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Traitement statistique des donn ees manquantes-Part III Missing not at random data (MNAR)

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Traitement statistique des données manquantes

Atelier Statistique de la SFdS – 10 et 11 mars 2021

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Part III
Missing not at random data (MNAR)

C. Biernacki

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It is an ongoing research work

(with its own notations, possibly differing from Part I/II)

Take home message

1. The missing data pattern may convey some information on clustering
2. Embed the missingness mechanism directly within the clustering modeling step

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

6 Concluding remarks

7 Practical activity

Ignorable *vs.* non ignorable model

A missing mechanism is ignorable if likelihoods can be decomposed as

$$L(\theta, \psi; \underbrace{y^{\text{obs}}, c}_{\text{observed data}}) = L(\psi; c|y^{\text{obs}}) \times L(\theta; y^{\text{obs}})$$

Some simple algebra show that this occurs when missing mechanism is not MNAR

Inference of θ

"If the missing mechanism is ignorable then likelihood-based inferences for θ from $L(\theta; y^{\text{obs}})$ will be the same as likelihood based inference for θ from $L(\theta, \psi; y^{\text{obs}}, c)$." ([Little and Rubin, 2002] Section 6.2)

- M(C)AR is ignorable
- MNAR is not ignorable

Clustering: model-based approach

- Partition with K clusters: $Z = (z_1|\dots|z_n)^T \in \{0,1\}^{n\times K}$ where
  - $z_i = (z_{i1},\dots,z_{iK}) \in \{0,1\}^K$
  - $z_{ik} = 1$ if y_i belongs to cluster k, $z_{ik} = 0$ otherwise
- Mixture model: $y_1,\dots,y_n$ are i.i.d. from the mixture

$$f(y_i;\pi,\theta) = \sum_{k=1}^{K} \pi_k f_k(y_i;\theta_k)$$

  - $\pi_k = P(z_{ik} = 1)$, $\pi = (\pi_1,\dots,\pi_K)$
  - $f_k(.;\theta_k)$: pdf of the k-th component parametrized by θ_k, $\theta = (\theta_1,\dots,\theta_K)$
    - **continuous data**: $f_k(.;\theta_k) = \phi(.;\mu_k,\Sigma_k)$ is the d-variate **Gaussian distribution** with mean vector μ_k and covariance matrix Σ_k
    - **categorical data**: the features are independent conditionally to the group membership i.e. $f_k(.;\theta_k) = \prod_{j=1}^{d} f_{kj}(.;\theta_{kj})$, where $f_{kj} = \prod_{\ell=1}^{\ell_j} (\theta_{kj}^{\ell})^{y_{ij}^{\ell}}$ is the **multinomial distribution** with $\theta_{kj} = (\theta_{kj}^{\ell} = \mathbb{P}(y_{ij}^{\ell} = 1|z_{ik} = 1))_{\ell=1,\dots,\ell_j}$
    - **mixed data**: the features are independent conditionally to the group membership, $f_k(.;\theta_k)$ is the product of univariate Gaussian and multinomial distributions
    - Can also be extended to other cases (count data with Poisson distributions for instance)

Question we address in this work

Which distribution $\mathbb{P}(c|y,z;\psi)$ to propose in this clustering context?

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

6 Concluding remarks

7 Practical activity

Proposed zoology of MNAR models in clustering

$$\mathbb{P}(c_i|y_i, z_{ik} = 1; \psi) = \prod_{j=1}^{d} \mathbb{P}(c_{ij}|y_i, z_{ik} = 1; \psi)$$

- $\boxed{\text{MNAR}y^k z^j}$, with $\psi = (\alpha, \beta)$ where $\alpha = (\alpha_{11}, \ldots, \alpha_{1d}, \ldots, \alpha_{K1}, \ldots, \alpha_{Kd})^T \in \mathbb{R}^{Kd}$ and $\beta = (\beta_{11}, \ldots, \beta_{1d}, \ldots, \beta_{K1}, \ldots, \beta_{Kd})^T \in \mathbb{R}^{Kd}$

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj} + \beta_{kj} y_{ij}),$$

  with ρ the cdf of any continuous distribution (logit, probit)

- $\boxed{\text{MNAR}yz}$, $\boxed{\text{MNAR}y^k z}$, $\boxed{\text{MNAR}yz^j}$

- $\boxed{\text{MNAR}y}$, $\boxed{\text{MNAR}y^k}$

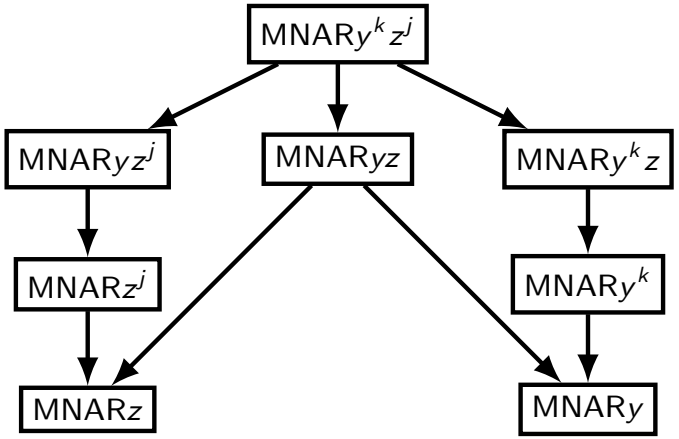
  $\psi = (\beta_{11}, \ldots, \beta_{1d}, \ldots, \beta_{K1}, \ldots, \beta_{Kd})^T$

