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Do children with mathematical learning disabilities use the inversion principle to solve three-term arithmetic problems?: the impact of presentation mode

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Running head: The inversion principle in children with mathematical learning disabilities

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Conflicts of interest

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Credit Authorship contribution statement

Amélie Lubin: Conceptualization, Data curation, Formal analysis, Methodology, Project administration, Software, supervision, validation, visualization, writing - original draft, writing- review & editing

Laetitia Kana: Conceptualization, Formal analysis, Investigation, Methodology, Software, writing- review & editing

Nelly Ducloy: Conceptualization, Investigation, Methodology, Resources, Supervision, writing- review & editing

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1. Introduction

When children solve an arithmetic problem, they must rely not only on procedural knowledge (knowledge of the steps to be taken to solve a problem: "know how") but also on conceptual knowledge (understanding and using conceptual principles in arithmetic: "know why") (Dowker, 2005). Studies on the development of mathematical cognition have long focused on procedural knowledge. More recently, studies have also focused on conceptual knowledge by examining both how participants solve problems and their understanding of conceptual principles (Crooks & Alibali, 2014). Several conceptual principles exist, such as identity, negation, commutativity, inversion, associativity and equivalence, and all of these principles involve distinct developmental patterns (Robinson et al., 2017). Among these principles, the inversion principle is one of the most complex in typically developing (TD) children (Robinson et al., 2017). Given that approximately 1% to 10% of school-age children have difficulties acquiring arithmetic skills (Devine et al., 2013; Fischer, 2009; Lafay et al., 2015), it is important to better understand the nature of these difficulties in children with mathematical learning disabilities (MLD) to develop and implement effective care. Thus, this study aims to investigate a particular conceptual principle: the inversion principle in school-age children with MLD.

The inversion principle applied to additive and subtractive operations is the ability to understand that addition is the opposite of subtraction and vice versa. The principle also involves understanding that the two operations are closely interrelated. Adding a particular number b to a set a can be cancelled by subtracting the same number b ($a + b - b = a$). This principle makes it possible to reduce cognitive effort by substituting a much more complex calculation mechanism. Due to this cognitive shortcut, complex arithmetic operations (e.g., 3-term problems) can be

solved without calculation. This principle is important for learning mathematics. Many studies have focused on the development of this inversion principle in TD children (see Baroody et al., 2009; Verschaffel et al., 2012 for special issues on this topic). These studies showed discrepancies regarding the age at which the principle develops (Gilmore & Papadatou-Pastou, 2009), leading to the emergence of two perspectives. The first perspective states that the inversion principle appears late in the development process. This position is supported by Piaget (1952; Piaget & Moreau, 2001), who believed that children can grasp this principle only from the concrete operations stage at approximately 6-7 years of age and can explain it at approximately 10 years. Thus, authors have assumed that understanding this concept requires recognizing addition and subtraction problems as complementary (Baroody, 1999; Canobi, 2004, 2005; Canobi et al., 1998). According to these authors, the complementarity between addition and subtraction is not obvious for children. Children have great difficulties extracting information from the resolution of a first operation, such as "a + b = c", to resolve another complementary operation, such as "b – c = a". For a long period of time, children will solve the two operations separately without perceiving the link between them. According to these authors, understanding this complementarity is a conceptual achievement that is accessible from 6-7 years of age.

The other perspective is supported by Bryant et al. (1999), Gilmore and Bryant (2006), Klein and Bisanz (2000), Lubin et al. (2015), Rasmussen et al. (2003), Sherman and Bisanz, (2007), and Starkey and Gelman (1982) and suggests that children can start using inversion in a quantitative manner as early as preschool age. In these studies, the authors used a more direct method to demonstrate the presence of the inversion principle in children. Inverse problems such as "a + b – b = ?" are compared to standard problems such as "a + b – c = ?". If calculation is used, inverse problems yield similar performance to standard problems of equivalent quantities, but if the inversion principle is used, inverse problems yield better performances (see Bisanz and Lefevre, 1990, who proposed this original method). Indeed, children who use the inversion principle will

achieve better accuracy and resolve inverse problems faster than they can resolve standard problems because inverse problems require no calculation. In inverse problems, adding and subtracting the same quantity must result in the unchanged original quantity; therefore, resolving such problems does not require calculation (Klein and Bisanz, 2000). In these studies, the authors seek to determine whether children are able to use this principle in 3-term problems without asking the children to explain how they proceeded. The children's achievement is tested, but their explicit understanding of the abovementioned interrelationship is not assessed. As Gilmore and Papadatou-Patsou (2009) noted, although this may seem procedural, many researchers argue that conceptual knowledge of the inversion principle directly underlies the use of the conceptual shortcut. The discrepancy between these two perspectives can most likely be explained by the fact that the definition of inversion is not consensual and refers to different concepts that do not precisely question the same levels of inversion. The first perspective focuses on the explicit understanding of the inversion principle by using complementary additive and subtractive operations in children over 6-7 years of age, while the second focuses on a more implicit understanding of the inversion by using this principle when younger children solve 3-term problems. Whereas some authors suggest using different terms for these perspectives, as they rely on different principles such as the addition-subtraction complement principle for the first and the inversion principle for the second (Baroody et al., 2009; Verschaffel et al., 2012), we use a single term. Indeed, both principles underlie the same mathematical structure (as suggested by Van den Heuvel-Panhuizen & Treffers, 2009) but seem to refer to different understanding levels of the inversion principle.

Studies conducted with younger children require the use of concrete and imaged material, which seems to facilitate the use of the inversion principle. For instance, many studies have shown an early understanding of this principle among children when objects are used rather than symbolic material such as digits (Bryant et al. 1999; Gilmore & Bryant, 2006; Klein & Bisanz, 2000; Rasmussen et al., 2003). In their meta-analysis of more than 700 subjects (14 independent

studies), Gilmore and Papadatou-Pastou (2009) revealed the importance of the way in which a problem is presented. When the problem is contextualized (with concrete material, pictures or words), the difference between inverse and standard problem solving becomes more important. In contrast, this difference is smaller when the problem is presented in a symbolic form with digits. Contextualizing the problem therefore promotes the success of TD children in inverse problems and helps them identify and use the inversion principle.

