



# On the Nature of Psychophysical Correlations

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## On the Nature of Psychophysical Correlations

### Abstract.

This article proposes a strategy of decision based on Bell-type experimental tests for the long-standing question about the nature of psychophysical correlations. We will first briefly discuss the main traditional answers to this question and some more recent suggestions developed within the framework of a generalised quantum theory. The proposed strategy of decision is developed within the framework of the full formalism of (generalized) quantum theory with continuous variables observables. It appeals to a signalling condition in order to test the possible existence of a direct, causal “interaction” between bodily and mental processes, to a Bell-type condition for testing the (non-)local nature of the psychophysical correlations and to a general method for implementing the essential notion of complementary observables in the psychosomatic domain.

### I. Introduction: the psychophysical correlations.

The existence of psychophysical correlations has been well established. For example, brain imaging techniques have shown that visual perception is correlated with the activation of a multitude of specialized brain areas that deal with the different aspects of the perceived object - information about its orientation, its shape, its colour or its state of movement (Koch 2006). Even more, referring to the binocular rivalry experiences, some authors have claimed that *conscious* visual perception would be correlated to the activation of specific areas of the visual system (Leopold and Logothetis 1996). Indeed, as shown by the neurologist Damasio, *the whole body* is involved in the very making of consciousness by accounting to the role of emotions: emotions, which are generated in extra-cortical structures from the information bearing *on the whole body*, are inserted (in the cortex) to the cognitive and motor tasks and then give rise to “the feeling of what happens” (Damasio 1999).

In our daily life we experience this close relation between our thoughts or the emotions they give rise to and our body reaction. For example, one can observe that “negative” emotions,

like fear, anger, anxiety or hostility are generally correlated to palpitation of the heart, elevation of blood pressure or a disturbance of the respiratory system. Another striking illustration of this radical inseparability between mental and bodily processes is provided by the so-called “placebo effect”, where a pharmacologically inert substance such as a sugar tablet or a saline solution can produce the same physiological effect as that of a genuine pharmacologically active substance (such as an analgesic or an anxiolytic). This phenomenon, which has been observed in rigorous, double blind experiments, essentially relies on the physician's belief in the treatment and the patient's faith in the physician, that is, on purely psychological factors (Janssen 2006).

The idea according to which the psychic and the somatic features of a same person are strongly correlated has been explored for a long time by ancient Greek physicians and, more recently, by psychosomaticians since Freud's works on hysteria (Freud 1895) and Alexander's proposition to develop a holistic conception of the individual (Alexander 1950). Psycho-neuro-endocrino- immunology systematically explores the correlations between the whole psychic life (being conscious or not) and the physiological processes, the metabolism, the endocrin and immunological mechanisms of the body -some of its main results are reported in Janssen's previously quoted book. However, in spite of the mountain of evidence that shows precisely the existence of strong psychophysical correlations, the question of their nature is still problematic: Can these correlations between two seemingly inhomogeneous substances or processes be explained in terms of “*causal interactions*”? If yes, which particular causal patterns are involved and how and where do these interactions take place ? If not, could they be understood in terms of *quantum-like entanglement* or are they due to a *common cause*? And how are these different modes of explanation related?

Though the concept of causality plays a central role in any attempt to describe or to

explain phenomena, no universal definition susceptible to be applied for the mind-body correlations have been provided up to now (see, for example (Kistler 2004)). For example, the deductive-nomological model of causality (Hempel 1965) reduces the causal relation to a logical deduction on the basis of the existence of general laws, but the problem is that, in spite of some interesting attempts (see section II), no reliable laws governing the psychophysical relation have been proposed. Nevertheless, some conceptions of causality, possibly in combination with other ones, can be used to deal with the difficult question of the nature of psychophysical correlations. For example, the probabilistic conception of causality (according to which the occurrence of the “cause” increases the probability of the occurrence of the “effect”<sup>1</sup>) is not sufficient to characterize the notion of causal relation between two series of events since it only proves the existence of correlations between them. However, the extent to which two series of events are correlated can provide valuable information on these correlations. According to this line of thought, let us mention the very fruitful use of Bell statistical test for deciding the non-separability of any kind of systems, being microscopic (Aspect 1982), macroscopic (Aerts 1984, 2000) or even mental -regarding, in particular, the property of contextuality of concepts used in a sentence (Aerts 2011 a and b). Another useful conception that can be used is Lewis’ counterfactual model (1986) which asserts that two events c and e are causally dependent if and only if we can say that “c were to occur e would occur; and if c were not to occur e would not occur”. The counterfactual condition is obviously not sufficient to characterize a causal interaction between two events since, for example, it is satisfied even if their correlations are due to a common cause, but it is nevertheless a *consequence* of the existence of such a causal

