



HAL
open science

Compact MILP formulations for the p-center problem

Zacharie Alès, Sourour Elloumi

► **To cite this version:**

Zacharie Alès, Sourour Elloumi. Compact MILP formulations for the p-center problem. ISCO (International Symposium on Combinatorial Optimization) 2018, Apr 2018, Marrakesh, France. hal-03503279

HAL Id: hal-03503279

<https://hal.science/hal-03503279>

Submitted on 27 Dec 2021

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Compact MILP formulations for the p -center problem

Zacharie Ales¹, Sourour Elloumi¹

ENSTA-ParisTech / UMA, 91762 Palaiseau, France
 Laboratoire CEDRIC, Paris, France
 {zacharie.ales, sourour.elloumi}@ensta-paristech.fr

We consider N clients $\{C_1, \dots, C_N\}$ and M potential facility sites $\{F_1, \dots, F_M\}$. Let d_{ij} be the distance between C_i and F_j . The objective of the p -center problem is to open up to p facilities such that the maximal distance (called *radius*) between a client and its closest selected site is minimized.

This problem is very popular in combinatorial optimization and has many applications. We refer the reader to the recent survey [1]. Very recent publications include [5] which provides heuristic solutions and [2] on an exact solution method.

Let \mathcal{M} and \mathcal{N} respectively be the sets $\{1, \dots, M\}$ and $\{1, \dots, N\}$. The most classical formulation, denoted by (P_1) , for the p -center problem (see for example [3]) considers a variable R equal to the value of a radius, the binary variables y_j equal to 1 if and only if F_j is open and the binary variables x_{ij} equal to 1 if and only if C_i is assigned to F_j .

$$(P_1) \left\{ \begin{array}{ll} \min R & (1a) \\ \text{s.t. } \sum_{j=1}^M y_j \leq p & (1b) \\ \sum_{j=1}^M x_{ij} = 1 & i \in \mathcal{N} \quad (1c) \\ x_{ij} \leq y_j & i \in \mathcal{N}, j \in \mathcal{M} \quad (1d) \\ \sum_{j=1}^M d_{ij} x_{ij} \leq R & i \in \mathcal{N} \quad (1e) \\ x_{ij}, y_j \in \{0, 1\} & i \in \mathcal{N}, j \in \mathcal{M} \\ r \in \mathbb{R} \end{array} \right.$$

Constraint (1b) ensures that no more than p facilities are opened. Each client is assigned to exactly one facility through Constraints (1c). Constraints (1d) link variables x_{ij} and y_j while (1e) ensure the coherence of the objective.

A more recent formulation of the p -center problem, denoted by (P_2) , was proposed in [4]. Let $D^0 < D^1 < \dots < D^K$ be the different d_{ij} values $\forall i \in \mathcal{N} \forall j \in \mathcal{M}$. Note that, if many distances d_{ij} have the same value, K may be significantly lower than $M \times N$. Let \mathcal{K} be the set $\{1, \dots, K\}$. Formulation (P_2) is based on the variables y_j , previously introduced, and one binary variable z^k , for each $k \in \mathcal{K}$, equals to 1 if and only if the optimal radius is greater than or equal to D^k :

$$(P_2) \begin{cases} \min D^0 + \sum_{k=1}^K (D^k - D^{k-1}) z^k & (2a) \\ \text{s.t. } 1 \leq \sum_{j=1}^M y_j \leq p & (2b) \\ z^k + \sum_{j: d_{ij} < D^k} y_j \geq 1 & i \in \mathcal{N}, k \in \mathcal{K} & (2c) \\ y_j, z^k \in \{0, 1\} & j \in \mathcal{M}, k \in \mathcal{K} \end{cases}$$

Constraints (2c) ensure that if no facility located at less than D^k of client C_i is selected, then the radius must be greater than or equal to D^k .

This formulation has been proved to be tighter than (P_1) . However, its size strongly depends on the value K (*i.e.*, the number of distinct distances d_{ij}).

In this work we first prove that a large part of constraints (2c) are redundant and can be removed without affecting the quality of the linear relaxation. This leads to a formulation (CP_1) with $\mathcal{O}(\min(NM, NK))$ constraints instead of $\mathcal{O}(NK)$.

Then, we introduce (CP_2) which is the most compact formulation currently known for this problem. It is obtained by replacing the K variables z^k of (CP_1) by a unique variable r which represents the index of a radius. We prove that the linear relaxation of (CP_1) is stronger than the one of (CP_2) .

We besides introduce an iterative algorithm which enables us to reduce the number of clients and facilities as well as to compute strong bounds which significantly reduce the size of formulations (P_2) , (CP_1) and (CP_2) .

Finally, the efficiency of the iterative algorithm and the proposed formulations are compared in terms of quality of the linear relaxation and computation time over instances from OR-Library.

References

1. Hatice Calik, Martine Labbé, and Hande Yaman. *p-Center Problems*, pages 79–92. Springer International Publishing, Cham, 2015.
2. Hatice Calik and Barbaros C Tansel. Double bound method for solving the p-center location problem. *Computers & Operations Research*, 40(12):2991–2999, 2013.
3. M Daskin. Network and discrete location analysis. *ed: John Wiley and Sons, New York*, 1995.
4. Sourour Elloumi, Martine Labbé, and Yves Pochet. A new formulation and resolution method for the p-center problem. *INFORMS Journal on Computing*, 16(1):84–94, 2004.
5. D. Ferone, P. Festa, A. Napolitano, and M. G. C. Resende. On the fast solution of the p-center problem. In *2017 19th International Conference on Transparent Optical Networks (ICTON)*, pages 1–4, July 2017.