Some studies have shown that performance in resolving inverse problems is not necessarily related to calculational skills (Bryant, et al. 1999; Canobi, 2004; Gilmore & Bryant, 2006; Gilmore and Papadatou-Pastou, 2009; Rasmussen et al., 2003; Sherman & Bisanz, 2007, Watchorn et al., 2014). For instance, Gilmore and Bryant (2006) identified three groups of children aged 6 to 9 years: a group with good performance in calculation and inversion, a group with poor performance in calculation and inversion, and a group with good performance in inversion but poor performance in calculation. These profiles were also found in Gilmore and Papadatou-Pastou's (2009) meta-analysis. These results suggest that calculational skills do not seem to condition the use of the inversion principle and that conceptual knowledge is not necessarily closely linked to procedural knowledge. Because the inversion principle makes it possible to reduce cognitive effort by replacing a much more complex calculation mechanism, one may wonder whether children with MLD are able to identify and use this principle, thereby avoiding calculation.

MLD is a neurodevelopmental disorder that indicates abnormalities in brain structure and function leading to severe delays in mathematical areas (DSM-5; 5th ed., American Psychiatric Association, 2013). According to prevalence studies, MLD affects 1% to 10% of children (Devine et al., 2013; Fischer, 2009; Lafay et al., 2015). The DSM-5 describes MLD as a specific learning disorder with impairment in mathematics that results in difficulties mastering number

sense, arithmetic facts, calculation and mathematical reasoning. These difficulties are persistent and interfere with academic performance. However, they cannot be defined as an intellectual or physical disability, as a psychiatric pathology or as resulting from precarious psychosocial conditions or educational deficiencies. Nevertheless, pure MLD seems very rare, and it is often associated with written language disorders (Shalev et al., 1997; Von Aster & Shalev, 2007). In the DSM-5, MLD does not appear as a single disorder but as a learning disability among other concerns. In addition to difficulties in processing numerical quantities, children with MLD often have extensive difficulties acquiring arithmetic skills. Children with MLD show procedural difficulties in calculation, which are often linked to the use of immature calculation strategies (Geary et al., 1991, 2000) and a lower mastery of arithmetic facts (Barouillet et al., 1997; Geary et al. 2000). However, as mentioned above, full mastery of arithmetic can be achieved only through a genuine understanding of the relationships between arithmetic operations (i.e., conceptual principles) to develop a flexible and adaptive application of mathematical knowledge (Baroody, 2003).

Very few studies have been conducted in this field among children with MLD, and these studies have not always been consistent. For example, a case study conducted by Hittmair-Delazer et al. (1995) involving an adult patient with MLD showed that despite impaired procedural computational skills, the patient had preserved conceptual knowledge. However, only the principle of commutativity was tested. Moreover, because the patient's impairment was due to a disease, it cannot be considered as MLD but rather acalculia. A study conducted among children showed no differences between children with MLD and TD children in tasks measuring conceptual principles such as commutativity or inversion (Russell & Ginsburg, 1984). However, other studies have highlighted examples of lower capacity for mastering these two principles in children with MLD (Andersson, 2010; Jordan et al., 2003). In these studies (Andersson, 2010; et al., 2003; Russell & Ginsburg, 1984), conceptual principles were tested along with other

mathematical tasks. In addition, the two principles were not distinguished separately in the analyses (the score included the success rates for both principles). The authors used the methodology developed by Baroody (1999) to test the inversion principle by presenting problems of complementarity (for instance, 23 + 14 = 37; what about 37 - 14?). The problems were presented in a symbolic written form and read aloud by the experimenter. The child was required to solve the problems orally within a limited time of 5 seconds. To our knowledge, no study has specifically addressed this inversion principle with the paradigms used in younger TD children, such as a comparison of 3-term inversion and standard problems. In addition, many studies have highlighted the difficulty of children with MLD in mastering and manipulating numerical symbolic materials such as digits (De Smedt & Gilmore, 2011; Kucian & Von Aster, 2015; Landerl et al., 2004, 2009; Rousselle & Noël, 2007). To our knowledge, the impact of presentation mode has also not been investigated in this area.

This literature review showed that the use of the inversion principle is not necessarily determined by children's arithmetic skills (Bryant, et al. 1999; Canobi, 2004; Gilmore & Bryant, 2006; Gilmore & Papadatou-Pastou, 2009; Rasmussen et al., 2003; Sherman & Bisanz, 2007; Watchorn et al., 2014). In addition, studies on the inversion principle in children with MLD are scarce and incomplete and show discrepant results (Andersson, 2010; Jordan et al., 2003; Russell & Ginsburg, 1984). Studies of TD children indicate that the presentation mode of an inverse problem affects children’s performance, and the contextualization of the problem helps them respond more accurately (Bryant et al. 1999; Gilmore et al. 2010; Gilmore & Bryant, 2006; Gilmore & Papadatou-Pastou, 2009; Klein & Bisanz, 2000; Rasmussen et al., 2003). Comparative studies of TD children and children with MLD are essential to identify the strengths and weaknesses of children with MLD with the aim of designing and implementing effective care. The aim of this study is therefore to investigate whether 10-year-old children with MLD are able to use conceptual knowledge, such as the inversion principle, to more easily solve 3-term

arithmetic problems without resorting to calculation. More specifically, recognizing that the presentation mode of problems has an impact on TD children (Gilmore and Papadatou-Patsou, 2009), we also assess the impact of presentation mode on children with MLD. We investigate whether, despite difficulties in arithmetic, children with MLD can use this inversion principle by comparing their results with those of TD children in solving 3-term problems that allow (inverse problem) or do not allow (standard problem) the use of this principle. In addition, we assess the impact of presentation mode by introducing these problems either in a symbolic mode with digits or in a picture mode. We use these two modes because Gilmore and Bryant's study (2006) showed that the effect of the type of material was significant in a group of children with good inversion performance but poor computation performance. We hypothesize that the picture mode should help children with MLD identify and use the inversion principle. Thus, if children with MLD, despite their low arithmetic skills, have preserved conceptual skills, they should solve inverse problems more easily than standard problems, especially in picture mode. On the other hand, if these abilities are affected, then children with MLD should achieve lower results than TD children for all problems regardless of presentation mode.

