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6 **Linear and nonlinear profiles of weak behavioral and neural differentiation of**
7 **numerical operations in children with math learning difficulties**
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43 solving
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Data availability statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Author Contributions

L. Chen, T. Iuculano, and V. Menon designed the research; L. Chen, T. Iuculano, P. Mistry, J. Nicholas, Y. Zhang analyzed the data; L. Chen, T. Iuculano, P. Mistry, and V. Menon wrote the manuscript.

Conflict of interest disclosure

The authors declare no conflict of interests.

Ethics approval statement

Informed written consent was obtained from legal guardians of study participants and all study-protocols were approved by the Stanford University Review Board.

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Research highlights

- Children with low math abilities showed less differentiated performance on addition and subtraction operations and their low efficiency stemmed from weak evidence-accumulation during problem solving.
- Children with low math abilities showed less differentiated neural representations between the two operations in cortical areas important for quantity judgement, cognitive control, attentional processing, semantic, and phonological processing.
- Less differentiated neural representations between two operations were specific to children with math learning difficulties.
- Children with math learning difficulties also showed less differentiated neural network-level representations.

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4 **Abstract**
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7 Mathematical knowledge is constructed hierarchically during development from a basic
8 understanding of addition and subtraction, two foundational and inter-related, but semantically
9 distinct, numerical operations. Early in development, children show remarkable variability in
10 their numerical problem-solving skills and difficulties in solving even simple addition and
11 subtraction problems are a hallmark of math learning difficulties. Here, we use novel quantitative
12 analyses to investigate whether less distinct representations are associated with poor problem-
13 solving abilities in children during the early stages of math-skill acquisition. Crucially, we
14 leverage dimensional and categorical analyses to identify linear and nonlinear neurobehavioral
15 profiles of individual differences in math skills. Behaviorally, performance on the two different
16 numerical operations was less differentiated in children with low math abilities, and lower
17 problem-solving efficiency stemmed from weak evidence-accumulation during problem-solving.
18 Children with low numerical abilities also showed less differentiated neural representations
19 between addition and subtraction operations in multiple cortical areas, including the fusiform
20 gyrus, intraparietal sulcus, anterior temporal cortex and insula. Furthermore, analysis of multi-
21 regional neural representation patterns revealed significantly higher network similarity and
22 aberrant integration of representations within a fusiform gyrus-intraparietal sulcus pathway
23 important for manipulation of numerical quantity. These findings identify the lack of distinct
24 neural representations as a novel neurobiological feature of individual differences in children's
25 numerical problem-solving abilities, and an early developmental biomarker of low math skills.
26 More generally, our approach combining dimensional and categorical analyses overcomes
27 pitfalls associated with the use of arbitrary cutoffs for probing neurobehavioral profiles of
28 individual differences in math abilities.
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Introduction

Mathematical knowledge is constructed hierarchically from symbolic representations of quantity and rules to manipulate them by adding and subtracting items to and from numerical sets. Although the symbolic representations of these operations differ only minimally in their perceptual format, they differ considerably at the cognitive-semantic level (Campbell & Alberts, 2009). Crucially, knowledge of basic addition and subtraction problems lies at the core of successful acquisition of more complex mathematical skills during development, and poor performance on these two basic arithmetic operations is a defining phenotypical and clinical feature of learning disabilities and math learning difficulties (MLD) more generally. Here, we use novel quantitative analyses to investigate whether two foundational arithmetical operations, namely addition and subtraction, share overlapping neural representations, and whether an inability to form distinct representations is associated with poor problem-solving abilities in children during the early stages of math-skill acquisition. A key aspect of our approach is that we combined dimensional and categorical analyses to uncover linear and nonlinear neurobehavioral profiles of heterogeneity in MLD.

Addition and subtraction differ minimally in surface format and are highly inter-related operations, by virtue of the fact that subtraction is the mathematical inverse of addition. Perceptually the two operations are virtually identical as they differ only by a single vertical line: '+' vs. '-'. However, they are highly dissimilar in terms of cognitive processes and efficiency (Barrouillet et al., 2008). Behavioral studies have shown that while single-digit addition problems are typically solved by memory retrieval (Barrouillet et al., 2008; Thevenot et al., 2007; Thevenot & Barrouillet, 2020) or fast procedural strategies (Ashcraft, 1992; Y. Chen & Campbell, 2018), related subtraction problems are far less likely to be solved by direct retrieval, and they place greater demands on working memory and cognitive control, particularly in children with weaker problem-solving skills (Caviola et al., 2014; Hayashi et al., 2000). Typically-developing children initially use inefficient strategies such as finger counting, slow mental counting and eventually learn to retrieve solutions to simple addition problems from memory with high efficiency; however, they continue to solve subtraction problems with more elaborate algorithmic procedures, such as counting and multi-step calculation (Barrouillet et al., 2008; G. Peters et al., 2014). While typically developing children engage distinct strategies when starting to master addition and subtraction problems, children with poor math abilities continue to rely on laborious procedural computations for both operations (Ostad, 1999). These behavioral findings suggest that the two operations rely on dissimilar cognitive processes, but the extent to which they engage distinct brain representations, and the underlying neurobehavioral sources of individual variability are poorly understood.

Neuroimaging studies have shown that children with poor math abilities show aberrant brain responses and connectivity during the processing of both addition and subtraction problems (Ashkenazi et al., 2012; Iuculano et al., 2015; Lowe, 2011; Rosenberg-Lee et al., 2015; Rykhlevskaia, 2009). Surprisingly, despite slower and less accurate performance, children with low math skills show significant hyper-activity in multiple parietal, occipito-temporal and prefrontal regions during arithmetic problem solving (De Smedt et al., 2011; Iuculano et al., 2015; L. Peters & De Smedt, 2018) as well as hyper-connectivity between parietal and prefrontal cortices (Rosenberg-Lee et al., 2015). Yet, although extant brain imaging studies have identified the anatomical and functional bases of deficits in children with different levels of math

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4 difficulties, it remains unclear how these aberrancies may relate to behavioral difficulties in
5 arithmetic processing, and for different types of numerical problems and operations. Examining
6 patterns of neural representations, rather than activations, could potentially provide a
7 neurobiological mechanism to probe sources of heterogeneity in math processing, and thereby
8 identify novel biomarkers of impairments in children at the lower end of the distribution of
9 abilities. Furthermore, it is unknown whether aberrant patterns of neural representations exist
10 along a continuum of math abilities or whether children with most severe difficulties represent
11 information in a fundamentally different manner. To date, few studies have examined individual
12 variation in children with low math skills and the neural mechanisms associated with these
13 differences, creating a significant challenge to our understanding of heterogeneity in math
14 problem-solving skills during its formative stages, particularly in children falling at the lower
15 end of the distribution of abilities. Notably, in order to develop effective interventions for those
16 with low math skills, the neurobiological basis of the deficits need to be well characterized using
17 robust quantitative approaches. Thus far, research into these causes has been hindered by
18 variable selection/cut-off criteria used to define individuals with lower math abilities (Geary,
19 2011, 2013). To address this challenge, we leverage both dimensional and categorical analyses to
20 probe neural representations underlying distinct numerical operations in the brains of children
21 with poor math abilities, using multiple levels of analyses (**Figure 1**).

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27 The first aim of our study was to investigate behavioral and cognitive profiles associated with
28 arithmetic problem-solving involving addition and subtraction operations in children with low
29 math abilities (**Figure 1A&B**). We used both dimensional and categorical approaches to
30 determine whether children with low proficiency are impaired on both operations, and whether
31 performance on the two operations is less differentiated in children with poor math skills
32 compared to their typically developing (TD) peers (Bruyer & Brysbaert, 2011). In addition to
33 overt behavioral measures, we evaluated latent decision-making processes by jointly modelling
34 accuracy and reaction time using a hierarchical drift diffusion model (HDDM) in which a drift-
35 process accumulates evidence over time until it crosses one of the two response boundaries
36 (Froehlich et al., 2016; Oganian et al., 2016; Ratcliff & McKoon, 2008; Ratcliff & Smith, 2004).
37 Drift-rate, the speed with which the accumulation process approaches the decision boundaries,
38 represents the relative evidence for or against a particular response (Wiecki et al., 2013), and
39 previous work has suggested that a lower drift-rate could indicate worse performance on math
40 tasks involving addition problem solving (Iuculano et al., 2020). Therefore, we investigated
41 whether the rate of evidence accumulation to a decision threshold was a potential mechanism
42 underlying weak problem-solving skills and tested the hypothesis that, compared to their peers
43 with higher math skills, children with poor math skills would show a lower speed of evidence
44 accumulation, for both addition and subtraction operations.

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50 The second aim of our study was to characterize neural representations between addition and
51 subtraction operations, and to test the hypothesis that children with poor problem-solving
52 abilities show weak neural differentiation between the two distinct operations. Different from
53 mapping brain activation levels, neural representational similarity (NRS) analysis assesses
54 whether cognitive processes share similar neural features, and identifies brain areas that are most
55 sensitive to distinctions between mental states evoked by specific task conditions (Kriegeskorte
56 et al., 2008; Kriegeskorte & Kievit, 2013). Neural representational similarity reflects similarities
57 in population-based coding and is therefore ideal for examining neurocognitive processes
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4 underlying addition and subtraction problems, given their close perceptual similarity but
5 semantic dissimilarity. A previous study found that children with MLD tended to show less
6 differentiated neural representations between addition problems of different levels of difficulty
7 (Ashkenazi et al., 2012). Whether such lack of differentiation extends to distinct numerical
8 operations is currently not known. We hypothesized that children with low math abilities would
9 show less differentiated neural representations between two operations because they are likely to
10 engage inefficient strategies for both (Ostad, 1999). An alternative hypothesis is that if children
11 with lower math abilities engaged entirely different cognitive processes for addition and
12 subtraction problems (Barrouillet et al., 2008; G. Peters et al., 2014), they would show more
13 differentiated neural representations than their TD peers. Here, we test these competing
14 hypotheses to gain insights into neurocognitive processes associated with basic problem-solving
15 skills in children with MLD.
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20 In an advance over previous work, we used both dimensional and categorical approaches to
21 characterize linear and nonlinear relationships of math skills and neural representations of
22 addition and subtraction, the two arithmetic operations during a crucial age for math skill
23 acquisition in children (**Figure 1C&D**). A dimensional approach was used to assess linear
24 changes along a continuum of arithmetic abilities, while a combination of dimensional and
25 categorical approaches was used to assess distinct profiles in children at the lower end of the
26 distribution of abilities, including those with MLD, in contrast to the profiles observed in their
27 TD peers. This two-pronged approach was used to identify brain systems that show weaker- or
28 stronger-than-expected profiles in children with MLD.
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32 The third and final aim of our study was to probe disruptions in communication of neural
33 representations across brain regions in children at the lower end of the distribution of abilities,
34 including those with MLD (**Figure 1E&F**). In a further advance over previous research in the
35 field, we examined representational similarity at a network level and determined impairments in
36 co-occurring patterns of deficits across multiple brain regions (Anzellotti & Coutanche, 2018;
37 Pillet et al., 2020). This approach was used to characterize the organization of multivariate
38 representational networks in children with low math skills and determine specific pathways of
39 impaired communication. Using an NRS-based network analysis, we specifically examined
40 whether: (a) weak differentiation of neural representations at the regional level is also
41 manifested at the network level; and (b) integration of neural representations in the ventral (i.e.,
42 number form system) and dorsal (i.e., quantity information system) pathways (Battista et al.,
43 2018; Evans et al., 2015; Menon, 2014) are aberrant in children with poor math abilities.
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48 We hypothesized that children with poor math abilities would show less differentiated
49 behavioral, cognitive, and neural representational profiles between addition and subtraction
50 problems. We further hypothesized that aberrant neural representations would be detected in
51 distributed brain areas, including parietal areas associated with visuo-spatial attention, temporal
52 lobe regions involved in language and semantic processing, as well as prefrontal cognitive
53 control systems, consistent with a multicomponent model of math learning disabilities and
54 developmental dyscalculia (Fias et al., 2013; Iuculano, 2016). We demonstrate that our novel
55 quantitative approaches provide a more comprehensive understanding of how distinct numerical
56 operations are represented in the brains of children, elucidate linear and nonlinear profiles of
57 neural representations associated with individual differences in children's math abilities, and
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4 reveal weak operation-specific pruning of distributed neural circuits. Our findings are relevant
5 for understanding both typical and atypical development of problem-solving skills and has the
6 potential to inform targeted interventions in children with math learning difficulties.
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9 **Method**

