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1 Algorithms to reduce the computational cost of vector Preisach 2 model in view of Finite Element analysis

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9 Abstract

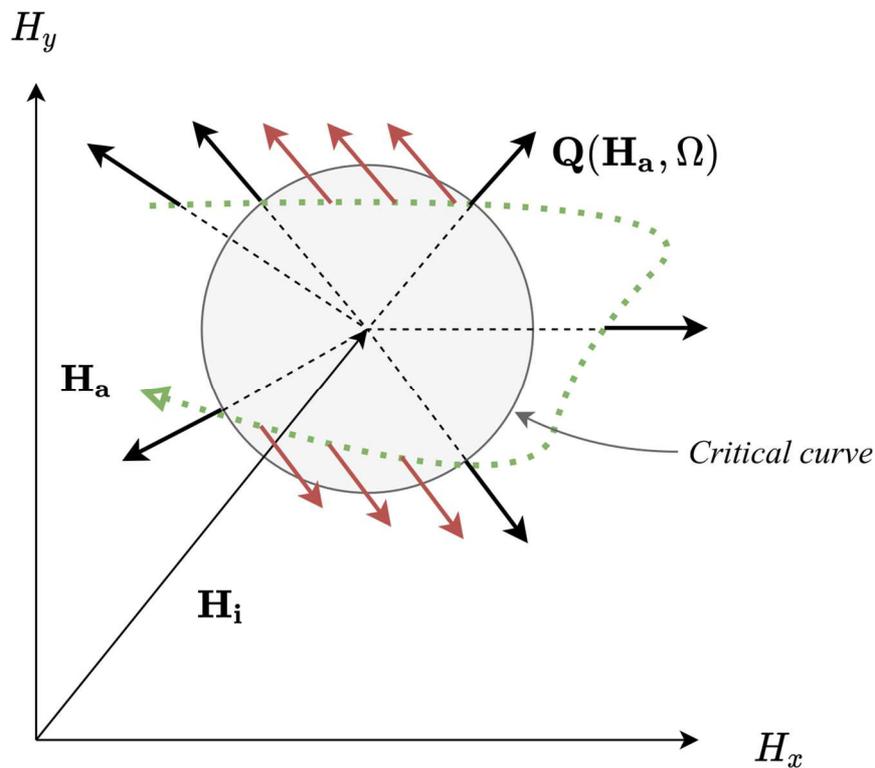
10 The purpose of this work is to devise algorithms to reduce the memory consumption of the
11 vector Preisach model in view of its usage in Finite Element analysis. Four algorithms, which
12 all implement a vector Preisach hysteresis model, are presented and critically compared
13 theoretically and by numerical experiments taken on with two materials and three signals.
14 Several strategies are presented to reduce both the memory occupation and the computational
15 cost of several orders of magnitude.

16 Introduction

17 The hysteresis phenomena of the magnetic materials can be modelled using different
18 approaches depending on the scale length of the component taken into account. The magnetic
19 hysteresis at ultra-millimeter scale length, such as the components involved in the electrical
20 machines, power converters, permanent magnets, magnetic recording and magnetic sensors,
21 is in general represented using phenomenological models. About that some of the most used
22 and applied are the Jiles Model [1-4], the Play Model [5-8], the Equivalent Ellipse Model [9]
23 and the Preisach Model [10-12]. The latter one, called also the Classical Scalar Preisach
24 Model, can be generalized for the representation of vector hysteresis to the 2-d and 3-d cases
25 as indicate in a series of previous papers [13-17], hereafter called Vector Hysteresis Model to
26 distinguish it from the Preisach-Mayergoyz model [11] and its improvements [24-26]. This
27 vector approach is based on the definition of a vector mathematical operator, called for
28 convenience here vector hysteron, described in the H-space by a closed and convex critical
29 surface. Each vector hysteron has a unique critical surface, described by a suitable set of
30 parameters, indicated here with the parameter vector Ω . The magnetization state vector of
31 the hysteron can be denoted by the unitary and dimensionless vector $\mathbf{Q}(\Omega, \mathbf{H})$. This means
32 that the direction of the unit magnitude magnetization is a single-valued function of the
33 applied magnetic field outside the critical surface and a multivalued function determined by
34 the magnetization history inside the critical surface. For clarity we refer here to the case of
35 two dimensions, where the hysteron critical surface become to a critical curve. In particular
36 we refer to the circular hysteron as described in the figure 1. The components of Ω , i.e. the
37 model parameters, are the components of the interaction field (H_{ix} , H_{iy}) that identify the
38 hysteron centre and the value of the parameter u that is the hysteron radius, as described in
39 more detail in the references above mentioned and in the following. The hysterons are

40 distributed in the H-plane, and their density distribution can be described by a dimensionless
 41 function $P(\Omega)$. The rules of this generalized vector model of hysteresis are:

- 42 • the normalized component of the magnetization for each hysteron has unit
 43 magnitude everywhere;
- 44 • for fields inside the critical surface the magnetization is frozen in the direction that
 45 it had just before it entered the critical surface, and it remains constant until it
 46 exits the critical surface;
- 47 • when exiting the critical surface the irreversible magnetization instantly rotates so
 48 as to align itself along a new direction, from the hysteron centre to the vertex of the
 49 applied magnetic field. This behaviour is corresponding to the Barkhausen
 50 jumps that occur in the magnetic materials.
- 51 • the total magnetization is the vector sum of the magnetization due to all the vector
 52 hysterons.



53
 54 Figure 1. An example in 2-D of hysteron and of the rules of magnetization change with the applied field for a
 55 particular applied field \mathbf{H}_a trajectory. \mathbf{H}_i is the center of the hysteron. Frozen unitary magnetization vectors are
 56 depicted in red color.

57 In previous papers [13-19] some general properties of the vector hysterons are shown. In
 58 particular we have proved in rigorous mathematical way that is possible define some classes
 59 of hysteron where:

- 60 • the critical surfaces of the vector hysterons are equipotential surfaces
- 61 • the lines of force of the M-field of a vector hysteron are always straight lines.
- 62 • the critical surface of the vector hysterons must obey to a necessary and sufficient
 63 conditions in order to satisfy the conservative properties of vector fields.

- 64 • the critical surface of the vector hysterons must satisfy an additional necessary and
65 sufficient condition for the congruency with the second principle of the
66 thermodynamics.
67 • any assembly of vector hysterons defined as above obeys to the saturation property, to
68 the losses property, to the deletion and the congruency property.

69 The generalization of the Preisach method to the representation of magnetic hysteresis in 3-d
70 has proved to be precise, efficient and robust in the reproduction of the state of magnetization
71 of many magnetic materials widely used in many industrial and aerospace applications.
72 However, the use of this methodology involves some disadvantages, which did not allow a
73 full application to the numerical analysis of 3-d electromagnetism problems with the use of
74 Finite Element techniques [28]. Among these disadvantages it is worth mentioning the
75 considerable memory space necessary to take into account the state of each hysteron, which
76 can change according to the magnetic field applied.

77 In consideration of the fact that the numerical representation of the vector hysteresis, if not
78 appropriately corrected, requires the definition of hundreds of thousands of hysterons in
79 complicated geometries such as those referring to electromagnetic applications in the
80 industrial and aerospace fields mentioned, it appears evident that each method of reducing the
81 number of hysterons or grouping them can have a significant effect on the duration and
82 memory requirement in numerical processing. It has to be mentioned that recent works have
83 been devoted to acceleration of the Preisach-Mayergoyz model by using Everett function [22-
84 23, 27]. However the ideas developed in these works cannot be transposed (at least not
85 straightforwardly) to the Vector Hysteron Model.

86 In this paper we present a new strategy to reduce the computational resources required to
87 simulate a vector Preisach model, in view of its implementation on Finite Element analysis
88 (FEA). The first purpose of this work is to reduce of several orders of magnitude memory
89 requirements. Unfortunately, the economy in memory resources comes with a proportional
90 increase of the computational time. Hence, the second (but not less important) point which is
91 addressed in this work is the reduction of the computational time.

92 This article is organized as following. First, the proposed algorithms are presented, and
93 compared with the classical algorithm described in the paper [20] which is extremely greedy
94 in terms of memory occupation. Then two strategies to reduce the computational times are
95 presented: the first one is based on filtering out a significant part of hysterons. The second
96 strategy is based on the generation of reduced models, depending on the applied magnetic
97 field. The proposed algorithms are compared with respect of several signals representative of
98 magnetic materials for electrical machine cores and of magnetic components for power
99 electronics. Conclusions and perspective of future works conclude the article.

100 **Materials and Methods**

101 Hereafter are presented several algorithms which implement the vector Preisach model in a
102 2D space:

- 103 • Basic algorithm: this is a quite standard algorithm which implements Preisach model.
104 The state of each hysterons is stored explicitly;
105 • Pooled algorithm: this is a variation of the basic algorithm where only the unit
106 polarization of frozen hysterons is stored;

- 107 • Memoryless algorithm: in this algorithm, the state of the model is represented by the
- 108 past and present applied magnetic field;
- 109 • Incremental algorithm: this algorithm is a variant of the memoryless algorithm, which
- 110 is theoretically more computationally efficient and more suitable for further
- 111 improvements.

112 It is assumed that critical surfaces of hysterons are circles. For each hysteron, the radius and
 113 the center of the crucial surface are denoted respectively by u_i and \mathbf{H}_i . The number of
 114 hysterons and of computational points (that is, of elements which compose a ferromagnetic
 115 region) is indicated respectively by N and M . It can be foreseen that in practical FEA,
 116 $N=10^5$ and $M=10000$ are realistic values (for instance, see [29, 30] for academic and
 117 practical 2d examples, and [31] for a practical 3d example). It is important to keep in mind
 118 these orders of magnitudes, in order to analyse the computational cost of these algorithms in
 119 view of their application in FEA.

120 Basic algorithm

121 In the basic algorithm, the state of each hysteron is represented by its unit polarization \mathbf{Q}_i at
 122 any time, and in all computational points. At any time step, for each computational point n
 123 and for each hysteron i , the vector \mathbf{r}_{ni} is computed:

$$124 \quad \mathbf{r}_{ni} = \mathbf{H}\mathbf{a}_n - \mathbf{H}_i \quad (1)$$

125 where $\mathbf{H}\mathbf{a}_n$ is the magnetic field externally applied in the n^{th} computational point.
 126 If $\|\mathbf{r}_{ni}\| > u_i$ or if the hysteron get frozen at this time step (that is, if the hysteron was not
 127 frozen in the previous time step and $\|\mathbf{r}_{ni}\| \leq u_i$), the value of the unit polarization is updated:

$$128 \quad \mathbf{Q}_{ni} \leftarrow \frac{\mathbf{r}_{ni}}{\|\mathbf{r}_{ni}\|} \quad (2)$$

129 Otherwise, if the hysteron is already frozen the unit magnetization is unchanged. Notice that
 130 in this case \mathbf{Q}_{ni} has the value, which it had since the hysteron got frozen. In the continuous
 131 case, the total polarization in the point n is computed as [32]:

$$132 \quad \mathbf{J}_n = \alpha \iiint \mathbf{Q}_n(H_{xi}, H_{yi}, u) P(H_{xi}, H_{yi}, u) dH_{xi} dH_{yi} du \quad (3)$$

133 After discretisation, the former equation writes:

$$134 \quad \mathbf{J}_n = \alpha \sum_{i=1}^N \mathbf{Q}_{ni} w_i \quad (4)$$

135 where w_i is the weight of the i^{th} hysteron defined by $P(\mathbf{Q})$, and α is a scaling coefficient
 136 corresponding to the saturation polarization. The other relevant quantities are defined as
 137 usual:

138
$$\mathbf{M} = \frac{1}{\mu_0} \mathbf{J} \quad ; \quad \mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0 \mathbf{H} + \mathbf{J} \quad (5)$$

139 When this algorithm is used in FEA, the size of the state is of at least MN vectors, in that N
 140 vectors have to be stored for each of the M computational points. Just to give an order of
 141 magnitude, if two double precision values are required for each vector (that is, 16 bytes), the
 142 memory occupied by the set of unit magnetization vectors \mathbf{Q}_{ni} only would be of 16 Gb in our
 143 reference case ($N = 10^5$, $M = 10000$). As for the computational time, it is dominated by the
 144 time required to upgrade all hysterons in all points, that is $O(MN)$. To wrap up, the space
 145 and time complexity of the basic algorithm are both $O(MN)$. This algorithm is taken as
 146 reference for the algorithms presented hereafter.

