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XAI: a Natural Application Domain for Fuzzy Set Theory

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Abstract. As digital systems cover all personal and professional activities, artificial intelligence is now everywhere. In this context, it is crucial for systems and decisions to be understandable for humans, in a human-in-the-loop process. This global objective is known as *eXplainable Artificial Intelligence* (XAI). In this chapter, we argue that fuzzy logic is a key concept for XAI as it offers a theoretical framework that is closer than many others to human cognition, human reasoning and human intuitions. We exemplify the many advantages fuzzy logic offers to the XAI domain.

Keywords: Fuzzy Logic, Artificial Intelligence, Explainable AI (XAI), Computing With Words, Machine Learning, Approximate Reasoning.

1 Introduction

Explainable Artificial Intelligence (XAI) has been at the core of many developments in Artificial Intelligence in the recent years, following the DARPA incentives¹ to not only produce more explainable artificial intelligence models while maintaining good learning performances, but also to enable the user to easily interact with intelligent systems. Several related concepts, such as interpretability, accountability, understandability, transparency or expressiveness are inherent in the capacity of intelligent systems to be explainable.

Along the same lines, the guidelines for Artificial Intelligence presented by the European Commission in April 2019² point out the importance of transparency of data, systems and Artificial Intelligence business models, as well as the necessity for AI systems and their decisions to be explained in a manner adapted to the stakeholder concerned. This has become a crucial issue as AI can be considered as a pervasive methodology, present in most digital systems, covering almost all personal and professional activities. To cite a single example, in 2019, the journal *ComputerWeekly*³ established that “*Almost one-third*

¹ <https://www.darpa.mil/program/explainable-artificial-intelligence>

² <https://ec.europa.eu/digital-single-market/en/news/ethics-guidelines-trustworthy-ai>

³ <https://www.computerweekly.com> (30 Apr 2019)

of UK bosses surveyed in the latest PwC global CEO survey plan to use artificial intelligence in their businesses – but explainable AI is critical”.

Indeed, among others, the understandability of decisions implies an increase in the decision trustworthiness, as is for instance the explicit aim of one of the first and now best-known explanation systems, LIME [61]. The understandability of decisions also implies a decrease in ethical problems, e.g. those associated with diversity (gender, culture, origin, age, etc), as it opens the way for a deep analysis of the considered model: as such, it relates to the notion of *fair AI* (see for instance [7]). Besides, another beneficial outcome of such models is the robustness of decisions. The interpretability of decisions requests better interactions with the user, raising the question of the design of interfaces dedicated to explainable systems [50].

The capacity of an AI system to meet these interpretability requirements lies in all its components. At the origin, data must be easily apprehended through their description or their visualisation, and with the help of an explanation of the characteristics used. An AI model is then applied to these data in order to extract knowledge. Its description must be clear and explanations on how the results are obtained must be available. The expressiveness of the resulting information and its legibility by the user are also part of the explainability of the AI. As a consequence, the concepts of explainability, understandability, expressiveness and interpretability must be considered as closely intertwined; among others the explainability of the model is based on its transparency and includes its expressiveness. Numerous surveys describing the state-of-the-art of this wide field have been recently published, see for instance [10, 39, 55, 7].

This chapter studies the issue from the point of view of fuzzy set theory; it argues that the latter, as elaborated as early as the 1970s by Lotfi Zadeh [73] and its numerous developments and applications since then, can be seen as having taken into account explainability considerations from its very beginning, in its motivations and principles: fuzzy set-based methods are natural solutions to the construction of interpretable AI systems, because they provide natural language-like knowledge representations, with a capacity to manage subjective and gradual information which is familiar to human beings. The easiness of interactions with the user and the transparency of features are qualities of fuzzy systems involved in their explainability; expressiveness and interpretability have been extensively investigated in the construction and analysis of fuzzy intelligent systems. In addition, fuzzy set theory and its developments consider human cognition to be the source of inspiration for the proposed methods and tools, naturally leading to legible outputs, that follow the same way of thoughts.

This chapter proposes to review some of the aspects that make the domain of fuzzy set theory and its developments appropriate tools to deal with the issues of XAI, mainly dealing with the capacity of an intelligent system to explain how it obtains results, and to provide the user with easily understandable outcomes, in light of the fuzzy paradigm.

The chapter is organised as follows: Section 2 describes the key concept called *Computing With Words*. It relies on user interactions based on using natural

language, both regarding the inputs and the outputs, which obviously makes systems easily legible. Section 3 discusses the application of fuzzy logic to model natural human ways of reasoning, providing outputs that are interpretable for the users. Section 4 is dedicated to the most visible part of Artificial Intelligence nowadays, namely machine learning, and discusses several fuzzy approaches that make it possible to increase the expressiveness and thus the explainability capacities of machine learning. Section 5 discusses some conclusions.

2 Computing With Words

Computing With Words is a key concept proposed by Zadeh [78–80] that aims at allowing to compute and reason using words instead of numbers. It allows to cope with imprecision, when for instance we know that a person is *tall* without knowing precisely her size or when precision is not necessary. For instance, even if we know precisely the heights of the people, it can be the case that we are looking for *tall* people without designing such a characteristic in a crisp manner. This thus provides flexibility and/or robustness to the systems. Beyond this flexible knowledge representation discussed in this section, Computing With Words integrates a reasoning component that mimics common sense reasoning familiar to human beings, as discussed in Section 3.

We introduce below the main concepts and formal tools before presenting some of the main applications, namely fuzzy databases and fuzzy summaries, including gradual patterns.

2.1 Formal Tools

The fuzzy set theory underlies Computing With Words by providing a formal framework: the key idea is that most of the objects that are manipulated in natural language and computation are not members of a set or not, but they rather *gradually belong* to sets.

For instance, firms can be categorised according to their size,

but the definition of categories depends on the considered country rules or domain of activity. As a result, a firm with 2 employees could then be considered as a micro-company (or Small Office Home Office a.k.a. SOHO) with no doubt, while one with 10 employees could still be considered as fairly micro but may rather be considered a small-size company, depending on the environment.