10 **Participants**

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14 A total of forty-six children in their 2nd or 3rd grade of schooling (ages 7 to 9) were recruited
15 from multiple school districts in the San Francisco Bay area. Informed written consent was
16 obtained from the legal guardian of the child and all study protocols were approved by the
17 Stanford University Review Board. All participants were volunteers and were treated in
18 accordance with the American Psychological Association “Ethical Principles of Psychologists
19 and Code of Conduct”. The participants had no history of medical, neurological or psychiatric
20 illness. All participants had Full-scale IQ (FS-IQ) scores > 80 (range: 84-128; **Table 1**), as
21 assessed by the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999). Math
22 skills of children were assessed by the *Numerical Operations (NumOps)* subtest of the Wechsler
23 Individual Achievement Test Second Edition (WIAT-II; Wechsler, 2001) given that weak
24 arithmetic ability represents one of the most distinctive behavioral features of impaired problem-
25 solving skills in children, and it is a hallmark of MLD (Butterworth, 2005). On this test, a wide
26 range of math skills was observed (range: 77-131). One participant with an invalid *NumOps*
27 subtest score due to administrator error was then excluded, resulting in a final sample of 45
28 children (25 Females; mean age = 8.37 years old).
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34 Using a norm-based criterion, namely below-grade-level scores (the 25th percentile. i.e., a
35 standardized score below 90) on the WIAT-II *NumOps* subtest, 21 children were classified as
36 having math learning difficulties (MLD group), while 24 children whose scores were at or above
37 90 on the same test formed the TD group (**Table S1**). The 25th percentile was chosen based on
38 the extant literature of cohorts identified as having math learning difficulties (Lambert &
39 Spinath, 2018; Schwartz et al., 2018; Skeide et al., 2018; Swanson et al., 2018; Tolar et al., 2016;
40 Wong & Chan, 2019). We also chose this criterion to ensure an appropriate *N* of children falling
41 within the low-end of the distribution of abilities (*N* = 21) in order to conduct categorical
42 analysis of nonlinearity-effects associated with individual differences in math problem-solving
43 skills, and to ensure that individuals with low math abilities did not have comorbid disabilities in
44 general cognitive functions, such as low IQ or impaired working memory compared to TD. The
45 MLD and TD groups did not differ on age, FSIQ, Verbal and Performance IQ, and standardized
46 Working Memory (WM) measures (Working Memory Test Battery for Children, WMTB-C;
47 Pickering & Gathercole, 2001). The MLD had significant lower *NumOps*, *Math Reasoning* and
48 *Word Reading* scores than the TD group (see **Table S1**).
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53 **Standard assessments of math and reading abilities**

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55 Children’s mathematical and reading abilities were assessed using the WIAT-II (Wechsler,
56 2001). This achievement battery includes nationally standardized measures of academic skills
57 and problem-solving abilities for Grades K to 12, which are normed by grade and time of
58 academic year (Fall, Spring, or Summer). The *Numerical Operations* was used to assess
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4 participants' mathematical skills and assign children into MLD or TD groups, and measures
5 number writing and identification, rote counting, number production and simple addition,
6 subtraction, multiplication and division. Additionally, the *Mathematical Reasoning* subtest was
7 also administered (see **Table 1**). This subtest measures counting, geometric shape identification,
8 and single- and multi-step word problem-solving involving time, money, and measurement. In
9 this subtest, students are given unlimited time to solve written math problems and the problems
10 are organized with increased progression of task difficulty. The participant is required to solve
11 problems with whole numbers, fractions or decimals, interpret graphs, identify mathematical
12 patterns, and solve problems of statistics and probability. The WIAT-II (i.e. *Word Reading* and
13 *Reading Comprehension* subtests) was also used to assess reading abilities in both cohorts.
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17 **Experimental procedures**

18 *Functional MRI (fMRI) tasks*

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22 The fMRI experiment examined single-digit arithmetic problem-solving skills in children and
23 consisted of one run of addition and one run of subtraction. Within each run, there were four task
24 conditions: (i) Complex arithmetic, (ii) Simple arithmetic, (iii) Symbol-finding and (iv) Passive
25 fixation/rest. In the Complex addition task, participants were presented with an equation
26 involving two addends and were asked to indicate, via a button press, whether the presented
27 answer was correct (e.g. "3 + 4 = 8"). The first operand ranged from 2 to 9, the second from 2 to
28 5 (tie problems, such as "5 + 5 = 10", were excluded), and correct answers appeared in 50% of
29 the trials. Incorrect answers deviated by ± 1 or ± 2 from the correct sum. The Simple addition
30 task was identical except that one of the addends was always '1' (e.g. "3 + 1 = 4"). In the
31 Complex subtraction task, the first operand ranged from 3 to 14 and the second operand from 2
32 to 5. In the Simple subtraction task, the first operand ranged from 2 to 14 and the second operand
33 was always '1'. As in the addition task, incorrect answers deviated by ± 1 or ± 2 from the actual
34 difference, with the constraint that actual and presented difference was always greater than zero.
35 All subtraction problems were the inverse of addition problems and matched on problem size. In
36 the symbol-finding condition, participants were asked to decide whether the digit "5" was
37 present in a string of symbols, e.g., "3 @ 5 (9" or "4 (7 @ 2". In the passive fixation block-
38 periods the symbol "*" appeared at the center of the screen and participants were asked to focus
39 their attention on it.
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46 During the task, stimuli were presented in a block fMRI design in order to optimize signal
47 detection (Friston et al., 1999). In each task, stimuli were displayed for 5 seconds with an inter-
48 trial interval of 500 ms. There were 18 trials for each task condition, broken into 4 blocks of 4 or
49 5 trials (2 blocks of 4 trials and 2 blocks of 5 trials, resulting in a total of 18 trials), thus each
50 block lasted either 22 or 27.5 seconds. The total length of each experimental run was 6 minutes
51 and 36 seconds. The order of blocks was randomized across participants with the following
52 constraints: in every set of four blocks, all conditions were presented, and the Complex and
53 Simple arithmetic condition-blocks were always separated by either a symbol-finding (not
54 examined here) or a passive-fixation condition-block. We focused on the contrast between
55 Complex and Simple arithmetic conditions, rather than the symbol-finding condition, to best
56 equate (and thus control for) low-level perceptual, motor and decision-making processes
57 involved in arithmetic problem solving. Previous research has shown that 'N+1' and 'N-1'
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4 arithmetic problems are solved by incremental, or decremental, counting (Campbell & Metcalfe,
5 2007) with higher accuracy and faster reaction times relative to more complex addition (and
6 subtraction) problems (Cho et al., 2011). Hence, this choice of contrast allowed us to best isolate
7 processes involved in arithmetic problem-solving across addition and subtraction tasks,
8 reflecting overall arithmetic problem-solving efficiency with different strategies in children.
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10 11 ***Behavioral performance on mental arithmetic tasks***

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14 We determined accuracy (%) and Reaction Times (RTs) associated with performance of addition
15 and subtraction problem-solving tasks during fMRI scanning in each participant. To overcome
16 accuracy-speed trade-off issues (Bruyer & Brysbaert, 2011), we used composite efficiency
17 scores (ES) calculated as: accuracy % / mean RTs of correct responses, to examine task
18 performance so higher ES values reflect better performance. We used this measure because a
19 speed-accuracy trade-off has been systematically reported to be a phenotypic characteristic of
20 poor math performance across a range of math abilities, including MLD (Butterworth, 2005;
21 Geary et al., 1991; Iuculano et al., 2008; G. Peters et al., 2014; L. Peters & De Smedt, 2018;
22 Rosenberg-Lee et al., 2015).
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25 26 ***Cognitive modeling of behavior***

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29 The addition and subtraction verification processes (2-choice) were modeled as a drift diffusion
30 process, in which evidence accumulates over time resulting in a forced-choice selection, when a
31 decision threshold is reached. We developed a hierarchical drift diffusion model (HDDM) to
32 determine the drift rate, decision threshold, response bias, and non-decision time. The drift rate
33 parameter characterizes evidence accumulation, with higher values characterizing a greater
34 proportion of correct responses, and higher absolute values of the drift rate characterizing faster
35 responses. The decision threshold parameter captures the degree of evidence required to
36 conclusively evaluate the answers. For each individual, the threshold, bias, and non-decision
37 time parameters were allowed to vary for addition and subtraction, and for simple versus
38 complex problems. The drift rate was hierarchically inferred by adding an item-response model
39 that allowed the drift rate to be a combination of item-level difficulty that was inferred at a group
40 level (TD and MLD) individually for each unique item, and individual level ability, separately
41 for addition and subtraction. The model was implemented within a Bayesian inference
42 framework using JAGS (Plummer, 2003).
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47 ***Functional MRI data acquisition***

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50 Functional brain images were acquired on a 3T GE Signa scanner (General Electric, Milwaukee,
51 WI) using a custom-built head coil at the Stanford University Lucas Imaging Center. Cushions
52 were placed around participants' heads to minimize head movement. A total of 29 axial slices
53 (4.0 mm thickness, 0.5 mm skip) parallel to the anterior commissure-posterior commissure line
54 and covering the whole brain were acquired using a T2* weighted gradient echo spiral-in/spiral-
55 out pulse sequence (Glover & Lai, 1998) with the following parameters: TR = 2 sec, TE = 30 ms,
56 flip angle = 80°, 1 interleave. The field of view was 20 cm, and the matrix size was 64 x 64,
57 providing an in-plane spatial resolution of 3.125 mm. To reduce blurring and signal loss from
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4 field inhomogeneity, an automated high-order shimming method based on spiral acquisitions was
5 used before acquiring fMRI scans (Kim et al., 2002).
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7 ***Functional MRI data preprocessing*** 8 9

10 Data were analyzed using SPM12 (<http://www.fil.ion.ucl.ac.uk/spm>). The first 5 volumes were
11 discarded to allow for signal equilibration. Images were reconstructed, by inverse Fourier
12 transform, for each of the time points into 64 x 64 x 28 image matrices (voxel size 3.125 x 3.125
13 x 4.5 mm). Images were first realigned to the first scan to correct for motion and slice acquisition
14 timing. A linear shim correction was applied separately for each slice during reconstruction using
15 a magnetic field map acquired automatically by the pulse sequence at the beginning of the scan
16 (Glover & Lai, 1998). Translational movement in millimeters (x, y, z) was calculated based on
17 the SPM12 parameters for motion correction of the functional images in each subject. To correct
18 for deviant volumes resulting from spikes in movement, we used de-spiking procedures similar
19 to those implemented in AFNI (Cox, 1996). Deviant volumes were identified as having either
20 total displacement from the initial volume greater than half of a voxel size (1.562 mm) or change
21 in global signal greater than 5%. The frame-wise displacement was then computed as the square
22 root of the sum of both translational and rotational displacement by first converting rotational
23 displacement from degree to millimeters, assuming a brain radius of 65mm. Deviant volumes
24 were then interpolated using the two adjacent scans. Furthermore, translational movement
25 parameters (x, y, z), rotational movement parameters (roll, pitch, yaw), and framewise
26 displacement did not differ between MLD and TD groups for either addition or subtraction tasks
27 (all $ps > .05$, see **Table S6**). No participants had more than 0.5 mm frame-to-frame mean
28 displacement. After the interpolation procedure, images were spatially normalized to standard
29 stereotaxic space (based on the Montreal Neurologic Institute – MNI – coordinate system),
30 resampled every 2 mm using *sinc* interpolation, and smoothed with a 6mm full-width half-
31 maximum Gaussian kernel to decrease spatial noise prior to statistical analysis.
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38 ***Functional MRI data analysis*** 39 40

