. two successive beats of this clock between 0 seconds and 1 second. The Lorentz equations give the following results for these two beats:

With respect to (K): $t_0 = 0$ and $t_1 = 1 / \sqrt{1 - v^2/C^2}$

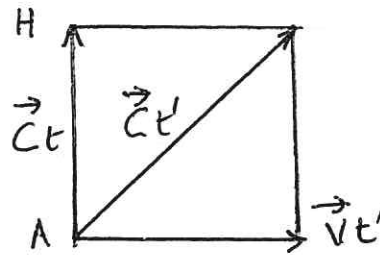
Here the clock is animated by a speed relative to the system (K) and the interval between the two beats is no longer 1 second but $1 \text{ second} \times 1 / \sqrt{1 - v^2/C^2}$.

d) Another explanation based on the movement of light :

A light point A in motion emits light which arrives on a screen (H), parallel to the motion of point A;

Diagram n° 10

we have $C^2 t'^2 = C^2 t^2 + v^2 t'^2$
 $C^2 t^2 = C^2 t'^2 + v^2 t'^2$



which leads to $t = t' \sqrt{1 - v^2/C^2}$ or $t' = t / \sqrt{1 - v^2/C^2}$

It can be seen that the speed of light plays an important role in space-time. It also plays a role as a limiting speed for the Lorentz equations, because if $C < v$ then the equations no longer make sense.

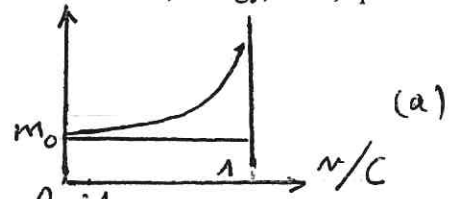
9) Relationships between: mass, energy, time, speed and volume:

Diagram n°11 : (all 7 diagrams)

To remind, we summarize on the following graphs the links between mass, energy, time, speed and volume (this is a small reminder) :

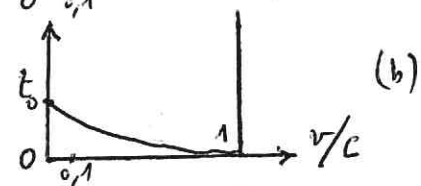
• a) Mass and speed: $m' = m_0 / \sqrt{1 - v^2/C^2}$

The mass increases with the speed
 C the speed of light is a limiting case



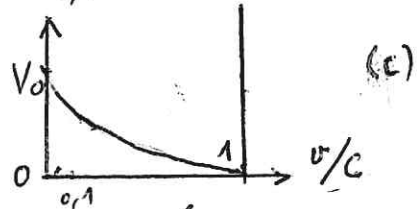
• b) Time and velocity: $t' = t_0 \sqrt{1 - v^2/C^2}$

Time decreases with speed



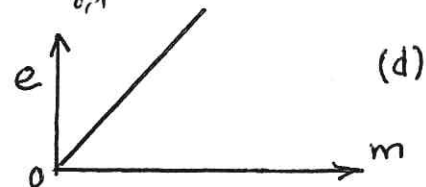
• c) Volume and velocity: $v' = v_0 \sqrt{1 - v^2/C^2}$

Like time, volume (space) shrinks with speed
 time and space being mixed, in in motion, they shrink with speed.



• d) Mass and energy: $e = m \cdot C^2$.

e = energy and m = mass



• e) Time and mass (see void below)

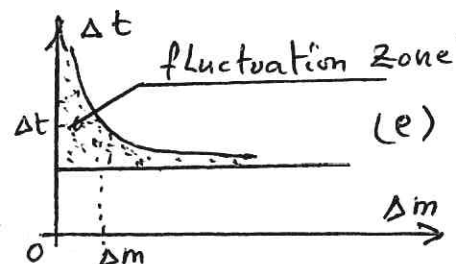
Heisenberg's principle :

Let ΔE = uncertainty of the energy of a particle and

Δt = uncertainty of the lifetime of this particle.

The principle is written :

h : is Planck's constant = $6.62 \cdot 10^{-24}$ joules/s



The vacuum pulls the strings: In the world of the infinitely small during a fluctuation, the vacuum lends the energy necessary for the creation of particles for an extremely brief moment, then, the particles return to the vacuum giving back the borrowed energy. But the product of the lifetime Δt of the particles by the energy given up

ΔE cannot exceed $h/4\pi$, i.e.

$$\Delta E \cdot \Delta t < (h/4\pi)$$

$$\Delta E = \Delta m C^2 \text{ and } \Delta E \cdot \Delta t = \Delta m \cdot C^2 \cdot \Delta t < (h/4\pi)$$

$$\text{then: } \Delta t < (h/4\pi C^2 \cdot \Delta m)$$

$$\Delta t < (\epsilon h/\Delta m) \text{ with } \epsilon = 1/4\pi C^2, \Delta t \Delta m < \epsilon h. \text{ if } m \rightarrow +\infty, \Delta t \rightarrow 0$$

It is obvious that if mass increases, space-time bends and therefore time tends to shrink

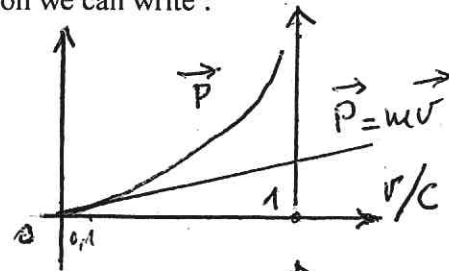
in another way, from the two equalities $t' = t_0 \sqrt{1 - v^2/C^2}$ and $m' = m_0/\sqrt{1 - v^2/C^2}$, we can write that $m' \cdot t' = m_0 \cdot t_0$ thus

$m \cdot t = \text{constant}$; i.e. if m increases, time tends to shrink, we can conclude that: the more the mass increases, the more the space-time curves and by identification we can write :

$$m \cdot t = h / 4\pi C^2$$

• f) Relative quantity of motion :

$$P = mv/\sqrt{1 - v^2/C^2}$$



• g) Work and energy :

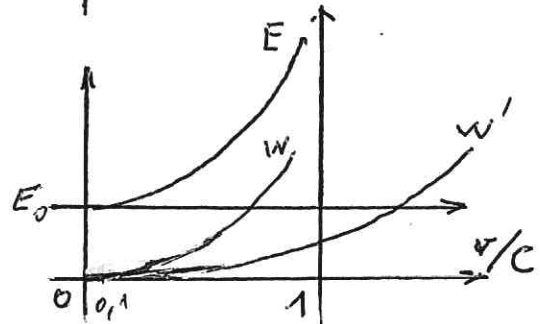
$$W = mC^2 \cdot (1/\sqrt{1 - v^2/C^2} - 1)$$

W = work or energy of relative motion.

E = relative energy $E = m \cdot C^2/\sqrt{1 - v^2/C^2}$

E and W are similar in form, but grow at a higher rate

$$W' = \frac{1}{2}(mv^2)$$



10) The void :

The concept of the vacuum is difficult to define, however in physics the vacuum can be considered as the absence of all matter and elementary particles in a spatial area.

Matter is built on the vacuum:

The vacuum is everywhere, not only in the space between particles, but also within matter itself, at the heart of atoms. Each atom consists of a nucleus around which quasi-punctal electrons revolve in the distance, and thus almost 99.99% of the atom is made of vacuum.

Not an absolute vacuum:

Einstein did rehabilitate the vacuum as noted earlier by definitively discarding the idea of the ether concept that was trying to dominate among scientists; but he denies the existence of the absolute vacuum, i.e. an empty space without fields and elementary particles. The physical object is not in 'space', but this physical object has a spatial extent so that the concept of empty space loses its meaning.

Cosmological vacuum :

Superclusters of galaxies resemble each other in filaments, forming regions hundreds of millions of light years wide that are almost empty of galaxies. In these regions there is at least dark energy, the origin of the expansion of the universe.

For quantum physics :

The vacuum is not space stripped of all matter and is at a temperature of absolute zero with

complete darkness. Quantum physics has therefore further complicated the definition of the vacuum and it foresees many effects appearing in the vacuum: it is then called a quantum vacuum. This quantum vacuum is the seat of spontaneous and fleeting materialisations of particles and antiparticles (or virtual and ephemeral particles) which annihilate immediately after their creation. The vacuum is full of energy, fields and particles in a virtual state and plays a key role in quantum

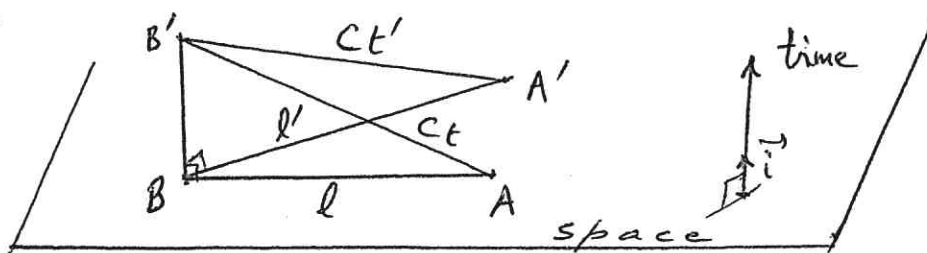
physics, susceptible to fluctuations and pulling the strings of matter behind the scenes. According to current theories, it is the vacuum that gave birth to matter just after the big bang and then gave birth to the universe. The quantum vacuum is indeed empty of real particles but full of virtual particles.

According to quantum physics, the absolute vacuum is not of this world.

11 Formula of invariance:

Find Poincaré's formula by geometric construction in a 4-dimensional space assuming that a plane represents a 3-dimensional space and the perpendicular to this plane represents time.

Diagram n°12



$$C^2t^2 - l^2 = C^2t'^2 - l'^2 = ((ivt)^2 = s^2 \text{ then}$$

$$s^2 = l^2 - C^2t^2 = l'^2 - C^2t'^2)$$

This is the invariant distance (Poincaré's formula)

III) SCHEMATISATION OF GENERAL RELATIVITY

A) Topology of the space-time Universe :

A way of seeing the Universe: Sketches

"A short sketch tells me more than a long report" are words attributed to Napoleon Bonaparte. Indeed, without long and tedious theories or mathematical formulas that are not accessible to everyone, a simple diagram can explain a lot. Moreover, one of the qualities worthy of esteem of the theory of general relativity is its simplicity, which can be summed up as follows:

matter = curvature

or in another way; the heaviest body makes a kind of hollow in its surroundings to attract the lighter bodies into this hollow.

One fact that is currently accepted is the expansion of the Universe, which I also call the expansion of the "space-time sphere". The diagram of this sphere that we are going to see is a representation of the Universe that serves to scrutinise space-time and to draw consequences and conclusions.

1) Hyper-sphericity of the Universe :

Instead of using the term hyper-sphere in a 4-dimensional space, to simplify and to facilitate the schematisation, we will use the term sphere in a 3-dimensional space and a 4th dimension will represent the time starting from the original point, centre of the sphere inflating in all directions. Recall that the 4-dimensional hyper-sphere of diameter O and radius R is the locus of points in 4-dimensional space located at distance R from O .

It is a variety of dimension 3, i.e. the hyper sphere minus one point is topologically equivalent to the usual space. Just as the shadow of a sphere on a plane is a disc (projection) the shadow of a hyper sphere on a hyper plane

(space) is a ball of dimension 3.

Considering the homogeneity, the isotropy and the symmetry of the distribution of matter on a large scale after the Big Bang, as well as the expansion which, in principle, took place in all places in space (everything was in one point) and identically in all directions and in all senses, and this from the time $t = 10^{-43}$ seconds (Planck's time), the hyper spherical form of the Universe appears to be the closest and most plausible to this great blow of creation. On a sphere, all the points are equivalent with a constant curvature (it is necessary to pay attention concerning " the big bang " it is an abuse of language, it is not like an explosion in space, it looks rather like a grandiose appearance, abrupt, violent and sudden).

When the radius of curvature (there is a nuance between radius of curvature and radius of the sphere and I prefer to say radius of curvature) tends towards infinity, the sphere tends to become an endless plane. In what follows we will often speak of a space-time sphere (or hyper-sphere) or a space-time plane.

Homogeneity and isotropy, these two postulates combined, form the cosmological principle which translates a large-scale curvature and whatever the point in space-time considered, the physical properties must be the same.

According to the theory of general relativity, the distribution of matter in the Universe cannot be of exact validity of Euclidean geometry. Since matter is considered to be uniformly distributed on a very large scale, then the Universe should necessarily be spherical. But in reality and in detail, matter is irregularly distributed, so the Universe deviates from the purely spherical form and

becomes quasi-spherical, with indentations and depressions (see diagram of the pierced and crumpled fractal space on page).

Locally the Universe is quasi-Euclidean, but is not homogeneous and is not isotropic either, on a large scale it is homogeneous, isotropic, symmetrical and quasi spherical.

There is a uniform, large-scale, homogeneously distributed population of galaxies with the same curvature everywhere; furthermore, space-time is not identical to itself and is therefore dynamic and expanding at all times.

The diagrams that will be presented in this second part are only the reflections of what was presented in the first part, as a summary of the theory of general relativity with some suggestions and some rectifications.

The age of the Universe:

Based on globular cluster analyses, a range of ages for the Universe from 12 to 14 billion years has been provided.

T = 13.8 billion years: is the official age determined by the Planck satellite science team; this figure is therefore taken as the basis and reference for the calculations; C.T = 300,000 x 13.8.10⁹. AL (Light Years) is the distance travelled by light from the original point.

2) Veracity of sphericity :

a) The distance of galaxies :

Galaxies are moving away from us in all directions. They are all moving away from each other in all directions. The most distant ones are systematically moving faster than the nearest ones. The galaxies in the Virgo Cluster, 50 million light years away from our solar system, are moving away faster and faster at a current distance of 1100 km/s, which continues to increase. The further away the galaxy is, the greater its apparent speed (to us).

This can corroborate and confirm the sphericity of the Universe, in fact, it can be demonstrated that on an inflating spherical balloon, given a point A taken arbitrarily (the observer is at this point A) and two other points B and C such as AC > AB, point C moves away faster than point B in relation to A under the sole effect of the inflation of the spherical balloon, inflation at constant speed.

We consider a spherical balloon inflating with radius R increasing with time from the centre O to infinity at a constant speed V

At the moment where the radius is equal to Ro, the points A, B et C are on the positions Ao, Bo and Co.

We pose :

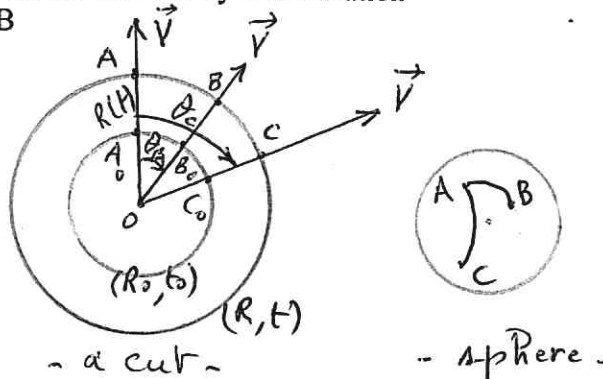
vb = speed of removal of point B with respect to A due to the speed V of inflation (V=constant means that R grows in a constant way)

vc = velocity of removal of point C from A due to the velocity V of inflation

θb = angle corresponding to the distance AB

θc = angle corresponding to distance AC

diagram 13:



AB = R.θB

AoBo = R0.θb(t), grows with time

vb = (AB - AoBo) / (t - to)

vc = (AC - AoCo) / (t - to)

and V = (R - R0) / (t - to)

vb = (AB - AoBo) / (t - to) = (θb.R - θb.R0) / (t - to) = θb.(R - R0) / (t - to) = θb.V

Similarly for point C, we have $v_c = \theta_c \cdot V$

So we have this: $AC > AB$ then $\theta_c > \theta_b$ and then $v_c > v_b$

In general, whatever the point M on the sphere, we always have $v_m = \theta_m \cdot V$, with v_m = speed of distance from M and θ_m angle corresponding to the distance AM (or the arc AM).

The further away from A on the sphere, the greater the speed. Point C flees from point A faster than point B. The swelling velocity V which causes the recession velocities of points B and C with respect to A is always considered to be constant.

Galaxies move away from each other with increasing velocities as the distances between them increase

This observation, proven by telescopes, indicates two results:

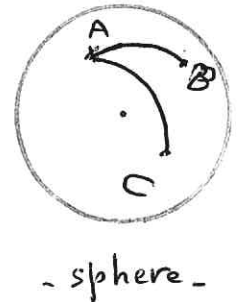
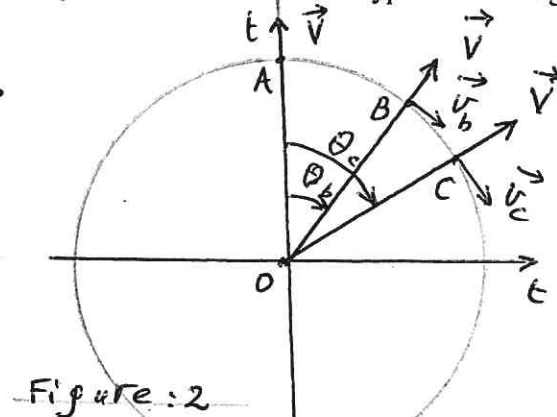
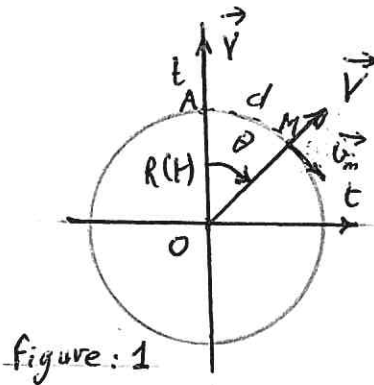
- 1- the possible hyper-sphericity of the Universe
- 2- the extension of this sphere Universe at constant speed.

Galaxies and matter entities are therefore moving away from each other at speeds proportional to their mutual distances under the effect of an expansion of the Universe at constant speed and not necessarily at accelerated speed.

The first measurements of galaxy outflows began in 1920. Hubble published the following results: out of 41 galaxies observed, 36 were moving away; the only ones that were approaching, such as M31 in the constellation Andromeda, are part of the local cluster. From a certain distance onwards, they all flee, and the further away they are, the faster. When the distance is too great, the findings become imprecise.

In 2003, observations of 11 distant supernovae confirmed the hypothesis that galaxies are far apart.

Diagram 14:



From the centre O (the original point), the balloon inflates. The radius R increases with time t. The points A, B, and C are carried away by the space-time membrane under the effect of the inflation in the same direction as the radii at the speed $V = R(t) / t$.

We are fixed on A and we look at B and C which are moving away at respective speeds v_b and v_c .

We have as data : $AB = \theta_b \cdot R(t)$ and $AC = \theta_c \cdot R(t)$

v_b = velocity of flight of point B with respect to A: $v_b = (AB/t) = \theta_b \cdot (R(t)/t) = \theta_b \cdot V$

v_c = leakage velocity of point C with respect to A: $v_c = (AC/t) = \theta_c \cdot (R(t)/t) = \theta_c \cdot V$

Conclusion : $\theta_c > \theta_b$: If C is further from A than B then $v_c > v_b$: figure 2

In a general way (figure 1), if v is the escape velocity of any point M with respect to A, we always have $v = \theta V$

$d = AM$; $d = \theta \cdot R(t)$, we have $v = (d/R(t)) \cdot V = (d/R(t)) \cdot (R(t)/t) = (d/t) = d \cdot H_0$, with $H_0 = (1/t)$. Here we find Hubble's formula

b) The expansion of the Universe is not necessarily accelerated:

In 1998 two astronomy teams :

The "supernova cosmology project" and

The high Zsupernovae search team

led respectively by Saul Perlmutter and Brian P. Schmidt; have come to the unexpected result that the current expansion of the Universe appears to be accelerating. They found that the rate of of galaxies from the Milky Way is increasing over time.

This phenomenon of galaxies moving away from the Milky Way was later confirmed by observations from space telescopes as mentioned: Ref: <http://Fr wikipedia. org/wiki/ Expansion of the Universe .www futura-sciences.com./.../physics. Acceleration - expansion-universe .>

What has been confirmed can no longer be disputed, but to draw a direct conclusion that the current expansion of the Universe is accelerating is not entirely accurate, unless the acceleration of the expansion of the Universe is the name given to the phenomenon that sees the speed of recession of galaxies relative to our own increase with time. It is the galaxies that are moving away at an accelerating rate due to the constant speed expansion of the Universe and not the expansion of the Universe that is accelerating.

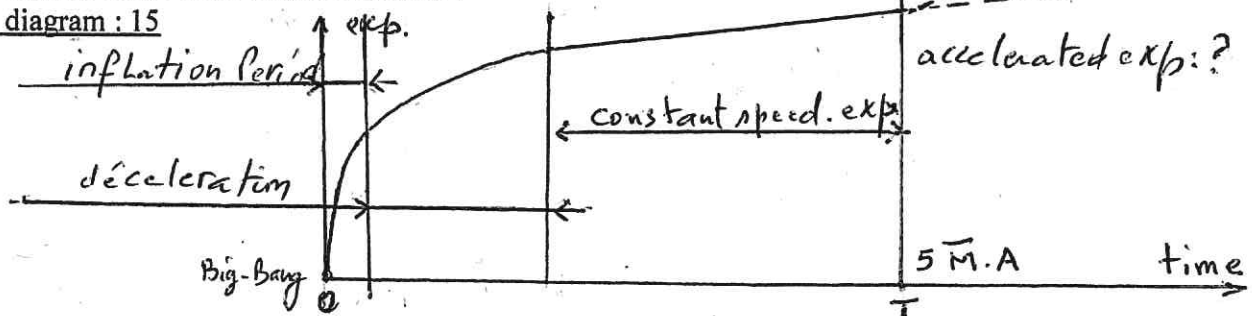
It is enough to see the small demonstration above concerning the inflating sphere and the effect of this inflation at constant speed V on the distances with time of points B and C from A;

As noted earlier, galaxies move away from each other at speeds proportional to their mutual distances under the effect of the constant speed of expansion (of course if it is spherical).

This confirmed result from both teams can only provide evidence for the expansion of the Universe at constant speed and not necessarily for an accelerated expansion of the Universe.

In examining the observations, a hypothesis was put forward; "The expansion would have decelerated following cosmic inflation and then accelerated after 5 to 8 billion years". Proposed Scheme of the Accelerated Universe :

diagram : 15



In view of this diagram, one is forced to ask the following questions: The deceleration that followed inflation is understandable; as the expansion of the Universe following this period of inflation caused a decrease in temperature and pressure and thus a decrease in the speed of the expansion.

But the next acceleration of the expansion in question from time T onwards is caused by what?

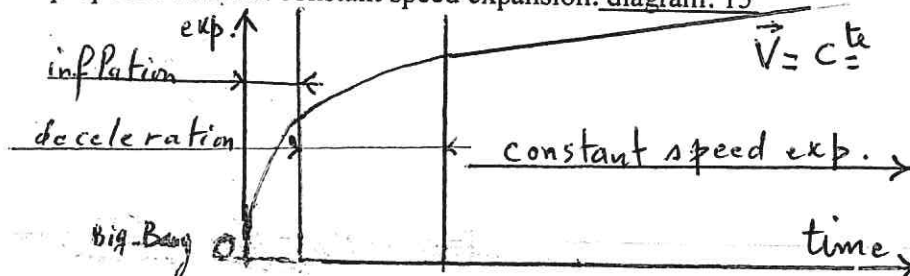
By an extra dose of dark energy?

And where does this energy come from?

We still don't know anything about this form of dark energy that makes the universe expand and opposes gravitation.

I propose another scheme that is more in harmony with the theory of general relativity, because the conclusion drawn by the two teams, namely that the expansion of the Universe is accelerating, does not fit in well with the said theory. The constant speed of the expansion of the Universe fits better with it.

The proposed scheme: constant speed expansion: diagram: 15'



We can therefore conclude on this point by noting the following:
Friedmann's equations which gave us a Universe whose expansion remained at constant speed still retain their validity and robustness and in my opinion Friedmann was not wrong as has been claimed.

c) Hubble's formula :

We pose :

V = constant speed of the ballooning Universe (of the expansion)

\vec{v} = velocity of the galaxies away from our solar system

d = distance from observer A (i.e. us) to the galaxy M which is moving away from us.

H_0 = Hubble constant, H_0 is the inverse of the time taken to travel the distance d .

At the speed \vec{v} , $H_0 = 1/t$ and $\vec{v} = H_0.d$

The constant can be written as follows: $H_0 = 100. h. \text{ km/s/Mpc}$, where h = the expansion rate.

Recall that: 1 Megaparsec (Mpc) = 3.261564. million light years (AL).

From $\vec{v} = \theta V$ or $\vec{v} = H_0.d$, we can write $d = \theta.R$

Here are some indications concerning the escape of galaxies

values in 2011:

$v = \text{min}65, \text{mean } 73 \text{ max } 77$

If we take $v = 73. \text{ km/s/Mpc}$

A galaxy located at 1 Mpc from observer A moves away at a speed of 73 km /s; at 10 Mpc the speed of distance is 730 km/s; at 100 Mpc the speed of distance is 7300 km/s: we verify here the Hubble constant. $H_0 = (d/v) = (1 \text{ Mpc}/73) = (10 \text{ Mpc} / 730) = (100 \text{ Mpc}/7300) = \text{Cte}$

But as we have already noticed, the interpretation of galaxy motions becomes improper at very large distances (see figure 14).

Some results: During the second half of the 20th century, the results on the outward velocity corresponding to H_0 are estimated at values between 50 and 90 km/s.

In May 2011, the Hubble Space Telescope gave a value of 72km/s/Mpc with an uncertainty of 8 km /s/Mpc

WMAP satellite: from 2003 data: 71km/s/Mpc with an uncertainty of 4km /s/Mpc; then for the year 2006: 70.4 km/s/Mpc

For 2008: 71.9 km/s/Mpc

In August 2006, the Chandra X-ray telescope gave this result: 77 km/s/Mpc with uncertainty 11km/s/Mpc.

In March 2013, the Planck telescope gave this result: 67.8 km/s/Mpc, i.e. at 1 Mpc, the galaxy is running away at an apparent speed of 67.8 km/s/Mpc.

Generally, according to specialists and telescopes, for 1 Mpc the determined escape velocity is between 60 and 80 km/s

Galaxies move away from us at radial velocities; $\vec{v} = d.(V/R)$: The velocity of a galaxy is proportional to its distance from the observer, this is Hubble's law.

d) If the universe is completely flat (Euclidean):

If space-time is absolutely flat and with the observation of the $v_c \dots v_b$ telescopes, in this case one asks the question: what phenomenon will move points B and C away from the relatively fixed point A?

If space-time is absolutely flat, if it stretches in all directions or if it is stable, points A, B and C keep their same positions and $v_b = v_c = 0$ which is contrary to what we have assumed. So the space-time can only be quasi-Euclidean to have $v_c \gg v_b$

e) Poincaré's conjecture :

A closed, edgeless and simply connected 3-dimensional space is necessarily hyper spherical (to be solved)

A topological space of which there is no partition (in closed or open parts), non empty and said connected. It is a smooth space, without any fault, without holes or tears. For our sphere (hyper sphere) with 4 dimensions, the fourth one represents the rays coming from the centre representing the time (to see on the diagrams

f) The sphere is the form adopted by Riemann : for his model and also by Einstein for his. This leads us to think that the choice of the hyper sphere is not bad.

g) Another conjecture : to be taken into account: the Universe which inflates has a beginning, it is thus finite, moreover it is unlimited, it is then hyper spherical (to solve).

3) Curvature of space-time :

The curvature of space-time depends on the density of matter.

According to the " WMPA " : (Wilkinson Microdegree Anisotropy Prob), the critical density of the Universe is close to unity so space-time is almost flat. After inflation, the Universe swells disproportionately until it becomes almost flat because it was well curved. For space-time to be flat to infinity, it is necessary that $\Omega = 1$ exactly (Ω represents the ratio between the density of the Universe and the critical density). Now WMPA, after seven years of observations in January 2010, gives a value of between 0.991 and 1.173: $0,991 < \Omega < 1,173$.

Ω is therefore close to 1, but not exactly, the average value of these two limits is : $(0.991 + 1.173)/2 = 1.082$; Ω is close to 1 but on the positive side.

The Universe is almost flat and the curvature is very slightly positive and therefore excessively small. The radius of the curvature of the Universe is then excessively immense (the radius tends towards infinity).

The latest results of the European satellite Planck: are unexpected, in fact, it affirms that the curvature is completely null, we are thus in the presence of a Euclidean geometry where space-time is similar to a sheet of paper. This does not contradict its sphericity; the range of existence (see definition later) can be totally Euclidean from the inside and two parallel lines will always remain parallel until infinity, i.e. until they close in on themselves.

Among other results, Planck gives the following:

- *Luminous matter (stars and others) 1%
- *Ordinary dark matter5%
- *Dark matter that has never been observed directly (neutron matter, protons, electrons)25%
- *Dark matter and energy (hypothetical, this may be the one be the one that is facing gravitation and that drives expansion).....69%

(see latest news from the cosmos : Hubert Reeves; edition du seuil 2002)

4) Dark energy:

It has been found indirectly that there is (perhaps) an energy in the cosmos called "dark energy" which counterbalances gravitation in its tendency to bring galaxies closer together and thus slow

down expansion.

The resultant = (dark energy - gravitation), causes the expansion of the Universe to proceed at a constant speed (and I maintain my idea: constant and not accelerated).

The concept of "dark energy" is therefore born from this observation. It looks like an antigravitational pressure, a mysterious energy that maintains the expansion of the Universe. It was not known and still is not known, but its existence is not suspected either. Some link it to the vacuum, which is not correct if vacuum means cosmological vacuum, because the latter did not precede time and space-time was born in its entirety.

There may be a link between this dark energy that promotes expansion and Einstein's famous repudiated constant. This cosmological constant that Einstein wanted to insert in his formula in order to make the Universe static and that was abandoned by himself so that he could reconcile himself with the theory of expansion; this constant could describe the resultant that causes the expansion and in turn the dark energy in question: (Dark energy - Gravitation = A); A is Einstein's repudiated constant.

The reign of this energy certainly began before the birth of galaxies, i.e. before gravitation and its relative decrease caused by inflation would have favoured the birth of structures and then galaxies from the beginning.

5) Metamorphosis :

Metamorphosis is a feature of nature, it occurs at the moment of birth in amphibians, insects, animals (including us) and even in plants of all kinds. It could be present at the first time of the birth of the cosmos.

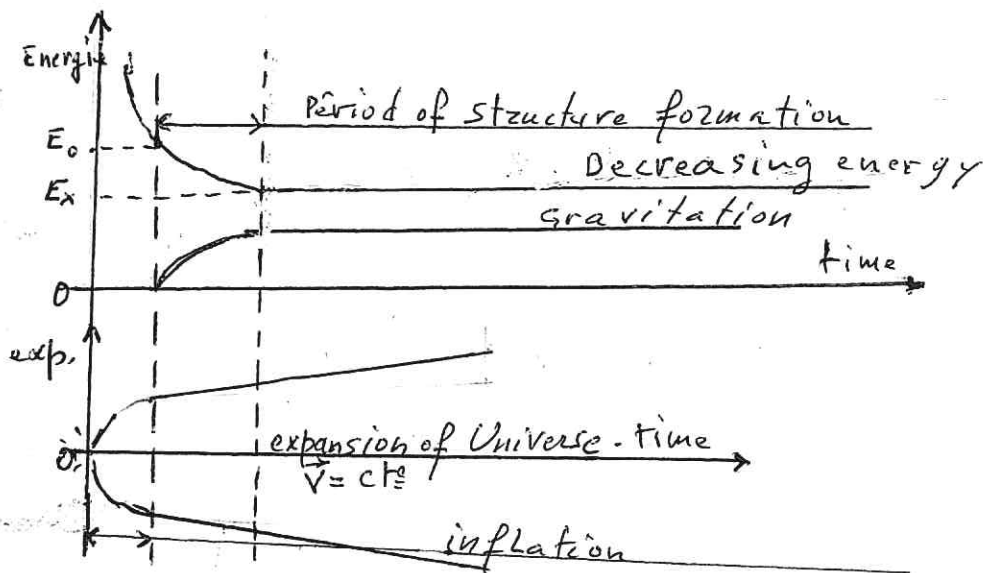
The metamorphosis lies in this: gravity was born with the advent of structures and the formation of lumps just after inflation. At the very beginning, the dark energy that inflated the space-time hypersphere decreased in intensity in the face of inflation; this decrease in intensity favoured the birth of gravitation, which locates itself on the lumps (this is the metamorphosis). During this period of inflation, nuclear chain reactions rapidly transform a large part of the hydrogen into heavier elements. This is in contradiction with the observations because hydrogen forms 3/4 of the matter in the Universe today. A constraint must have been placed on the entire primordial Universe for an unfavourable environment to have prevented this process from taking place; an intense background of homogeneous radiation will solve this problem: a very high temperature that destroys the nuclei of elements other than hydrogen as quickly as they are formed and hydrogen remains in the majority. This radiation will then decrease with the expansion of the Universe until it reaches 2.736K

K according to the COBE satellite;

Over time, matter clumps around small irregularities and so matter attracts matter by gravitation. Clusters of galaxies are formed, connected by filaments like a rag. Theoretically, the distribution of matter in the Universe is assumed to be almost homogeneous.