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\beta_{kj} y_{ij})$$

- $\boxed{\text{MNAR}z}$, $\boxed{\text{MNAR}z^j}$

  $\psi = (\alpha_{11}, \ldots, \alpha_{1d}, \ldots, \alpha_{K1}, \ldots, \alpha_{Kd})^T$

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj})$$



Overview of the proposed MNAR models

	Effect on the variable j		Effect on the class membership k		Nb parameters	
	Depends on j	Depends on k	Depends on j	Depends on k	Continuous	Categorical
MNAR$z^j y^k$	✓	✓	✓	✓	$2Kd$	$K(d + \sum_{j=1}^{d}(\ell_j - 1))$
MNARyz^j	✓	✗	✓	✓	$(K+1)d$	$Kd + \sum_{j=1}^{d}(\ell_j - 1)$
MNAR$y^k z$	✓	✓	✗	✓	$K(d+1)$	$K(1 + \sum_{j=1}^{d}(\ell_j - 1))$
MNARyz	✓	✗	✗	✓	$(K+d)$	$K + \sum_{j=1}^{d}(\ell_j - 1)$
MNARy	✓	✗	✗	✗	d	$\sum_{j=1}^{d}(\ell_j - 1)$
MNARy^k	✓	✓	✗	✗	Kd	$K \sum_{j=1}^{d}(\ell_j - 1)$
MNARz	✗	✗	✗	✓	K	K
MNARz^j	✗	✗	✓	✓	Kd	Kd

Terminology in the sequel:

- MNARz, MNARz^j: the only effect of missingness is on the class membership
- MNAR$y*$: all the other models which considers the effect of the missingness depending on the variable
- MNAR$*$: all the models

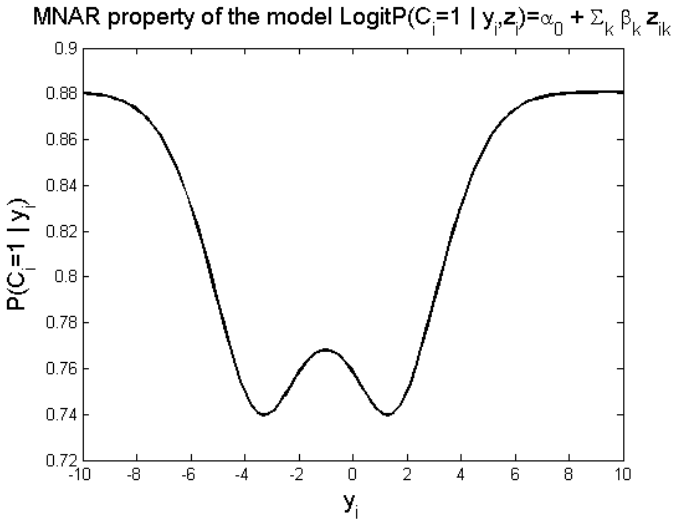
MNARz analysis: it depends on y through z!

$$P(c_{ij} = 1|y_i; \theta, \psi) = \sum_{k=1}^{K} P(c_{ij} = 1|y_i, z_{ik} = 1; \psi) P(z_{ik} = 1|y_i; \theta)$$

Example of a univariate Gaussian model with the three components

$$0.2N(\cdot; 0, 1) + 0.3N(\cdot; 1, 2) + 0.5N(\cdot; 2, 3)$$

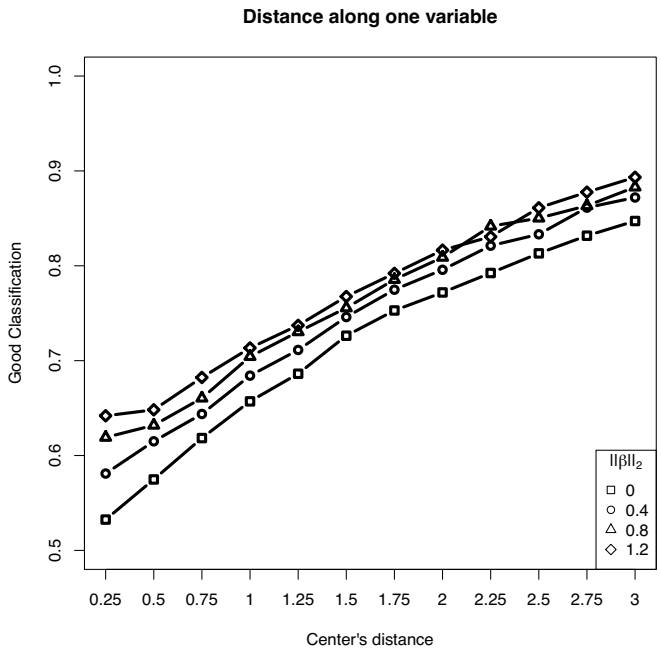
and with parameters of the logit expression: $\alpha_0 = 1, \beta_1 = 1, \beta_2 = -1, \beta_3 = 1$



MNARz analysis: pattern c gives information on partition z!