41 ***1st-level analyses*** Task-related brain activation in response to each condition (complex
42 arithmetic, simple arithmetic, symbol-finding, fixation) and each task (addition, subtraction) was
43 first modeled at the individual subject-level using the General Linear Model (GLM)
44 implemented in SPM12. For each subject we modeled task-related regressors as boxcar functions
45 corresponding to the epochs during which each condition was presented, and for each task, and
46 convolved with a hemodynamic response function. The six head motion parameters generated in
47 the realignment procedure were also included in the model as regressors of non-interest. Voxel-
48 wise contrasts and *t*-statistics images were then generated by contrasting Complex versus Simple
49 problems in each task. We focused on the contrast of Complex versus Simple since behavioral
50 research suggests that the simple ($n \pm 1$) trials are solved by incremental or decremental counting
51 (Campbell & Metcalfe, 2007), and performance on this task is characterized by higher accuracy
52 and faster reaction times compared to complex problems (Cho et al., 2012; Iuculano et al., 2014;
53 Rosenberg-Lee et al., 2011; Young et al., 2012). Moreover, because stimuli in the Simple
54 condition have the same format as in the Complex condition, it provides a high-level control for
55 sensory and number processing, as well as decision-making and response selection. All trials
56 were included in the analysis to ensure similar number of trials across participants.
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6 **Multi-voxel Representational Similarity Analyses (MRSA)** A whole brain searchlight MRSA
7 (Kriegeskorte et al., 2008) was used to assess the similarity of spatial activity patterns associated
8 with two arithmetic tasks: addition and subtraction at the individual subject level. At each voxel,
9 a 6mm radius sphere was used to define a searchlight region. The Pearson correlation coefficient
10 between *t*-scores for addition and subtraction problems was then computed for each region after
11 removing mean brain activity. Then, correlation coefficients were normalized using Fisher's r-to-
12 Z transform: $Z = 0.5 * \ln((1+r)/(1-r))$ and entered into group-level analyses. First, we used the
13 NRS z-score in a one-sample correlational analysis to identify brain regions showing multivoxel
14 representation similarity between addition and subtraction that was associated with individual
15 differences in math skills using *NumOps* scores. In order to ensure an independent analysis for
16 the nonlinear approach, we then investigated brain regions showing different relationships of
17 NRS and *NumOps* between the MLD and TD groups to characterize any nonlinear patterns
18 between NRS and *NumOps*. Significant NRS clusters were determined using a voxel-wise height
19 threshold of $p < .005$, FDR-corrected for multiple spatial comparisons at $p < .01$ (cluster extent
20 87 voxels), based on Monte Carlo simulations (Cho et al., 2012; Iuculano et al., 2014).
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25 **Multi-region network similarity analysis** In order to further investigate how neural
26 representations were related to individual differences in math skills in children at a network
27 level, we conducted a series of analyses based on the NRS score from the local regions which
28 showed significant correlations – in terms of less differentiated neural representations – and math
29 skills across individuals (**Table 2 and Figure 3**). We characterized the multi-region network
30 similarity in MLD and TD groups separately. Based on the regional NRS of the same set of
31 identified regions from the GLM results, we first calculated the Manhattan Distance of these
32 brain regions across all subjects with MLD or across all subjects in the TD group, and then
33 converted the Distance matrix into a similarity matrix by normalization, $1-(D-\min(D))/\text{range}(D)$,
34 in which *D* is the distance matrix. We then used independent sample t-tests to examine group
35 difference (MLD vs TD) in the mean values of the lower triangles of the multi-region similarity
36 matrix. To test the hypothesis of deficits in ventral-dorsal integration in MLD, we selected three
37 regions from the GLM results for this analysis: (i) the posterior fusiform gyrus (pFG) as a seed,
38 and examined the similarity of the right pFG with a (ii) dorsal target superior parietal
39 lobe/intraparietal sulcus (SPL/IPS) and a (iii) ventral target superior anterior temporal cortex
40 (sATC) as a comparison control). We chose these regions as they are core regions for
41 representing the number form, manipulation of numerical quantity, and semantic knowledge of
42 numerical operations (Fias et al., 2013; Iuculano, 2016; Menon, 2014; L. Peters & De Smedt,
43 2018). The significance testing was conducted by subsampling and permutation. In each
44 permutation, we subsampled 80% of subjects in the MLD and TD groups separately, and
45 conducted the same multi-region similarity analysis and the hierarchical clustering based on the
46 similarity matrix. We recorded the distance of pFG with SPL/IPS and sATC in MLD and TD
47 groups. After 10,000 permutations of this procedure, we established distributions of pFG-
48 SPL/IPS and pFG-sATC distances in both MLD and TD. Finally, we conducted a two-way
49 ANOVA to examine the interaction between Group (MLD vs. TD) and Target (dorsal vs.
50 ventral).
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Results

Performance on addition and subtraction operations is correlated with individual differences in math abilities

We investigated the relation between performance on addition and subtraction tasks, undertaken during fMRI scanning, and *Numerical Operations (NumOps)* subscores of the WIAT-II, a standardized measure of math abilities (see **Table 1**) (Wechsler, 2001). Behavioral performance was assessed using efficiency scores (Iuculano et al., 2008), a composite measure obtained by dividing accuracy by mean reaction time (RT) in each participant (Bruyer & Brysbaert, 2011). Efficiency scores in both addition ($r = 0.447, p < .01$) and subtraction ($r = 0.299, p < .05$) were correlated with *NumOps*, suggesting that children with lower math abilities performed worse on both arithmetical operations (**Figure 2A**). Although the overall difference in slopes was not significant ($p > 0.05$), children at the lower end of performance tended to show smaller differences in efficiency between the two operations. Additional analyses with accuracy and RT measures, separately, are shown in the Supplementary Information (see **Figure S1** and **Tables S1 & S2**).

Less differentiated performance in children with MLD

Next, we used a categorical approach to investigate whether children with MLD show poorer performance overall, when compared to TD controls, and crucially, whether they show less differentiated performance between the addition and subtraction operations (**Table S3**). A two-way mixed ANOVA on performance efficiency with between-subject factor Group (MLD vs. TD) and within-subject factor Operation (Addition vs. Subtraction) revealed a significant interaction between Group and Operation ($F(1,43) = 4.647, p < 0.05$). Main effects of Group ($F(1,43) = 11.22, p < .01$) and Operation ($F(1,43) = 8.062, p < 0.01$) were also significant. Further analysis revealed that in the MLD group, there was no significant difference between performance on addition and subtraction ($t(20) = 0.3085, p = 0.76$), whereas the TD controls showed significantly better performance in the addition task compared to the subtraction task ($t(23) = 4.376, p < 0.001$) (**Figure 2B**). It is unlikely that the lack of statistical significance in the MLD group was due to sample size because the sample size of $n = 21$ (i.e. MLD group) had a large *a-priori* power = 95% assuming the same effect size in TD ($d = 0.83$), at the $\alpha = 0.05$ level.

Decision-making associated with addition and subtraction problem solving is impaired in children with low abilities

Next, we investigated latent decision-making processes associated with arithmetic problem solving by combining hierarchical drift diffusion modeling (Ratcliff & McKoon, 2008; Ratcliff & Smith, 2004) with item-response based modeling to account for differential item level difficulty. We found that *NumOps* scores were positively correlated with the drift rate on addition ($r = 0.341, p < 0.05$) and subtraction ($r = 0.347, p < 0.05$) problems. No other model parameters – decision threshold, non-decision time, or response bias – showed a significant correlation with *NumOps* (**Table S4**). Comparison of children with MLD and their TD peers revealed that the drift rate on addition and subtraction problems differed in the TD group ($M_{Add} =$

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4 0.91 vs. $M_{\text{Sub}} = 0.80$, $p < 0.05$), but not in the MLD group ($M_A = 0.68$ vs. $M_S = 0.60$, $p > 0.05$;
5 *a-priori* power = 46% at $\alpha = 0.05$ assuming a similar effect size in TD as $d = 0.43$) (**Figure S2**).
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8 ***Less differentiated neural representations in children with lower math abilities***

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10 We next sought to determine whether low math abilities are associated with weaker
11 differentiation between neural representations for addition and subtraction problems, using a
12 whole-brain searchlight algorithm (Ashkenazi et al., 2012; Bugden et al., 2019; Misaki et al.,
13 2009). We found that *NumOps* scores were negatively correlated with NRS between addition and
14 subtraction problems in multiple parietal-temporal-prefrontal areas, including intra-parietal
15 sulcus, posterior superior temporal sulcus and anterior temporal cortex, and anterior insula
16 (**Table 2** and **Figure 3A-H**). No brain regions showed a positive correlation between NRS and
17 *NumOps* scores. These results suggest that more distinct NRS is a hallmark of higher math skills.
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20 ***Less differentiated neural representations in children with MLD***

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22 A categorical approach was used to further demonstrate that children identified as MLD in our
23 study did show aberrant neural representations in the same set of regions that were revealed in
24 the dimensional analysis. Two-sample *t*-tests between the MLD and TD groups showed a
25 significant group difference in each of the parietal, temporal and prefrontal cortex regions
26 identified above, with the MLD group showing significantly higher NRS values between
27 addition and subtraction problems (all $ps < .05$; **Figure 3I** and **Table S5**).
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30 ***Nonlinear profile of less differentiated neural representations in children with MLD***

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32 Next, we investigated whether any brain regions showed nonlinear profiles of neural
33 representations in children with MLD, compared to TD controls. This analysis allowed us to
34 identify brain areas that showed worse- or better-than-expected profiles in children with MLD.
35 Adding group membership (MLD, TD) as a moderator (**Figure 4A**), we found a significant
36 interaction between Group and *NumOps* scores on NRS in the left superior part of the anterior
37 temporal cortex (sATC; **Figure 4B**). Additionally, the independent whole-brain NRS analysis,
38 which examined the interaction between Group and *NumOps* on NRS, identified two other brain
39 regions: the left dorsolateral prefrontal cortex (dlPFC) and intraparietal sulcus (IPS) showing a
40 moderation effect of group on the relationship between *NumOps* scores and NRS values (height
41 threshold $p < .005$; FDR-corrected for multiple spatial comparisons at $p < .01$). Specifically, we
42 found that lower *NumOps* scores were associated with higher NRS values in the MLD group
43 only ($ps < 0.05$; **Figure 4C&D**), but no such relationship was found in the TD group. More
44 surprisingly, the relationship between *NumOps* and NRS was positive in the TD group in left
45 dlPFC ($p < .001$).
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52 ***Aberrant multi-region network similarity in children with MLD***

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54 As noted above, we found a significant relationship between math abilities and NRS values in
55 multiple brain regions. We extended this analysis to determine whether aberrant neural
56 representations at the local level are also manifested at the network level. We first noted that
57 when we averaged NRS values across the eight brain regions shown in **Figure 3** and tested their
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4 association with *NumOps*, we observed a significant negative relationship: $r(43) = -0.82$, p
5 $< .001$. This result is not surprising given the linear relationship between *NumOps* scores and
6 NRS values found in individual brain regions. The consistent pattern across these brain regions
7 can be seen in **Figure 5A&B**. Furthermore, we also observed that the multi-region NRS values
8 across individuals seemed to be more similar (or consistent) in the MLD group, i.e., the
9 individual lines showed similar patterns around the group averaged line, compared to the TD
10 group. This observation suggested that NRS across multiple brain regions (i.e., multi-region
11 network similarity) could provide a novel aspect of neural representational similarity at a
12 network level.
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16 To further quantify the consistent patterns of NRS across multiple brain regions and its
17 relationship to arithmetic abilities, we examined whether children with MLD showed different
18 patterns of network-level similarity compared to TD controls. To calculate the multi-region
19 network similarity, we first derived a distance matrix between regional NRS values across
20 participants in the MLD and TD groups separately. We then converted the group-specific
21 distance matrices to a similarity measure wherein '0' indexed lowest similarity (i.e., largest
22 distance) and '1' highest similarity (i.e., shortest distance; for more details see **Method**). We
23 found that children with MLD showed higher multi-region network similarity compared to TD
24 children (**Figure 5C&D**). Permutation tests with 10,000 subsampling procedures revealed a
25 significant difference in the multi-region network similarity matrix between MLD and TD
26 groups ($M_{\text{MLD}} = 0.66$, $M_{\text{TD}} = 0.43$; $p < .001$; **Figure 5E**).
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31 *Aberrancies in neural representations in the ventral-dorsal pathway in children with MLD*

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33 To further investigate the structure of multi-region network similarity, we applied hierarchical
34 clustering to the similarity matrix in the MLD and TD groups, separately. This analysis revealed
35 that the multi-region representations between the posterior fusiform gyrus (pFG) and Superior
36 Parietal Lobule/Intra-Parietal Sulcus (SPL/IPS) were clustered at a higher level of hierarchy in
37 children with MLD compared to TD children. In contrast, multi-region representations between
38 the pFG and the anterior temporal cortex were clustered at a higher level of hierarchy in TD
39 children. Using permutation testing across 10,000 subsamples, we confirmed that neural
40 similarity between pFG and SPL/IPS representations was significantly lower in the MLD group,
41 compared to the TD group, while similarity between the pFG and anterior temporal cortex
42 showed a reverse pattern ($p < .001$; **Figure 5F**).
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47 **Discussion**