The existence of matter and therefore gravity corroborates the existence in the past of an inflationary period that has lowered this unknown: dark energy

diagram:16



B): Scheme constructions :

1) The void, nothingness or elsewhere :

We cannot start with zero, but with almost zero and before almost zero is the unknown? At the beginning, in the vicinity of point O, space-time, matter, unlimited energy, unlimited pressure, unlimited density, all condensed into a single point; we can consider that almost zero and infinity were merged.

Outside the point there was nothingness or elsewhere (elsewhere indicates a place other than the one where we are), that is to say neither matter, nor energy; neither space, nor time, nor space-time, in short, there was non-being. The universe is drawn from nothingness. Today, the expansion of the Universe continues to gain on the terrain of nothingness and not on that of empty space as we imagine. Empty space without time is meaningless. How to conceive of nothingness or elsewhere? (I don't know). The nothingness or perhaps "elsewhere" is outside the cone of light of special relativity.

The existence of other universes parallel to ours has even been proposed, and this is in fact only speculation and imagination, because several universes form only one.

According to some theories and particularly according to Alan Gush's concept (1980), the vacuum was the engine of cosmic inflation, of the beginning of creation. This concept of inflation was invented to help the accepted cosmological model, which suffers from some anomalies and irregularities, and especially to correct and solve the problem of the cosmic horizon. Until now the concept of inflation remains accepted by the astrophysical community despite the fact that it is based on the vacuum considered as the engine of this inflation. If vacuum means empty space then this becomes inadmissible; one must look for another explanation linked to nothingness or elsewhere or something else rather than to the vacuum linked to space which cannot exist before the big bang. If we admit that the vacuum was the engine of cosmic inflation, we should also admit that this vacuum (i.e. space devoid of everything) existed before the big bang of creation, which is contrary to the theory, unless vacuum means nothingness or elsewhere.

According to the current theory, the quantum vacuum had all the qualities required to trigger such an expansion. This theory remains fragile and without a strong foundation for the following reasons:

First of all, there is ambiguity concerning the concept of the quantum vacuum; until today nobody has presented a clear-cut definition. The concept remains unclear and the question has not yet been decided. Moreover, this vacuum, which is still ill-defined, generated the universe, so who generated it? To this day, there is no evidence to support the theory that inflation is due to the quantum vacuum.

Concerning the cosmological vacuum, Einstein rehabilitated it in 1905 (by giving the coup de grâce to the ether), as being the absence of all particles, of all radiation and devoid of all matter; this is the

atomistic definition of the vacuum. There are corners in the Universe where there is barely 0.1 atom/cm^3 . For comparison, air contains $10^{23}/\text{cm}^3$. There is no absolute vacuum in the Universe, because there are always magnetic waves, photons, and scattered massless particles and especially fossil radiation which has always been present throughout the Universe since the birth of the Universe and the lumps (we will see the fossil radiation in more detail in the proposed diagrams). The vacuum is the space that contains nothing (it is neither nothingness nor elsewhere) and it has never preceded matter. The space and time that form the space-time appeared with its singularity "the matter" at the same moment and the nothingness (or the elsewhere) made place to them. The geometric properties of this space-time are conditioned by the matter and energy it contains, so it is useful to know the state of these. Indeed, the velocities of the stars (the matter) being too small compared to the speed of propagation of light, we can assume that the matter (the stars) is relatively immobile, although this speed of $300,000 \text{ km/s}$ is not at all important compared to the size of the Universe.

Due to the influence of gravitational fields and the large-scale distribution of matter, it can be stated, as already noted, that there is no validity of Euclidean geometry, but it can only be local

Our sun, for example, has little influence on the metrics of its surrounding space, and at our level our world is irregularly curved in detail, but almost Euclidean, like the surface of a pond at rest: from a distance it is flat, and up close we see curves and ripples. If this is so, the average density of matter would be almost zero, but if the density deviates a little from zero, then it can only be quasi-euclidean.

2) The patterns of departure, space-time spheres and great circles:

The big bang of creation or more precisely the sudden appearance or hot big bang was achieved at any point in the Universe, since everything was merged into one point. The diagram that I propose is a section of this point which evolves in a sphere (or hyper sphere) which inflates, of radius R which ends towards the infinite. This cut which passes by the centre gives a large disc whose edge is a large circle of radius R , intersection of this sphere and a plane passing by the centre O of this one. The aim is not to describe or explain the transformations that occurred just and after the original explosion, but simply to go through, grasp, and perceive in a simple form in the past and in the future and to draw conclusions.

We will now try to reason by recurrence: The speed of light coming from distant stars and galaxies being constant, this light reaches us only after a very long time. When we see the nearest galaxy $M 31$, in the constellation of Andromeda in this case, we see it with a delay of 2.3635 million years, at this moment, it is no longer where we see it, how it is now? The sun, we see it with a delay of 8 minutes. If we look at our neighbourhood, the car parked in front of me a few metres away, I see it with a delay of a few thousandths of a second and by recurrence, everything I see, I see in the past, including everything around me.

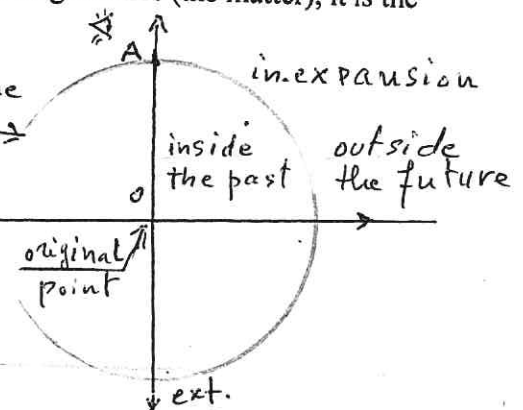
I see in the past, because the speed of light is constant. Seeing is therefore looking into the past, i.e. inside the space-time sphere or the space-time ball (see diagram n° 17).

A sphere of radius R in extension that started to swell from almost zero.

We are on this sphere, on a space-time membrane that is swelling. The interior is the past that once existed and is no more (it has entered into nothingness). The outside of the sphere is the future which is not yet of this world (it is still in nothingness). The intersection of these two nothingnesses is the circle (the present), it is the space-time membrane with its singularities (the matter), it is the Universe

diagram n° 17 :

The sphere in extension; the present is the space-time membrane, this circle is a section of the expanding sphere through a plane passing through the origin O



A sphere (or hyper sphere) of radius R in extension . We are on this sphere, on a space-time membrane, in the Universe.

The interior is the past (nothingness or elsewhere); the exterior is the future (nothingness or elsewhere) and the sphere in extension is the present. The intersection of the two nothingnesses is the present, the intersection of the past which no longer exists and the future which does not yet exist is the present universe.

Proven proposition: The speed of light being constant, then everything I see, I see in the past inside the space-time hyper-sphere.

Consequence: We are in the universe on the space-time membrane, at the edge of the present time: on the space-time membrane. This space-time membrane which corresponds to the present time and which is situated between the past which no longer exists and the future which does not yet exist, that is to say between the two nothingnesses. The intersection of the two nothingnesses (the past and the future) is not nothingness but it is the present space-time, at any moment one leaves a nothingness to enter another nothingness (or elsewhere).

Past space-time: is represented by the ball or whatever is inside the sphere. The concentric circles 1, 2, 3,n represent the continuous evolution of space-time (continuous evolution of the space-time membrane). It is therefore a space-time continuum. A is an arbitrary point. We are at point A on the last sphere (or on the plane because the radius remains infinite) still expanding; see diagram n° 19.

Time: it is represented by the half-right lines coming from the centre O and directed in all the directions towards the outside of the sphere (or of the orthogonal lines to the plane, because R tends towards infinity). These are therefore rays which start from the centre O towards the outside, so that any point on the surface of the sphere (or spherical membrane) is also a point which belongs to one of these rays.

Each point on the surface of the sphere is therefore a mixture of space (the spherical membrane) and time (the rays), see diagram 19.

Each point of the plane (R is infinite) is a mixture of space and time (the normals of the plane): to speak of a plane or a sphere is here identical because R is infinite.

We therefore say that each point of the spherical membrane is a mixture of space and time. We obtain the space-time sphere (or space-time plane)

The range of existence: see diagram n° 18:

The present space-time is this membrane; the more this membrane advances in the direction of time, i.e. towards the external nothingness (it swells), the more we move away from our present towards another more recent one, we always leave the first one to the internal nothingness.

We are therefore facing the past, inside the sphere juxtaposing the membrane and very far from the centre O, thus on a range clearly limited on one side by the membrane and not clearly limited on the other side. It is as if we turn our backs on the membrane while looking inwards, i.e. towards the past.

We live and look in the past with our backs to the membrane, but the further we look, the more our gaze enters the past. Our gaze can only always be directed towards the interior of the space-time sphere, in the opposite direction to the direction of time, i.e. towards the position of the centre O, the original centre which is no longer. Everything we see is part of the past. The direction of the time axis (or axes) is directed outwards or towards the space-time that does not yet exist, towards the future space-time, so the time component of the 4-dimensional space-time must be imaginary.

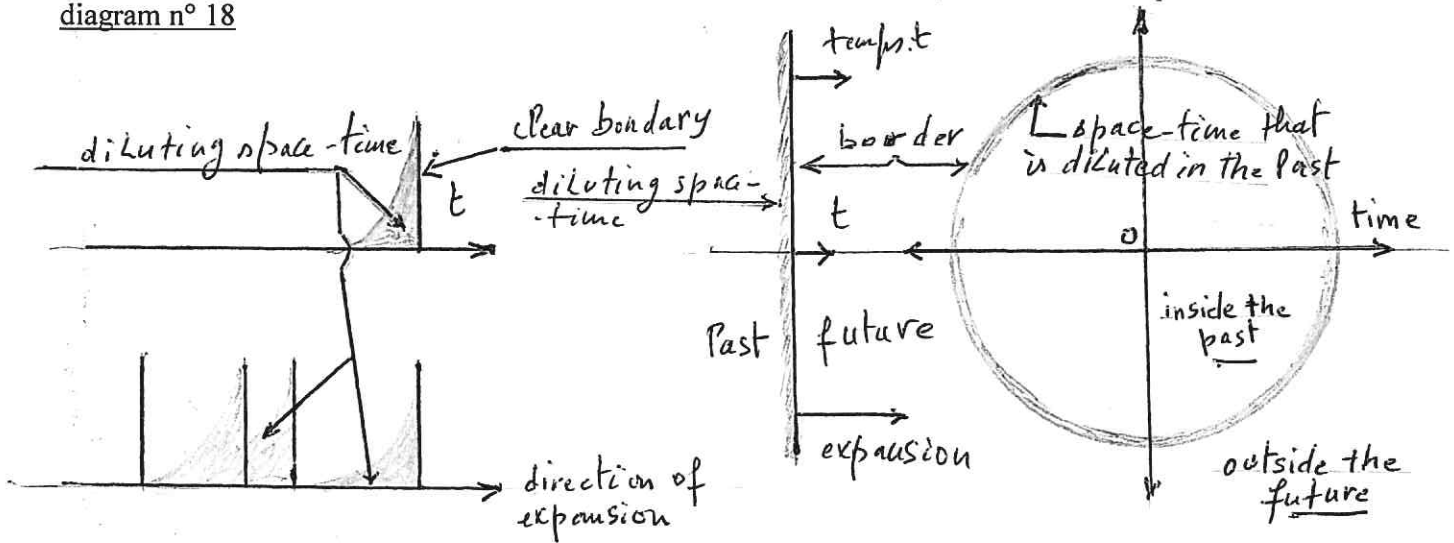
The other 3 dimensions of space

are in front of us in the past, i.e. inside and directed towards the apparent world (the past), in the opposite direction of time whose non-apparent unit vector (i) can only be imaginary. The further we look, the more our view escapes from the surrounding space-time towards a past universe, which no longer exists.

For example, quasars do not exist in the vicinity of our galaxy, they are billions of light years away, we see them when the Universe was young, but even further away, we no longer see them. So their existence is relative to a limited period of time, the Universe being changing, they have probably changed their nature and are currently very far away and at the same level as us, i.e. on the space-time membrane.

We are now on the present space-time membrane (or on the plane membrane), at the intersection of the past which no longer exists and the future which does not yet exist, that is to say and as I have already noticed, at the intersection of two nothingnesses, between yesterday and tomorrow or between two microscopic lapses of time, past and future, adjacent to each other (and us between the two). This is the only component of time that allows us to grasp the variation of space-time, the other three components cannot allow us to do so, we see them as constant in front of us.

We exist on a range of past space-time, limited as mentioned, on one side by a clear boundary where the future begins (the present) and on the other side, the boundary is diluted in the past: see diagram n° 18

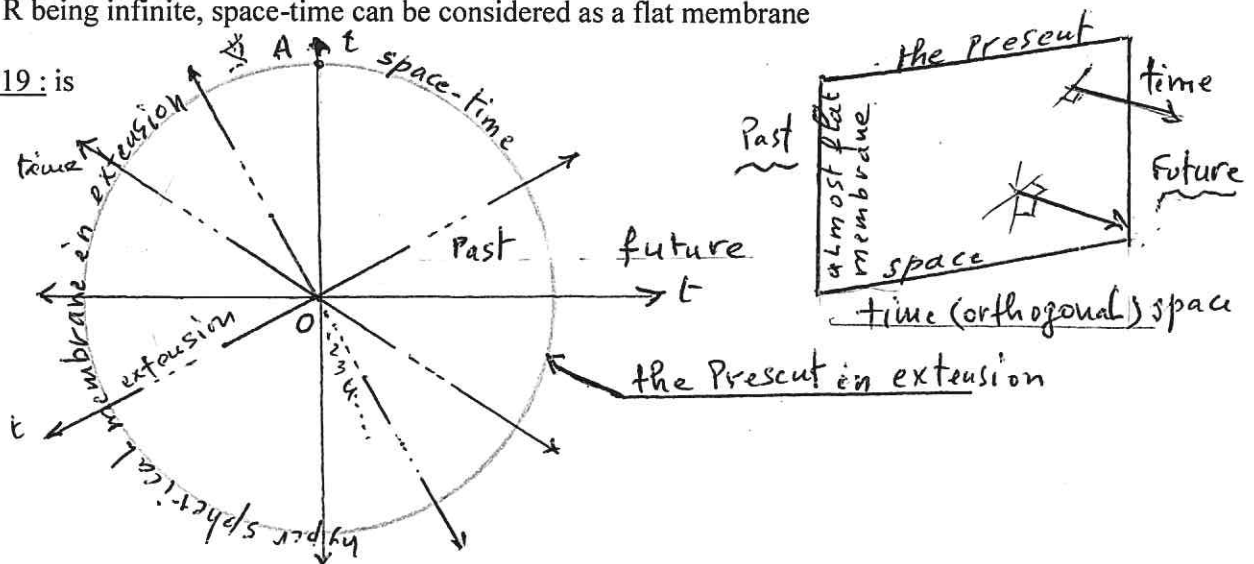


We are on the range of existence that is diluted in the past. On one side there is a clear border: the present

On the other side, space-time is diluted in the past

The radius R being infinite, space-time can be considered as a flat membrane

diagram n° 19 : is



The concentric spheres: 1, 2, 3, 4,n represent the continuous evolution of the expansion of space-time, a space-time continuum. Time is represented by the half-lines oriented towards the outside of the sphere; these are rays starting from the origin O (the original point). Any point on the surface (spherical or plane) is therefore a mixture of time and space.

Attention, when we say that the Universe is at 13,8 billion AL, of the original point O, in truth this original point is fictitious (it also everywhere in the Universe) and the diagrams are also fictitious, they do not exist in reality because the great blow of the creation (the big bang) was carried out in any point of the Universe, since all was in a single point O. What I schematize only serves to explain an idea; moreover, the whole model that describes the Universe only serves that purpose.

The exact reality can only be determined by the creator of the universe.

Travel by train in the last cabin:

A traveller occupying the last cabin of the train and looking through the window in the opposite direction to the train's progress. He sees the railway tracks, the trees along the tracks, the passers-by and all the objects in his field of vision flash before his eyes but as they move away.

The distance between the two is in time and space. The membrane corresponding to space and time is the window through which he looks. This window, which moves in the opposite direction to the gaze, moves in the same way as the space-time membrane moves in nothingness, i.e. in the direction of time (in the direction of the train's movement).

The expansion of the Universe makes that we advance while moving backwards, without seeing the front (the future, outside the space-time sphere), but only the back (the past, inside the space-time sphere).

The gaze of observer A is always directed towards the past, i.e. towards the interior of the sphere (in the opposite direction to the movement of the train).

For the space-time sphere, the axis of time is directed towards the external nothingness (or

elsewhere), i.e. towards the future and the unit vector of time is in the nothingness, in the elsewhere, we do not see it and thus it can only be imaginary which does not contradict Minkowski's choice to take an axis of time with an imaginary unit vector i such that $i^2 = -1$.

The plot of the curve :

The diagram n° 20: represents the section of the space-time sphere by a plane passing by the origin O, thus following a large disc of contour a large circle in continuous extension (it is the large circle which interests us); a space-time continuum. We are at point A taken arbitrarily as we have noticed because on a sphere (or on a plane here $R = \text{infinity}$) all points are equal. We are going to consider that our view (or our telescope) is going to focus on only one direction, thus on only one line which corresponds to this great circle. The points B, C, D, E, F, etc..... are the positions on this great circle of the singularities of space-time, they are thus stars, galaxies, clusters of galaxies, let us say the matter quite simply). On the sphere, there are an infinite number of them. These singularities move away from each other as a result of the swelling of this membrane (or the stretching of this quasi-planar membrane), i.e. as a result of the expansion of the Universe.

The singularities: A, B, C, D, E, etc., are located at the same time and on the same great circle as A (that we observe): we say that these singularity points are at the same level as A: on the same great circle.

To see a bright object located at point B, at the same level as A is impossible, we can only see it in

Bo on the OB radius, because it would take time for the light to reach us, as a consequence of the constant speed of light. The time taken for this light to reach us from point Bo is the same as the time taken by B to travel the path BoB

In addition, the light has to travel through past space-time, so it is only within the sphere that the light can move. The time taken by the shining object to move from Bo to B is the same as the time taken by the light to move from Bo to A.

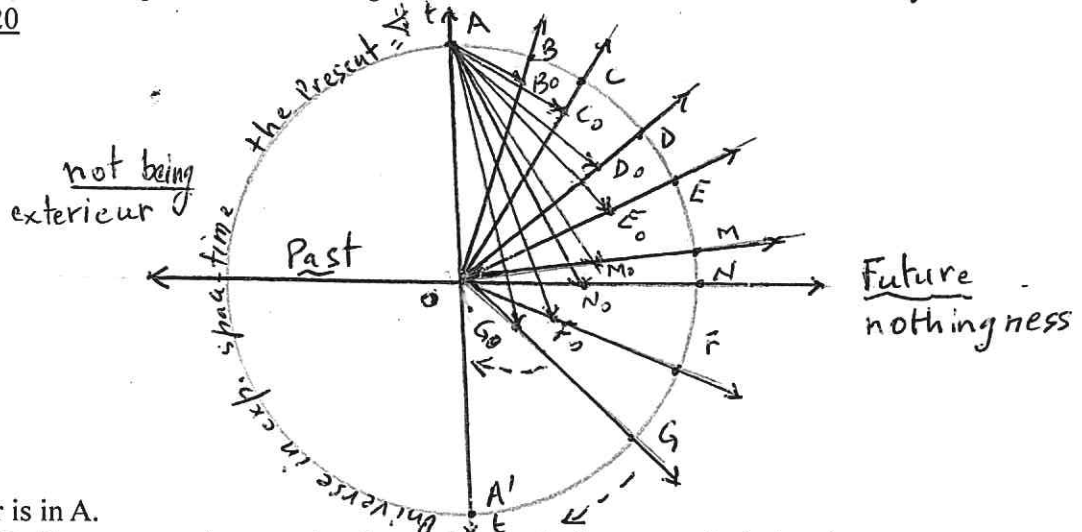
Similarly for point C, located further away than B, it can only be seen in Co on the OC ray but such that $CCo \gg BBo$.

ditto for all the other points of the semi great circle AA' and we continue to the furthest point A', diametrically opposed to A (we see A' in O), to obtain in the end a continuous curve, image of the great circle carrying the points in question. We obtain a half cardioid $\delta(t)$, with t in radian from zero, (position OA' to πR (position OA'). The observer at A can only see everything on the semi great circle AA' on the curve $\delta(t)$, so he sees the semi great circle in the past, i.e. on the curve $\delta(t)$. The further back one sees, the further one follows this curve to the original point O, which is the image of A' on this curve. At the very beginning of time, A and A' were merged at this point O.

Our view follows this curved line in past time and any point of the semi great circle can only be seen on this curve: see diagram 19.

Observer A, sees all the points of the semi great circle on the semi cardioid continuously.

Diagram : 20



The observer is in A.

Points B, C, D, E, etc. are at the same level as A, i.e. on the last great circle (on the space-time membrane). One can only see point B in Bo, point C in Co, point D in Do etc... One obtains the points Bo, Co, Do, etc..., images of points B, C, D, etc... on a continuous curve. The whole semicircle AA' can be seen on this curve, the set of points Bo, Co, Do, etc...

The radius OA is out of the view of the observer (the observer cannot see himself in the past). This curve is the light path curve.

3) The light path curve : $\delta(t)$

it is the image curve of the great circle (diagram n° 21)

$B_0B = z(t) = z(d/R)$: a function of d (d represents the distance of B from A), a function representing the space-time shift between the actual position of point B (position on the space-time

$$OB_0 = \delta(t) \text{ and } B_0B = z(t)$$

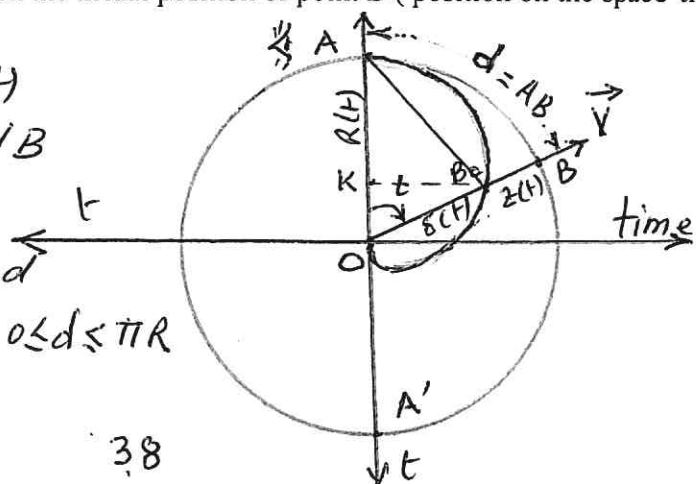
d = distance between A and B

R = infinite radius

t = angle corresponding to d

$$d = t \cdot R; 0 \leq t \leq \pi \text{ and } 0 \leq d \leq \pi R$$

$$B_0B = z(t) = z(d/R)$$



sphere or great circle) and the image B_0 of B on the curve $\delta(t)$, when the latter emitted the luminous flash towards A. The function $z(t)$ representing the shift increases with t. The further away the point B is from A, the larger $z(t)$ is.

For simplicity, let us consider $R = 1$

$\delta(t) = 1 - z(t)$, this is the function that represents the light path

The function $z(t)$ must verify: $z(0) = 0$; (at point A) and $z(\pi) = 1$; (at point A', diametrically opposite A).

The curve of $\delta(t)$ intersects the ox axis at a single point: $\delta(\pi/2) = z(\pi/2) = 1/2$

$AK = 1 - \cos t = k \cdot z(t)$; the constant k to be determined:

The points $z(0) = 0$ and $z(\pi) = 1$, give $k = 2$ and $z(t) = (1 - \cos t)/2$ then

$\delta(t) = 1 - z(t) = (1 + \cos t)/2$, it is the only function which in the great circle verifies $z(0) = 0$ and $z(\pi) = 1$

The curve $\delta(t) = R \cdot (1 + \cos t)/2$ is unique. It represents the set of image points of the set of points of the great circle. Everything that is not visible on the great circle, it can be seen on this curve. This is the light path curve.

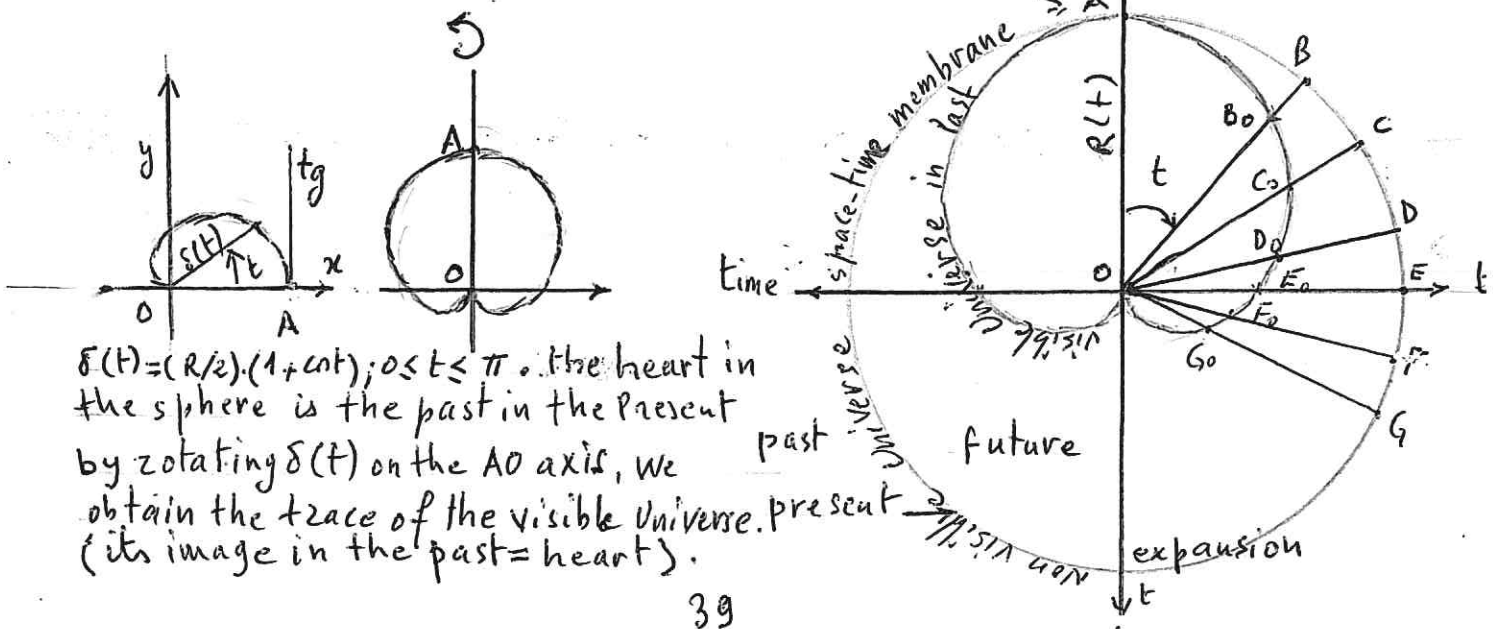
$z(t)$ represents the shift towards the light signal emitted since a given time and received today. Observer A, sees the distant light sources B, C, D, E, etc... located currently on the great circle (or on the space-time sphere, at the same level as A), he sees them respectively in the past, i.e. on positions: B_0, C_0, D_0, E_0 , etc...

There is thus a backward shift which leads us to choose an appropriate function $\delta(t)$ which also verifies: $0 < \delta(t) < \pi R$

This means that this curve carrying the images B_0, C_0, D_0, E_0 , etc... of points B, C, D, E, etc. is inside the great circle (inside the space-time sphere), in the past.

To obtain all the image points located on the space-time membrane sphere, or more precisely all the

image stars and matter of the Universe, one makes a rotational movement of the cardioid $\delta(t)$ around the axis AO and one obtains "a heart" image of the sphere (or hyper sphere) drawn in the past, see diagram n° 22.



All the points of the half-circle AA' can be seen on the cardioid curve $\delta(t)$; similarly, on the other side, the same thing can be seen: dotted curve, all the points of the circle can also be seen on the cardioid $\delta(t)$ and the whole sphere can be seen on the "heart".

The angle t can also represent the time spent (t) in the negative direction, inside the sphere, a time that goes backwards, since the further you look, the further you go back in time:
 $0 \leq t \leq \pi R$. (R is infinite). The further away from A on the great circle, the more the image points move back towards the centre O. $\delta(t)$ thus indicates the path of light in the past 4-dimensional space-time. It should be remembered that the time that currently flows, represented in the diagrams by the rays, from the centre O outwards, touches all the points of the space-time membrane sphere and any point of the sphere is therefore a mixture of space and time.

$\delta(t)$ is the consequence of the recoil of time caused by the constant speed of light. When I look live into the depths of the Universe, my view follows this curve $\delta(t)$, it does not follow the AO ray because I cannot see myself in the past. Only the points located on the AO ray are excluded from my view: see diagram n°20.

If one makes a complete turn of the great circle following the fixed points A and A', (one makes a 360° turn to look everywhere) one sees the totality of the space-time sphere and inside, the curve $\delta(t)$ swivels around OA making a past space-time "pear".

All the points of the space-time sphere are seen on the pear; all the Universe has for image the pear and this because of the constant speed of the light: see diagram n°21

4) The path of light and the radius of the Universe:

The curve $\delta(t)$ is on a very large scale (R tending to infinity) a straight line normal to the quasi-planar space-time membrane. The length of this line $\delta(t)$ is :

$$L = C.T = \int_0^{\pi R} \sqrt{\delta^2 + \delta'^2} dt = 2R, \text{ with } \delta(t) = R.(1 + \cos t)/2$$

C : is the speed of light and T : is the time taken by light to travel from the original point O to our present position, which is the age of the Universe, 13.8 billion years.

The radius of the space-time sphere is then $R = L/2$

$L = 13.8$ billion light years (AL) which represents the path of light since the beginning of creation and not the radius of the Universe, which is $R = 6.9$ billion light years (AL);

We verify that $R.C = V.L$; $v = t.V$. $t = d/R$ then $v = (d/R).V$

with : V : average speed of inflation of the space-time balloon (the spherical space-time membrane)

R : radius of the space-time membrane or radius of curvature of the Universe.

v : velocity of a point B away from A (A is assumed to be fixed), this is the velocity of B which is due solely to the swelling of the spherical space-time membrane.

C : speed of light.

T : the time taken by light from the original centre O to point A or age of the Universe.

t: the angle (OA, OB) which also indicates time.
 L: distance travelled by light from the origin O: $L = C.T + 2R$
 d: distance AB, distance in Megaparsec that separates us (A) from a light source, i.e. a star or galaxy located at point B. It is the real distance not apparent on the great circle (or sphere).
 z: the offset of the observed star between its real non-apparent position and its visible non-real position.

Note: The furthest point from us in the Universe is A' located at a distance $17.R$ AL (from the observer at A) i.e. 22.10^9 AL and the extent of the Universe is $27.R$ i.e. $43.96.10^9$ AL.

5) Actual distance and apparent distance:

Let d = the actual real distance (the Universe changes all the time) not visible on the space-time membrane (on the space-time sphere)

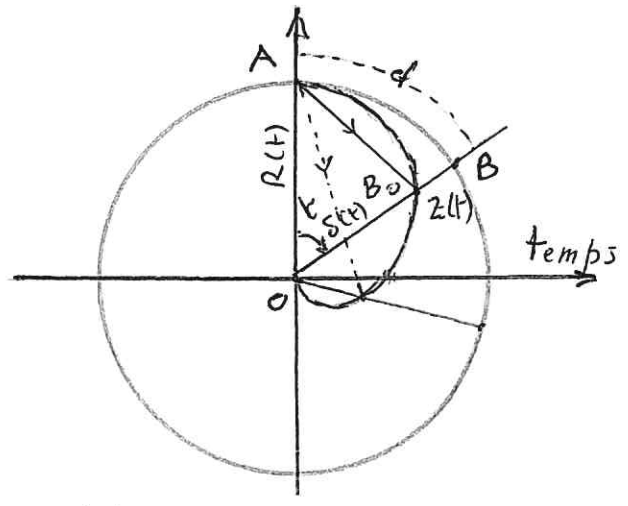
l = the visible distance (in the past) not real, shifted to the past on the cardioid, light path $\delta.(t)$

d = AB and l = ABo

$\delta.(t)$ = a function of the angle t,

t is the time

diagram :23



$\delta.(t)$ = a function of t; angle t is time

B is any point taken on the space-time sphere (on the great circle) and B0 its visible image on the cardioid $\delta.(t)$

OBo = $\delta.(t)$ and BBo = z(t)

Curvilinear abscissa ABo = $l = \int_0^\theta \sqrt{\delta'^2 + \delta^2} dt = R \cdot \sqrt{2} \cdot \sqrt{1 - \cos \theta}$

with $d = \theta R$ for $t = \theta$ fixed
 $l = R \cdot \sqrt{2} \cdot \sqrt{1 - \cos \frac{d}{R}}$ and $d = R \cdot \text{arc. cos}(1 - \frac{l^2}{2R^2}) = 2R \cdot \text{arc. sin.} \frac{l}{2R}$

The real non-apparent distance can be obtained as a function of the apparent distance l :
 $d = 6,9.10^9 \text{ arc. cos}(1 - l^2/R^2)$

Examples :

*For the galaxy M31 (NGC 224) in the constellation Andromeda, currently at an apparent distance (in the past) : $l = 2.3616.10^6$ AL ; the actual non-apparent distance is about the same (it is relatively close to us)

For a star located at 1 Mpc (i.e. $3.261564.10^6$ AL); the real current non-apparent distance is $3.29718.10^6$ AL (I speak of real current distance because distances vary just like the Universe). But looking further back, the difference becomes noticeable and right with the passage of time.