Draw Bayes error of a MNARz model with two components and 20% of missing data

$$\pi_k = 0.5,\ \|\mu_2 - \mu_1\| \text{ varies},\ \Sigma_1 = \Sigma_2 = \mathbf{I}, |\beta_2 - \beta_1| \text{ varies}$$



Both μ_k and β_k act on the Bayes error

Reinterpretation of the MNARz and MNARz^j models as MAR

Commonly used in Machine Learning
[Jones, 1996, Little and Rubin, 2002, Josse et al., 2019]

Mixture model for Y^{obs} and Bernoulli distribution for C
$\Leftrightarrow$ MAR mixture model for $\tilde{Y}^{\text{obs}} = (Y^{\text{obs}}|C)$

For example,

$$Y^{\text{obs}} = \begin{pmatrix} ? & 2.6 & 5 \\ \text{blue} & 1.9 & 4 \\ \text{red} & 2.3 & ? \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

then $\tilde{Y}^{\text{obs}}$ is expressed as

$$\tilde{Y}^{\text{obs}} = \begin{pmatrix} ? & 2.6 & 5 & 1 & 0 & 0 \\ \text{blue} & 1.9 & 4 & 0 & 0 & 0 \\ \text{red} & 2.3 & ? & 0 & 0 & 1 \end{pmatrix}.$$

Proposition 1: in terms of maximum likelihood

The maximum likelihood estimate associated to the dataset $\tilde{Y}^{\text{obs}}$ under MAR model is the one associated to the dataset Y^{obs} under MNARz or MNARz^j models.
$\Rightarrow$ can be extended to other estimation strategies

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

6 Concluding remarks

7 Practical activity

Continuous and count data (1)

Previous works: [Teicher, 1963] (without NA), [Miao et al., 2016] (for MNAR data)

Identifiability for a mixture model with MNAR data
$\Leftrightarrow$ Mixture/MNAR parameters are uniquely determined from available information

Proposition 2: identifiability for continuous and count data

Assume that

1. The marginal mixture $\sum_{k=1}^{K} \pi_k f_k(y_i; \theta_k)$ is identifiable
2. There exists a total ordering $\preceq$ of $\mathcal{F}_j \times \mathcal{R}$, for $j \in \{1, \ldots, d\}$ fixed, where $\mathcal{F}_j = \{f_{1j}, \ldots, f_{Kj}\}$ and $\mathcal{R} = \{\rho_1, \ldots, \rho_K\} = \{\rho(.; \psi_1), \ldots, \rho(.; \psi_K)\}$. The total ordering is s.t. $\forall k < \ell$, $F_k = \rho_k f_{kj} \preceq F_\ell = \rho_\ell f_{\ell j}$ implies

$$\lim_{u \to +\infty} \frac{\rho_\ell(u) f_{\ell j}(u)}{\rho_k(u) f_{kj}(u)} = 0$$

Then the mixture model with one of the MNAR$*$ mechanisms is identifiable up to label swapping

Continuous and cound data (2)

Is the total ordering checked for classical distributions ?

f_k	Gaussian		Poisson	
ρ_k	Probit	Logit	Probit	Logit
MNAR$z^j y^k$ MNAR$y^k z$ MNARy^k	✓	generic idenfiability	✓	generic idenfiability
MNARyz^j MNARyz MNARy MNARz MNARz^j	✓	✓	✓	✓

Generic identifiability: all not-identifiable parameter choices lie within a proper subvariety, and thus form a set of Lebesgue zero measure

Categorical data

Previous work: [Allman et al., 2009] (without NA)
Recall that for categorical data: conditional independence of the features given the group membership holds i.e. $f_k(\cdot;\theta_k) = \prod_{j=1}^{d} f_{kj}(\cdot\,;\theta_{kj})$

Proposition 3: identifiability for categorical data

Assume that $d \geq 2\lceil \log_2 K \rceil + 1$ and $f_k(\cdot;\theta_k) = \prod_{j=1}^{d} f_{kj}(\cdot\,;\theta_{kj})$

✓ Then the mixture model with MNARz or MNARz^j mechanism is identifiable up to label swapping

✗ The mixture model with one of the MNAR$y*$ mechanisms is not identifiable

Mixed data

$f_k(\cdot;\theta_k) = \prod_{j=1}^{d} f_{kj}(\cdot\,;\theta_{kj})$, thus identifiability of mixed data directly follows from Proposition 2 for continuous variables and from Proposition 3 for categorical variables