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49 We examined whether addition and subtraction, the two foundational numerical operations, are
50 represented differently in children with low math abilities, compared to their TD peers. Both
51 dimensional and categorical approaches revealed less differentiated neural representations in
52 children with low math abilities in multiple neurocognitive systems including the parietal visuo-
53 spatial attention, lateral temporal lobe language-semantic, and prefrontal cognitive control
54 systems. Furthermore, a nonlinear profile characterized by a worse-than-expected profile of
55 deficits in children with MLD was detected in parietal attention and anterior temporal cortex
56 language-semantic systems. Analysis of multi-region network similarity patterns revealed weak
57 communication of neural representations across pathways linking the fusiform gyrus with
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4 parietal regions important for manipulation of quantity and numerical problem-solving.
5 Together, these findings point to weak pruning of local and distributed neural circuits in children
6 with poor math abilities, and identify less differentiated neural representations as a potential
7 neurobiological signature of MLD.
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10 ***Weak behavioral differentiation between numerical operations in children with low***
11 ***mathematical skills***
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14 The first aim of our study was to investigate behavioral performance and decision-making
15 associated with problem-solving involving addition and subtraction operations in children with
16 low math abilities. Not surprisingly, children with poor math abilities showed lower efficiency in
17 solving addition and subtraction problems. Critically, children with lower math skills were
18 similarly impaired in problem-solving efficiency of both operations. In contrast, compared to
19 their TD peers who performed better on addition than subtraction problems, children with MLD
20 showed no such distinction. Children with low math abilities showed less differentiated
21 performance on the two numerical operations, suggesting the engagement of similar – and
22 inefficient – computational – strategies for both addition and subtraction operations/problems.
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26 Computational modeling of latent decision-making processes during problem-solving revealed
27 that lower math abilities were related to slower drift rate (v) for both addition and subtraction
28 problems on a continuous scale. At a group level, TD children, compared to children with MLD,
29 showed better differentiation on drift rate (v) between addition and subtraction problems. This
30 suggests that decision-making processes between operations are better differentiated in TD
31 children compared to the MLD group. Our results indicate that lower problem-solving efficiency
32 in children with impaired math abilities may stem from weak evidence-accumulation, partly on
33 account of poor item discrimination, leading to impaired decision-making. In TD children, the
34 rate of evidence accumulation differed significantly between addition and subtraction problems,
35 and therefore, it is reasonable to assume that at this developmental stage, TD children may rely
36 on different strategies, processes, or representations for solving addition and subtraction
37 problems (Barrouillet et al., 2008; Geary et al., 1991; Jordan et al., 2003; Ostad, 1999), and
38 children with MLD did not show such a distinction. Further studies with larger samples and
39 analysis of strategy-use are needed to elaborate on the observed patterns.
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44 Previous research has suggested that children with poor math abilities fail to use retrieval
45 strategies even for simple addition problems, and they adopt suboptimal strategies for both
46 operations (Ostad, 1999). During the early stages of formal math learning, children use a wide
47 range of inefficient strategies, such as finger counting, verbal counting and some forms of
48 decomposition, for solving both operations (Barrouillet et al., 2008; Siegler, 1987; Siegler &
49 Shrager, 1984). With increased practice, operation-specific shifts take place, and by 3rd grade,
50 neurotypical children start applying more efficient retrieval strategies for addition problems
51 while continuing to rely on more laborious mental manipulation strategies for subtraction
52 problems (Barrouillet et al., 2008; Geary et al., 1991; Jordan et al., 2003; Ostad, 1999). Our
53 findings converge on these observations, and suggest that children with low math abilities may
54 rely on similar, indistinct, and suboptimal cognitive processes for solving the two distinct
55 operations.
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4 *Neural representations are less distinct in children with low math abilities*
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7 The second aim of our study was to determine the neural underpinnings related to weak
8 behavioral differentiation in children with poor math abilities. Specifically, we examined
9 whether children with low math abilities showed weak neural differentiation during problem-
10 solving associated with the two numerical operations. We found that children with low math
11 abilities displayed less distinct neural representations in posterior parietal, prefrontal and lateral
12 temporal cortices including the intraparietal sulcus (IPS), anterior insula, fusiform gyrus, and
13 anterior temporal cortex. Interestingly, no brain region showed lower neural representational
14 similarity, i.e. more differentiated representations, in children with weak math abilities. These
15 results provide novel insights into the neurobiological basis of behavioral learning difficulties in
16 math problem-solving, highlighting a distinctive brain-based feature of impairment in relation to
17 two foundational arithmetical operations, deficits in which are known to be a defining
18 phenotypic feature of MLD.
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22 It is noteworthy that the format of numerical problems presented to participants was perceptually
23 identical across the two problem types: the two strings of presented problems differ only by a
24 single vertical line ‘+’ vs. ‘-’. Thus, as expected, no individual differences or aberrancies in
25 neural representations were found in primary visual cortex, consistent with the view that the lack
26 of distinct representations arises at the semantic level, rather than at the low-level perceptual
27 processing.
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31 Less differentiated neural representations between addition and subtraction problems associated
32 with lower math skills were observed in the IPS/SPL, a brain region that plays a critical role in
33 quantity judgement and manipulation (Dehaene et al., 2003; Holloway et al., 2013; Schel &
34 Klingberg, 2016). The IPS/SPL is a critical locus of numerical processing deficits in children and
35 adults with MLD (Arsalidou & Taylor, 2011; Ashkenazi et al., 2012; Chang et al., 2016; De
36 Smedt et al., 2011; Houdé et al., 2010; L. Peters & De Smedt, 2018; Rosenberg-Lee et al., 2015).
37 Less differentiated representations associated with lower math skills were also detected in the
38 fusiform gyrus. The fusiform gyrus in the ventral-occipital cortex is involved in high-level visual
39 processing of complex visual objects, including words and numbers (Cantlon et al., 2009; Shum
40 et al., 2013; Vogel et al., 2017). Taken together, this pattern of weak neural differentiation of
41 addition and subtraction problems in the dorsal (IPS/SPL) and ventral visual stream (fusiform
42 gyrus) is consistent with impaired core-systems for representing and manipulating numerical
43 quantity, highlighting a less tuned representation of these problems in terms of their core features
44 of symbols processing and numerosity manipulation.
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49 Weak differentiation was also observed in the anterior temporal cortex, a region crucial for
50 integrating cross-modal semantic information (L. Chen et al., 2017; Lambon Ralph et al., 2016),
51 and for processing abstract concepts (Binney et al., 2016; Hoffman et al., 2015; Rodd et al.,
52 2010; Sabsevitz et al., 2005). Lesions in the anterior temporal cortex are associated with
53 semantic dementia and furthermore specifically with deficits in arithmetic problem-solving
54 (Cappelletti et al., 2012; Julien et al., 2008). These observations point to aberrant representations
55 in brain areas important for processing semantic knowledge in children with MLD.
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4 Children with low math abilities also showed weak differentiation in brain regions involved in
5 phonological processing, including the superior temporal sulcus, superior temporal gyrus, and
6 supramarginal gyrus (De Smedt et al., 2011; Prado et al., 2014). Differential engagement of
7 language-related systems have been reported in relation to distinct arithmetical operations,
8 reflecting task and operation-specific specializations (Archibald et al., 2013; De Smedt et al.,
9 2011; Hecht et al., 2001; L. Peters & De Smedt, 2018; Prado et al., 2014). Critically,
10 verbalization during arithmetic problem solving has been linked to language systems (Zarnhofer
11 et al., 2013). Higher neural representational similarity between addition and subtraction problems
12 within language-processing regions may therefore stem from similar reliance on immature and
13 effortful verbally-mediated strategies for both types of arithmetic problems. We suggest that
14 children with low math abilities may rely on the same language-based strategies to solve both
15 types of problems, consequently resulting in higher neural representational similarity, and lower
16 neural differentiation, across the two operations.
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21 Children with low math abilities also showed higher neural representational similarity in the
22 anterior insular cortex, a region important for cognitive control during cognition in general, and
23 numerical problem-solving in particular (Supekar & Menon, 2012). The strength of causal
24 signals from the anterior insular cortex to parietal and prefrontal regions has been associated with
25 better performance during arithmetic problem-solving (Supekar & Menon, 2012). Meta-analysis
26 of brain imaging studies points to the insula as a region showing a consistent profile of deficits in
27 children with low math abilities (Arsalidou et al., 2018). Compared to addition problems,
28 subtraction problems require more effortful processing, resulting in different levels of cognitive
29 control demands for the two operations (Caviola et al., 2014; Hayashi et al., 2000). Our results
30 indicate, for the first time, that children with low math abilities may not appropriately engage
31 this prefrontal control region for operation-specific numerical problem-solving.
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36 In sum, both dimensional and categorical approaches revealed that neural patterns associated
37 with the two basic operations were less distinct in children with poor math abilities across
38 multiple functional brain systems, further supporting the hypothesis of dysfunctions in multiple
39 functional brain systems in the neurobiological characterization of MLD (Fias et al., 2013;
40 Iuculano, 2016). Notably, our findings suggest that neural dysfunction in MLD arises not just at
41 the level of task-related activation (Iuculano et al., 2015; Rosenberg-Lee et al., 2015), but also at
42 the level of multivariate pattern representations in multiple brain systems (Iuculano, 2016;
43 Iuculano et al., 2015). This pattern of enhanced neural representational similarity is consistent
44 with hyperactive responses observed in children with MLD (Jolles et al., 2016; Rosenberg-Lee et
45 al., 2015), and together points to weak operation-specific tuning of neural circuits as a putative
46 mechanism of poor math abilities in these children. More generally, the distributed nature of the
47 deficits uncovered here provides further support for a multi-componential deficit model of math
48 difficulties, including MLD (Fias et al., 2013; Iuculano, 2016).
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53 ***Nonlinear, worse-than-expected, profile of weak neural representations in children with MLD***

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55 The next aim of our study was to characterize nonlinear profiles of neural representations in
56 children integrating dimensional and categorical approaches. While most brain areas showed a
57 continuous linear profile of neural representational similarity associated with poor math skills,
58 the left anterior temporal cortex and left IPS showed a nonlinear profile characterized by a
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4 significant negative relation in the MLD group and no variation in the TD group (**Figure 4**).
5 These results suggest that children with MLD show a unique neural profile, characterized by
6 weaker than expected differentiation of problem representations in two left hemisphere regions
7 important for semantic knowledge of numerical operations (Julien et al., 2008) and their
8 manipulation (Cappelletti et al., 2012). Our findings identify a novel locus of impairments in
9 children with MLD and are noteworthy because they converge on studies of patients with
10 semantic dementia (Julien et al., 2008) which have suggested that aberrancies in the anterior
11 temporal cortex result in difficulty with identifying arithmetic signs as well as conceptual
12 understanding of quantity.
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16 Our analysis also revealed a worse than expected profile in the dorsolateral prefrontal cortex.
17 However, in this case, while the MLD group showed a negative relationship between neural
18 representational similarity and math abilities, as seen in the parietal-temporal-prefrontal areas
19 consistently identified above, this relationship was instead positive in the TD group. This result
20 suggests that children with MLD engage the dorsolateral prefrontal cortex differently from TD
21 children, and further highlights a dissociation in representations encoded by this brain region.
22 Notably, children at the lowest and highest end on the ability scale show similar overlap in
23 representations, but for entirely different reasons. The dorsolateral prefrontal cortex plays a key
24 role in selective manipulation of information in working memory (Barbey et al., 2013; Curtis &
25 D'Esposito, 2003). One interpretation of our findings is that children with high math abilities
26 engage the dorsolateral prefrontal cortex with similar efficiency for both operations, whereas
27 children with MLD at the lowest end of the abilities (**Figure 4C**) engage this region with a
28 similar level of inefficiency for both operations. This interpretation is also consistent with
29 previous findings showing that adults with better arithmetic skills display higher neural
30 representational similarity between addition and subtraction operations in the dorsolateral
31 prefrontal cortex compared to typically-developing children (Chang et al., 2015).
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37 Together, these results identify brain areas with a nonlinear profile, characterized by worse-than-
38 expected aberrations in neural representations, and thus, areas of particular vulnerability in the
39 most severe cases of children with poor math skills.
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41 ***Less differentiated neural representations at the network level in children with MLD***

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44 The final goal of our study was to characterize network similarity across brain regions
45 (Anzellotti & Coutanche, 2018; Pillet et al., 2020) showing lower differentiation in neural
46 representational similarity values as a function of math abilities. We used a novel computational
47 approach to determine multi-region representational similarity patterns, and assessed whether
48 these patterns could differentiate children with MLD from their TD peers. Analysis of network
49 similarity revealed that children with MLD showed higher multi-region representational
50 similarity compared to their TD peers, suggesting that less differentiated neural representations
51 are manifested not only at the regional level (**Figure 3I**) but also at the network level (**Figure**
52 **5E**). Notably, this analysis also highlights weaker integration of representations between the
53 fusiform gyrus and intraparietal sulcus in children with MLD. These brain regions are crucial for
54 representing and manipulating numerical quantity (Ansari, 2008; Cantlon et al., 2009), and the
55 functional connectivity between these two regions has been shown to be associated with the
56 successful acquisition of numerical abilities (Battista et al., 2018; Evans et al., 2015). In contrast,
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4 children with MLD showed higher integration of neural representations between the right
5 fusiform and the left anterior temporal cortex, likely reflecting the greater need to access
6 semantic representations for solving both problem types. More generally, these results highlight
7 impairments in co-occurring patterns of deficits across multiple brain regions in children with
8 MLD (Fias et al., 2013).
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10 11 *A developmental model of operation-specific changes in neural representation with age and* 12 *math skills* 13

