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► **To cite this version:**

Cyril Karamaoun, Haribalan Kumar, Médéric Argentina, Didier Clamond, Benjamin Mauroy.
Curvature-induced motion of a thin Bingham layer in airways bifurcations. 2023. hal-03500343v2

HAL Id: hal-03500343

<https://hal.science/hal-03500343v2>

Preprint submitted on 24 Nov 2023 (v2), last revised 14 Jun 2024 (v3)

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Curvature-induced motion of a thin Bingham layer in airways bifurcations

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(Dated: November 24, 2023)

The mucus on the bronchial wall forms a thin layer of a non-Newtonian fluid protecting the lung by capturing inhaled pollutants. Due to the corrugation of its interface with air, this layer is subject to surface tensions affecting its rheology. This physical system is analyzed via the lubrication theory and 3D simulations. We characterize the mucus nonlinear behavior and show that surface tension effects can displace overly thick mucus layers in the airway bifurcations. Thus, this movement can disrupt the mucociliary clearance and break the layer thickness homogeneity.

As one of the central organ of respiration, the main functions of the lung are to bring oxygen from ambient air to blood and to extract carbon dioxide from the body. The large exchange surface between the air and the blood, about 75–100 m² [? ?] is connected to the ambient air by a space-filling and multi-scale network of airways. To perform its function, the lung relies on several physical processes and on its tree-like geometry. The transport of oxygen and carbon dioxide in the lung is the most studied physical process [? ?]. However, other physical processes are involved and some of them protect the integrity of the organ [?].

One of these mechanisms relies on a layer of non-Newtonian fluid coating the airways walls: the bronchial mucus [?]. The lung is a potential entry point for external contaminants – dust particles, chemicals, bacteria, viruses – that are constantly inhaled. The mucus traps contaminants and is transported towards to the larynx, where it is either expelled or swallowed. Two main phenomena are responsible for the mucus displacement [?]. First, ciliated cells are located in the bronchial epithelium and, by beating metachronously [?], the cilia push the mucus toward the trachea. This phenomenon is called *mucociliary clearance* [? ?]. Second, during cough [? ?] or high ventilation rates [?], exhaled airflows can apply to the mucus a shear stress high enough to induce its displacement. The efficiency of the protection by the mucus is dependent on the proper functioning of these two phenomena. Pathologies such as asthma, chronic obstructive pulmonary disease (COPD) or cystic fibrosis can impair the mucociliary clearance, leading to major respiratory symptoms and to infections [? ? ?]. The mucociliary clearance and the air-mucus interaction have been explored thoroughly [? ? ? ? ? ? ? ?].

Other physical phenomena, such as gravity [?] and surface tensions [?], can affect the mucus transport.

The role of surface tensions on the air-mucus interface remain not well understood as of today. The large airway curvatures suggest that the multi-scaled structure of the bronchial tree, together with surface tensions and mucus rheology, can affect the transport of the mucus. However, in most studies, the mucus is modeled as a Newtonian fluid [? ? ? ? ?].

The surface tensions induce (Laplace) pressure jumps Δp_L across curved interfaces ($\Delta p_L = 2\gamma\kappa$, γ the coefficient of surface tension, κ the mean curvature of the interface). The distribution of this pressure can be evaluated in an auto-similar tree that models the bronchial tree [? ? ? ? ? ?]. This model is a bifurcating tree with branches as perfect cylinders. The size of the branches decreases homothetically at each bifurcation by a factor $h = 2^{-1/3} \simeq 0.79$. In this model, the airways are indexed by their generation i representing the number of bifurcations from the airway to the root of the tree (trachea). The radius of the airways in the i -th generation are $r_i = h^i r_0$, with r_0 the radius of the root of the tree. Assuming that air-mucus interface and airways have the same curvature, the Laplace pressure of the i -th generation is $p_{L,i} = -\gamma/r_i$. This pressure decreases with the generation index, the curvature effects tending to push the layer toward the deeper parts of the tree. The resulting stress in a layer of thickness τ that coats the bifurcation between generations i and $i+1$ can be evaluated to $\sigma_i \simeq \gamma \frac{h-1}{r_i^2} \frac{\tau}{2}$, see [? , section I]. This stress is larger in the small bifurcations, as it increases with the generation index. However, a more detailed analysis is needed to evaluate, whether or not, the curvature effects can move the mucus. Therefore, detailed bifurcations shapes, more realistic mucus rheology and mucus hydrodynamics must be included in the model.

