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Thomas Louail, Marc Barthelemy

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A dominance tree approach to systems of cities

Thomas Louail, CNRS, Géographie-cités, Paris

Marc Barthelemy, CEA, IPhT & CAMS, Paris

Motivation

What are the **simple and standard methods** to describe the **evolution of the spatial organisation of systems of cities, at different scales?**

A general problem

Capturing the spatial organization of a 'marked point process'
e.g. cities with population size, trees with height

2 distributions : the spatial distribution of the locations (x_i) of the points
and the statistical distribution of their marks -- here their population sizes (P_i)

For systems of cities, there exists two distinct bodies of literature

1. Characterization of the distribution of city sizes P_i
(e.g. see Cottineau C, *Plos One*, 2017)
2. Many methods and coefficients to summarize spatial point patterns x_i
(see e.g. Velázquez et al., *Ecography*, 2016)

> Characterizing a spatial hierarchy requires considering x_i and P_i together

Okabe and Sadahiro's approach (1996)

Proposed a **non-parametric** method to capture the notion of spatial hierarchy

Built upon the **Voronoi tessellation** and the notion of **local centers**

A node is a local center if its population size is larger than the one of its neighbors

Recursive procedure: which are the new local centers among the local centers? etc.

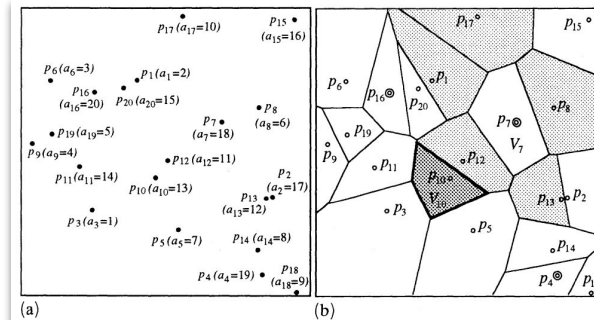
Environment and Planning A 1996, volume 28, pages 1533-1552

An illusion of spatial hierarchy: spatial hierarchy in a random configuration

A Okabe, Y Sadahiro

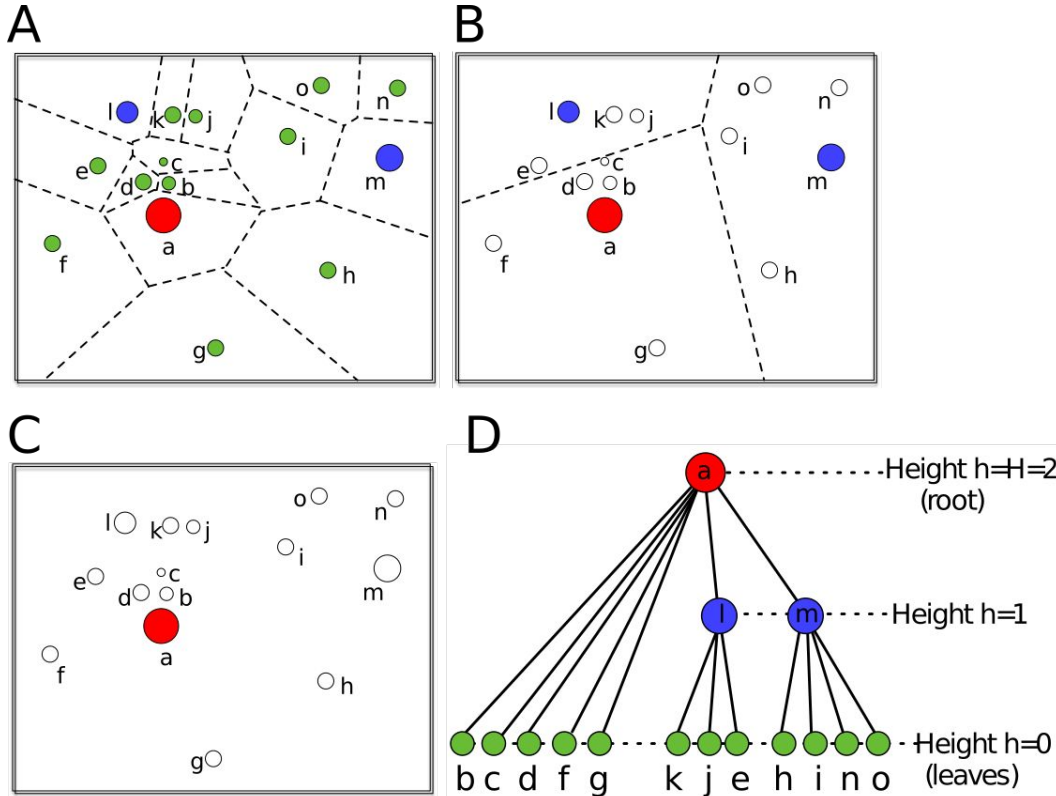
Department of Urban Engineering, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, Japan 113

