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Traitement statistique des données manquantes-Part III Missing not at random data (MNAR)

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Traitement statistique des données manquantes

Atelier Statistique de la SFdS – 10 et 11 mars 2021

—

Part III Missing not at random data (MNAR)

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Merci à : F. Antonazzo, C. Boyer, G. Celeux, Q. Grimonprez, J. Jacques, J. Josse,
C. Keribin, V. Kubicki, F. Laporte, M. Marbac, A. Sportisse, J. Vandaele, V. Vandewalle

It is an ongoing research work
(with its own notations, possibly differing from Part I/II)

Take home message

- 1 The missing data **pattern** may convey some information on clustering
- 2 **Embed the missingness mechanism** directly within the clustering modeling step

Outline

- 1 Introduction
- 2 A model-based MNAR clustering approach
- 3 Identifiability
- 4 Inference procedures
- 5 Medical study illustration
- 6 Concluding remarks
- 7 Practical activity

Ignorable vs. non ignorable model

A missing mechanism is ignorable if likelihoods can be decomposed as

$$L(\theta, \psi; \underbrace{y^{\text{obs}}, c}_{\text{observed data}}) = L(\psi; c | y^{\text{obs}}) \times L(\theta; y^{\text{obs}})$$

Some simple algebra show that this occurs when missing mechanism is not MNAR

Inference of θ

“If the missing mechanism is **ignorable** then likelihood-based inferences for θ from $L(\theta; y^{\text{obs}})$ will be the same as likelihood based inference for θ from $L(\theta, \psi; y^{\text{obs}}, c)$.”
([Little and Rubin, 2002] Section 6.2)

- M(C)AR is ignorable
- MNAR is not ignorable

Clustering: model-based approach

- **Partition with K clusters:** $Z = (z_1 | \dots | z_n)^T \in \{0, 1\}^{n \times K}$ where
 - $z_i = (z_{i1}, \dots, z_{iK}) \in \{0, 1\}^K$
 - $z_{ik} = 1$ if y_i belongs to cluster k , $z_{ik} = 0$ otherwise
- **Mixture model:** y_1, \dots, y_n are i.i.d. from the mixture

$$f(y_i; \pi, \theta) = \sum_{k=1}^K \pi_k f_k(y_i; \theta_k)$$

- $\pi_k = P(z_{ik} = 1)$, $\pi = (\pi_1, \dots, \pi_K)$
- $f_k(\cdot; \theta_k)$: pdf of the k -th component parametrized by θ_k , $\theta = (\theta_1, \dots, \theta_K)$
 - **continuous data:** $f_k(\cdot; \theta_k) = \phi(\cdot; \mu_k, \Sigma_k)$ is the d -variate **Gaussian distribution** with mean vector μ_k and covariance matrix Σ_k
 - **categorical data:** the features are independent conditionally to the group membership i.e.

$$f_k(\cdot; \theta_k) = \prod_{j=1}^d f_{kj}(\cdot; \theta_{kj}), \text{ where } f_{kj} = \prod_{\ell=1}^{\ell_j} (\theta_{kj}^{\ell})^{y_{ij}^{\ell}}$$
 is the **multinomial distribution** with $\theta_{kj} = (\theta_{kj}^{\ell} = \mathbb{P}(y_{ij}^{\ell} = 1 | z_{ik} = 1))_{\ell=1, \dots, \ell_j}$
 - **mixed data:** the features are independent conditionally to the group membership, $f_k(\cdot; \theta_k)$ is the product of univariate Gaussian and multinomial distributions
 - Can also be extended to other cases (count data with Poisson distributions for instance)

Question we address in this work

Which distribution $\mathbb{P}(c|y, z; \psi)$ to propose in this clustering context?