*For a star located at an apparent distance $l = 1$ billion AL corresponds to an actual non-apparent distance $d = 1.0008$ billion

*For $l = 2$ billion AL corresponds to $d = 2.007$ billion AL

*For $l = 8$ billion AL it corresponds to $d = 8.533$ billion AL

*For $l = 10$ billion AL it corresponds to $d = 11.850$ billion AL

* For a star located at an apparent distance $l = 12.5$ billion AL, we calculate the actual distance $d = 15.639$ billion AL.

The farthest point A' (diametrically opposite A) is at a distance $d = 17.R = 21.666.10^9$.AL . In

general $l \leq d$

The difference in the distances of the locations from A (the observer) is notable, worthy of note and should be pointed out to mark the effect of the light speed shift.

If we note $BB_0 = z(t)$, the shift of the star B caused by the late arrival of light, $z(t)$ varies between A and OA' , i.e.: $0 \leq z(t) \leq R$

We can therefore write: $l = \sqrt{2} R \sqrt{1 - \cos \theta}$ with $0 \leq \theta \leq \pi$ and

$$z = R(1 - \cos \theta)^{1/2}; \text{ which can also be written } z = l^2/4R$$

Examples:

For light coming from the galaxy M31 in the constellation Andromeda, the shift is $z = 202.07 \text{ AL}$.

By the time we see the galaxy, it has already moved away from its true position by 202.07 AL.

For the centre of our galaxy, the Milky Way, which is 30,000 AL away from our earth, the offset is approximately equal to

$z = 1.087 \cdot 10^{-6} \text{ AL}$; a very small distance compared to the size of the galaxy and the universe. This means that the centre of our galaxy is extremely close to us if we compare it to the rest of the galaxies in the Universe.

Our solar system has at the same time moved parallel to the same distance and a supposed flash from the centre would have to travel a path of length $l = 2 \sqrt{Rz}$

For a star observed at 12.5 AL, the value of z is : $5.661 \cdot 10^{-9} \text{ AL}$.

It should be noted that an exact and fixed measurement cannot be given, as space-time varies continuously. These calculations allow us to understand the effects of the shift caused by the constancy of the speed of light and by the plasticity of space-time.

The plasticity of space-time and the deformations of light paths due to gravitation make all distance measurements uncertain. Distances are variable with time and also with space. We recall once again that, contrary to Newtonian ideas, certainty is not part of this world.

6) Leakage rate v and swelling rate V :

According to Hubble's observations, the stars move away from A (A is the observer) at speeds proportional to their respective distances from A.

The relation $\delta(t)$ and the relations derived from it will allow us to prove the previous proposition.

We have seen that A, a fixed point on an inflating balloon and two other points B and C on the same balloon such that $AB \leq AC$

we have seen that under the effect of the sole inflation of the balloon at constant speed V , we always have $v_b \leq v_c$

On the space-time membrane Universe, these points B and C which are moving away have their images B_0 and C_0 on the curve $\delta(t)$ which are also moving away at speeds v_{b0} and v_{c0} such that $v_{b0} \leq v_{c0}$.

These two points B_0 and C_0 appear to the Hubble observer to be moving away at respective velocities relative to A (Hubble position): $v_{b0} = l_{b0}/t$ and $v_{c0} = l_{c0}/t$. l_{b0} = distance from A to B_0 (B_0 is the trace of B on $\delta(t)$) and l_{c0} = distance from A to C_0 (C_0 is the trace of C on $\delta(t)$). In general, a Best galaxy seen fleeing on the cardioid in B_0 and its escape velocity is $v = (l/t) = (1/t) \cdot l$; $(1/t)$ is the Hubble constant.

We have the following relationship which follows from

$$\delta(t) = R \cdot (1 + \cos t) / 2; \text{ It is also easily demonstrated that :}$$

$$\dot{\delta}(t) = R \cdot (1 - v^2/C^2)$$

$$L = \frac{2 \cdot R}{C} = \frac{CT}{C}; \quad C = 2V; \quad R \cdot C = V \cdot L; \quad V \cdot = R/t \text{ and}$$

$$l = R \sqrt{2(1 + \cos \theta)}; \quad \text{for } t = \theta \text{ fixed}$$

$$\text{For } t = T \text{ we have } (l/t) = (l/T) = (C/2R); \quad \text{then } v = k \cdot l \text{ with}$$

$$k = (C/2R) = (3/13,8).10^{-4}/AL$$

$$v = k.l \quad \text{with} \quad k = 0.2173913.10^{-4}/AL$$

Although called a "constant", this parameter varies with time and therefore with R. This parameter, which describes the rate of expansion of the Universe, is fraught with uncertainty if one wants to determine it empirically by means of satellite observations, since the observations and measurements of the distances of galaxies are sensitive and difficult to assess. Calculations according to the above equations give a figure for the escape velocity v 70.903 km/s / Mpc

In conclusion, we state :

The escape velocity of a star from an observer fixed at A is proportional to its distance from A. This result from $\delta(t)$ is therefore the same as that confirmed by Hubble's observations. These observations therefore reinforce equation $\delta(t)$ and the relations that follow from it.

The following relations resulting from $\delta(t)$ are also verified:

*Knowing the constant k , we can determine the age of the Universe, in fact: $k = (C/2R) = C/CT) = 1/T$ and $T = 1/k = 13.8.10^9$ A

* $v = (C/\sqrt{R})\sqrt{z}$: The escape velocity v is proportional to the square root of the offset z ; we can also write

$z(t) = v^2.(R/C^2) = 0.076666.v^2$: the offset $z(t)$ is proportional to the square of the escape velocity.

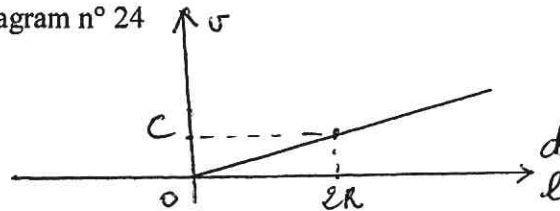
It is also shown that $\delta(t) = R.(1 - v^2/C^2)$: here we see that $\delta(t)$ is also a function of the escape velocity and if $v = 0$ then $\delta = R$ and if

$v = C$ (limit case) then $\delta = 0$

We have seen that v is also proportional to the real distance of non-apparent distance d and in the same proportion as l that is to say :

$v = k.d$ and $v = k.l$; see the following figure : diagram n° 24

diagram : 24



The conformity of the relation $v = k.l$ with the results of the observations of the telescopes can be checked: see the following examples:

1) a star located at 1 Mpc, i.e. $l = 3.261564.10^6$ AL, the escape velocity is $v = k.l = 70.903$ km/s

The escape velocity according to WMAP observations in MAY 2011 is $v = 71$ km/s.

In 2008 the value of v is 71.9 km/s/ Mpc

In 2006 the value of v is 70.4 km/s/Mpc

2) For a cluster of galaxies located in the constellation Virgo at 50 million light years, $l = 50.10^6$

AL, the escape velocity is $v = k.l = 1087$ km/s and the escape velocity according to WMAP observations in 2011 is 1100 km/s

With these examples, it is clear that the calculations according to the proposed relations lead to the same results as the empirical observations of the telescopes in earth orbits. This supports and confirms the veracity of $\delta(t)$ and the resulting relations.

should also be noted that according to specialists and telescopes in earth orbits and also according to the equations quoted above deriving from $\delta(t)$, the escape velocity v of a star with respect to our solar system, always remains between 60 km/s and 80 /s for a distance of 1 Mpc.

For very large distances, the proposition that the escape velocity of a star is proportional to its

distance from the observer A is still valid.

At very large distances, the escape velocity is no longer imprecise as claimed, but still remains proportional to the distance from the observer A without exceeding the value of C; The limit C of the escape velocity v is located at point A', which is the furthest from A, as we will see in the following examples:

*For a star located at an apparent distance (on the cardioid) $l = 1$ billion AL = 306.601 Mpc; the escape velocity is: $v = k.l = 21.739$ km/s
This corresponds to a real non-apparent distance (on the great circle) $d = 1.008$ billion AL = 306.8/46 Mpc; with $vd = 21.756$ km/s.

*For a star located at an apparent distance $l = 2$ billion AL = 613.202 Mpc, the escape velocity is $v = 43.478$ km/s. This corresponds to a real, non-apparent distance: $d = 2.007$ billion AL = 615.349 Mpc; with $vd = 43.630$ km/s.

*For a star located at an apparent distance $l = 8$ billion AL = 2453 Mpc; the escape velocity is $v = 173.912$ km/s. This corresponds to a real non-apparent distance $d = 8.533$ billion AL = 2616.23 Mpc; with $vd = 185.499$ km/s.

*Finally, for an apparent distance $l = 13.8$ billion AL (the trace of the farthest point A', theoretically one should see it at the original point O) = 4231.100Mpc, the escape velocity is $v = 300,000$ km/s. This corresponds to the real non-apparent distance $d = 21.666$ billion AL + 664 mPC with vd (theoretical) = 471000 km/s.

A' is the farthest limit point, diametrically opposite to A, which also corresponds to the limit speed C.

The star at point A' is the furthest from A; the speed $v = C$ is a limit never reached. It is as if A' is fleeing in the opposite direction to A at speed $C/2$. The resultant of A going in the opposite direction at speed $C/2$ and A' in the opposite direction to A at speed $C/2$ is the limit C.

In summary, it can be said that at very great distances the escape velocity is not imprecise, but always remains proportional to the distance from the observer A; without exceeding the speed limit of light. This limit C, which cannot be reached, is located at A', the furthest point from (A' is theoretically seen in the past at the original point O).

With empirical measurements the imprecision remains and since Hubble, debates and conflicts have divided the actors on the estimation and value of the constant k (or H_0 as it is usually called).

Other writing for $\Delta(t)$:

...(t) can also be written as a function of v (escape velocity of a considered star); this velocity which increases with the distance of the star from A is written:

$$v = k.l = (C/2R).l = C\sqrt{(1-\cos t)}/2 = C\sqrt{z(t)}/R.$$

From these equalities we can write: $\Delta(t) = O\beta_0 = R.(1-(v^2/C^2))$ (1)

from (1) we also write $t = \arccos(1-(2v^2/C^2))$ and $d = R.t$

then $d = R.\arccos(1-(l^2/2R^2))$.

How to translate proportionality:

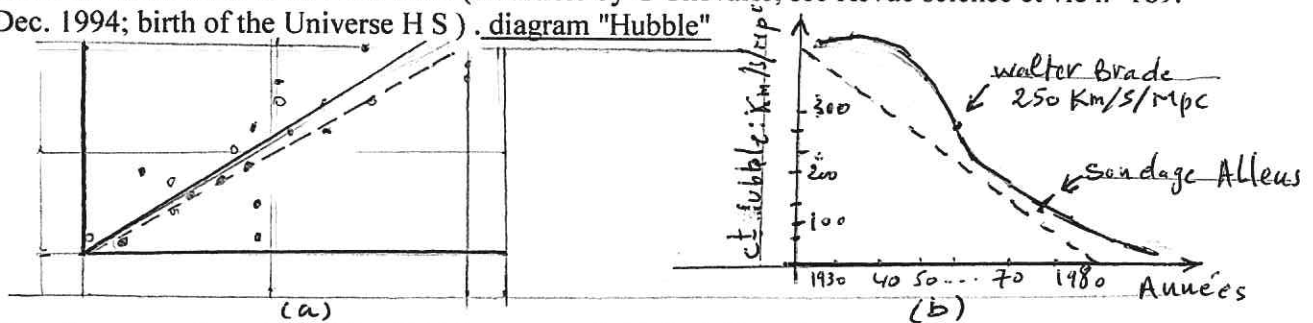
Proportionality is expressed as a straight line and the Hubble constant is the slope of this line. The point O, intersection of the two axes (distance axis and speed axis) means that the constant is at the time of the measurement. The inverse of the constant ($1/k$) designates the duration of the expansion which can be assimilated to the age of the Universe. One often used $v = H_0.d$ with $H_0 = (1/t)$ with t = the duration of the expansion and d = the distance of the observer A to the star without specifying if it is in the past time or not. We must always specify the difference between the real

non-apparent distance (d) and the apparent observed distance (l).

If Hubble's law is a straight line passing through the origin O , this does not mean that the expansion operates only from our position (position of observer A) but it is always assumed that Hubble's law would be established whatever the position of A . This is based on the fact that we consider the Universe as a whole, totally homogeneous and isotropic on a large scale at any instant. Do not forget that homogeneity and isotropy are hypotheses that have never been verified; but also without simplifying hypotheses one does not advance in physical science and especially in cosmology. It may also be noted that the equations of general relativity, which describe the interactions of space-time and matter and which also explain that curvature is always related to the gravitational field, can provide the age of the Universe in another way, depending on the density of matter. Other scientific methods independent of our subject can also do this (the analysis of radiation from old stars in the depths of the Universe

7) History: Hubble's constant:

The diagram below is original (figure (a)); it was published by Hubble himself in 1929. In this diagram, the black circles represent individual nebulae and the circles represent groups of nebulae. The solid line is for the circles, it materializes the relation of proportionality between the distance which separates us from the visualized star (in abscissa) and the speed of escape (in ordinate). The dotted line is for the circles. These lines represent Hubble's law. The controversy about its exact value is far from over. (cf. article by G Chevalier; see Revue science et vie n° 189. Dec. 1994; birth of the Universe H S) . diagram "Hubble"



Over the years, the value of the Hubble constant initially followed a strongly decreasing curve to stabilise today at about 70km/s/Mpc (see M Rowan-Robinson cf note 11_ sciences et vie n°189) : figure (b)

8) The speed V of expansion (or swelling) of the Universe:

We have seen that the speed of expansion is $V = C/2$. We want to prove that this speed is empirically verified. We know that according to the observations of specialists in the field, the escape velocity of a star with respect to our position (point A : from the earth) for a distance of 1 Mpc is generally between 60 km/s and 80 km/s (already mentioned above): we have $v = (C/2 \cdot R) = (C/2) \cdot (1/R)$; on the other hand $v = \theta V = (d/R) \cdot V = R \cdot \arccos(1 - l^2/2R^2) \cdot V/R = V \cdot \arccos(1 - l^2/2R^2) \cdot (1)$ for $l = 1 \text{ Mpc} (1 \text{ Mpc} = 3.61564 \cdot 10^6 \cdot \text{AL})$; we have : $\arccos(1 - l^2/2R^2) = 1/R = 4.72669 \cdot 10^{-4}$

By identification in (1) we have : $V = C/2$

Examples :

For 1 Mpc and for $V = 150.000 \text{ km/s}$ we have $v = 70,9 \text{ km/s}$

For 2453 Mpc or 8 billion AL we have $v = 173912 \text{ km/s}$

The proposed diagrams and curve give us, as indicated by the writing above: $v = C/2$ and this supports the validity of these formulas. During inflation, the speed of the expansion is supposed to be close to the speed of light (a little lower).

9) The speed V of the expansion of the Universe (the speed of inflation)

9) **cannot exceed C during the inflation period:**

Even during the inflation period, the speed V of inflation cannot exceed the speed of light C , in fact during inflation:

*If $V > C$ the light is no longer in the space-time, i.e. it is no longer carried by the space-time membrane; in what and on what will it advance?

*If $V = C$ the light advances with the space-time membrane and in this case we will not have any darkness and the light floods the Universe.

*It remains $V < C$: the light which starts from the original point O_d must follow a path different from that of point A (taken arbitrarily) it is the path of light $\Delta(t)$ located in the past (in the space-time sphere); a path curved in such a way that this light arrives at point A at the same moment as the space-time membrane. In this case we will have darkness (the sky is always black) V is therefore less than C .

During inflation, the speed of expansion V was apparently a little less than C . This speed decreased with the expansion to half of C . If the speed of expansion during the period of inflation was greater than C , then it must be accepted that any explanations for this period are not within the scope of the theory of general relativity and C is not exceeded by any other speed.

10) Shrinkage :

A stick of length l moves at speed v in the length direction. If v approaches C , the speed of light, the stick decreases in length on the side of the direction of motion. Space shrinks and bends.

11) Bundle of light lines:

In the manner of a field, the lines " light paths " advance and follow each other continuously from the origin O the present one in A , this one continues to advance on the axis OY : $OA_1, OA_2, OA_3, \dots, OA_n, \dots$

These lines $\Delta(t)$ start from O and arrive on the space-time membrane in A .

When the space-time sphere swells, the pear (or heart) set of image points of this sphere swells at the same time inside the sphere (in the past). When the sphere swells, everything moves in a continuous way.

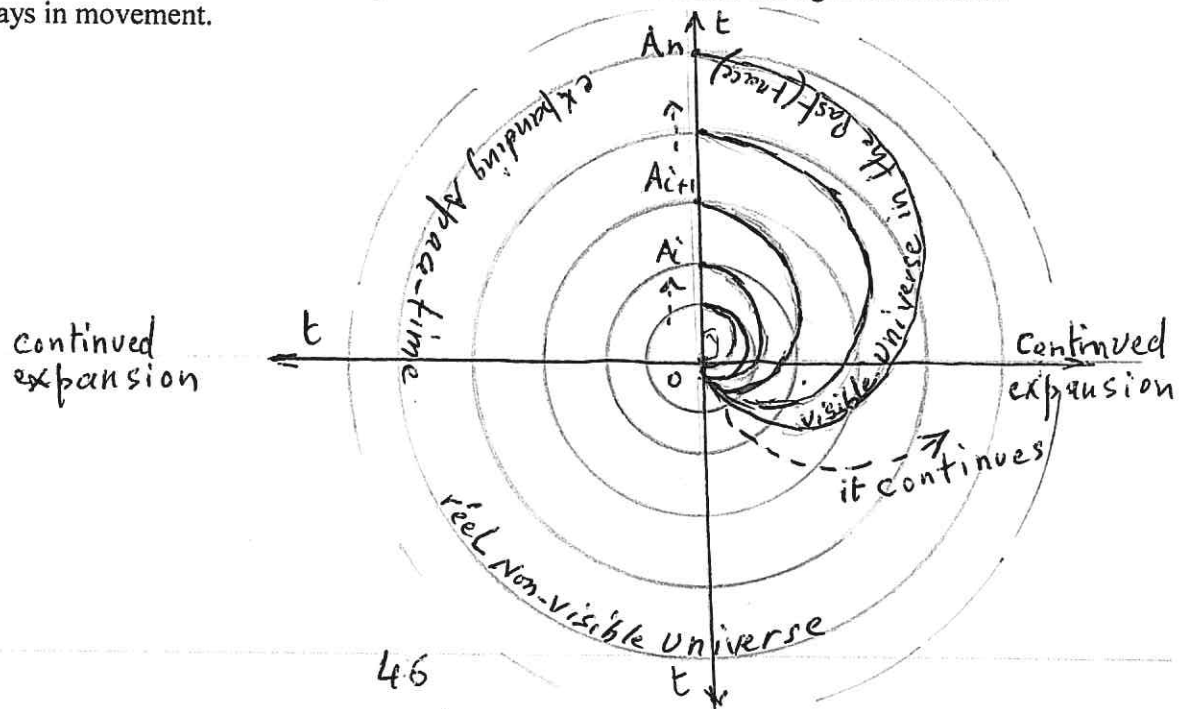
Figure 25

Bundle of continuous lines: the paths of light.

Like a field, the lines advance and follow each other continuously from the origin O (the original point) to the present in A , which continues to advance on the axis OY and also on the circle: $\dots O A_i, O A_{i+1}, O A_{i+2}, \dots O A_{i+n} \dots$

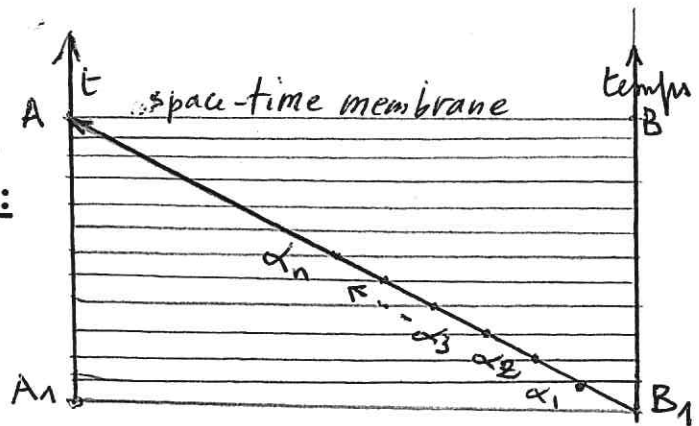
The lines $\dots(t)$ start from O and arrive on the space-time membrane in A and the light advances on these curves always in movement.

Diagram : 25



12) Displacement of fossil photon:

diagram : 26



The parallel lines represent the space-time membrane moving in the direction of the arrow: time (the radius r of the space-time sphere is infinite).

The photon always remains on the space-time membrane and it is the displacement of this one which makes the photon appear to change position: see the diagram above: the light leaves B_1 towards A_1 to find A_1 in A ;

The photon released in B_1 remains always on the same sphere in extension (or on the same plane in extension) until its arrival in A but without travelling, it is the case of the fossil radiation: see figure above, B is immobile relatively to the 3 dimensions of space, but varies only on time, thus on the space-time. The photon takes successively and continuously the positions on the inflating membrane, represented by the concentric circles (or parallel planes, because the radius R of the inflating balloon which is the Universe tends towards infinity) until the point A and respectively also, B_1 takes in a continuous way the positions $B_{11}, B_{12}, B_{13}, \dots, B_{1n}$, on the same inflating membrane represented by the same concentric circles (or parallel planes since R tends towards infinity) and on the axis of time until B .

More precisely, it can be explained as follows;

When B_1 is on position B_{11} then the photon is on position α_1

When B_1 is on position B_{12} then the photon is on α_2

When B_1 is on position B_{13} then the photon is on α_3

etc..... until α_n arrives at A and B_1 arrives at B continuously. The same approach is generalized for the other points: C, D, E, F , etc....

In summary, the photons have travelled from B_1 to A_1 , but they only meet A_1 at A . If we look further, we see C in C_1 , and again D in D_1 etc....

These hatches represent the successive and continuous positions of the space-time membrane containing the photons;

The photon does not travel, it is always on the same space-time membrane which continues to swell. It is stable in 3 dimensional space and what varies is the fourth dimension which is time.

Light and photon :

Time does not pass for a photon, it does not age. The space and the distance crossed by the light mean nothing for the photon. It does not travel, it has been and will always remain on the space-time membrane which carries it with it as a result of its inflation (see the following diagram).

A big circle for each direction:

It should be noted that each direction has a great circle and we have arbitrarily taken only one for the analysis of the sphere, in short, we cut the sphere by a plane passing through the origin and we take the edge of the resulting disc which is a circle for the analysis.

13) Fossil radiation :

The age of the Universe could be determined by knowing the age of certain stars (by analysing their lights). It could also be determined by knowing the age of certain atoms as they begin to decay over time. It could also be based on the extension and distances of galaxies. In any case, specialists put forward approximate ages of 14 billion years (in the calculations we took 13.8 billion years). The fossil radiation that Arno and Wilson discovered was radiation for which there is no known source in the universe. It is a pale glow that is evenly distributed in the Universe and no corner of the Universe is completely empty. This faint glow comes to us as a millimetre wave radio, detectable with appropriate antennas in all directions of the sky. It is the world's oldest 'residue' and what telescopes show us is the world's oldest image. Earth-orbiting telescopes have detected a strange, glowing substance in the shape of a flat elliptical pattern on which we see tiny temperature differences indicating fluctuations, this means that the cosmological background is not at the same temperature. There are minute structural differences consisting of striations, folds and wrinkles.

Temperature :

The current temperature of the Universe is 2.736°k (absolute zero is about $-270^{\circ}\text{c} = 0^{\circ}\text{k}$); this is the temperature of the uniform radiation that was put into the Universe before 3000°k . There is a relationship between the temperature and the concentration of photons in this radiation; specialists give us the figure of 411 photons per cm^3 , the rest of this volume is occupied by photons emitted by the stars of the Universe.

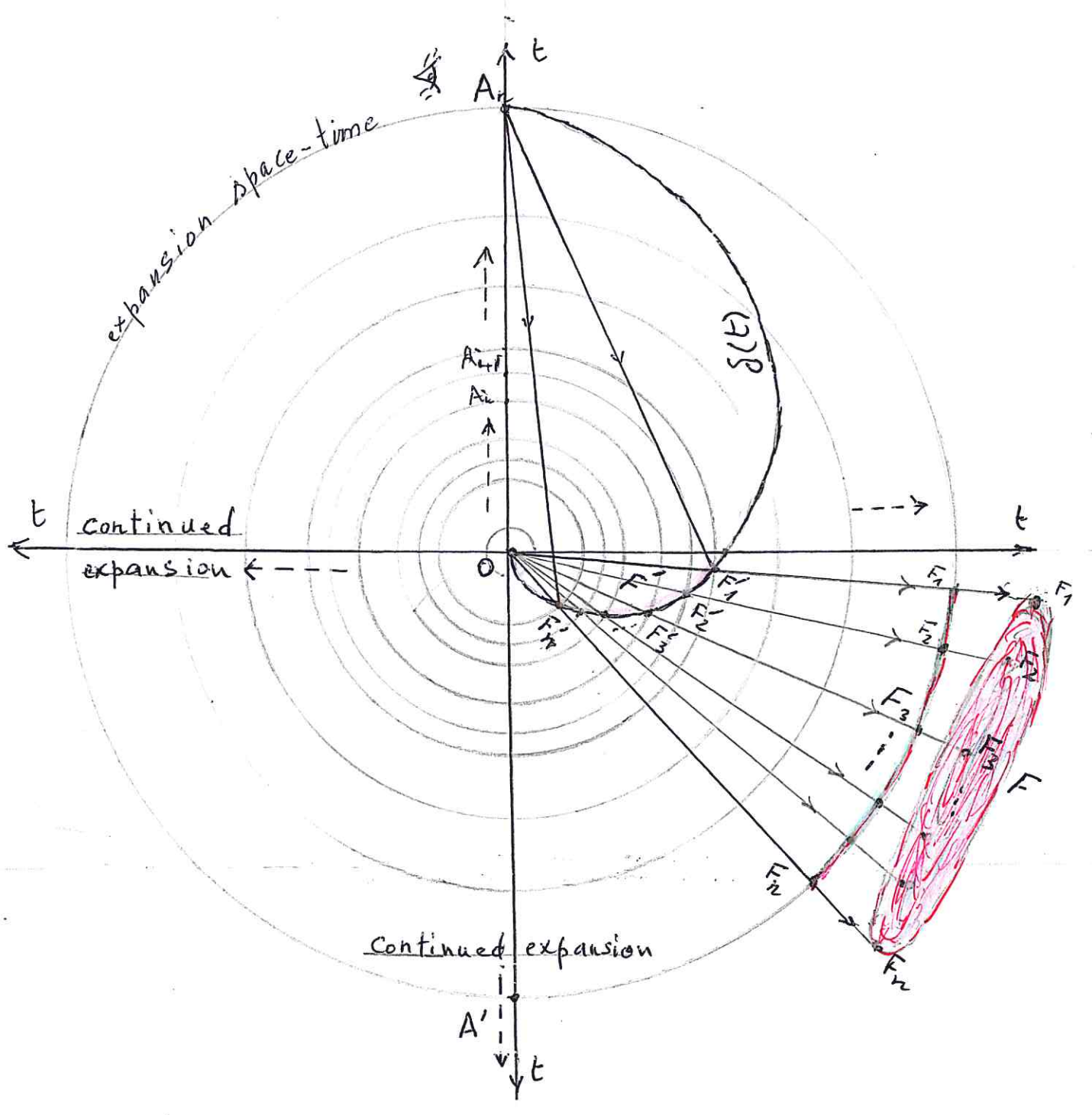
The temperature of the Universe has continued to fall since the great blow of creation. The Hubble Earth Orbiting Telescope has seen a galaxy 12 billion light years away that is bathing in the Universe at a temperature of 7.6°k ; this temperature has therefore dropped to the current temperature of 2.736°k , a drop predicted by the theory of general relativity.

In Figure 27:

Each point of the observed glowing figure belongs to a different concentric circle or to a different space-time membrane and what we see as an elliptical shape is not the whole Universe, but only a small part of it. We see the part (F') image of (F), F is at the same level as the observer in A. Each point of this diffuse background has its own time and therefore its own space-time.

Diagram : 27





The radiation currently located at (F) at the same level as (A) is seen at (F') on the cardioid $S(t)$. If the observer in (A) looks at the opposite side (he makes a 180° turn), the radiation (K) is seen in (K') on ... (t). The two regions, separated by a very large distance, are physically similar since they have made the same historical journey (both regions have the same age, they can look alike).

F: cosmic microwave background; not visible on the same level as A

F': image in the past F is seen in F' in the past

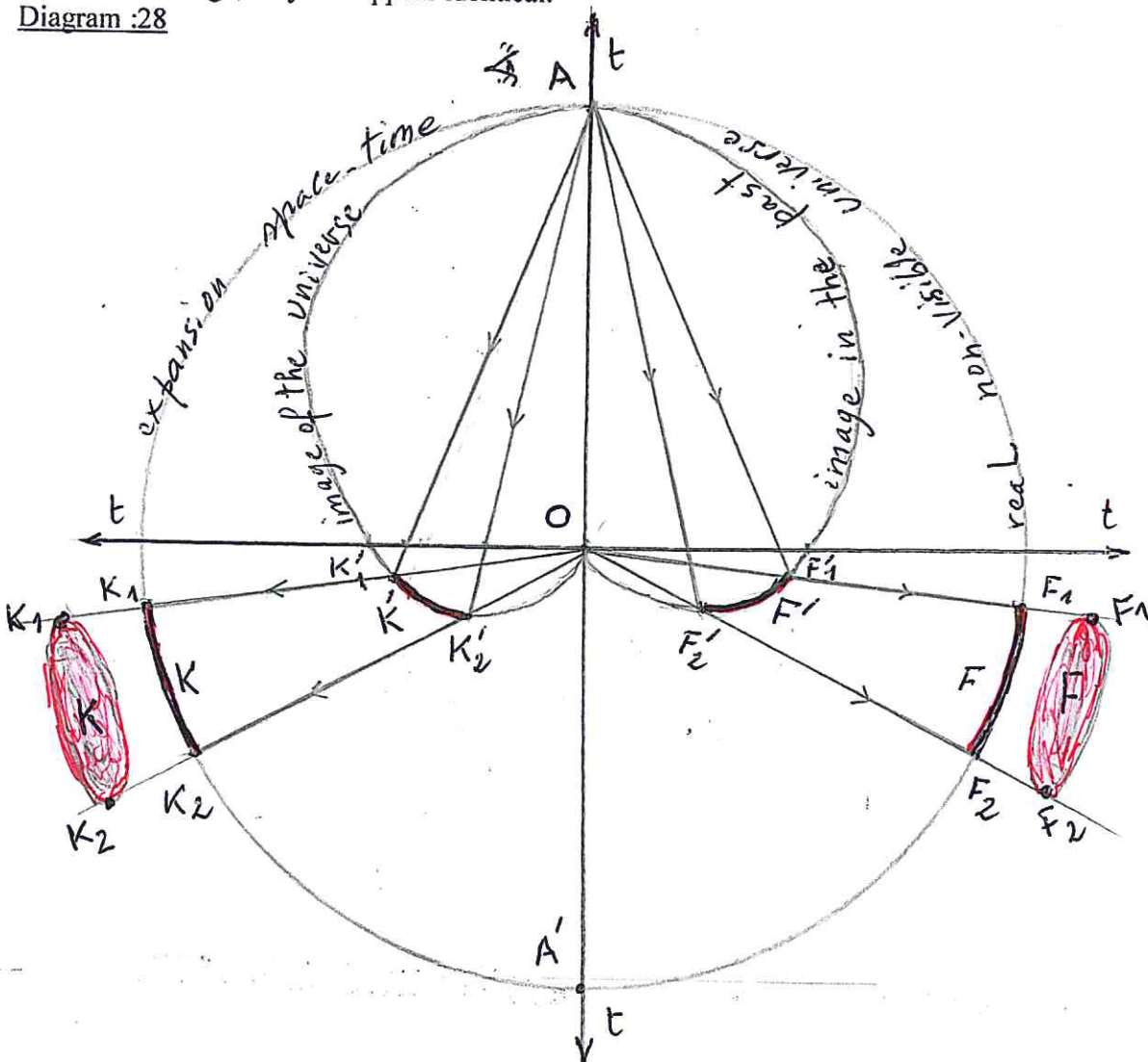
The concentric circles in continuous extension represent the space-time membrane in continuous extension. The fissile ray currently located in F, at the same level as A, is seen in F', its image on the cardioid(t). The points of (F') are on different circles, i.e. on different space-time, so it is possible that they are physically different.