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

6 Concluding remarks

7 Practical activity

EM algorithm: looks simple

The **expected complete log-likelihood** knowing the observed data and a current value of the parameters can be **decomposed into two parts**

$$\begin{aligned} Q(\theta, \psi, \pi; \theta^r, \psi^r, \pi^r) &= \mathbb{E}[\ell_{\text{comp}}(\theta, \psi, \pi; y, z, c) | y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r] \\ &= Q_y(\theta, \pi; \theta^r, \psi^r, \pi^r) + Q_c(\psi; \theta^r, \psi^r, \pi^r) \end{aligned}$$

$$Q_y(\theta, \pi; \theta^r, \psi^r, \pi^r) = \sum_{i=1}^{n} \sum_{k=1}^{K} (\tau_{ik})^r \log(\pi_k) + \sum_{i=1}^{n} \sum_{k=1}^{K} (\tau_{ik})^r E_{iy}^r(\theta)$$

$$Q_c(\psi; \theta^r, \psi^r, \pi^r) = \sum_{i=1}^{n} \sum_{k=1}^{K} (\tau_{ik})^r E_{ic}^r(\psi)$$

where for $i = 1, \ldots, n$ and $k = 1, \ldots, K$,

$$\begin{aligned} E_{iy}^r(\theta) &= \mathbb{E}\left[\log(f_k(y_i; \theta_k)) \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r\right] \\ E_{ic}^r(\psi) &= \mathbb{E}\left[\log(\mathbb{P}(c_i \mid y_i, z_{ik} = 1; \psi)) \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r\right] \\ (\tau_{ik})^r &= \mathbb{P}(z_{ik} = 1 \mid y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r) \quad \propto \quad \pi_k^r f_k(y_i^{\text{obs}}; \theta_k^r) \mathbb{P}(c_i \mid y_i^{\text{obs}}, z_{ik} = 1; \psi^r) \end{aligned}$$

EM algorithm for MNARz and MNARz^j

MNARz, MNARz^j : needs some computations but still simple.

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj}) \qquad (\textbf{independent of } y) \qquad (\Delta)$$

- Gaussian data : $(y_i | z_{ik} = 1; \theta^r) \sim \mathcal{N}(\mu_k, \Sigma_k)$
  - $\mathbb{P}(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r) = \mathbb{P}(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r)$ **using** (Δ) and

$$\left(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r\right) \sim \mathcal{N}\left((\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r\right)$$

    where $(\tilde{\mu}_{ik}^{\text{mis}})^r$ and $(\tilde{\Sigma}_{ik}^{\text{mis}})^r$ only depend on μ_k^r, Σ^r and y_i^{obs}
    $\Rightarrow E_{iy}^r(\theta) = \mathbb{E}\left[\log(f_k(y_i; \theta_k)) \mid y_i^{\text{obs}}, z_{ik} = 1, ; \theta^r\right]$ easy to compute (classical formulae)
  - **Using** (Δ)

$$E_{ic}^r(\psi) = \log(\mathbb{P}(c_i \mid z_{ik} = 1; \psi)) = \sum_{j=1}^{d} c_{ij} \log \rho(\alpha_{kj}) + (1 - c_{ij}) \log(1 - \rho(\alpha_{kj}))$$

  - **Using** (Δ)

$$(\tau_{ik})^r \propto \pi_k^r \phi(y_i^{\text{obs}}; (\mu_{ik}^{\text{obs}})^r, (\Sigma_{ik}^{\text{obs,obs}})^r) prod_{j=1}^{d} \rho(\alpha_{kj}^r)^{c_{ij}} (1 - \rho(\alpha_{kj}^r))^{1-c_{ij}}$$

EM algorithm for MNARz and MNARz^j

Recall that: $\left(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r\right) \sim \mathcal{N}\left((\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r\right)$.

- $\boxed{\text{E-step}}$: for $k = 1, \ldots, K$ and $i = 1, \ldots, n$, compute $(\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r, (\tau_{ik})^r$ and

$$(\tilde{y}_{i,k})^r = (y_i^{\text{obs}}, (\tilde{\mu}_{ik}^{\text{mis}})^r) \qquad \tilde{\Sigma}_{ik}^r = \begin{pmatrix} 0_i^{\text{obs,obs}} & 0_i^{\text{obs,mis}} \\ 0_i^{\text{mis,obs}} & (\tilde{\Sigma}_{ik}^{\text{mis}})^r \end{pmatrix}$$

