

# Spin-squeezed states for metrology

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Spin-squeezing is a well-established “quantum technology”, where well-designed correlations in an ensemble of two-level systems reduce the statistical uncertainty of spectroscopic experiments. This paper reviews some important advances in the field, with emphasis on the author’s contributions concerning in particular the fundamental limits imposed by decoherence. Building on the material presented in the first part, new ideas and some promising developments are outlined in the last section.

## I. INTRODUCTION AND OVERVIEW

### A. Collective spin, spin squeezing and quantum gain

A quantum system with two energy levels  $E_a$  and  $E_b$  is formally equivalent to a spin 1/2. Starting from  $N$  such identical systems, for example atoms, one can form a collective spin operator  $\vec{S}$  whose three components in first quantization are

$$\hat{S}_x + i\hat{S}_y = \sum_{i=1}^N |a\rangle\langle b|_i; \quad \hat{S}_z = \frac{\sum_{i=1}^N (|a\rangle\langle a|_i - |b\rangle\langle b|_i)}{2} \quad (1)$$

In a state of the system in which each atom is in a superposition of the two levels, the collective spin precesses around the  $z$ -axis at the frequency  $\omega_{ab} = (E_b - E_a)/\hbar$ . Whether one wants to measure the unperturbed frequency  $\omega_{ab}$  of a given atomic transition as in an atomic clock, or whether one is interested in the change of this frequency induced by an external field, for example a magnetic field, spin precession is at the heart of many atomic sensors. Such sensors, even after eliminating all sources of technical noise, remain limited by the quantum noise intrinsic to the state in which the collective

spin has been prepared. If, for example, the system is in the eigenstate of  $\hat{S}_x$  with maximum eigenvalue  $N/2$ , tensor product of the state  $(|a\rangle + |b\rangle)/\sqrt{2}$  for each atom shown on the left in figure 1, the uncertainty on the angular position of the collective spin is given by

$$(\Delta\phi)_{NC} = \frac{\Delta S_y}{\langle S_x \rangle} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}. \quad (2)$$

On the contrary, in a spin-squeezed state<sup>1</sup>, shown on the right in figure 1, the uncertainty on one spin component orthogonally to the mean spin is reduced thanks to quantum correlations among the atoms. If the reduced uncertainty is on the  $S_y$  component as in figure 1, the accuracy  $\Delta\phi$  on the angular position of the collective spin is increased. We then define a spin squeezing parameter  $\xi^2$ ,

$$\xi^2 \equiv \frac{N\Delta S_{\perp}^2}{|\langle \vec{S} \rangle|^2} \quad (3)$$

noise-to-signal squared ratio in a spectroscopy experiment where the phase  $\phi$  is to be determined, which we want to be as small as possible, and which allows to quantify the metrological gain of a spin-squeezed state with respect to an uncorrelated one. In fact, we have<sup>2</sup>

$$\Delta\phi = \xi(\Delta\phi)_{NC} = \frac{\xi}{\sqrt{N}} \quad (4)$$

The definition of  $\xi^2$  in (3) can be generalized for non-Gaussian states<sup>3</sup> more complex than the spin-squeezed state represented in figure 1, for which the information on the precession phase  $\phi$  must be extracted from an observable  $\hat{X}$  which is in general a nonlinear function of the three components of the collective spin<sup>4-6</sup>, with the same meaning as indicated by equation (4).

### B. Non-linear evolution and non-destructive quantum measurement

In order to generate the correlations that change a factorized state into a spin-squeezed state, an elegant method consists of a unitary evolution with a nonlinear Hamiltonian, e.g.<sup>1</sup>