147 **Pooled algorithm**

148 The “pooled” algorithm is a variant of the basic algorithm where only the state of frozen
 149 hysterons is stored. This algorithm allows an economy of memory, in that the space
 150 complexity is $O(fMN)$, where $f < 1$ is the average fraction of frozen hysterons, which
 151 cannot be foreseen in advance. Provided that the extra computational burden to handle the
 152 pool of frozen hysterons is negligible, the time complexity is unchanged, that is $O(MN)$.

153 **Memoryless algorithm**

154 In the “memoryless” algorithm, the state of each computational point is represented by the set
 155 of the present and all past values of the applied magnetic field $\mathbf{H}\mathbf{a}_n$ which have been applied
 156 to the computational point. At each time step, vector \mathbf{r}_{ni} is computed for each hysteron, and
 157 for each computational point (Equation 1). If the hysteron is not frozen, then the unit
 158 magnetization \mathbf{Q}_{ni} is computed by using Equation 2. Otherwise, in order to know the unit
 159 polarization it is mandatory to go back in time up to the time step when the hysteron got
 160 frozen. Finally, the total polarization is computed by using Equation 4.

161 This algorithm allows a dramatic reduction of the required memory, because for each
 162 computational point it is necessary to store only the past values of the applied magnetic field,
 163 hence the space complexity is $O(MT)$ where T is the number of time steps. It can be easily
 164 foreseen that in practice $T \ll N$ by several orders of magnitude (for instance, see [29-31] for
 165 practical values of T), hence this algorithm has a dramatic advantage in terms of memory
 166 occupation with respect of the basic algorithm. As for the computational time, it depends on
 167 the state of hysterons (frozen / unfrozen). Basically, unfrozen hysterons cost no more than in
 168 the basic algorithm, whereas the cost of frozen hysterons is much higher, and increases with
 169 the number of time steps. Hence, the time complexity is bounded by $O(MN(1 + fT))$.
 170 However, it is evident that the price to pay to reduce the memory occupation is an increase of
 171 computational time.

172 **Incremental algorithm**

173 In the “memoryless” algorithm, hysterons with “huge” radius u_i are extremely expensive, on
 174 one hand because they are likely to be frozen for most of time steps, and on the other hand

175 because the computational effort required to come back in time up to the instant when they
 176 got frozen (if any) is much higher with respect of “small” hysterons. Moreover, in most of
 177 time steps, the algorithm will repeat the same computation, which will provide the same unit
 178 magnetization of many previous time steps. The idea behind the “incremental” algorithm is to
 179 split the total polarization as the sum of the contribution of frozen and unfrozen hysterons:

$$180 \quad \mathbf{J}_n = \mathbf{J}_n^{(u)} + \mathbf{J}_n^{(f)} \quad (6)$$

181 For each time step and for each computational point, only $\mathbf{J}_n^{(f)}$ is stored in the state, and
 182 upgraded at any time step. Four cases must be distinguished, depending on the state of the
 183 hysteron at the last (= present) and at the second-to-last (= previous) time step:

- 184 1. The hysteron is unfrozen, and was already unfrozen at the previous time step: the unit
 185 polarization of this hysteron, computed by Equation 2, is added to obtain $\mathbf{J}_n^{(u)}$.
- 186 2. The hysteron is frozen, and was already frozen at the previous time step: there is
 187 nothing to do, because in this case the contribution of the hysteron is already taken
 188 into account by $\mathbf{J}_n^{(f)}$.
- 189 3. Transition unfrozen \rightarrow frozen: the unit polarization of the hysteron, computed by
 190 Equation 2, is added to $\mathbf{J}_n^{(f)}$.
- 191 4. Transition frozen \rightarrow unfrozen: this is the most complex case because the unit
 192 polarization of the *hysteron back in time when it got frozen* has to be retrieved, and
 193 subtracted from $\mathbf{J}_n^{(f)}$. Then, the present unit polarization of the hysteron is computed
 194 by Equation 2 and added to $\mathbf{J}_n^{(u)}$.

195 One observes that, from the point of view of computational time, hysterons in the case (1)-(4)
 196 cost no more than in the basic algorithm. Indeed, hysterons which are and remain frozen
 197 (case 2) do cost even less than in the basic algorithm. In particular, huge hysterons which got
 198 frozen and unfrozen very seldom has a marginal cost with respect of the memoryless
 199 algorithm. Conversely, hysterons which fall in the case (4) require more computational time,
 200 but not more than in the case of memoryless algorithm. As for the memory requirement, this
 201 algorithm is marginally more expensive than the memoryless algorithm, because for each
 202 computational point the vector $\mathbf{J}_n^{(f)}$ has to be stocked, in addition to the present and past
 203 values of the applied magnetic field $\mathbf{H}\mathbf{a}_n$.

204 To wrap up, it is expected that the space complexity of the incremental algorithm is
 205 substantially the same as the memoryless algorithm $O(M(1+T))$, while the time complexity
 206 writes $O(MN(1+f_u T))$, where f_u is the fraction of hysterons which undergo the transition
 207 frozen \rightarrow unfrozen at each time step. If $f_u \ll f$, then the time complexity of the incremental
 208 algorithm should be intermediate between the basic and the memoryless algorithm. In this
 209 case, the incremental algorithm is the best candidate to replace the basic algorithm in FEA.
 210 Conversely, if at any time step the fraction of frozen hysterons f is low with respect of the
 211 number of hysterons which get unfrozen ($f_u > f$), the memoryless or even the pooled
 212 algorithms could have better performances.

213 To summarize, the time and space complexity of the four algorithms introduced beforehand
 214 are summarized in table 1. One observes that the complexity depends upon some parameters
 215 (namely, the number of time steps T , the fraction of frozen hysterons f and the fraction of
 216 hysterons which got unfrozen f_u) which are impossible to predict with no *a priori*
 217 information, and which depends on the simulated signals and on the set of hysterons which
 218 model magnetic materials.

219 It is important to observe that space complexity of the basic and pooled algorithms scales
 220 with the number of hysterons N , whereas the one of memoryless and incremental algorithms
 221 scales with the number of time steps T . Hence, from the point of view of memory
 222 occupation, these algorithms are expected to behave in a very different way. In particular, the
 223 memoryless and incremental algorithms are foreseen to be more suitable for FEA, when a
 224 large number of elements have to be considered.

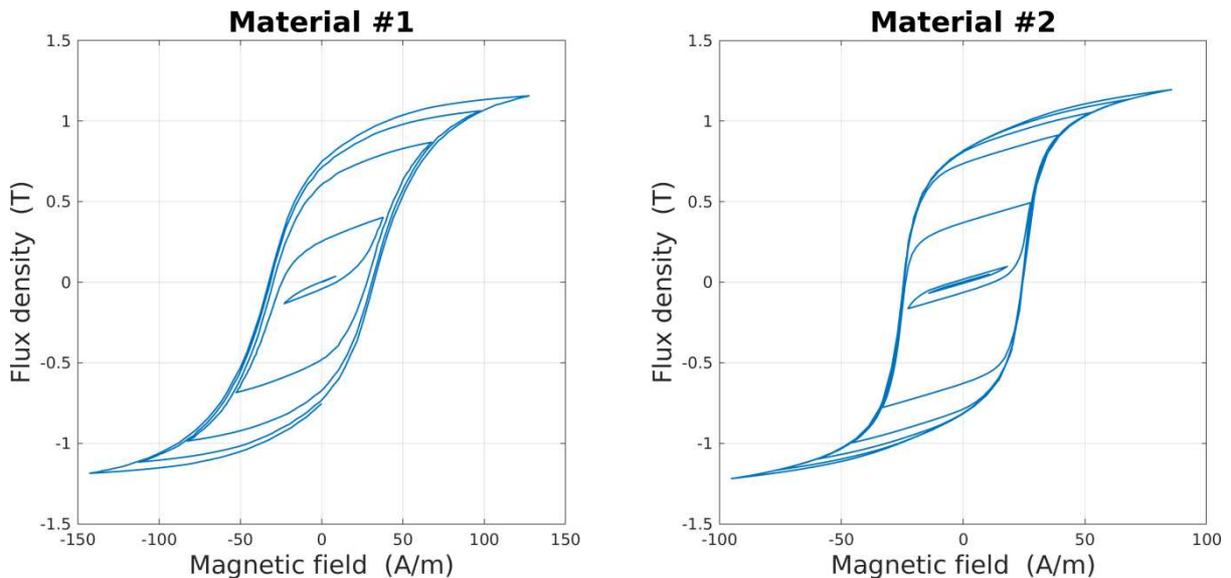
225 Table 1: Time and space complexity of algorithms.

Algorithm	Basic	Pooled	Memoryless	Incremental
Time complexity	$O(MN)$	$O(MN)$	$O(MN(1+fT))$	$O(MN(1+f_uT))$
Space complexity	$O(MN)$	$O(fMN)$	$O(MT)$	$O(M(1+T))$

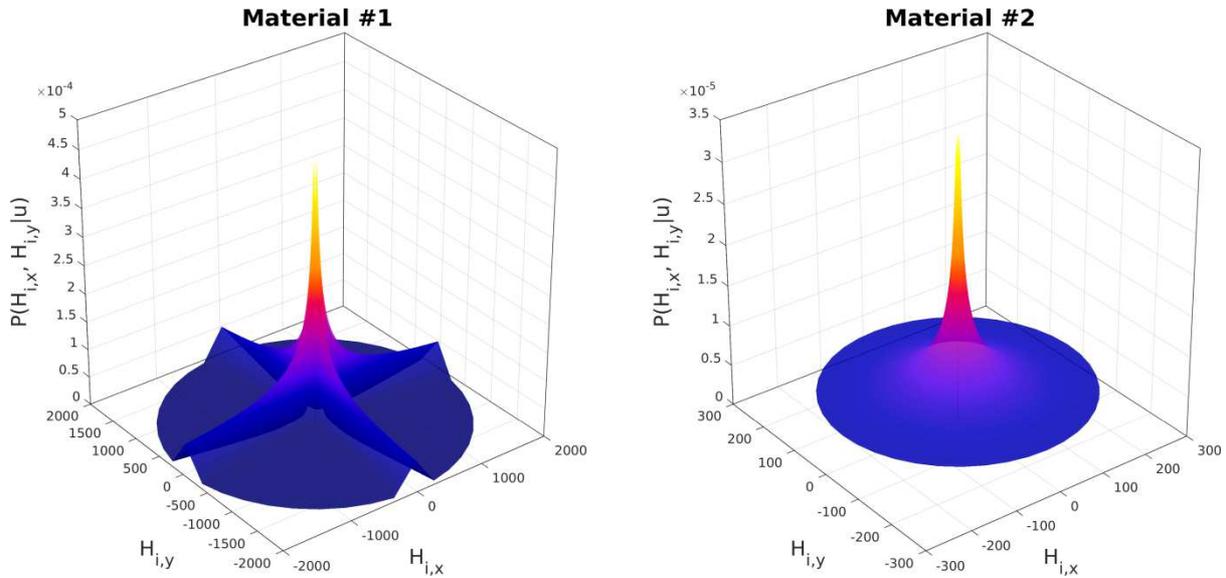
226 Magnetic materials used in simulations

227 The algorithms have been tested with two fictive isotropic magnetic materials (hereafter
 228 named material #1 and material #2, or simply #1 and #2). These materials are characterized
 229 by $N_1 = 134000$ and $N_2 = 150000$ hysterons respectively. The scalar magnetization loops for
 230 these two materials are depicted in Figure 2. Their magnetic behaviour is very similar to the
 231 electrical steel FeSi non oriented grain, with two different grade of differential magnetic
 232 permeability, lower for the #1 and higher for the #2. At present time, moving is not
 233 implemented in our algorithms; hence it is not possible to simulate realistic anisotropic
 234 materials. The distribution $P(\Omega)$ for these materials is depicted in Figure 3.