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15 Our findings inform putative developmental models of how distinct neural representations may
16 develop across ages and different levels of math abilities (**Figure 6**). Behavioral studies have
17 characterized distinctive strategies used to solve addition and subtraction operations at different
18 developmental stages. School-age children apply efficient retrieval strategies to solve addition
19 problems 65% of the time – with lower rates in children with poor math abilities – while only
20 19% of subtraction problems are solved using this strategy by the 3rd grade (Barrouillet et al.,
21 2008). Consistent with these behavioral results we found less distinct neural representations
22 between addition and subtraction problems children with high math abilities. In contrast, adults
23 solve 76% of addition problems by retrieval and use this strategy during subtraction 58% of the
24 time (Campbell & Xue, 2001). These differences in retrieval-rates suggest that with
25 development, there is a shift from effortful counting to efficient fact retrieval, leading to a
26 convergence of problem-solving strategies across the two operations. In this case, we would
27 predict increased overlap in neural representations between the two operations in adults. Thus,
28 we predict that, paradoxically, children with poor math abilities and adults would show similar
29 levels of neural representational similarity across distributed frontal, parietal, and temporal
30 regions, but for different reasons (**Figure 6**). Specifically, in children with poor math abilities,
31 similar NRS patterns reflect the use of inefficient strategies for both operations, while in
32 proficient adults, similar NRS patterns reflect the use of efficient strategies for both operations
33 (Chang et al., 2015). Furthermore, our data also hints at the possibility that linear versus non-
34 linear patterns of neural representational similarity may reflect different sub-types of math
35 difficulties, such that children following a linear trajectory may be classified as having a
36 developmental delay, while non-linear profiles may reflect more severe forms of math
37 difficulties, including children commonly classified as having severe forms of learning
38 disabilities. Future studies using longitudinal designs are needed to assess how neural
39 representations change over time heterogeneous groups of children with MLD.
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47 **Conclusions** 48

49 The present study identifies distinct neural representations as a novel neurobiological feature for
50 individual differences in math abilities, and a potential neurobiological marker of poor math
51 skills in an inability to form distinct neural representations for different numerical operations.
52 Less differentiated neural representations for addition and subtraction problems associated with
53 poor arithmetic abilities were evident in widely-distributed cortical regions related to quantity
54 representations and their manipulation, as well as in prefrontal regions involved in cognitive
55 control, and those engaged in the semantic and phonological aspects of language processing. Our
56 findings identify a novel distributed locus of information processing and representational deficits
57 in children at the lower end of the distribution of math abilities. Moreover, network-level
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4 analysis revealed that poor mathematical skills were associated with not only less differentiated
5 representations at the regional level but also with manifested overlapping representations at the
6 network level. Critically, the present study is the first to show that failure to properly
7 differentiate between arithmetical problem-types may be a hallmark of math difficulties.
8 Notably, lack of representational differentiation was not evident in perceptual, but rather in
9 cognitive-semantic processing brain systems. The approach and methods developed here may be
10 useful for future studies of neural representational features in other learning disabilities, such as
11 developmental dyslexia. Finally, our approach provides a template for probing typical and
12 atypical developmental changes associated with cognitive problem-solving and skill acquisition,
13 overcoming pitfalls associated with use of arbitrary cutoffs for probing neurobehavioral profiles
14 of heterogeneity in learning disabilities.
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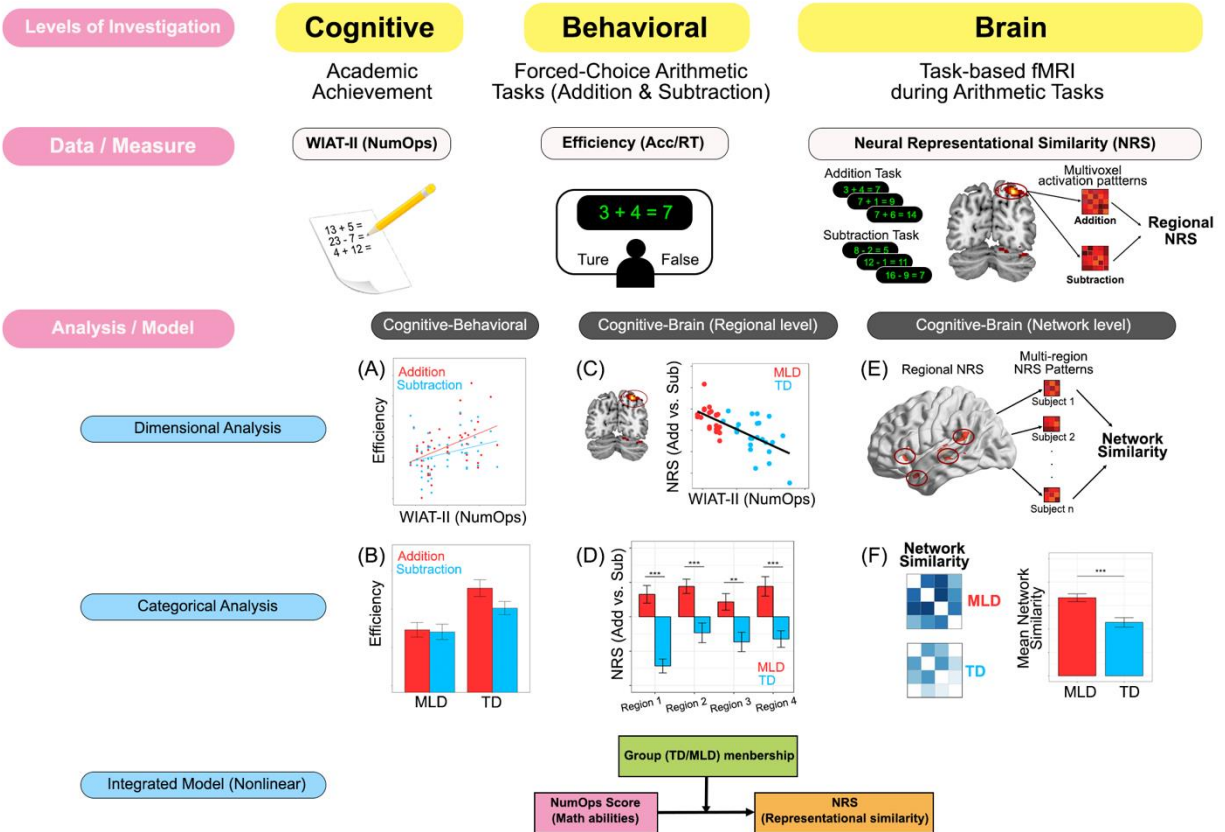


Figure 1. Multi-level analytical framework for investigation individual differences in behavioral, cognitive and neural profiles of differentiation between distinct numerical operations. (A, B) Analysis of cognitive-behavioral relationship using dimensional and categorical approaches. **(C, D)** Analysis of cognitive-brain relationships using dimensional and categorical approaches. **(E, F)** Inter-regional network similarity analysis to probe integration of neural representations in pathways important for manipulation of numerical quantity.

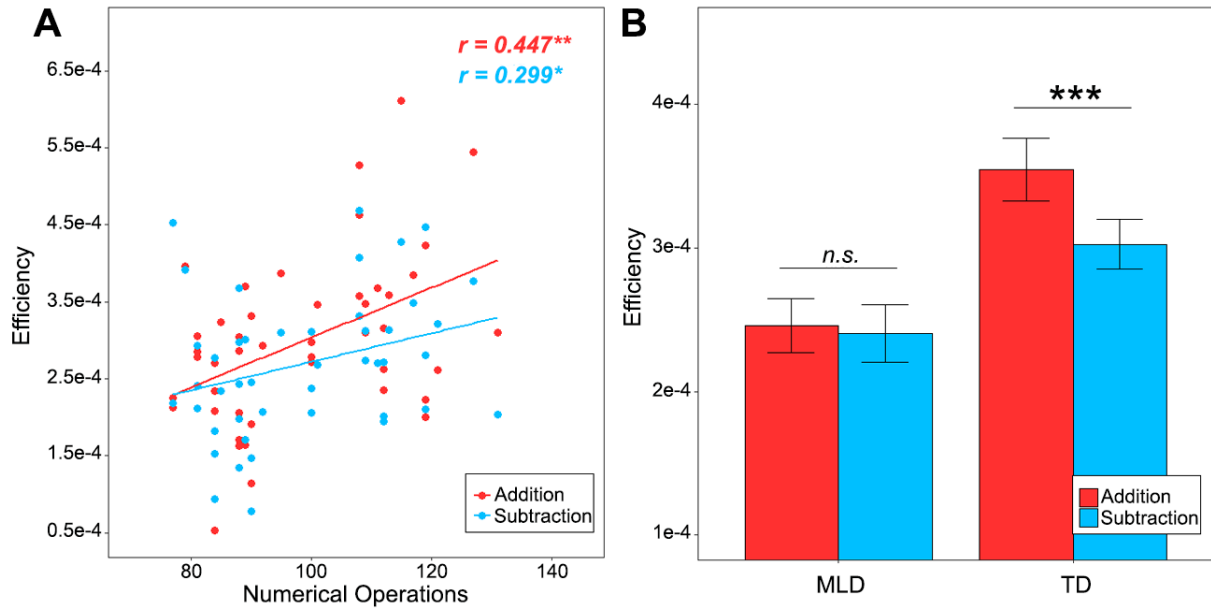


Figure 2. Behavioral performance during problem-solving involving addition and subtraction operations. (A) *Numerical Operations* subtest scores were significantly correlated with performance efficiency (accuracy/reaction time) in both addition and subtraction tasks; (B) Lower efficiency in children with MLD compared to TD children in both addition and subtraction tasks. A significant difference between addition and subtraction operations was observed in TD, but not in MLD children.

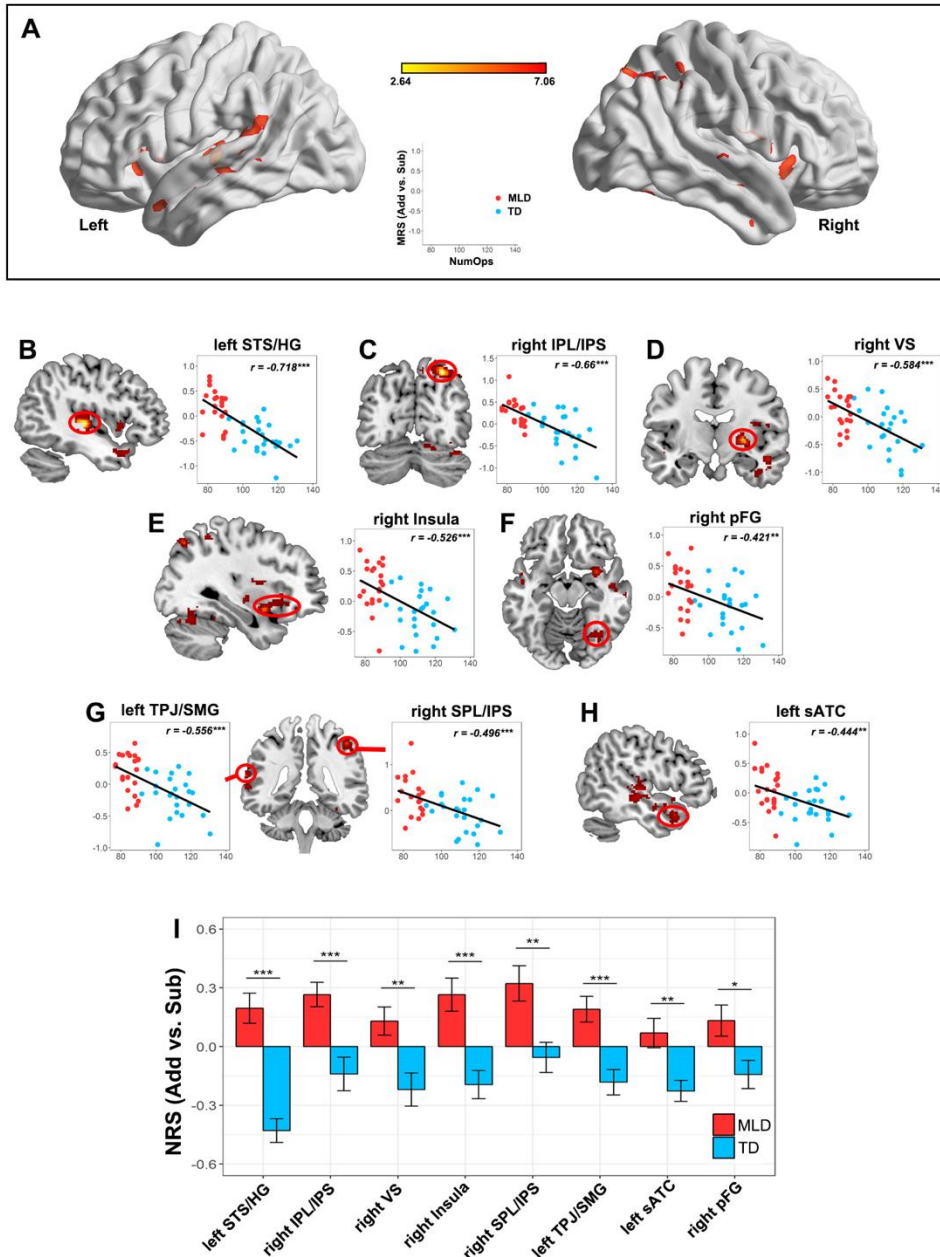


Figure 3. Higher Neural Representational Similarity (NRS) in children with low math abilities. (A) Brain regions showing significant negative correlation between NRS and *Numerical Operations* (*NumOps*) subscores of the WIAT-II. (B-H) Children with low math abilities show higher NRS in multiple parietal, temporal and prefrontal cortical regions (I) Children with MLD show higher NRS than TD children (ROIs were chosen to visualize the MRS difference in a categorical approach). *Note:* Statistical testing and analysis were conducted using a stringent threshold (height, $p < 0.005$; FDR-corrected for cluster extent $p < 0.01$); to facilitate visualization activation maps are shown at $p < 0.01$ (height) (FDR-corrected for cluster extent $p < 0.01$). STS/HG = Superior Temporal Sulcus and Heschl's gyrus; IPS = Intraparietal Sulcus; VS = Ventral Striatum (putamen/pallidum); pFG= posterior Fusiform Gyrus; TPJ/SMG = Temporoparietal Junction and Supramarginal Gyrus; SPL/IPS = Superior Parietal Lobule and Intraparietal Sulcus; and sATC = superior Anterior Temporal Cortex.