The mucus is a complex viscoelastic fluid, potentially thixotropic [?]. Its rheological properties depend on the individual, the localization of the mucus in the bronchial tree and the environment – air humidity, temperature, etc. Mechanical constraints also influence the mucus behavior, and one of its core properties is to present a yield

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Quantity	Value (ref value)	Notation	Ref
Lungs' geometrical data			
trachea radius	0.01 m	r_0	[?]
reduction factor	$(1/2)^{\frac{1}{3}} \sim 0.79$	h	[? ? ?]
Mucus properties			
surface tension	0.03 Pa m	γ	[?]
mucus density	1000 kg.m ³ (water)	ρ	[?]
mucus viscosity	10 ⁻³ - 10 Pa s (1)	μ	[?]
mucus yield stress	10 ⁻² - 10 Pa (0.1)	σ_y	[?]
mucus layer thickness (healthy)	5 - 30 μ m (10)	τ	[?]
cilia induced mucus velocity	10 - 500 μ m s ⁻¹ (50)	v_{cilia}	[? ? ?]

TABLE I. Data range of our model parameters and their default values used in this paper, shown between parentheses.

stress σ_y under which it behaves like a solid material [? ? ? ? ?]. This characteristic induces that the inner shear stresses in the fluid have to overcome σ_y in order for the mucus to behave like a fluid of viscosity μ . The typical healthy thickness τ of the mucus layer is of order 10 μ m [?]. To understand under which conditions the surface tensions can be large enough to overcome the mucus yield stress, we modelled the mucus as a Bingham fluid. This approach has already brought rich insights [? ?]. Moreover, it captures the layer nonlinear dynamics, which cannot be well represented with a Newtonian fluid.

According to Table ??, the Reynolds number is small in the mucus layer, $\text{Re} \sim \rho v_{\text{cilia}} \tau / \mu < 0.015$. Thus, the fluid mechanics can be approximated via the Stokes equations. Denoting \mathbf{u} the fluid velocity and p the fluid pressure, the momentum and mass conservation equations are

$$\begin{aligned}
\rho \frac{\partial \mathbf{u}}{\partial t} - \nabla \cdot \Sigma + \nabla p &= 0 && \text{in the layer} \\
\nabla \cdot \mathbf{u} &= 0 && \text{in the layer} \\
\Sigma \cdot \mathbf{n} - p \mathbf{n} &= p_L \mathbf{n} && \text{at air-fluid interface } \mathcal{L}_t \\
\mathbf{u} &= 0 && \text{at airway wall } W \\
\frac{d\mathbf{x}}{dt} &= (\mathbf{u}(\mathbf{x}, t) \cdot \mathbf{n}(\mathbf{x}, t)) \mathbf{n}(\mathbf{x}, t) && \mathbf{x} \in \mathcal{L}_t \\
p_L &= -2\gamma\kappa(\mathbf{x}, t) && \mathbf{x} \in \mathcal{L}_t
\end{aligned}$$

The vector $\mathbf{n}(\cdot, t)$ is the normal to the air-Bingham fluid interface \mathcal{L}_t at time t and κ is the mean curvature of this interface. The normals are oriented towards the air medium. The characteristic thickness of the layer along these normals is denoted τ . Σ is the viscous stress tensor. The air-Bingham fluid interface \mathcal{L} is a free surface and a geometrical point \mathbf{x} of its surface is moving with the normal component of the Bingham layer velocity, $(\mathbf{u}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x})) \mathbf{n}(\mathbf{x})$. Mucociliary clearance could be modeled as a slip boundary condition on the wall W . However, in order to isolate the sole effects of surface tensions, the clearance is not accounted for in the model and a non-slip boundary condition is assumed on W .