$$H_{NL} = \hbar\chi\hat{S}_z^2 \quad (5)$$

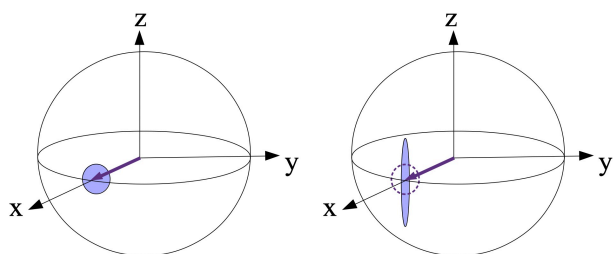


FIG. 1. Collective spin prepared in the  $\hat{S}_x$  eigenstate of eigenvalue  $N/2$  without correlation between the atoms (left) and in a spin-squeezed state (right), represented on the Bloch sphere of radius  $N/2$ . The blue-colored part schematically represents the uncertainty on the spin components orthogonal to the mean spin, due to their quantum nature.

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which represents a Kerr-type nonlinearity for the atoms. It can result from short-range interactions between cold atoms in a Bose-Einstein condensate<sup>7,8</sup>, but also, in an ensemble of atoms without interaction, from their collective interaction with an electromagnetic field in an optical cavity<sup>9</sup>. In both cases it is possible to control the value of the constant  $\chi$ . For condensates, in the simple case where the states  $|a\rangle$  and  $|b\rangle$  play symmetric roles and atoms in different states do not interact with each other,  $\chi = (d\mu/dN)/\hbar$  where  $\mu = \rho g$  is the chemical potential and  $\rho$  the atomic density for each of the two components. The coupling constant  $g = 4\pi\hbar^2 a_s/m$  is proportional to the  $s$ -wave scattering length  $a_s$  between cold atoms.

Another method to create a spin-squeezed state from an uncorrelated state consists in performing a quantum non-demolition measurement on a component of the collective spin. In general, this is done by coupling the system  $\mathcal{S}$  to be measured to a second quantum system  $\mathcal{P}$  and, after some interaction time between the two systems, by performing a destructive measurement on  $\mathcal{P}$ . In order to measure non-destructively the  $\hat{S}_z$  component of the collective atomic spin, one can take as a secondary system the Stokes spin  $\hat{P}$  of the light<sup>10</sup>, and use the Faraday interaction between the two spins<sup>11,12</sup>

$$H_{\text{Faraday}} = \chi \hat{S}_z \hat{P}_z. \quad (6)$$

### C. Overview of the experimental results and application perspectives

Using the two methods mentioned in the previous section, spin-squeezed states have been obtained in the laboratory and the demonstration of their metrological gain has been done in several systems<sup>13</sup>, which include Bose-Einstein gas condensates<sup>14,15</sup>, cold atoms interacting with an electromagnetic field<sup>16–18</sup> for which a metrological gain  $\xi^{-1}$  of the order of 10 has been measured with  $5 \times 10^5$  atoms<sup>18</sup>, and hot atomic vapors<sup>19</sup>. These remarkable results are proofs of principle. The transition to applications in metrology should take place if quantum gain becomes easily accessible from a technological point of view or if in some cases it is indispensable. Among the various atomic sensors, the clocks that already in 1999 had reached and observed the quantum limit in the case of independent atoms<sup>20</sup>, could be the first to benefit from spin squeezing techniques. In general, the smaller the number  $N$  of atoms that we use, and here we have in mind the evolution towards miniaturisation of atomic sensors, the greater the relative importance of quantum noise and the potential utility of spin-squeezed states.

## II. QUANTUM METROLOGY AND DECOHERENCE

Quantum metrology aims to improve the accuracy of measurements by reducing their “fundamental” statistical uncertainty given by quantum fluctuations. This is

achieved by preparing the physical system of interest in a state with particular quantum correlations. For this approach to make sense from a practical point of view, two conditions seem necessary. The first is that the statistical uncertainty on the variable, obtained from a measurement with independent atoms in an imparted time, is dominated by quantum fluctuations and not by other sources of classical noise. The second is that the decoherence is sufficiently weak not to destroy the quantum correlations we want to put to use. Mathematically, if we fix the time  $T$  of the measurement, the second condition is automatically verified in the limit  $\gamma \rightarrow 0$  where  $\gamma$  is the decoherence rate in the system.