235



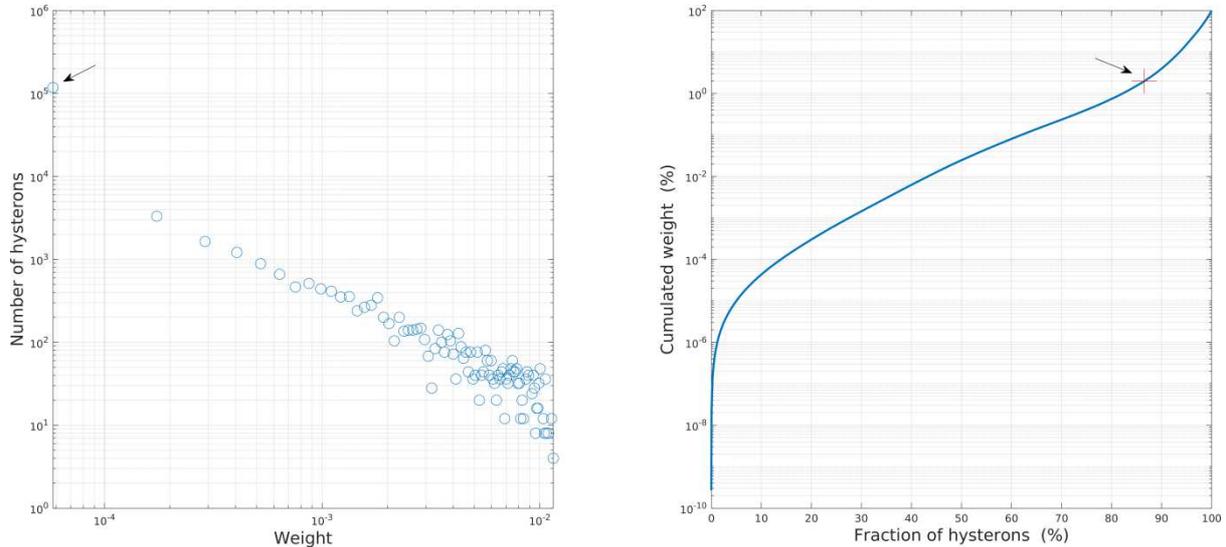
236 Figure 2. Scalar magnetizing loops for materials #1 and #2.
 237



238
239 Figure 3. Distribution $P(\Omega)$ for the materials #1 and #2 for a fixed value of the radius of hysterons u .

240 **Simplification of the set of hysterons**

241 The number of hysterons in vector Preisach models is generally very high (of the order of 10^3
242 – 10^5 hysterons). However, most of hysterons have a very small weight. In Figure 4 (left) it is
243 depicted the histogram of the weights of hysterons for the material #1 used in this work. It
244 can be observed that a huge number of hysterons have a very small weight, and hence their
245 contribution to the total magnetization is small. For instance, in Figure 4 (right) it is depicted
246 the cumulate distribution of the weights of hysterons with respect of the fraction of hysterons,
247 in the case of material #1. It can be observed that the 86 % of hysterons with the smallest
248 weights taken all together contribute to only 2 % of the overall weight.
249



250
251 Figure 4. Left: histogram of the weights of hysterons of material #1. Right: cumulated distribution of the
252 weights of hysterons. The arrow indicates the point corresponding to the threshold weight w^* for $p = 2\%$ (right),
253 and the class to which hysterons with weight $w < w^*$ belong (left).

254
255 In order to simplify the set of hysterons, let's define "threshold" weight w^* such that:

256
$$\sum_{w_i < w^*} w_i = p \quad (7)$$

257 where p is the fraction of hysterons which will be discarded. Let \mathbf{J}^* be the polarization
 258 computed by trunking the sum of Equation 4 so as to neglect hysterons, the weight of which
 259 is smaller than w^* :

260
$$\mathbf{J}^* = \alpha \sum_{w_i \geq w^*} \mathbf{Q}_i w_i \quad (8)$$

261 The maximal absolute error on the approximated magnetization \mathbf{J}^* will be at most αp ,
 262 which is much smaller than usual experimental error:

263
$$\|\mathbf{J} - \mathbf{J}^*\| = \alpha \left\| \sum_{w_i < w^*} \mathbf{Q}_i w_i \right\| \leq \alpha \sum_{w_i < w^*} \|\mathbf{Q}_i\| w_i = \alpha p \quad (9)$$

264 A similar bound can be devised for the relative error with respect of the maximum value of
 265 the polarization:

266
$$e_{max} = \frac{\max \|\mathbf{J} - \mathbf{J}^*\|}{\max \|\mathbf{J}\|} \leq \frac{\alpha \sum_{w_i < w^*} \|\mathbf{Q}_i\| w_i}{\alpha \sum \|\mathbf{Q}_i\| w_i} = p \quad (10)$$

267 This bound is sharp, and the equality holds for very high magnetic fields, where all unit
 268 polarizations \mathbf{Q}_i are practically aligned. Notice that it is impossible to devise a practical error
 269 bound for the relative error on the polarization itself (that is, for $\|\mathbf{J} - \mathbf{J}^*\| / \|\mathbf{J}\|$).

270 One observes from Equation 9 that by simply truncating the sum in Equation 8 so as to
 271 neglect hysterons with the lowest weight, the saturation polarization is reduced
 272 proportionally. This gives rise to the highest error when the material is closed to saturation,
 273 which is an undesirable behaviour (Figure 5). A possible mitigation strategy is to rescale the
 274 prefactor α in Equation 8 so as to conserve the maximum polarization, that is:

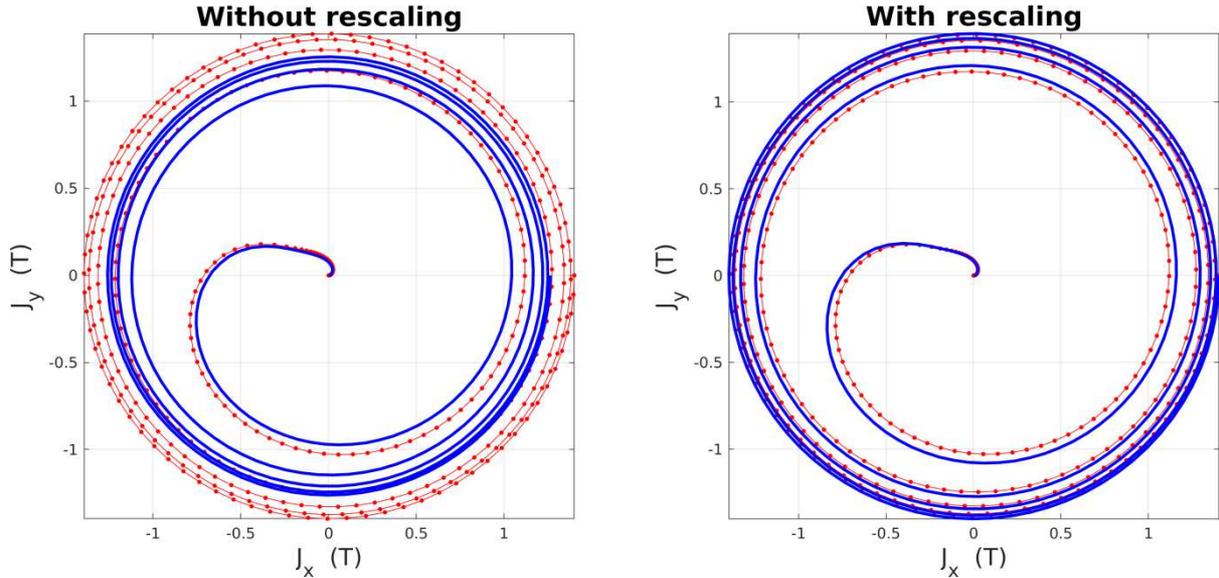
275
$$\alpha^* = \frac{\alpha}{\sum_{w_i \geq w^*} w_i} \quad ; \quad \mathbf{J}^* = \alpha^* \sum_{w_i \geq w^*} \mathbf{Q}_i w_i \quad (11)$$

276 It is observed that most of removed hysterons have a large radius. In the case of material #1
 277 the average radius of hysterons $\{u_i\}$ is lowered from 1319 A/m to 89.7 A/m for the simplified
 278 set (with $p = 2\%$). The case of material #2 is similar: under the same hypothesis, the average
 279 radius is lowered from 180.6 A/m to 28 A/m. When using a simplified set of hysterons it can
 280 be foreseen that the fractions f and f_u will be lower with respect of the original set. Notice
 281 that this contributes to reduce the computational complexity for the memoryless and

282 incremental algorithms (Table 1). Similar considerations apply also to the position of
 283 hysterons $\|\mathbf{H}_i\|$, as it can be easily understood by looking at Figure 3.

284 In the case of material #1, by removing the 86 % smallest hysterons, it is possible to reduce
 285 of 86 % the computational cost and the price to pay for this approximation is an error of at
 286 most 2 %. The outcome with material #2 is similar: by removing the 73 % smallest hysterons
 287 contribute to only 2 % of the overall weight. Of course the amount of the gain depends
 288 strongly on the material, and on how it has been identified. Nevertheless, this simple
 289 procedure provides an effective and general strategy to reduce the computational cost of
 290 Preisach model, whatever the implemented algorithm.

291



292
 293
 294
 295

Figure 5. Polarization simulated with a simplified model of material (material #2, $p = 10\%$) without rescaling (left, Equation 8) and with rescaling (right, Equation 11). Dotted line = reference, simulated with the original, not simplified model.

296 Generation of reduced models of the material

297 Even by using the simplification above-mentioned strategy, the number of hysterons remains
 298 very high. If it were possible to further reduce the number of hysterons which have to be
 299 handled at any time, the computational cost would be reduced proportionally. Hereafter is
 300 exposed a strategy based on the creation of reduced models of the material, which are valid
 301 while $\mathbf{H}\mathbf{a}$ lies in a certain neighbourhood of the \mathbf{H} -plane. This strategy achieves the objective
 302 (of reducing the computational cost) at the cost of a moderate approximation, and eventually
 303 a few additional computations which can be run off-line. The strategy will be exposed by
 304 making reference to the incremental algorithm, but in principle it could be applied also to
 305 other algorithms.

306 The idea is the following: let $\mathbf{B}(\mathbf{H}\mathbf{a}_0, \rho) = \{\mathbf{h} : \|\mathbf{h} - \mathbf{H}\mathbf{a}_0\| < \rho\}$ be a ball of radius ρ centred in
 307 a certain value of the applied magnetic field $\mathbf{H}\mathbf{a}_0$, and assume that $\mathbf{H}\mathbf{a} \in \mathbf{B}(\mathbf{H}\mathbf{a}_0, \rho)$. Under
 308 this condition, a reduced set of hysterons can be used in the computation, instead of the full
 309 set of hysterons. For each domain of validity $\mathbf{B}(\mathbf{H}\mathbf{a}_0, \rho)$, the becoming of each hysteron can
 310 be one of the following (Figure 6a):

- 311 i. hysterons which are “far enough” from the domain of validity are clustered together
 312 to form a tiny subset of N_c heavyweight hysterons,

- 313 ii. hysterons which may get frozen / unfrozen, or which are simply too close to the
 314 domain of validity are not modified,
 315 iii. hysterons which are frozen, and will remain frozen $\forall \mathbf{H}\mathbf{a} \in \mathbf{B}(\mathbf{H}\mathbf{a}_0, \rho)$ will be
 316 removed from the set of hysterons.

317 The procedure to create reduced models is controlled by two parameters, the signification of
 318 which will be clarified hereafter:

- 319 • N_C = number of clusters (= heavyweight hysterons),
- 320 • $h_{min} > \rho$ = threshold distance for clustering.

321 Let $r_i = \|\mathbf{H}_i - \mathbf{H}\mathbf{a}\|$ the distance between the centre of i^{th} hysteron and the applied magnetic
 322 field. All hysterons which fulfil the condition:

$$323 \quad u_i > r_i + \rho \quad (12)$$

324 are removed, because they are (and remain) frozen $\forall \mathbf{H}\mathbf{a} \in \mathbf{B}(\mathbf{H}\mathbf{a}_0, \rho)$, and hence they fall in
 325 the case 2 of the incremental algorithm. We recall that with the incremental algorithm,
 326 hysterons which are and remain frozen during a time step are accounted by the term $\mathbf{J}_n^{(f)}$,
 327 hence they give no contribution until they get defrozen. All hysterons which fulfil the
 328 following condition:

$$329 \quad r_i > u_i + h_{min} \quad (13)$$

330 are clustered together into N_C new heavyweight hysterons. The rationale is that “far”
 331 hysterons are necessarily unfrozen, and it is not necessary to take into account all of them
 332 individually. Instead, they are taken into account collectively into a N_C hysterons, basing on
 333 their unit polarization, computed with respect of the centre of the validity domain $\mathbf{H}\mathbf{a}_0$
 334 (Figure 6). In this work, clustering is performed by using the k-means algorithm [21], but
 335 other clustering algorithms could fit as well.

