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Fixed-time sliding mode control with mismatched disturbances

Emmanuel Moulay [a], Vincent Léchappé [b], Emmanuel Bernuau [c], Michael Defoort [d], Franck Plestan [e]

[a] *XLIM (UMR CNRS 7252), Université de Poitiers, 11 bd Marie et Pierre Curie, 86073 Poitiers Cedex 9, France*

[b] *Laboratoire Ampère (UMR CNRS 5005), INSA de Lyon, 20 Avenue Albert Einstein, 69100 Villeurbanne, France*

[c] *INRAE, AgroParisTech, UMR 1145 Joint Research Unit for Food Process Engineering, 91300, Massy, France*

[d] *LAMIH (UMR CNRS 8201), INSA Hauts-de-France, Polytechnic University Hauts-de-France, 59313 Valenciennes, France*

[e] *LS2N (UMR CNRS 6004), École Centrale de Nantes, 1 Rue de la Noë, 44321 Nantes Cedex 3, France*

Abstract

The aim of this article is to develop new second-order fixed-time sliding mode controllers able to tackle mismatched disturbances. Contrary to what is done in the literature, the proposed sliding-mode control strategy does not use disturbance observers because the sliding surface possesses robustness properties. The results are applied to define a new fixed-time differentiator.

Key words: Sliding mode control, fixed-time stability, mismatched disturbances, fixed-time differentiator

1 Introduction

Sliding mode control (SMC) has been developed since the seminal work of Utkin [1] in the 70s, see for instance [2] for more details. Recently, the problem of robust fixed-time SMC was treated in [3,4] where fixed-time stability developed in [5–7] ensures that all solutions converge to the equilibrium point in a fixed-time. However, the proposed controllers were rather complex to implement due to a problem of singularity avoidance coming from the fixed-time stability strategy. In [8], simple robust controllers are proposed by using state-dependent variable exponent coefficient in the sliding surfaces and the controllers.

All the above mentioned articles deal with matched disturbances which are disturbances acting on the control input channel and are the most common disturbances encountered in control applications. Mismatched disturbances acting on other channels than the control channel also appear in automatic control for instance in [9–23] and it can model the DC–DC buck power converter systems [24]. In the previous works, the control problems are solved by using observers to estimate mismatched disturbances. In our article, we use a fixed-time SMC strategy adapted from [8] which is known to be robust in order to reject the mismatched disturbance of a second-order system without using disturbance observers. This renders this new strategy easier to tune. To the best of the authors' knowledge, this is the first time that a mismatched disturbance is rejected by using a fixed-time sliding surface known for its robustness properties. Thus, we adapt the sliding-mode controller developed in [8] in order to be used with a mismatched disturbance. The results are tested in simulation on the double integrator in order to show the efficiency of the proposed method. Finally, the proposed control strategy is applied to develop a new fixed-time differentiator, the reader may refer to [25] for more details on differentiators.

The article is organized as follows. After some preliminaries given in Section 2, the main results are presented in Section 3 and tested on the double integrator with matched and mismatched disturbances. They are also applied to develop a new fixed-time differentiator. Finally, a conclusion is addressed in Section 4.

Email addresses: emmanuel.moulay@univ-poitiers.fr (Emmanuel Moulay), vincent.lechappe@insa-lyon.fr (Vincent Léchappé), emmanuel.bernuau@agroparistech.fr (Emmanuel Bernuau), michael.defoort@uphf.fr (Michael Defoort), franck.plestan@ec-nantes.fr (Franck Plestan).

2 Preliminaries

In the following, denote e the constant such that $\ln(e) = 1$. Let us recall some results on finite-time stability and fixed-time stability. Consider the following ordinary differential equation

$$\begin{aligned} &\dot{x}(t) = f(x(t)), \qquad x(t) \in \mathbb{R}^n \\ &x(0) = x_0 \end{aligned} \tag{1}$$

with f a locally essentially bounded function.

Definition 1 *[26,27] The origin is globally finite-time stable for system (1) if it is Lyapunov stable and for all $x_0 \in \mathbb{R}^n$ there exists $T(x_0) \geqslant 0$ such that, for any $x(\cdot)$ solution of (1) with $x(0) = x_0$, $\lim_{t \to T(x_0)} \|x(t)\| = 0$, i.e. $\|x(t)\| \equiv 0$ for all $t \geqslant T(x_0)$. Such a function T is called a settling-time function.*

Definition 2 *[6] System (1) is globally fixed-time stable if:*

(1) it is globally finite-time stable;
(2) there exists a globally upper bounded settling-time function T, i.e. there exists a constant $\mathbf{T} > 0$, independent of the initial condition, such that for all $x_0 \in \mathbb{R}^n$, $T(x_0) \leqslant \mathbf{T}$.