In this chapter, we claim that when considering explainable systems, this point of *gradual belonging* is crucial so that decisions can take into account the fact that every object or situation cannot always be categorised in a binary manner, as *belongs/does not belong*. This is the case in particular when complex systems are considered where there are very little simplistic black-and-white visions. The same idea holds for all domains. For instance, a system could not easily justify that it does not recommend a medicine to an infant of weight 19.8kg because it is for children over the weight of 20kg.

This points out a critical question between explainability and confidence in a system that would mimic human intelligence.

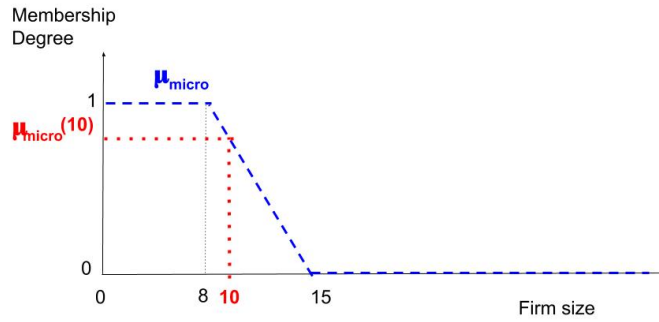


Fig. 1. Illustration of the fuzzy set *micro*

Fuzzy Sets Formally, a fuzzy set A is defined over the universe U by its membership function

$$\mu_A : U \rightarrow [0, 1]$$

that can be considered as extending the classical binary characteristic function of a set: whereas the latter allows only values 0 or 1, the membership function allows for the whole gradual range between 0 and 1.

For instance, the fuzzy set *micro* can be defined over the universe of company sizes $U = \mathbb{N}^+$ by the membership function

$$\mu_{micro}(u) = \begin{cases} 1 & \text{if } 0 < u < 8 \\ \frac{-u + 15}{7} & \text{if } 8 \leq u < 15 \\ 0 & \text{if } u \geq 15 \end{cases}$$

Such a fuzzy set can be represented graphically, as illustrated by Figure 1.

In this example, a firm of size 10 is considered as being fairly micro, with a degree $\mu_{micro}(10) = \frac{5}{7}$. By considering such fuzzy sets, it is then possible to cope with the ambiguity of the term over the world, as micro companies are not officially considered of the same size in US compared to Europe. Using such gradual membership also allows to mitigate a difference of one employee in the number of employees: classic binary set definitions are such that a firm of size 199 and a firm of size 200 belong to two different categories (medium and large respectively). The fuzzy set theory allows to make such crisp, and therefore somehow arbitrary, boundaries more gradual, and thus easier to understand.

Fuzzy sets defined on numbers can represent various types of information, as for instance:

- fuzzy intervals: a profession is said to be gender-balanced if the gender rate ranges between 40% and 60%⁴, or

⁴ <https://cutt.ly/8t3Kt3m>

- fuzzy numbers: almost 10% of employees working in AI in French companies are women⁵.

The definition of the fuzzy sets to be used in the system is equivalent to the definition of their membership functions. Three types of methods to design them can be mentioned: first, this design can rely on user expertise, so that for instance the formal definition of *micro firms* correspond to the user point of view. This makes it possible to personalise the whole system, adjusting its components (here the fuzzy sets) to her own representation. A second method to design fuzzy sets relies on cognitive considerations, see for instance [48]: it can be shown that fuzzy sets representing the imprecise notion *around x*, where x is a numerical value, actually do not vary so much with each individual user, and that they can be defined based on cognitive studies. It has been shown that they mainly depend on x magnitude, last significant digit, granularity and a notion of cognitive saliency. A third method to design fuzzy set consists in extracting them automatically from possibly available data, so that the fuzzy sets match the underlying distribution of the data. Such an approach can be applied in a supervised paradigm (see e.g. [53]) or a non-supervised one (see e.g. [65]).

As for classical sets, fuzzy sets are provided with operators such as union, intersection, difference, etc that are extended from crisp membership in $\{0,1\}$ to gradual membership ranging in $[0,1]$, based on operations on their membership functions [45].

Linguistic Variables and Fuzzy Partitions One of the central concepts for Computing With Words is the *linguistic variable* [74]. A linguistic variable is defined by a name, a universe and a set of fuzzy sets. For instance *firm size* can be defined over the universe \mathbb{N}^+ with the elements $\{\text{micro, small, medium, large}\}$. These fuzzy elements are meant to cover all values of the universe as a partition would do. In order to convey the possibility of imprecision, fuzzy sets can overlap in the thus so-called *fuzzy partition*.

Fuzzy partitions can be defined in two main manners. First, a partition P over the universe U can be defined as a set of fuzzy sets $\{p_1, \dots, p_n\}$ with membership functions $\{\mu_{p_1}, \dots, \mu_{p_n}\}$ such that

$$\forall u \in U, \quad \sum_{i=1}^n \mu_{p_i}(u) = 1$$

In such a definition, all fuzzy sets cross so that the sum of the membership degrees is 1 for any element that thus has total membership. For instance, we may consider $P = \{\text{micro, small, medium, large}\}$ with membership functions represented by Figure 2.

Another way to define a partition P over the universe U is to consider that:

$$\forall u \in U, \quad \exists i \in \llbracket 1, n \rrbracket \text{ such that } \mu_{p_i}(u) > 0$$

⁵ <https://cutt.ly/tt3KoM3>

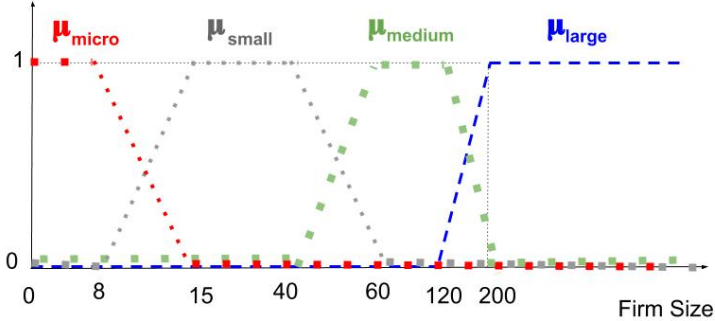


Fig. 2. A fuzzy partition

This definition leaves more flexibility for setting the membership functions that build the partition. Membership functions can be built by an expert of the domain, on the basis of a subjective assessment, or automatically derived from real data, by means of supervised learning or optimisation techniques (see e.g. [16]). In the first case, the partitions are generally of the first type, while they can be of the second type when obtained automatically.