The concentric circles in continuous extension, represent the space-time membrane in continuous extension too. The fossil radiation currently located in (F), at the same level as the observer in (A), is seen in (F') its image. The respective images F'1, F'2, F'3, F'n of F1, F2, F3,.....Fn are on different times, i.e. on different space-time.

On the figure n°27 : each point belongs to a different level of the others, to a different circle (we always remember that the circles represent the spherical membrane space-time in continuous extension); then if we see two disjoint regions which seem to us to have the same physical properties, they are not it in reality, because we see them on different times, on different concentric circles: I have two photographs in my hand, mine when I was 40 years old and my father's when he was also the same age, by seeing them, should I draw the conclusion that I was born on the same day as my father?

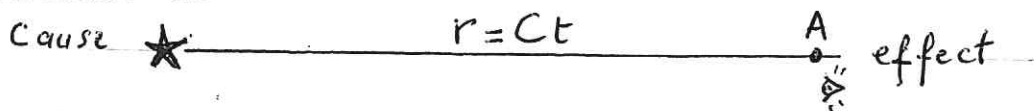
The two disjointed regions that appear identical may also belong to the same concentric circle, i.e. at the same space-time membrane, there the two regions could have the same historical path, also have the same age, they can appear identical.

Diagram :28



14) Cosmological horizon :

A lamp is turned on, the illumination of the surrounding objects is not instantaneous; it would take a very short time for the light to fill the entire surroundings. A flash on the surface of the sun arrives the Earth after 8 minutes and at Jupiter after 40 minutes, and a flash on the surface of a star requires a time depending on the distance the light travels to arrive at a given point. The radius of the causal sphere increases with time. There is a time delay between the cause (the source of the light flash) and the effect (the arrival of the signal), because the speed of light is constant. There is then a causal sphere of radius $r = C.t$



The cosmological horizon can be defined as the limit of the universe observable from a given point (the Earth or observer A).

The most distant signals come from the cosmic microwave background and the region from which they originate is about 12.5 light years away. This figure defines the cosmological distance: it is the length of the curved line (almost to the original point O): see Figure 29.

Horizons exist because light has a finite speed. There are stars whose radiation has not yet reached us, because the speed of light C is very small compared to the size of the universe.

A problem concerning the cosmological horizon that was discovered almost by chance; two distinct cosmological regions but with almost identical physical aspects / how to explain this harmonisation without interaction?

At any moment, there will be regions in the Universe that have not yet seen each other, regions that cannot see each other cannot in principle influence each other. In spite of the separation, we can see that they have similar physical characteristics: same temperatures, same pressures, same concentrations of matter, etc..

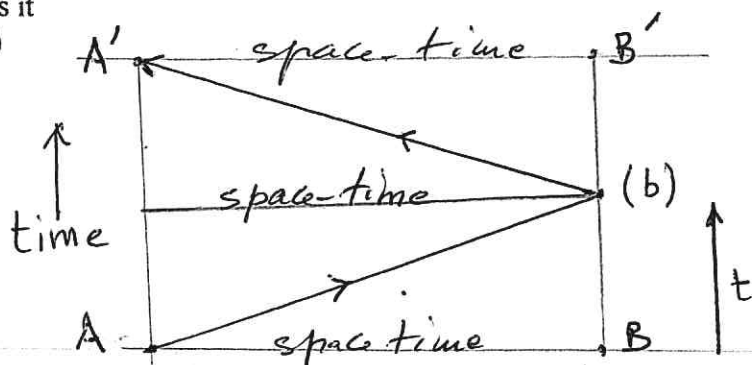
.how to explain that disconnected regions could have exchanged information at a certain period so that they could appear similar today.

For this problem some answers have been put forward by the remarks and suggestions on diagrams n° 27 and n° 28.

In addition the horizon is a horizon perceived in the past and not seen directly. The points seen belong to different time-spaces: see diagram n° 29.

NB: Time changes, therefore space-time changes and consequently space also changes;

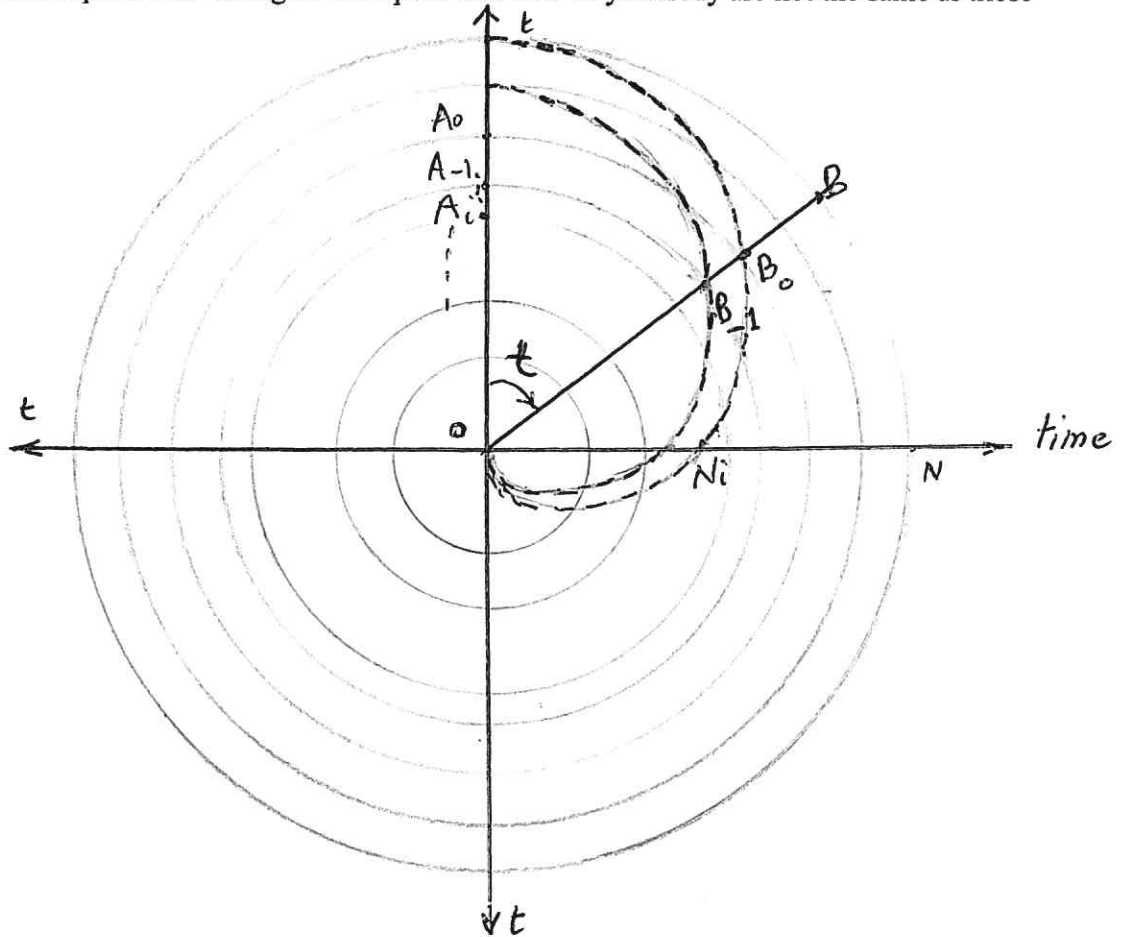
he starts from A and finds B in (b)
when he returns to A, he finds it
in A' (time and space change)



A mobile starts from point A to point B. In a Euclidean space, when the mobile returns to point A, it always finds it at the same place, For space-time, it is different: time has changed and therefore space too since space and time are only one thing: space-time. The swelling or expansion of the space-time membrane has caused the space-time position of points A and B to change as well.

The parallel straight lines (or planes) represent the space-time continuum. It is not only time that changes, the whole space-time changes. The space and time of yesterday are not the same as those of today

diagram 29:



*Cosmic horizon: The distance between observer A and the distant star B (the arc AB), this distance is related to the epoch where its cosmic time is the same as that of observer A. The flash starts from B when it was in B_0 and also when A was in A_0 . The flash and the observer in A_0 arrive at the same time in A. The flash has made the path B_0A (curvilinear distance) : $B_0A = H = C.t$: this is the cosmic horizon . Moreover $AB_0 < AB$ with $AB_0 = R.(2(1-\cos t))^{1/2}$

On the other hand, answers based on the cosmological inflation of the first phase of creation can be challenged because under the effect of the extension, matter changes positions. When a star sends out a flash, its light describes a straight line (the path of light $\mathcal{L}(t)$ is a straight line in space-time,) \times but if we assume a time independent of a space considered as Euclidean, $\dots(t)$ would be a real straight line); on the scale of the Universe $\dots(t)$ is a straight line; $\mathcal{L}(t)$ is in fact a cardioid. To have all the light paths, we rotate $\mathcal{L}(t)$ around the axis OA to obtain a pear inside the space-time sphere

Let a galaxy be located at a distance of x light years from observer A. Its light takes x years to reach

On the other hand, the k_0 point (at the same level as the observer when he was in A_0), he does not see it, this point is not in his horizon, he will see it only when he will be in A_1 and the point in question will be at that time in k_1 . The points to the right of the A_0O curve (outside the cardioid) are not yet accessible to him.

The cosmological horizon (the whole surface of the heart except the radius OA) widens with time, i.e. with the displacement of point A (the observer) on the axis OY , i.e. also with the expansion of space-time (the membrane space-time sphere) .

See diagram: The observer is currently in A_0 ; he can see all the points (or stars or galaxies) located on the cardioid curve (light path curve) A_0O : this is the horizon in question. All that is on A_1O , is not yet accessible, for example the points ...2 or ...3, the observer meets them only when he is on the position A_1 ; currently they are inaccessible and all that is on the right of the curve A_0O is not yet part of the accessible; it is still outside the cosmological horizon. On the other hand, all the points on the A_1O curve have already been seen and therefore all the points to the left of A_0O have already been seen (the hatched part has been seen).

The cosmic horizon represented by the curve A_0O can only be variable and varies with time. The regions already seen are in the past and the regions not yet seen are in the future, we will see them later. The horizon is twice the length of the radius of the Universe and increases proportionally with the age of the Universe.

How to determine the age of the Universe: (a small idea)

According to the current theory, the age of the Universe is determined by studying the shape of its main sequence (Hertzsprung and Russel diagram: a diagonal on which the majority of the stars are found), this method is used to try to find the age of the oldest star clusters.)

Age can also be determined by nuclear physics using the evolution times of the atomic nuclei of uranium-235 (which contains 143 neutrons and evolves over one billion years) and uranium-238 (which contains 146 neutrons and evolves over 6 billion years).

Estimates based on nuclei are imprecise but independent of other methods.

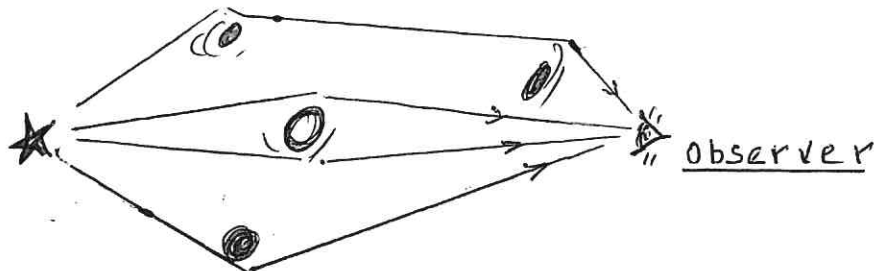
The age of the Universe can also be determined from the Hubble constant $H_0 = 70\text{km/s/Mpc}$ and $(1/H_0) = 14.10^9 \text{ M.A}$

In general, the estimates are globally consistent with nearby values.

15) Luminous distortions and the extent of the Universe:

Gravitation is also a reflecting mirror: so far we have disregarded the influence of matter and gravitation on the path of light, but in reality in front of obstacles and under the attraction of gravitation the light ray deviates and can take multiple paths. The obstacles between a light source and us at like galaxies can act as mirrors, reflecting lenses, therefore, the Universe as we have already noticed is full of light distortions; the images of a single light source can appear to reach us from several places; thus, gravitation can act as mirrors, reflecting lenses (already seen: Einstein's corona) diagram n°31

diagram n°31 :



If you line the walls of a room with mirrors, it gives the illusion of a larger room through the play of

multiple reflections on the walls. You have the impression of seeing infinity and it could well be so of cosmic space. This multi-connectedness would create additional paths in the Universe for the light rays that reach us from distant galaxies and it is impossible to recognise and distinguish the real images from the phantom images.

We may be living in a cosmic optical illusion, giving us the impression that we belong to an infinitely large Universe, when in reality it is not large enough. The sky seems to be more full of stars than it actually is. In spite of all the light that pervades the Universe, darkness persists; this darkness is also another proof of the Universe's extension.

16) Visible and real Universe according to the current theory:

According to the current theory, the optical illusion or topological mirage makes the real Universe smaller than the

visible Universe (lecture by Jean Pierre Luminet on the crumpled Universe, on you tube.....).

The topological mirage makes the volume of real space equal to 80% of the volume of the observed space (lecture by Professor J.P.Luminet).

Space is curved by energy and matter, so there is a cosmological horizon that limits the view. The cosmological horizon is a sphere with a radius of 53 billion AL, whereas the real extent of the Universe is estimated at 43 billion AL;

The mirage does not necessarily increase the distances, but it does multiply the bright elements in space. Another argument can contradict this point of view by insisting on the fact that the real Universe is larger than the observable, in fact, at the moment when we receive the flash of any light source in space, this source has already moved away from its perceived position; therefore the real distance at the moment of the arrival of the said flash is greater than the distance observed in the past.

In the Universe, distance measurements are not constant, they vary and increase with time and the Universe observed in the past is smaller than the actual one seen now: The galaxy M31 in the constellation of Andromeda of apparent distance $l = 2.3616 \cdot 10^6 \text{ AL}$, its actual non-apparent distance is $d = 2.3635 \cdot 10^6 \text{ AL}$.

17) Different levels of curvature :

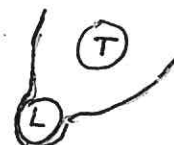
There are different levels of curvature (or hollows), as can be seen in the following figures. One passes from the hollow of the large bolide which wanders in space, to the hollow of the moon, to that of the earth, to that of the sun; to that of the centre of the galaxy, to that of the galaxy, to that of the galactic group etc.....

The trough is the region of space around a mass (matter) with an intense gravitational field that holds the satellites of that mass. The trough or curvature depends on the mass (planet, star, galaxy) and the curvature takes a multitude of forms: the trough of the galaxy contains the sun and its trough; that of the sun contains the earth, Jupiter, Saturn, Mars, etc. with their respective troughs and that of the earth contains the moon and its trough and so on everywhere: see figure n°32

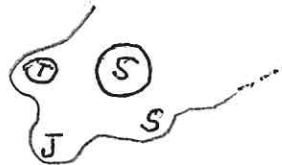
At the level of the whole Universe, the hollows are the manifestation of gravitation which opposes expansion. Two hypotheses: either this expansion will continue indefinitely, or it will slow down and then reverse to the initial state (this is the opinion of some). A third possibility is that the matter in the galaxies will be swallowed by the black holes and pass into nothingness, space-time will end up like a rag with a hole in it, crumpled and folded in on itself and then return to nothingness.

Figure 32: The different levels of curvature:

a) The moon and its hollow in the hollow of the Earth



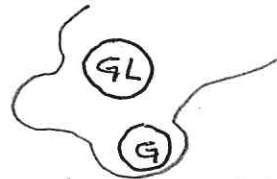
b) The Earth and its trough, Jupiter and its trough, Mars and its trough etc. are in the trough of the Sun



c) In the hollow of the black hole of our galaxy, billions of stars gather to form the galaxy, of course including the sun and its hollow



d) In the hollow of the local group (the galaxies of the local group are approaching each other to form in the future, perhaps a single gigantic galaxy) our neighbours and our own galaxy are gathered



e) This last diagram tries to summarize the 4 preceding ones, i.e. a; b; c; and d:



NB: The drawings are made on a plane to explain an idea, the reality is made in a space-time with 4 dimensions

Note :

The overall shape of space-time finds its rules only in irregularities and fragmentation, comparable to snowflakes and bronchioles. Irregularities manifest themselves in hollows, craters, eminences and protuberances, and this is how it is for the billions of stars and galaxies. In addition to this, space-time is subject to gigantic gravitational waves that cause matter to wobble and jerk. Under this effect, the galaxies float like a flag under the effect of a violent wind.

IV DOPPLER EFFECT: ADEQUACY OF THE RELATIONSHIP $\delta(t) = R.(1-v^2/C^2)$:

1) spectrum and velocity of distance : Light is an electromagnetic radiation which is broken down into fringes of different wavelengths (or different frequencies). Spectra are made up of lines corresponding to a limited number of wavelengths obtained by electrical excitations of certain gases. These are electronic transitions: when an atom is excited, an electron can move from a layer n to a deeper one (n-1) and thus to a lower energy state and emit fluorescent radiation, then the atom de-excites and returns to its ground state. The energy produced by the photon is $AE = E_n - E_{n-1} = h.C/\lambda$ with

- $h = 6.62.10^{-34}$ /s: Planck's constant
- $\mu = C/\lambda$ the frequency of radiation
- λ = wavelength of the radiation
- C = speed of light

The Doppler-Fizeau effect determines the shift in the frequencies of an observed light wave between transmission and reception measurements. The relativistic calculation is performed when the distance between transmitter and receiver varies with time. Whether the receiver is fixed and the source is moving or vice versa, the Doppler effect is always symmetrical and depends only on the relative speed between the transmitter and the receiver.

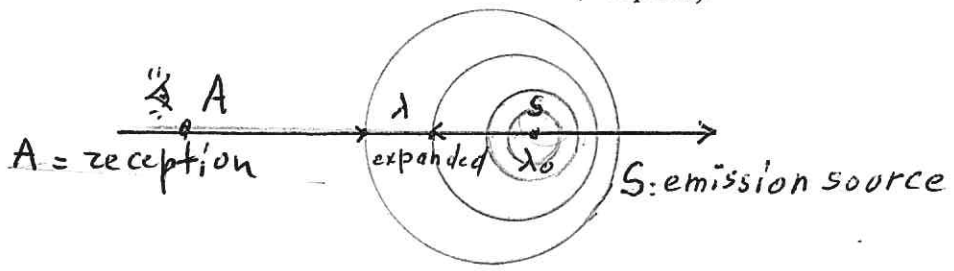
The relationship between the relative frequencies of the transmitter and receiver remains the same:

$$Fr((reception)) = \sqrt{(1-v/C).(1+v/C)} . Fr(source) \quad (1)$$

The receiver receives the wave with a frequency $Fr(reception)$ and the source transmits the wave at the frequency $Fr(source)$.

With v = relative speed between source and receiver, it is an algebraic speed, i.e. it is positive if there is distance and negative if there is approach. The receiver is of course our observer (A) and the source is the galaxy or the distant star. The displacement of the source does not affect the celerity of the light, but dilates the wavelength(reception) and thus acts on $Fr(reception)$.

Figure 33:



- C: light velocity
- V: relative speed source - observer, its sign is positive when the source and the observer move away from each other.
- T: duration of the period

λ : wavelength during a period T, and we have $\lambda = C.T$

The source (S) moves away from (A) by a distance $v.T$ and the wavelength is changed by this relative velocity v of $\Delta\lambda = v.T$

then we have: $\lambda + \Delta\lambda = \lambda + v.T = \lambda + v.(\lambda / C)$ donc $\Delta\lambda = v.(\lambda / C)$
 then $(\Delta\lambda / \lambda) = (v / C)$

The received wavelength is dilated by $\Delta\lambda$ with respect to the emitted one because of the distance of the galaxy and thus the extension of the Universe.

Example: if $\Delta\lambda = 1\text{\AA}$ and $\lambda = 6000\text{\AA}$, the relative speed of the source is $v = 50\text{km/s}$

Remember that 1\AA (angstrom) = 10^{-10} metres

The speed of the galaxy can therefore be calculated from the spectrum by doing :

~~$\mu = (\Delta\lambda / \lambda) = (\lambda - \lambda_0) / \lambda_0 = v / C$ with~~

.....

μ : Doppler shift : when we receive light from a distant galaxy and because of the expansion of the Universe between where it was put .

λ : wavelength measured by the observer (A)

λ_0 ; wavelength of the source

v : speed of the moving source (star or galaxy)

C ; celerity of light

The Doppler formula remains valid since the escape velocity of the galaxy v remains small in front of the celerity C of light.

Using $v = k.l$ with $k = C/2R$ and $l =$ the distance between the observer and the light source.

$2R$; being the distance travelled by the light from the original point O ,

$$2R = 13.8 \cdot 10^9 \cdot \lambda$$

The relationship between the frequencies of transmission and reception (l) can also be written as follows:

$$F(\text{reception}) = \sqrt{(1 - l/2R) \cdot (1 + l/2R)} \cdot F(\text{source}) \text{ or also}$$

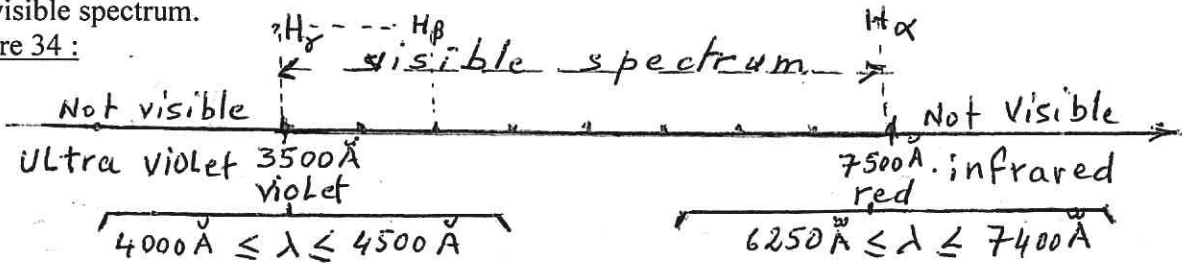
$$F_r(\text{recep}) = \sqrt{(1 - 0.0724637 \cdot 10^{-9} \cdot l)(1 + 0.0724637 \cdot 10^{-9} \cdot l)} \cdot F(\text{source})$$

i.e. the ratio of the frequencies $F(\text{recep})/F(\text{source})$ is only a function of $l =$ distance of the source from the observer.

2) The hydrogen line :

White light is composed of radiation with wavelengths between 3500A and 7500A. This is called the visible spectrum.

Figure 34 :



Near 3500Å is violet and below that is ultraviolet, not visible to the naked eye. Nearly 7500Å is red and beyond that is infrared, also not visible. The spectrum also has black lines which correspond to the wavelengths absorbed when white light with a continuous spectrum passes through a gaseous body. The photons are absorbed by the atoms of the gas through which they pass and these photons disappear leaving black lines on the initially continuous spectrum.

The hydrogen spectrum is the set of wavelengths present in the light that hydrogen is capable of emitting.

We are interested in hydrogen because it is the simplest and most common atom in the Universe, and it has the simplest spectrum.

The first lines of hydrogen are located in the visible range of the spectrum:

H_{α} : 1st Hydrogen line : ... $\lambda_{H\alpha} = 6562\text{\AA}$: it is red

H_{β} : is blue with $\lambda_{H\beta} = 4861\text{\AA}$

H_{γ} : is purple with $\lambda_{H\gamma} = 4340\text{\AA}$ etc.....

to the line with $\lambda = 3646\text{\AA}$, the latter being the Balmer limit. The wavelengths corresponding to red are

$$6250\text{\AA} \leq \lambda \leq 7400\text{\AA}$$

For violet:

$$4000\text{\AA} \leq \lambda \leq 4500\text{\AA}$$

and for visible light $4000\text{\AA} \leq \lambda \leq 7400\text{\AA}$

Note that by taking into account the complementarity between emission and absorption spectra, the

monochromatic chemical species constituting the emission sources can be determined.

We therefore consider the hydrogen line with $\lambda_0 = 6562.8 \text{ \AA}$

$\mu = (\lambda - \lambda_0) / \lambda_0 = v / C$, $v =$ speed of the galaxy, v is positive

$v = (\lambda_{\text{measured}} - 6562.8 \text{ \AA}) / 6562.8 \text{ \AA} \cdot C$ (2)

It was the measurement of λ that allowed Hubble to show that the Universe is expanding.

3) Adequacy of the algebraic expressions derived from ... (t) with the Doppler effect :

Determination of the distance (l) between source and receiver using the equality $v = k \cdot l$

Using the equality $v = k \cdot l$ with $k = C / 2R = C / 13.8 \cdot 10^9 \text{ AL} = 0.2173913 \cdot 10^{-4} / \text{AL}$

We can write $\Delta \lambda / \lambda_0 = \dots 1/2R$ with i.e. $\Delta \lambda = \lambda - \lambda_0$ i.e.

$(\lambda_{\text{measured}} - 6562.8 \text{ \AA}) / (6562.8 \text{ \AA}) = 1. \text{ AL} / 13.8 \cdot 10^9 \text{ AL}$ (3)

Knowing the received (or measured) wavelength, we can determine the visible distance (l) between the transmitter and the receiver $l = 2R \mu = 13.8 \cdot 10^9 \text{ AL} \cdot \mu$

Equality (3) can be verified.

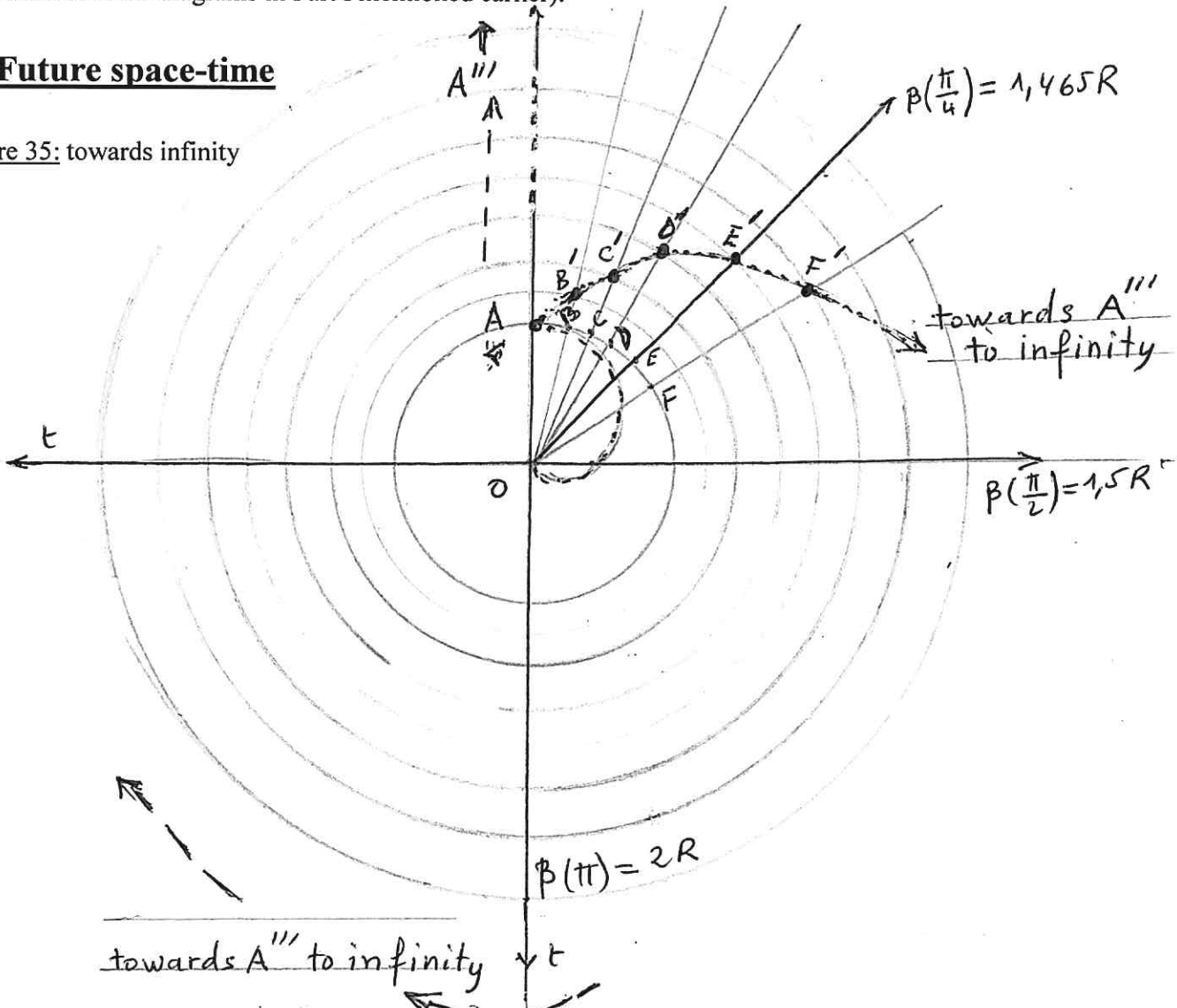
if $l = 1 \text{ Mpc}$ (1 Mega parsec), then $\lambda_{\text{measured}} = 6564.435 \text{ \AA}$ and the escape velocity is $v = (1.55033 / 6562.8) \times 300000 \text{ km/s} = 70.904 \text{ km/s}$

This result supports and confirms the formula (2) and once again the equality $k = C / 2R = 0.2173913 \cdot 10^{-4} / \text{AL}$ and consequently the validity of $\beta(t)$ i.e. that :

IF the Universe is quasi-hyper-spherical in a 4-dimensional space, its observable image (in the past) is a "quasi-hyper-heart" in another 4-dimensional space adjacent to the first one. (see construction of the diagrams in Part I mentioned earlier).

V) Future space-time

Figure 35: towards infinity



Previously we have studied the case where we have received light; now we are the source of light. We send a ray from point a, from our position on the earth, to point B (see diagram n°35; this is a fictitious and imaginary case because our view is towards the past). The light ray can only meet B in B', outside on another advanced space-time membrane sphere, i.e. in the future, and if we send it to a point C further away than B, it will meet the latter in C' more advanced than B' and so on until point A' diametrically opposed to A which will meet it at point A'.

We use the same technique of construction of the path of light of the curve ... (t) of the past space-time (inside the sphere). We obtain a future light path curve .

A supposed observer at point B sees the light beam coming to him from A, the transmitting position, at that moment, point A, has already advanced to another position \ddot{A} at the same level as B, i.e. on the same great circle where B is located (or on the same space-time sphere). It is as if light had been sent from \ddot{A} , but with an advance of $z(t)$.

In order to respect the shift of the light caused by its constant speed, as we have already done (this time it is point A which is backward with respect to B), we add the quantity $z(t)$ to the unit instead of subtracting it.

The equation of the curve to be constructed is: $\beta(t) = R.(1+z(t)) = R.(1+(1-\cos t/2)) = (3 - \cos t) \cdot R/2$

~~cos t/2~~

Such a function is increasing on the interval: $0 \leq t \leq \pi \dots R$

Theoretically if the light continues straight on this path, the flash returns to A when it is in A''' (see figure), it returns to A after a very long time of billions of billions of years and after having travelled a very large distance of billions of billions of light years.

On a large scale this curve is a straight line of length equal to C.T

where C is the speed of light and T is the time taken for the distance covered by the flash of light

.It is summarized as follows: when we receive a flash (from far or near) it is from the past, it comes from inside the space-time sphere on the membrane spheres, but when we send one, we send it

towards the future, towards the outside of the current space-time membrane on the expanding membranes.

The curve shown is virtual and simply serves to clarify the problem of the extension of the Universe and the displacement of light. It also serves to schematise the theory of general relativity.

VI) A universe wave :

See figure 36:

We are situated with our backs to the wall (the wall is here the space-time membrane) looking towards the interior of the sphere, towards the past; this wall marking the end of the past and the beginning of the future, on the interior surface (or on the interior space very close to the surface) of the space-time sphere in extension, in the range of existence which is diluted in the past (see the hatched and dark part)

As I have already noted, we are at the intersection of two nothingnesses: the past that no longer exists and the future that does not yet appear. The intersection of these two nothingnesses is therefore our present; in front of us is the Universe in the past.