2. Method

2.1 Participants

Sixty-four children participated in this study. The group of children with mathematical learning disabilities (MLD) included 32 children (mean age in years; months ± SD (in months): 10; 4 ± 10; 16 girls) educated in the special school[1] of the Child Neurological Rehabilitation Unit of the Learning Disorders Reference Centre of the Kremlin-Bicêtre Hospital (France). These

[1] This school hosts children with severe specific learning disorders in day hospitals for one or two years to provide intensive and multiple reeducations (speech therapy, psychotherapy, psychomotor therapy, occupational therapy, orthoptic therapy) and specific education. Each year, 35 students living in the Paris region (France) are divided into 5 classrooms with a small number of students (7 students per classroom). Students have a curricula adapted to their academic skills provided by a teacher of the National Education that works closely with the therapists on site. When the students have progressed and learned to read (in general after one or two years), they then return to the ordinary school system with accommodations, sometimes in special classes for children with specific learning disorders.

children had all been diagnosed with learning disabilities in mathematics (neurodevelopmental disorders, DSM-5; 5th. Ed., American Psychiatric Association, 2013) by the Child Neurological Rehabilitation Unit of the Learning Disorders Reference Centre. Their disorder was persistent, their academic skills were well below the level expected for their age, and these difficulties could not be explained by intellectual or physical disability, psychiatric pathology, precarious psychosocial conditions or educational deficiencies (see Appendix A for more information about their comorbidities and their mathematical performances). All of them received logical-mathematical support in their follow-up at the Centre. The group of typically developing (TD) children included 32 children (mean age in years; months ± SD (in months): 10;1 ± 9; 16 girls) with a developmental trajectory within the norm who were educated in the ordinary school system in the same urban zone in the Paris region (France)[2]. All families and children provided written consent. This protocol was approved by the ethics committee of Bicêtre Hospital (CPP 16-003).

2.2 Procedure

Each child was assessed individually by a trainee psychologist in a quiet room for 30 minutes. A calculation fluency test and 3-term arithmetic problems were administered.

Calculation fluency (French translation of the Woodcock-Johnson et al. subtest "math fluency", 2001)

To assess the calculation performance of the two groups, the children performed a written *calculation fluency* test in which they had to solve as many one-digit addition, subtraction and multiplication problems as they could in less than 3 minutes. The number of correct answers was recorded.

[2] In these schools, approximately 25 students per classroom follow the national curriculum of their academic level provided by a teacher of the National Education.

Three-term arithmetic problems

Twenty problems were presented on the computer with Microsoft PowerPoint software. Among these problems, 10 were inverse problems ($a + b - b$) and 10 were standard problems ($a + b - c$). The total result of each problem could not exceed 24: term a was between 11 and 20, term b was between 12 and 22, and term c was between 9 and 19 (see all the problems in Appendix B).

Two presentation modes were used (10 problems in each presentation mode): a symbolic mode and a picture mode. The children began with either the symbolic mode or the picture mode. Within each mode, two presentation orders were possible: the child could start with either an inverse problem or a standard problem. Then the two types of problem were alternately presented. In the symbolic mode, the problems were presented on the screen as a literal expression with Arabic digits (Figure 1A). The experimenter orally stated the problem. In the picture mode, the problems were contextualized as a story presented orally and illustrated on the screen using 4 images showing a sequence of events with the addition and removal of marbles in a bag (Figure 1B). The child saw the picture on the screen, and the experimenter told him or her a brief story at the same time: "Imagine that there are 18 marbles in this bag. One child adds 14 marbles to this bag, then just after that, another child takes 14 marbles out of the same bag. How many marbles do you think are left in the bag?"

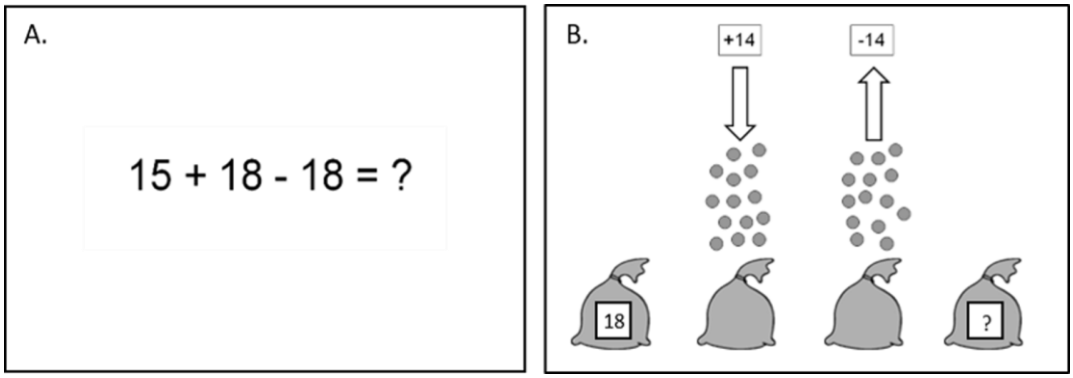

Figure 1: Examples of an inverse problem presented in **A.** as the symbolic mode and **B.** as the picture mode.

For both modes, two two-term training problems ($a + b =$) were presented in advance to familiarize the child with the task and material. Then, the child was informed that the problems would appear on the screen and that he or she must also respond orally and as quickly as possible. If the child could not answer after two minutes, the screen moved to the next problem. For each problem, the child's oral response and response time were recorded. At the end of the test, to obtain information about the strategy used, but without overloading the experiment for the children with MLD, the experimenter asked the child what strategy he or she used to solve the last inverse problem and standard problem.

The children performed all the tests. The different tests were presented in the order "3-term arithmetic problems in picture mode, 3-term arithmetic problems in symbolic mode and calculation fluency test" or in the order "3-term arithmetic problems in symbolic mode, 3-term arithmetic problems in picture mode and calculation fluency test".

3. Results

Statistical analyses show that the two groups of children were comparable in terms of age ($t_{(62)} = 1.05$, $p = .29$) and sex distribution (X^2 (1, N = 64) = 0.06, $p = .80$) (Table 1). However, as expected, children with MLD (m = 30.6, SD = 11.5), due to their disorder, exhibited much poorer performance than TD children (m = 56.3, SD = 23.05) on calculation fluency ($t_{(62)} = -5.57$, $p < .001$).