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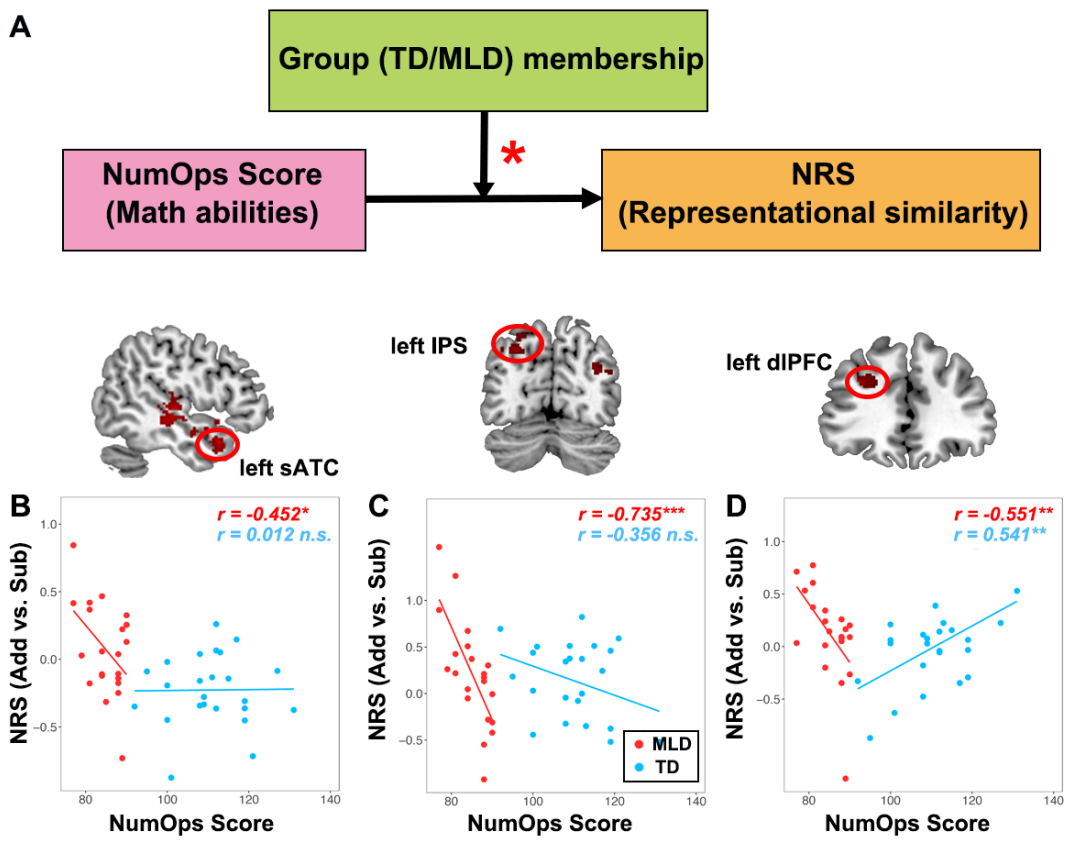


Figure 4. Nonlinear relationship between math ability and Neural Representational Similarity (NRS). (A) Model to test effect of group membership on the relationship between *NumOps* and NRS. (B-D) NRS in the superior part of the left anterior temporal cortex (sATC), left intraparietal sulcus (IPS), and left dorsolateral prefrontal cortex (dIPFC) was associated with lower math skills in children with MLD, but not in TD children.

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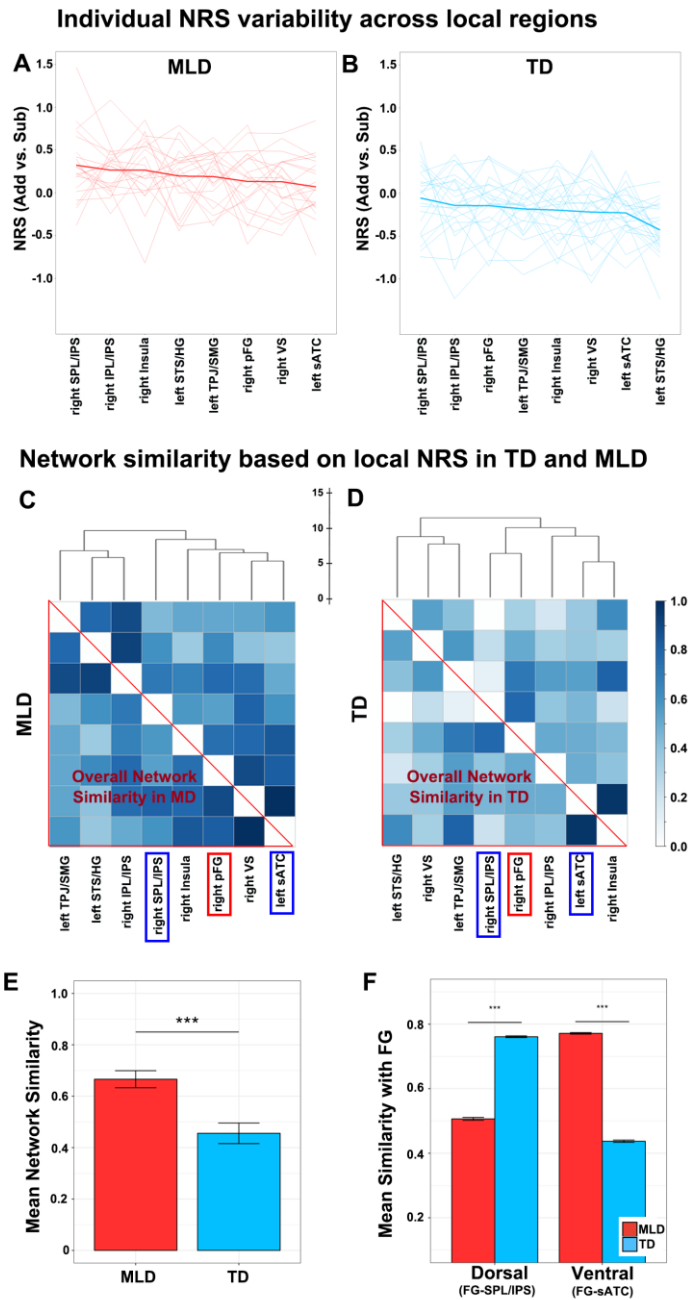


Figure 5. Aberrant inter-regional network similarity in children with MLD. (A, B) Individual and group-averaged NRS across brain regions in children with MLD and TD children. **(C, D)** Network similarity matrix based on NRS in eight brain areas that showed higher regional NRS in MLD, compared to the TD group. **(E)** The average Network similarity was higher in MLD compared to TD children; **(F)** NRS similarity between the fusiform gyrus and intraparietal sulcus (FG-IPS) nodes was lower in children with MLD, compared to TD children. In contrast, NRS similarity between fusiform gyrus and anterior temporal cortex (FG-sATC) nodes was higher in children with MLD, compared to TD children. Error bars and significance were estimated using subsampling and permutation testing.

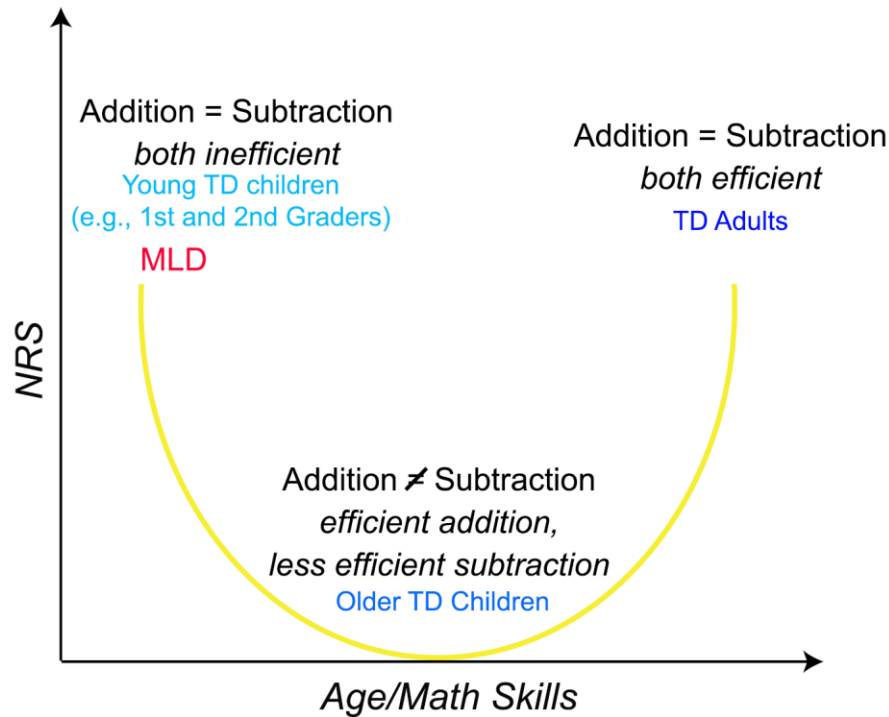


Figure 6. Proposed developmental model of operation-specific shifts in neural representations with age and math skills. This model predicts that there is a shift from effortful counting to efficient fact retrieval, leading to a convergence of problem-solving strategies across the two operations. Therefore, we would predict increased overlap in neural representations between the two operations in adults. Paradoxically, children with poor math abilities would also show high levels of neural representational similarity across distributed frontal, parietal, and temporal regions as in adults, because they tend to use inefficient strategies for both operations.

Table 1. Demographic and cognitive profiles of the whole sample and the TD/MLD subgroups.

	The whole sample			TD	MLD
	N	Range	Mean (SD)	Mean (SD)	Mean (SD)
Gender (Female/Male)	25F/20M			12F/12M	13F/8M
Grade (2 nd /3 rd)	15/30			7/17	8/13
Age	45	7.14-9.92	8.37 (0.64)	8.40 (0.64)	8.34 (0.65)
WASI					
VIQ	45	79-138	110 (12.47)	111.88 (12.46)	107.86 (12.43)
PIQ	45	77-145	108.38 (12.8)	109.38 (12.71)	107.24 (13.12)
FSIQ	45	84-128	110.2 (10.5)	111.83 (10.26)	108.33 (10.69)
WIAT-II					
Word Reading	45	77-130	106.29 (11.26)	110.38 (9.36)	101.62 (11.64)
Numerical Operations	45	77-131	98.73 (15.01)	110.75 (9.6)	85.00 (4.29)
Reading					
Comprehension	45	88-124	106.73 (10.06)	109.04 (10.19)	104.10 (9.45)
Math Reasoning	45	50-130	104.69 (14.78)	109.17 (15.64)	99.57 (12.16)
WMTB-C					
Digit Recall	45	73-145	106.47 (16.57)	107.42 (16.3)	105.38 (17.22)
Block Recall	44	64-114	93.55 (11.72)	96.21 (11.39)	90.35 (11.6)
Count Recall	45	56-117	85.38 (18.82)	88.04 (20.03)	82.33 (17.31)
Digit Backward Recall	45	70-131	96.29 (16.53)	100.42 (15.4)	91.57 (16.88)

Notes: WASI = Wechsler Abbreviated Scale of Intelligence; WIAT-II = Wechsler Individual Achievement Test (Second Edition); WMTB-C = Working Memory Test Battery for Children.

Table 2. Brain regions showing significant effects of NRS associated with *NumOps* scores and Group (MLD vs. TD)* *NumOps* interaction.