3 Main results

Consider the following uncertain nonlinear second-order system

$$\begin{cases} \dot{x}_1 = x_2 + d_1 \\ \dot{x}_2 = f(x) + g(x)u + d_2 \end{cases} \tag{2}$$

with $x = (x_1, x_2) \in \mathbb{R}^2$ the state, $u \in \mathbb{R}$ the control input, f and g continuous functions such that $f(0) = 0$, $g(x) \neq 0$ for all $x \in \mathbb{R}^2$, d_1 a mismatched disturbance such that $|d_1(t)| < \delta_1$ and d_2 a matched disturbance such that $|d_2(t)| < \delta_2$. The equilibrium trajectory of system (2) is given by $(0, -d_1(t))$. The second order systems with mismatched disturbances have been studied for instance in [19].

The main objective of this subsection is to design a sliding variable leading to global robust fixed-time stabilization of system (2) towards the equilibrium trajectory. Consider the sliding variable with a state-dependent variable exponent coefficient given by

$$s(x) = x_2 + \beta |x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}} \operatorname{sgn}(x_1) \tag{3}$$

with $\lambda_1 > 0$, $\mu_1 > 0$ such that $\theta_1 = \frac{\lambda_1}{1+\mu_1} > 1$, $\beta > \delta_1 e^{\frac{\lambda_1}{2e}}$ and the following new controller inspired from Proposition 2 in [8]

$$\begin{aligned} u(x) = -g(x)^{-1}\Big[& f(x) + k_1 \operatorname{sgn}(s)|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} \\ & + \varphi(|x_1|)(x_2 + k_2 \operatorname{sgn}(s))\Big] \end{aligned} \tag{4}$$

with $\lambda_2 > 0$, $\mu_2 > 0$ such that $\theta_2 = \frac{\lambda_2}{1+\mu_2} > 1$ and

$$\varphi(z) = \frac{\beta\lambda_1}{1+\mu_1 z^2}\left(\frac{2\ln(z)}{1+\mu_1 z^2} + 1\right) z^{\frac{\lambda_1 z^2}{1+\mu_1 z^2}+1}.$$

Let us state a first technical lemma.

Lemma 1 *For $z \geqslant 0$, we have $\varphi(z) > 0$ when $z > 1$ and $|\varphi(z)| \leqslant 2\beta\lambda_1(1+\mu_1)$ when $0 \leqslant z \leqslant 1$.*

Proof. The first statement is obvious. When $0 \leqslant z \leqslant 1$, we write

$$\varphi(z) = \frac{\beta\lambda_1}{(1+\mu_1 z^2)^2}\left(2z\ln(z) + z + \mu_1 z^3\right) z^{\frac{\lambda_1 z^2}{1+\mu_1 z^2}}.$$

We see that $0 \leqslant \frac{\beta\lambda_1}{(1+\mu_1 z^2)^2} \leqslant \beta\lambda_1$ and $0 \leqslant z^{\frac{\lambda_1 z^2}{1+\mu_1 z^2}} \leqslant 1$. Now denoting

$$\psi(z) = 2z\ln(z) + z + \mu_1 z^3$$

we see that $\psi'(z) = 2\ln(z) + 3 + 3\mu_1 z^2$. Denoting z_0 the unique solution of $\psi'(z) = 0$, we see that ψ is decreasing on $(0, z_0)$ and increasing on $(z_0, 1)$. Therefore it leads to $|\psi(z)| \leqslant \max\{1+\mu_1, |\psi(z_0)|\}$. But, given that $\psi'(z_0) = 0$, we have

$$\begin{aligned} \psi(z_0) &= z_0\left(2\ln(z_0) + 1 + \mu_1 z_0^2\right) \\ &= z_0\left(-3 - 3\mu_1 z_0^2 + 1 + \mu_1 z_0^2\right) \\ &= -2z_0(1+\mu_1 z_0). \end{aligned}$$

Finally, because $0 \leqslant z_0 \leqslant 1$, we see that $|\psi(z_0)| \leqslant 2(1+\mu_1)$ and we get the result. ■

We can now state the first theorem.