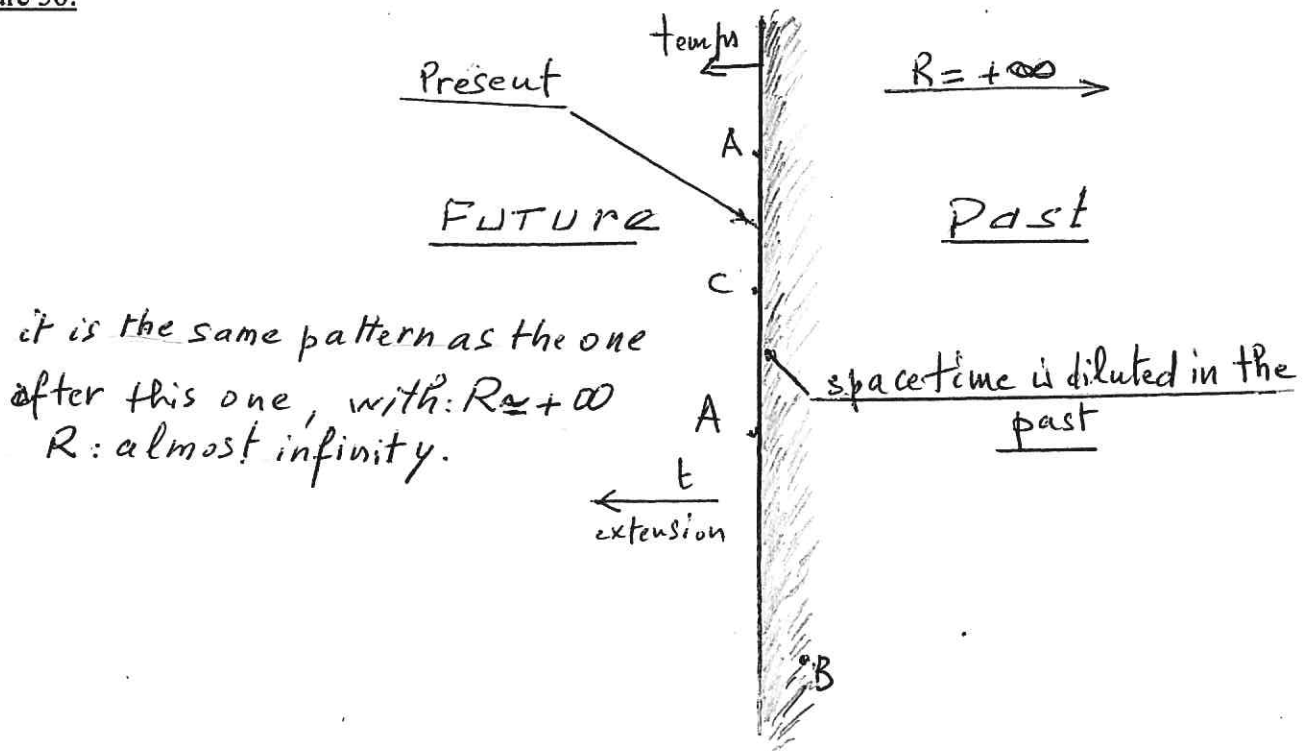
The interaction between the gravitation which tries to slow down the expansion, gravitation due to the visible and non visible matter and the dark energy (still unknown) which favours the expansion, makes that the space-time membrane is everywhere deformed (curvatures due to the gigantic gravitational waves) and often torn (black holes seen as real holes in the space-time fabric): A torn, crumpled balloon, with holes everywhere.

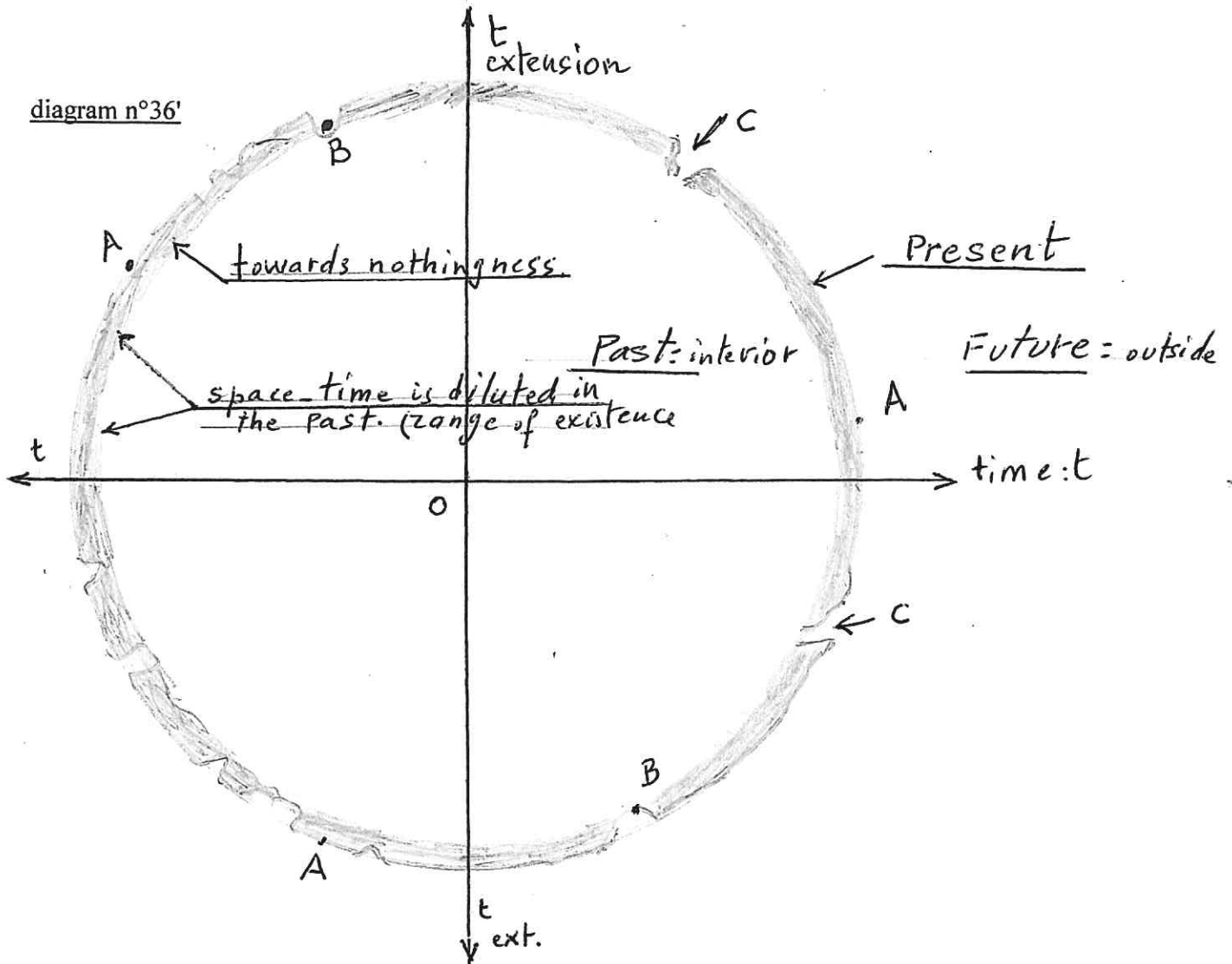
Nevertheless, it is the resultant force of expansion that prevails and our universe resembles a wave or rather a spherical wave (or plane wave since the radius is infinite) which advances on the axis (axes in all directions) of time. This wave started from the original point O (everything was concentrated in this point O) and propagates indefinitely. A crumpled, inflating balloon of radius R tending towards infinity. It is like throwing a pebble into a pond of calm water, the thrown pebble causes a circular wave which continues to propagate on the surface of the pond.

Between the observer A (our position for example) and the most distant point of the Universe A', the distance is not equal to zero (we cannot pass through nothingness) but it is equal to $\pi R = 21,98.10^9$ AL and the extent is $43,96.10^9$ AL (already mentioned)

In the Universe, galaxies float like flags as a result of gigantic gravitational waves and this explains the deformations of the space-time membrane (the hyper-spherical universe wave). Time differs from one point to another on the space-time membrane and therefore so is space and therefore this crumpled balloon could have a fractal character like the edge of a cloud;

Figure 36:





*The Universe has this crumpled and perforated shape, because time and space in different places are not the same (space-time is not the same everywhere: a wave

*In B, time lags behind that of A. In C, time stops 'black hole'. The quasi-planar membrane is crumpled and perforated and advances in the direction of time, like a wave. In A, B, or C, time is not the same.

We are on the membrane, but inside the sphere we can only see the pas
Figure 36':

a quasi-planar membrane with a hole in it (R is infinite).

VII) Close to the original point :

1) General relativity and quantum physics :

General relativity establishes that gravitation exerted by masses acts on the structure of space-time. This one cannot be conceived without content, it does not admit the vacuum. General relativity therefore describes the expansion of the Universe as a dilation of space-time (a balloon that inflates indefinitely), but does not include its origin. The origin is considered as an insoluble mathematical singularity and everything would have come out of this singularity point.

General relativity and quantum physics, which tries to explain the world of the infinitely small, remain incompatible because the former ignores everything about the fluctuations of a flat and identical space.

There are therefore two different conceptions that explain two different worlds.

General relativity wants to explain the Universe and the infinitely large and quantum physics tries to explain the world of the infinitely small which is considered immutable and whose space is pre-existent.

Each of these two conceptions is incomplete and each belongs to a different mathematical framework. Moreover, space-time is not quantifiable like matter and gravitation is not quantified like the other interactions (the other three forces: electromagnetic force, strong nuclear force and weak nuclear force).

Each of the two theories is incomplete, but each operates in its own domain: quantum mechanics acts in a flat and unchanging space while general relativity acts in a geometric "plastic" space.

To bring them together and marry them in order to have a single unitary theory, it would be necessary to make reciprocal concessions on both sides, i.e. on the one hand to reduce geometry to matter and we end up with the superstring theory or the theory of branes and on the other hand to reduce geometry to matter and we end up with the theory of quantum gravity loops: there, space itself is quantized and is formed of grains or particles of space gravity.

The marriage finally leads the way to the string theory with a space of 13 dimensions.

Matter is no longer independent of time and we end up with an infinite number of non-refutable theories, whereas a scientific model can only be refuted by observations.

These theories eliminate the singularity of the beginning, which means that the Universe should start from zero, whereas it cannot start from a zero-dimensional point; the big bang of creation is a singularity that the theory of general relativity cannot do without, although it is an aberration. It is an aberration of the theory in which everything becomes infinite (temperature, pressure, curvature,...). This aberration is the limit of the domain of the general relativity theory; if we extrapolate the equations of the theory at this point, we arrive at absurdities and we cannot go any further.

So attempts have been made to unify the two theories, but without success until now. What is the point of unifying them and for what purpose?

The infinitely large and the infinitely small are two different worlds, so they are explained by two different theories. Science has always continued to make good progress in the present situation, i.e. in the situation where each theory keeps its own independent mathematical framework.

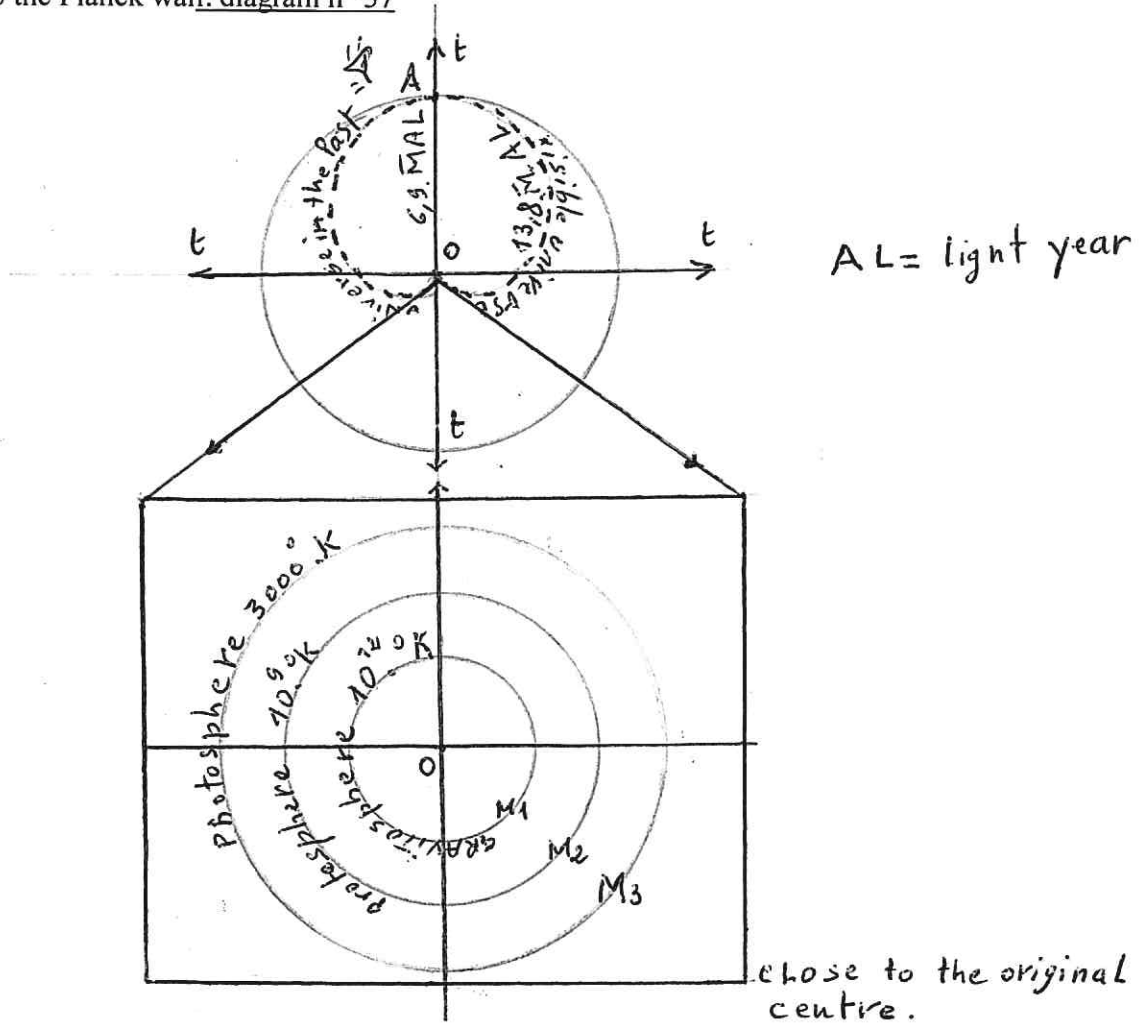
2) The genetic code and the universe :

The tiny grain contains the information necessary for the development of generations of living beings (we humans, animals, insects, plants, etc.). Just as there is a genetic code in life, we can imagine a code at the origin of the universe. My personal convictions are that there is a will that precedes everything and that acts with discernment on a "universe genetic code" expressing an

order, a balance, a law, a growth, and a complexity that totally escapes us.
 Order, balance, law, complexity, harmony, beauty and most of all consciousness (man)?
 Where do they come from?

3) The infinitely small (a small idea): anne

Very close to the Planck wall: diagram n°37



M3: photosphere :

M3: Fringe between opacity and transparency: the fossil beam of photons detected by Pansia and Wilson was emitted by M3, when the temperature was about 30000°K. The hydrogen was ionised.

M2: protosphere :

M2: An intense neutrino fossil beam was emitted from the protosphere. We do not yet have the means to detect this radiation.

M1: Gravisphere :

M1: Beyond M1, the temperature is higher than 10^{12} °K. Fossil graviton radiation filled the Universe, but was not yet detected.

Beyond M1:

- _ at 10^{-32} seconds: the Universe is a soup of elementary particles: electrons and quarks.
- _ at 10^{-43} seconds: the Universe goes from the size of an atom to the size of a

I have found this data in several documents and journals that say more or less the same thing about the beginning of creation. Is this reality overcoming fiction?

In any case, this is the proposal that has so far won general agreement.

NB: The more the radius of the Universe increases (i.e. the more the expansion continues) the more the temperature of the Universe decreases and tends towards absolute zero without reaching it:

$t \longrightarrow 0^\circ\text{K}$

Currently $t^\circ = 2.736^\circ\text{K}$

No corner in the Universe is empty. Fossil photons are still there and maybe also fossil photons and fossil gravitons. The absolute vacuum does not exist in the Universe: at least 411 fossil photons per cm^3 . Fossil photons are carried by the space-time membrane and so they travel throughout the Universe since M3. Those of the sun are 1000 times more energetic, but the rays γ . which are also very energetic photons are absorbed by the core of the sun.

SCHEMATISATION OF GENERAL RELATIVITY:

PART II

TENSORS AND RESTRICTED RELATIVITY

Contents :

The focus this time is on two other supports of the theory of general relativity, namely: tensors and special relativity.

In the 1st work "schematization of general relativity, space-time and gravitation", the visualization of the inflating space-time hypersphere (expansion of the Universe) in a 4-dimensional space being inaccessible, then I had projected it on a space with 3 dimensions; This projection is a swelling sphere and the movement due to the continuous swelling could only represent the evolution of time which goes in all directions;

thus each point of the swelling sphere is a mixture of space and time (see the constructions of the diagrams of the 1st part);

The space-time Universe being represented by the sphere, its visible past is represented by the heart as indicated by the constructions of the diagrams in Part I schéma :

On the other hand, we recall that Einstein made space-time itself an actor and a theatre under the effect of masses and energy and it is its deformations that guide everything that is mobile. This deformed space-time was explained by Riemannian geometry, which replaced a three-dimensional space with absolute and independent time. The substitute is then a 4-dimensional space-time and is equivalent to a pseudo-Riemannian variety.

Riemannian geometry gives itself a scalar product on each tangent space and pseudo-Riemannian geometry gives itself a non-degenerate bilinear form on each tangent space. The space-time of general relativity is then a 4-dimensional pseudo-Riemannian variety.

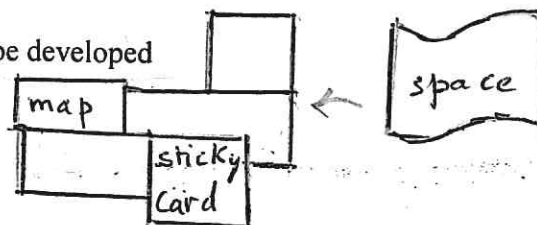
In the theory of varieties, each point belongs to a neighbourhood with a coordinate system call "map". The map is a portion of the variety, analogous to a portion of a vector space. In the neighbourhood of this point, there is an observer.

Special relativity therefore applies to the level of this observer, i.e. to the level of the map, a level considered as flat, pseudo-Euclidean, equivalent to a reference frame with coordinates. All these maps are then glued together to describe the whole variety and to obtain an atlas. The gluing is done at the level of the overlapping zone like terrestrial geographic maps; The earth sphere is a Riemannian variety and the square neighbourhood (assumed to be square for simplicity) of an observer considered as flat, is drawn on a small flat square map: there is a bijection between the map and the neighbourhood of the observer; the set of all the maps then constitutes the atlas representing the Riemannian variety: the earth

The role of the tensor is to translate the curvature or the inclination between the said map and its neighbour. The tensor explains the curvatures and inclinations at the overlaps, i.e. at the places where the maps are joined. It then translates the declivities on the 4 coordinates (it is a 4x4 matrix) which are difficult to represent but easy to conceive.

NB: As it is not possible to represent a 4-dimensional space, the following popularised diagram which does not take into account complex data, simplifies the cards one next to the other diagram :we glue the cards next to each other :

Tensor and special relativity are therefore the main themes to be developed



I) TENSORS :

1) Preamble:

The theory of tensors in mathematics and physics is very vast, it was made in physics within the framework of the mechanics of continuous media, in order to describe the deformations undergone by the constraints of bodies and to define the state of the tensions at all points. The media can be liquids, viscous materials, gases, magnetic fields, gravitational fields and other continuous media. We allow ourselves to expand a little in order to be able to see an analogy between these different environments and space-time considered also as a continuous variety. General relativity uses tensors to rigorously describe space-time as a four-dimensional curved variety. Tensors generalise the scalars and vectors that are used in differential geometry and continuum mechanics as we have noticed. In a continuous medium, scalars can express pressures, tensions and temperatures and vectors can express forces, velocities, magnetic fields and also gravitational fields. These scalars and vectors are special tensors, but insufficient to express other constraints in an n-dimensional space (for the case of space-time $n = 4$). In a Euclidean space, a scalar is the data of a number and the representation of a vector requires 3 numbers and in a general way, a constraint can be represented by a matrix which can express a tension deforming the surrounding medium.

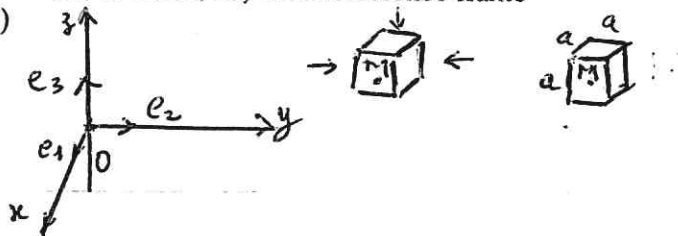
2) Popularisation: example and construction:

force vectors and tensors:

We can construct a tensor represented by a 3x3 matrix for a continuous medium in a 3 dimensional space

Let M be a point located in a 3-dimensional Euclidean space, this point M is in the middle of a very small cube with edge $dx = a$ (M is a cubic point); (O, e_1, e_2, e_3) is the reference frame

It is assumed: $e_i \perp$ at face i with $i = (1,2,3)$



\vec{F}_j force vector on face $j = (1,2,3)$

$\vec{F}_j = (f_{1j}, f_{2j}, f_{3j})$ and F_{ij} = the component along e_i of the force vector on face j

the surface of each facet is $ds = a^2$

the stress on the surface ds is defined by $\vec{\sigma} = dF/ds$

we can then define 9 stress components $\sigma_{ij} = dF_{ij}/a^2$

The state of stress is therefore described by the matrix or tensor $T_{ij}(M)$ at point M

$$T_{ij}(M) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

This tensor is defined for a point M located in an infinitesimal volume a^3 in a static medium, generally this tensor is balanced and symmetrical i.e. $\sigma_{ij} = \sigma_{ji}$, i.e. also there is at least one

orthonormal basis of space in which the tensor is a diagonal matrix.

$$T_{ij} = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$$

$\alpha_1, \alpha_2, \alpha_3$ are called principal constraints

It is insisted that the tensors are sufficient to represent the deformation states of material areas. Generally speaking, tensors are linear operators, linear forms, bilinear and multilinear forms and linear applications, which are particular tensors in a Euclidean space. Mathematically, the tensor is defined from a generally chosen four-dimensional vector space that does not depend on a particular coordinate system and this is necessary in order to express physical laws that do not depend on the chosen coordinate system either; but the numerical quantities and results obtained from the physical descriptions depend on the coordinate system used

3). Tensors in mathematics :

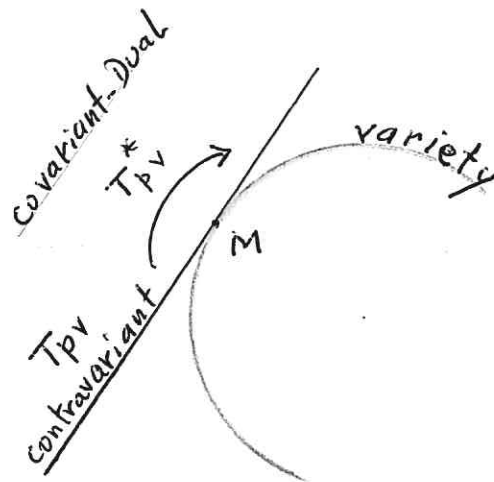
Mathematical reminders are necessary to approach tensors in general relativity.

3-1) multilinear application:

General relativity uses invariant structures, independent of the coordinate system used, and the mathematical tool best suited to this is the tensor or tensor field;

For any point M of a linear variety V, one can construct tangent spaces T_pV and cotangent spaces T^*pV to this linear variety V.

The elements of the tangent space T_pV are said to be contravariant and the elements of the cotangent space (or dual space) are covariant (or covectors). These two vector spaces are used to construct tensors:



A tensor is a multilinear application: let V = a vector space of dimension n over R .
 The dual vector space V^* is formed by all linear applications defined on V and dimension of V is equal to n ; $\dim V = n$

The elements of V and V^* are therefore called contravariant vectors and covariant vectors respectively.

$T(V_1^*, V_2^*, \dots, V_p^*, V_1, V_2, \dots, V_q) = s$; is a tensor associated to a scalar s

Here we have p vectors and q covectors

T is linear on each variable. When we change the reference frame, it is this tensor that defines the correspondence between the contravariant and covariant components. If g is the metric tensor, g being a matrix, we can write:

$g \cdot (\text{contravariant component}) = \text{covariant component}$ and vice versa

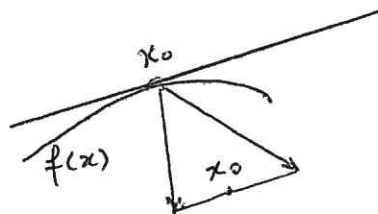
In the theory of special relativity, changes of reference frame modify the vector bases and the tensor is used to ensure that certain physical measurements do not vary, which means that a physical law does not change when looked at in another way.

3-2) differential geometry :

NB: Some notions of differential geometry are recalled in order to have a clear and limpid view on tensors in general relativity. The main physical application of differential geometry is in the theory

of general relativity where it allows the modelling of the curvatures of space-time. Differential calculus consists in approximating the neighbourhood of a point x_0 of a function $f(x)$ by a simpler affine function.

Enlargement to the neighbourhood of x_0



Differential application: see references:

*precise of mathematics. analysis 1, R- Gouffier

*André Warusfel MPSI PCS (2nd edition- Dunod collection)

Vector field :

For convenience we take $n = 4$ (dimension of space-time)

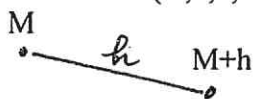
Let U be an open of R^n

Definition: we call vector field on U , any application V

$$\begin{array}{ccc} V : U & \longrightarrow & R^n \\ M & \longrightarrow & V(M) \end{array}$$

M is a point of U , and the vector $V(M)$ in R^n corresponds to it. V is a vector field of class C^1 on U ; and the components of V are (v_1, v_2, v_3, v_4) which are also of class C^1 .

The components v_i for $i = (1, 2, 3, 4)$ after a small displacement h



are: $v_i(M+h_j) = v_i(M) + \sum (\partial v_i / \partial x_j)(M) \cdot h_j + \|h\| \cdot \epsilon_i(h)$

with when h , $\epsilon_i(h) \rightarrow 0$ when $h \rightarrow 0$

on the first component, for example, this is: $v_1(M+h_1) = v_1(M) + (\partial v_1 / \partial x_1) \cdot h_1 + (\partial v_1 / \partial x_2) \cdot h_2 + (\partial v_1 / \partial x_3) \cdot h_3 + (\partial v_1 / \partial x_4) \cdot h_4 + \|h\| \cdot \epsilon(h)$ with $\epsilon(h) \rightarrow 0$ when $h \rightarrow 0$.

The general equation is also written as follows: $\vec{V}(M+\vec{h}) = \vec{V}(M) + dV_M \cdot \vec{h} + \|\vec{h}\| \cdot \epsilon(\vec{h})$ with $\epsilon(\vec{h}) \rightarrow 0$ when $\vec{h} \rightarrow 0$.

This quantity dV_M is a linear application called differential of \vec{V} in M and is written as a matrix J in the canonical basis of R^4 ; $J = (\partial V_i / \partial x_j)(M) = \text{Jacobian Matrix of } V \text{ in } M$

$$J = \begin{bmatrix} \partial V_1 / \partial x_1 & \partial V_1 / \partial x_2 & \partial V_1 / \partial x_3 & \partial V_1 / \partial x_4 \\ \partial V_2 / \partial x_1 & \partial V_2 / \partial x_2 & \partial V_2 / \partial x_3 & \partial V_2 / \partial x_4 \\ \partial V_3 / \partial x_1 & \partial V_3 / \partial x_2 & \partial V_3 / \partial x_3 & \partial V_3 / \partial x_4 \\ \partial V_4 / \partial x_1 & \partial V_4 / \partial x_2 & \partial V_4 / \partial x_3 & \partial V_4 / \partial x_4 \end{bmatrix}; \vec{V}(M+\vec{h}) = \vec{V}(M) + J \vec{h} + \|\vec{h}\| \cdot \epsilon(\vec{h}) \text{ with } \epsilon(\vec{h}) \rightarrow 0$$

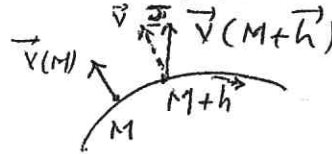
h is the vector representing the infinitesimal displacement in R^4
In simpler writing:

New vector = old vector + tensor \times small displacement

Note that this Jacobian matrix is the matrix of first order partial derivatives of the vector function V . Between the point M and the point $(M+\vec{h})$ there is a curvature of length h ; the vector $\vec{V}(M+\vec{h})$ is

the vector at the point $(M+\vec{h})$ after this curvature. \vec{V} is said to be differentiable at M . \vec{V} is differentiable in M if there exists a linear application J such that: $\vec{V}(M+\vec{h}) = \vec{V}(M) + J \cdot \vec{h} + \epsilon(\vec{h})$; $\epsilon(\vec{h})$: a negligible quantity. This linear application J is unique, it is called the differential of V in M , when $h \rightarrow 0$

$J = dV_M$: which is a linear application

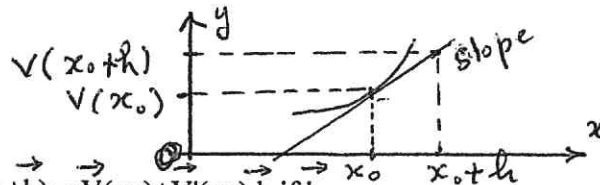


For each component v_i , $i = (1,2,3,4)$, there exists a curvature (i) and V is a function of the variations of all components (i) of the point M . The Jacobian J is a generalization that indicates the inclination or curvature of the vector field V relative to each component (i).

$$v_i(M+h) = v_i(M) + \left(\frac{\partial v_i}{\partial x_1} \cdot \frac{\partial v_i}{\partial x_2} \cdot \frac{\partial v_i}{\partial x_3} \cdot \frac{\partial v_i}{\partial x_4} \right) \cdot \begin{pmatrix} h_1 \\ \vdots \\ h_4 \end{pmatrix} + \|\vec{h}\| \cdot \epsilon(\vec{h})$$

with $\epsilon(\vec{h}) \rightarrow 0$ when $\vec{h} \rightarrow 0$

To simplify the ideas further, another example is the derivative of a real function with real variables which is a tensor.



$V'(x_0) = \lim_{h \rightarrow 0} (V(x_0+h) - V(x_0)) / h$, or $V(x_0+h) = V(x_0) + V'(x_0) \cdot h$ if $h \rightarrow 0$
This is an affine approximation of \vec{V} in x_0 ; it is a differential of the function \vec{V} in x_0 .

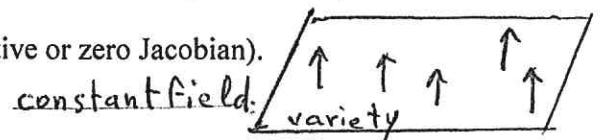
$$\vec{V}(x_0+h) = \vec{V}(x_0) + (\text{tensor}) \cdot \vec{h} = V(x_0) + (\text{inclination} \times h) = V(x_0) + (\text{slope}) \cdot h$$

This slope determines the declivity of the curve, here the tensor = slope. For the previous example, the gradient of the 4-dimensional space is caused (gradients in all 4 dimensions) by the Jacobian J .

*In a space-time, in a vacuum, at rest with no curvature and no singularity, i.e. no matter or energy, the vector at point M is the same as that at point $(M+h)$ and the matrix in question is the null matrix:

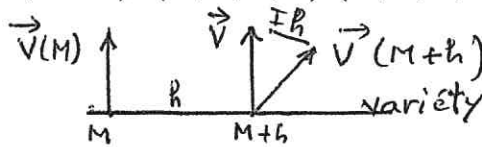
$$\vec{V}(M+h) = \vec{V}(M) + 0 \cdot \vec{h} = \vec{V}(M)$$

The field vectors remain parallel, no tensions (zero derivative or zero Jacobian).



The presence of matter or energy puts the space-time under tension, so the components of the matrix (the tensor) are not all zero. The simple displacement makes the presence of matter or energy felt, in effect, :

$\vec{V}(M+h) = (x_1+h_1, x_2+h_2, x_3+h_3, x_4+h_4) = (x_1, x_2, x_3, x_4) + (h_1, h_2, h_3, h_4) = V(M) + I;h$ with I = the identity matrix



The vector field or vector field is a function that associates a vector to each point of a Euclidean space or more generally of a differential variety. Fields are often used to model the directions of velocities and magnetic or gravitational forces that evolve point by point.

3-3) The gradient is a tensor: the gradient is a vector that indicates how a physical quantity varies in space. It represents the variation of a function with respect to the variations of the various parameters; it is a generalisation of the notion of derivative for a function with several variables: f

being a continuous and derivable function, we can construct a gradient vector of a scalar function $f(x, y, z)$ in R^3

$$\text{Grad} f = \vec{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

The direction of the gradient corresponds to the line of greatest slopes at the point considered. This is the direction that should be followed from that point to see f vary the most. The gradient is therefore a generalisation of the derivative which indicates the direction and sense of variation of the function. The modulus of the gradient gives the slope of the surface at the point considered.