Fuzzy Quantifiers Fuzzy set theory also enables to represent another type of imprecise concepts, using so-called *fuzzy quantifiers*, that for instance allow to model the linguistic quantifiers *a few* or *most*, beyond the classical mathematical quantifiers \forall and \exists . To that aim, the fuzzy quantifiers are modelled through their membership functions. Absolute fuzzy quantifiers, like *at least 20* or *almost 5*, are distinguished from relative ones, like *most*: the first ones are defined over the universe of natural numbers, the second ones over the $[0, 1]$ universe of proportions. For instance the two quantifiers *few* and *most* can be defined by the membership functions shown in Figure 3.

These fuzzy quantifiers allow to describe situations with words instead of crisp numbers whose details can for instance make difficult to cope with the given message, or even undermine the message. We can for instance feel uncomfortable with a sentence providing information about *more than 96.3%* of the data, and rather replace it with *almost all data*.

2.2 Examples of Methodological Utilisation

In this section, we present some applications based on fuzzy sets presented above. These applications provide users with tools that help them to better understand and deal with their data. As many data are stored in databases, the first part is thus devoted to fuzzy databases, before presenting methods for data summarisation.

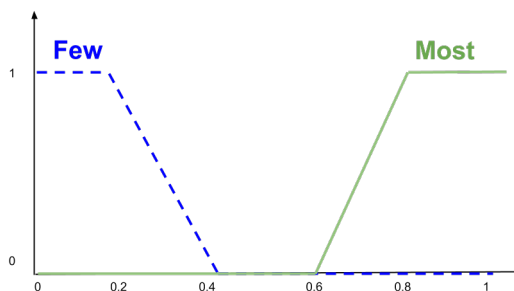


Fig. 3. Example of fuzzy quantifier membership functions

	2011	2012	2013	2014	2015	2016	2017
1 to 9 employees	16.5	15.8	15.4	14.8	14.1	14.2	12.6
10 to 49 employees	20.2	20.5	20.5	19.6	21.5	20.0	20.4
50 to 249 employees	21.3	19.3	20.5	20.1	20.6	21.0	19.3
250+ employees	21.5	20.7	20.6	19.7	19.7	19.2	19.3

Table 1. Gender pay gap, measured in %, for all employees by business size & year UK, ASHE, 2011-2017 (<https://cutt.ly/4t3KqP8>)

Fuzzy Databases As databases are a key component in many numerical systems, they are a key element to consider in order to provide explainable systems.

The literature provides many works on fuzzy databases [56], especially for the relational model which has been extensively considered. In such systems, imprecision is considered for both data and/or queries. Flexible queries can thus be of different forms [72]: they can allow users to propose either “crisp” or “fuzzy” terms in the queries themselves and can rely on “crisp” or “fuzzy” data. In flexible queries, fuzzy terms (words) are integrated within the various clauses of extended query languages, as for instance SQL_f for relational databases [15, 66].

Such extensions provide ways to better get information from raw data. For instance, Table 1 reports the gender pay gap depending on the business size in 2017 in UK, as published within the 2017 Annual Survey for Hours and Earnings from the Office for National Statistics⁶. From such data, if the user wants to extract the years and business sizes when the pay gap has been *lower than 14%*, then she will not come out with “2015 for businesses of size 1 to 9 employees” because it was 14.1. This result is likely to appear as non explainable, as 14.1 is so close to the requested threshold. With fuzzy queries, the condition *lower than 14%* can be made flexible so as to gradually consider the situations as described above with fuzzy membership functions.

As relational databases have been extended to multidimensional databases and NoSQL engines, works have proposed models and query languages for these frameworks. [47] proposes fuzzy operators for OLAP databases which are meant

⁶ <https://cutt.ly/4t3KqP8>

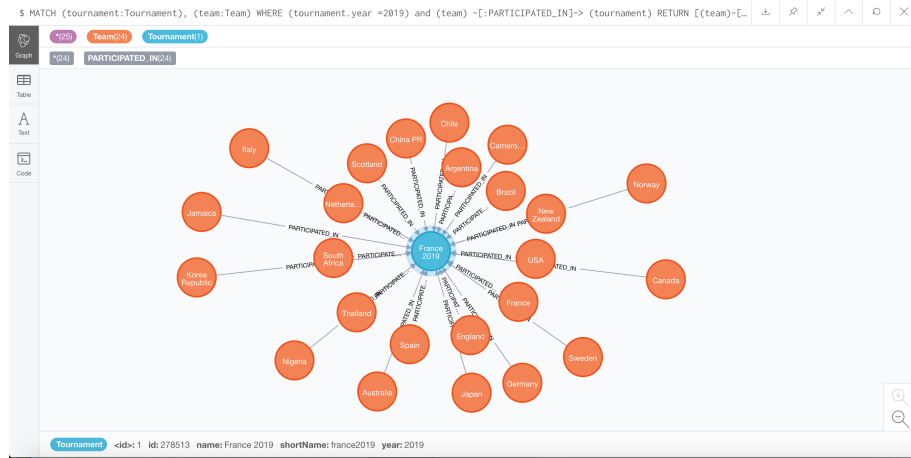


Fig. 4. Nodes, properties and relationships labels in a property graph database [57]

to help decision makers to navigate through OLAP cubes. Such cubes provide aggregated data structured by measures analysed over dimensions, as for instance the *number of sales* as a measure and *category, time, location* as dimensions. Fuzzy queries can be expressed, like navigating through the *number of products of category Outdoor being sold in Winter near Paris* where *Winter* can be defined in a fuzzy manner, so as the *Outdoor category* and *near Paris*.