Theorem 1 *If $k_1 > \delta_3 = \delta_2 + 2\beta\lambda_1(1+\mu_1)(\delta_1 + k_2)$ and $k_2 > \delta_1$, the closed-loop system (2)–(3)–(4) reaches the equilibrium trajectory $(0, -d_1(t))$ in a fixed time satisfying*

$$\begin{aligned} T(x_0) \leqslant & \frac{1}{(k_1 - \delta_3)(\theta_2 - 1)} + \frac{1}{k_1 e^{\frac{-\lambda_2}{2e}} - \delta_3} \\ & + \frac{1}{(\beta - \delta_1)(\theta_1 - 1)} + \frac{1}{\beta e^{\frac{-\lambda_1}{2e}} - \delta_1}. \end{aligned} \tag{5}$$

Proof. One has

$$\begin{aligned}\dot{s} =& f(x) + g(x)u(x) \\ &+ \frac{\beta\lambda_1|x_1|\,(x_2+d_1)}{1+\mu_1 x_1^2}\left(\frac{2\ln|x_1|}{1+\mu_1 x_1^2}+1\right)|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}} + d_2 \\ =& -k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}}\operatorname{sgn}(s) + d_2 + \varphi(|x_1|)(d_1 - k_2\operatorname{sgn}(s))\end{aligned} \tag{6}$$

Now, considering the Lyapunov function $V(s) = s^2$, we see that

$$\dot{V}(s) \leqslant 2|s|\left(-k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} + \delta_2 + \varphi(|x_1|)(\operatorname{sgn}(s)d_1 - k_2)\right). \tag{7}$$

When $|x_1| > 1$, we have $\varphi(|x_1|) > 0$, therefore

$$\dot{V}(s) \leqslant 2|s|\left(-k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} + \delta_2 + \varphi(|x_1|)(\delta_1 - k_2)\right). \tag{8}$$

But given that $k_2 > \delta_1$, we get $\varphi(|x_1|)(\delta_1 - k_2) < 0$ which in turn yields

$$\dot{V}(s) \leqslant 2|s|\left(-k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} + \delta_2\right). \tag{9}$$

When $|x_1| \leqslant 1$, we have $|\varphi(|x_1|)| \leqslant 2\beta\lambda_1(1+\mu_1)$ and thus

$$\dot{V}(s) \leqslant 2|s|\left(-k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} + \delta_2 + 2\beta\lambda(1+\mu_1)(\delta_1 + k_2)\right). \tag{10}$$

Finally, we see that

$$\dot{V}(s) \leqslant 2|s|\left(-k_1|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}} + \delta_3\right) \tag{11}$$

where $\delta_3 = \delta_2 + 2\beta\lambda_1(1+\mu_1)(\delta_1+k_2)$. By using Theorem 2 in [8], one deduces that system (6) starting at $s(0) = s_0$ reaches the sliding surface $\{s = 0\}$ in a fixed-time satisfying $T(s_0) \leqslant \frac{1}{(k_1-\delta_3)(\theta_2-1)} + \frac{1}{k_1 e^{\frac{-\lambda_2}{2e}} - \delta_3}$. From (3), one has

$$\dot{x}_1 = -\beta|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}}\operatorname{sgn}(x_1) + d_1. \tag{12}$$

By using one more time Theorem 2 in [8], it is deduced that $x_1(t)$ starting at $x_1(0) = x_{10}$ reaches the origin in a fixed-time satisfying $T(x_{10}) \leqslant \frac{1}{(\beta-\delta_1)(\theta_1-1)} + \frac{1}{\beta e^{\frac{-\lambda_1}{2e}} - \delta_1}$. Finally, the closed-loop system (2)–(3)–(4) reaches the equilibrium trajectory $(0, -d_1(t))$ in a fixed time satisfying $T(x_0) = T(s_0) + T(x_{10})$. ■

Remark 1 *Let us remark that if we want to use the controller given in Proposition 2 of [8]*

$$\begin{aligned}u(x) =& -g(x)^{-1}\Bigg[f(x) + k|s|^{\frac{\lambda_2 s^2}{1+\mu_2 s^2}}\operatorname{sgn}(s) \\ &+ \frac{\beta\lambda_1|x_1|x_2}{1+\mu_1 x_1^2}\left(\frac{2\ln|x_1|}{1+\mu_1 x_1^2}+1\right)|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}}\Bigg]\end{aligned} \tag{13}$$

with $\theta_2 = \frac{\lambda_2}{1+\mu_2} > 1$, $k > \delta_2 e^{\frac{\lambda_2}{2e}}$ then we have to prove that $\frac{\beta\lambda_1|x_1|d_1}{1+\mu_1 x_1^2}\left(\frac{2\ln|x_1|}{1+\mu_1 x_1^2}+1\right)|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}}$ is bounded, i.e.