336 For each cluster C , the original hysterons are replaced by a single heavyweight hysteron. A
 337 desirable property of this new hysteron would be that it gives exactly the same contribution
 338 as the set of clustered hysterons when $\mathbf{H}\mathbf{a} = \mathbf{H}\mathbf{a}_0$, that is:

$$339 \quad \sum_{i \in C} w_i \mathbf{Q}_i = w_{HW} \mathbf{Q}_{HW} \Leftrightarrow \mathbf{Q}_{HW} = \frac{1}{w_{HW}} \sum_{i \in C} w_i \mathbf{Q}_i \quad (14)$$

340 where w_{HW} and \mathbf{Q}_{HW} are respectively the weight and the unit polarization of the
 341 heavyweight hysteron for $\mathbf{H}\mathbf{a} = \mathbf{H}\mathbf{a}_0$. The weight w_{HW} of the heavyweight hysteron is the
 342 sum of the weights of clustered hysterons:

$$343 \quad w_{HW} = \sum_{i \in C} w_i \quad (15)$$

344 It is observed that Equation 14 is verified with excellent approximation if the centre \mathbf{H}_{HW} of
 345 the heavyweight hysteron is set to the barycentre of the original hysterons. In fact, Equation
 346 14 writes:

347

$$\mathbf{Q}_{HW} = \frac{\mathbf{Ha}_0 - \mathbf{H}_{HW}}{\|\mathbf{Ha}_0 - \mathbf{H}_{HW}\|} = \frac{\sum_{i \in C} w_i \frac{\mathbf{Ha}_0 - \mathbf{H}_i}{\|\mathbf{Ha}_0 - \mathbf{H}_i\|}}{\sum_{i \in C} w_i} \quad (16)$$

348 For the sake of simplicity, and without loss of generality, assume that $\mathbf{Ha}_0 = 0$; then
 349 Equation 8 writes:

350

$$\mathbf{H}_{HW} = \frac{\sum_{i \in C} w_i \mathbf{H}_i \frac{\|\mathbf{H}_{HW}\|}{\|\mathbf{H}_i\|}}{\sum_{i \in C} w_i} \square \frac{\sum_{i \in C} w_i \mathbf{H}_i}{\sum_{i \in C} w_i} \quad (17)$$

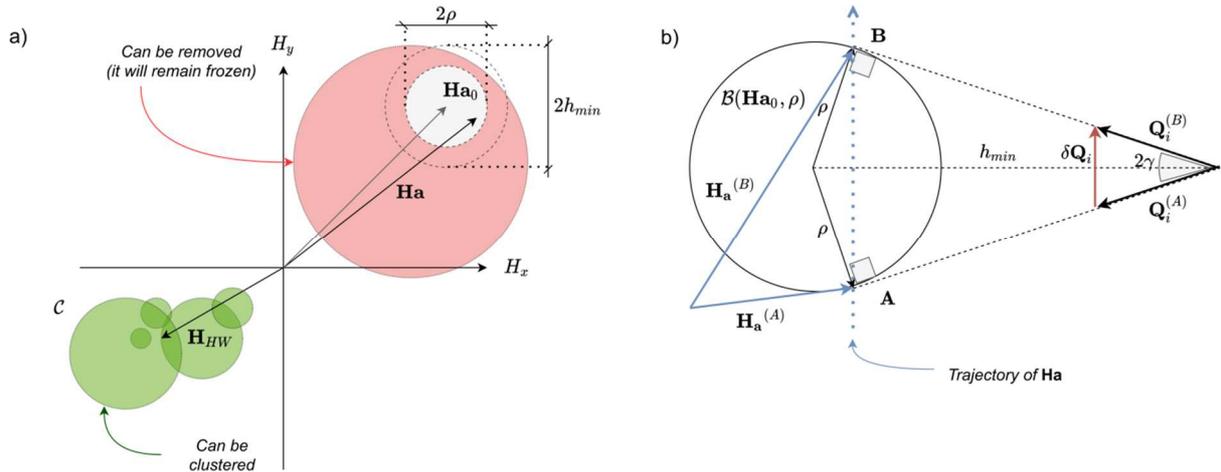
351 Finally, the radius of the new hysteron plays no role because clustered hysterons are unfrozen
 352 by construction.

353 The threshold distance $h_{min} > \rho$ controls the approximation error, by avoiding to cluster
 354 unfrozen hysterons which are “too close” to the validity domain. The rationale is that when
 355 the applied field \mathbf{Ha} lies in the validity domain, the variation of the unit polarization \mathbf{Q}_i of
 356 any hysteron is bounded by (Figure 6b):

357

$$\|\delta \mathbf{Q}_i\| < 2 \sin \gamma = \frac{2\rho}{h_{min}} \quad (18)$$

358 In practice one would like to cluster as many hysterons as possible (that is, to select a value
 359 of h_{min} close to ρ), but on the other hand small values of h_{min} could compromise the
 360 accuracy of the approximation. The other parameter which has an influence on the accuracy
 361 is of course the number of clusters N_C : the bigger N_C , the higher the accuracy.



362

363 Figure 6. a) Sketch of the reduction algorithm. The red hysteron, which is and will remain frozen for any \mathbf{Ha}
 364 within the validity domain, is removed (case iii). Green hysterons, which have similar unit magnetization, are
 365 clustered into a new heavyweight hysteron (case i). The remaining hysterons are not modified (case ii). b)
 366 Maximum variation of $\delta \mathbf{Q}_i$ when the applied magnetic field \mathbf{Ha} lies inside the validity domain, which happens
 367 when trajectory of the applied magnetic field moves from $\mathbf{H}_a^{(A)}$ to $\mathbf{H}_a^{(B)}$.

368 **Implementation**

369 All those algorithms have been implemented in Matlab/Octave language¹. Needless to say,
370 this language can eventually provide high performances, provided that computations are
371 vectorised. Vectorization is easy in the case of the basic algorithm, whereas it appears to be
372 quite difficult – if not impossible – for other algorithms, in particular for the incremental
373 algorithm. For this unfortunate reason, precautions are mandatory when measuring time
374 complexity. In this work, time complexity has been evaluated by using the two following
375 quantitative indicators:

- 376 • The number of computations of unit magnetization (Equations 1 and 2) is tracked, in
377 order to get an estimate of the number of elementary operations executed by each
378 algorithm,
- 379 • In computer programs used to measure execution time, vectorization has been
380 purposely avoided, so as to achieve a fair comparison between algorithms. This
381 precaution is key: for instance, in one of our tests the execution times for the basic
382 algorithm in the case of vectorized and not vectorized codes are respectively 0.3 and
383 4.8 seconds, whereas the number of elementary operations is strictly identical.

384 Nevertheless, in our opinion the most reliable indicator of time complexity is from far the
385 number of execution of elementary operations. Hereafter, execution times are provided only
386 for the sake of completeness, but data is interpreted basing on the number of elementary
387 operations only.

388 Several strategies can be devised to handle model reduction. A whole set of reduced models
389 can be generated off-line (once for each material), or on-the-fly during simulations. For a
390 given pool of reduced models, the search of the most appropriate model for each
391 computational point can be done in several ways (linear search, oct-tree search, etc.). By the
392 way, some points belong to more than a single validity domain, hence several reduced
393 models could be used. This could help to mitigate the discontinuity which happens when a
394 point “jumps” from a validity domain to another. In this work we have implemented a simple
395 strategy, based on a dynamical set of reduced models which is populated during the
396 simulation. For each computational point, a single reduced model is employed, and it is
397 selected by linear search.

398 Similarly, the strategy to handle the “history” of the material can be implemented in several
399 ways, and it is prone to optimization. Notice that, in view of utilisation of Preisach model in
400 FEA, one is obliged to compute and store somewhere the past values of the magnetic field.
401 However, it is not guaranteed that Finite Element software interfaces allow to look back for
402 the past values of the magnetic field during the resolution; and whenever this could be
403 possible, this is likely to come with an additional computational cost. In other words: in
404 principle computing and storing the past values of the magnetic field during FEA does not
405 increase the overall computational cost, but in practice retrieving this information by using
406 existing Finite Element software could be costly, or complicate (if not impossible).
407 In this work, the magnetic field at all the time steps, and for all computational points, is
408 stocked within the software component which models the material, so as to avoid the
409 complexity of interfacing with Finite Element software. The price to pay is of course a
410 duplication of data, because the past magnetic field in magnetic materials has to be stocked

¹ More precisely, programs have been developed by using Matlab 2020b on a standard laptop running Linux.

411 twice (once in the component which models magnetic materials, and the other on the FEA
 412 side). The search back in time for the unit magnetization of frozen hysterons is performed by
 413 linear search.

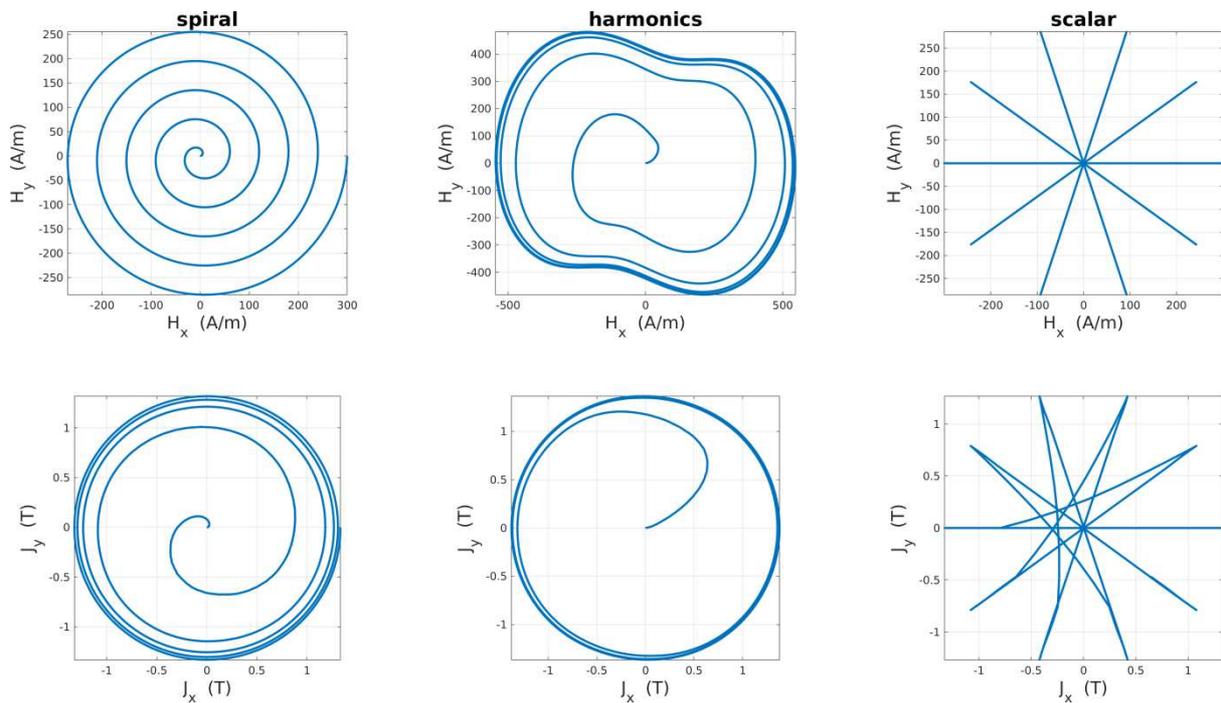
414 Results and Discussion

415 Simulated signals

416 We performed several tests by using as excitation magnetic field \mathbf{H}_a the following signals:

- 417 1. Spiral: the magnitude of the magnetic field increases linearly, while its orientation
 418 rotates at a constant rate. This highlights the vector behaviour of the material,
- 419 2. Periodic signal with 3rd harmonic: this signal is representative of the kind of signals
 420 which can be encountered in some real electrical appliances, notably in the T-joints of
 421 transformers,
- 422 3. Pseudo-scalar signal: this signal excites the material in several directions, but in each
 423 direction the excitation is delivered in a “scalar fashion”, that is with no modification
 424 of the direction of the field.

425 Hereafter these signals will be denoted “spiral”, “harmonics” and “scalar” respectively. For
 426 each signal, the magnetic field \mathbf{H}_a and the computed polarization \mathbf{J} (all algorithms provide
 427 exactly the same result) are depicted in Figure 7 for the material #1. Each signal is discretized
 428 in $T = 400$ time steps.