Received 7 March 1994; in revised form 11 November 1994



(a) The configuration $[P, A]$ of activity places ($P = \{p_1, \dots, p_{20}\}$) with their attribute values, $A = \{a_1, \dots, a_{20}\}$, or the 0th level configuration $[P^{(0)}, A^{(0)}]$. (b) The Voronoi diagram V generated by the set P of activity places in (a). (The densely shaded region indicates the Voronoi polygon V_{10} of p_{10} and the double circles indicate the 1st-level local centers; the lightly shaded regions are the Voronoi polygons adjacent to V_{10} .)

Elaborating on Okabe & Sadahiro's work



- A. Identify the local centers from the initial Voronoi tessellation
- B. Draw new Voronoi polygons by considering these local centers only
- C. Determine the local centers, etc.

- D. In the end we obtain a dominance tree representing the spatial hierarchy of the system. **Each node is characterized by its height h in the tree**

Toy models

Distribution of locations x_i : uniform distribution (spatial Poisson process)
in the square $[-1, +1]^2$

Distribution of marks : m_i

Defining a model is specifying how to match these two distributions

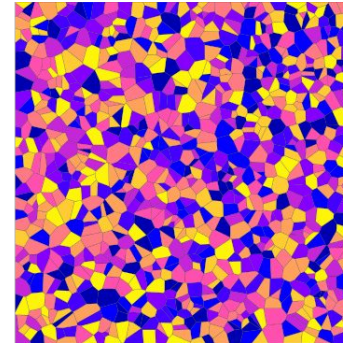
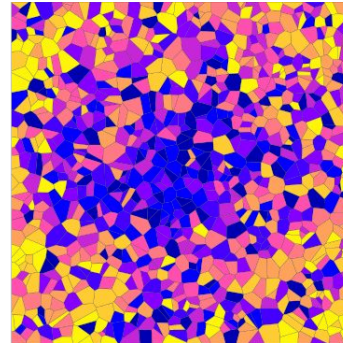
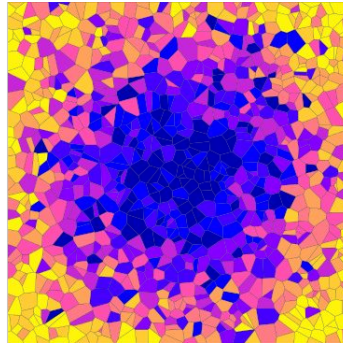
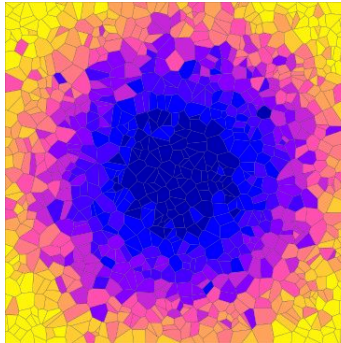
Deterministic