Table 1: Descriptive statistics for the MLD and TD groups

		MLD	TD
n		32	32
Gender		16 girls/16 boys	16 girls/16 boys
Age in years; months (SD in months)		10; 4 (10)	10; 1 (9)
Calculation fluency (average number of correct responses ± SD)		30.6±11.5	56.3±23.5
Average percentage of correct responses (ACC ± SD)			
Symbolic mode			
	Inverse problem	59.4±41.7	91.3±16
	Standard problem	24.4±24.2	70.6±28.3
Picture mode			
	Inverse problem	85±28.7	86.9±26.7
	Standard problem	20±23.8	71.9±30.4
Average response time in sec (RT ± SD)			
Symbolic mode			
	Inverse problem	21.5±24.8	7.6±11.4
	Standard problem	33.4±17.8	17±13.2
Picture mode			
	Inverse problem	7.4±11.3	6.8±10
	Standard problem	34.6±21.8	15.9±12.6
Strategy used (on 32)			
Inverse problem			
	symbolic	inversion: 19; calculation: 13	inversion: 25; calculation: 7
	picture	inversion: 27; calculation: 5	inversion: 26; calculation: 6
Standard problem			
	symbolic	Left-to-right: 24; right-to-left: 8	Left-to-right: 21 ; right-to-left: 11
	picture	Left-to-right: 22; right-to-left: 10	Left-to-right: 22; right-to-left: 10
Finger counting		19	4

3.1 Percentage of correct responses (ACC) for the 3-term arithmetic problems

The analysis of ACC (average percentage of correct responses) was performed using a generalized linear mixed model, more precisely a logistic binomial regression where the probability of finding the right answer to a given problem depends on the presentation mode of the problem (symbolic mode or picture mode), on its type (inverse or standard problem) and on the group of children (MLD or TD). The mixed part of the model consists of the presence of a random intercept in the model, corresponding to the intragroup variability between children. The analysis was performed with R software (R Core Team, 2021) and the lme4 package (Bates et al., 2015).

The analysis indicated a three-way interaction for Group x Presentation mode x Type of problem, Wald *z-value = 4.0* ($p < .001$), and appreciable intragroup variability (sd = 1.38 on the logit scale) (Figure 2A). Because the interpretation of the main effects was affected by the interaction, we focused only on the analysis of this interaction (see appendix C for all tests). Our results showed that the presentation mode affected the performance of children with MLD only on inverse problems (p<.001, odds ratio (OR) = 5.6, 95% confidence interval (CI) = 3.1–10.6 ~~d =0.62~~), with children with MLD performing better on inverse problems when the problems were presented in picture mode rather than in symbolic mode. We also observed that in symbolic mode, children with MLD achieved lower performance than TD children on inverse problems (p<.001, OR = 14.6, 95% CI = 5.4–41.8) and standard problems ($p < .001$, OR = 15.6, 95% CI = 6.4-40.5). On the other hand, in picture mode, children with MLD achieved performance similar to TD children on inverse problems ($p = 0.4$, OR = 1.5, 95% CI = 0.5-4.2) but not on standard problems ($p < .001$, OR = 23.2, 95% CI = 9.3-61.6). Inverse problems were always completed more successfully than standard problems by both groups of children and under both presentation modes (symbolic and picture) (MLD symbolic: $p < .001$, OR = 7.2, 95% CI = 4.1-12.8; MLD

picture: $p < .001$, OR = 54.9, 95% CI = 27.7-114.7; TD symbolic: $p < .001$, OR = 6.7, 95% CI = 3.3-14.5; TD picture: $p <.001$, OR = 3.6, 95% CI = 1.9-7.0).

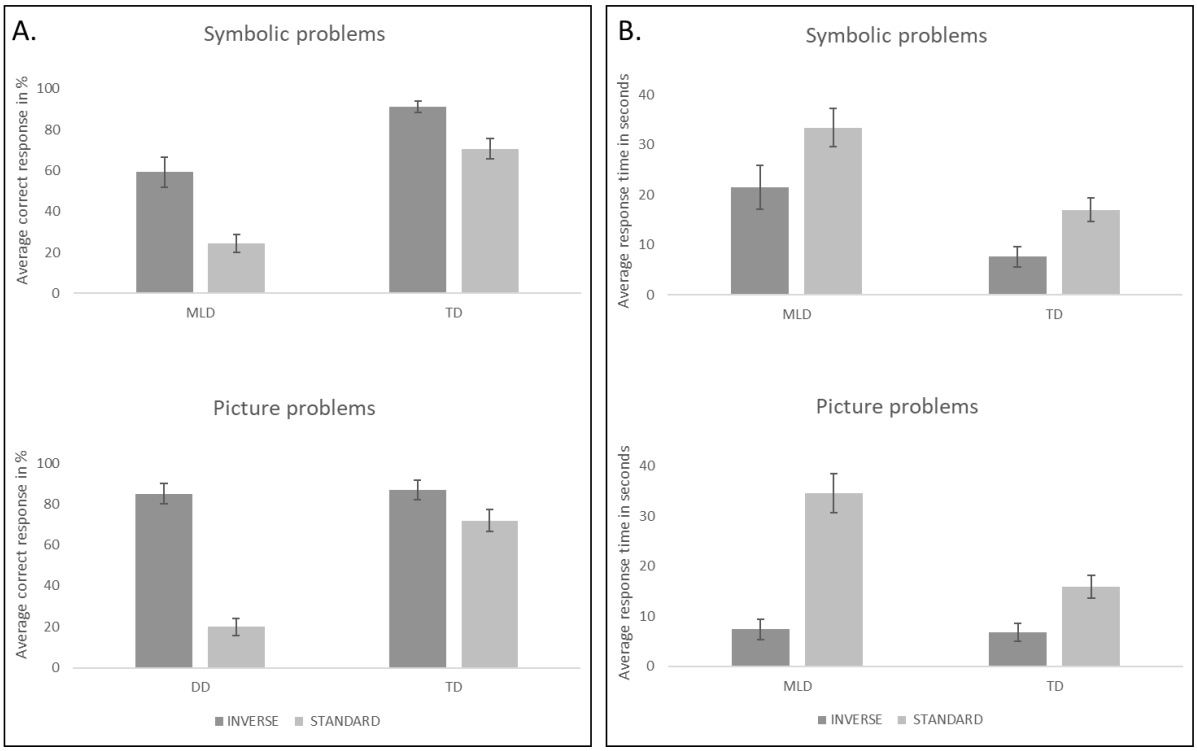

Figure 2. A. Average percentage of correct responses (ACC) and **B.** Average response time (RT) among each group (MLD *vs* TD), according to problem type (inverse *vs* standard) and presentation mode (symbolic *vs.* picture). The error bars are standard errors.

3.2 Response time (RT) for the 3-term arithmetic problems

The analysis of RTs[3] was performed using a linear mixed model. The analysis was performed with R software (R Core Team, 2021) and the lme4 and lmerTest packages (Kuznetsova et al., 2017).