429

430 Figure 7. Excitation magnetic field \mathbf{H}_a and the corresponding polarization \mathbf{J} simulated with material #1 for
 431 the signals “spiral”, “harmonics” and “scalar”.

432 Comparison of algorithms

433 The time and space complexity of the four algorithms introduced have been measured
 434 “experimentally” by simulating the excitation of an isotropic magnetic material by the three

435 signals depicted in Figure 7. Simulations have been performed on a single computational
 436 point by using materials #1 and #2. For all simulations, the number T_{OP} of elementary
 437 operation, the execution time T_{CPU} and the required memory T_{MEM} have been tracked and
 438 reported in Table 2. We stress that the reported computational times are not representative of
 439 a “true” implementation, and are provided for completeness only. In these simulations the
 440 materials are simulated “as such”, that is neither simplification nor reduction strategies are
 441 employed.

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449 Table 2: Statistics of the simulations with several excitation signals ($M = 10^6$ elementary operations). No
 450 simplification / reduction are used. Symbols #1 and #2 indicate the material used in simulations.

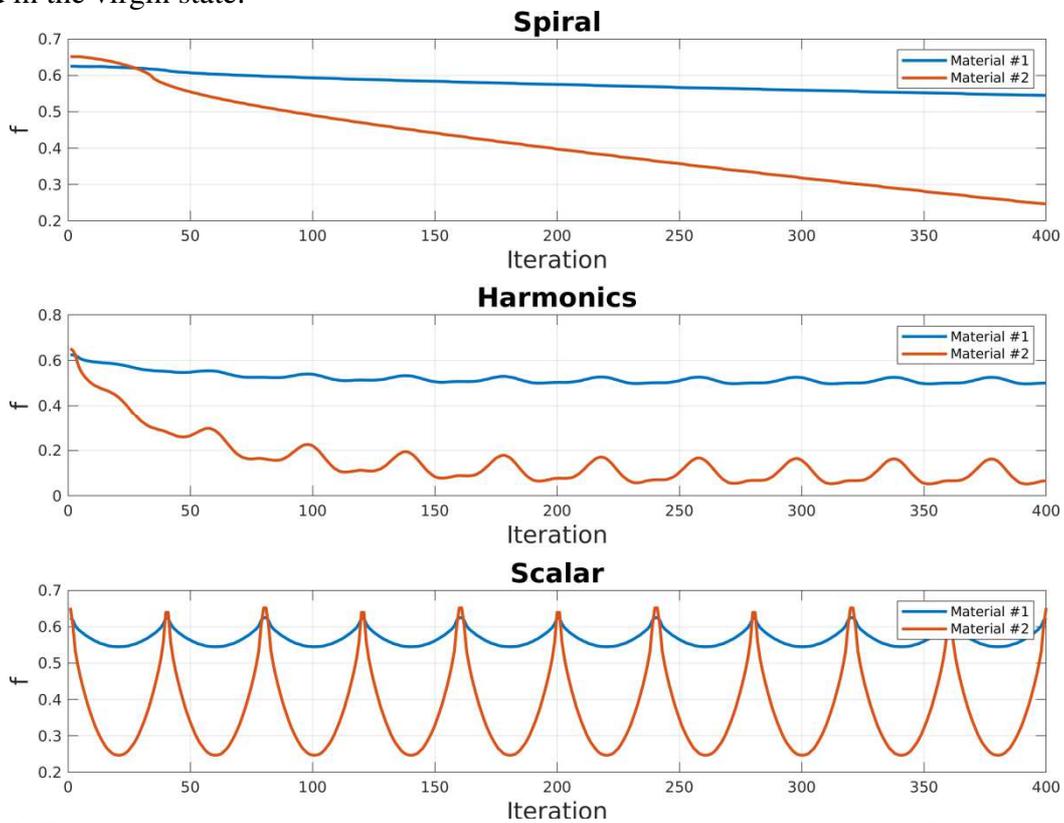
Algorithm		Basic	Pooled	Memoryless	Incremental	
“spiral”						
Time complexity	T_{op}	#1	53.7 M	53.7 M	5841 M	111 M
		#2	60.2 M	60.2 M	3701 M	133 M
	T_{cpu}	#1	84.7 sec	1931 sec	522 sec	90 sec
		#2	66.0 sec	1345 sec	358 sec	260 sec
Space complexity	T_{mem}	#1	2.27 Mb	1.75 Mb	1.61 kb	2.3 kb
		#2	2.55 Mb	0.89 Mb	1.77 kb	2.5 kb
“harmonic”						
Time complexity	T_{op}	#1	53.7 M	53.7 M	4949 M	112 M
		#2	60.2 M	60.2 M	640 M	128 M
	T_{cpu}	#1	77.0 sec	1561 sec	464 sec	98 sec
		#2	29.7 sec	337 sec	144 sec	207 sec
Space complexity	T_{mem}	#1	2.27 Mb	1.61 Mb	1.61 kb	2.3 kb
		#2	2.55 Mb	0.24 Mb	1.77 kb	2.5 kb
“scalar”						
Time complexity	T_{op}	#1	53.7 M	53.7 M	5580 M	111 M
		#2	60.2 M	60.2 M	2586 M	130 M
	T_{cpu}	#1	83 sec	1852 sec	508 sec	85 sec
		#2	58 sec	1120 sec	285 sec	227 sec
Space complexity	T_{mem}	#1	2.27 Mb	2.01 Mb	1.61 kb	2.3 kb
		#2	2.55 Mb	2.34 Mb	1.77 kb	2.5 kb

451 Space complexity: the difference in memory occupation between the first two algorithms
 452 (basic and pooled) and the last two ones is striking (memoryless and incremental). The

453 difference is of the same order of magnitude of the ratio between the number of hysterons
 454 and the number of time steps. For instance, for material #1 one has:

455 Material #1 :
$$\frac{N}{T} = \frac{134000}{400} = 335 \sim 987 = \frac{2.27 \text{ Mb}}{2.3 \text{ kb}} \quad (19)$$

456 Conversely to the other algorithms, the memory occupation of the pooled algorithm is not a
 457 constant value, because it depends on the fraction f of frozen hysterons, which varies with
 458 the particular time step, the signal and of course the material (Figure 8). The occupied
 459 memory T_{mem} reported in Table 2 have been tracked at the last time step, but they are well
 460 representative of the general trend, apart for the signal “scalar”. With the material #1 the
 461 average fraction f of frozen hysterons is of about 50- to 60- % at any iteration, and for all
 462 signals. Conversely, in the case of material #2 the fraction f of frozen hysterons spans in a
 463 much broader range [0.05 ; 0.65]. It is observed that higher excitation field values are
 464 correlated with smaller fraction of frozen hysterons, whereas the higher value is generally
 465 found in the virgin state.



466 Figure 8. Fraction of frozen hysterons as a function of the iteration for the signals “spiral”, “harmonics” and
 467 “scalar”.
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469
 470 **Time complexity:** the number of elementary operations for the basic and the pooled methods
 471 is identical. Even by taking into account the additional burden of handling the pool of frozen
 472 hysterons, these two algorithms have fundamentally the same time complexity. The huge
 473 difference in the measured computational time reported in Table 2 is only a matter of
 474 implementation: indeed when both algorithms are vectorised, they exhibit similar
 475 computational times². The much larger number of elementary operations required by the

² More precisely, the maximum measured execution time for the vectorized versions of basic and pooled algorithms are respectively 2.4 sec and 3.2 sec for material #1.

476 memoryless algorithm is representative of the fraction of frozen hysterons (Figure 8). Frozen
 477 hysterons are very expensive in terms of computational time, because the memoryless needs
 478 to go back in time to retrieve the time step when they got frozen. Conversely, the incremental
 479 algorithm is competitive, in that its computational time is of the same order of magnitude of
 480 the basic algorithm. This difference is due to the fact that, even if about 60 % of hysterons are
 481 frozen at any time step, only a small fraction of them undergo the transition frozen \rightarrow
 482 unfrozen. The huge difference in the number of operations T_{op} between the memoryless and
 483 incremental algorithms confirm the hypothesis that in general $f_u \ll f$. However, it must be
 484 pointed out that the material and the signal have both a strong influence; for instance, it is
 485 observed that the incremental algorithm over the memoryless one is more pronounced for
 486 material #1 than for material #2.

487 To summarize, the theoretical predictions on the time and space complexity are supported by
 488 experimental data. The pooled algorithm has a moderate advantage over the basic algorithm
 489 in terms of space complexity, and the time complexity is practically the same for these two
 490 algorithms. Memoryless and incremental algorithms have both a huge advantage from the
 491 standpoint of space complexity, but from the standpoint of time complexity the incremental
 492 algorithm seems to be more competitive than the memoryless algorithm.

493 **Influence of the simplification of the set of hysterons**

494 In order to analyse the effect of simplification of the set of hysterons, we repeated the
 495 simulations by truncating the set of hysterons so as to discard respectively 1 %, 2 %, 5 % and
 496 10% of the original set of hysterons. The results are analysed in terms of accuracy and from
 497 the standpoint of time and space complexity.

498 As for the accuracy, two indicators e_{max} and e_{rel} have been evaluated:

$$499 \quad e_{max} = \frac{\max \|\mathbf{J}^* - \mathbf{J}\|}{\max \|\mathbf{J}\|} \times 100 \% = \frac{\max \|\mathbf{M}^* - \mathbf{M}\|}{\max \|\mathbf{M}\|} \times 100 \% \quad (20)$$

$$500 \quad e_{rel} = \max_{\|\mathbf{M}\| \geq 1 \text{ A/m}} \left(\frac{\|\mathbf{M}^* - \mathbf{M}\|}{\|\mathbf{M}\|} \right) \times 100 \% \quad (21)$$

501 where \mathbf{J} is the polarization computed with the original set of hysterons, whereas \mathbf{J}^* is
 502 computed by using a simplified set of hysterons (similar definitions hold for \mathbf{M} and \mathbf{M}^*).
 503 Both of all are relative errors: e_{max} is normalized with respect of the maximum value of the
 504 magnetization, whereas e_{rel} is a “pure” relative error where small values have been discarded
 505 so as to get rid of numerical noise (the threshold value $\|\mathbf{M}\| \geq 1 \text{ A/m}$ has been chosen
 506 arbitrarily). The results obtained with rescaling (Equation 11) and without rescaling
 507 (Equation 8) are summarized in Table 3 (see also Figure 5).

508
 509 Accuracy: it can be observed that the theoretical prediction of the error bound for e_{max} is
 510 verified (Equation 10). In particular, Equation 10 holds sharply as long as no rescaling is
 511 applied, while an improved accuracy is observed when the rescaling is used; the higher the
 512 value of p , the higher the improvement in the accuracy. However, one must keep in mind that
 513 the purpose of rescaling is to conserve the maximum polarization of the material: hence an
 514 eventual improvement obtained by using rescaling depends on the material and on the

515 excitation signal. If the material is going to be saturated, an improved accuracy for high fields
516 can be foreseen. Otherwise, the effectiveness of the rescaling cannot be predicted; in this
517 case, rescaling may even degrade the accuracy. As anticipated beforehand, it is not possible
518 to provide a theoretical bound for the relative error e_{rel} , which is by intrinsically higher than
519 e_{max} .