- $\boxed{\text{M-step}}$: for $k = 1, \ldots, K$, compute

$$\pi_k^{r+1} = \frac{1}{n}\sum_{i=1}^n (\tau_{ik})^r \qquad \mu_k^{r+1} = \frac{\sum_{i=1}^n (\tau_{ik})^r (\tilde{y}_{k,i})^r}{\sum_{i=1}^n (\tau_{ik})^r}$$

$$\Sigma_k^{r+1} = \frac{\sum_{i=1}^n \left[(\tau_{ik})^r \left((\tilde{y}_{i,k})^r - \mu_k^{r+1})((\tilde{y}_{i,k})^r - \mu_k^{r+1})^T + \tilde{\Sigma}_{ik}^r\right)\right]}{\sum_{i=1}^n (\tau_{ik})^r}$$

For ψ^{r+1}: maximization of $Q_c(\psi; \theta^r, \psi^r, \pi^r)$ over ψ with a **Newton-Raphson algorithm** (classical procedure for link functions of interest)

An EM algorithm can also be easily derived for categorical data

Not EM algorithm for MNAR$y*$

MNAR$y*$: needs approximations

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj} + \beta_{kj} y_{ij}) \qquad (\textbf{not independent of } y)$$

- Gaussian data :
  - $(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1, c_i)$:
    ✗ not classical if ρ is Logit, ✓ truncated Gaussian distribution if ρ is Probit
  - No closed form of $E_{ic}^r(\psi)$ neither for Probit nor for Logit:

$$E_{ic}^r(\psi) = \sum_{j=1}^{d} c_{ij} \int_{\mathcal{Y}_{ij}^{\text{mis}}} \log(\rho(\alpha_{kj}+\beta_{kj} y_{ij}^{\text{mis}})) \frac{\rho(\alpha_{kj}^r + \beta_{kj}^r y_{ij}^{\text{mis}})^{c_{ij}} \mathbb{P}(y_{ij}^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r)}{\int_{\mathcal{Y}_{ij}^{\text{mis}}} \rho(\alpha_{kj}^r + \beta_{kj}^r x)^{c_{ij}} \mathbb{P}(x \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r) dx} dy_{ij}^{\text{mis}}$$
$$+ (1 - c_{ij}) \log(1 - \rho(\alpha_{kj} + \beta_{kj} y_{ij}^{\text{obs}}))$$

    ✗ not concave function if ρ is Logit
  - No closed form of $(\tau_{ik})^r$ neither for Probit nor for Logit

- In the Gaussian case, there is **no closed form** [Pirjol, 2013]
- SEM easier? random drawing instead of expectation

SEM algorithm for MNAR$y*$

- SE-step: draw the missing data $((y_i^{\text{mis}})^{r+1}, z_i^{r+1}) \sim (. \mid y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r)$

  Use of **One**-Gibbs sampling:
  - $(y_i^{\text{mis}})^{r+1} \sim (\cdot \mid y_i^{\text{obs}}, z_i^r, c_i; \theta^r, \psi^r)$:

    ✗ not classical if ρ is Logit, ✓ truncated Gaussian distribution if ρ is Probit
  - $z_i^{r+1} \sim (\cdot \mid y_i^{r+1}, c_i; \theta^r, \psi^r, \pi^r)$: draw the membership k of z_i^{r+1} from the **multinomial distribution** with probabilities

$$\mathbb{P}(z_{ik} = 1 | y_i^{r+1}, c_i; \theta^r, \psi^r, \pi^r) = \frac{\mathbb{P}(c_i | y_i^{r+1}, z_{ik} = 1; \psi^r)\mathbb{P}(y_i^{r+1} | z_{ik} = 1; \theta^r)\pi_k^r}{\sum_{h=1}^K \mathbb{P}(c_i | y_i^{r+1}, z_{ih} = 1; \psi^r)\mathbb{P}(y_i^{r+1} | z_{ih} = 1; \theta^r)\pi_h^r}$$

  Let $Y^{r+1} = (y_1^{r+1} | \ldots | y_n^{r+1})$, $Z^{r+1} = (z_1^{r+1} | \ldots | z_n^{r+1})$ be the imputed matrix and the partition

- M-step: for $k = 1, \ldots, K$, compute
  - π_k^{r+1} with the proportion of rows of Y^{r+1} belonging to class k
  - $\mu_k^{r+1}, \Sigma_k^{r+1}$ with the mean and covariance matrix of rows of Y^{r+1} belonging to class k
  - ψ^{r+1} with a Newton-Raphson algorithm

Summary for algorithms

	EM			SEM		
	Gaussian		Categorical	Gaussian		Categorical
MNARz MNARz^j	✓		✓	✓		✓
	Probit	Logit		Probit	Logit	
MNAR$y*$	no closed form	no closed form, optim. pb	not ident.	✓	require algorithms as SIR (costly)	not ident.

What about model selection?

Can select between MCAR and MNAR$*$ with any information criterion (BIC, ICL)

Even if the missing mechanism is ignorable for MCAR...