If we consider $f(x, y, z)$ and (e_1, e_2, e_3) a basis of R^3

In Cartesian coordinates, the variation df is written : $df = \left(\frac{\partial f}{\partial x} \right) dx + \left(\frac{\partial f}{\partial y} \right) dy + \left(\frac{\partial f}{\partial z} \right) dz$
this is the variation of f , when we go from $M(x, y, z)$ to $M'(x+dx, y+dy, z+dz)$ or $MM' = dM = (dx, dy, dz)$

$$df = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \text{grad} f \cdot dM$$

In cylindrical coordinates: $dM = (dr, r d\phi, dz)$ and

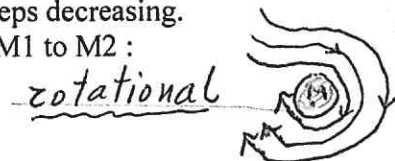
$$\vec{\nabla} f = \left(\frac{\partial f}{\partial r}, \frac{1}{r} \frac{\partial f}{\partial \phi}, \frac{\partial f}{\partial z} \right)$$

$$df = \left(\frac{\partial f}{\partial r} \right) dr + \left(\frac{\partial f}{\partial \phi} \right) d\phi + \left(\frac{\partial f}{\partial z} \right) dz$$

A vector field is said to be a gradient field if there is a function f such that at any point: $\vec{v} = \text{grad} f$, it is necessary and sufficient that $df = V_x dx + V_y dy + V_z dz$ (in Cartesian coordinates). Since V is a gradient field, any function U such that $\vec{V} = -\text{grad} U$ is called a scalar potential from which the field v is derived;

properties :

- The surfaces $U = Cte$ are called equipotential surfaces. The field lines are normal to the equipotential surfaces; the field is directed towards the decreasing potentials.
- U is defined to within one constant and the choice of the constant depends on the initial conditions.
- A field line does not close in on itself, because the potential keeps decreasing.
- The circulation $C(V, M_1 M_2)$ is independent of the path from M_1 to M_2 :



$$C(\vec{V}, M_1 M_2) = \int_{M_1 M_2} \vec{V} \cdot d\vec{M} = \int_{M_1 M_2} -\text{grad}U \cdot d\vec{M} = \int_{M_1 M_2} -dU = U_{M_1} - U_{M_2}$$

3-4) The rotational is a tensor: see figure above

It expresses the tendency of the vector field lines around a point. The rotational is a differential operator which transforms a vector field into another vector field; to a field A, we make it correspond to another field: $\text{rot } \vec{A} = \nabla \wedge \vec{A}$.

$$\text{rot } \vec{A} = \nabla \wedge \vec{A} = (\partial A_x / \partial y - \partial A_y / \partial x) \vec{i} + (\partial A_x / \partial z - \partial A_z / \partial x) \vec{j} + (\partial A_y / \partial z - \partial A_z / \partial y) \vec{k}$$

Example: the eye of a tornado and the vector field is the wind speed.

In a 3-dimensional space in Cartesian coordinates, the rotational of a field $F(F_x, F_y, F_z) = \text{Rot } \nabla \wedge F$. On a related domain, a continuous field \vec{f} , derives from a scalar potential U such that: $\vec{f} = -\text{grad}U$.

3-5) The Laplacian is a tensor: Δ

It is a differential operator which combines and links the static description of a field described by its gradient to the dynamic effects (the divergence) of this field in space, it is defined as follows: $\Delta \phi$.

$$\Delta \phi = \nabla \cdot (\nabla \phi) = \text{div}(\text{grad} \phi) \text{ with}$$

$$\nabla \phi = \begin{bmatrix} \partial \phi / \partial x \\ \partial \phi / \partial y \\ \partial \phi / \partial z \end{bmatrix} \text{ and } \nabla \cdot (\nabla \phi) = \begin{pmatrix} \partial / \partial x & \partial / \partial y & \partial / \partial z \end{pmatrix} \begin{bmatrix} \partial \phi / \partial x \\ \partial \phi / \partial y \\ \partial \phi / \partial z \end{bmatrix} = \partial^2 \phi / \partial x^2 + \partial^2 \phi / \partial y^2 + \partial^2 \phi / \partial z^2 = \text{div}(\text{grad} \phi).$$

as a second derivative, Δ translates an acceleration and the gradient in 3 dimensions translates the temporal variation.

for example the heat equation in a homogeneous medium is written: $\partial T / \partial t = \alpha \cdot \nabla^2(T)$

With α = the constant, T : the temperature and t : the time

The Laplacian is a differential operator for vector fields, it has many similarities with the scalar Laplacian. In a Euclidean field in Cartesian coordinates, the Laplacian of a vector field \vec{A} has as its component the Laplacians of the components of \vec{A} .

4) Tensors in General Relativity :

4-1) Einstein's notations and summations :

This is a notation intended for tensor calculus; it is considered a shortcut for manipulating equations involving coordinates, vectors, covectors and matrices.

N.B.: see details at :

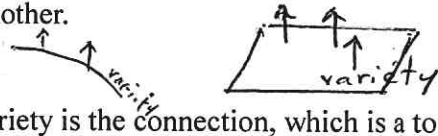
*http://fr.wikipedia.org/wiki/convention_de_la_summation

*summation convention following repeated indices: <http://www.math.unicaen.fr/choi/html/>

*https://en.wikipedia.org/wiki/summation_inabstract-index

4-2) Parallel transport or translation :

Intuitively, a vector field can be said to be parallel if all its derivatives are zero, i.e. it is constant. A parallel vector field allows one vector at a point x to be identified with another at a point y, such vectors are said to be transported or translated from one another.

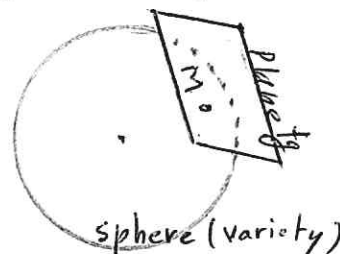
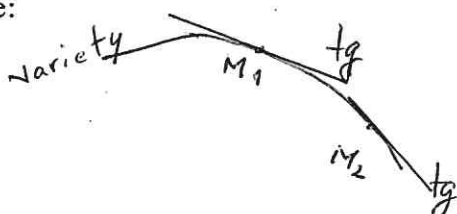


One way of transporting vectors along the curves of the variety is the connection, which is a tool for parallel transport.

4-2-a) Affine connection :

In differential geometry, an affine connection is a geometric operation defined on a differential variety that connects neighbouring tangent spaces and thus allows tangent vector fields to be derived as if they were functions defined on the variety and taking their values in a single vector space.

4-2-b) Example: we can make a tangent affine plane roll on a sphere from one point to another. figure:



The comparison of tangent vectors at different points of a variety cannot be defined. The affine connection provides a solution by using parallel transport and conversely parallel transport allows to define an affine connection.

5) Christoffel symbols:

Or Christoffel coefficients or connection coefficient. Used to determine the geodesics of Riemannian varieties, they are basic tools used in the framework of general relativity to describe the handling and writing are long, tedious and even boring.

The following development by Roland Fortunier seems to me to be the clearest, briefest and most explanatory: "Elements of tensor calculus, Roland Fortunier, Centre Georges Charpac):

<https://www.ensma.fr/roland-fortunier>. action of matter and energy on the transformation of space-time. Their handling and writing are long, tedious and even boring.

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<https://www.ensma.fr/roland-fortunier>.

6) Riemannian structures and tensors:

One recalls that a variety is a space of points (plane surfaces, spherical surfaces, etc...); one locates the points with parameters or the coordinates are x_i with $i = 1, 2, 3, \dots, n$; n is the dimension of the variety ($n = 4$ for our case).

There are various structures of non-Euclidean spaces, but the simplest and most interesting case is the Riemannian space.

General relativity assumes that space-time can only be Riemannian, because it is characterised by a certain special curvature that takes the desired shape.

The curvature of a spherical surface in R^3 is usually measured by the quantity: $Q = k/R^2$, where R is the radius of the sphere and k is a constant. This notion of curvature is attached to the surface of the sphere, but the curvature of space-time in the Riemannian sense is not such a curvature; It is rather a deformation inside this space-time which does not need external dimensions to exist (in the first part, I used the concept of "space-time membrane" in order to facilitate the schematization, this membrane corresponds to the space-time which advances and the range of existence corresponds to the very close past which is diluted in the past time).

The deformation is defined by a space-time curvature tensor called the Riemann-Christoffel tensor, which depends only on the derivatives of the components of the metric tensor of the Riemannian space-time.

The curvature results, as has often been noted, from the presence of matter or energy. This curvature follows the environment of the matter, i.e. it is localized on it. It is weak in one place of the Universe and intense in another and everything depends on the concentration of matter



Einstein assumed that space-time is a Riemannian space and this assumption proved to be very successful. The theory of general relativity is essentially based on the geometry of Riemann spaces whose coordinate system is well suited to curved spaces and which involves an important component, the tensor. It is introduced from the notion of connection and whose physical reality does not change and the laws remain invariant by change of reference frame. The tensor represents

the components of each point of the space-time considered as almost resembling the Riemann space. It should also be noted that Riemann space is a space of points, just like ours, and each point is marked by n coordinates ($n = 4$ for the case of space-time and $i = 1, 2, 3, 4$)

6-1) The Riemann tensor :

In differential geometry, the Riemann tensor is the most common way of expressing the curvature of Riemannian varieties or more generally of a variety with an affine connection with or without torsion (for example rolling a plane on a sphere).

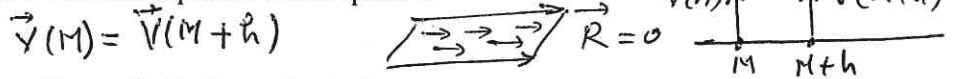
In a curved space, let Γ be a geodesic; as noted, the quantity ($h \times$ tensor) causes the declivity of the geodesic.



Let \vec{v} be a vector in M orthogonal to the neighbourhood of this point considered as flat. When M arrives on M' following the displacement of infinitesimal length (h), \vec{v} is transformed into \vec{v}' , \vec{v}' being also orthogonal to the flat neighbourhood of M', transformed from M by the displacement (h). The difference \vec{R} of the vectors \vec{v} and \vec{v}' constitutes the Riemann tensor: $\vec{R} = \vec{v}' - \vec{v}$

It expresses the curvature of the geodesic Γ between points M and M'. There are three possibilities:

- If $\vec{R} = 0$, then no curvature and \vec{v} remains equal to \vec{v}' : flat space :



- If $\vec{R} > 0$ then the curvature is positive: elliptical or spherical space



- If $\vec{R} < 0$ then the curvature is negative: hyperbolic space



\vec{R} also expresses the derivative of the function or the gradient or also the Jacobian of the vector field \vec{v}

\vec{R} also expresses the declivity of the surface containing the points M and M'

Two geodesics in a curved space in the vicinity of a point M cannot maintain parallelism and the Riemann tensor here expresses the evolution of the geodesics with respect to each other; the more curved the space, the closer or further apart the geodesics become. The tensor determines how far apart or how close these geodesics are.

For any vector field (u, v, w) we have : $R(\vec{u}, \vec{v}) \vec{w} = \nabla_{\vec{u}} \cdot \nabla_{\vec{v}} \vec{w} - \nabla_{\vec{v}} \nabla_{\vec{u}} \vec{w} - \nabla_{(\vec{u}/\vec{v})} \vec{w}$

The Riemann vector R is projected onto a set of coordinates. This tensor which takes into account 3 vectors and a linear form is thus a quadratic application. The Riemann tensor is written in local Cartesian coordinates:

$R_{xyxy} = 1/2(\partial^2 g_{xx} / \partial y^2 + \partial^2 g_{yy} / \partial x^2$

g_{xx} and g_{yy} are the coefficients of the metric in Riemann coordinates, i.e. local Cartesian coordinates.

A Riemann space is a variety to which a metric (u_i) is attached on a coordinate system and the metric is defined by the quadratic form $ds^2 = g_{ij} u_i u_j$, this metric is called Riemannian if :

- * $g_{ij} = g_{ji}$ symmetry
- * $\det(g_{ij}) \neq 0$
- * invariance by coordinate change
- * g_{ij} are of class C^2

6-2) Ricci tensor :

a brief idea :

We know that in the framework of general relativity, the gravitational field is interpreted as a deformation of space-time. This deformation is expressed by the Ricci tensor. This is the Laplacian of the Riemannian metric tensor in the case of Riemannian varieties. This tensor combines and relates the static description of the field to the dynamic effects of this field in space-time.

It is a second-order tensor that plays an important role in the Einstein equation (the main equation of general relativity). It is also fundamental in differential geometry. It is obtained from the Riemann curvature tensor and can also be expressed from the Christoffel symbols which represent

the evolution of the basis vectors from one point to another in space-time due to the curvature of the latter.

The Ricci tensor, R_{ik} is formed from the inverse metric g^{ij} and the covariant Riemann tensor R_{ijkl} by the relation :

$$R_{ik} = g^{mn} R_{mink}$$

This is the Laplacian of the Riemann metric tensor in the case of Riemannian varieties.

7) Levi-Civita connection:

In a pseudo-Euclidean space, the determinant of the metric tensor is not always positive and in the case of Minkowski space, the 4-dimensional space-time of special relativity, the determinant is negative. The determinant of the tensor in question can then be expressed using the Levi-Civita symbol.

8) The Einstein tensor :

In differential geometry, the Einstein tensor expresses the curvature of a pseudo-Riemannian variety. In general relativity, it appears in the Einstein field equation to describe how the gravitational field is affected by the presence of matter or energy; it is a second-order tensor that can be represented by a 4x4 matrix. It is deduced from the Ricci tensor by the formula:

$$\text{with: } G_{\mu\nu} = R_{\mu\nu} - 1/2 (R \cdot g_{\mu\nu})$$

$G_{\mu\nu}$ = Einstein tensor

$R_{\mu\nu}$ = Ricci tensor

$g_{\mu\nu}$ = the Riemannian space-time metric

R = the scalar constant = the trace of the Ricci tensor

In two dimensions, the formula reduces to :

$$G_{xx} = R_{xx} - 1/2 \cdot (R g_{xx}) \quad \text{or} \quad G_{yy} = R_{yy} - 1/2 \cdot (R g_{yy})$$

We have $G_{xx} = G_{yy} = 0$, the Einstein tensor of a surface is zero

8-1) Einstein field equation :

Published by A. Einstein for the 1st time on November 25, 1915, it describes how matter (and or) energy modify the geometry of space-time.

This created curvature of the geometry around a source of matter (or energy) is interpreted as the gravitational field of this source. The motion of objects in this field is described by its geodesic equation.

The Einstein field equation:

$$R_{\mu\nu} - 1/2 \cdot (R \cdot g_{\mu\nu}) + \Lambda g_{\mu\nu} = k \cdot T_{\mu\nu} \quad \text{with :}$$

$R_{\mu\nu}$ = Ricci tensor

R = scalar curvature

$g_{\mu\nu}$ = the metric tensor of signature (+, -, -, -)

A = cosmological constant (Einstein believed in a static universe)

$T_{\mu\nu}$ = energy-impulse tensor

k = Einstein's constant, $k = (8\pi G)/C^4$; G is the gravitational constant, $G = 6.6742 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

C = is the speed of light in vacuum $\sim 299792458 \text{ ms}^{-1}$

This equation is a tensor equation relating a set of symmetric 4 x 4 tensors.

This equation also allows us to know the metric tensor g given a matter and energy distribution expressed as an energy-impulse tensor.

This equation is complex because the Ricci tensor R and the scalar curvature R depend on the metric g.

A = Einstein's constant which allowed him a static universe, after Hubble he resigned himself and admitted that $A = 0$

$$G_{\mu\nu} = R_{\mu\nu} - 1/2 (R g_{\mu\nu}) = (8\pi G/C^4) \cdot T_{\mu\nu}$$

It should also be noted that the value of $A = 0$ is controversial today (how to explain the existence of a dark energy not yet proven).

8-2) Properties of the Einstein equation :

a*) conservation of energy and motion : $\nabla_{\nu} T_{\mu\nu} = 0$

b*) Einstein's equation gives rise to non-linear partial differential equations and this distinguishes the theory of general relativity from other theories (electromagnetism, quantum mechanics).

c*) The solutions of Einstein's equations are the metric tensors of space-time. These tensors describe the structure of space-time, but are very difficult to solve because of the non-linearity of the field equations; one must then proceed by approximations because there are no known complete solutions.

d*) Astronomical techniques claim that a non-zero value of Einstein's constant A is necessary to explain certain observations in the cosmos; the existence of a non-zero cosmological constant is equivalent to the existence of an as yet unidentified energy in the universe.

e*) in the case where the region (case of special relativity) is empty, i.e. the energy-impulse tensor $T = 0$, then the Minkowski metric applies and the distances are measured by the metric:

$ds^2 = C^2 \cdot dt^2 - dx^2 - dy^2 - dz^2$ with :

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

NB: to be reviewed in part II in special relativity

8-3) example 1: Schwarzschild metric: space around a spherical mass: the metric is already seen in part I. The spherical coordinates: $(x_1, x_2, x_3, x_4) = (t, r, \theta, \varphi)$.

In the framework of general relativity, it is a solution of the Einstein equation. It describes the geometry of space-time when it is deformed by the gravitational field of a spherical, static, non-rotating and uncharged mass located in a vacuum (it can be a star, a planet, a black hole etc.). The metric that describes the curvature is an infinitesimal displacement interval such as :

$$ds^2 = (-1 - (2GM/RC^2))C^2 \cdot dt^2 + (1 - (2GM/RC^2))^{-1} dr^2 + r^2 \cdot d\theta^2 + r^2 \sin^2 \theta \cdot d\varphi^2$$

with : (t, r, θ, φ) which are the spherical coordinates of a point p in space-time, with also

t : time coordinates

r : radial coordinates

θ : latitude angle of the point in radian

φ : angle longitude of the point in radian
 ds : space-time interval of an infinitesimal displacement
 C : speed of light
 G : gravitational constant

M : mass of the spherical object

$t = 0$ means that the metric does not vary with time and therefore static, the curvature is always the same in all places of space surrounding the spherical mass; the absence of the mixed term $do \cdot dt$ indicates that there is no rotation in space-time.

If $dr = 0$ and $dt = 0$, then r and t are constants and the metric reduces to $r^2(d\theta^2 + \sin^2\theta d\varphi^2)$; this quantity is nothing other than a distance on a sphere of radius r in Euclidean space.

8-4) example 2: Kerr metric: around a rotating body: specific metric for black holes

The spherical coordinates are:

$(x_1, x_2, x_3, x_4) = (t, r, \theta, \varphi)$

$$g_{\mu\nu} = \begin{bmatrix} (1 - 2Mr/\Sigma) & 0 & 0 & Z \\ 0 & -\Sigma/\Delta & 0 & 0 \\ 0 & 0 & -\Sigma & 0 \\ Z & 0 & 0 & 3 \end{bmatrix}$$

with: $\Sigma = r^2 + a^2 \cos^2\theta$

$\Delta = r^2 - 2Mr + a^2$

$Z = (4Mr \sin^2\theta) / \Sigma$ and

$3 = ((r^2 + a^2 + 2a^2 M r \sin^2\theta) \sin^2\varphi) / \Sigma$

8-5) example 3: metric FLRW: (already mentioned in part 1 : Friedman, Lemaitre, Robertson and Walker).

which allows to describe a space-time of homogeneous and isotropic geometry.

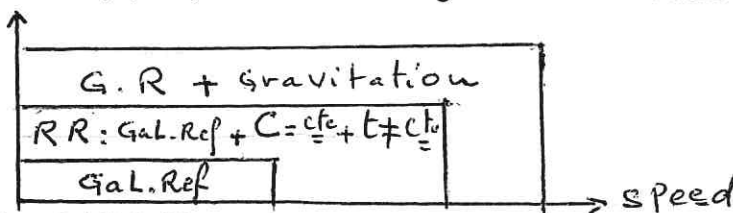
II) SPECIAL RELATIVITY :

1) Preamble: Special relativity is a theory developed by Albert Einstein in 1905. It is in fact the Galilean frame of reference reshaped and tailor-made to fit another form, a form that goes with the theory of general relativity. It is an operation of reworking the Galilean frame of reference which is based on three principles:

- 1) admit the Galilean reference frame (Ref. Gal).
- 2) the speed of light (C) is constant
- 3) no absolute time

We can retain the writing of special relativity (RR) under the following formula: $RR = \text{Ref. Gal} + (C = cte) + \text{Non absolute time (1)}$

We schematise ;



The laws of physics take the same form in all Galilean reference frames, this is thus modified by Einstein to be extended to non-inertial reference frames: it is the invariance of the physical laws by change of inertial reference frame; by virtue of the principle of equivalence, the form of the laws remains conserved. If a relationship between parameters exists in one reference frame, the same relationship between different values of these same parameters exists in the others

It turns out that for the transformations considered, the tensors have precisely these properties, i.e. if we manage to write a law in general relativity in the form of a relation between tensors, if it is true in a system of coordinates, (any reference frame) then it is true in any other system. Special relativity is used to draw all the physical consequences of general relativity and of the principle that the speed of light is constant. It also considers only the case where the observer is in a restricted and inertial reference frame, the other reference frames are the object of studies of general relativity. Special relativity therefore preserves the neighbourhood of the observer's location; it works for all speeds, but not in regions of space subject to a very large gravitational field. The gravitational interaction can only be treated in the framework of general relativity, which plays the most important role, except of course at the first moments of the big bang, which are not part of the analysis related to general relativity.

The principle of general relativity states that the laws of physics are the same everywhere, at any time and in any reference frame. The length of a vector is invariant to all transformations of the coordinates, i.e. the laws do not change when observed from another place or at another time. On the train or on the ground, a stone falls in the same way, even on the moon but with a lower acceleration. On a boat moving at a speed of 20 km/h and me on it moving at 5 km/h, the result for me is 25 km/h (speed additivity). These two examples quoted above are the basis of the principle of the Newtonian theory.

But in the vicinity of the speed of light, the speeds do not add up. For Michelson and Morley, a flash emitted from the earth moves in relation to the universe at the same speed C as in relation to the earth; there is no additivity of speed when this one is close to C.

As noted in special relativity, time and space are no longer absolute, but variable and depend on the speed of travel. The only absolute reference is the speed of light C. There is a contraction of lengths and a dilation of durations when approaching C; it is as if everything depends on the speed of

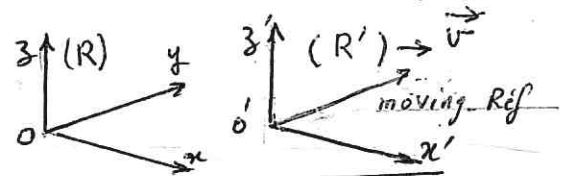
displacement of the measuring instruments used. The aim is to describe the laws of physics terminated by a Galilean observer even at very high speed (speed of light). It can be said that special relativity is mixed with classical physics under normal conditions of life.

2) Special relativity and Lorentz transformation:

Already mentioned in part 1, but briefly recalled here:

One considers two reference frames, one is fixed and the other is animated by a movement at the speed v , Einstein's hypotheses lead to the Lorentz transformations which express the coordinates (x, y, z, t) of a given event in the fixed reference frame (R) as a function of the coordinates (x', y', z', t') of the same event in the mobile reference frame (R') .

$$\begin{cases} C \cdot t = \gamma (C t' + \beta \cdot x') \\ x = \gamma (x' + \beta C t') \\ y = y' \\ z = z' \end{cases}$$



with: $\beta = v/C$ and $\gamma = 1/\sqrt{1 - \beta^2}$

If we use hyperbolic functions of parameter θ , θ being the angle of rotation of the Minkowski space defined by $\tan \theta = \beta = (v/C)$

soit $\theta = \text{arc tanh}(v/C) = \text{arctanh}(\beta)$;

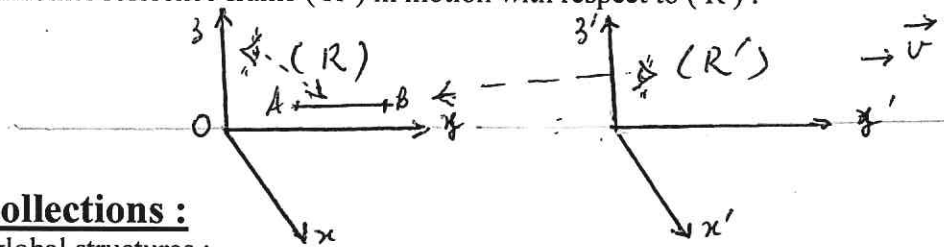
we obtain $\gamma = \sqrt{1 - \beta^2} = 1/\sqrt{1 - \tanh^2 \theta} = \cosh \theta$ and

$$Ct = Ct' \cosh \theta + x' \sinh \theta$$

$$x = Ct' \sinh \theta + x' \cosh \theta$$

The transformations that preserve the orientations of space-time are the Lorentz transformations. These transformations are the basis of the theory of special relativity; they are used to express the results of the Michelson and Morley experiment by a contraction. These expressions take the form of a real rotation of lengths in the direction of motion.

Simultaneity: These 2 motions A and B are simultaneous in the reference frame (R) , they are not simultaneous in another reference frame (R') in motion with respect to (R) .



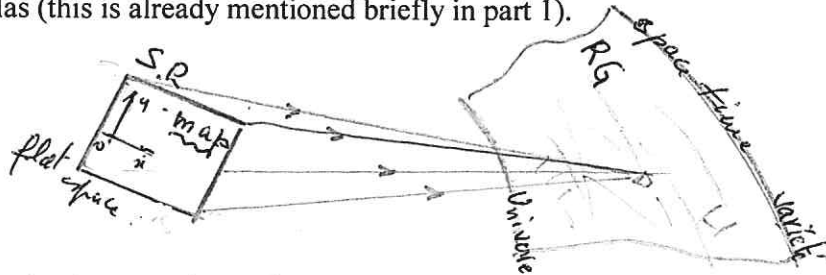
3) The Recollections :

3-1) Local and global structures :

Modern approaches to general relativity are based on the concept of variety; and in differential topology, a variety is a geometric concept obtained by recollecting vector space openings. The notion of variety reflects the idea that space can be curved and have a more complicated topology, but which can also be locally assimilated to R^n (special relativity), assimilated does not mean that the metric is the same.

The representation of gravitation as a curved space-time is modelled by a continuous, connected, 4-dimensional Lorentzian pseudo-Riemannian variety and is a separate space; other quantities are generally represented by tensors. Special relativity always applies to a neighbourhood, and to the neighbourhood of each point of space-time, where the observer is located; one can consider that

there is a system of coordinates (or maps) which constitutes the local representation. The laws of special relativity apply locally at each point and the map can resemble the flat space-time; the description of the Universe (or the total space-time) requires the recollection of all the maps constituting an Atlas (this is already mentioned briefly in part 1).



A map is an application between 2 sets, between the object which can be a territory and its representation on a sheet of paper (geographical maps)

$$f : (\text{Territory}) \longrightarrow (\text{geographical map})$$

$$x \longrightarrow f(x)$$

The function f is a bijection; everything seen on the territory is drawn on the map.

Measurements in physics are always made in a relatively small region, which is why local structures in this region are studied in general relativity. The Universe is considered to be a quasi hyper-spherical membrane and its surface constitutes a variety that resembles R^4 .

Comparing the Universe to a swelling grape cake whose grapes are moving away from each other as galaxies move away from each other is not a good comparison (Hubert Reeves' comparison) because the cake is full, whereas the variety is anything but full. Not everything that is full is a variety and not everything that does not look like R^4 locally is also a variety. On the other hand, one can compare the Universe in its expansion to a speckled balloon which inflates and whose speckles move away from each other like the galaxies.

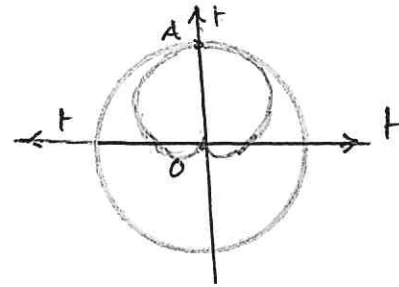
The variety can be a straight line, a curve, a flat surface or a curved surface, a toric surface, a cylindrical surface, a sphere, a cone without the top, etc... The main thing is that it does not have to be solid. In geometry and differential topology, a variety is a mathematical object, obtained geometrically by recollecting vector space openings, the complete variety is then constituted by connecting without discontinuity all local regions. In a general way, in differential geometry the connection is a tool which carries out the parallel transfer. The variety can be curved and its topology will then become more complicated, but can be locally assimilated to a Euclidean space R^4 .



The Universe is a hyper sphere and the membrane without edge is the variety, it constitutes the surface of this quasi hyper sphere which locally looks like R^4 . We glue the openings together and we obtain a variety; the connection is made as it was noticed before without discontinuity. The cardioid image of the Universe drawn in the past is a variety if we exclude the original centre O ; this goes well with the theory of general relativity, the world of the infinitely small and beyond the Planck wall, is not part of the Universe and is not part of the competence of the theory of general

relativity: we exclude the original centre of the cardioid, the cardioid without the centre is a variety that is to say that the past Universe without the original point (where there was a big bang) is a variety.

The origin O is a singularity



The sphere and the heart without the point O are varieties

4) Special relativity: mass and energy, acceleration and gravitation:

To begin with, let us recall that Einstein's famous equation

$E = m \cdot C^2$ is a consequence of special relativity. Mass can be transformed into energy and conversely, energy can materialise into particles.

4-1) contraction and dilation (mentioned in part 1): An object of length 1 meter and width 0.5 meter, if the object moves at the speed 75% of C in the direction of the length, the width does not change, only the length decreases by about 2/3 of a meter. This means that only the dimension measured in the direction parallel to the movement decreases.

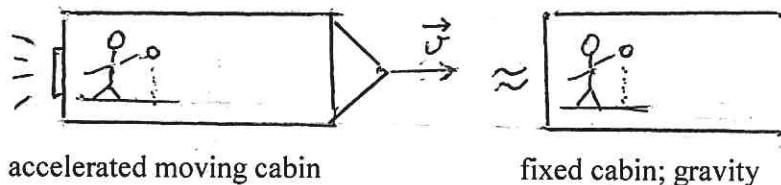
The mass of an object also changes if its speed increases a lot. The faster a body moves, the more massive it is and the more energy is needed to accelerate it. If its velocity approaches C, its mass becomes enormous and to reach C, the energy will be infinite which is impossible to provide and we will never reach C. This reasoning does not apply to photons which have zero mass and can therefore travel at speed C. What I have just noted is also one of the consequences of special relativity. At the level of general relativity, gravity can pierce space-time in the case of a condensation of matter, which leads to a black hole. Some scientists consider that the name "black hole" is an abuse of language, because the gravitation is so strong that this super-massive star does not let anything escape; in my opinion, it is a real hole in the fabric of space-time that returns to nothingness (everything that is captured returns to nothingness). This black hole can generate waves that distort space-time: these are the gravitational waves that travel through the universe.

Between gravitation and acceleration, there is an equivalence which is at the origin of the theory of general relativity.

4-2) principle of equivalence: diagram :

Basic principle:

When a proposition is demonstrated in an accelerated system, the conclusion drawn is also valid in a system subject to gravitation and vice versa. This is a direct consequence of the equivalence principle.

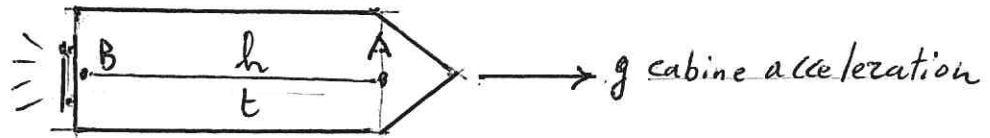


In motion, it is as if space-time is transported into the cabin: it is also as if the cabin has its own space-time independently of the outside. The principle of equivalence states that an accelerated system and a system subject to gravity are equivalent; that is, the laws of mechanics are the same in both systems.

2 examples:

Example 1: Richard Feynman demonstrates that various measurable effects can be deduced from

the equivalence principle with the following demonstration on gravitational shift.
In an accelerated cabin:



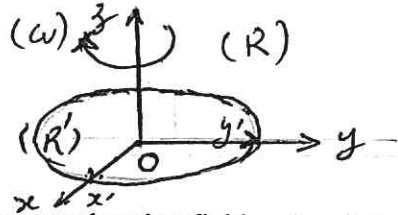
$\vec{V} = \vec{g} \cdot t = \vec{g} \cdot (h/C)$ with $t = h/C$ is the time required for a flash of light at point A to travel the distance h to reach receiver B

Receiver B moves relative to transmitter A so that the frequency offset $\Delta \lambda$ is written as :

$$\Delta \lambda(\text{measured}) = \Delta \lambda(\text{transmitted}) + \Delta \lambda(\text{transmitted}) \cdot (v/C) = \Delta \lambda(\text{transmitted}) \cdot (1 + g \cdot h/C^2)$$

The signal received at B has a different frequency than the one transmitted at A. According to the equivalence principle, a light emitted in a gravitational field will suffer the same fate as in an acceleration field. A shift in wavelengths between emission and reception is observed; this is the gravitational shift; when the receiver B moves closer to the transmitter A, a blue shift is obtained and when B moves away from A a red shift is obtained (not to be confused with the Doppler effect).

2nd example For reference frames where there is an acceleration field, the square of the interval ds^2 will have a more complicated form than when considering only inertial reference frames.



(R) : fixed reference frame (x, y, z)

(R') : reference frame rotating around oz, subject to an acceleration field and ω is the angular rotation speed directed on oz. A circle in the plane (o, x, y) of (R) centred on oz can also be considered as a circle in the plane (o, x', y') of (R').