←

Tunable

→

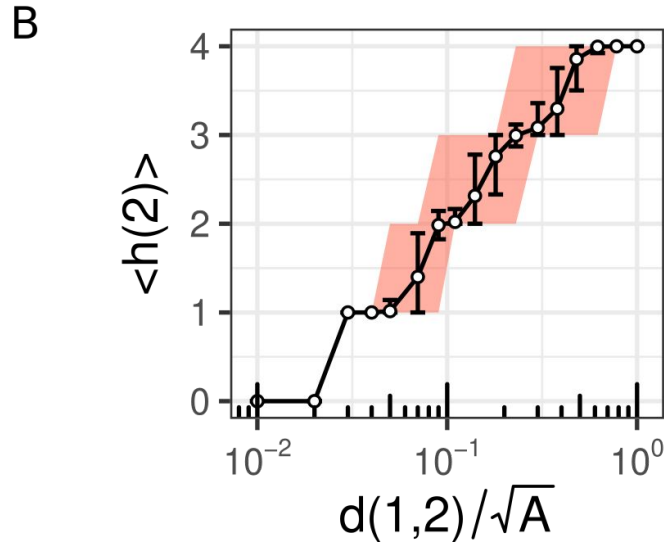
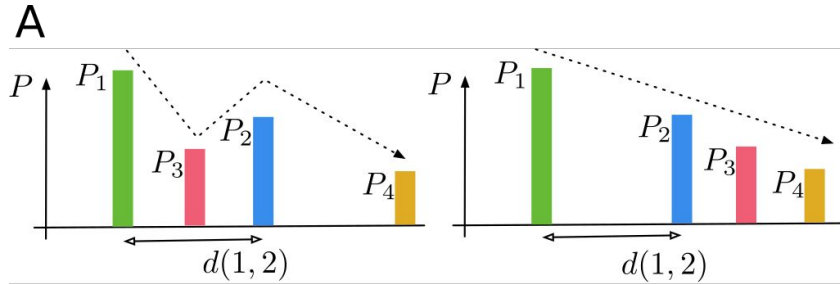
Random



Deterministic: the largest marks (from blue to yellow) are attributed to the cells the closest to the center.

Tunable: transition from Deterministic to Random

Height of a point in the dominance tree



A. 2 simple organizations

(left) 'polycentric': P_3 is located between P_1 and P_2 , allowing P_2 to reach a higher height in the tree before being 'absorbed' by P_1

(right) 'monocentric': there is no local maxima, as each city is dominated by its neighbor, except for P_1

B. For a given point pattern (x_i) , the average height reached by the point x_2 with the second largest population P_2 increases as the distance between x_1 and x_2 increases

> The height of a point in the dominance tree carries more information than its rank alone

Application: French and US systems of cities (1880-2010)

INSEE population data of French municipalities (since 1876)

Compilation of US cities populations between 1790 and 2010 (every 10 years)