More recently, NoSQL databases have emerged to cope with very large databases. Some of them specifically address the need to represent and query graph data. In this case, data and links are represented by two families of objects: the so-called *nodes* and *relationships* which have labels and types. These two families of objects are similar to any graph structure. But in property graphs, so-called properties can be defined over nodes and relationships. For instance, persons may be described by their name, birth date, etc.

Consider the example of the dataset describing 2019 women’s world cup [57], in this database, we have⁷: Persons, Teams participating in Matches from Tournaments. Figure 4 represents the teams participating in the 2019 tournament in France.

It has been shown that such data engines allow to process complex queries that would exceed the performance limitations of the relational databases. This is especially the case when queries require too many “join” operations (as for instance to retrieve persons playing in teams involved in matches from specific tournaments).

These engines are provided with their own query language that has been extended to allow fuzzy queries such as *retrieve the teams participating in most of tournaments* [26, 59].

⁷ Data can be retrieved from <https://neo4j.com/sandbox/>.

Fuzzy Linguistic Summaries Pursuing the same goal of making information as understandable as possible, fuzzy summaries have been proposed in the 1980s [71, 44] (see [21, 60] for surveys). They summarise data using so-called *protoforms*, defined as sentence patterns to be filled with appropriate values depending on the considered data, based on fuzzy sets for expressing concepts and quantifiers.

Such fuzzy summaries can be illustrated by sentences such as *Most women are less-paid than men* or *Few large firms display a low gender pay gap*. The second one is an instantiation of the protoform defined as

$$Qy \text{ are } P$$

where

- Q stands for a fuzzy quantifier (e.g. Few),
- y are the objects to be summarised (e.g. large firms),
- and P is a possible value, such as in *low gender pay gap*.

The quality of linguistic summaries can be assessed by many measures (see for instance [27] for a survey), the seminal one being the *degree of truth*, denoted T that computes the extent to which the (fuzzy) cardinality of the data satisfying the fuzzy set P satisfies the fuzzy quantifier Q . Formally

$$T(Qy's \text{ are } P) = \mu_Q \left(\frac{1}{n} \sum_{i=1}^n \mu_P(y_i) \right)$$

where n is the number of objects (y_i) that are summarised and μ_P and μ_Q are the membership functions of the summariser P and quantifier Q , respectively.

In order to extract such summaries from large data sets, efficient methods have been proposed [33, 67].

Extended Linguistic Summaries Other types of patterns can help to capture and extract knowledge, as for instance gradual patterns [42, 32, 46] that are of the form *the more/less X, the more Y*. For instance, sadly, the following gradual pattern holds⁸: *the higher the level of responsibility, the less the proportion of women*. Such patterns can take a fuzzy form, when X and Y represent membership degrees to fuzzy linguistic variables [4], as in “the more the age is *middle-aged*, the more the gender pay gap is *large*”.

Such gradual patterns can be extended to more complex protoforms or more complex data, thus helping to better describe, understand and explain phenomena. For instance, protoforms can be extended to take into account the fact that some contexts can strengthen a pattern [20]: *the younger, the lower the salary, all the more as employees are women*.

They can also help to understand and explain how the spatial dimension impacts some phenomena: spatial gradual patterns have been proposed to recommend decisions in public health policies by explaining the spatial dynamic of

⁸ <https://www.insee.fr/fr/statistiques/3363525>

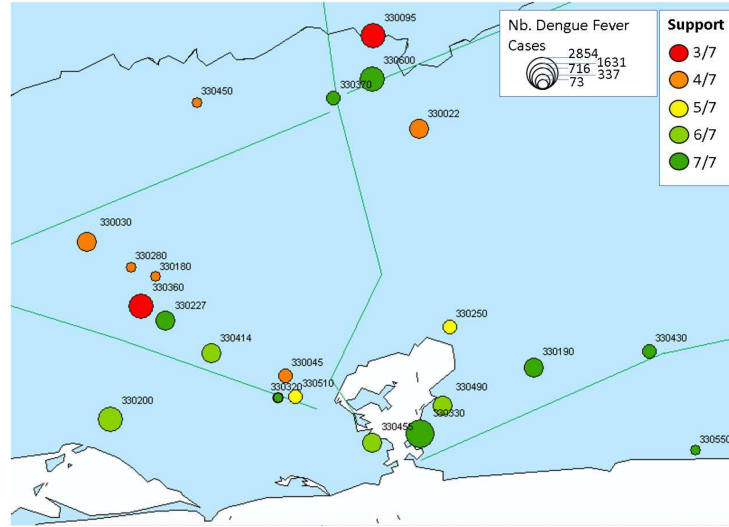


Fig. 5. Gradual Map (Temperature, \uparrow), (Mobility, \downarrow), (NbCases, \downarrow) [3]

potentially avoidable hospitalisations [58] or to describe epidemiological dengue fever data from Brazil, by mining for gradual patterns in a geographical information system [3]. In the latter work, patterns like “*the higher the temperature, the lower the mobility, the lower the number of cases*” can be extracted and information can be plotted on so-called gradual maps, as shown on Figure 5. Here, it can be seen that emerging epidemic dengue fever mostly appeared in cities located around the coast (displayed in green on the map).

Protoforms can also be extended from complex data, such as time series [1, 43] or property graphs (introduced above), as in [64].

From such property graph data sets, various summaries and gradual patterns can be extracted. For instance⁹ “*the higher the number of matches played, the higher the number of won tournaments*”. Exceptions may be retrieved, such as “*the higher the number of matches played, the higher the number of won tournaments, unless the tournament is not in final draw*”.

Computing With Words conclusion Whatever the considered type of data or protoform, the linguistic summaries that fuzzy set theory allows to output share the property of being very easy to understand, by all kinds of user, from data or model experts to naive users who want to explore available data. As flexible database queries, this example of Computing With Word application illustrates the benefits of this framework in the eXplainable Artificial Intelligence context.

⁹ These summaries are given as examples and have not been extracted from the data set.