...need to model c to compare a MCAR and a MNAR model

CAUTION

- It is just a selection between several proposed MNAR models
- It is not deciding if missingness procedure is "genererically" MNAR or not

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

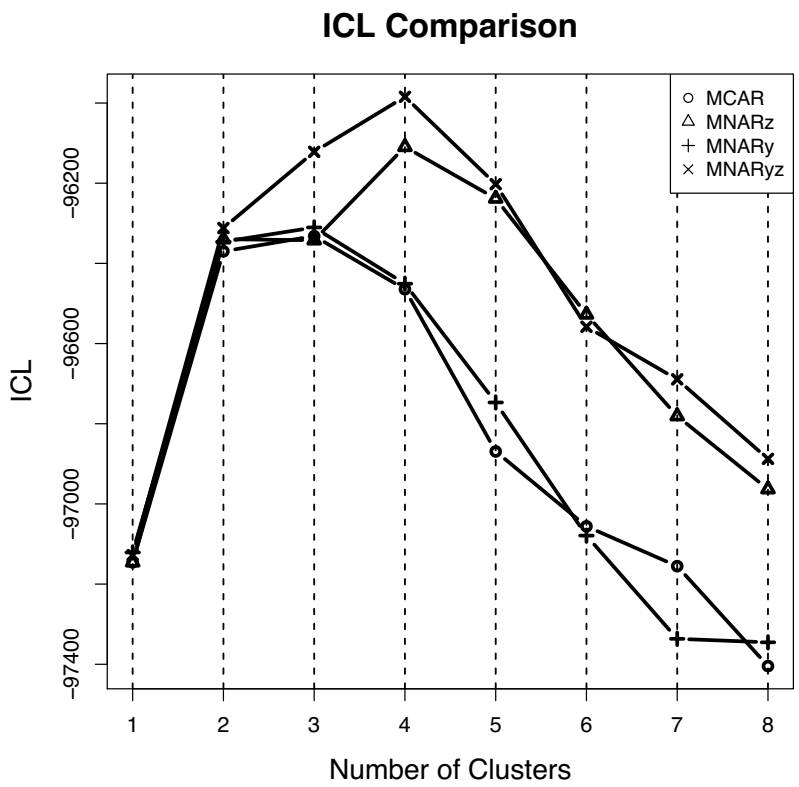
6 Concluding remarks

7 Practical activity

Hospital Data: continuous features case

- Number of patients: $n = 5\ 146$
- Number of features: $d = 7$
  - Age
  - Size
  - Weight
  - Cardiac frequency
  - Hemoglobin concentration
  - Temperature
  - Minimum Diastolic and Systolic Blood Pressure
- Percentage of missing data: 6.4%

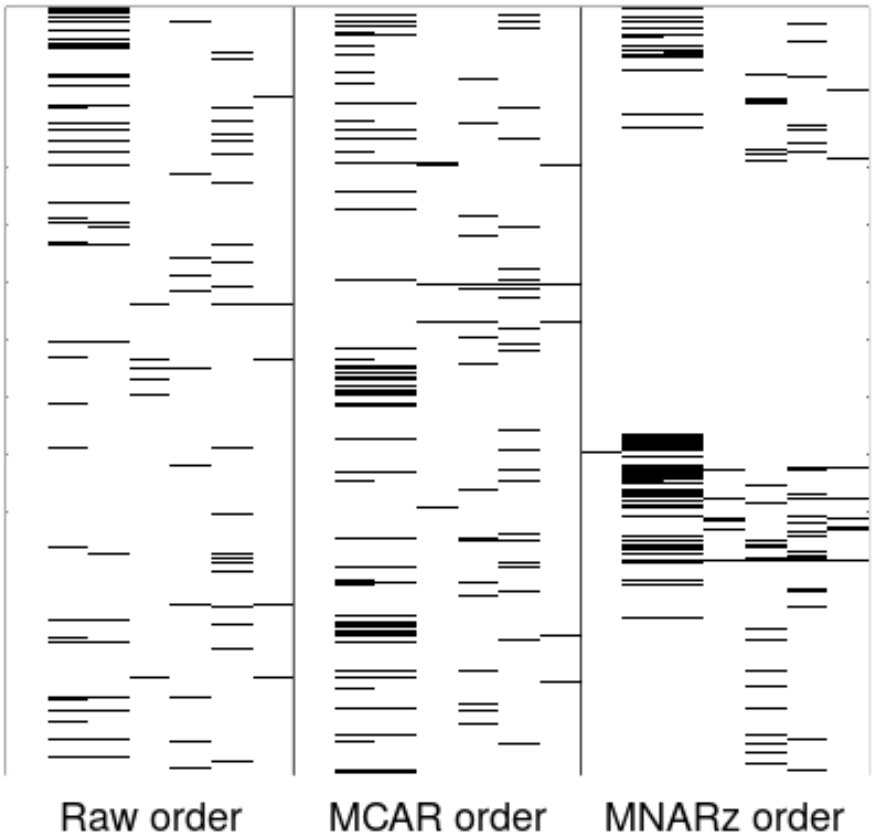
ICL comparison



- MCAR, MNARy and MNARz are equivalent until $K = 3$
- MNARz and MNARyz clearly indicate presence of an additional cluster ($K = 4$)