The Bingham fluid constitutive equations are

$$\begin{cases} \Sigma = \left(\mu + \frac{\sigma_y}{\dot{\gamma}} \right) \dot{\Gamma} & \text{for } \sigma > \sigma_y \\ \dot{\Gamma} = 0 & \text{for } \sigma \leq \sigma_y \end{cases}$$

with $\dot{\Gamma} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^t)$ the rate of strain tensor and with $\sigma = \sqrt{\frac{1}{2} \Sigma : \Sigma}$ and $\dot{\gamma} = \sqrt{\frac{1}{2} \dot{\Gamma} : \dot{\Gamma}}$ the second invariants of, respectively, the stress tensor and the rate of strain tensor.

The mucus layer being generally thin compared to the mean curvature radius of the bronchi wall, planar lubrication theory applies [? ?] (see [? , section II]). Our results show that the viscous stresses tangential to the wall dominate inside the fluid. These stresses are proportional to the interface curvature gradient $\nabla_{\xi} \kappa$, computed in the curvilinear coordinates system ξ . The normal stress is dominated by the Laplace pressure. Our theory uncovers a characteristic Bingham-like number $B = \frac{\sigma_y r^2}{2\gamma\tau}$, with r the characteristic radius of curvature of the interface, typically the radius of the airway. The fluid layer is liquid if $B < \|\nabla_{\xi} \tilde{\kappa}\|$, where $\nabla_{\xi} \tilde{\kappa} = r^2 \times \nabla_{\xi} \kappa$ (over tildes denote quantities normalised by r).

A proportion $e = \max\left(0, 1 - B/\|\nabla_{\xi} \tilde{\kappa}\|\right)$ of the layer near the wall is liquid [? ?]. Above this liquid layer, the Bingham fluid is solid. The liquid layer drags the solid one. The surface tension homogenizing the layer thickness, we assume hereafter that τ is constant and that the layer curvature equals that of the airways (this analysis assumes the airways to be perfectly smooth). Thus, we do not account for large fluid accumulations, clots or plugs [? ? ? ?]. Under these conditions, for the i -th generation and to the leading order, the velocity field $\mathbf{v}_{\text{st},i}$ (averaged over the layer thickness) is

$$\begin{aligned}
\mathbf{v}_{\text{st},i} &= -f(e_i) \frac{\gamma\tau^2}{\mu r_i^2} \nabla_{\xi} \tilde{\kappa}, \quad f(e) = e^2 \left(1 - \frac{e}{3}\right), \\
e_i &= \max\left(0, 1 - \frac{B_i}{\|\nabla_{\xi} \tilde{\kappa}\|}\right).
\end{aligned} \tag{1}$$

The vector $-\frac{\gamma\tau^2}{\mu r_i^2} \nabla_{\xi} \tilde{\kappa}$ being the velocity in the case of a Newtonian fluid, our results show the presence of a prefactor $f(e_i)$ in the case of a non-Newtonian fluid. This dimensionless prefactor fully characterizes the non-Newtonian behavior and depends nonlinearly on the Bingham number B_i in the i -th generation.