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Table 3: Number of remaining hysterons and error obtained when using simplified sets of hysterons with rescaling (without rescaling). Symbols #1 and #2 indicate the material used for the corresponding line of the table.

p		1 %	2 %	5 %	10 %
Nb. of hysterons	#1	23888	18033	12129	8614
	#2	50205	41044	30996	24266
“spiral”					
e_{max}	#1	0.83 % (0.83 %)	1.5 % (1.5 %)	3.5 % (3.5 %)	6.0 % (6.0 %)
	#2	0.57 % (0.93 %)	1.0 % (1.9 %)	2.2 % (4.9 %)	4.0 % (9.7 %)
e_{rel}	#1	0.94 % (0.95 %)	1.8 % (1.8 %)	4.2 % (4.3 %)	7.4 % (7.6 %)
	#2	0.77 % (1.45 %)	1.4 % (2.9 %)	3.2 % (7.1 %)	6.2 % (13 %)
“harmonic”					
e_{max}	#1	0.83 % (0.80 %)	1.5 % (1.5 %)	3.3 % (3.2 %)	5.6 % (5.6 %)
	#2	0.56 % (1.00 %)	0.99 % (2.0 %)	2.1 % (5.0 %)	3.8 % (9.9 %)
e_{rel}	#1	0.94 % (0.95 %)	1.8 % (1.7 %)	4.2 % (4.4 %)	7.4 % (7.5 %)
	#2	0.77 % (1.5 %)	1.4 % (2.7 %)	3.2 % (7.1 %)	6.2 % (13 %)
“scalar”					
e_{max}	#1	0.85 % (0.21 %)	1.6 % (0.67 %)	4.1 % (2.9 %)	7.8 % (7.6 %)
	#2	0.90 % (0.92 %)	1.5 % (1.9 %)	3.2 % (4.8 %)	5.5 % (9.7 %)
e_{rel}	#1	1.2 % (0.23 %)	1.9 % (1.1 %)	5.0 % (8.0 %)	9.6 % (22 %)
	#2	1.7 % (2.2 %)	2.6 % (3.7 %)	5.2 % (6.4 %)	9.2 % (10 %)

531
532 Computational cost: as illustrated in Figure 9, the number of hysterons, and hence the
533 computational cost, can be greatly reduced by neglecting even a small fraction p of the total
534 weight. The total number T_{OP} of elementary operations, and the occupied memory T_{MEM} (at
535 the last iteration only) for the different signals and algorithms are reported in Table 4
536 (material #1) and Table 5 (material #2).

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Table 4: Computational cost when using simplified sets of hysterons (material #1). T_{op} = number of operations ($M = 10^6$ elementary operations), T_{mem} = occupied memory.

Algorithm	p	Basic	Pooled	Memoryless	Incremental
“spiral”					
T_{op}	0 %	53.7 M	53.7 M	5841 M	111 M
	1 %	9.6 M	9.6 M	375 M	20.6 M
	2 %	7.2 M	7.2 M	184 M	15.6 M
	5 %	4.9 M	4.9 M	49 M	10 M
	10 %	3.5 M	3.5 M	19 M	7 M
T_{mem}	0 %	2.27 Mb	1.75 Mb	1.61 kb	2.3 kb
	1 %	408 kb	85 kb	1.61 kb	2.3 kb
	2 %	308 kb	32 kb	1.61 kb	2.3 kb
	5 %	208 kb	3.7 kb	1.61 kb	2.3 kb
	10 %	148 kb	3.0 kb	1.61 kb	2.3 kb
“harmonic”					
T_{op}	0 %	53.7 M	53.7 M	4949 M	112 M
	1 %	9.6 M	9.6 M	102 M	20 M
	2 %	7.2 M	7.2 M	14 M	15 M
	5 %	4.9 M	4.9 M	5.6 M	10 M
	10 %	3.5 M	3.5 M	3.8 M	7.0 M
T_{mem}	0 %	2.27 Mb	1.61 Mb	1.61 kb	2.3 kb
	1 %	408 kb	32 kb	1.61 kb	2.3 kb
	2 %	308 kb	3.6 kb	1.61 kb	2.3 kb
	5 %	208 kb	2.5 kb	1.61 kb	2.3 kb
	10 %	148 kb	2.3 kb	1.61 kb	2.3 kb
“scalar”					
T_{op}	0 %	53.7 M	53.7 M	5580 Mb	111 M
	1 %	9.6 M	9.6 M	254 M	34 M
	2 %	7.2 M	7.2 M	89 M	26 M
	5 %	4.9 M	4.9 M	7.5 M	10 M
	10 %	3.5 M	3.5 M	4.4 M	7.1 M
T_{mem}	0 %	2.27 Mb	2.01 Mb	1.61 kb	2.3 kb
	1 %	408 kb	289 kb	1.61 kb	2.3 kb
	2 %	308 kb	209 kb	1.61 kb	2.3 kb
	5 %	208 kb	141 kb	1.61 kb	2.3 kb

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	10 %	148 kb	103 kb	1.61 kb	2.3 kb
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Table 5: Computational cost when using simplified sets of hysterons (material #2). T_{op} = number of operations ($M = 10^6$ elementary operations), T_{mem} = occupied memory.

Algorithm	p	Basic	Pooled “spiral”	Memoryless	Incremental
T_{op}	0 %	60.1 M	60.2 M	3701 M	133 M
	1 %	20.1 M	20.1 M	48.8 M	41.4 M
	2 %	16.5 M	16.5 M	32.4 M	33.7 M
	5 %	12.4 M	12.4 M	22.0 M	25.4 M
	10 %	9.7 M	9.7 M	16.7 M	19.9 M
T_{mem}	0 %	2.55 Mb	0.89 Mb	1.77 kb	2.5 kb
	1 %	855 kb	85.0 kb	1.77 kb	2.5 kb
	2 %	700 kb	2.3 kb	1.77 kb	2.5 kb
	5 %	529 kb	2.3 kb	1.77 kb	2.5 kb
	10 %	415 kb	2.3 kb	1.77 kb	2.5 kb
“harmonic”					
T_{op}	0 %	60.1 M	60.2 M	640 M	128 M
	1 %	20.1 M	20.1 M	20.7 M	40.4 M
	2 %	16.5 M	16.5 M	16.7 M	33.0 M
	5 %	12.4 M	12.4 M	12.6 M	24.9 M
	10 %	9.7 M	9.7 M	9.8 M	15.7 M
T_{mem}	0 %	2.55 Mb	0.24 Mb	1.77 kb	2.5 kb
	1 %	855 kb	2.3 kb	1.77 kb	2.5 kb
	2 %	700 kb	2.3 kb	1.77 kb	2.5 kb
	5 %	529 kb	2.3 kb	1.77 kb	2.5 kb
	10 %	415 kb	2.3 kb	1.77 kb	2.5 kb
“scalar”					
T_{op}	0 %	60.1 M	60.2 M	2586 M	130 M
	1 %	20.1 M	20.1 M	22.1 M	41.1 M
	2 %	16.5 M	16.5 M	17.7 M	33.5 M
	5 %	12.4 M	12.4 M	13.2 M	25.3 M
	10 %	9.7 M	9.7 M	10.3 M	19.8 M
T_{mem}	0 %	2.55 Mb	2.34 Mb	1.77 kb	2.5 kb
	1 %	855 kb	500 kb	1.77 kb	2.5 kb
	2 %	700 kb	395 kb	1.77 kb	2.5 kb
	5 %	529 kb	299 kb	1.77 kb	2.5 kb
	10 %	415 kb	241 kb	1.77 kb	2.5 kb

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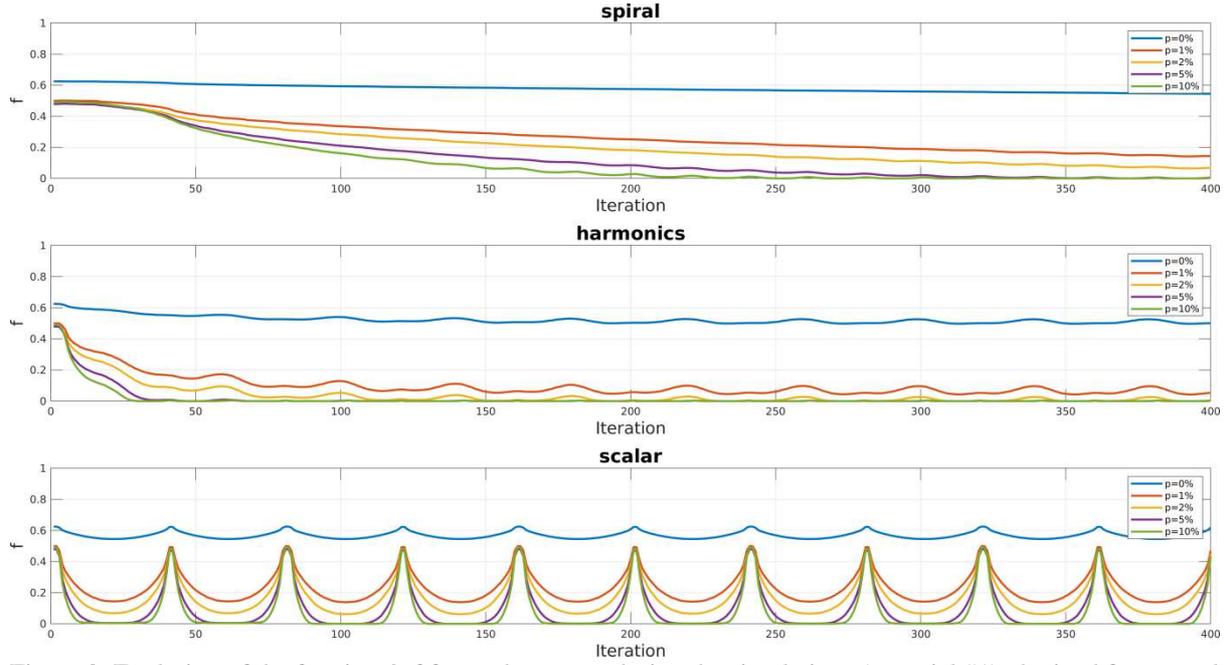
The required memory as well as the number of operations in the basic algorithm scales linearly with the number of hysterons, hence the reduction of hysterons is of course beneficial from both points of view. All algorithms take benefit from the simplification of the set of hysterons, but the improvement is particularly remarkable for the memoryless algorithm, for which a decrease of the computational time of nearly two orders of magnitude is observed. This is due to the fact that the simplification procedure seems to remove mostly hysterons which are frozen for the most of time. Thus the fraction f of frozen hysterons is strongly decrease (Figure 9), and hence the time complexity of the memoryless and pooled algorithms is decreased as well (Table 1).

The benefit is less pronounced for the incremental algorithm, but still it is clearly visible. Surprisingly, for $p=0.05$ and $p=0.1$ the incremental algorithm generally has a computational cost higher than the memoryless algorithm. This can be explained by the fact that the incremental algorithm needs to evaluate the unit magnetization at the current time step, but also at the previous time step³. Hence, when the fraction of frozen hysteron f is low enough so that it becomes comparable to f_u , the memoryless algorithm becomes even more competitive than the incremental one. From the standpoint of the occupied memory, the memoryless and incremental algorithms take no benefit from the simplification of the set of hysterons; nevertheless, these algorithms still outperform both the basic and the pooled algorithms.

Notice that this effect depends strongly on the type of signals, and it is unsurprisingly more pronounced in the case of the signal “harmonics”, which comes quickly to a steady state where most of the hysterons are unfrozen. Conversely, in the case of the signal “scalar” where the applied magnetic field vanishes several times, this effect is somehow reduced, but still it is clearly visible.

From the standpoint of FEA, this effect could make the pooled and memoryless algorithms competitive with respect of the basic algorithm. However, it has to be noticed that the virgin state is problematic because of the high fraction of frozen hysterons.

³ This is necessary to determine whether hysterons undergo a transition frozen/unfrozen. Unfortunately the unit polarization at the previous step cannot be stored in memory, otherwise all benefits of the algorithm from the standpoint of memory occupation would be wasted.



603 Figure 9. Evolution of the fraction f of frozen hysterons during the simulations (material #1) obtained for several
 604 values of p . Even small values of p reduce significantly the fraction f of frozen hysterons.
 605

606 Incremental algorithm with reduced models

607 Reduced models have been devised to be employed mainly with the incremental algorithm.
 608 At each step of the algorithm, and for each computational point, a reduced model is searched
 609 in a pool which is enriched dynamically – that is, if no suitable reduced model is found in the
 610 pool, a new model is created on-the-fly and added to the pool.