3 Fuzzy Approximate Reasoning

Approximate Reasoning [76, 9], that aims at performing more natural reasoning than classical logic, offers another domain where fuzzy set theory developments, in the form of fuzzy logic theory, provide valuable tools for interpretability and explainability of decisions.

This section presents the general principles of this approach and then describes in more details several tools for fuzzy approximate reasoning: it discusses the Generalised Modus Ponens, which is the inference rule offered by fuzzy logic, and gives a short presentation of other inference rules that allow to model gradual, analogical and interpolative reasoning and, therefore, to increase the legibility of the reasoning as well. It finally discusses some examples of utilisation of these formal tools.

3.1 General Principles

Formal logic provides a theoretical framework to model inference and reasoning; among others, it aims at guaranteeing that all reasoning steps are correct, free from any error and bias. Yet classical logic is binary, insofar as any assertion can only be *true* or *false*, without any intermediate or other truth value. Now limiting the possibilities to these two exclusive values actually remains far from the natural way of thinking for human beings, who intuitively may use degrees¹⁰, such as *rather false*, *more or less true*, or who may answer *I do not know*. Fuzzy logic, as a generalisation of multi-valued logic [37], aims at providing a theoretical framework to model reasoning with such extended *truth degrees*. The latter are of course related to the notions of membership degrees and fuzzy sets presented in the previous section.

As such, fuzzy logic offers a formal framework that is closer to the natural way of thinking of human beings and, therefore, more easily legible and understandable: from its very introduction by Zadeh [76], fuzzy logic can be seen as a tool for explainable artificial intelligence.

3.2 Generalised Modus Ponens

Modus Ponens The classic Modus Ponens inference rule, also named implication elimination in the natural deduction or sequent calculus inference systems, can be sketched as follows

$$\frac{A \implies B \quad A}{B}$$

¹⁰ Another increase of expressiveness, beside the use of degrees, is to consider modalities, such as *I believe it is true* or *it ought to be true*. Formalising reasoning with these modalities is the aim of modal logic [11], which is out of scope of this chapter.

It means that from the pieces of knowledge that the rule *if A, then B* holds and that *A* holds as well, the piece of knowledge that *B* holds can be inferred.

Let us for instance consider the rule that characterizes the French demographic balance statistics¹¹ in 2019: *if age is between 50 and 60 years old, the proportion of women is between 53.4% and 56.8%*. When observing people aged 53, the rule premise $A = \textit{age is between 50 and 60 years old}$ holds and thus allows to infer that the rule conclusion *B* holds, i.e. that the proportion of women among people of that age is between 53.4% and 56.8%.

Fuzzy extension The modus ponens rule can be considered as strict, both in terms of inference and representation. Indeed, it first requires that the implication premise is exactly satisfied and does not allow for any tolerance: in the previous example, if the considered age is 49, then condition *A* is considered not satisfied and the inference process does not take place. Thus no new piece of knowledge can be derived. This can be considered as difficult to interpret for a human being, who may intuitively expect that the implication rule applies to some extent and that the proportion is not far from this interval¹². Approximate reasoning aims at performing more natural reasoning and among others at allowing to perform inference even in such a case where the observation A' does not perfectly match the rule premise *A*.

Another difficulty with classical logic is that it only allows for precise, binary, concepts to be used, as for instance the mathematical condition "between 50 and 60" or exact value, e.g. "equals 53.4". It does not allow to take into account imprecise concepts, such as "young people" or "slightly above balance". Such fuzzy concepts are however easier to understand and to process for human beings, as discussed in Section 2.

Fuzzy logic proposes to generalise binary logic on both points, regarding representation and inference tools. Regarding the representation issue, the notion of fuzzy set and fuzzy linguistic, as well as fuzzy quantifiers, as presented in Section 2.1, can be used to model imprecise concepts. Regarding the inference process, the Generalised Modus Ponens [75] implements a solution to approximate reasoning and can be sketched as:

$$\frac{A \implies B \quad A'}{B'}$$

The main difference with classical Modus Ponens is that A' may be different from *A* and still allow to infer a piece of knowledge B' . However, B' is then usually different from *B*, to take into account the difference between *A* and A' .

¹¹ <https://www.insee.fr/en/statistiques/2382597?sommaire=2382613>

¹² Actually, the proportion of women among people aged 49 is 50.5%, according to the same source.

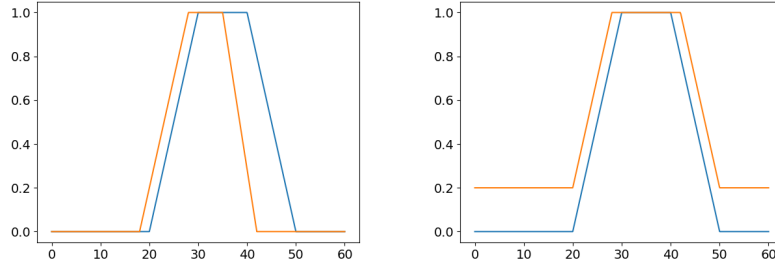


Fig. 6. Illustration of the Generalised Modus Ponens with Lukasiewicz operators: (left) rule premise A in blue, observation A' in red; (right) rule conclusion B in blue, output B' in red.

Formally, the membership function of the output is defined from that of the rule premise, rule conclusion and observation as

$$\forall y \in \mathcal{Y}, \mu_{B'}(y) = \sup_{x \in \mathcal{X}} \top(\mu_{A'}(x), \mathcal{I}(\mu_A(x), \mu_B(y)))$$

where \top is a so-called t-norm, i.e. a conjunctive operator, and \mathcal{I} a fuzzy implication operator. This equation means that a value y has a high membership degree to B' if there exists (sup operator) at least one value x that both (\top conjunctive operator) belongs to the observation A' and can imply (\mathcal{I} implication) y when considering the rule $A \implies B$.