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Proposed zoology of MNAR models in clustering

$$\mathbb{P}(c_i | y_i, z_{ik} = 1; \psi) = \prod_{j=1}^d \mathbb{P}(c_{ij} | y_i, z_{ik} = 1; \psi)$$

- $\text{MNAR}_{y^k z^j}$, with $\psi = (\alpha, \beta)$ where $\alpha = (\alpha_{11}, \dots, \alpha_{1d}, \dots, \alpha_{K1}, \dots, \alpha_{Kd})^T \in \mathbb{R}^{Kd}$ and $\beta = (\beta_{11}, \dots, \beta_{1d}, \dots, \beta_{K1}, \dots, \beta_{Kd})^T \in \mathbb{R}^{Kd}$

$$\mathbb{P}(c_{ij} = 1 | y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj} + \beta_{kj} y_{ij}),$$

with ρ the cdf of any continuous distribution (logit, probit)

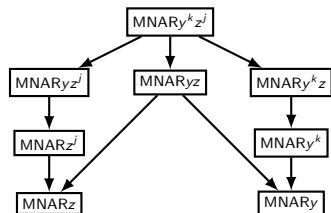
- MNAR_{yz} , $\text{MNAR}_{y^k z}$, MNAR_{yz^j}

- MNAR_y , MNAR_{y^k}
 $\psi = (\beta_{11}, \dots, \beta_{1d}, \dots, \beta_{K1}, \dots, \beta_{Kd})^T$

$$\mathbb{P}(c_{ij} = 1 | y_i, z_{ik} = 1; \psi) = \rho(\beta_{kj} y_{ij})$$

- MNAR_z , MNAR_{z^j}
 $\psi = (\alpha_{11}, \dots, \alpha_{1d}, \dots, \alpha_{K1}, \dots, \alpha_{Kd})^T$

$$\mathbb{P}(c_{ij} = 1 | y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj})$$



Overview of the proposed MNAR models

	Effect on the variable j		Effect on the class membership k		Nb parameters	
	Depends on j	Depends on k	Depends on j	Depends on k	Continuous	Categorical
MNAR $z^j y^k$	✓	✓	✓	✓	$2Kd$	$K(d + \sum_{j=1}^d (\ell_j - 1))$
MNAR $y z^j$	✓	✗	✓	✓	$(K + 1)d$	$Kd + \sum_{j=1}^d (\ell_j - 1)$
MNAR $y^k z$	✓	✓	✗	✓	$K(d + 1)$	$K(1 + \sum_{j=1}^d (\ell_j - 1))$
MNAR yz	✓	✗	✗	✓	$(K + d)$	$K + \sum_{j=1}^d (\ell_j - 1)$
MNAR y	✓	✗	✗	✗	d	$\sum_{j=1}^d (\ell_j - 1)$
MNAR y^k	✓	✓	✗	✗	Kd	$K \sum_{j=1}^d (\ell_j - 1)$
MNAR z	✗	✗	✗	✓	K	K
MNAR z^j	✗	✗	✓	✓	Kd	Kd

Terminology in the sequel:

- MNAR z , MNAR z^j : the only effect of missingness is on the class membership
- MNAR y^* : all the other models which considers the effect of the missingness depending on the variable
- MNAR $*$: all the models

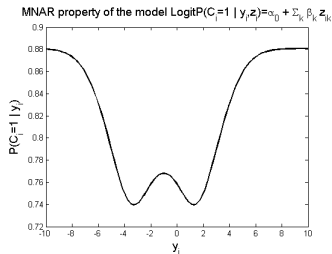
MNARz analysis: it depends on y through z !

$$P(c_{ij} = 1 | y_i; \theta, \psi) = \sum_{k=1}^K P(c_{ij} = 1 | y_i, z_{ik} = 1; \psi) P(z_{ik} = 1 | y_i; \theta)$$

Example of a univariate Gaussian model with the three components

$$0.2N(\cdot; 0, 1) + 0.3N(\cdot; 1, 2) + 0.5N(\cdot; 2, 3)$$

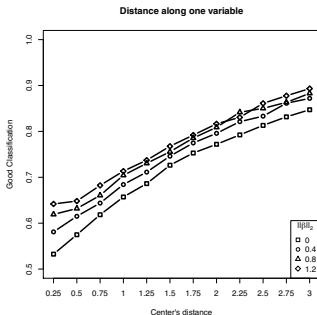
and with parameters of the logit expression: $\alpha_0 = 1, \beta_1 = 1, \beta_2 = -1, \beta_3 = 1$



MNAR_z analysis: pattern c gives information on partition z !