We found a three-way interaction for Group x Presentation mode x Type of problem, Wald *z-value = 4.1, p* < .001 (Figure 2B). The intragroup standard deviation is estimated to 11.2 seconds, with a residual deviation equal to 16.9 seconds (see appendix C for all tests). Our results

[3] All the RTs in seconds, regardless of the accuracy of the responses, were included in the RT analyses.

show that the presentation mode affected the RTs of children with MLD only for inverse problems. Children with MLD were faster in responding to inverse problems when these problems were presented in picture mode rather than in symbolic mode ($p < .001$, beta = 14.2, 95% CI = 10.5-17.9). In addition, children with MLD were consistently slower than TD children on standard problems in both modes (symbolic: $p <= .001$, beta = 16.4, 95% CI = 9.8-22.9; picture: $p < .001$~~3~~, beta = 18.7, 95% CI =12.1-25.3) and on inverse problems in symbolic mode (p<.001, beta = 13.9, 95% CI = 7.3-20.5), but they were as quick to respond as TD children on inverse problems in picture mode ($p = 0.9$). Finally, it should be noted that the inverse problems were consistently solved faster than the standard problems by both groups of children and in both presentation modes (symbolic and picture) (MLD symbolic: $p < .001$, beta = 11.8, 95% CI = 8.2-15.6, MLD picture: $p < .001$, beta = 27.2, 95% CI = 23.5-30.9; TD symbolic: $p < .001$, beta = 9.4, 95% CI = 5.7-13.1; TD picture: $p < .001$, beta = 9.1, 95% CI = 5.4-12.8).

3.3 Correlations between the scores of inverse problems and calculation fluency in all children.

To analyze whether the children's performance on inverse problems was related to computational performance, we also performed correlations between the calculation fluency score and the ACC for inverse problems in both presentation modes. The results show that performance in inverse problems correlated with the score achieved for calculation in symbolic mode ($r = .44$, $p < .001$) but not in picture mode ($r = .12$, $p > .05$).

3.4 Strategies used when solving inverse problems.

When the children finished the problems, they were asked to explain the strategy they used to solve the last inverse problem and the last standard problem. Each response was coded after the data collection was complete according to strategies identified in previous studies (Bisanz & LeFevre, 1990; Klein and Bisanz, 2000; Robinson et al., 2006; Watchorn et al. 2014). For the reliability of

the coding system, two coders independently coded 10% of the participants' responses. Interrater reliability (the number of agreements divided by the number of comparisons) was 96.4. Regarding inverse problems, we coded each response according to one of the following 3 types of strategy: the inversion strategy (the child clearly stated that calculation is not necessary), the calculation strategy (the child explained the calculation with addition and subtraction) and another strategy (strategy other than the inversion and calculation strategies) (Table 2).

Table 2. Types of strategies used by the children

	Example of an inverse problem 17 + 16 - 16
Inversion	Use of an inversion shortcut: “I counted that 16 minus 16 is 0. If we do not remove any number then the answer is 17”
Calculation	Description of the calculation strategy: “I add 16 to 17. This means 33 minus 16, so we still have 17"
Other	Any other strategy: e.g. : “I just guessed!”
	Example of a standard problem: 15 + 18 - 14
Left-to right	Adding before subtracting: “15 plus 18 is 33. 33 minus 14 is 19”
Right-to-left	Subtracting before adding: “18 minus 14 is 4. 4 plus 15 is 19”
Other	Any other strategy: e.g. : “I don’t know”

The results show that the distribution of strategies used by the children with MLD and the TD children did not differ in the symbolic mode (X^2 (1, N = 64) = 2.62, p = .11) or in the picture mode (X^2 (1, N = 64) = 0.11, p = .74). Children in both groups mainly used the inversion strategy (although this was less obvious for the children with MLD in the symbolic mode).

Concerning standard problems, we also coded each response according to one of the following 3 types of strategy: the left-to-right strategy (the child adds the first 2 numbers and then subtracts them according to the order of the problem), the right-to-left strategy (based on the concept of associativity, the child first solves the subtraction of the last 2 numbers and then adds it to the first, but not according to the order of the problem) and another strategy (strategy other than the left-to-right and right-to-left strategies) (Table 2). The results show that the distribution of strategies used by the children with MLD and the TD children did not differ in symbolic mode (X^2 (1, N = 64) = 0.67, p = .41) or picture mode (X^2 (1, N = 64) = 0, p = 1). Children in both groups mainly solved these problems in a linear way from left to right as presented, first adding and then subtracting.

Finally, we identified the number of children who counted on their fingers. The children with MLD used their fingers more frequently than the TD children to solve 3-term arithmetic problems (X^2 (1, N = 64) = 15.27, p < .001)

4. Discussion

4.1 Findings and interpretation

The aim of this study was 1) to investigate whether 10-year-old children with MLD were able to use the inversion principle in 3-term arithmetic problems as TD children do and 2) to assess the impact of presentation mode on solving this type of problem. For this purpose, children with MLD and TD children completed 3-term arithmetic problems that allowed (inverse problem) or did not allow (standard problem) the use of the inversion principle in two presentation modes (symbolic or picture). As expected, the results show that children with MLD had more difficulties solving 3-

term problems than TD children did. However, when the problem was contextualized with pictures, children with MLD seemed to solve~~d~~ a 3-term arithmetic problem as well as the TD children based on the inversion principle. Moreover, although children with MLD had difficulties operationalizing and solving arithmetic problems, when the problem was contextualized, these children were able to identify and make good use of the conceptual shortcut of inversion and thus achieved a similar performance to that of TD children.

In this type of paradigm, the use of this principle is revealed by better performance and greater speed in solving inverse problems than standard problems. We note that, in picture mode, children with MLD had better performance (more than 80% of ACC) and were faster (approximately 7 seconds) in responding to inverse problems than standard problems, which reveals the use of the inversion principle. As expected due to their disorder, if we look at the performance of children with MLD on standard problems, they reached only the 20% level for ACC and took quite a while to answer (more than 30 seconds). This result allows us to better perceive the calculation difficulties of children with MLD regardless of presentation mode. In addition, the difference between inverse and standard problems was even more pronounced among children with MLD, highlighting their substantial difficulties in solving arithmetic problems. Moreover, children with MLD were able to use the inversion principle to achieve a level of performance similar to that of TD children, but only in picture mode. However, we observed that in symbolic mode, children with MLD attained approximately 60% for ACC when solving inverse problems. It seems that children are sometimes able to perceive and use the inversion principle even if it is less obvious and less automatic when the problem is presented only with digits. Contextualization allows children with MLD to reduce cognitive load, enabling them to focus on alternative strategies. The freed resources allow children to analyze the problem and identify the conceptual shortcut that does not require calculation. Our results support those of Russell and Ginsburg (1984), who argued that the

conceptual capacities of children with MLD were preserved in inversion and commutativity. The results contradict those of Andersson (2010) and Jordan et al. (2003), who claimed the opposite.