Figure 6 shows an example of result that can be obtained when considering fuzzy subsets defined on real numbers, i.e. $\mathcal{X} = \mathcal{Y} = \mathbb{R}$, with the Lukasiewicz operators $\top(u, v) = \max(u + v - 1, 0)$ and $\mathcal{I}(u, v) = \min(1 - u + v, 1)$. The left part of the figure shows the rule premise A in blue and the observation A' in red, that only partially matches A ; the right part shows the rule conclusion B in blue and the obtained output B' in red: although A' is not equal to A , an informative inference can be performed. B' can be observed to be a fuzzy subset similar to B , although with a larger kernel (set of values with membership degrees equal 1). The main difference is that, for B' , all values in the universe have a non-zero membership degree: the minimal value is 0.2. This is interpreted as an uncertainty level, expressing that, because the observation only partially matches the rule premise, the rule does not completely apply. Therefore, some values not implied by the rule may not be excluded, due to the fact that the rule premise only partially accounts for the observation. However, the rule still provides some information, whereas a classical logic inference process would not allow to draw any conclusion from the observation.

3.3 Other Reasoning Forms

Generalised Modus Ponens makes it possible to perform more natural reasoning than the strict classical logic Modus Ponens, however it can be considered that

it also suffers from some limitations: this section briefly presents other types of approximate reasoning variants, namely gradual, analogical and interpolative reasoning.

Gradual Reasoning In the case where there is no match at all between the observation and the rule premise, the Generalised Modus Ponens usually outputs a conclusion with full uncertainty: it thus expresses that the rule does not allow to perform any informative inference, which can be considered as justified insofar as the rule does not actually apply to the observation. Gradual reasoning has been proposed as a method to take into account other principles to guide inference, and in particular monotonicity constraints¹³: it considers the case where the rule *if A, then B* is actually not to be understood as a pure implication, but in the form *the more A, the more B*. As such, it is a reasoning, logical, counterpart to the gradual patterns discussed in the previous section.

Gradual reasoning requires a monotonous behaviour of the inference result with respect to the rule premise: it takes into account the relative position of the observation with respect to the premise. Considering, e.g. fuzzy sets defined on the universe $\mathcal{X} = \mathbb{R}$, it allows to get different results when the observation is on the left or on the right of the premise. Roughly speaking, if the observation is shifted with respect to the rule premise, then the inferred result should also be shifted with respect to the rule conclusion, which is not the case with Generalised Modus Ponens, as illustrated on Figure 6. Indeed, the result B' is symmetrical with respect to the rule conclusion B , not taking account the fact that the observation A' is on the left of the rule premise A .

For the example given previously, regarding gender balance, one can consider that the actual rule knowledge is of the form *the higher the age, the higher the proportion of women*, with the reference proportion given in the previous rule. Considering gradual reasoning would then lead to different results in the case where the observations A' is *age is around 48* or *age is around 52*.

There exist several formal expressions to model this type of natural reasoning, which are not developed in this chapter. The interested reader is for instance referred to [69, 12].

Analogical Reasoning Another natural way of thinking, called analogical reasoning (see for instance [8]), stems from the philosophical concept of analogy, regarded as a process to transfer information from a domain to another one on the basis of similarities. Analogy is widely used by human beings to make decisions, use metaphors or understand complex systems. It became popular in artificial intelligence in the form of the so-called case-based reasoning. A first fuzzy version of case-based reasoning was proposed in [13], followed by many derived versions. More generally, fuzzy analogical reasoning relies on a comparison between the observation and the rule premise. Schematically, from a "rule"

¹³ This notion of monotonicity is here understood as a global constraint across the universes, and not in terms of truth values, as is sometimes the case [34].

that links A and B and from the observation, it outputs B' such that B' is to B what A' is to A . As such, it does not explicitly consider an implication relation between A and B and it does not perform logical inference: rather, it exploits comparison measures, to assess the similarity between the observation and the rule premise and to build the output with an equivalent similarity relation to the rule conclusion.

This approach has also been developed as a method of approximate reasoning, with many variants, see [29, 17, 19, 30] to name a few. Another form of reasoning based on similarities was introduced as *similarity-based reasoning* by [63] and developed in [35]. It can be observed that, to some extent, gradual reasoning is also related to analogical reasoning, because of its integration of a relative position comparison. Other approaches [12] combine even more explicitly the analogical similarity principle with the Generalised Modus Ponens inference process.

Interpolative Reasoning A derived case of analogical reasoning concerns an extension of classic interpolation, very useful in the case of sparse data, to make decisions in the interval between two existing cases: it also makes it possible to perform inference even when no rule applies to the considered observation. It relies on the comparison of the observation with the premises of other rules that differ.

When the available information is imprecise, fuzzy interpolative reasoning enables the user to make a decision in a region of the universe where there is no explicit data, considering both a graduality in decisions with respect to inputs and an analogy with existing data [18, 22, 6].

Fuzzy Reasoning Conclusion In all cases, the underlying motivation for fuzzy reasoning approaches is to model natural ways of thinking, so that a user can easily understand the performed inference and thus the obtained result, while still offering guarantees about the reasoning validity. The latter come from the proposed formalisation, using a reference formula for building the output and predefined parameters (e.g. conjunctive or implication operators or comparison measures).

3.4 Examples of Methodological Utilisation

The formal tools described in the previous subsection lead to many different types of applications. In the framework of machine learning, they open the way for the successful fuzzy rule-based systems, presented in Section 4.1. Two other examples of methodological utilisation are presented here.

Fuzzy Ontologies Reasoning tools are of particular use when knowledge is represented using ontologies, for instance in the framework of description logic [5]. Fuzzy logic has been integrated into ontologies [24], so as to benefit from its advantages regarding imprecise or vague knowledge representation, suitable to

model and process concepts as they are handled by human beings and they thus improve the interactions with users. Fuzzy description logic [68] provides solutions to instance checking, relation checking, subsumption, consistency, in an imprecise environment. It has in particular proved to be highly useful in the semantic web.

Another development of fuzzy set theory and fuzzy logic, not detailed in this chapter, is the notion of possibility theory and possibilistic logic [77]: they make it possible to deal with uncertain knowledge, whereas fuzzy logic is more dedicated to the case of imprecise knowledge. Possibilistic description logic [28, 49] is based on possibilistic logic and, in particular, allows to represent and manage certainty about instances and relations. It is based on gradual and subjective uncertainties and evaluates the non-statistical confidence assigned to a piece of information. It can be used in multisource and heterogeneous information environments.