### A. Scaling laws of the quantum gain

If, as we have said, decoherence plays an important role in the use of entangled states, it equally plays an important role in their preparation, determining in general a fundamental limit to the quantum gain that can be obtained. Starting from an uncorrelated, eigenstate of  $\hat{S}_x$  of eigenvalue  $N/2$  with  $N \gg 1$ , unitary evolution with the nonlinear Hamiltonian (5) generates at an optimal time  $\chi t \simeq N^{-2/3}$  a spin-squeezed state for which  $\xi^2 \simeq N^{-2/3}$ . This very favourable scaling law of quantum gain for spin-squeezed states becomes even more favorable for non-Gaussian states obtained by prolonging the unitary evolution for longer times until the maximum limit  $\xi^2 = N^{-1}$  is reached.<sup>21–23</sup> However, very entangled states offering large quantum gain are in general more fragile to decoherence, which modifies the scaling laws of quantum gain<sup>5,6,24</sup>. In some cases one loses the scaling law completely and obtains a spin squeezing parameter that tends towards a constant  $\xi_{\text{min}}^2$  in the limit  $N \rightarrow \infty$ . If this indicates that one finds a classical scaling law for  $\Delta\phi$  in the light of the equation 4, the quantum gain can remain very large if  $\xi_{\text{min}}^2 \ll 1$ .

## III. FUNDAMENTAL LIMITS FOR SPIN SQUEEZING IN BOSE-EINSTEIN CONDENSATES

With colleagues we have studied the fundamental limits on spin squeezing that can be achieved in a Bose-condensed gas formed by  $N$  two-level atoms, usually two hyperfine levels in the fundamental state, using short-range van der Waals interactions between cold atoms. While this physical system accurately realizes the simple model (5) in the mean-field description<sup>7,8</sup>, an intrinsic source of decoherence for spin squeezing dynamics comes from the interaction of the condensate with thermally populated low-energy excitations in the gas, whose population fluctuates from one realization to another in the experiment even under identical conditions for the experimental parameters. In the framework of the multimodal microscopic description of Bogoliubov<sup>25</sup>, we have shown that at the thermodynamic limit  $N \rightarrow \infty$ , and for an

optimal interaction time  $t_{\text{opt}}$  which remains finite, the squeezing parameter tends towards a constant  $\xi_{\text{min}}^2$ .<sup>26</sup>

$$\xi^2(t_{\text{opt}}) \xrightarrow{N \rightarrow \infty} \xi_{\text{min}}^2, \quad (7)$$

which then constitutes a limit for the quantum gain. For a temperature above the chemical potential of the gas, but well below the critical condensation temperature, the limit  $\xi_{\text{min}}^2$  is well approximated by the initial non-condensed fraction in the gas. An important point is that for realistic values of the parameters, it is in principle possible to obtain values of  $\xi_{\text{min}}^2$  of the order of 1/900 corresponding to a gain of a factor 30 on  $\Delta\phi$  according to the formula (4) compared to the use of uncorrelated atoms.

A second source of fundamental decoherence that we have considered is given by particle loss, a consequence of an imperfect vacuum for one-body losses, and inherent to the metastable nature of ultracold gases for three-body losses. In the theoretical framework of an open system described by a master equation<sup>27,28</sup> we have shown that in the limit of  $N$  large the optimal spin squeezing is limited by a constant, as in (7), where  $\xi_{\text{min}}^2$  is in this case proportional to the fraction of atoms lost at time  $t_{\text{opt}}$ . Once spin squeezing has been prepared at time  $t = 0$ , its evolution, in the absence of interactions and in the presence of one-body losses of rate  $\gamma$ , is given by

$$\xi^2(t) - 1 = [\xi^2(0) - 1] e^{-\gamma t}. \quad (8)$$

Spin squeezing that lasts more than one second and is limited by losses according to equation (8) has been seen in<sup>29</sup>.