It seems to be an illustation of the effect of c through MNARz and MNARyz

Missing Pattern



It seems that MNARz modelling leads to a missing free cluster

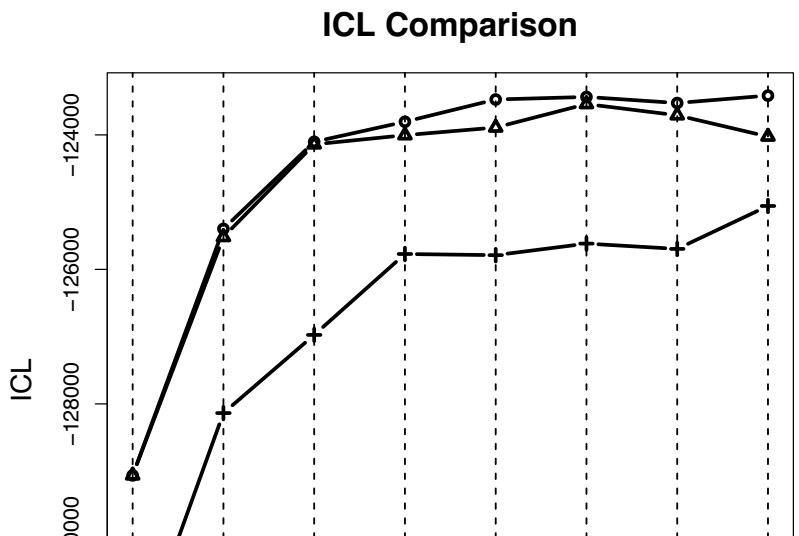
Hospital data: mixed features case

- Number of patients: $n = 5\ 146$
- Number of features: $d = 15$ (7 continuous and 8 categorical)
- Percentage of missing data: $\sim 4\%$

Model

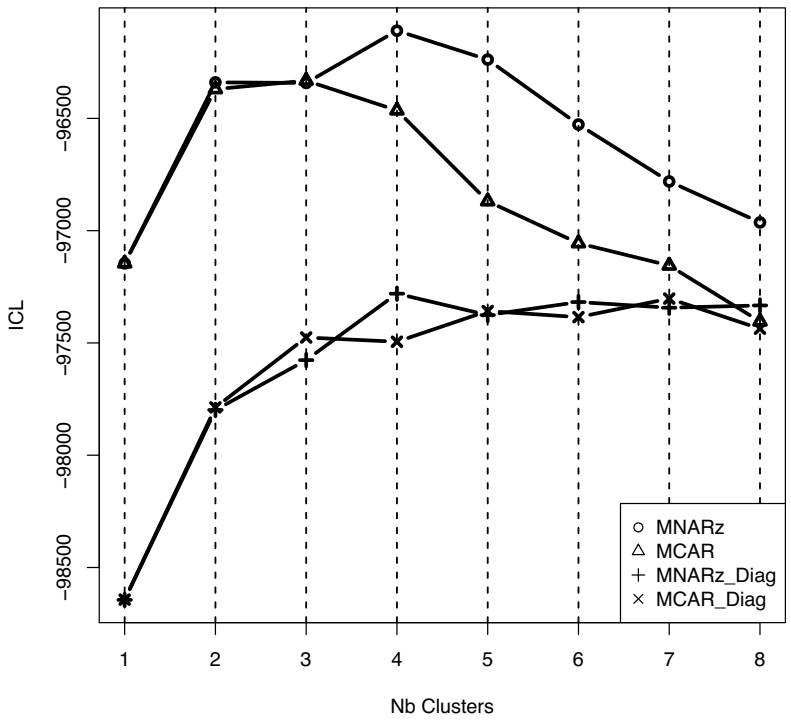
Conditional independence of the variables knowing the cluster

ICL comparison



- MCAR and MNARz are equivalent
- MNARyz seems really inappropriate
- Seems to miss the previous latent structure: requires a specific exploration. . .

Exploration 1: local independence is not relevant for this data set



Not accounting for possible conditional dependencies between the continuous variables is inappropriate for this dataset.