These results are also in line with previous studies conducted among the general population that highlight the impact of presentation mode on children's performance. However, in the present study, this effect of presentation mode was found only in children with MLD. We can explain this result by the fact that the TD children were 10 years old and that this effect of presentation mode is often found in younger children (Bryant et al., 1999) or children with lower calculational abilities (Gilmore & Bryant, 2006). In addition, to avoid disadvantaging the children with MLD, the problems we chose involved average numbers, and the final result did not exceed 24. It is possible that this effect could be found in TD children if they were presented with larger numbers. Nevertheless, the TD children seemed to use the inversion principle well in both modes because they were better and faster when solving inverse rather than standard problems. The results concerning the strategies used in both types of problems also point in this direction. The children with MLD reported the use of inversion strategies for inverse problems, as did the TD children, while for standard problems, the children with MLD reported left/right strategies as did TD children. Thus, the children with MLD seemed to be able to adapt their strategy based on the problems to be solved.

These results are consistent with those observed by Watchorn et al. (2014) in a symbolic task and the results of Gilmore and Bryant (2006). Some children with a low level of arithmetic are still able to use this principle. The use of the inversion principle therefore does not seem to be linked to children's arithmetic abilities, especially when the material used is imaged. In Gilmore and Bryant's (2006) study, it appeared that contextualization with pictures favored only the group of children who had weak results in arithmetic but who were able to use the inversion principle. Our results therefore add support for this observation. It appears that children with calculation deficits are able to use the inversion principle and that contextualization helps them identify this principle. The results of the correlations also point in this direction. Moreover, it appears that the link between

the use of inversion and calculation capacities is not perceptible in picture mode but is perceptible in symbolic mode. This result indicates that the inversion principle is not used only by children who are highly skilled in calculation.

Research on MLD shows that for many children with MLD, their difficulty does not necessarily stem from a lack of sense of numbers (understanding quantity; Butterworth, 2005; Piazza et al., 2010; Wilson & Dehaene, 2007) but rather from a lack of number sense access (difficulty accessing quantity from symbols; Lafay et al., 2017; Noël & Rousselle, 2011; Olsson et al. , 2016; Rousselle & Noël, 2007). Our results support this notion by highlighting the negative impact of symbolization on the ability to use the conceptual inversion principle. The symbolic mode, which uses only digits, does not allow children with MLD to perceive the cognitive shortcut they seem to grasp as well as the TD children. Children with MLD thus seem to have preserved inversion capacities when the task is concrete and moves away from the symbolic.

4.2. Educational and clinical implications

As suggested by Deruaz et al. (2020), it is important to collaborate between the fields of education and cognitive science to better care for children with MLD by designing remedial interventions or support devices for instance. Our results present interesting prospects for teaching the inversion principle to TD children and for the care of children with MLD.

Before they start school, children use mathematical concepts in their daily lives and develop informal mathematical knowledge that they must reorganize when numerical symbolization emerges (Lubin et al., 2015). In studies of TD children, a gap has been observed between intuitive and informal knowledge and the formal knowledge taught in schools (Bisanz et al., 2009). It seems that, similar to the two perspectives discussed previously in the introduction, we can highlight two relatively independent levels of inversion: an implicit and rather concrete/nonsymbolic first level that is acquired early and an explicit and symbolic second level that is acquired later. The transition

from the first to the second level seems difficult. Thus, it is essential for teachers and professionals who care for children with difficulties to create more explicit bridges between these different types of knowledge. Intervention studies have highlighted the benefits of teaching the inversion principle to TD children (Ching & Wu, 2019; Lai et al., 2008; Nunes et al., 2009, 2012; Robinson & Dubé, 2012; 2013). Recently, Ching and Wu (2019) showed that interventions with concrete representations were more effective than abstract representations for kindergarten children with lower inversion knowledge. Moreover, their results suggested that using a concreteness fading intervention (i.e., learning a new concept progressively with a physical model, then pictorial representations and then abstract representation) seemed promising to help children integrate the inversion principle.

We believe it is important to teach children explicit conceptual strategies that can enable them to solve arithmetic problems in a quicker and more effective manner. The study of Van Garderen et al. (2020) showed the importance of supporting the conceptual understanding on symbolic numerical magnitude thanks to an explicit intervention in children who struggle in mathematics. In view of our results, the passage through contextualization seems to be a way to connect the two levels, especially for children with MLD. Many studies have shown the importance of contextualization and manipulation in mathematics (for a meta-analysis, see Carbonneau et al., 2013; see also Bouck & Park, 2018 and Peltier et al, 2020 for children with mathematics disabilities). It therefore seems appropriate to explicitly teach the most vulnerable pupils the links between addition and subtraction by allowing them to experiment with the inversion principle through concrete and contextualized exercises and then gradually teaching them to relate this principle to numerical symbols. Moreover, because this conceptual principle seems to be preserved in MLD, it seems appropriate to make children with MLD aware of this shortcut strategy that they have at their disposal so that they can use this conceptual knowledge to solve complex calculations more effectively.

4.3 Limitations

Our study has some limitations, and the findings should be interpreted with caution. First, our sample size was small and heterogeneous. Indeed, we recruited children from the Child Neurological Rehabilitation Unit of the Learning Disorders Reference Centre. These children were schooled in day hospitals so that they could follow their schooling program in an adapted care center for their multiple and severe disorders. Although all the children were diagnosed with MLD, all of them also had reading/written expression disorders, and some had comorbid diseases, such as developmental language disorders, ADHD or writing disorders. It is argued that isolated MLD is very rare and that there are many types of MLD (Bartelet et al., 2014; Skagerlund & Träff, 2016). Twenty to 60% of children with MLD are reported to have associated learning difficulties, such as dyslexia (Dirks et al., 2008; Mayes and Calhoun, 2006; Morsanyi et al. 2018). Thus, it would be interesting to conduct further research by characterizing different profiles. Moreover, one could argue that differences between the curricula and organizations of this school and the ordinary school could explain the results. It would be interesting to perform the same study with children with MLD who are schooled in the ordinary school system.