#### IV. PROMISING DEVELOPMENTS FOR THE FUTURE

Research in the field of quantum technologies is abundant, and several promising avenues are currently being explored. For atomic clocks, two significant examples are the first demonstration of spin-squeezing on an optical transition<sup>30</sup> and the theoretical studies of spin-squeezing with atoms in optical lattices<sup>31,32</sup>. The purpose of this section is to outline some promising developments from the material we presented in the first part of the article. These ideas fit perfectly into the theoretical framework described so far, as they are based on either non-linear evolution (5) or quantum non-demolition Faraday interaction (6). However, due to the difference in the physical system used, the time scales involved or the type of quantum platform that can be obtained, the realization of each of them would represent a breakthrough from the state of the art and open the field to new applications.

*Storing spin-squeezing in a Mott-insulator phase* - We have already mentioned that the binary short-ranged van der Waals interactions in a Bose-Einstein condensate where each atom is prepared in a superposition of two internal states  $(|a\rangle + |b\rangle)/\sqrt{2}$ , generate the nonlinear Hamiltonian (5) that transforms the factorized initial state into

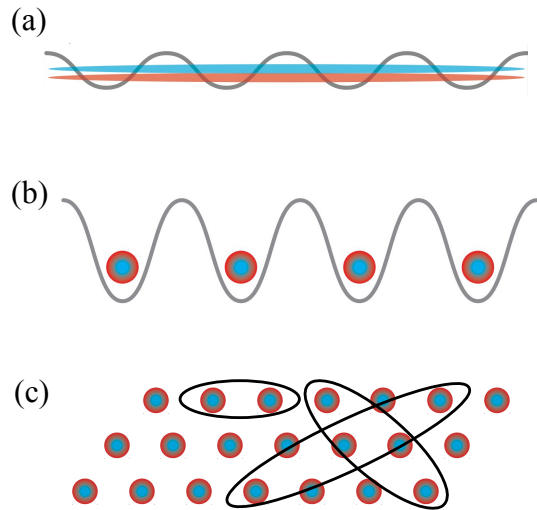


FIG. 2. (a) An optical lattice is adiabatically raised in an interacting two-component condensate. (b) The system enters the Mott-insulating phase with one atom per site at the optimal squeezing time, thus freezing the quantum correlations between atoms (figure adapted from<sup>31</sup>). (c) In the squeezed-Mott configuration, each atom is in a superposition of the two the two internal states  $|a\rangle$  and  $|b\rangle$  and is equally correlated to any other atom. The entanglement is then distributed between spatially separated and individually accessible modes.

a spin-squeezed state. Physically, since the interaction strength depends on the atoms internal state, the *phase of each atom*, represented by the  $\hat{S}_y$  component of the collective spin, correlates *via* the mean field shift with the *population difference* of the ensemble, represented by  $\hat{S}_z$ . For phase sensing or atom interferometry, the squeezing, that is hence generated in a superposition of  $\hat{S}_y$  and  $\hat{S}_z$ , can be transferred to the  $\hat{S}_y$  spin component, as in figure 1, by a simple rotation. To this physical picture we now add an optical lattice that is adiabatically raised in the gas, progressively bringing the interacting system from the superfluid to the Mott-insulator phase<sup>33</sup>. As long as there is a delocalised superfluid component, figure 2 (a), the spin-squeezing dynamics continues to create correlations among the internal states of the atoms. When the system enters the Mott-insulator phase with one atom per site, figure 2 (b), the condensate disappears, the van der Waals interactions are suppressed as the atoms are separated, but the correlations among the internal states of the atoms that were created in the superfluid phase remain. We have shown in this way that one can freeze a spin-squeezed state<sup>31</sup> or an even more entangled non-Gaussian state<sup>34</sup>, in a Mott-insulator configuration where each atom is a spin 1/2 that shares quantum correlations with all others, figure 2 (c).

In addition to the potential interest for atomic clocks<sup>35,36</sup>, such a configuration where quantum entanglement is shared between individually accessible spatial modes offers interesting prospects for quantum information and multiparameter quantum metrology<sup>37,38</sup>.