Data come primarily from the US Census Bureau

<https://github.com/cestastanford/historical-us-city-populations>

Both datasets are public and available for free



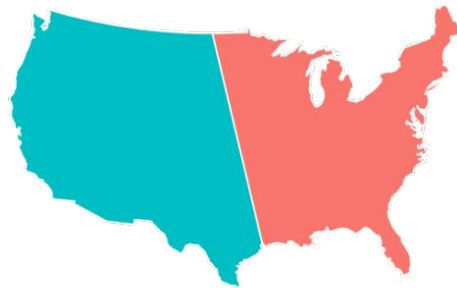
h=1



h=2

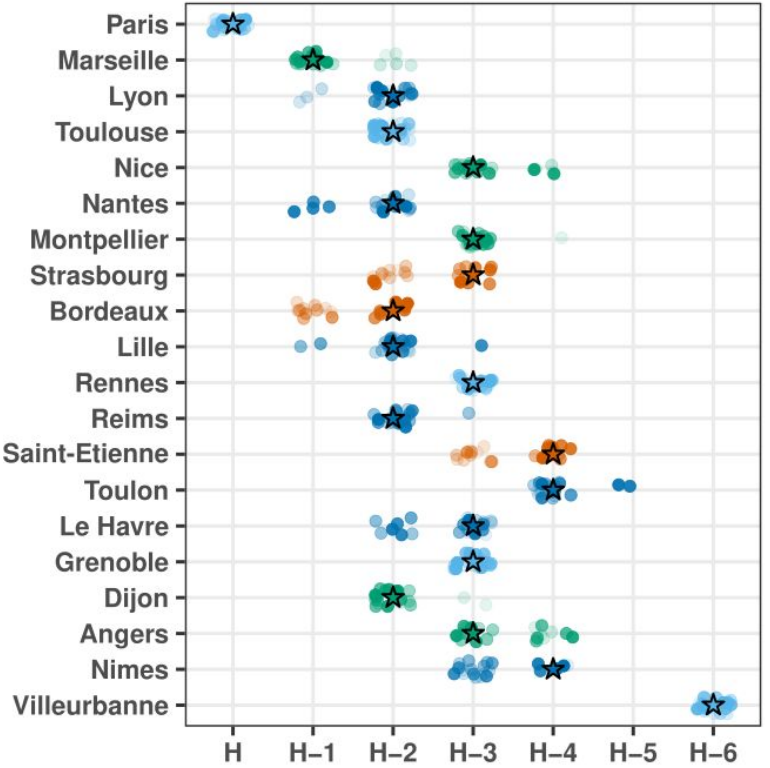


h=3



h=5

City trajectories



By construction the root of the tree is always the city with the largest population (NYC/Paris)
Non-monotonous trajectories of large cities in the US, with more height fluctuations

Typology of cities

Based on how the height of the city in the dominance tree evolved through time

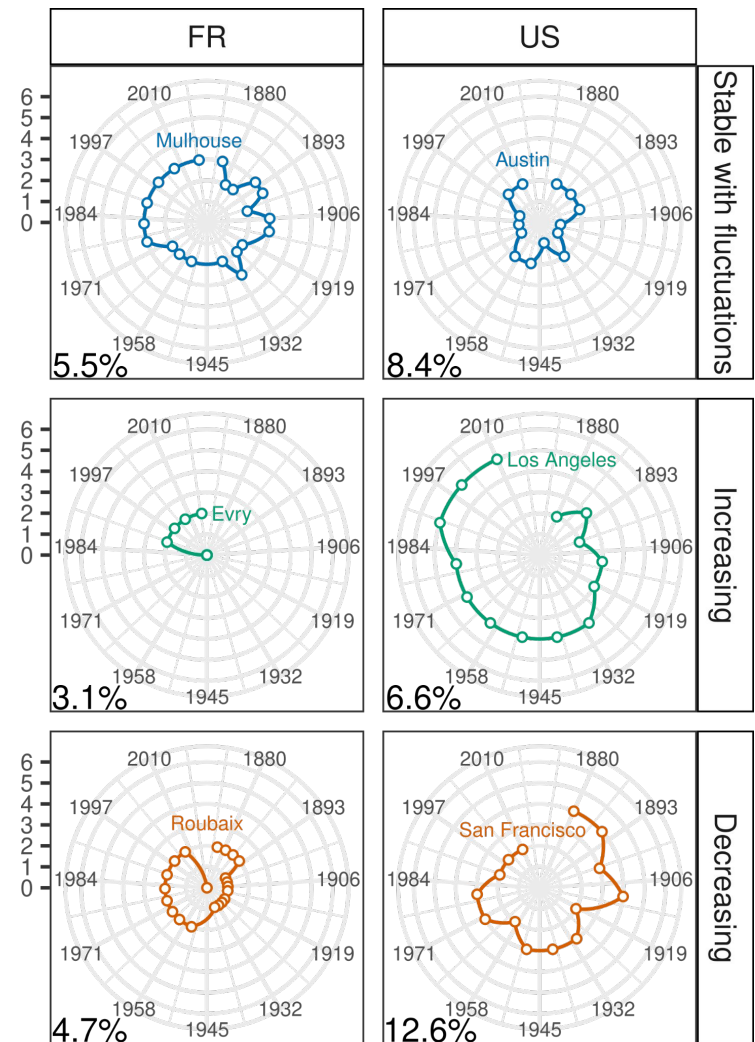
Stable

Stable with fluctuations

Increasing height

Decreasing height

The height-clock to visualize the height dynamics, inspired by M. Batty's rank clocks (*Nature*, 2006)