Draw Bayes error of a MNAR_z model with two components and 20% of missing data

$$\pi_k = 0.5, \|\mu_2 - \mu_1\| \text{ varies}, \Sigma_1 = \Sigma_2 = \mathbf{I}, |\beta_2 - \beta_1| \text{ varies}$$



Both μ_k and β_k act on the Bayes error

Reinterpretation of the MNAR_z and MNAR_z^j models as MAR

Commonly used in Machine Learning

[Jones, 1996, Little and Rubin, 2002, Josse et al., 2019]

Mixture model for Y^{obs} and Bernoulli distribution for C
 \Leftrightarrow MAR mixture model for $\tilde{Y}^{\text{obs}} = (Y^{\text{obs}}|C)$

For example,

$$Y^{\text{obs}} = \begin{pmatrix} ? & 2.6 & 5 \\ \text{blue} & 1.9 & 4 \\ \text{red} & 2.3 & ? \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

then \tilde{Y}^{obs} is expressed as

$$\tilde{Y}^{\text{obs}} = \begin{pmatrix} ? & 2.6 & 5 & 1 & 0 & 0 \\ \text{blue} & 1.9 & 4 & 0 & 0 & 0 \\ \text{red} & 2.3 & ? & 0 & 0 & 1 \end{pmatrix}.$$

Proposition 1: in terms of maximum likelihood

The maximum likelihood estimate associated to the dataset \tilde{Y}^{obs} under MAR model is the one associated to the dataset Y^{obs} under MNAR_z or MNAR_z^j models.
 \Rightarrow can be extended to other estimation strategies

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Continuous and count data (1)

Previous works: [Teicher, 1963] (without NA), [Miao et al., 2016] (for MNAR data)

Identifiability for a mixture model with MNAR data

⇔ Mixture/MNAR parameters are uniquely determined from available information

Proposition 2: identifiability for continuous and count data

Assume that

- 1 The marginal mixture $\sum_{k=1}^K \pi_k f_k(y_i; \theta_k)$ is identifiable
- 2 There exists a total ordering \preceq of $\mathcal{F}_j \times \mathcal{R}$, for $j \in \{1, \dots, d\}$ fixed, where $\mathcal{F}_j = \{f_{1j}, \dots, f_{Kj}\}$ and $\mathcal{R} = \{\rho_1, \dots, \rho_K\} = \{\rho(\cdot; \psi_1), \dots, \rho(\cdot; \psi_K)\}$. The total ordering is s.t. $\forall k < \ell, F_k = \rho_k f_{kj} \preceq F_\ell = \rho_\ell f_{\ell j}$ implies

$$\lim_{u \rightarrow +\infty} \frac{\rho_\ell(u) f_{\ell j}(u)}{\rho_k(u) f_{kj}(u)} = 0$$

Then the mixture model with one of the MNAR* mechanisms is identifiable up to label swapping

Continuous and count data (2)

Is the total ordering checked for classical distributions ?

f_k	Gaussian		Poisson	
ρ_k	Probit	Logit	Probit	Logit
MNAR $z^j y^k$				
MNAR $y^k z$	✓	generic idenfiability	✓	generic idenfiability
MNAR y^k				
MNAR $y z^j$				
MNAR $y z$				
MNAR y	✓	✓	✓	✓
MNAR z				
MNAR z^j				

Generic identifiability: all not-identifiable parameter choices lie within a proper subvariety, and thus form a set of Lebesgue zero measure

Categorical data

Previous work: [Allman et al., 2009] (without NA)