Using the non linearity of a degenerate Fermi gas - Interesting perspectives are also offered by degenerate gases of fermions in two spin states, in which one can control the strength of the interactions without inducing significant losses of atoms, and obtain at will: a gas of dimers formed by two strongly bonded atoms of opposite spin, a gas of weakly bonded Cooper pairs, or a strongly interacting gas in the so-called unitary regime<sup>39</sup>. To give an example, let us consider the following *Gedankenexperiment* inspired by our studies on phase coherence of a paired-fermions condensate<sup>40,41</sup>. A condensate of fermion dimers of opposite spin is initially prepared in a symmetric double-well in an uncorrelated state, where each dimer is symmetrically delocalized on the two wells (figure 3(a)-(b)). By raising the barrier between the two wells until they are completely separated, slowly enough not to excite the condensates, one actually prepares the initial state  $\langle \hat{S}_x \rangle = N/2$  where this time  $a$  and  $b$  refer to the two potential wells, and one can show that the state of the system evolves according to the Hamiltonian (5) where, as for atomic condensates,  $\chi = (d\mu/dN)/\hbar$ ,  $\mu$  being the chemical potential of the gas in each well. Note that here  $\hat{S}_z$  represents the difference between the number of pairs in the two wells,  $\hat{S}_x$  represents the macroscopic phase coherence between the two wells and, again, the quantity that is squeezed by the Hamiltonian (5) is a superposition of  $\hat{S}_z$  and  $\hat{S}_y$ . Similar experiments with bosonic condensates, aimed at using spin-squeezed states in an atomic interferometer, have been carried out<sup>42,43</sup>. In the case of the fermionic-pairs condensate, as shown for example in<sup>40</sup>, the largest value of  $\chi$ , hence the fastest spin squeezing dynamics, is obtained in the limit  $a_s \rightarrow 0^-$  where the chemical potential in each well tends towards the Fermi energy of an ideal gas of fermions  $\mu \rightarrow \epsilon_F$ . This paradoxical situation, where spin squeezing would be generated in a Fermi sea without interaction, is depicted in figure 3(c). In the homogeneous three-dimensional case, it is straightforward to compare the nonlinearities  $\chi_{\text{Bose}} = \frac{\rho g}{N}$  and  $\chi_{\text{Fermi}} = \frac{2}{3} \frac{\epsilon_F}{N}$  with  $\epsilon_F \propto \rho^{2/3}$  and it is seen that the nonlinearity is much larger for the ideal Fermi gas

$$\frac{\chi_{\text{Fermi}}}{\chi_{\text{Bose}}} \simeq \frac{\epsilon_F}{\rho g} \simeq \frac{1}{a_s \rho^{1/3}} \gg 1 \quad (9)$$

proportionally to the power  $-\frac{2}{3}$  of the parameter  $\sqrt{\rho a_s^3}$ , which typically holds  $10^{-3}$ - $10^{-2}$  in an atomic Bose-Einstein condensate<sup>44</sup>. If these considerations are very encouraging, a study at non-zero temperature is needed to draw conclusions on the best interaction regime to use with fermions and calculate the maximum spin squeezing that can be achieved with this method.

*Long-lived nuclear spin squeezing in a gas cell* - In the field of atomic sensors, spin squeezing and quantum technologies have a vocation to extend to room temperature gases or vapours in a cell<sup>45,46</sup>. Although the physical system is different from those discussed so far, once the average over the rapid movement of the atoms in the