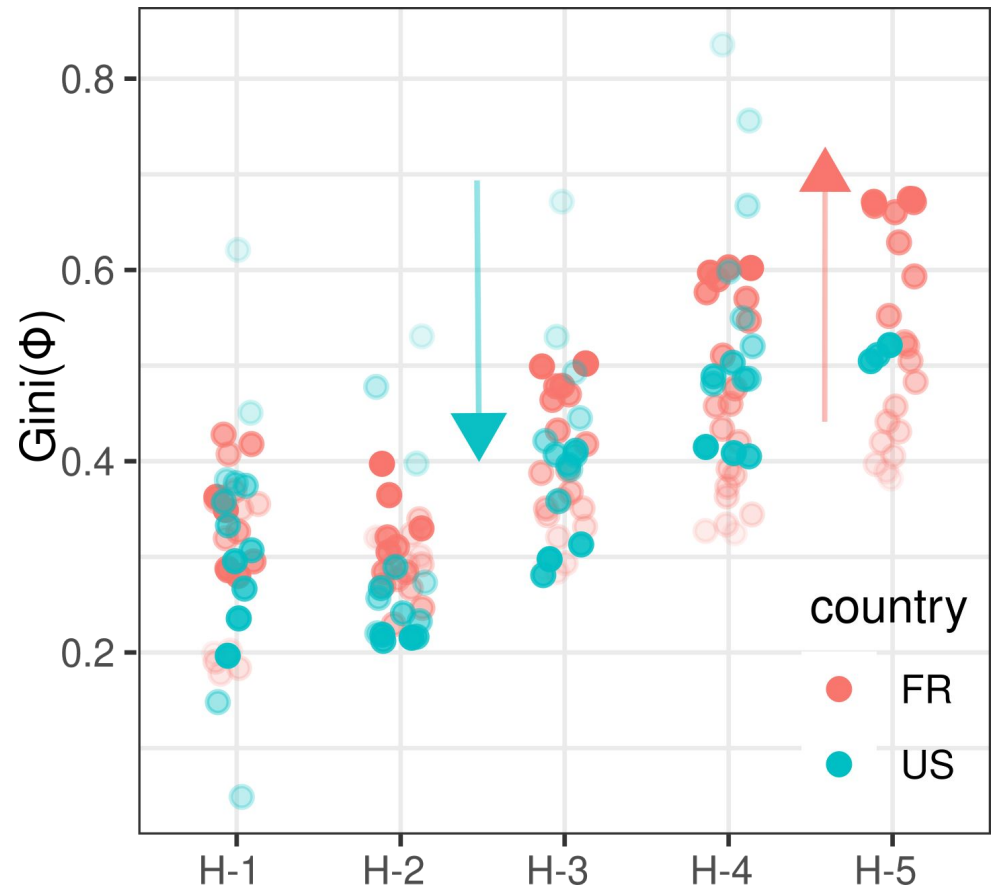
Basins of attraction

At a given height in the tree
we can compare
the basins of attraction
of the remaining cities

At each step a local maximum
– say city i –
contains a number of cities
in its Voronoi cell $V(i)$

$$\phi_i = \sum_{j \in V(i)} P_j$$

For both countries, populations sizes of attractions basins get more homogeneous at larger scales
In France, at all scales basins sizes get more unequal with time. It is the opposite in the US