We have : $x = x' \cos \omega t - y' \sin \omega t$ and $y = x' \sin \omega t + y' \cos \omega t$

In (R'), ds^2 is then written

$$ds^2 = C^2 - \omega^2(x'^2 + y'^2)dt^2 + (dx'^2 + dy'^2 + dz'^2) + 2\omega y' dx' dt - 2\omega x' dy' dt \quad (a)$$

Whatever t is, this expression cannot be reduced to Euclidean form; the geometry of (R') seen from (R) is not Euclidean geometry. It can be shown that the ratio of the circumference of a circle of (R') to its diameter is different from...

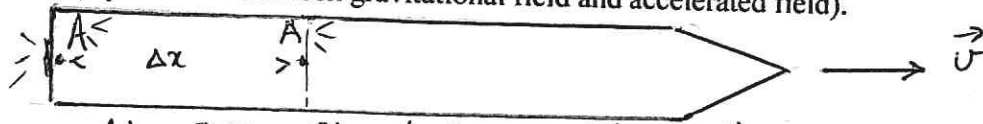
Based on the equivalence principle, formula (a) is valid in reference frames where a gravitational field prevails. The square ds^2 will have a more complicated form than when considering only inertial reference frames. In a non-inertial reference frame ds will be a differential quadratic form of coordinates (dx1, dx2, dx3, dx4) such that :

$$ds^2 = \begin{bmatrix} g_{11} & & & g_{14} \\ g_{21} & & & g_{24} \\ g_{31} & & & g_{34} \\ g_{41} & & & g_{44} \end{bmatrix} \cdot \begin{bmatrix} dx^1 \\ dx^2 \\ dx^3 \\ dx^4 \end{bmatrix}$$

5) Time dilation in special relativity :

It is difficult to define time, using it as a variable (t) in mathematical equations does not help. The sensation of time is linked to our awareness of any change; change includes everything that is movement (change of position in space), everything that is transformation (chemical reaction, ageing, etc.) and everything that is variation (successive unfolding of days, etc.) Change is linked to matter and therefore to space. Time changes and so does space because space and time are one and the same; today's space is not the same as yesterday's.

As we noticed before, each moment is the intersection of the past that is no longer and the future that is not yet, it is the intersection of two nothingnesses; the future continuously dislodges and replaces the past. Close to matter (or energy which is a transformation of matter) which curves space, time dilates, that is to say that the past (which is nothingness) does not rush to join the other nothingness (the future), the latter is not in such a hurry to replace the first one either. The term time dilation designates an effect of special relativity. A moving clock seems to slow down compared to a stationary clock. This phenomenon of slowing down of clocks extends in general relativity to clocks close to a massive body which will slow down compared to those which are far from it (equivalence between gravitational field and accelerated field).



the same flash A; $\Delta x = (\vec{v} \cdot \Delta t, \Delta t)$

Two lightning bolts in one place in a rocket advancing at speed \vec{v} . Let Δr be the time interval between two flashes measured in the rocket; these two flashes are emitted at the same place for the rocket. The rocket passes in front of a terrestrial observer who notes that the time interval between these two flashes is $\Delta t \neq \Delta r$. The two flashes are therefore separated by a space-time interval $\Delta x = (v\Delta t, \Delta t)$ for the earth observer and $\Delta x = (0, \Delta t)$ in the rocket.

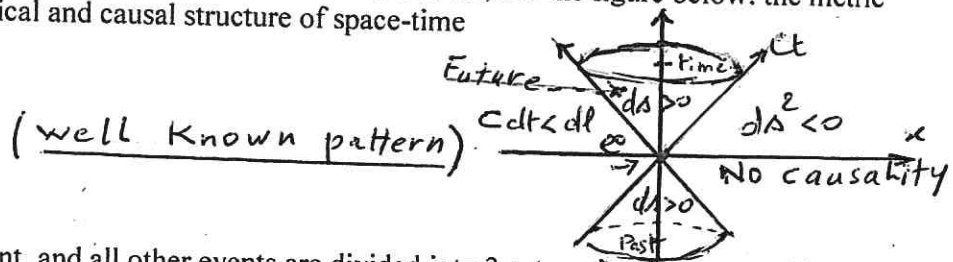
Special relativity states that the square of the space-time interval between these two events is the same in both reference frames (terrestrial frame and rocket frame) and so we have the following equality: $C^2\Delta r^2 = C^2\Delta t^2 - v^2\Delta t^2$ which gives

$$\Delta t = \frac{\Delta r}{\sqrt{1 - v^2/C^2}}; \text{ conclusion is then } \Delta t \geq \Delta r$$

This result can also be obtained by the Lorentz transformation

6) Minkowski's contribution :

An important enrichment for special relativity: Minkowski modelled special relativity with his light cone, this becomes one of the fundamental notions of special relativity which allows to distinguish between a past event, a future event and an inaccessible event, see the figure below: the metric describes the geometrical and causal structure of space-time



(e^0) : is the present event, and all other events are divided into 3 categories. Let us consider in a given reference frame, 2 events separated in space by the distance $dl = \sqrt{x^2 + y^2 + z^2}$ and in time by the magnitude dt . In special relativity, these two quantities are not invariant by change of reference frame, but the quantity $ds^2 = C^2 dt^2 - dl^2$ is.

1°- on the edge of the cone: $ds^2 = 0$, i.e. $C \cdot dt = dl$

The events are only joined by (e^0) if the message goes at the speed of light: $C \cdot t = l$ is the equation of the 3-dimensional edge of a cone of revolution in a 4-dimensional space.

2°- Inside the cone: $ds^2 > 0$ i.e. $C \cdot dt > dl$

Events can join (e^0) at speeds lower than the speed of light. There is a causal link between these events and (e^0) ; moreover $C \cdot dt = dl$, is the equation of the interior of a cone in a 4 dimensional space.

- The upper inner part of the cone contains all future events that can be joined from (e^0) .
- The lower inner part of the cone contains all past events that can be joined to (e^0) .

3° - The exterior of the cone: which corresponds to $ds^2 < 0$, i.e. $C \cdot dt < dl$ corresponds to the equation of the exterior of a cone in a 4 dimensional space. The events outside are said to be "elsewhere" with respect to "e^o").

*It should be noted that on the time axis (t), for an observer on earth and for an astronaut, time is the same. We recall at the end that in general relativity space-time is any curved space and in special relativity space-time is a flat Minkowski space.

6-1) Geometry in Minkowski space:

Minkowski space is a 4-dimensional pseudo-Euclidean affine space with an origin O and a 4-dimensional vector space modelling the spacetime of special relativity. This vector space has a non-degenerate symmetric bilinear form and is denoted by (u/v). or (e1, e2, e3, e4) is a basis of the vector space, with :

$U = (u1, u2, u3, u4)$, are the components of vector u in this basis and $v = (v1, v2, v3, v4)$ are the components of v in the same basis.

$(u/v) = tu.g.v = u1v1 - u2v2 - u3v3 - u4v4$ and also

$(u/v) = tugu = (u1)^2 - (u2)^2 - (u3)^2 - (u4)^2$

Two vectors are said to be orthogonal in Minkowski space if and only if $(u/v) = 0$ with :

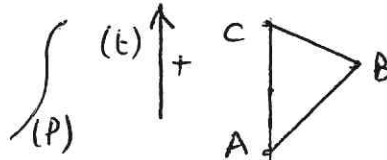
$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

6-2) Other particularities :

In a Euclidean space and for a triangle ABC we always have

$AC < AB + BC$ whereas in a Minkowski space: $AC \geq AB + BC$, because AB and BC are oriented towards the future

in euclidean space: $AC < AB + BC$



The trajectory of a material point in space-time defines the Universe line (p), this trajectory must go in the increasing direction of time.

7) Expression and metrics in special relativity:

In a Euclidean space, the square of the distance dl between two neighbouring points A and B is written as follows: $dl^2 = dx^2 + dy^2 + dz^2$. In special relativity and in a 4-dimensional space, the square of the distance between two events A and B is written as follows:

$ds^2 = C^2dt^2 - dl^2$, and also $ds^2 = dx^2 + dy^2 + dz^2 + d(i^2C^2t^2) = dl^2 + i^2C^2dt^2 = dl^2 - C^2dt^2$. The factor C^2

is imposed by the Lorentz transformations and the principle of special relativity, used to justify the invariance of ds by change of inertial reference frame. ds is a pseudo-metric defined by :

$ds^2 = C^2t^2 - dl^2$ or also $ds^2 = dl^2 - C^2dt^2$ according to the convention of the chosen sign of the

signatures : (+, -, -, -) or (+, -, -, -) . It should also be remembered that the invariance of the

interval ds in space-time by changing the reference frame is a central property of special relativity.

This invariance of ds can be stated as a founding axiom of the theory of special relativity, but it can

also be deduced directly from the original axioms of the said theory. We can write: $|\cdot dl/dt| = C$

then $ds^2 = C^2dt^2 - dl^2 = 0$ and if the two events are seen in another frame of reference, we will

always have $|dl'/dt'| = C$ and $ds'^2 = C^2 dt'^2 - dl'^2 = 0$ and $ds^2 = ds'^2$ thus invariance by change of reference;

The square ds^2 between the two events can be positive, negative or zero.

- 1) $ds^2 = C^2 dt^2 - dl^2 > 0$, we say that the square ds^2 is of the time type
- 2) $ds^2 = C^2 dt^2 - dl^2 < 0$, the square ds^2 is said to be of the space type
- 3) $ds^2 = C^2 dt^2 - dl^2 = 0$ we say that the square ds^2 is of the light type

This interval can be considered as a metric from which mathematical properties of special relativity and general relativity can be demonstrated.

*in special relativity: affine space and Minkowski space

*in general relativity: variety and curved space

In special relativity, we have seen that the infinitesimal space-time interval is written as follows:

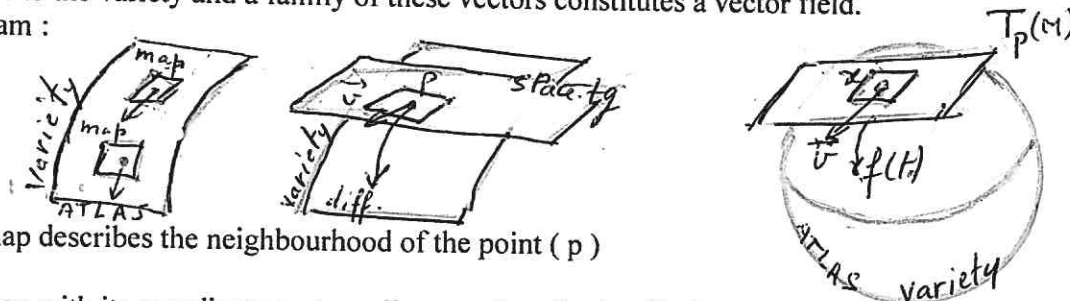
$ds^2 = C^2 dt^2 - (dx^2 + dy^2 + dz^2) = C^2 dt^2 - dl^2$. The metric of general relativity is defined from that of special relativity on the basis of the equivalence principle.

$ds^2 = \sum \sum G_{\mu\nu} dx^\mu dx^\nu$ or the coefficients of the metric G vary from one point to another in space-time according to the curvature. Einstein's writing convention is like this: $ds^2 = G_{\mu\nu} dx^\mu dx^\nu$

8) Pseudo-Riemannian variety and special relativity:

A Riemannian variety is a differential variety with an additional structure: a Riemannian metric to define the length of a path between two points of the variety. We can therefore perform differential and integral calculus operations. It is a topological variety where the space is possibly separated and can be locally assimilated to a Euclidean space. As we noticed before, the Riemannian variety fits better the space-time universe, we then call upon this kind of curved spaces, resembling locally affine spaces at the neighbourhoods of each point. On each of these points, one can lead tangent vectors to the variety and a family of these vectors constitutes a vector field.

Diagram :



The map describes the neighbourhood of the point (p)

The map with its coordinate system allows to describe locally the space-time around the point M1 or M2 of the variety.

The laws of special relativity apply locally at each point x in space-time. Around the point x of the variety, the region resembles a Minkowski space, i.e. a flat space-time. The measurements are made in a small region and the complete description of space requires several maps grouped together in an Atlas as we have noticed before (we would need an infinite number of maps for our galaxy

alone). Special relativity has therefore put an end to inertial reference frames for the description of physical phenomena and general relativity has eliminated all privileged reference frames.

We recall that the main tools used in this geometric theory of gravitation are tensor fields, defined on a pseudo-Riemannian variety representing our space-time. The space curved by gravitation is then modelled by a continuous, connected 4-dimensional pseudo-Riemannian variety constituting a separate space; the other physical factors are represented by tensors.

9) Metric of the Universe :

In Euclidean geometry and in a 3-dimensional space, we have pointed out that in Cartesian coordinates, the distance between two very close points is written: $dl^2 = dx^2 + dy^2 + dz^2$; and in

spherical coordinates, this distance is written: $dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$; where (dx, dy, dz) are the Cartesian coordinates of dl , and (dr, θ, ϕ) are the spherical coordinates of dl . In space-time the metric ds between two events is defined as follows: $ds^2 = C^2 dt^2 - dl^2$; the same dl as in Newtonian physics with dt = time difference between two events, i.e. the Minkowski metric in special relativity. For Einstein, who linked space and time into a single space-time and curved it, the metric is no longer defined in an absolute way but depends on the amount of matter and the distribution of energy in the universe. The character of the isotropy and homogeneity of the Universe on a large scale constrains the form of the metric. Thus, the metric deduced is called the FLRW metric (mathematical formula of Friedman, Lemaitre, Robertson and Walker; already mentioned in Part 1) and is written in the following form: $ds^2 = C^2 dt^2 - a(t)^2 ((dr^2 / (1 - kr^2) + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2))$. This can be seen as follows:

1) The temporal part does not change, i.e. the Universe evolves identically in all space; this is the principle of temporal equivalence.

2) The constant k can take one of three possible values:

-1, 0, +1,

* $k = -1$: the Universe is hyperbolic

* $k = 0$: the Universe is flat and infinite which can be described by Euclidean geometry.

* $k = +1$: the Universe is elliptical or spherical and closed on itself and can be described by Riemannian geometry, this is the case that interests us most..

$a(t)$: is the term of increase of the Universe; if it increases with time, the Universe expands accelerated and the distances in this Universe also expand accelerated; if it decreases with time they are compressed. If it is constant with time, then it reflects the expansion of the Universe in a constant way as I pointed out in Part 1, the parameters (a) and (k) must be determined by the observer.

10) The signature :

As I have noticed many times, the space-time of special relativity can be characterized by the elementary interval ds with:

$$ds^2 = C^2 dt^2 - dl^2 = C^2 dt^2 - (dx^2 + dy^2 + dz^2) = C^2 dt^2 - dx^2 - dy^2 - dz^2 .$$

ds is the distance between the two events (the two points) which are infinitely close. This interval is as I noticed invariant by change of reference frame in the Lorentz transformation; and the expression comprises a sign (+) of beginning for $C^2 dt^2$ and 3 signs (-) for the components (dx, dy, dz) , i.e. : (+, -, -, -) : this is the signature.

The Minkowski metric is written with the signature; (+, -, -, -) and $ds^2 = C^2 dt^2 - (dx^2 + dy^2 + dz^2)$

Another possibility is to choose the signature (-, +, +, +) and we have: $ds^2 = -C^2 dt^2 + dx^2 + dy^2 + dz^2$ and we recognize here the Poincaré invariance formula: $s^2 = l^2 - C^2 t^2$

This signature is called normal hyperbolic because it generalises the hyperbolic equation:

$Y^2 - X^2 = \text{Constant}$, since it takes the form of $ds^2 = C^2 dt^2 - dl^2 = \text{invariant quantity}$ whatever the reference frame. It is thus chosen to ensure the condition $ds^2 > 0$ and $C^2 > (dl^2/dt^2) = v^2$. The speed of light C is greater than the speed of any material particle (v); and because of this hyperbolic signature, space-time can resemble Euclidean space.

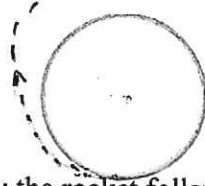
We have $ds^2 = (iCdt)^2 + dl^2 = -C^2 dt^2 + dx^2 + dy^2 + dz^2$. The interval with terms of the same signature : (-, +, +, +) is said to be elliptical. In general relativity, one can use any reference frame and any coordinate system to describe the metric; one could also give an expression of this same metric using the radial distance which would in this case be more complicated.

11) Geodesic: the shortest path over a curved surface:

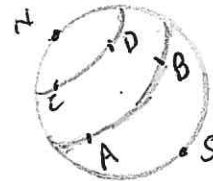
In geometry, a geodesic refers to the generalisation of a straight line on a surface. It is the shortest path between two events in space-time. Geodesics generally describe the movements of free

particles: a satellite in orbit follows a geodesic, but a rocket heading for the moon does not follow a geodesic because of the thrust force exerted by the rocket engine.

In Euclidean geometry, straight lines form the most common geodesics. On a sphere, geodesics are circles or parts of circles. The shortest path between points A and B is given by the smallest portion of the great circle passing through A and B. If A and B are respectively at the north and south poles, then there are infinitely many geodesics (infinitely many shortest paths between A and B).



Satelliteisation: the rocket follows a geodesic



When there is a field that modifies the trajectory, a probe follows a curved spatial route. One of the principles of general relativity is that in a space-time, masses and electromagnetic radiation, when subject only to gravitation, perform motions necessarily following geodesics. The displacement of the moon around the earth, the displacement of the earth around the sun, the displacement of the sun around the galactic centre, are inertial motions because the moon, the earth and the sun are subject only to the action of gravitation. If, for example, we want to calculate the trajectory of the earth using the geodesic equation of general relativity and taking into account the presence of the sun and the presence of other planets, we obtain an elliptical orbit covered according to Kepler's laws. Under the sole action of inertia and gravitation, a particle describes a geodesic of pseudo-Riemannian space-time. For a massive body, the motion in a gravitational field is more complicated, since this massive body creates its own field. The use of a low mass particle simplifies the study of the laws of motion, because the particle does not modify the field to be studied (negligible influence of the mass of the particle) and the trajectory of this particle in space-time is indeed a geodesic in Riemannian geometry, a geodesic is a curve which follows locally a minimal distance; more precisely a parametric curve μ , such that:

$$\begin{array}{ccc} \mu : (\text{interval } I) & \xrightarrow{\hspace{2cm}} & (\text{metric space } M) \\ \mu : & I & \xrightarrow{\hspace{2cm}} M \end{array}$$

The application μ is a geodesic if there exists a constant $k > 0$ such that $\forall t \in I$, there exists a neighbourhood J in I such that if t_1 and t_2 belong to J , we have $d(\mu(t_1), \mu(t_2)) = k |t_1 - t_2|$

A metric space is said to be geodesic if any two of its points are always connected by at least one geodesic.

If $k = 1$, $d(\mu(t_1), \mu(t_2)) = |t_1 - t_2|$; this is a natural parametrisation. The trajectories of particles in a gravitational field are described by the following differential system: $(d^2x^a/ds^2) + G_{\alpha\beta}(x^\gamma) dx^\beta/ds \cdot (-dx^\alpha/ds) = 0$ with $G_{\alpha\beta}$ denoting the coefficients defining the metric of the curvilinear coordinate system (x_1, x_2, x_3, x_4) .

12) Space-time is plastic and soft:

Space-time is flexible, its shape can change under pressure just like water (water is not elastic whereas space-time is). The propeller of the boat turns and moves the boat through the water, causing the water to swirl backwards; what happens with space-time and the earth turning and moving forward is something like this: a swirl whose flowing fluid is space-time itself.

Lense-Thirring effect:

Around the earth, space-time is driven by its rotation and is therefore not fixed. It is disturbed and

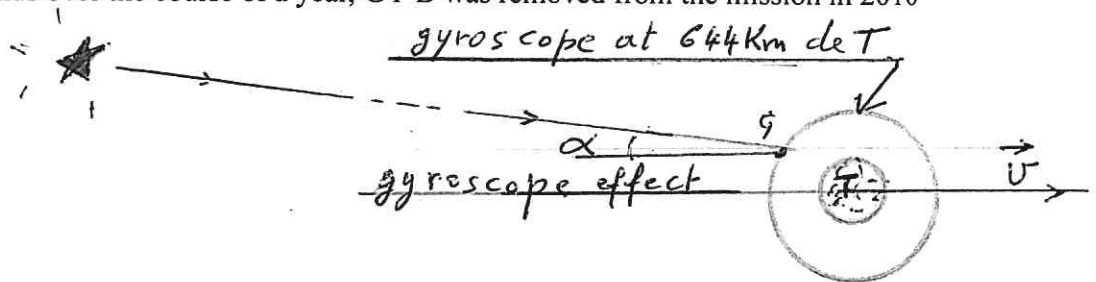
the earth appears to be in a viscous medium that has both a soft and a hard consistency. It is as if it were doughnuts in a honeyed medium: this is the lense-thiring effect. The lense-thiring effect is a small-scale astrophysical phenomenon predicted by the theory of general relativity to have a significant effect around rapidly rotating objects in a strong gravitational field. It is a relativistic correction to the gyroscopic precession of a body whose mass and angular velocity belong to an order of magnitude that escapes Newtonian mechanics. This effect was predicted in 1918 by the Austrian physicists Josef Lense and Hans Thiring in their work on general relativity. NASA confirmed the presence of a vortex of space-time around the Earth with the Gravity ProbB mission (GPB), launched in 2004. The satellite, orbiting at 644 km from Earth (G P B), used four ultra-precise gyroscopes launched in April 2004; and without this very high precision technology, we would never be able to confirm the Lense-Thiring effect, which Einstein himself believed to be impossible to achieve. We wanted to measure two key effects:

The first effect is the geodesic effect which consists of the deformation of space-time.

The 2nd effect is the amount of space-time that such an object, i.e. the earth, has affected by moving forward and turning on itself. The Lense-Thiring effect :

(Wikipedia.<http://fr.wikipédia.org/wiki/espace-temps>; Nasa announcements results of epic.space-time : 04 May 2011)

Around the rotating earth, space and time are pulled with the motion of its rotation (like water with the propeller of the boat). G.P.B used a device called " stellar tracker " which is oriented and pointed in the direction of a single star: reference guide star; it is the star " I M pegasus ", while being on polar orbit on the earth; to measure the deviation of the gyroscopes embarked. It is as if the earth advances at the speed v and turns around itself in this soft medium, behind it the space-time (soft medium) is under the torsion, thus reaction of a vortex detected by the gyroscope. The hands of the gyroscope describe a conical movement around a mean position: the proper axis of rotation in the case of a Euclidean space, the gyroscopes would always remain aligned with the star I M pegasse (guide star). But behind the earth, which moves and rotates and because of its mass and rotation, the direction of the axis of rotation of the G P B gyroscopes changes, this direction records an angle $\epsilon = 6.6$ milliseconds over the course of a year, G P B was removed from the mission in 2010



Another key aspect of general relativity was confirmed, namely that a clock on Earth runs more slowly than on a spacecraft orbiting 644 km from Earth. This experiment confirms the predictions of the theory of general relativity, which also helps to understand the energetic bursts of quasars and the rapid rotation of super-massive black holes.

NB: If we strip space of everything: all the molecules, atoms, photons, energy, in short all the particles that can exist; the rest is a soft plastic space, which can be bent or twisted.

13) STRATIFICATION AND HISTORICAL IMAGE OF THE UNIVERSE:

The study of subsoil strata is the basis of the geological sciences. Stratification is the layering of rocks and these layers are distinguished by their own characteristics according to their components. The colours give information on the nature of the materials and the thicknesses can give information on the paleoclimate, on the movement of the continental plates and on geological crisis. The lowest layers are the oldest. Another example is the thermal stratification of the groundwater: cold water at the bottom and warm water at the top.

The world changes and many things have disappeared forever; but the traces persist and are still

reasoning today. For the earth, which is about 4.5 billion years old, geologists distinguish between periods: the oldest is infernal and characterised by the heaviest elements (a metallic core that creates a magnetic field around the earth, protecting it from the solar winds). Subsequently, other periods followed: Archean (archaic earth resembling Titan), Proterozoic (2 billion years long), Phanerozoic, etc. until now.

To approach the Universe and its history, we can use the same approach, i.e. the mathematical theory of stratification. We see the Universe through the light that reaches us from stars and galaxies. The speed of light being constant, then these stars and galaxies observed far away are no longer where we see them (see part 1, construction of the diagrams). We cannot see the present Universe, we can only see its ancient images which can tell us about its history. Its images and its colours which seem to arrange themselves one behind the other in the depth of this Universe can tell us about the materials that make up the galaxies. The arrangement and superimposition in the depth can also tell us about the evolution and formation of stars and galaxies. The further out we see, the hotter it is, and even further out, the matter appears to be disintegrating. As with terrestrial geology, a theory of the stratification of the Universe is possible and can even enrich, marry and be part of the theory of general relativity.

SCHEMATISATION OF THE GENERAL RELATIVITY:

Dynamics of expansion Part III

1) Simulation:

simulate a situation that may be relevant to the conformity of the universe. We therefore took the supposedly hyper-spherical Universe as a model.

We then give ourselves a swelling sphere in three-dimensional space of radius $R(t)$, variable with time t from 0 (or almost zero) to infinity. It is as if the sphere is in a 4-dimensional space, the motion of the inflaton which represents time is this 4th dimension (see the constructions of the diagrams in the 1st part).

It is also assumed that the distribution of matter is symmetrical as proposed by the cosmological principle of uniformity and isotropy.

It is also assumed that this system is made by an approximation of a concentric layer of constant ρ density.

O is the centre of the swelling sphere (origin and start of the swelling). The distributed matter is subject to its own gravitation as explained in the previous chapters.

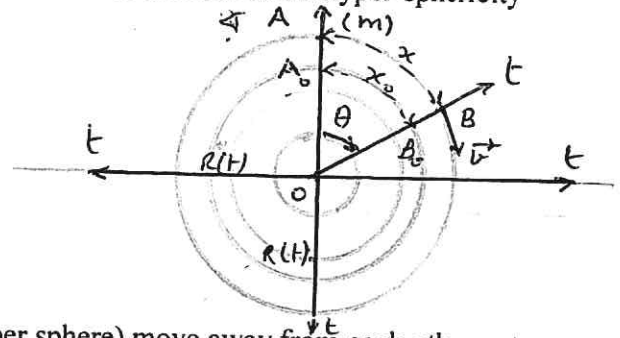
Let us now consider a point (A) of this swelling sphere, of mass (m) and location of the observer.

We have seen that the distance of a point (B) from point (A), the observer, under the sole effect of inflation, is achieved at the speed v and this speed is proportional to the distance $\vec{x} = \vec{AB}$

$\vec{v} = k \cdot \vec{x}$: Hubble's law and k is Hubble's constant. We have also mentioned and demonstrated that this distance at a speed proportional to the distance x corroborates the idea of the hyper-sphericity of the Universe.

Diagram :

$R(t)$ grows from 0 to $+\infty$ and $OA_0 < OA \iff x_0 < x$



We also mentioned that the points on the sphere (or hyper sphere) move away from each other not by their own movements, but only under the effect of the dilation of the spherical surface (for the Universe, it is the hyper spherical membrane of space-time) of radius $R(t)$ which grows with time.

For the construction of the diagrams, we have also considered a plane intersecting this swelling sphere by passing through the centre of the sphere O (the original point) and also through the point (A) of mass (m) and therefore also through the point (B). We obtain then, to recall, the diagram mentioned above: a large circle of radius $R(t)$ variable with the time t from 0 to $+\infty$ (from almost zero to $+\infty$).

By inflating, point (B) moves away from (A) and the angle $(OA,OB) = \theta$ remains always constant, but the distance x between A and B increases with $R(t)$.

$x = \theta \cdot R(t)$ and according to Hubble $\vec{v} = k \cdot \vec{x}$

\vec{v} : velocity of (B) with respect to (A) and x is the distance between them: i.e.

$$(dx/dt)/x = k(t) \quad (1)$$

varies with time t, despite being called the Hubble constant, k but in the very long term just like R(t). Since θ is constant, we write: (dde mechanics,

$$(dx/dt)/x = (dR(t)/dt)/R(t) = R'(t)/R(t) \quad (2)$$

((The following is partly based on an idea taken from a mechanics exercise, reference: The Earth, planet of the solar system, evolution of the cosmological horizon and the radius of the observable Universe within the framework of the standard model and the inflationary model)).

This point (A) of mass (m) is subjected to a radial attractive force:

$$m.R''(t) = (-G.M.m)/R^2(t) \quad (3) \quad \text{with :}$$

M = the mass of the sphere containing the matter: the Universe is unlimited but finite and has a mass.

G = the universal gravitational constant :

$$G = 6,6708 \cdot 10^{-11} \cdot \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$$

$$\text{Integrating (3), we obtain: } (m.R'^2(t))/2 - (G.Mm)/R(t) = K \quad (4)$$

K is a constant

$$\text{i.e.: kinetic energy + gravitational energy = total energy = K}$$

The evolution of the swelling will depend on the total energy, i.e. on the value of K, or in another way, the increase of R(t) will depend on K.

Hubble's law (2): $R'(t)/R(t) = k$ also allows to determine the variation of k with time as well as the value of the spectral shift knowing that the wavelengths are stretched by a factor R(t) during the expansion (see previous chapter II).

We also recall the result $k = C/2R(t) = C/RH$ with $RH = 2R(t)$ called Hubble radius, it is the curvilinear length OA represented by the function $\delta(t)$, path of light going from the original point (O) to the point (A) the observer (see chapter I on the construction of diagrams).

$$\text{The spectral shift } \Delta\lambda/\lambda = \vec{v}/C \quad (\text{result already seen}) ; \Delta\lambda/\lambda = k \cdot \lambda/c = (C/2R(t))(\lambda/c) = \lambda/2R(t)$$

$$x'/C = x/2R(t) \quad \text{and} \quad x'/x = C/2R(t) = k \quad \text{and}$$

$$\theta x = R(t) \quad \text{and} \quad \theta x' = R'(t) \quad \text{which gives} \quad x'/x = R'(t)/R(t) = k \quad \text{and}$$

$$k' = (R''(t)/R(t)) - k^2$$

$$\text{equality (4): } R(t)'^2/2 - G.M/R(t) = K$$

The dynamics of the swelling of the hyper sphere (or the sphere), depends on the value of K or in another way on the evolution of R(t).

$$\text{** 1) If } K = 0 \text{ then } R'(t) = (2G.M/R(t))^{1/2} \quad \text{and}$$

$$R(t) = (9G.M/2)^{1/3} \cdot t^{2/3} \quad \text{then} \quad R'(t) = 2/3(9.G.M/2)^{1/3} \cdot t^{-1/3}$$

The expansion continues indefinitely with a very low rate which tends to cancel out with $t \rightarrow \infty$ and $k = R'(t)/R(t) = 2/3t$

the Universe is almost flat and we obtain an estimate for the current age of the Universe to $= 2/3.k$
For $\vec{v} = 70 \text{ km/s/Mpc}$ we get $t = 13 \cdot 10^9 \text{ Years}$

$$k^2 = (R'(t)/R(t))^2 = (8\pi G \rho_e)/3 \quad \text{so} \quad \rho_e = 3k^2/8\pi.G \quad \text{or} \quad \rho_e = 10^{-29}/\text{cm}^3$$

The Universe is almost flat and these equations give an idea of the behaviour of the radius of the Universe and the Hubble constant.

Diagram:

**2) If $K < 0$: ($\rho_0 = \text{critical } \rho$, i.e. $\rho > \rho_0$)

The expansion is elliptical and solving (4) gives: $R(t) = R_0 (1 - \cos \omega t)$ and $t = (R_0/C)(\omega - \sin \omega t)$

which are the parametric equations of a cycloid or ω is the development parameter of the Universe ;
 $\omega \in [0, +\infty[$ and $R_0 = \text{a scale length } R_0 = GM/gC^2$

The expansion will stop, the radius of the Universe passing through a maximum $R(t) = 2R_0$ and then reversing into a contraction, ending in a large collapse.