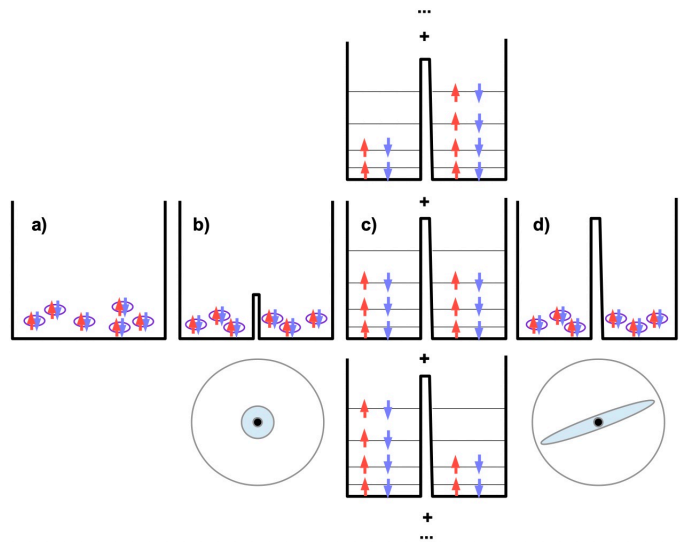


FIG. 3. Spin squeezing Gedankenexperiment with a condensate of fermion pairs at zero temperature in a double-well. (a)-(b) A condensate of fermions “dimers” is prepared in a delocalized coherent state on the two wells. (c) After raising adiabatically the barrier separating the two wells, the scattering length is set to the desired value, e.g.  $a_s \rightarrow 0^-$ , for the spin squeezing phase. In this case, squeezing is created in the ideal Fermi gas due to the  $N$ -dependence of the chemical potential:  $\epsilon_F \propto \rho^{2/3}$  in 3D. After the squeezing phase, the scattering length is restored to the initial positive value corresponding to a dimer condensate, obtaining a spin-squeezed state for the latter. In the lower parts of figures (b) and (d), we plot on the Bloch sphere the fluctuations of the collective spin constructed from the  $a$  and  $b$  modes for the center of mass of the dimers corresponding to the ground state in the left and right wells.

cell has been taken, it is possible to describe the internal degrees of freedom in terms of the collective spin and connect to the theoretical description presented in the article. An extreme system for its level of isolation and coherence time, measured in tens of hours, is the purely nuclear collective spin of a rare gas such as helium-3 in its ground state. A first theoretical study showed that exchange collisions between helium atoms in the ground state and in an optically accessible metastable state, currently used to polarize the nuclear spins<sup>47</sup>, allow quantum correlations and squeezing to be transferred to the nuclear spin<sup>48,49</sup>. More recently, it has been shown that spin squeezing schemes using a quantum non demolition measurement with a Faraday Hamiltonian of type (6), successfully realized for alkaline gas<sup>50</sup>, can be adapted to nuclear spin squeezing<sup>51-53</sup>, using spin-exchange collisions between the rare gas and an alkaline<sup>54,55</sup> or metastability exchange collisions in a pure helium-3 gas.<sup>56</sup> In the latter case, we have predicted that the best spin squeezing parameter  $\xi^2$  accessible<sup>53</sup> scales as  $\sqrt{\gamma_0/\gamma_m}$  where  $\gamma_m$  is the rate of metastability exchange collisions for a metastable helium atom, of the order of  $5 \times 10^6 \text{ s}^{-1}$  for a ground state helium pressure of 2 mbar, and  $\gamma_0$  is the de-

coherence rate of metastable atoms, dominated by their diffusion towards the cell walls, of the order of  $2 \times 10^4 s^{-1}$  for a centimetric cell without buffer gas. The numerous spectroscopic applications of helium nuclear spin, ranging from magnetometry to fundamental physics tests<sup>47</sup>, could thus in the future benefit from quantum noise control and stimulate new ideas and applications of quantum technologies.

## V. CONCLUSION

Spin squeezing and quantum metrology is a very active field, constantly fed by new ideas, extensive theoretical studies and surprising experimental discoveries. The transition from science to technology should take place when the quantum advantage outweighs the complexity of the experiments or, as we have said, if it is indispensable in some cases. An interesting example from this point of view is that of the squeezed states of light, extensively studied in the 1980s and now used in an interferometer to detect gravitational waves<sup>57</sup>.

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