Recall that for categorical data: conditional independence of the features given the group membership holds i.e. $f_k(\cdot; \theta_k) = \prod_{j=1}^d f_{kj}(\cdot; \theta_{kj})$

Proposition 3: identifiability for categorical data

Assume that $d \geq 2 \lceil \log_2 K \rceil + 1$ and $f_k(\cdot; \theta_k) = \prod_{j=1}^d f_{kj}(\cdot; \theta_{kj})$

✓ Then the mixture model with **MNAR_z** or **MNAR_z^j** mechanism is identifiable up to label swapping

✗ The mixture model with one of the **MNAR_y*** mechanisms is not identifiable

Mixed data

$f_k(\cdot; \theta_k) = \prod_{j=1}^d f_{kj}(\cdot; \theta_{kj})$, thus identifiability of mixed data directly follows from Proposition 2 for continuous variables and from Proposition 3 for categorical variables

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EM algorithm: looks simple

The **expected complete log-likelihood** knowing the **observed data** and a **current value of the parameters** can be **decomposed into two parts**

$$\begin{aligned} Q(\theta, \psi, \pi; \theta^r, \psi^r, \pi^r) &= \mathbb{E}[\ell_{\text{comp}}(\theta, \psi, \pi; y, z, c) | y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r] \\ &= Q_y(\theta, \pi; \theta^r, \psi^r, \pi^r) + Q_c(\psi; \theta^r, \psi^r, \pi^r) \end{aligned}$$

$$Q_y(\theta, \pi; \theta^r, \psi^r, \pi^r) = \sum_{i=1}^n \sum_{k=1}^K (\tau_{ik})^r \log(\pi_k) + \sum_{i=1}^n \sum_{k=1}^K (\tau_{ik})^r E_{iy}^r(\theta)$$

$$Q_c(\psi; \theta^r, \psi^r, \pi^r) = \sum_{i=1}^n \sum_{k=1}^K (\tau_{ik})^r E_{ic}^r(\psi)$$

where for $i = 1, \dots, n$ and $k = 1, \dots, K$,

$$E_{iy}^r(\theta) = \mathbb{E} \left[\log(f_k(y_i; \theta_k)) \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r \right]$$

$$E_{ic}^r(\psi) = \mathbb{E} \left[\log(\mathbb{P}(c_i \mid y_i, z_{ik} = 1; \psi)) \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r \right]$$

$$(\tau_{ik})^r = \mathbb{P}(z_{ik} = 1 \mid y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r) \propto \pi_k^r f_k(y_i^{\text{obs}}; \theta_k^r) \mathbb{P}(c_i \mid y_i^{\text{obs}}, z_{ik} = 1; \psi^r)$$

EM algorithm for MNAR_z and MNAR_z^j

MNAR_z, MNAR_z^j: needs some computations but still simple.

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj}) \quad (\text{independent of } y) \quad (\Delta)$$

■ **Gaussian data**: $(y_i \mid z_{ik} = 1; \theta^r) \sim \mathcal{N}(\mu_k, \Sigma_k)$

■ $\mathbb{P}(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1, c_i; \theta^r, \psi^r) = \mathbb{P}(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r)$ using (Δ) and

$$(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r) \sim \mathcal{N}((\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r)$$

where $(\tilde{\mu}_{ik}^{\text{mis}})^r$ and $(\tilde{\Sigma}_{ik}^{\text{mis}})^r$ only depend on μ_k^r, Σ^r and y_i^{obs}

$\Rightarrow E_{iy}^r(\theta) = \mathbb{E} \left[\log(f_k(y_i; \theta_k)) \mid y_i^{\text{obs}}, z_{ik} = 1, ; \theta^r \right]$ easy to compute (classical formulae)

■ **Using (Δ)**

$$E_{ic}^r(\psi) = \log(\mathbb{P}(c_i \mid z_{ik} = 1; \psi)) = \sum_{j=1}^d c_{ij} \log \rho(\alpha_{kj}) + (1 - c_{ij}) \log(1 - \rho(\alpha_{kj}))$$