Main effect of <i>NumOps</i> (linear)						
Region	MNI Coordinates			Max Z	Cluster Size	
	x	y	z			
Negative effect						
Left STS/ Heschl's gyrus	-42	-22	2	5.72	414	
Right IPL/IPS	26	-72	52	5.45	318	
Right Ventral Striatum	26	-10	2	4.79	316	
Right ventral insula	32	6	-12	4.14	200	
Right SPL/IPS	46	-40	52	3.79	112	
Left TPJ/SMG	-64	-44	22	3.74	192	
Left sATC	-50	12	-22	3.56	90	
Right cerebellum	38	-64	-24	3.5	274	
Right FG	36	-62	-12	3.21		
Positive effect						
No significant clusters						
Interaction between Group*<i>NumOps</i> (nonlinear)						
TD > MLD						
Left dlPFC	-30	36	28	3.74	97	
Left IPS	-28	-72	42	3.76	89	
Right Cerebellum	14	-50	-22	4.30	103	
MLD > TD						
No significant clusters						

Supplementary Information

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- II. Supplementary Results
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I. Supplementary Methods

Hierarchical Drift-Diffusion model (HDDM) of decision-making dynamics during arithmetic problem solving

A hierarchical drift diffusion model (HDDM) was applied to the distribution of reaction times from the numerical problem-solving fMRI task using the python toolbox “HDDM” (Ratcliff, 2014; Ratcliff & Smith, 2004). Drift diffusion models account for the decision process as a continuous sampling of information that accumulates over time until a decision threshold is reached, and have been widely used to analyze latent decision-making variables during forced-choice tasks (Froehlich et al., 2016; Oganian et al., 2016; Ratcliff & McKoon, 2008; Ratcliff & Smith, 2004). Output model parameters of interest include: the decision threshold ‘a’ characterizing the amount of information that needs to be accumulated for the decision; drift-rate ‘v’ representing the speed of evidence-accumulation over time; non-decision time ‘t’ representing the aspects of response time not included in active deliberation (e.g. stimulus encoding, motor response). The present study did not model response-bias ‘z’ as we had no reason to believe that an a-priori bias existed here.

HDDM is better suited to handle data with a smaller number of trials (Froehlich et al., 2016; Ratcliff & Smith, 2004), such as those from fMRI tasks. A Bayesian approach was used to estimate model parameters by assigning prior probability distributions to each parameter. Markov Chain Monte-Carlo (MCMC) sampling was then used to approximate the posterior distribution for each parameter at both the individual- and group- level. We initialized HDDM with five chains (5000 samples, first 200 discarded as burn-in each, for a total of 24000 samples and 1000 burn-in) and estimated the parameters ‘a’, ‘v’, and ‘t’ during each task-condition and in each group (MLD, TD). Model convergence was determined using the Gelman-Rubin statistic (Gelman & Rubin, 1992). We then used Bayesian hypothesis-testing and calculated the proportion of overlap between the posterior distributions of the parameters ‘a’, ‘v’, and ‘t’ for each group, for each type of operation, and for each difficulty level (complex vs. simple) of the problems. The HDDM parameters across two difficulty levels were then averaged separately for each group and operation, and then we examined the correlations of *NumOps* scores and HDDM parameters as well as the difference between MLD and TD groups on HDDM parameters.

II. Supplementary Results

Math abilities are positively correlated with accuracy on addition and subtraction problems

NumOps scores were correlated with accuracy in both the addition ($r = 0.308, p < .05$) and subtraction ($r = 0.414, p < .01$) tasks, and negatively correlated with the mean reaction times on correct trials in the addition task ($r = -0.38, p < .01$) but not with the mean RT in the subtraction task (**Figure S1A&C** and **Table 4**). These results suggest that addition and subtraction problem-solving skills are weaker in children with lower math abilities.

III. Supplementary Figures

Figure S1. Behavioral performance on addition and subtraction tasks: accuracy and mean reaction times (RT). (A) *Numerical Operations* subtest scores were significantly correlated with accuracy in both addition and subtraction tasks; (B) Lower accuracy in children with MLD compared to TD children in both addition and subtraction tasks, but no difference between the two operations in either MLD or TD groups; (C) *Numerical Operations* subtest scores were significantly correlated with mean RT in the addition task, but not in the subtraction task; (D) A significant difference of mean RTs between the addition and subtraction tasks was observed in TD, but not in children with MLD.

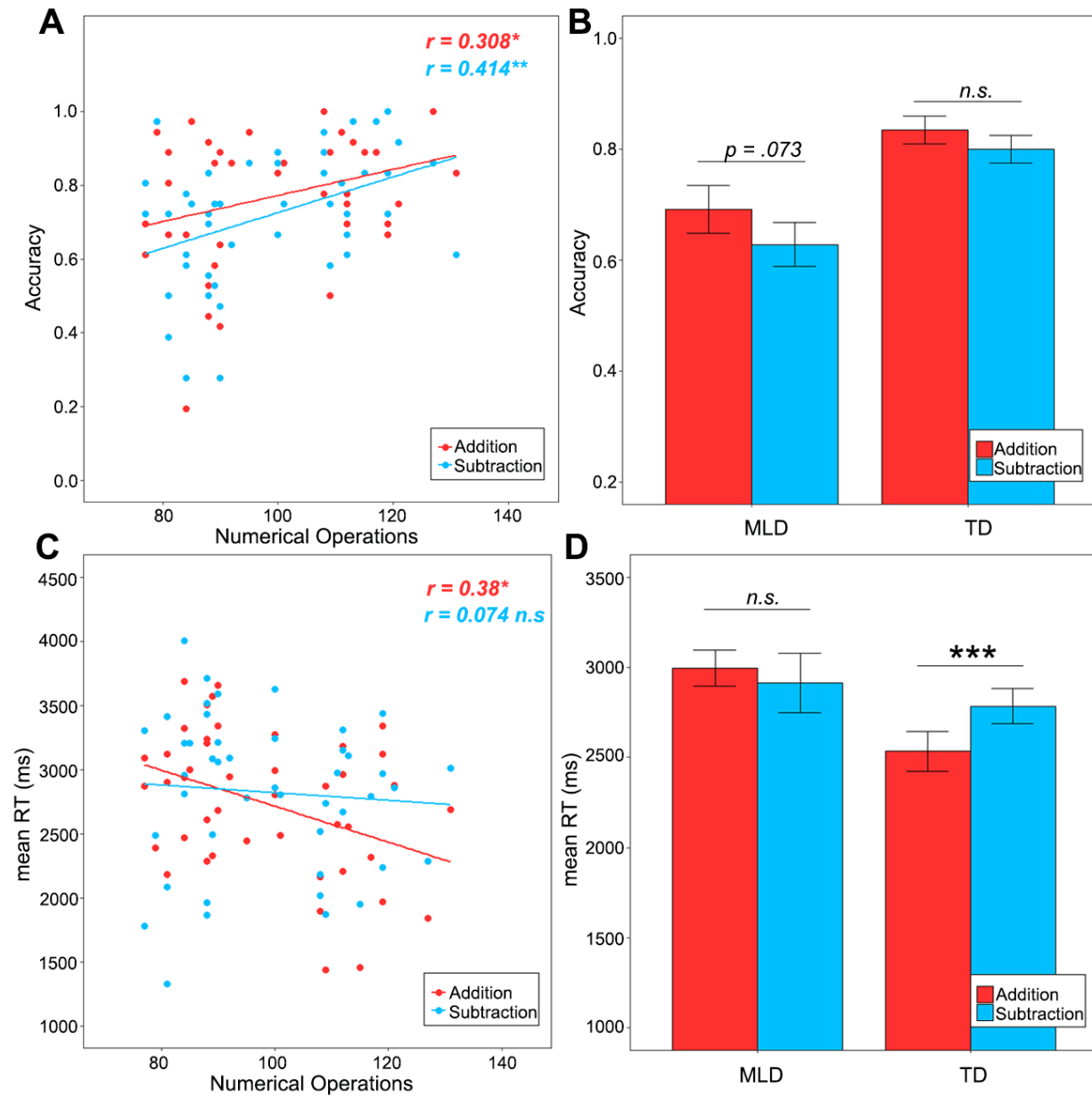
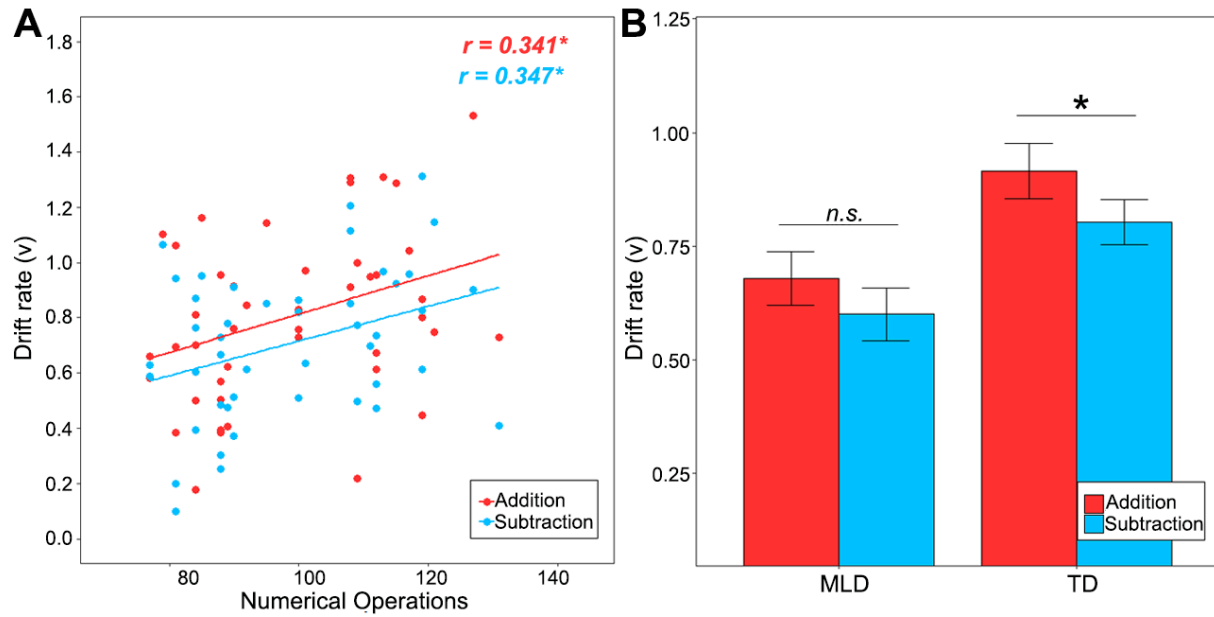


Figure S2. Drift rate in HDDM on addition and subtraction tasks. (A) *Numerical Operations* subtest scores were significantly correlated with drift rate (v) in both addition and subtraction tasks; (B) Lower drift rate in children with MLD compared to TD children in both addition and subtraction tasks, and a significant difference between drift rates of two operations in TD but not in MLD.



IV. Supplementary Tables

Table S1. Demographic and cognitive profiles of TD and MLD subgroups.

	TD	MLD			
	Mean (SD)	Mean (SD)	χ^2 value	<i>df</i>	<i>p</i> -value
Gender (Female/Male)	12F/12M	13F/8M	0.25	1	0.616
Grade (2 nd /3 rd)	7/17	8/13	0.10	1	0.751
			<i>t</i> -value	<i>df</i>	<i>p</i> -value
Age	8.40 (0.64)	8.34 (0.65)	-0.30	43	0.766
WASI					
VIQ	111.88 (12.46)	107.86 (12.43)	-1.08	43	0.286
PIQ	109.38 (12.71)	107.24 (13.12)	-0.55	43	0.582
FSIQ	111.83 (10.26)	108.33 (10.69)	-1.12	43	0.269
WIAT-II					
Word Reading	110.38 (9.36)	101.62 (11.64)	-2.80	43	0.008
Numerical Operations	110.75 (9.6)	85.00 (4.29)	-11.33	43	<.001
Reading Comprehension	109.04 (10.19)	104.10 (9.45)	-1.68	43	0.100
Math Reasoning	109.17 (15.64)	99.57 (12.16)	-2.27	43	0.028
WMTB-C					
Digit Recall	107.42 (16.3)	105.38 (17.22)	-0.41	43	0.686
Block Recall	96.21 (11.39)	90.35 (11.6)	-1.69	42	0.099
Count Recall	88.04 (20.03)	82.33 (17.31)	-1.02	43	0.316
Digit Backward Recall	100.42 (15.4)	91.57 (16.88)	-1.84	43	0.073

Notes: WASI = Wechsler Abbreviated Scale of Intelligence; WIAT-II = Wechsler Individual Achievement Test (Second Edition); WMTB-C = Working Memory Test Battery for Children; all the *p* values that were smaller than 0.05 were bolded.