Due to the scaling law between generations, the mean curvature gradients in the i -th generation is $\nabla_{\xi^i} \kappa_i = h^{-2i} r_0^{-2} \nabla_{\xi} \tilde{\kappa}$. This leads to a scaling law for the fluid velocity:

$$\mathbf{v}_{\text{st},i} = \left(\frac{1}{h^2}\right)^i f(e_i) \frac{\gamma\tau^2}{\mu r_0^2} \nabla_{\xi} \tilde{\kappa}. \tag{2}$$

In addition, the wall mean curvature $\tilde{\kappa}$ is obtained from an idealized 3D tree geometry, see Fig. ??A and [? , section IV]. The fluid velocity field in the 3D idealized tree

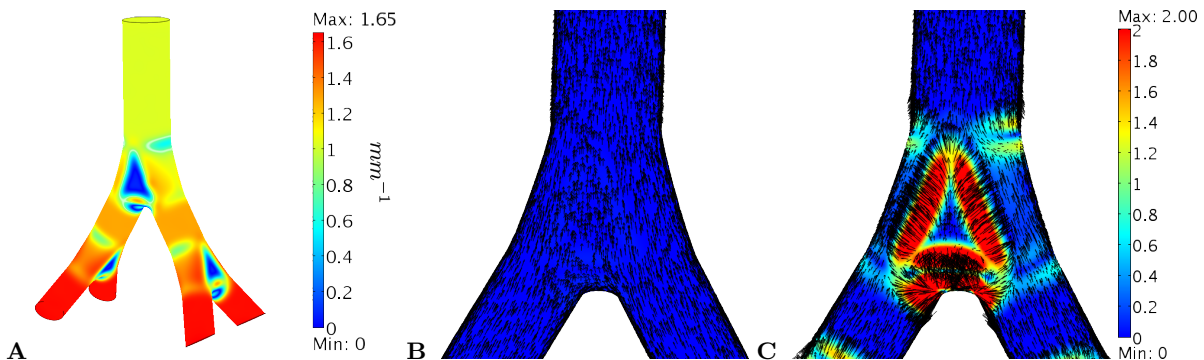


FIG. 1. **A**: The reference 3D geometry used with the lubrication theory. The geometry is rescaled to cover the whole scales of the bronchial tree bifurcations. The root branch radius is 1 mm. The branches size decreases with a ratio $h = (1/2)^{1/3}$ at the bifurcations. The branching angle is 60° and the angle between the two successive branching planes is 90° in accordance with the mean observed values [?]. The colors represent twice the non signed mean curvature field $|\kappa_0|$ (m^{-1}). **B** ($\tau = 10 \mu\text{m}$), **C** ($\tau = 75 \mu\text{m}$): Bingham fluid velocity fields $\mathbf{v}_{m,i}$ (arrows) and ratio α between the amplitudes of the velocity induced by the surface tension effects and of the velocity induced by the idealized mucociliary clearance, i.e. $\alpha = \|\mathbf{v}_{\text{st},i}\|/\|\mathbf{v}_{\text{cilia}}\|$ (colors). Here, the mother branch has a radius of 1 mm and the daughter branches a radius of $(1/2)^{1/3} \simeq 0.79$ mm. At physiological thickness (B), only the idealized mucociliary clearance is driving the motion of the mucus. For non healthy thickness (C), the idealized mucociliary clearance is strongly altered by the surface tension effects.

is obtained from the theoretical formula (??). Finally, the velocity induced by the mucociliary clearance is also evaluated in the 3D idealized tree. It is modeled as the gradient of a Laplacian field on the bifurcation walls, see [? , section V]. The resulting idealized clearance is tangential to the bifurcation walls and directed toward the larger airways [?]. Its amplitude is set to $50 \mu\text{m}\cdot\text{s}^{-1}$ [?].