■ **Using (Δ)**

$$(\tau_{ik})^r \propto \pi_k^r \phi(y_i^{\text{obs}}; (\mu_{ik}^{\text{obs}})^r, (\Sigma_{ik}^{\text{obs,obs}})^r) \text{prod}_{j=1}^d \rho(\alpha_{kj}^r)^{c_{ij}} (1 - \rho(\alpha_{kj}^r))^{1-c_{ij}}$$

EM algorithm for MNAR_z and MNAR_z^j

Recall that: $(y_i^{\text{mis}} | y_i^{\text{obs}}, z_{ik} = 1; \theta^r) \sim \mathcal{N}(\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r$.

- **E-step**: for $k = 1, \dots, K$ and $i = 1, \dots, n$, compute $(\tilde{\mu}_{ik}^{\text{mis}})^r, (\tilde{\Sigma}_{ik}^{\text{mis}})^r, (\tau_{ik})^r$ and

$$(\tilde{y}_{i,k})^r = (y_i^{\text{obs}}, (\tilde{\mu}_{ik}^{\text{mis}})^r) \quad \tilde{\Sigma}_{ik}^r = \begin{pmatrix} 0_{i_j^{\text{obs,obs}}} & 0_{i_j^{\text{obs,mis}}} \\ 0_{i_j^{\text{mis,obs}}} & (\tilde{\Sigma}_{ik}^{\text{mis}})^r \end{pmatrix}$$

- **M-step**: for $k = 1, \dots, K$, compute

$$\pi_k^{r+1} = \frac{1}{n} \sum_{i=1}^n (\tau_{ik})^r \quad \mu_k^{r+1} = \frac{\sum_{i=1}^n (\tau_{ik})^r (\tilde{y}_{k,i})^r}{\sum_{i=1}^n (\tau_{ik})^r}$$

$$\Sigma_k^{r+1} = \frac{\sum_{i=1}^n \left[(\tau_{ik})^r \left((\tilde{y}_{i,k})^r - \mu_k^{r+1} \right) \left((\tilde{y}_{i,k})^r - \mu_k^{r+1} \right)^T + \tilde{\Sigma}_{ik}^r \right]}{\sum_{i=1}^n (\tau_{ik})^r}$$

For ψ^{r+1} : maximization of $Q_c(\psi; \theta^r, \psi^r, \pi^r)$ over ψ with a **Newton-Raphson algorithm** (classical procedure for link functions of interest)

An EM algorithm can also be **easily derived for categorical data**

Not EM algorithm for MNAR_{y*}

MNAR_{y*}: needs approximations

$$\mathbb{P}(c_{ij} = 1 \mid y_i, z_{ik} = 1; \psi) = \rho(\alpha_{kj} + \beta_{kj}y_{ij}) \quad (\text{not independent of } y)$$

■ Gaussian data:

■ $(y_i^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1, c_i)$:

✗ not classical if ρ is **Logit**, ✓ truncated Gaussian distribution if ρ is **Probit**

■ No closed form of $E_{ic}^r(\psi)$ neither for Probit nor for Logit:

$$E_{ic}^r(\psi) = \sum_{j=1}^d c_{ij} \int_{\mathcal{Y}_{ij}^{\text{mis}}} \log(\rho(\alpha_{kj} + \beta_{kj}y_{ij}^{\text{mis}})) \frac{\rho(\alpha_{kj}^r + \beta_{kj}^r y_{ij}^{\text{mis}})^{c_{ij}} \mathbb{P}(y_{ij}^{\text{mis}} \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r)}{\int_{\mathcal{Y}_{ij}^{\text{mis}}} \rho(\alpha_{kj}^r + \beta_{kj}^r x)^{c_{ij}} \mathbb{P}(x \mid y_i^{\text{obs}}, z_{ik} = 1; \theta^r) dx} dy_{ij}^{\text{mis}} \\ + (1 - c_{ij}) \log(1 - \rho(\alpha_{kj} + \beta_{kj}y_{ij}^{\text{obs}}))$$