Table S2. Correlation matrix of cognitive assessments and in-scanner performance.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1 Age	-												
2 VIQ	-0.194	-											
3 PIQ	-0.136	0.145	-										
4 FSIQ	-0.212	0.741***	0.768***	-									
5 Word Reading	-0.277#	0.376*	0.041	0.262#	-								
6 Number Operations	0.063	0.057	0.189	0.162	0.319*	-							
7 Reading Comprehension	-0.122	0.569***	0.212	0.495***	0.691***	0.161	-						
8 Math Reasoning	-0.240	0.315*	0.494***	0.54***	0.302*	0.432**	0.237	-					
9 Mean Accuracy (Addition)	-0.024	0.152	0.008	0.105	0.394**	0.308*	0.283#	0.231	-				
10 Mean Correct RT (Addition)	0.053	0.050	0.074	0.083	-0.111	-0.38*	0.005	-0.21	-0.385**	-			
11 Mean Accuracy (Subtraction)	-0.011	0.199	0.158	0.237	0.267#	0.414**	0.307*	0.315*	0.697***	-0.461**	-		
12 Mean Correct RT (Subtraction)	0.100	-0.108	0.140	0.016	0.024	-0.074	0.079	-0.112	-0.157	0.585***	-0.144	-	
13 Efficiency (Addition)	-0.025	-0.028	-0.101	-0.082	0.267#	0.447**	0.089	0.307*	0.764***	-0.841***	0.647***	-0.486***	-
14 Efficiency (Subtraction)	-0.067	0.160	-0.053	0.073	0.157	0.299*	0.126	0.276#	0.536***	-0.676***	0.744***	-0.741***	0.747***

Notes: # $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$.

Table S3. In-scanner performance in addition and subtraction tasks.

	The whole sample	MLD	TD	<i>t</i> -value	<i>df</i>	<i>p</i> -value
	mean (SD)	Mean (SD)	Mean (SD)			
Addition						
Efficiency (Total)	2.99 X10 ⁻⁴ (1.09 X 10 ⁻⁴)	2.42 X10 ⁻⁴ (8.50 X 10 ⁻⁵)	3.49 X10 ⁻⁴ (1.04 X 10 ⁻⁴)	-3.73	43	0.001
Accuracy (Total)	0.77 (0.17)	0.69 (0.2)	0.83 (0.12)	-3.00	43	0.004
Mean RT (Total)	2730.77 (553.36)	2969.22 (467.69)	2522.13 (546.11)	2.93	43	0.005
Accuracy (Complex)	0.71 (0.2)	0.62 (0.22)	0.8 (0.13)	-3.41	43	0.001
Mean RT (Complex)	3018.23 (601.13)	3251.01 (523.76)	2814.54 (599.97)	2.58	43	0.013
Accuracy (Simple)	0.82 (0.19)	0.77 (0.22)	0.87 (0.15)	-1.89	43	0.065
Mean RT (Simple)	2481.97 (579.4)	2742.3 (535.34)	2254.18 (526.3)	3.08	43	0.004
Subtraction						
Efficiency (Total)	2.70 X10 ⁻⁴ (9.31 X 10 ⁻⁵)	2.35 X10 ⁻⁴ (9.50 X 10 ⁻⁵)	3.00 X10 ⁻⁴ (8.13 X 10 ⁻⁵)	-2.48	43	0.017
Accuracy (Total)	0.72 (0.18)	0.63 (0.18)	0.8 (0.13)	-3.71	43	0.001
Mean RT (Total)	2820.54 (599.61)	2880.04 (723.48)	2768.47 (476.01)	0.62	43	0.54
Accuracy (Complex)	0.64 (0.21)	0.52 (0.21)	0.75 (0.15)	-4.32	43	<.001
Mean RT (Complex)	3005.25 (665.91)	3050.91 (849.4)	2967.2 (479.27)	0.41	42	0.683
Accuracy (Simple)	0.8 (0.18)	0.74 (0.2)	0.85 (0.14)	-2.14	43	0.038
Mean RT (Simple)	2671.69 (612.65)	2750.21 (713.47)	2602.97 (514.63)	0.80	43	0.428

Table S4. Correlation matrix of numerical operation scores and HDDM parameters.

	1	2	3	4	5	6	7	8
1 Numerical Operations	-							
2 Drift rate (Addition)	0.341*	-						
3 Drift rate (Subtraction)	0.347*	0.571***	-					
4 Threshold (Addition)	-0.176	0.302*	0.175	-				
5 Threshold (Subtraction)	0.162	0.067	0.201	0.415**	-			
6 No decision time (Addition)	-0.078	0.188	0.035	0.061	0.208	-		
7 No decision time (Subtraction)	-0.181	-0.031	0.452**	0.315*	0.112	0.257#	-	
8 Response bias (Addition)	0.121	0.303*	0.25#	-0.011	0.008	0.203	0.034	-
9 Response bias (Subtraction)	-0.047	-0.162	0.018	-0.055	0.012	-0.061	0.233	-0.194

Notes: # $p < .10$; * $p < .05$; ** $p < .01$; *** $p < .001$.

Table S5. Group differences between MLD and TD on NRS in brain regions.

Brain Regions	MLD	TD	<i>t</i> -value	<i>df</i>	<i>p</i> -value
	Mean (SD)	Mean (SD)			
left STS/HG	0.2 (0.35)	-0.43 (0.3)	6.46	43	<.001
right IPL/IPS	0.27 (0.29)	-0.14 (0.42)	3.72	43	0.001
right VS	0.13 (0.33)	-0.22 (0.41)	3.11	43	0.003
right Insula	0.27 (0.39)	-0.19 (0.35)	4.17	43	<.001
right SPL/IPS	0.32 (0.41)	-0.06 (0.38)	3.2	43	0.003
left TPJ/SMG	0.19 (0.3)	-0.18 (0.32)	4.02	43	<.001
left sATC	0.07 (0.34)	-0.23 (0.26)	3.27	43	0.002
right pFG	0.13 (0.37)	-0.14 (0.35)	2.57	43	0.014

Table S6. In-scanner movement statistics in addition and subtraction tasks.

	The whole sample		TD	MLD	<i>t</i> -value	<i>df</i>	<i>p</i> -value
	mean (SD)	range	Mean (SD)	Mean (SD)			
Addition							
Range X	0.65 (0.53)	0.12-2.44	0.53 (0.38)	0.79 (0.64)	1.7	43	0.095
Range Y	1.08 (1.04)	0.22-5.76	0.87 (0.62)	1.32 (1.36)	1.47	43	0.148
Range Z	1.87 (1.48)	0.3-8.14	1.64 (1.05)	2.12 (1.84)	1.09	43	0.284
Range Roll	0.99 (0.74)	0.19-3.38	0.9 (0.57)	1.1 (0.9)	0.91	43	0.369
Range Pitch	2.26 (1.84)	0.43-9.15	2.18 (1.91)	2.35 (1.79)	0.32	43	0.753
Range Yaw	0.9 (0.66)	0.14-3.19	0.76 (0.55)	1.05 (0.74)	1.5	43	0.142
% Volume Repaired	1.32 (1.35)	0.5-5.53	0.98 (1.05)	1.7 (1.56)	1.82	43	0.076
Framework Displacement	0.12 (0.06)	0.04-0.29	0.12 (0.06)	0.13 (0.07)	0.91	43	0.366
Subtraction							
Range X	0.65 (0.79)	0.13-3.91	0.44 (0.29)	0.89 (1.08)	1.95	43	0.058
Range Y	1.01 (0.78)	0.17-3.45	0.86 (0.61)	1.18 (0.93)	1.4	43	0.169
Range Z	1.97 (1.63)	0.39-8.05	1.84 (1.26)	2.12 (2)	0.58	43	0.565
Range Roll	1.25 (1.45)	0.24-8.68	1.11 (1)	1.42 (1.86)	0.72	43	0.473
Range Pitch	2.64 (2.6)	0.51-12.88	2.13 (1.48)	3.22 (3.42)	1.42	43	0.162
Range Yaw	0.99 (1.27)	0.09-7.79	0.72 (0.41)	1.29 (1.78)	1.54	43	0.132
% Volume Repaired	1.59 (2.27)	0.5-13.57	1.28 (1.29)	1.96 (3.02)	1	43	0.322
Framework Displacement	0.13 (0.09)	0.03-0.47	0.12 (0.07)	0.14 (0.1)	0.75	43	0.458

Table S7. Demographic and cognitive profiles of 2nd and 3rd graders.

	2 nd grader	3 rd grader			
	Mean (SD)	Mean (SD)	<i>t</i> -value	<i>df</i>	<i>p</i> -value
Age	7.69 (0.29)	8.71 (0.47)	-7.64	43	<.001
WASI					
VIQ	112.2 (13.07)	108.9 (12.23)	0.83	43	0.409
PIQ	110.53 (13.48)	107.3 (12.54)	0.8	43	0.431
FSIQ	112.6 (9.43)	109 (10.94)	1.09	43	0.283
WIAT-II					
Word Reading	109.8 (9.24)	104.53 (11.9)	1.5	43	0.141
Numerical Operations	96.87 (11.67)	99.67 (16.53)	-0.59	43	0.561
Reading Comprehension	109 (9.43)	105.6 (10.32)	1.07	43	0.29
Math Reasoning	109.6 (11.43)	102.23 (15.8)	1.6	43	0.116
WMTB-C					
Digit Recall	102.67 (16.79)	108.37 (16.42)	-1.09	43	0.282
Block Recall	95.14 (10.58)	92.8 (12.32)	0.61	42	0.543
Count Recall	85.13 (15.54)	85.5 (20.51)	-0.06	43	0.952
Digit Backward Recall	92.13 (17.08)	98.37 (16.14)	-1.2	43	0.237

Table S8. In-scanner performance in addition and subtraction tasks for 2nd and 3rd graders.

	2 nd grader	3 rd grader	<i>t</i> -value	<i>df</i>	<i>p</i> -value
	Mean (SD)	Mean (SD)			
Addition					
Efficiency (Total)	2.93 X 10 ⁻⁴ (1.19 X 10 ⁻⁴)	3.02 X 10 ⁻⁴ (1.05 X 10 ⁻⁴)	-0.25	43	0.803
Accuracy (Total)	0.76 (0.18)	0.77 (0.17)	-0.08	43	0.934
Mean RT (Total)	2801.49 (563.5)	2695.42 (554.45)	0.6	43	0.550
Accuracy (Complex)	0.68 (0.26)	0.73 (0.17)	-0.76	43	0.451
Mean RT (Complex)	3024.76 (508.88)	3014.96 (650.55)	0.05	43	0.96
Accuracy (Simple)	0.85 (0.14)	0.81 (0.21)	0.65	43	0.518
Mean RT (Simple)	2605.12 (690.96)	2420.4 (516.82)	1.01	43	0.319
Subtraction					
Efficiency (Total)	2.67 X 10 ⁻⁴ (1.10 X 10 ⁻⁴)	2.71 X 10 ⁻⁴ (8.54 X 10 ⁻⁵)	-0.11	43	0.909
Accuracy (Total)	0.71 (0.19)	0.72 (0.17)	-0.23	43	0.819
Mean RT (Total)	2842.68 (517.72)	2809.47 (644.77)	0.17	43	0.863
Accuracy (Complex)	0.63 (0.26)	0.65 (0.19)	-0.3	43	0.766
Mean RT (Complex)	3014.58 (513.46)	3000.9 (734.35)	0.06	42	0.950
Accuracy (Simple)	0.8 (0.17)	0.8 (0.19)	-0.1	43	0.924
Mean RT (Simple)	2674.84 (576.14)	2670.11 (639.71)	0.02	43	0.981

Table S9. In-scanner movement statistics in addition and subtraction tasks for 2nd and 3rd graders.

	2 nd grader	3 rd grader	<i>t</i> -value	<i>df</i>	<i>p</i> -value
	Mean (SD)	Mean (SD)			
Addition					
Range X	0.79 (0.54)	0.58 (0.52)	1.23	43	0.225
Range Y	1.26 (1.44)	0.99 (0.79)	0.8	43	0.427
Range Z	2.47 (1.85)	1.56 (1.17)	2.01	43	0.051
Range Roll	1.22 (0.63)	0.87 (0.77)	1.51	43	0.139
Range Pitch	2.57 (1.72)	2.10 (1.90)	0.81	43	0.423
Range Yaw	0.96 (0.4)	0.86 (0.76)	0.45	43	0.656
% Volume Repaired	1.51 (1.41)	1.22 (1.33)	0.66	43	0.510
Framewise Displacement	0.15 (0.06)	0.11 (0.06)	1.66	43	0.104
Subtraction					
Range X	1.09 (1.22)	0.43 (0.30)	2.79	43	0.008
Range Y	1.01 (0.7)	1.00 (0.84)	0.04	43	0.968
Range Z	2.37 (1.82)	1.77 (1.52)	1.17	43	0.248
Range Roll	1.76 (2.11)	1.00 (0.93)	1.68	43	0.100
Range Pitch	3.39 (2.96)	2.26 (2.37)	1.38	43	0.174
Range Yaw	1.43 (1.87)	0.76 (0.77)	1.72	43	0.093
% Volume Repaired	2.03 (1.89)	1.37 (2.43)	0.92	43	0.363
Framewise Displacement	0.15 (0.09)	0.12 (0.08)	1.18	43	0.246

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Credit Author Statement

L. Chen, T. Iuculano, and V. Menon designed the research; L. Chen, T. Iuculano, P. Mistry, J. Nicholas, Y. Zhang analyzed the data; L. Chen, T. Iuculano, P. Mistry, and V. Menon wrote the manuscript.