Fuzzy control The theoretical principles for extended formal logic also founded the practical domain of fuzzy control [38]: similar principles are applied to control systems, using rules that define the desired behaviour and fuzzy inference definitions to derive the applied command for a given observed state of the system. Fuzzy control has numerous real world applications in many domains for instance including electrical household appliances (washing machines, fridges, air conditioning, vacuum cleaner), autonomous vehicles (smart metro, helicopter, train inclination in curves) or industrial applications (steel industry, cement works, paper mills) to name a few. Among these, smart transportation, internet of things and robotics are examples of domains where the need of explainable AI is the most important because the users request security and explanations about the functioning of the systems in which they are personally involved.

4 Fuzzy Machine Learning

Machine learning is nowadays the most visible part of Artificial Intelligence. Statistical machine learning provides very good results in classification or decision-making, in particular in the case of big data. Nevertheless the reasons why an object is classified in a given class or why a decision is made are not obvious to the user. On the opposite, fuzzy inductive learning based on fuzzy decision trees provides the explicit list of attributes involved in the result, which enables the user to understand the reasons of the decision. A fuzzy set-based knowledge representation offers the advantage of taking into account imprecise or linguistically expressed descriptions of the cases, which facilitates interactions with end users in allowing them to use their natural manner to describe situations.

These considerations have nevertheless their limits. It has been pointed out that a trade-off is necessary between explainability and accuracy [31] where it is often considered that graphical models or decision trees are more explainable than deep learning or random forests, although sometimes at the expense of a lower prediction accuracy. This principle also applies to the case of fuzzy models [25].

Experimental approaches to the interpretability of fuzzy systems have been investigated [2] in an attempt to cope with the subjective perception of explainability. Multi-objective optimisation strategies can be used to maximise the interpretability of fuzzy AI systems while looking for high accuracy [51].

This section discusses two examples of classification systems, namely fuzzy rule-based systems and fuzzy decision trees, as the unsupervised learning case of clustering.

4.1 Fuzzy Rule-based Systems

Fuzzy rule-based systems are historically the first fuzzy AI systems, developed in the 1970s after their inception by L.A. Zadeh [73]. They are recognised to provide an easy interaction with the user.

Rules have the form "if V_1 is A_1 and V_2 is A_2, \dots , then W is B ", for instance "If the temperature is low and the wind is speedy the the risk is high". V_1, V_2, \dots, W are linguistic variables associated with fuzzy modalities A_1, A_2, \dots, B belonging to fuzzy partitions of their respective universes, as introduced in Section 2.1. They can be processed to infer decision for a given observation using tools such as the Generalised Modus Ponens, as described in Section 3.2.

Such rules were originally elicited from experts, as it was the case for expert systems. They were later obtained automatically through machine learning. Modalities are tuned, either by a consensus among experts made possible thanks to aggregation methods available in fuzzy logic, or automatically by various methods such as genetic algorithms or evolutionary computation [36]. Techniques for rule pruning or rule merging participate in the reduction of complexity [40].

Various types of properties are involved in the explainability of fuzzy rule-based systems. They have in particular be decomposed into high level properties, such as compactness, completeness, consistency and transparency of the set of rules, as opposed to low level properties for the set of modalities associated with each attribute, namely the coverage of the universe, convexity of membership functions or distinguishability between modalities [81]. Measures of focus and specificity of information have also been proposed to evaluate the informativeness of information in such systems [70].

4.2 Fuzzy Decision Trees

Fuzzy decision trees constitute classifiers that are built by inductive learning from a set of training data. As their crisp counterpart, they are constructed from the root to the leaves, by successively selecting the most discriminant attribute regarding classes. However, each edge from a vertex is associated with one of the *fuzzy* modalities describing the attribute selected for the vertex [52, 54]. As it was the case for fuzzy rules, modalities can be defined by experts or automatically constructed, for instance by means of mathematical morphology [53].

A path from the root to a leaf of the fuzzy decision tree, that represents the decision for a given observation data point, is then associated with a series of

linguistic variables described by a fuzzy modality: it provides a linguistic rule which can be easy for the user to understand. Therefore fuzzy decision trees are deemed to be explicit. It must still be noted that their complexity, required to achieve better accuracy, may make difficult the understanding of reasons of a decision or a classification, in the case when the number of involved features is too big to be grasped by the user.

4.3 Fuzzy Clustering

Unsupervised machine learning also has been considered from the point of view of fuzzy tools, in particular for the topic of clustering. Indeed, in classic clustering, each object is assigned to a cluster, in such a way that the elements of each cluster are similar to each other. It can be observed that in many cases, the boundaries between clusters do not look natural, but somehow arbitrary and are difficult to justify.

Probabilistic approaches to clustering, among others using the reference Gaussian Mixture Model, have been proposed to integrate more flexibility. However, they consider a different cost function for the clustering task: they do not aim at optimising an objective function expressing a balance between intra-cluster homogeneity and inter-cluster separability, but aim at modelling the data underlying probability distribution. Even if the two aims are related, they are not identical.

Fuzzy clustering, as first introduced by [62] and widely developed since then (see e.g. [41, 23, 14]), preserves the initial cluster compactness/separability cost function while also enabling an object to belong to several clusters, at least to a certain extent. The degree of membership of objects to clusters defines fuzzy classes, with imprecise boundaries. A fuzzy clustering is therefore more explainable to the user than a crisp clustering.

5 Conclusion

This chapter illustrated that fuzzy models effectively participate in solutions to achieve Explainable AI, due to their underlying principles: the concepts, methods and tools of fuzzy logic on which they are based, among which in particular imprecision management and Computing With Words, are key components to design interpretable and legible systems with high expressiveness. The latter then naturally implement human-in-the-loop requirements and are currently used in a very large spectrum of applications: they for instance include instrumentation, transportation, sentiment analysis, fuzzy question answering systems, information quality, and many more.