In addition, we used the methodology employed by Gilmore and Bryant (2006) for the two presentation modes that seemed to exhibit the greatest contrast in children with poor calculation skills but good inversion skills. However, the picture mode also includes written numerical symbols. In one case, we had a contextualized problem illustrated with images and a story, while in the other case, we used only quantified problems stated verbally by the experimenter. However, this contextualization does not help children with MLD solve standard problems requiring calculation; it only helps them identify and effectively use the inversion principle.

Another potential limitation concerns our choice of asking the children about the strategy used at the end of the experiment, contrary to most studies. This may not reflect the real strategy used by the children during the experiment. Indeed, children often discover the strategy during the

problem solving session, so they might have used calculation on some problems and the inversion shortcut on others, which may skew the results. However, we made this choice to not overload the experiment for the children with MLD, for whom the arithmetical task was already very difficult.

One could argue that our design study includes a potential flaw. We previously highlight that contextualization can decrease the cognitive load of the MLD children. However, some studies indicated that working memory (WM) may be a factor in explaining individual differences in inversion shortcut use/understanding (Klein & Bisanz, 2000; Rasmussen et al. 2003). Moreover, the WM skills are often impaired in MLD children (Peng & Fuchs, 2016). But, we did not collect any measures of WM to control this in the study design. It would be fundamental to take into account this variable in further studies whose results could lead to reconsider the best possible remedial intervention: helping MLD children to improve their WM or making connections between implicit and explicit inversion thanks contextualization.

One could wonder whether the use of shortcut inversion strategies has to do with children's ability to approach the 3-term problem in a self-regulated way, allowing them to inhibit the tendency to immediately start solving the problem without any previous analysis and planning. Robinson & Dubé (2013) showed that children with weak inhibition skills had difficulties inhibiting a well-learned computational algorithm. Given that self-regulation skills are often impaired in children with MLD (Deng et al., 2020; Wang et al., 2012), it is possible that they have an implicit understanding of this concept but are not capable of using it in symbolic mode not because of a lack of conceptual understanding but rather a lack of self-regulation skills. The picture mode potentially helped them perceive the shortcut strategy by decreasing their cognitive load, leading them to better self-regulation. This yields new research perspectives.

Finally, one could argue that an age match/ability match design would be more suitable than the age matched design used here. We chose to perform an age-matched design because the aim of our research was to investigate whether 10-year-old children with MLD were able to use the inversion principle to more easily solve 3-term arithmetic problems without resorting to calculation

and to explore the impact of the presentation mode. However, this type of design could be relevant to investigate the nature of the difficulties in terms of deficit or delay, and this could be the subject of future research.

5. Conclusion

Our study is the first to investigate the inversion principle in children with MLD using a paradigm that contrasts 3-term inversion and standard arithmetic problems in two presentation modes. It appears that 10-year-old children with MLD are as capable as TD children of solving a 3-term arithmetic problem based on the inversion principle when the problem is contextualized. The mastery or lack of mastery of conceptual principles in arithmetic MLD has not been well documented. These results call for further research to assess the different principles while modulating the presentation mode to observe whether levers can be identified and to guide care actions that can be implemented. Our results provide interesting perspectives on the care of children with MLD and encourage the implementation of intervention studies to observe the benefit of explicit and contextualized teaching of the inversion principle on the performance of children with MLD.

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Appendix A. Mathematical performances of the children with MLD collected during their clinical follow-up

The children with MLD were schooled in the same special school in the Child Neurological Rehabilitation Unit of the Learning Disorders Reference Centre of the Kremlin-Bicêtre Hospital (France) for severe learning disorders. All had specific learning disorders with impairment in reading, written expression and mathematics, and most of them had comorbidities such as developmental language disorders (75%), ADHD (16%) or writing disorders (12.5%). All of them received logical-mathematical support in their follow-up at the hospital. At the special school entry, they performed a standardized mathematical achievement battery, either the Neuropsychological Test Battery for Number Processing and Calculation in Children (ZAREKI-R; Von Aster & Dellatolas, 2006, French version calibrated on French children between 6 and 11.6 years) or the Test for the Diagnosis of Basic Skills in Mathematics Test (TEDI-MATH battery, Van Nieuwenhoven et al., 2001; French version calibrated on French children between 4 and 9 years) when their level was too low to perform the ZAREKI-R and the arithmetic subtest of the K-ABC (KAB-C battery, Kaufman & Kaufman, 1993; French version calibrated on French children between 2.6 and 12 years). As shown in the table below, they presented severe difficulties in mathematical tests: the mean score deviated -2.2 SD from the norm on the ZAREKI-R battery, and there is an approximately 2-year delay in the TEDI-MATH battery and in the arithmetic subtest of the KAB-C

MLD Participants	Age (years;months)	Gender (Boy, Girl)	Deviation from the norm in SD at the ZAREKI-R	Delay in years and months on the TEDI-MATH	Delay in years and months on the arithmetic subtest of KABC
1	10;6	B	-2.21	/	2;6
2	10;6	B	/	4;0	2;9
3	12;3	B	-3.29	/	4;9
4	9;6	B	/	1;6	2;0
5	10;10	G	-1.96	/	1;7
6	11;11	G	-1.74	/	/
7	11;6	G	-3.54	/	3;5
8	10;6	G	-1.86	/	1;9
9	10;6	G	-1.10	/	1;9
10	10;7	G	/	1;6	2;1
11	9;6	B	/	2;6	1;5
12	9;6	G	/	2;6	1;8
13	9;4	B	-2.18	/	1;3
14	11;3	G	/	/	2;2
15	11;9	G	-1.67	/	3;5

Appendix A. (continued)

MLD Participants	Age (years;months)	Gender (Boy, Girl)	Deviation from the norm in SD at the ZAREKI-R	Delay in years and months on the TEDI-MATH	Delay in years and months on the arithmetic subtest of KABC
16	9;5	B	/	2;0	1;7
17	9;5	G	/	1;6	1;10
18	9;8	B	/	3;0	2;4
19	9;7	B	/	2;0	/
20	10;6	B	/	/	3;0
21	9;4	B	/	2;0	1;8
22	9;6	B	/	2;0	2;8
23	10;10	G	/	/	2;11
24	10;6	B	/	2;0	2;1
25	11;5	G	-3.58	/	3;0
26	10;9	G	/	/	1;3
27	10;1	G	/	1;6	2;1
28	9;7	G	/	1;6	1;5
29	11;11	B	-2.46	/	2;0
30	9;6	B	/	2;6	1;10
31	9;8	G	/	1;6	1;11
32	10;2	B	-1.13	/	1;9
		Mean	**-2.4**	**2;0**	**2;1**