Many common lung pathologies induce a thickening of the layer [?]. Thus, in addition to healthy mucus layer thickness, other thicknesses compatible with mucus pathophysiology were tested. Typical velocity outputs of this study are presented in Fig. ?? (spatial distribution) and Fig. ??A (averaged amplitude). Our results show that after a threshold generation, the curvature gradients are able to overcome the yield stress of the Bingham fluid. This implies that, from this generation to deeper ones, the fluid exhibits a liquid behavior and can be displaced. For healthy mucus ($\tau = 10 \mu\text{m}$), this threshold is around the 9th generation, see Fig. ??A. In this case, the velocity magnitude is negligible relatively to that induced by the clearance. For thicker layers, the threshold generation decreases and the velocity increases, eventually becoming larger than the magnitude of the velocity induced by the idealized clearance, see ??A. Thus, the thicker the mucus, the more the idealized clearance is impaired, see Fig. ??. Eventually, the effects of the clearance disappear and the velocity fields is driven by curvatures only (Fig. ??C).

The disruption of the clearance depends on the layer thickness and the position in the bifurcation. The convergence (or divergence) of the predicted velocity field indicates possible mucus accumulation (or depletion) at different spots of the bifurcation, see Fig. ??C. A local accumulation increases the risk of bronchial obstruction. Conversely, a local depletion reduces the protection of

the epithelium, making it more susceptible to external contaminants and physicochemical stresses [?]. Furthermore, our model suggests that local mucus accumulation should develop first in the parts of the airways the closest to the bifurcation zone, see Fig. ??C. On the other hand, the carina of the bifurcation, i.e. the meeting point of the two small airways, is more susceptible to mucus depletion, since it has a relatively low curvature (Fig. ??A). Notice that inhaled particules are more likely to deposit at the carina [? ?], and mucus overproduction might counterintuitively increase the risk of epithelial damage near the carina [?].

In order to quantify how the curvature gradient is opposing the clearance in a bifurcation, we project the velocity field $v_{\text{st},i}$ (induced by the surface tension) on the local direction of the idealized clearance. Then, we average this velocity component over the layer thickness and over the bifurcation to define $V_{\text{st},i}$, see [? , section III]. The results are shown in Fig. ??B and C. In all the cases, the averaged Bingham fluid velocity is opposed to that induced by the idealized clearance. Moreover, the non-Newtonian effects are driven by the Bingham number $B_i = \frac{\sigma_y r_i^2}{2\gamma\tau}$ only, as shown in Fig. ??C. For $B_i < 0.1$, the fluid adopts a liquid behavior in the whole tree and opposes the most to the clearance. For $B_i > 0.5$, the velocities drop drastically and the clearance is not anymore disrupted; in this case, the Bingham fluid is no longer fully liquid and eventually behaves as a solid.

These results agree with observations, as an increased thickness of the mucus layer in pathological conditions has been associated with a disturbance of the clearance [? ? ?]. Moreover, our analysis highlights the importance of the mechanisms of control of the mucus thickness, in particular in the bifurcations [?].