$$\lim_{x_1\to\infty}\frac{\beta\lambda_1|x_1|d_1}{1+\mu_1 x_1^2}\left(\frac{2\ln|x_1|}{1+\mu_1 x_1^2}+1\right)|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}} < +\infty. \tag{14}$$

It leads to the condition $\lambda_1 \leqslant \mu_1$ because $|x_1|^{\frac{\lambda_1 x_1^2}{1+\mu_1 x_1^2}}$ behaves as $|x_1|^{\frac{\lambda}{\mu}}$ far from the origin $x_1 = 0$. However, we have the condition $1 + \mu_1 < \lambda_1$ coming from Theorem 2 in [8] which is incompatible with the condition $\lambda_1 \leqslant \mu_1$. This is the reason why we developed the new controller (4) *which is discontinuous in the sliding variable but able to manage the mismatched disturbance.*

Remark 2 *With Theorem 1, we obtain that $x_2(t)$ tracks $-d_1(t)$ in a fixed time. Thus, we obtain a new fixed-time estimator of the mismatched disturbance without using observers.*

Consider now the following new controller inspired from Proposition 3 in [8]

$$\begin{aligned}u(x) = -g(x)^{-1}\Big[&f(x) + k_1\operatorname{sgn}(s) + k_2|s|^{\alpha}\operatorname{sgn}(s) \\ &+ \varphi(|x_1|)(x_2 + k_3\operatorname{sgn}(s))\Big]\end{aligned} \tag{15}$$

with $\alpha > 1$.

Theorem 2 *If $k_1 > \delta_3 = \delta_2 + 2\beta\lambda_1(1+\mu_1)(\delta_1+k_3)$, $k_2 > \delta_3$ and $k_3 > \delta_1$ then the closed-loop system* (2)–(3)–(15) *reaches the equilibrium trajectory $(0, -d_1(t))$ in a fixed time satisfying*

$$\begin{aligned}T(x_0) \leqslant& \frac{1}{(k_2-\delta_3)(\alpha-1)} + \frac{1}{k_1-\delta_3} \\ &+ \frac{1}{(\beta-\delta_1)(\theta_1-1)} + \frac{1}{\beta e^{\frac{-\lambda_1}{2e}} - \delta_1}.\end{aligned} \tag{16}$$

Proof. Similarly to the proof of Theorem 1, we get, with $V(s) = s^2$:

$$\dot{V}(s) \leqslant 2|s|\left(-k_1 - k_2|s|^{\alpha} + \delta_3\right) \tag{17}$$

where $\delta_3 = \delta_2 + 2\beta\lambda_1(1+\mu_1)(\delta_1+k_3)$. If $|s| > 1$, we see that $\dot{V} \leqslant 2|s|(\delta_3 - k_2|s|^\alpha)$. But given that $|s| > 1$ and $\delta_3 > 0$, we get

$$\dot{V}(s) \leqslant 2|s|(\delta_3|s|^\alpha - k_2|s|^\alpha) \tag{18}$$
$$\leqslant 2(\delta_3 - k_2)V(s)^{\frac{1+\alpha}{2}} \tag{19}$$

which yields, as long as $|s(t)| > 1$, $1 \geqslant V(s(t))^{\frac{1-\alpha}{2}} \geqslant (1-\alpha)(\delta_3 - k_2)t$. This inequality shows that $s(t)$ reaches the set $\{V \leqslant 1\}$ in a time $T_1 \leqslant \frac{1}{(1-\alpha)(\delta_3-k_2)}$. If $|s| \leqslant 1$ we get

$$\dot{V}(s) \leqslant 2|s|(\delta_3 - k_1) \leqslant 2(\delta_3 - k_1)V^{\frac{1}{2}}. \tag{20}$$

Therefore, as long as $s(t) > 0$, $V(s(t))^{\frac{1}{2}} \leqslant 1 + (\delta_3 - k_1)t$, which yields that $s(t)$ reaches 0 in a time $T_2 \leqslant \frac{1}{k_1-\delta_3}$. Finally, we conclude that, regardless of its initial condition, $s(t)$ reaches 0 in a fixed-time $T(s_0) \leqslant T_1 + T_2$. The end of the proof is similar to the one of Theorem 1. ■