611 The parameters which control the creation of new reduced models are the size ρ of the
 612 validity domain, the threshold distance h_{min} and the number of clusters N_C . The size ρ of the
 613 domain of validity $B(\mathbf{H}\mathbf{a}_0, \rho)$ of reduced models depends on the applied magnetic field in
 614 the current $\mathbf{H}\mathbf{a}_n$ and the second-to-last one $\mathbf{H}\mathbf{a}_{n-1}$, and on the magnitude of the applied field:

$$615 \quad \rho = \max\left\{\rho_{min}, \frac{2\pi\|\mathbf{H}\mathbf{a}\|}{N_\rho}, \|\mathbf{H}\mathbf{a}_n - \mathbf{H}\mathbf{a}_{n-1}\|\right\} \quad (22)$$

616 A minimal size $\rho_{min} = 20 \text{ A/m}$ is imposed in order to avoid the unnecessary creation of
 617 reduced models with tiny validity domain. The parameters N_ρ represents the minimal
 618 number of reduced models used to cover closed cycle of the applied magnetic field. The
 619 validity domain of new reduced models is centred in $\mathbf{H}\mathbf{a}_n$, so Equation 22 guarantees that
 620 $\mathbf{H}\mathbf{a}_n$ and $\mathbf{H}\mathbf{a}_{n-1}$ belong to the validity domain. The threshold distance has been chosen
 621 proportionally to $\mathbf{H}\mathbf{a}_n$:

$$622 \quad h_{min} = \chi\|\mathbf{H}\mathbf{a}\| \quad (23)$$

623 For each set of parameters (N_ρ , χ , N_C), we compute the errors e_{max} and e_{rel} defined in
 624 Equations 20 and 21 respectively. Also, it is interesting to track the total number of reduced

625 models and the occupied memory for the pool of reduced models. Some statistics are
 626 presented in Table 6 (material #1) and Table 7 (material #2) for the reference set of
 627 parameters: $N_p = 40$, $\chi = 1.2$, $N_c = 32$. Simulations have been run by using a simplified set
 628 of hysterons with $p = 2\%$. The total number of elementary operations T_{OP} , and the “net”
 629 number which does not include the creation of reduced models (RM) are provided. Notice
 630 that in principle reduced models could be generated off-line once for all.

631
 632 Table 6: Statistics of the simulation by using the incremental algorithm with reduced models (material #1).
 633 Simulation parameters are: $N_p = 40$, $\chi = 1.2$, $N_c = 32$ and $p = 2\%$ ($M = 10^6$ elementary operations).

	“spiral”	“harmonics”	“scalar”
e_{max}	2.55 %	1.00 %	0.77 %
e_{rel}	2.60 %	1.04 %	0.57 %
T_{op}	4.50 M	2.36 M	3.84 M
T_{op} (generation of RM excluded)	2.98 M	1.30 M	2.69 M
T_{op} incremental algorithm without RM	15.60 M	14.74 M	15.29 M
T_{op} basic algorithm	7.23 M	7.23 M	7.23 M
Nb reduced models	113	71	101
Average Nb of hysterons	2083	1392	2633
Occupied memory	262 kb	286 kb	181 kb

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Table 7: Statistics of the simulation by using the incremental algorithm with reduced models (material #2).
 Simulation parameters are: $N_p = 40$, $\chi = 1.2$, $N_c = 32$ and $p = 2\%$ ($M = 10^6$ elementary operations).

	“spiral”	“harmonics”	“scalar”
e_{max}	1.9 %	1.0 %	1.5 %
e_{rel}	9.5 %	8.5 %	3.9 %
T_{op}	8.56 M	3.4 M	7.0 M
T_{op} (generation of RM excluded)	4.26 M	0.78 M	3.65 M
T_{op} incremental algorithm without RM	33.70 M	33.0 M	33.5 M
T_{op} basic algorithm	16.46 M	16.46 M	16.46 M
Nb reduced models	113	71	101
Average Nb of hysterons	2545	967	6026
Occupied memory	656 kb	661 kb	356 kb

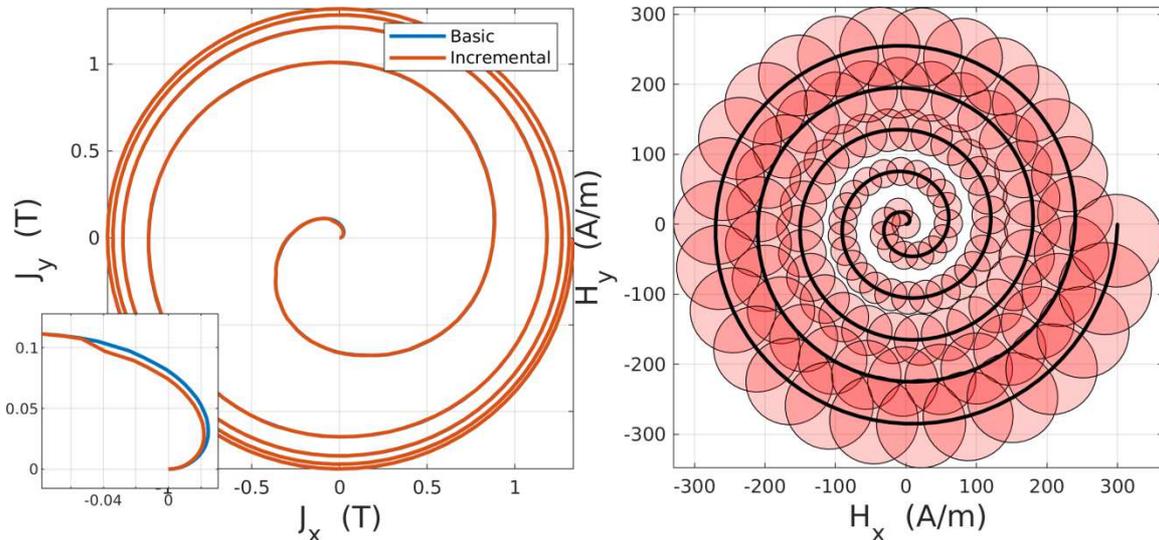
642
 643 In Figure 10 is depicted the magnetization \mathbf{M} for the “spiral” signal, computed with material
 644 #1 by using the basic algorithm (reference) and the incremental algorithm with reduced
 645 models. One observes that the error is quite acceptable, and it is mainly located close to the
 646 virgin state (see the zoom in the inset). For this signal, 113 reduced models have been
 647 created. Reduced models are composed of 2083 hysterons in average for the material #1 (to
 648 be compared with the 18033 hysterons of the simplified set, and with the 134000 of the
 649 original set). The number of hysterons depends on the domain of validity of reduced models:
 650 the highest number ($N = 4158$) is found close to the virgin state, whereas for
 651 $\|\mathbf{H}_a\| > 100$ A/m the number of hysterons for each reduced model is steadily under the
 652 average. A similar trend is observed for the material #2. However, it is worth observing that

653 the couple material/signal strongly determines the extent of the economy obtained by using
 654 reduced models.

655 Depending on the signal and on the material, the memory required to store the pool of
 656 reduced models is of about 500 kb (order of magnitude). The occupied memory may appear
 657 rather high, but we stress that the pool of reduced models is common to all computational
 658 points. In order to appreciate the practical consequences of memory occupation of the pool of
 659 reduced models, algorithms should be plugged into a FEA software, where a high number of
 660 points have to be computed at each time step, with a substantial scaling economy.

661 The number of elementary operations is reduced with respect of the incremental algorithm
 662 when reduced models are not used, and it is usually much lower with respect of the basic
 663 algorithm. The gain is even bigger if one takes out the computations required to create
 664 reduced models, which can eventually done off-line. However, one should keep in mind that
 665 the number of elementary operations is not fully representative of execution time, in that the
 666 algorithms which have to be implemented for the incremental algorithm and to handle the
 667 pool of reduced models are much more complex with respect of the basic algorithm.
 668 Nevertheless, data reported in Tables 6 and 7 suggest that the incremental algorithm with
 669 reduced models may indeed be competitive with the basic algorithm in terms of both space
 670 and even time complexity.

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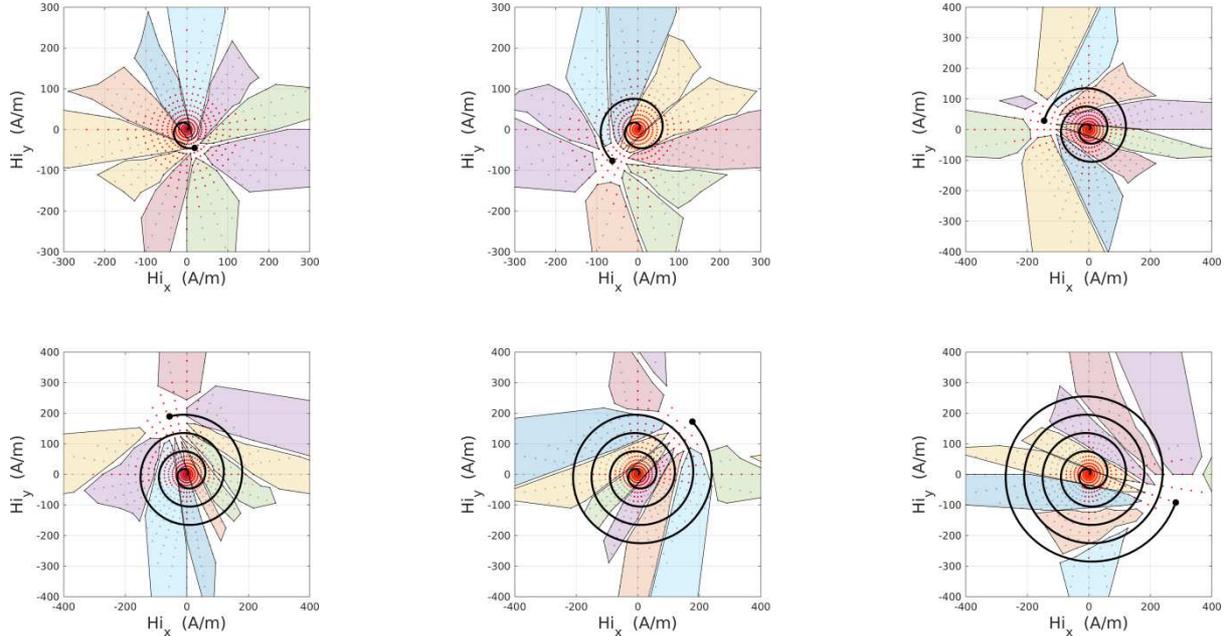


672 Figure 10. Left: magnetization computed by using the basic algorithm and the incremental algorithm which
 673 makes use of reduced models ($N_p = 40$, $\chi = 40$, $N_C = 32$). The inset shows a zoom of the \mathbf{J} -plane close to the virgin
 674 state, where a moderate discrepancy between the two algorithms can be observed. Right: validity domain of the
 675 reduced models.
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677 The effect of clustering at several time steps are depicted in Figure 11. Clusters are depicted
 678 by using patches of different colours. One observes that clusters correspond to different
 679 “sectors” of the plane, which converge toward the computational point (in order to have a
 680 readable image, the number of clusters have been purposely reduced to generate this figure).
 681 An “exclusion zone” where no hysteron is clustered can be observed in the neighbourhood of
 682 the computational point. The extent of the exclusion zone is determined by the parameter χ ,
 683 and it is motivated by the necessity of reducing the error on unit polarization (Equation 18).
 684 Qualitatively similar results are obtained by using the two other signals. A very small
 685 variability in the statistics is observed by repeating the simulations with the same parameters.
 686 This variability, which is barely observable, is explained by the random initialization of the
 687 k-means algorithm, and doesn’t alter the overall conclusions. Conversely, the number of
 688

689 reduced models is solely determined by the signal and the set of parameters used to drive
 690 simulations, namely ρ_{min} and N_p .

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693 Figure 11. Clusters of hysterons at several time steps ($N_c = 10$) for the signal “spiral” (material #1). Red spots
 694 represent hysterons which have been conserved in the reduced model; grey spots represent the original
 695 hysterons (a single spot may correspond to several hysterons with different radius).

696 The effect of parameters χ and N_c on the results are presented hereafter. When χ , and thus
 697 h_{min} , is increased, the error on the unit polarization decreases (Equation 18). Hence the
 698 computed error decreases, while the number of average hysterons of reduced models and the
 699 number of elementary operations both increase (Table 8).

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701 Table 8: Effect of variations of the parameter χ . Simulation parameters are: $N_p = 40$, $N_c = 32$ and $p = 2$ %. The
 702 number of operations T_{op} include the generation of reduced models ($M = 10^6$ elementary operations).