✗ not concave function if ρ is **Logit**

■ No closed form of $(\tau_{ik})^r$ neither for Probit nor for Logit

■ In the Gaussian case, there is **no closed form** [Pirjol, 2013]

■ **SEM easier?** random drawing instead of expectation

SEM algorithm for MNAR_y*

- **SE-step**: draw the missing data $((y_i^{\text{mis}})^{r+1}, z_i^{r+1}) \sim (\cdot | y_i^{\text{obs}}, c_i; \theta^r, \psi^r, \pi^r)$

Use of **One-Gibbs sampling**:

- $(y_i^{\text{mis}})^{r+1} \sim (\cdot | y_i^{\text{obs}}, z_i^r, c_i; \theta^r, \psi^r)$:
 - ✗ not classical if ρ is **Logit**, ✓ **truncated Gaussian distribution** if ρ is **Probit**
- $z_i^{r+1} \sim (\cdot | y_i^{r+1}, c_i; \theta^r, \psi^r, \pi^r)$: draw the membership k of z_i^{r+1} from the **multinomial distribution** with probabilities

$$\mathbb{P}(z_{ik} = 1 | y_i^{r+1}, c_i; \theta^r, \psi^r, \pi^r) = \frac{\mathbb{P}(c_i | y_i^{r+1}, z_{ik} = 1; \psi^r) \mathbb{P}(y_i^{r+1} | z_{ik} = 1; \theta^r) \pi_k^r}{\sum_{h=1}^K \mathbb{P}(c_i | y_i^{r+1}, z_{ih} = 1; \psi^r) \mathbb{P}(y_i^{r+1} | z_{ih} = 1; \theta^r) \pi_h^r}$$

Let $Y^{r+1} = (y_1^{r+1} | \dots | y_n^{r+1})$, $Z^{r+1} = (z_1^{r+1} | \dots | z_n^{r+1})$ be the imputed matrix and the partition

- **M-step**: for $k = 1, \dots, K$, compute
 - π_k^{r+1} with the proportion of rows of Y^{r+1} belonging to class k
 - $\mu_k^{r+1}, \Sigma_k^{r+1}$ with the mean and covariance matrix of rows of Y^{r+1} belonging to class k
 - ψ^{r+1} with a Newton-Raphson algorithm

Summary for algorithms

	EM		SEM			
	Gaussian	Categorical	Gaussian	Categorical		
MNAR _z MNAR _{z^j}	✓	✓	✓	✓		
	Probit	Logit	Probit	Logit		
MNAR _{y*}	no closed form	no closed form, optim. pb	not ident.	✓	require algorithms as SIR (costly)	not ident.

What about model selection?

Can select between MCAR and MNAR* with any information criterion (BIC, ICL)

Even if the missing mechanism is ignorable for MCAR...

... need to model c to compare a MCAR and a MNAR model

CAUTION

- It is just a selection between several proposed MNAR models
- It is not deciding if missingness procedure is “genererically” MNAR or not

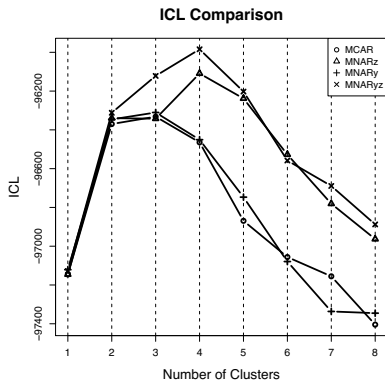
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Hospital Data: continuous features case

- Number of patients: $n = 5\,146$
- Number of features: $d = 7$
 - Age
 - Size
 - Weight
 - Cardiac frequency
 - Hemoglobin concentration
 - Temperature
 - Minimum Diastolic and Systolic Blood Pressure
- Percentage of missing data: 6.4%