Some guidelines are provided below in order to tune the parameters of controller (15):
1. choose δ_1 and δ_2 according to the maximum values of disturbances d_1 and d_2;
2. take $\lambda_1 > 0$ and $\mu_1 > 0$ such that $\theta_1 = \frac{\lambda_1}{1+\mu_1} > 1$;
3. take β such that $\beta > \delta_1 e^{\frac{\lambda_1}{2e}}$;
4. take $k_3 > \delta_1$; 5. take $k_1 > \delta_3$ and $k_2 > \delta_3$ with $\delta_3 = \delta_2 + 2\beta\lambda_1(1+\mu_1)(\delta_1+k_3)$;
6. take $\alpha > 1$.
The guidelines for the tuning of controller (4) can be obtained in a similar way.

Example 1 *The results of Theorem 1 and Theorem 2 have been tested in simulation on a double integrator, i.e. taking $f = 0$ and $g = 1$ in system (2). The following disturbances have been chosen $d_1(t) = \sin(5t)$ and $d_2(t) = 2\sin(10t)$ so $\delta_1 = 1$ and $\delta_2 = 2$. The Euler solver with a fixed step of 10^{-4}s has been used. The parameters of both controllers are as follows: $\lambda_1 = 3$, $\mu_1 = 1$, $\beta = 5.5$, $k_1 = 30$, $\lambda_2 = 4$, $\mu_2 = 1$, $k_2 = 2$ for controller (4) and $\lambda_1 = 3$, $\mu_1 = 1$, $\beta = 5.5$, $k_1 = 39$, $k_2 = 20$, $k_3 = 2$, $\alpha = 3$ for controller (15). This set of parameters verifies the conditions of Theorem 1 and Theorem 2. The results of the simulations with the initial condition $x(0) = [1, 3]^T$ are displayed on Figure 1(a)-(b). For comparison purpose, one has implemented the fixed-time controller given in [6]. One can observe that after a reaching phase, the state x_1 converges to 0. Note that the dynamics of x_2 tracks the unknown mismatched disturbance $-d_1$. The upper bound of the settling-time (5) given in Theorem 1 is equal to 30s and the upper bound of the settling-time (16) given in Theorem 2 is equal to 8.5s. One can see that this settling-time is much smaller in the simulation, around 1s for controller (4) and around 2s for controller (15). Let us notice that the fixed-time controller for the second order system proposed in [6] and displayed on Figure 1(c) is not able to manage the mismatched disturbances d_1, contrary to the proposed controllers (4) and (15) displayed on Figures 1(a)-(b) respectively.*

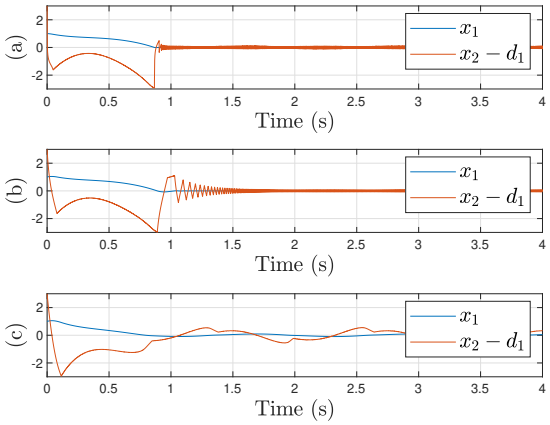


Fig. 1. (a) Double integrator with sliding surface (3) and controller (4); (b) Double integrator with sliding surface (3) and controller (15); (c) Double integrator with controller [6].

Example 2 *Let us consider a differentiable signal $h(t)$ defined on $[0, \infty)$ which is differentiable with bounded time derivative, i.e. $|\dot{h}(t)| \leqslant \delta_1$. The objective here is to estimate in fixed-time the time derivative of this signal. To achieve this objective, let us consider the following fixed-time differentiator:*

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 \\ \dot{\hat{x}}_2 &= u(\hat{x}_1 - h, \hat{x}_2) \end{aligned} \tag{21}$$

where $u(\hat{x}_1 - h, \hat{x}_2)$ is defined in (4) or (15) with $f = 0$ and $g = 1$. Defining the error $e = \hat{x}_1 - h$, one obtains

$$\begin{aligned} \dot{e} &= \hat{x}_2 - \dot{h} \\ \dot{\hat{x}}_2 &= u(e, \hat{x}_2) \end{aligned} \tag{22}$$