*Note./= unavailable data; ZAREKI-R (von Aster & Delatollas, 2006); TEDI-MATH (*Van Nieuwenhoven et al., 2001); K-ABC (Kaufman & Kaufman, 1993)

Appendix B: the 20 three-term arithmetic problems

Problem type	Symbolic mode	Difference between the addends b and c	Picture mode	Difference between the addends b and c
Inverse problem	11 + 22 – 22 = 11	/	16 + 17- 17 = 16	/
Standard problem	15 + 18 – 14 = 19	4	14 + 19 -16 = 17	3
Inverse problem	17 + 16 – 16 = 17	/	20 + 12 – 12 = 20	/
Standard problem	19 + 13 – 10 = 22	3	18 + 14 – 11 = 21	3
Inverse problem	13 + 15 – 15 = 13	/	12 + 21 – 21 = 12	/
Standard problem	11 + 22 – 19 = 14	3	20 + 12 – 9 = 23	3
Inverse problem	15 + 18 – 18 = 15	/	18 + 14 – 14 = 18	/
Standard problem	17 + 16 – 13 = 20	3	12 + 21 – 18 = 15	3
Inverse problem	19 + 13 – 13 = 19	/	14 + 19 – 19 = 14	/
Standard problem	13 + 15 – 12 = 16	3	16 + 17 – 15 = 18	2

Appendix C: Statistical analyses on the three-term problems

Analysis of ACC: The three tables exposed the different 2 to 2 comparisons.

Table C1 – Comparison of MLD and TD Children for Each Type and Presentation Mode Combination

Type of Problem	Presentation Mode	OR (TD)	95% CI	z-value	p
Standard	Symbolic	15.6	6.4–40.5	5.9	$4\ 10^{-9}$
Standard	Picture Mode	23.2	9.3–61.6	6.6	$4\ 10^{-11}$
Inverse	Symbolic	14.6	5.4–41.8	5.2	$2\ 10^{-7}$
Inverse	Picture Mode	1.5	0.5–4.2	0.8	0.4

Note. MLD children are the reference group: ORs greater than 1 correspond to TD children performing better for the problem; OR = odds ratio; CI = confidence interval

Table C2 – Comparison of Symbolic and Picture Mode for Each Group and Type of Problem Combination

Group	Type of Problem	OR (Picture mode)	95% CI	z-value	p
TD	Standard	1.1	0.6–1.9	0.3	0.8
TD	Inverse	0.6	0.3–1.3	-1.4	0.2
MLD	Standard	0.7	0.4–1.3	-1.0	0.3
MLD	Inverse	5.6	3.1–10.6	5.5	$4\ 10^{-8}$

Note. The reference presentation mode is Symbolic mode: ORs greater than 1 correspond to better performance for Picture mode; OR = odds ratio; CI = confidence interval

Table C3 – Comparison of Standard and Inverse Types for Each Group and Mode of Presentation Combination

Group	Mode of Presentation	OR (Inverse)	95% CI	z-value	p
TD	Symbolic	6.7	3.3–14.5	5.1	$4\ 10^{-7}$
TD	Picture Mode	3.6	1.9–7.0	3.7	$2\ 10^{-4}$
MLD	Symbolic	7.2	4.1–12.8	6.8	$8\ 10^{-12}$
MLD	Picture Mode	54.9	27.7–114.7	11.1	$2\ 10^{-28}$

Note. The reference type of problem is the Standard problem: ORs greater than 1 correspond to better performance for the Inverse problem; OR = odds ratio; CI = confidence interval

Analysis of RT: The three tables exposed the different 2 to 2 comparisons.

Table C4 – Comparison of MLD and TD Children for Each Type and Presentation Mode Combination

Type of Problem	Presentation Mode	Beta (seconds)	95% CI	z-value	p-value
Standard	Symbolic	16.4	9.8–22.9	4.8	$4\ 10^{-6}$
Standard	Picture Mode	18.7	12.1–25.3	5.5	$2\ 10^{-7}$
Inverse	Symbolic	13.9	7.3–20.5	4.1	$8\ 10^{-5}$
Inverse	Picture Mode	0.6	-6.0–7.1	0.2	0.9

Note. The reference group here is the TD group: positive beta values correspond to longer response times for MLD children; Beta is the estimated effect of belonging to the MLD group and is an estimate of the difference in response time, in seconds, between the MLD and TD groups. For comparison, the residual deviance of the response time is estimated to be 20.3 seconds. The significance of beta is tested by a Wald test.

Table C5 – Comparison of Symbolic and Picture presentation modes for Each Group and Type of Problem Combination

Group	Type of Problem	Beta (seconds)	95% CI	z-value	p-value
TD	Standard	1.1	-2.6–4.8	0.6	0.6
TD	Inverse	0.8	-2.9–4.5	0.4	0.7
MLD	Standard	-1.2	-4.9–2.5	-0.7	0.5
MLD	Inverse	14.2	10.5–17.9	7.5	$1\ 10^{-13}$

Note. The reference mode is Picture Mode: positive beta values correspond to longer response times for Symbolic Mode; Beta is the estimated effect of Picture presentation mode; it is an estimate of the difference of response time, in seconds, between Symbolic and Picture presentation mode. For comparison, the residual deviance if the response time is estimated to be 20.3 seconds. The significance of beta is tested by a Wald test.

Table C6 - Comparison of Standard and Inverse Types for Each Group and Mode of Presentation Combination

Group	Mode of presentation	Beta (seconds)	95% CI	z-value	p-value
TD	Symbolic	9.4	5.7–13.1	5.0	$8\ 10^{-7}$
TD	Picture Mode	9.1	5.4–12.8	4.8	$2\ 10^{-6}$
MLD	Symbolic	11.8	8.2–15.6	6.3	$5\ 10^{-10}$
MLD	Picture Mode	27.2	23.5–30.9	14.4	$1\ 10^{-43}$

Note. The reference type of problem is the Inverse type. Positive beta values correspond to longer response times for Standard problems.

Beta is the estimated effect of the Standard problem type; it is an estimate of the difference in response time, in seconds, between Inverse and Standard problems. For comparison, the residual deviance of the response time is estimated to be 20.3 seconds. The significance of beta is tested by a Wald test.

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