Exploration 2: mixture model bias *vs* missing model bias

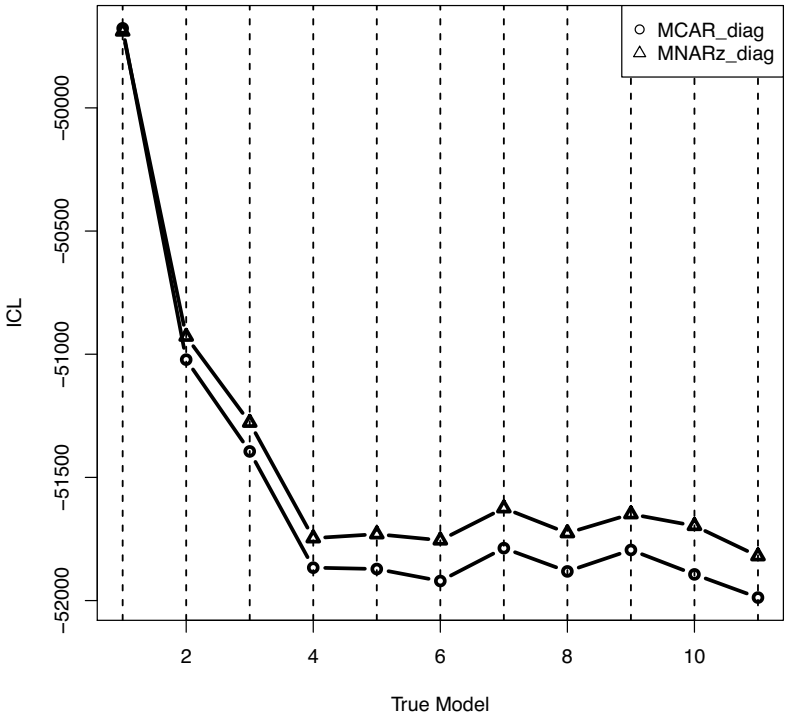
A simulated data set with the following parameters:

- Mixture model: varying from diagonal to non diagonal hypotheses
  - 2 clusters, dimension 7
  - 5 000 individuals
  - $\pi_1 = 0.3$, $\pi_2 = 0.7$, $\mu_1 = (0,0,0,0,0,0,0)$, $\mu_2 = (2,2,2,2,2,2,2)$
  - Covariance matrices: $r \in \{1, \ldots, 10, \infty\}$ ($r = \infty$ is the diagonal case)

$$\Sigma_k^{(r)} = \begin{pmatrix} 1 & 0.5^r & 0.25^r & \ldots & . & . & . \\ 0.5^r & 1 & 0.5^r & \ldots & . & . & . \\ 0.25^r & 0.5^r & 1 & \ldots & . & . & . \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ . & . & . & \ldots & 0.25^r & 0.5^r & 1 \end{pmatrix}$$

- Missingness model: MNAR hypothesis
  - Proportions of missingness: 0.001 for cluster 1 and 0.06 for cluster 2

Results from exploration 2



- The mixture model bias can not be compensated by the (unbiased) missing mechanism modeling
- It illustrates again the fact that information (on the latent partition) conveys by data is much more important than information conveys by the pattern

Outline

1 Introduction

2 A model-based MNAR clustering approach

3 Identifiability

4 Inference procedures

5 Medical study illustration

6 Concluding remarks

7 Practical activity

Summary

- Interest to put a model on c
- Interest of the simple but meaningful model MNARz
- Link between our models and usual methods

Ongoing works

- Deeper analysis of the previous results with doctors...
- Implement the proposed models/algo. in the Mixmod software[a]
- Address the trade-off between biased mixture model and biased missingness mechanism in particular for the mixed data case

[a] http://www.mixmod.org

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Run provided R files

Cours_clustering_Christophe.html

1 Introduction et notations
1.1 Données manquantes
1.2 Classification non supervisée
2 Cas continu
3 Cas catégoriel

Classification non supervisée avec données manquantes non-ignorables

A.S.

07 mars 2021

1 Introduction et notations

1.1 Données manquantes

Soit $Y \in \mathbb{R}^{n \times d}$ le jeu de données. Nous supposons qu'il contient des données manquantes. La **matrice de motif des données manquantes** permet d'indiquer où sont les données manquantes. Cette matrice est notée $C \in \mathbb{R}^{n \times d}$: $c_{ij} = 1$ si Y_{ij} est manquant, $c_{ij} = 0$ sinon. Les valeurs des variables observées (resp. des valeurs manquantes) pour l'individu i sont regroupées dans le vecteur y_i^{obs} (resp. y_i^{mis}). Dans le contexte de la classification non supervisée, le but est d'estimer une partition (inconnue) pour les données Y en K groupes. Cette partition est notée $Z = (z_1 | \dots | z_n)^T \in \{0,1\}^{n \times K}$ avec $z_i = (z_{i1}, \dots, z_{iK})^T \in \{0,1\}^K$ et où $z_{ik} = 1$ si y_i appartient à la classe k, $z_{ik} = 0$ sinon. Dans ce contexte, les données manquantes ne sont pas seulement les valeurs y_i^{mis} mais également les étiquettes de la partition z_i.

Les garanties théoriques et ainsi le choix des méthodes de traitement des données manquantes reposent généralement sur des hypothèses concernant les causes de l'absence de données. Rubin (1976) a décomposé ces causes en trois **mécanismes de données manquantes**. Les données sont dites:

- manquantes complètement aléeatoirement (MCAR) si la probabilité qu'une observation soit manquante est la même pour toutes les observations:

$$\mathbb{P}(c_i \mid y_i, z_i; \psi) = \mathbb{P}(c_i; \psi), \forall y_i,$$

où ψ est le paramètres du mécanisme de données manquantes.

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