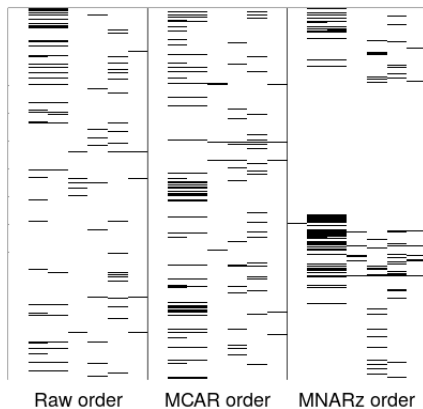
ICL comparison



- MCAR, MNARy and MNARz are equivalent until $K = 3$
- MNARz and MNARyz clearly indicate presence of an additional cluster ($K = 4$)

It seems to be an illustration of the effect of c through MNARz and MNARyz

Missing Pattern



It seems that MNARz modelling leads to a missing free cluster

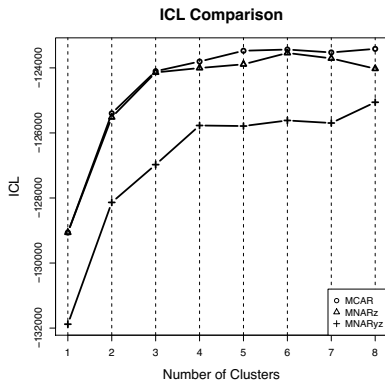
Hospital data: mixed features case

- Number of patients: $n = 5\,146$
- Number of features: $d = 15$ (7 continuous and 8 categorical)
- Percentage of missing data: $\sim 4\%$

Model

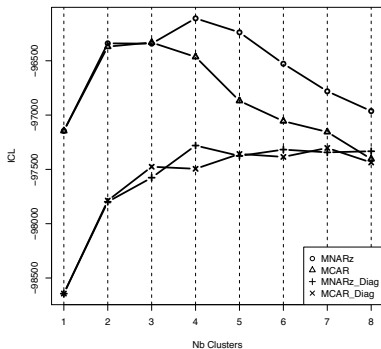
Conditional independence of the variables knowing the cluster

ICL comparison



- MCAR and MNARz are equivalent
- MNARyz seems really inappropriate
- Seems to miss the previous latent structure: [requires a specific exploration...](#)

Exploration 1: local independence is not relevant for this data set



Not accounting for possible conditional dependencies between the continuous variables is inappropriate for this dataset.

Exploration 2: mixture model bias vs missing model bias

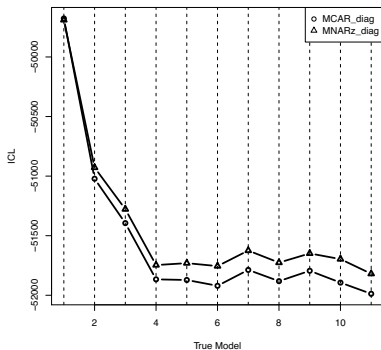
A simulated data set with the following parameters:

- Mixture model: **varying from diagonal to non diagonal hypotheses**
 - 2 clusters, dimension 7
 - 5 000 individuals
 - $\pi_1 = 0.3$, $\pi_2 = 0.7$, $\mu_1 = (0, 0, 0, 0, 0, 0, 0)$, $\mu_2 = (2, 2, 2, 2, 2, 2, 2)$
 - Covariance matrices: $r \in \{1, \dots, 10, \infty\}$ ($r = \infty$ is the diagonal case)

$$\Sigma_k^{(r)} = \begin{pmatrix} 1 & 0.5^r & 0.25^r & \dots & \cdot & \cdot & \cdot \\ 0.5^r & 1 & 0.5^r & \dots & \cdot & \cdot & \cdot \\ 0.25^r & 0.5^r & 1 & \dots & \cdot & \cdot & \cdot \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \cdot & \cdot & \cdot & \dots & 0.25^r & 0.5^r & 1 \end{pmatrix}$$

- Missingness model: **MNAR hypothesis**
 - Proportions of missingness: 0.001 for cluster 1 and 0.06 for cluster 2