**3) If $K > 0$: $\rho < \rho_0$

The expansion will continue indefinitely, it is hyperbolic and we have :

$$R(t) = R_0 (\text{ch } \omega t - 1) \quad \text{and} \quad t = (R_0/C)(\text{sh } \omega t - \omega t)$$

$\text{ch } \omega t$ and $\text{sh } \omega t$ are the hyperbolic cosines and sines

**4) Case where the Universe is dominated by radiation :

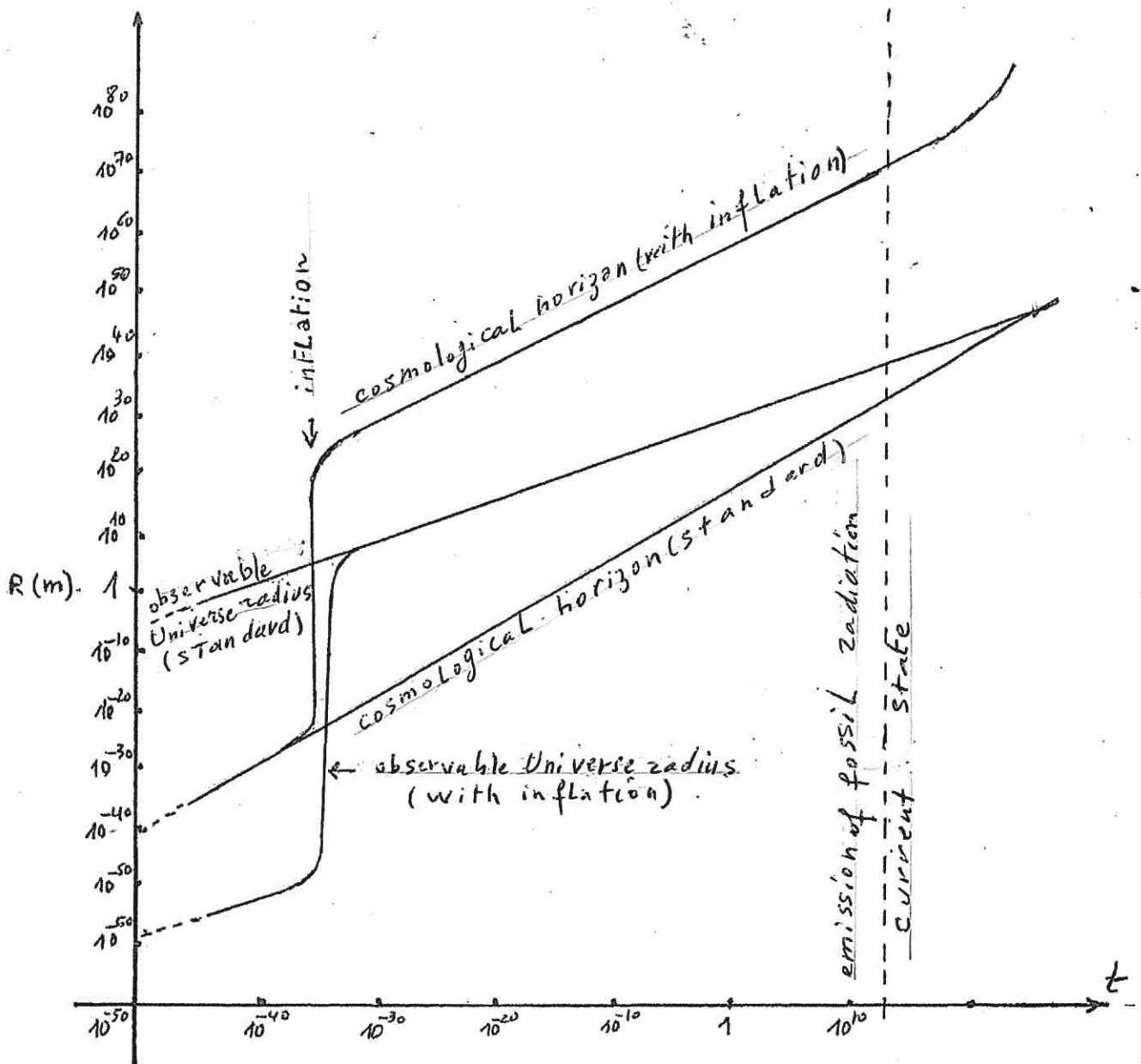
$m \cdot R''(t) = - (GMm/R^2(t))$ This gives : $R''(t) = 4\pi GA/3$; $A = C^2$
 then we have : $R'(t) = (8\pi GA/3 C^2) t$, $1/R^2(t) + C^2$

ON is in the case of small values of $R(t)$ and t and $1/R^2(t)$ overrides the value of the constant K we obtain:

$$R(t) = (32 \cdot \pi \cdot GA / C^2)^{1/4} \sqrt{t}$$

the scale factor varying between \sqrt{t} during the phase of the radiative universe, the temperature varies in $..1/\sqrt{t}$

This law of variation summarises the thermal history of the Universe at its beginning. Recourse to inflation: see following diagram :



evolutions of the cosmological horizon and the radius of the observable Universe in the framework of the Standard Model and the inflationary model (after Guth and Steinhardt in 1988).

2) CURVATURE AND PLASTICITY:

Mathematical models of the Universe consider it to be fluid and flexible, whose shape can change under "pressure" (existence of matter or energy) and this seems to be confirmed by the available observations: see the Lense-Thirring effect mentioned earlier in Part II).

Let us imagine that we manage to remove from space-time all forms of matter and energy: we remove atoms, photons, radiation, the earth, galaxies, etc. we remove everything, absolutely everything, what is left?

What is left is an empty space-time; this empty space, as the theory of general relativity tells us, is elastic, bendable, subject to curvature and even to torsion. It is a plastic space and the contraction of this space is always accompanied by the dilation of time (time tends to stop when pressed).

As we mentioned before, the theory of general relativity is based on the geometry of Riemann spaces called curved spaces and in Riemannian geometry the curvature is determined by a tensor, introduced from the notion of connection (see "tensors" mentioned previously in part II). To connect means to join and attach a part of space to another neighbouring part.



Mathematically we define minimum and maximum values of the curvature χ . These values are called respectively the main curvatures:

$$\text{average curvature} = \chi_m = (\delta_{max} + \delta_{min})/2$$

$$\text{Gaussian curvature} = \delta_G = \delta_{max} \cdot \delta_{min}$$

$$\text{total curvature} = \delta_t = \int_S \delta_G ; S = \text{Total area oriented.}$$

STRUCTURES AND SCALES OF MAGNITUDE:

Each order of magnitude has its own physics, which means that there are other systems and

structures whose details and laws still escape our knowledge: I quote some examples:

Example 1:

When you open the tap, you can see the water column descending under the effect of gravitation. But if we apply a powerful microscope to examine the movements of certain points within the water column, we could see, over a short period of time, regions that rise and then fall again, certainly according to physical laws specific to these microscopic regions.

NB: do not confuse these microscopic regions with the world of quanta.

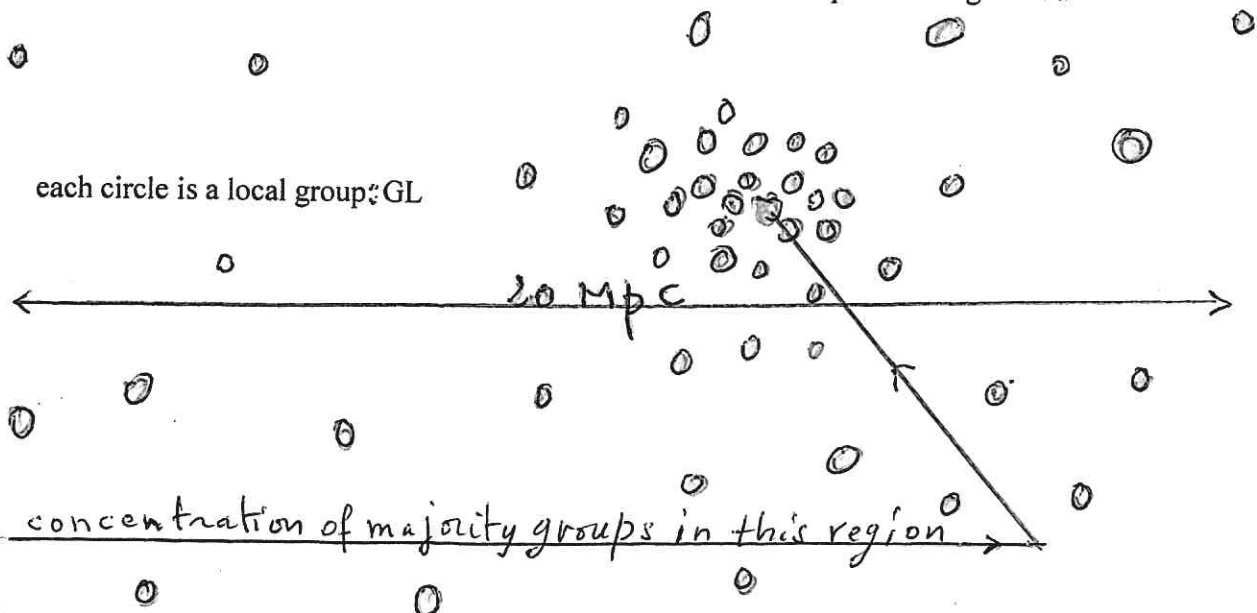


Example 2 and comparison with the Universe:

In a river, in a river or in the restless ocean, if we go to the infinitely microscopic, we will see the same thing; small eddies in spite of the river current, movements of points independent of the apparent major movements. There are certainly laws that govern and determine the organisation of the movements of this microscopic infinity that seems independent.

This is similar to the cosmological scale, as the motions of the moon, the earth, Jupiter and its satellites, Saturn and its satellites, and the sun itself, which follows its geodesic around the centre of the galaxy, are all part of the infinitely small relative to the motions of the galaxy, the local group, the cluster of galaxies and the rest of the universe. It is relatively similar to the microscopic infinity of the water column in the tap, in which points move independently of the gravitation that makes the water flow downwards, or to the small whirlpool carried along by the river current. Each order of scale and magnitude has its own law of gravitation, its own curvature and its own plasticity. Despite the expansion of the Universe, we can see that the Andromeda Galaxy, our Milky Way and the other galaxies in our local group are all converging towards their common centre of gravity (the galaxies are moving towards each other) and like this, this should happen in many other corners of the Universe. At all scales, gravity drives the dynamics and affects the distribution of matter. At very large scales, homogeneity seems to prevail, while heterogeneity is found at smaller scales. The further down the scale, the more the density contrasts become apparent; there is therefore a transition between homogeneity on a very large scale and heterogeneity on a smaller scale. Our galaxy, the Milky Way, with a diameter of 10,000 AL, neighbouring the Magellanic Clouds and a lesser neighbour, the Andromeda Galaxy, with its two satellites, is only part of a system of about twenty galaxies, the local group. This local group of size 1 Mpc is in its turn an element of the "local super-ama"; this ensemble is itself centred on the dominant galaxy cluster: "the virgin cluster" located at 10 Mpc and this ensemble has a diameter of about 30 Mpc and comprises about 1 million local groups.

Diagram: The distribution of galaxy groups within a radius of 20 Mpc is homogeneous/



The concentration of majority groups in this region indicated by the arrow represents the local superama

The circles (O) represent the local groups

This non-uniform structure is centred on the dominant ama. The concentration decreases from the centre outwards. Observations have identified the presence of intergalactic ionised gas at temperatures of the order of 100 million degrees which may be a residue of primordial gas.

Superclusters are the residue of the initial distribution of matter. They are the beginnings of large-scale spatial distributions and beyond that the quality of measurements degrades. The spectral shift of light is then used to directly access the distance of a distant object.

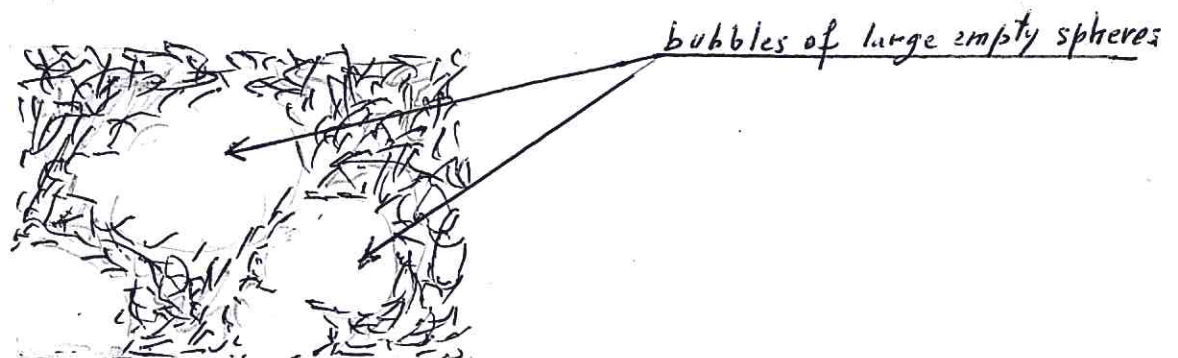
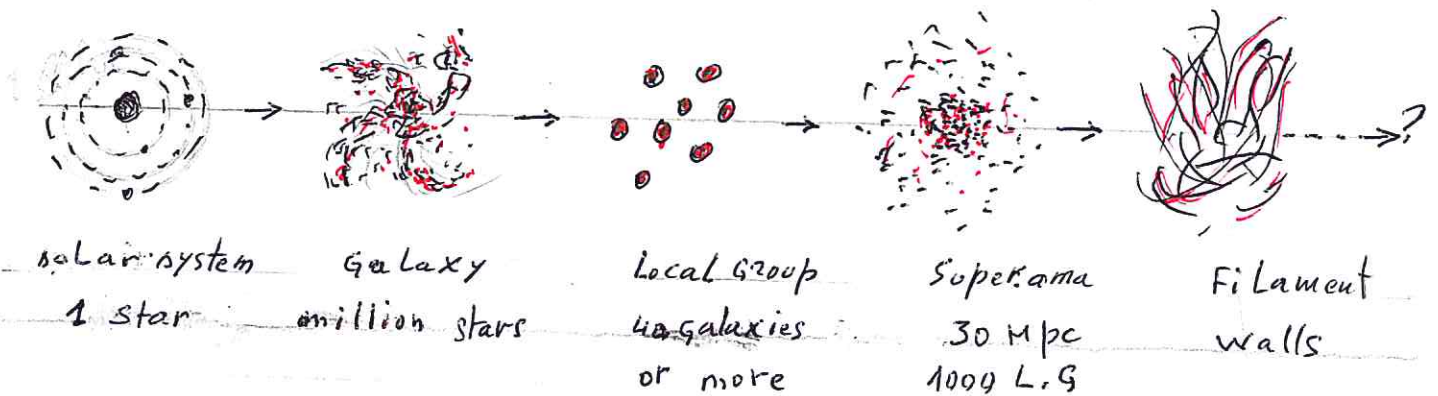
At very large scales the appearance is very heterogeneous. The Universe appears like a sponge made up of large empty spheres (bubbles) of 50 to 60 Mpc in diameter, with the galaxies located on

the walls and the intersections of the bubbles being the most concentrated.

The existence of large walls of dimensions close to 100 Mpc is evidence that the Universe does not have a homogeneous distribution at these scales. Computer simulations lead to the formation of large filamentary structures at this level, but beyond that, it is unknown.

We have noted that homogeneity and isotropy form the large-scale cosmological principle, so constant large-scale curvature allows us to construct a plausible model. Homogeneity and isotropy allow a plausible model to be built, meaning that the physical properties are the same at any point in space at any given time. But if the distribution is indeed heterogeneous at all scales, which is possible, the standard model of galactic system formation can be questioned. We must also remember that the view from the Earth is only a small part of the sky

Diagram (summary):

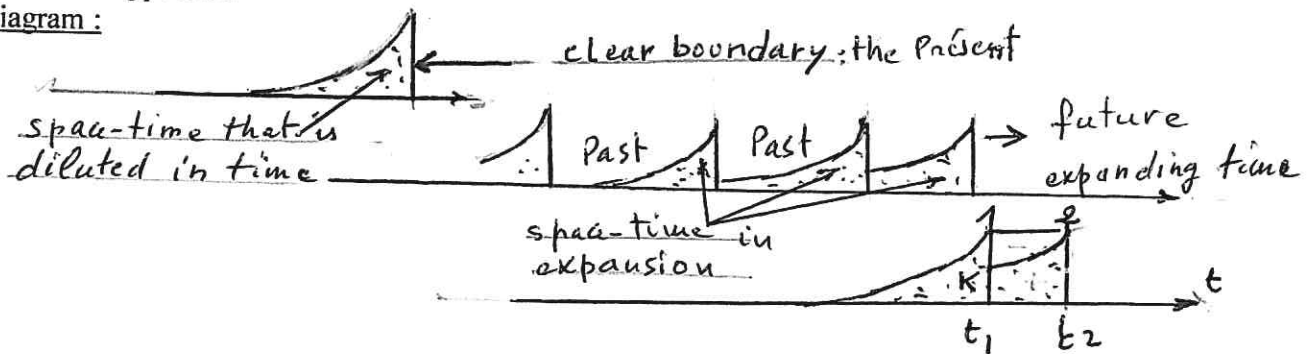


3) DYNAMICS AND EXPANSION :

The expansion of the Universe results in the dilution of space-time (see diagram below).

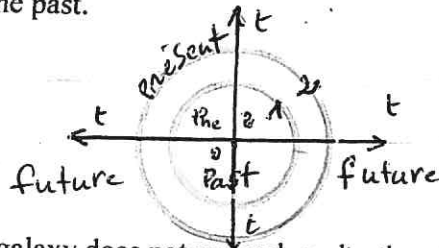
Intergalactic distances are constantly increasing, but the sizes of matter (atoms, molecules, particles, material objects, etc.) remain unchanged, including the distances between planets and their stars and the entire interior of galaxies. It is only intergalactic space that is expanding. The interior of the galaxy and the distances between stars remain unchanged. The almost empty galactic bubbles are growing and galaxies are forming due to the gravitational effects between these huge bubbles. Matter condenses to form galaxies despite the expanding space-time. Condensation and expansion are linked as two opposites. Expansion and condensation are like empty space and matter as dialectical opposites. Matter and expanding space produce each other, they are

Diagram :



between t_1 and t_2 , creation of the rectangle $(1,2,t_1,t_2)$ with dilution (loss in the past): dark zone $(1,2,k)$; creation prevails over loss and the Universe tends towards enlargement and expansion

The surface of the hyper-sphere in (2), the space membrane, encompasses and exceeds that in (1) which belongs to the past.



twist black hole

The interior of the galaxy does not expand, under the effects of gravitation and the torsion caused by the nucleus (the black hole), the local space remains outside the extension. Everything rotates around the centre and moves towards it; it is like holding an elastic cloth pulled from the four sides and in the middle you pinch at this middle point by rotating the hand around this point, the small region near the middle point remains under pressure without expanding even if the rest of the cloth is stretched (this small region around the middle point is the galaxy).

Mathematically : the identification between gravitation and geometry required a framework, that of a 4-dimensional space-time with pseudo-Riemannian metrics (this has been developed in the previous chapters: metrics and tensors). We recall that the measurement of the infinitesimal distance between two infinitely close points and the integration allow us to evaluate the space-time interval between these two points. The metric is generalised by a sum of all the 2 to 2 products and each of these products is assigned a coefficient which itself depends on the position, the time, the curvature and the environment of the 2 points. These coefficients called "metric potentials" become the essential variables of the theory of gravitation. They define the curvature properties of space-time and the curvature is revealed by the operation of the 'parallel transport' of a vector.

4) MATTER IS ENERGY (in special relativity) :

One of the consequences of the theory of special relativity is the following result, demonstrated by Albert Einstein: $E = m.C^2$

with E = the energy (in joules) and m = the mass of matter (in kg).

We consider a point of mass (m) which, by means of a force \vec{F} reaches the speed \vec{v} . For simplicity, we also consider that this force \vec{F} is collinear with \vec{v} :



The quantity of motion is defined by $\vec{Q} = m.\vec{v}$

The energy E acquired by the material point of mass (m) is equal to the work W of the force $\vec{F} = m.\vec{a}$. \vec{a} is the acceleration of the material point

$$dW = \vec{F}.d\vec{x} = m.\vec{a}.d\vec{x} = m.(d\vec{v}/dt).d\vec{x} = m.(d\vec{x}/dt).d\vec{v} = m.v.dv$$

For Newton who did not know that the mass (m) also depends on the velocity \vec{v} :

$$W = \int_0^v m.v.dv = 1/2.m.v^2 = E_c \text{ : kinetic energy}$$

But taking into account Lorentz's relativistic expression, the mass (m) varies with the velocity \vec{v} and $m = m_0 / (1 - v^2/c^2)^{1/2}$; m_0 is the mass of the material point at rest and the force \vec{F} and the velocity \vec{v} are not necessarily collinear.

The variation of the total relative energy is written $dE_{rt} = d(\vec{Q}.\vec{v})$, then

$$d(\vec{Q}.\vec{v}) = \vec{v}.d\vec{Q} + \vec{Q}.d\vec{v} \text{ (a)}$$

$$dE_{rt} = dE_c + dE_0 \text{ with}$$

dE_c = variation of the kinetic energy

dE_0 = variation of the energy at rest then

$$\text{Thus } \vec{Q} = m.\vec{v} \text{ and } d\vec{Q} = (m.\vec{v})'.d\vec{v} = (m'.v + m).d\vec{v} =$$

$$= \left[\left[\frac{m_0}{\sqrt{1 - v^2/c^2}} \right]'.v + \frac{m_0}{\sqrt{1 - v^2/c^2}} \right].d\vec{v} = m_0 d\vec{v} / (1 - v^2/c^2)^{3/2} \text{ then}$$

$$E_c = \int_0^v \vec{v}.d\vec{Q} = \int_0^v (m_0 v d\vec{v}) / (1 - v^2/c^2)^{3/2} = \left[\frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \right]_0^v = m_0 c^2 \left[\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right]$$

$$E_c = \left(\frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \right) - m_0 c^2 \quad \text{so} \quad E_c = E_{rt} - E_0 \quad \text{that is to say :}$$

(relative total energy - energy at rest)

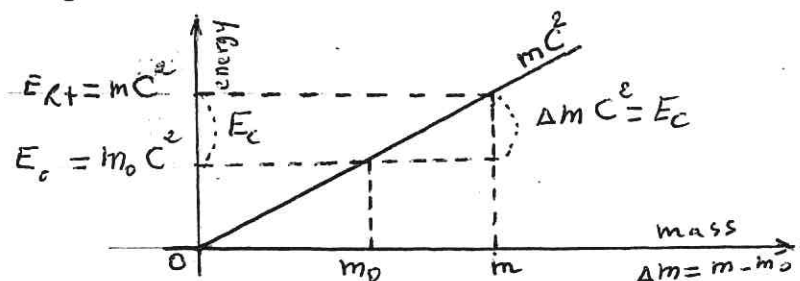
We see that the kinetic energy of a particle considering the relativistic expression is given by :

$$E_c = E_{rt} - E_0 = (\text{relative total energy} - \text{rest energy})$$

$$E_c = m.C^2 - m_0.C^2 = (m - m_0).C^2 = \Delta m.C^2$$

Any particle has an energy due to its rest mass ($m_0.C^2$) or an energy ($m.C^2$) if it is moving at speed v . The difference between $m.C^2$ and $m_0.C^2$ gives the kinetic energy.

Diagram :



Inertia and energy stored in matter :

When we move a material object, we feel that it resists. The heavier the object, the more it resists. This resistance to movement is due to stored and trapped energy. The inertia of the object is a consequence of the fact that the object is a concentration of energy. We can therefore say that inertia is an emergent property of an accumulation of energy trapped in matter.

$E = m_0 \cdot C^2$ means that a particle of mass (m_0) isolated and at rest in a reference frame possesses an energy (E) called mass energy.

According to the theory of special relativity any immobile system of mass (m_0) has a mass energy $E = m_0 \cdot C^2$. For Einstein, mass is also considered as a measure of the energy of a body. The equation $E = m_0 \cdot C^2$ makes it possible to calculate the energy that will be released in a nuclear fusion reaction of a bomb by knowing only the mass of the fuel; here matter disappears by being transformed into

energy. So can we say that matter and energy are the same thing, but in different states?

It is from $C = \text{constant}$ (we always remember that C is the speed of light), that Einstein managed to demonstrate the equation $E = m \cdot C^2$

V) HOLONIC STRUCTURE :

ref: see wikipedia. [https://fr.wikipédia.org/wiki/holon_\(philosophy\)](https://fr.wikipédia.org/wiki/holon_(philosophy)) . The novelist Arthur Koestler's book: "The Horse in the Locomotive" published in 1967.

Holon: something that is both a whole and at the same time a part of something larger and containing it. A holon is a complete whole included in another holon of which it is a constituent.

It is a word that does not appear in the dictionary but it belongs to Arthur Koestler, the famous novelist who used it in his book "The Horse in the Locomotive". In this book, the writer explains the origin of human violence while criticising Darwinism and behaviourism.

The dimensions of holons range from subatomic particles to the universe. The holon is influenced by the sets that constitute it, while at the same time it influences them and the holon that contains it. Information flows in both directions: this is bidirectionality. And when the bi-directionality of the information flows is compromised, then the system collapses; the assemblies no longer recognise their dependencies on their subordinates and the subordinates no longer recognise the organisinauthority of the assemblies.

Example 1:

In the biological field, cancers can be seen as such collapses. A hierarchy of holons is called holarchy and the holarchical model can be seen as an attempt to modify and modernise perceptions of natural hierarchy. If holons cease to exist, then the holons of which they were a part inevitably cease to exist too.

Example 2:

If atoms cease to exist, then the molecules made up of those atoms disappear, but the reverse is not, i.e. if the molecules cease to exist, the atoms remain. This is based on the principle of the fundamental and the significant; that is, if the fundamental disappears (the atom) then the significant disappears (the molecule).

Example 3:

The individual in the family, the family in the community, the community in the city, the city in the nation, the nation in humanity, humanity in all living beings, etc. If individuals no longer recognise the family (no more flow of information), the family breaks up and disappears. Similarly for the family, if it is destroyed, the information flow is compromised. It can be seen here that the individual is the foundation for the family (the significant) and the family is the fundamental for the significant, which is the community. In general, if the fundamental

disappears, then the significant also disappears and the reciprocal, as we have seen before, is not always true. On this endless chain everything can be fundamental and everything can also be significant, from the infinitely small to the infinitely large to the Universe.

The extremities of the holarchic system are in fact only laid down by the human mind at the moment, but then scratched off because we do not yet have the totality of reality (I believe that this limit, which is "total reality", is forever inaccessible) and these extremities remain open on both sides.

Karl Popper teaches us in his book "The Logic of Scientific Discovery", a book that has shaken up the scientific world since its publication almost ninety years ago in Germany, he teaches us this: What the human mind knows and will know in truth at a point in space and time is a similarity and

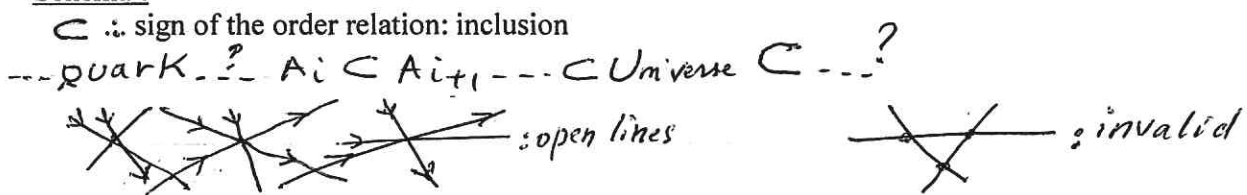
that it continues to approach reality without ever reaching it, in other words, the human quest for knowledge is a never-ending day, with countless views ahead but no possibility of reaching the end. We are conditioned to apprehend the world on our own scale and as soon as the dimensions go beyond our daily lives, we lose interest, and then when we turn to the infinitely large or the infinitely small, in both cases we lose the pedals.

Some physicists have sought a theory of everything, i.e. to reconcile the quantum theory and the theory of general relativity by starting from the microcosm to the macrocosm and this is not right for at least two reasons:

- 1-) The fundamental and the significant are not the same thing.
- 2-) The microcosm is open dimensional and there are laws that work on low systems and stop working at higher levels.

Finally, we can notice that the concept of "holon" is currently being emphasised again for a new system of organisation of nature

Schema :



A_i is a complete entity i , A_i is itself composed of chains of entities without extremities. The chain of inclusions can be seen as a holon line without ends and each holon which is itself an assembly of holons can also be seen as intersection of several holon lines.

The lines are always open and no loops, i.e. no returns for the lines because the inclusion is an order relation.

6) GRAVITATIONAL WAVES :

This is an oscillation in the curvature of space-time that propagates at the speed of light from its point of formation. Gravitational waves are produced by accelerated masses or by gigantic shocks. These waves do indeed have a physical existence and were observed by the US NASA in 2016. The wave created (or the disturbance) is not felt instantaneously from its starting point to another, because its speed is constant ($C = 300,000$ km/s).

When a mass moves, the sun for example, the change should be felt on the earth after about 8.33 minutes. As I noticed before, galaxies float like flags under the effects of gigantic gravitational waves as a result of grandiose shocks and large perturbations.

Gravitational waves were already mentioned at the beginning of the 20th century by Poincaré (in 1905) and by Einstein (in 1916) and their problem was discussed at length. Einstein himself changed his mind several times on the subject. Gravitation being represented by a curvature of space-time, due to the mass of matter, this gravitation is represented by the tensor $G_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} \cdot R g_{\mu\nu} = \left(\frac{8\pi G}{c^4} \right) \cdot T_{\mu\nu}$$

Einstein's formula already quoted before

and the wave equation is written : $\Delta h^{\alpha\beta} = (-1/c^2) \cdot \frac{\partial^2 h^{\alpha\beta}}{\partial t^2} = 0$

Mathematically this wave equation is written in exactly the same way as Maxwell's equation for electromagnetic waves, so there is a similarity and conformity between gravitational waves and electromagnetic waves.

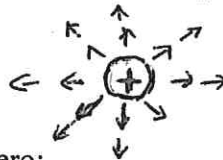
N B : A short reminder about electromagnetic waves:

Electromagnetic waves are perturbations of electric and magnetic fields, they are produced by charged and accelerated particles. The variations of the electric and magnetic fields are linked by

Maxwell's four equations:

1) an electric charge generates an electric field around itself:

$$\vec{\nabla} \cdot \vec{E} = \rho$$



Gauss's Law :

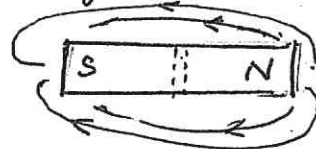
$\rho =$ charge density

2) The divergence of the magnetic field is zero:

$$\vec{\nabla} \cdot \vec{B} = 0$$

what comes out of the magnet comes back in

example: a magnetic field generated by a magnet

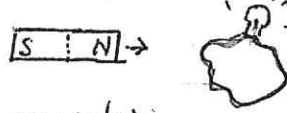


Gauss's Law

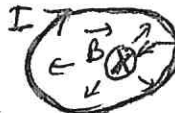
These two equations indicate the structure of the field

3) The rotational: If the field B varies, the rotational gives the shape of the field. If we vary a flux $\Delta\phi$ in a circuit, then there is creation of current (induced current):

Faraday's law: $\text{rot } \vec{E} = -\mu \frac{\partial \vec{B}}{\partial t}$: for the rotational exist, \vec{B} must vary
 (1) is also written: $\vec{\nabla} \times \vec{E} = -\frac{1}{c} \cdot \frac{\partial \vec{B}}{\partial t}$



4) Current \vec{E} is caused by a magnetic field \vec{B} :



moving magnet \Rightarrow current in the wire

Ampere-Maxwell's law:

the field only appears if \vec{B} varies

$$\vec{\nabla} \times \vec{B} = \frac{1}{c} (4\pi \vec{J} + \frac{\partial \vec{E}}{\partial t}); \vec{J} = \text{current density}$$

The four equations explain all electromagnetism and from 3) and 4), we obtain the d'Alembert equations:

$$\begin{cases} \Delta \cdot \vec{E} = -\epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \\ \Delta \cdot \vec{B} = -\epsilon\mu \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \end{cases} \text{ two identical equations}$$

These two equations describe a wave propagation in space in a form of the operator ∇ . The general wave equation is then written:

$$\Delta \cdot \vec{V} = \frac{1}{c^2} \cdot \frac{\partial^2 V}{\partial t^2} = 0$$

And the gravitational wave equation is written in the same way, it is written as we mentioned before:

$$\Delta h^{\alpha\beta} = - \frac{1}{c^2} \cdot \frac{\partial^2 h^{\alpha\beta}}{\partial t^2} = 0$$

NB: Δ differential operator = the Laplacian
 $\vec{\nabla}$ vector differential operator : nabla

Intergalactic wave and intragalactic wave:

we can consider that there are two kinds of waves in the universe:

The first are the intergalactic waves caused by the collisions of galaxies and their black holes. These waves travel through an expanding space-time; this expansion of space-time does not totally diminish the gigantic intensities of the waves, which nevertheless remain important and even manage to make the galaxies that are in their path float.

The second waves are intra-galactic waves and of course within galaxies, in a space-time that does not expand (because of the gravitation that keeps the galaxy together against the expansion of the Universe). The waves are also caused here by the disturbance of matter (moving stars, collisions, bursts, etc.). These are the ones that were detected in 1916 by NASA for the first time. All this makes space-time turbulent and the Universe can look like a turbulent and turbulent ocean. Everything in the Universe moves and creates disturbances as we have noticed many times.

The path of the wave :

In my opinion and what seems to me the most logical, we can only detect the intra-galactic waves, those that reach us from the Milky Way. The others, those from outside the galaxy, are so intense that they are undetectable even when they make our galaxy float. The intra-galactic waves that can be detected come from the galaxy (the milky way) they make the same journey as light to reach us. The same reasoning applied to light applies to the gravitational wave.

The gravitation of the sun's mass holds the solar system together against intra-galactic wave disturbances and the gravitation of the central black hole's mass holds all the stars in the galaxy together and in harmony against inter-galactic wave disturbances.

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