There are still many research avenues in fuzzy logic, and it seems clear that, as XAI is developing, the role of fuzzy logic will be growing. Even when it is not called fuzzy logic, the concepts of graduality or management of imprecision, are indeed of crucial importance to comply with the needs. Although currently too much reduced to deep learning, artificial intelligence is a very large domain

that most likely needs to take into account complex cognitive paradigms so as to achieve its explainability aim and its friendly acceptance by users. The general framework of fuzzy set theory and fuzzy logic can be seen as a key concept that offers many tools to progress in this direction.

The authors do not pretend to present an exhaustive list of fuzzy logic-based solutions to the difficult paradigm of explainable AI, but they rather describe various ways to integrate expressiveness, transparency and interpretability in smart devices or artificial intelligence-based decision support systems, to comply with the requirements of explainable artificial intelligence. Fuzzy artificial intelligence should not be limited to fuzzy rule bases, as it is often the case. Likewise, explainable AI must not be restricted to the interpretability of statistical machine learning or statistical data science.

The purpose of this chapter is to open the door to a multiplicity of solutions linking Computing With Words and, more generally fuzzy logic, with XAI and to show that many more developments can be expected.

Let us point out that the human component of all AI systems, be it the user, the expert, the patient or the customer, is carefully taken into account in all fuzzy models, whose main mission is to mimic natural language and common sense reasoning, with a capacity to deal with subjective information and qualitative uncertainty. This is a reason why many women feel interested in the field and the gender balance is very good in the fuzzy community.

Biographies

Bernadette Bouchon-Meunier entered the Ecole Normale Supérieure Cachan in the Mathematics main stream and discovered that a new option on Computer Science had just been created. Finding this option appealing, she chose to follow these courses in addition to courses in Mathematics and she received Master of Science degrees in Computer Science in 1970 and in Mathematics in 1972. She discovered Artificial Intelligence in 1969 under the guidance of the French Artificial Intelligence pioneer, Jacques Pitrat. She then earned a Ph.D. in Applied Mathematics in 1972 and a D. Sc. in Computer Science from the University of Paris in 1978. She was hired as a full-time researcher at the National Centre for Scientific Research in 1972 and started to work on information processing and decision support systems. In 1973, she discovered by serendipity the recent concept of fuzzy set created by L.A. Zadeh and it changed the direction of her research. Her professional life would not have been the same without the discovery of this promising paradigm and her immersion in the emerging fuzzy community, led by L.A. Zadeh, who was a true mentor to her.

She began to be the head of a research group in 1979 in the Paris VI-Pierre et Marie Curie University and, in 1990, she was promoted to director of research by the National Centre for Scientific Research. At the end of her career, she was the head of the department of Databases and Machine Learning in the Computer Science Laboratory of the University Paris 6 (LIP6). She is now director of research emeritus. She is the co-executive director of the IPMU International

Conference held every other year, that she created with R.R. Yager in 1986. She is the Editor-in-Chief of the International Journal of Uncertainty, Fuzziness and Knowledge-based Systems that she founded in 1993. She is also the (co)-editor of 27 books and the (co)-author of five. She supervised 52 PhD students, including 15 female students.

She is currently the President of the IEEE Computational Intelligence Society (2020-2021) and the IEEE France Section Computational Intelligence chapter vice-chair. She is an IEEE Life Fellow, an International Fuzzy Systems Association Fellow and an Honorary Member of the EUSFLAT Society. She received the 2012 IEEE Computational Intelligence Society Meritorious Service Award, the 2017 EUSFLAT Scientific Excellence Award and the 2018 IEEE CIS Fuzzy Systems Pioneer Award.

The friendly and dynamic community working on fuzzy sets has certainly been a breeding ground for her research from the early years, as has been later the broader Computational Intelligence community, and more particularly the IEEE Computational Intelligence Society. The latter is very active in terms of scientific leadership, and committed to the principles of diversity and support to all its members, especially women. Bernadette Bouchon-Meunier was the founding chair of the IEEE Women in Computational Intelligence committee, whose purpose is to develop, promote, organise and lead activities to ensure equal opportunities for both genders in the life of the IEEE Computational Intelligence Society and in the arena of computational intelligence.

Anne Laurent is Full Professor at the University of Montpellier, LIRMM lab. As the head of the FADO Research group, she works on open data, semantic web and data mining. She is particularly interested in the study of the use of fuzzy logic to provide more valuable results, while remaining scalable. Anne Laurent has been the head of the Computer Science Department at Polytech Montpellier Engineering School at the University of Montpellier, which prepares a 5-year Masters in computer science and management. She is currently Vice-President at University of Montpellier delegated to open science and research data. She also heads the Montpellier Data Science Institute (ISDM) and the high-performance computing center (MESO@LR). Anne has (co-)supervised 17 Phd thesis, among which 6 female candidates.

Interested in all subjects taught in high school, entering the field of CIS was quite natural as it was the best way to open multiple studies and career opportunities. It was then quite natural to stay in it for several reasons. Anne has met many high level and committed women who helped her and pushed her. All these women served as examples, which is a key point that Anne tries to reproduce with kids and teenagers (at Coderdojo Montpellier that aims at *Enabling young people worldwide to create and explore technology together*), students and young colleagues.

Marie-Jeanne Lesot is an associate professor in the department of Computer Science Lab of Paris 6 (LIP6) and a member of the Learning and Fuzzy Intelligent systems (LFI) group. Her research interests focus on fuzzy machine learning with an objective of data interpretation and semantics integration; they include

similarity measures, fuzzy clustering, linguistic summaries and information scoring. She is also interested in approximate reasoning and the use of non classical logics, in particular weighted variants with increased expressiveness that are close to natural human reasoning processes. Marie-Jeanne Lesot has (co-)supervised 15 Phd thesis, among which 3 female candidates.

When encountered during her studies, the domain of Computational Intelligence appeared to her as an optimal combination of theoretical challenges with practical considerations, taking a human-into-the-loop position: users are put in the centre of the solution design process, and considered as source for inspiration so as how to deal with the considered issues, and, as such, as incentive for integrating cognitive points of view.

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