A dominance tree approach to systems of cities

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Motivation

What are the *simple* and *standard methods* to describe the *evolution of the spatial organisation of systems of cities, at different scales*?
A general problem

Capturing the spatial organization of a ‘marked point process’
e.g. cities with population size, trees with height

2 distributions : the spatial distribution of the locations \( (x_i) \) of the points
and the statistical distribution of their marks -- here their population sizes \( (P_i) \)

For systems of cities, there exists two distinct bodies of literature

1. Characterization of the distribution of city sizes \( P_i \)
   (e.g. see Cottineau C, *Plos One*, 2017)
2. Many methods and coefficients to summarize spatial point patterns \( x_i \)
   (see e.g. Velázquez et al., *Ecography*, 2016)

> Characterizing a spatial hierarchy requires considering \( x_i \) and \( P_i \) together
Okabe and Sadahiro’s approach (1996)

Proposed a **non-parametric** method to capture the notion of spatial hierarchy

Built upon the **Voronoi tessellation** and the notion of **local centers**

A node is a local center if its population size is larger than the one of its neighbors

Recursive procedure: which are the new local centers among the local centers? etc.
Elaborating on Okabe & Sadahiro’s work

A. Identify the local centers from the initial Voronoi tessellation

B. Draw new Voronoi polygons by considering these local centers only

C. Determine the local centers, etc.

D. In the end we obtain a dominance tree representing the spatial hierarchy of the system. Each node is characterized by its height $h$ in the tree
Toy models

Distribution of locations $x_i$: uniform distribution (spatial Poisson process) in the square $[-1, +1]^2$

Distribution of marks: $m_i$

Defining a model is specifying how to match these two distributions

Deterministic: the largest marks (from blue to yellow) are attributed to the cells the closest to the center.
Tunable: transition from Deterministic to Random
Height of a point in the dominance tree

A. 2 simple organizations

(Left) ‘polycentric’: P3 is located between P1 and P2, allowing P2 to reach a higher height in the tree before being ‘absorbed’ by P1

(Right) ‘monocentric’: there is no local maxima, as each city is dominated by its neighbor, except for P1

B. For a given point pattern \((x_i)\), the average height reached by the point \(x_2\) with the second largest population P2 increases as the distance between \(x_1\) and \(x_2\) increases

> The height of a point in the dominance tree carries more information than its rank alone
Application: French and US systems of cities (1880-2010)

INSEE population data of French municipalities (since 1876)

Compilation of US cities populations between 1790 and 2010 (every 10 years)
Data come primarily from the US Census Bureau
https://github.com/cestastanford/historical-us-city-populations

Both datasets are public and available for free
By construction the root of the tree is always the city with the largest population (NYC/Paris). Non-monotonous trajectories of large cities in the US, with more height fluctuations.
Typology of cities

Based on how the height of the city in the dominance tree evolved through time

**Stable**

**Stable with fluctuations**

**Increasing height**

**Decreasing height**

The height-clock to visualize the height dynamics, inspired by M. Batty’s rank clocks (*Nature*, 2006)
Basins of attraction

At a given height in the tree we can compare the basins of attraction of the remaining cities.

At each step a local maximum – say city $i$ – contains a number of cities in its Voronoi cell $V(i)$.

$$\phi_i = \sum_{j \in V(i)} P_j$$

For both countries, populations sizes of attractions basins get more homogeneous at larger scales in France, at all scales basins sizes get more unequal with time. It is the opposite in the US.
Possible future directions

> City definitions (as always)

> Beyond Voronoi: generalisation of the method to any tessellation, e.g. administrative boundaries

> Beyond Euclidean distance, e.g. road distance, travel time distance

A **non-parametric method** useful for studying spatial correlations in a simple way

Could support the **comparison of the dynamics of different urban systems**
A city’s height in the tree is not directly determined by its population/rank, it depends also on its location.