Results from exploration 2



- The mixture model bias can not be compensated by the (unbiased) missing mechanism modeling
- It illustrates again the fact that information (on the latent partition) conveyed by data is much more important than information conveyed by the pattern

Outline

- 1 Introduction
- 2 A model-based MNAR clustering approach
- 3 Identifiability
- 4 Inference procedures
- 5 Medical study illustration
- 6 Concluding remarks**
- 7 Practical activity

Summary

- Interest to put a model on c
- Interest of the simple but meaningful model MNAR z
- Link between our models and usual methods

Ongoing works

- Deeper analysis of the previous results with doctors. . .
- Implement the proposed models/algo. in the Mixmod software^a
- Address the **trade-off between biased mixture model and biased missingness mechanism** in particular for the mixed data case

^a<http://www.mixmod.org>

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Run provided R files

Cours_clustering_Christophe.html

1 Introduction et notations

- 1.1 Données manquantes
- 1.2 Classification non supervisée
- 2 Cas continu
- 3 Cas catégoriel

Classification non supervisée avec données manquantes non-ignorables

A.S.

07 mars 2021

1 Introduction et notations

1.1 Données manquantes

Soit $Y \in \mathbb{R}^{n \times d}$ le jeu de données. Nous supposons qu'il contient des données manquantes. La **matrice de motif des données manquantes** permet d'indiquer où sont les données manquantes. Cette matrice est notée $C \in \mathbb{R}^{n \times d}$, $c_{ij} = 1$ si Y_{ij} est manquant, $c_{ij} = 0$ sinon. Les valeurs des variables observées (resp. des valeurs manquantes) pour l'individu i sont regroupées dans le vecteur y_i^{obs} (resp. y_i^{mis}). Dans le contexte de la classification non supervisée, le but est d'estimer une partition (inconnue) pour les données Y en K groupes. Cette partition est notée $Z = (z_1 | \dots | z_n)^T \in \{0, 1\}^{n \times K}$ avec $z_i = (z_{i1}, \dots, z_{iK})^T \in \{0, 1\}^K$ et où $z_{ik} = 1$ si y_i appartient à la classe k , $z_{ik} = 0$ sinon. Dans ce contexte, les données manquantes ne sont pas seulement les valeurs y_i^{mis} mais également les étiquettes de la partition z_i .

Les garanties théoriques et ainsi le choix des méthodes de traitement des données manquantes reposent généralement sur des hypothèses concernant les causes de l'absence de données. Rubin (1976) a décomposé ces causes en trois **mécanismes de données manquantes**. Les données sont dites:

- manquantes complètement aléatoirement (MCAR) si la probabilité qu'une observation soit manquante est la même pour toutes les observations:

$$\mathbb{P}(c_i | y_i, z_i; \psi) = \mathbb{P}(c_i; \psi), \forall y_i,$$

où ψ est la paramètre du mécanisme de données manquantes.

References



Allman, E. S., Matias, C., Rhodes, J. A., et al. (2009).

Identifiability of parameters in latent structure models with many observed variables.
The Annals of Statistics, 37(6A):3099–3132.



Jones, M. P. (1996).

Indicator and stratification methods for missing explanatory variables in multiple linear regression.
Journal of the American statistical association, 91(433):222–230.



Josse, J., Prost, N., Scornet, E., and Varoquaux, G. (2019).

On the consistency of supervised learning with missing values.
arXiv preprint arXiv:1902.06931.



Little, R. J. and Rubin, D. B. (2002).

Statistical Analysis with Missing Data.
Wiley.



Miao, W., Ding, P., and Geng, Z. (2016).

Identifiability of normal and normal mixture models with nonignorable missing data.
Journal of the American Statistical Association, 111(516):1673–1683.



Pirjol, D. (2013).

The logistic-normal integral and its generalizations.
Journal of Computational and Applied Mathematics, 237(1):460–469.



Teicher, H. (1963).

Identifiability of finite mixtures.
The annals of Mathematical statistics, pages 1265–1269.