In the absence of clearance, the fluid in the bifurcation

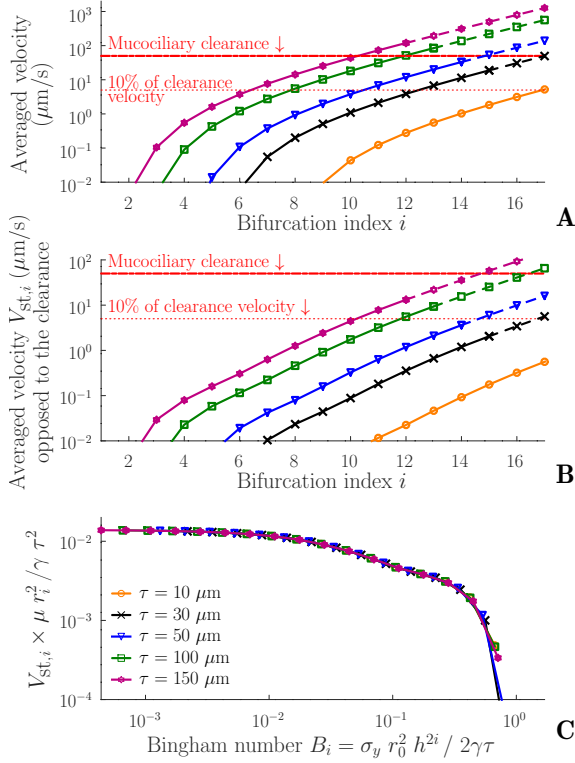


FIG. 2. Velocity patterns with parameters values $\sigma_y = 0.1$ Pa, $\mu = 1$ Pa s and the clearance amplitude is $50 \mu\text{m s}^{-1}$. The dashed parts of the curves correspond to generations where the model hypothesis $\tau/r_i \ll 1$ loses its validity (i.e. $\tau/r_i > 10\%$). Our results show that the curvature effects can disrupt mucociliary clearance for pathological thicknesses of the layer ($\tau \geq 50 \mu\text{m}$) preferentially in the medium and small airways. **A**: Bingham layer velocity amplitude (curvature effects only) averaged over the bifurcation and layer thickness. **B**: Component of the Bingham layer velocity (curvature effects only) in the direction of the idealized clearance and averaged over the bifurcation and layer thickness. The average velocity induced by curvature effects is always opposed to the clearance. **C**: Averaged velocity opposed to the clearance, normalized by the velocity of a Newtonian fluid in the same configuration. The normalized velocity depends only on the Bingham number B_i .

is globally driven by capillarity toward the small airways, see Fig. ??B. This movement is opposed to that induced by the clearance. The curvature gradients vanishing in cylindrical airways, further mucus displacement along the small airways can only result from a non constant thickness of the layer. Then, surface tension effects tend to homogenize the layer thickness, up to a point where the shear stress falls below the yield stress, resulting to local accumulation of mucus. Thus, our analysis suggests that the clearance can also compensate the curvature effects on the mucus in bifurcations. Moreover, the curvature gradients trigger the displacement of the layer mainly

along the wall. This suggests that they contribute to maintain a layer with constant thickness. However, when the layer is too thick, the curvature gradients become too strong for the clearance to counteract these effects.

Our analysis uncovers curvature effects on a Bingham fluid layer coating the airways walls of a bifurcation. Our results suggest that a pathological thickening of the bronchial mucus layer can counteract and, potentially, disrupt the mucociliary clearance. However, the curvature effects on the mucus remain an intricate phenomenon and the model does not account for the whole complexity of the bronchial mucus layer.

The mucus rheology is still not fully understood [?], and mucus properties vary largely between persons and depend on the environment [? ?]. Thus, our results are mainly qualitative and not exhaustive. However, a Bingham model for the mucus accounts for its main properties (viscosity, yield stress) [? ?] and our predictions are consistent with lung pathophysiology [? ? ?].

The mucus thickness likely depends on the airway location in the bronchial tree [?]. Moreover, bronchi are not perfect cylinders and their wall curvature is not constant. These phenomena, which are not accounted for in our model, can affect the layer thickness.

Our work shows that curvature effects can displace thick layers of Bingham fluid in airways bifurcations. For lung pathologies, our results indicate that curvature effects probably play a major role in the disruption of the mucociliary clearance, if mucus is in excess. Uncovering this new phenomena paves the way to a better understanding of the bronchial mucus dynamics in pathological lungs. Future studies should now include more realistic models of the bronchial tree and air-mucus interface along with other biophysical phenomena, such as gravity [?], to compare their respective effects. On a broader perspective, our work suggests that “hidden” physical processes could be triggered by some pathologies and affect strongly the organs function.

ACKNOWLEDGMENTS

This work has been supported by the Mission pour l’interdisciplinarité du CNRS, the Agence Nationale de la Recherche (the VirtualChest project, ANR-16-CE19-0014; the IDEX UCA JEDI, ANR-15-IDEX-01), the Académie des Systèmes Complexes de l’Université Côte d’Azur and the association Vaincre La Mucoviscidose (RF20190502489).

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