It yields system (2) with $d_2 = 0$ and $d_1 = -\dot{h}$. In the following simulations, the signal has been chosen as $h(t) = 0.2\cos(\frac{1}{5}t)$ so $\delta_1 = 1$ and $\delta_2 = 0$. The Euler solver with a fixed step of 10^{-5}s has been used. The parameters of u are as follows: $\lambda_1 = 2.5$, $\mu_1 = 1$, $\beta = 2$, $k_1 = 150$, $\lambda_2 = 2.2$, $\mu_2 = 1$, $k_2 = 3$ for scheme (4) and $\lambda_1 = 2.5$, $\mu_1 = 1$, $\beta = 2$, $k_1 = 150$, $k_2 = 45$, $k_3 = 1.1$, $\alpha = 3$ for scheme (15). This set of parameters verifies the conditions of Theorem 1 and Theorem 2. Furthermore, an additional uniformly bounded random measurement noise of magnitude 10^{-2} is considered in the following simulations. The results of the simulations with the initial condition $\hat{x}(0) = [1, 1]^T$ are displayed on Figure 2 and 3. One can observe that after a reaching phase, the state $\hat{x}_1$ converges to h. Note that the dynamics of $\hat{x}_2$ converges to

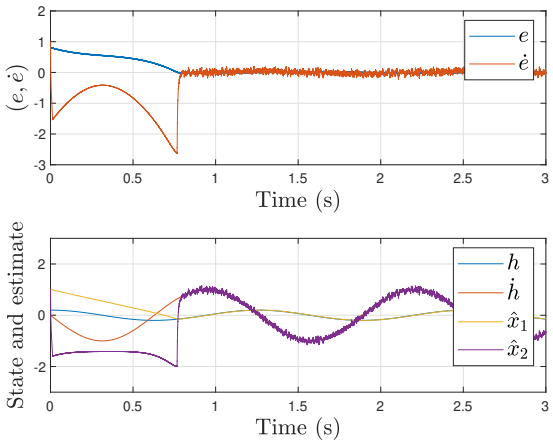


Fig. 2. (a) Fixed-time observer with sliding surface (3) and scheme (4).

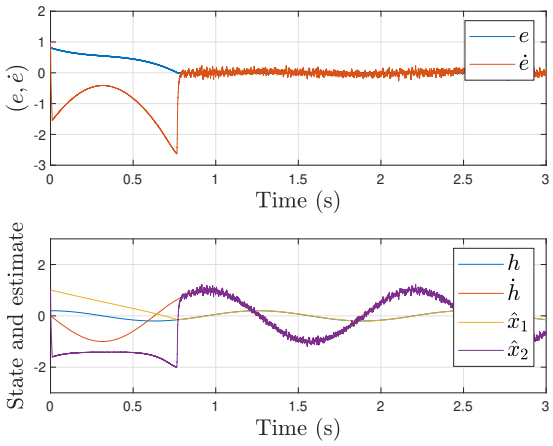


Fig. 3. Fixed-time observer with sliding surface (3) and scheme (15)

$\dot{h}$ *in fixed time. The upper bound of the settling-time* (5) *given in Theorem 1 is equal to* 8s *and the upper bound of the settling-time* (16) *given in Theorem 2 is equal to* 8s. *One can see that this settling-time is much smaller in the simulation, around* 0.9s *for both schemes.*
It is worth mentioning that there exist several finite time differentiators (see for instance [28]). Nevertheless, the convergence time, grows unboundedly with respect to the initial differentiation error. Recently, an arbitrary order fixed-time differentiator, which can been seen as a generalization of the Levant's differentiator, has been introduced in [29]. For comparison purpose, the root mean square error (RMSE) after the reaching phase for our proposed differentiator (22) *and [29] is as follows:* $2.84e-05$ *when using* (22) *with* u *given in Theorem 1,* $3.06e-05$ *when using* (22) *with* u *given in Theorem 2,* $4.72e-05$ *when uisng [29]. It can be seen that our proposed differentiators achieve better accuracy in the presence of measurement noise than the first order fixed-time differentiator proposed in [29].*

4 Conclusion

A new sliding mode controller is proposed in order to get fixed time-convergence of second order systems in presence of unknown but bounded matched and mismatched perturbations. It is applied to develop a new fixed-time differentiator. The fixed-time controller is based on the design of a robust sliding surface and does not require any disturbance observer which simplifies the solution. For future works, a high order SMC or implicit SMC strategy could be used for reducing the chattering when dealing with robust fixed-time stabilization.

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