Material	χ	“spiral”		“harmonics”		“scalar”	
		#1	#2	#1	#2	#1	#2
e_{max}	1	3.0 %	2.2 %	1.6 %	1.0 %	1.7 %	1.5 %
	1.2	2.7 %	1.9 %	1.6 %	1.0 %	1.7 %	1.6 %
	1.5	2.4 %	1.7 %	1.6 %	0.98 %	1.6 %	1.5 %
	2	2.2 %	1.3 %	1.5 %	0.99 %	1.6 %	1.6 %
	3	1.8 %	1.0 %	1.5 %	1.00 %	1.6 %	1.5 %
e_{rel}	1	53 %	11 %	9.3 %	10 %	10 %	4.4 %
	1.2	11 %	9.1 %	9.2 %	8.6 %	8.6 %	3.7 %
	1.5	10 %	11 %	6.7 %	6.7 %	6.1 %	3.3 %
	2	16 %	4.8 %	3.8 %	4.2 %	4.1 %	2.7 %
	3	4.3 %	2.3 %	2.6 %	2.17 %	2.8 %	2.6 %
T_{op}	1	4.33 M	8.19 M	2.22 M	3.30 M	3.66 M	6.72 M
	1.2	4.50 M	8.56 M	2.36 M	3.40 M	3.84 M	7.03 M
	1.5	4.79 M	9.17 M	2.58 M	3.51 M	4.12 M	7.47 M
	2	5.31 M	10.3 M	3.07 M	3.8 M	4.64 M	8.34 M
	3	6.56 M	13.3 M	4.65 M	4211	5.97 M	10.89 M
Average number of	1	1829	2131	1132	830	2347	5285
	1.2	2083	2545	1392	967	2633	6026

hysterons	1.5	2495	3252	1657	1226	3082	7175
	2	3207	4683	2335	1777	3853	9118
	3	5247	9178	4438	4211	5651	14314

703 Unsurprisingly, it is observed that the error is lowered by increasing the number of clusters
704 N_c (Table 9). The number of operations is not substantially modified, because the increase
705 in the number of hysterons which have to be handled is quite negligible with respect of the
706 average number of hysterons of reduced models. For high values of N_c we seldom
707 experience problems with the convergence of the k-means algorithm. These problems, which
708 could be solved by tweaking the parameters of the clustering algorithm (k-means), don't
709 seem to affect the accuracy of the results.

710
711 Table 9: Effect of the number of hysterons on the accuracy Simulation parameters are: $N_p = 40$, $\chi = 1.2$ and $p =$
712 2% . [*] indicates problems with the convergence of the k-means algorithm.

Material	N_c	"spiral"		"harmonics"		"scalar"	
		#1	#2	#1	#2	#1	#2
e_{max}	8	5.59 %	2.36 %	2.11 %	1.38 %	3.15 %	1.78 %
	16	3.45 %	2.1 %	1.71 %	1.08 %	1.64 %	1.57 %
	32	2.68 %	2.0 %	1.63 %	1.04 %	1.68 %	1.54 %
	[*] 64	2.24 %	1.9 %	1.63 %	0.98 %	1.64 %	1.53 %
e_{rel}	8	143 %	154 %	9.4 %	17 %	19 %	4.0 %
	16	84 %	8.6 %	15 %	9.1 %	10 %	3.9 %
	32	10 %	16 %	7.3 %	8.9 %	8.5 %	3.9 %
	[*] 64	10 %	9.3 %	8.7 %	8.7 %	8.5 %	3.7 %

713

714 Memoryless algorithm with reduced models

715 The generation of reduced models, which was devised to accelerate the incremental
716 algorithm, can be modified in order to be used with other algorithms. To this aim, it is
717 enough to leave all frozen hysterons in the reduced models, including those who are and
718 remain frozen $\forall \mathbf{Ha} \in \mathbf{B}(\mathbf{Ha}_0, \rho)$ (case iii, Figure 6). The result of simulations taken on by
719 using the memoryless algorithm are reported in tables 10 (material #1) and 11 (material #2).

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722 Table 10: Statistics of the simulation by using the memoryless algorithm with reduced models (material #1).
723 Simulation parameters are: $N_p = 40$, $\chi = 1.2$, $N_c = 32$ and $p = 2\%$ ($M = 10^6$ elementary operations).

	"spiral"	"harmonics"	"scalar"
e_{max}	2.21 %	1.55 %	1.61 %
e_{rel}	6.95 %	2.00 %	2.37 %
T_{op}	181 M	9.3 M	85 M
T_{op} (generation of RM excluded)	179 M	7.0 M	83 M
T_{op} memoryless algorithm without RM	184 M	14 M	89 M
T_{op} basic algorithm	7.23 M	7.23 M	7.23 M
Nb reduced models	151	133	109
Average Nb of hysterons	3919	1360	4888
Occupied memory	267 kb	294 kb	268 kb

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Table 11: Statistics of the simulation by using the memoryless algorithm with reduced models (material #2). Simulation parameters are: $N_p = 40$, $\chi = 1.2$, $N_c = 32$ and $p = 2\%$ ($M = 10^6$ elementary operations).

	“spiral”	“harmonics”	“scalar”
e_{max}	1.71 %	1.00 %	1.53 %
e_{rel}	6.52 %	1.40 %	3.17 %
T_{op}	41.2 M	5.9 M	6.6 M
T_{op} (generation of RM excluded)	18.0 M	0.58 M	2.6 M
T_{op} memoryless algorithm without RM	32.4 M	16.7 M	17.7 M
T_{op} basic algorithm	16.5 M	16.5 M	16.5 M
Nb reduced models	151	133	109
Average Nb of hysterons	2229	666	4239
Occupied memory	662 kb	663 kb	660 kb

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The obtained results are deceiving: in general, the usage of reduced models is quite ineffective in reducing the number of elementary operations, with the only exception of the signal “harmonics” simulated with material #2. This can be explained by the necessity of handling all frozen hysterons when using the memoryless algorithm.

The case of the pooled algorithm has not been explored in this work. However, one can foresee an additional complexity which consists in handling the transition from a reduced domain to another, which is far from being a simple task. Most importantly, the main issue with this algorithm is the memory occupation, which is not solved by using reduced models.

740 **On the saturation and congruency properties**

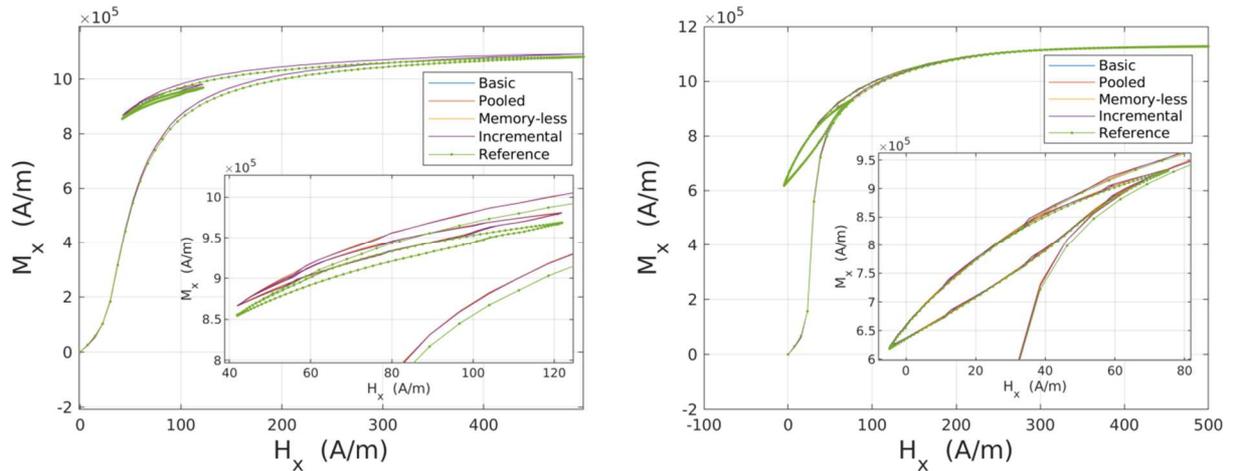
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The four algorithms presented in this work compute exactly the same result (up to rounding errors due to floating point arithmetic); hence they don’t alter at all the properties of congruency and the behaviour of the material in the case of saturation (see [15] for a detailed explanation of these properties).

Conversely, simplification of the set of hysterons and reduced models introduce approximations. Simplification of the set of hysterons ends up in creating a “lightweight” model which has the same behaviour of the original model, up to an acceptable approximation (Equation 10). Hence it preserves the congruency property, which is intrinsic to the Preisach model. The value of the polarization at saturation can be modified to a little extent, unless the weights are renormalized (Equation 11).

As for reduced models, it has to be observed that the approximation concerns only the contribution of unfrozen hysterons; hence the congruency property and the behaviour of the material at saturation are not modified. By the way, the approximation introduced by using reduced models is more effective precisely when the material is saturated, because clustered hysterons are “far” from the applied field \mathbf{H}_a , thus the angle γ is very small (Equation 18).

These observations are confirmed by numerical simulations. For both materials, we simulated the saturation, followed by three minor loops. The obtained results obtained with all algorithms are plotted in Figure 12, together with the reference case (computed by using the basic algorithm with the original set of hysterons, without using reduced models). There is of course an approximation, but one observes that there is no effect on the congruency property, nor on the saturation.



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 763 Figure 12. Signal with minor loops for the material #1 (left) and for the material #2 (right). Each loop is
 764 repeated three times. One observes that all four algorithms provide the same result, which is close to the
 765 reference one, and verify the congruency property (inset: all loops are perfectly superposed). Algorithms run on
 766 simplified materials ($p = 2\%$) with renormalization of weights; the incremental algorithm makes use of reduced
 767 models ($\chi = 1.2$, $N_c = 32$, $N_p = 40$).

768 Conclusions

769 In this work four algorithms, which all implement the Vector Hysteresis Model, are presented
 770 and critically compared. Apart from the basic algorithm, which is classical, the three other
 771 algorithms (named pooled, memoryless and incremental) all aim at reducing the memory
 772 occupation in view of simulating a real model with Finite Element analysis (FEA). The
 773 pooled algorithm is a variant of the basic algorithm, which allows a considerable economy of
 774 memory at the price of a more complex implementation. Conversely, the memoryless and
 775 incremental algorithms reduce of several orders of magnitude the memory occupation, at the
 776 price of a much higher computational cost. In order to reduce to an acceptable level the
 777 computational cost of these two algorithms, the generation and usage of reduced models is
 778 devised. Moreover, a simplification procedure which allows to dramatically reduce the
 779 number of hysterons of a given model has been implemented.

780 The time and space complexity of all algorithms are predicted theoretically and validated
 781 through numerical experiments on two artificial magnetic materials and three different vector
 782 excitations (signals). The obtained data clearly demonstrate that the pooled algorithm
 783 requires a very high amount of memory when materials are excited with a low magnetic field,
 784 because in this case, a very high fraction of hysterons are frozen. This kind of situation is
 785 typically found in the first time steps of FE simulations, where the material is in the virgin
 786 state. Conversely, the first time steps are very cheap in the case of memoryless and
 787 incremental algorithms. Hence, one could devise a computational strategy which consists in
 788 starting the simulation by using the memoryless or the incremental algorithm, then switch to
 789 the pooled algorithm when the fraction of frozen hysterons is low enough.

790 The usage of the simplification procedure, which could indeed be considered as a step of the
 791 identification procedure, is mandatory in that it is extremely effective in reducing both the
 792 computational cost and memory consumption. With the materials used in our numerical
 793 experiments, a very large part of the original hysterons are removed at the price of a very low
 794 error (of the order of a few percent), which is usually compatible with the practical
 795 experimental error, and with the intrinsic uncertainties in the material characterization.

796 Finally, the usage of simplified models together with the incremental algorithm achieve a
797 reduction of several orders of magnitudes of the memory occupation, and also a reduction of
798 the number of elementary operations required to simulate the material. We must warn that the
799 presented results on the time complexity are strongly influenced by the present
800 implementation of the algorithms, which is not at all optimized for speed, especially in as
801 much as the incremental algorithm is concerned. However, if the theoretical results should be
802 confirmed in practice, the incremental algorithm with reduced models could be a game-
803 changer in the usage of the Vector Hysteresis Model in FEA simulation of large electrical
804 systems.

805 While waiting for an optimized implementation of the algorithms, other paths can be devised
806 to reduce the computational cost of the memoryless and incremental algorithms. The main
807 point is that with these algorithms the state of the material is represented by its full history,
808 that is by all the past values of the applied magnetic field. If one could devise a strategy to
809 truncate the history which has to be taken into account, this would reduce proportionally the
810 computational time, which in these algorithms is dominated by the search back in time of the
811 time step when hysterons got frozen. Another point which deserves investigation is how to
812 handle the case of periodic signals. This is particularly relevant to simulate the steady state of
813 electrical systems by using standard harmonic formulations or the harmonic balance
814 technique.

815 **Conflicts of Interest**

816 The authors declares that there is no conflict of interest regarding the publication of this
817 paper.

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