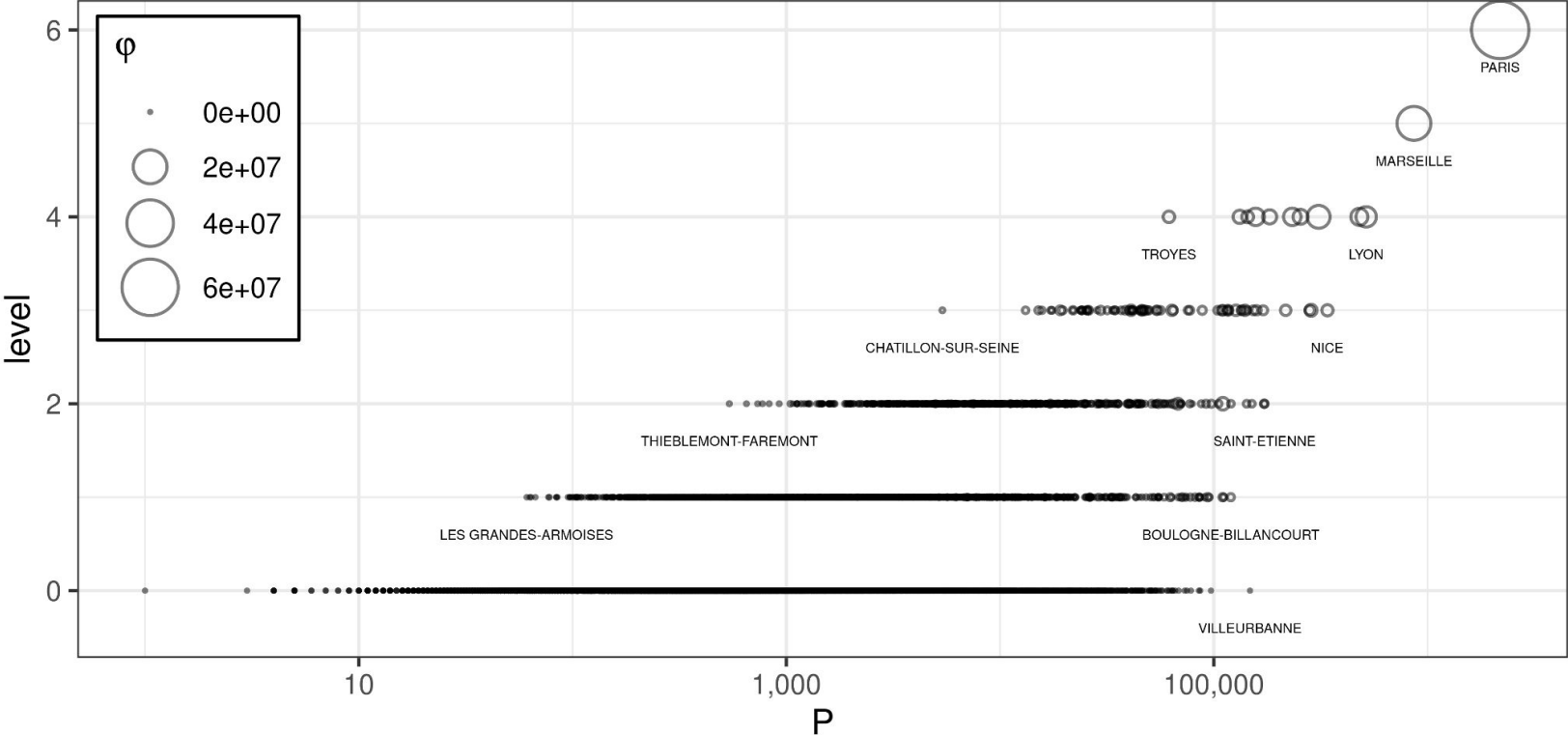
Possible future directions

- > City definitions (as always)
- > Beyond Voronoi: generalisation of the method to any tessellation, e.g. administrative boundaries
- > Beyond Euclidean distance, e.g. road distance, travel time distance

A **non-parametric method** useful for studying spatial correlations in a simple way

Could support the **comparison of the dynamics of different urban systems**

Height and rank/population size



A city's height in the tree is not directly determined by its population/rank, it depends also on its location