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<sup>1</sup> The probabilistic conception can indeed be seen as a weaker version of the deductive-nomological conception, where the concept of deterministic laws has been replaced by that of probabilistic laws. probabilistic laws.

interaction and its falsification can then be used to deny its existence. Similarly, the process conception of causality, which obviously applies in physics (as transfer of matter, energy or information which satisfies the relativist causality principle) is highly problematic regarding the mind-body relation –see section II.1. However, the possibility of “signalling”, that is, of existence of causal interactions between two systems, whatever is its particular form, can be proved or discarded in as much as it can have measurable consequences (as shown by the condition (C) of section IV).

Actually, the long-standing question about the nature of the psychophysical correlations has received very different responses, each of them relying on specific philosophical presuppositions about the nature of mind and matter, and about the possible primacy of one of them. Several presentations of some of the main traditional propositions to explain the psychophysical correlations have been provided –for example, in Kim’s book on Philosophy of Mind (Kim 2006, chap. VII). An accurate analysis of these questions that upholds the irreducibility of the phenomenal aspect of mental states (conscious states) to material processes has been provided by Chalmers (1996), by Bitbol (2008) and in Nagel’s famous paper (1974) “What is it like to be a bat?” Some recent propositions developed within the framework of the quantum paradigm have been explored by Atmanspacher (2003) –see also the reference (Uzan 2013) for a deepening of some of these propositions. This paper will try to move this debate forward by proposing a theoretical framework for representing psychophysical states and wherein the question of the nature of the psychophysical correlations can be decided on the basis of Bell-type statistical tests.

Section II will survey the traditional propositions to deal with the mind-body problem, propositions that will be classified according to whether or not they rely on a concept of mental

or psychophysical causation, and their extension within a quantum framework will be mentioned. In particular, we will mention a suggestion made by Atmanspacher and Primas (Atmanspacher 2003) on the basis of a theorem shown by Landau (1987, 2001) in order to justify the non-classical character of the psychophysical correlations from the existence of complementary observables in each of the material and the mental domain. We will show that exploring this interesting suggestion gives rise to four questions regarding 1) the possible existence of psychophysical “interactions”, 2) the possibility of defining couples of complementary observables in the somatic and the mental domains, 3) the theoretical framework wherein appropriate, continuous variables observables can be defined and 4) the practical implementation of such an idea, which is based on statistical tests.

Section III will set the quantum-like theoretical framework wherein psychosomatic states can be represented as continuous and derivable functions of the features of the individual –in fact, of the eigenvalues of a finite number of somatic and mental observables capable to describe her/his global psychosomatic state. This theoretical framework is a generalized form of quantum theory where any a priori reference to the material world has been relaxed. In spite of some differences which will be mentioned in section III.1, the proposed formalism is developed along the line of thought followed by several authors for modelling a wide range of phenomena beyond the strict field of physics for example in psychology of perception, for dealing with ambiguous stimuli (Amanspacher H, Römer H, Walach H 2002) (Atmanspacher H, Filk T, Römer H 2009), or in the field of decision theory, for dealing with the paradoxes of the classical theory of decision (Aerts et al. 2011 a, b). A method to define couples of complementary (continuous variables) observables will then be provided by appealing to the general idea of complementarity between “substance” observables and “process” observables (Römer 2006).

Section IV will first demonstrate, within the theoretical framework presented here, a “signalling” condition for generalized systems and for continuous variables observables provided their spectrum is bounded by 1. Then, the usual Bell-type condition for characterizing entangled states will be extended within this theoretical framework. These two conditions will be used in section V to propose a procedure of decision for the question of the nature of psychophysical correlations, a question which, as will be shown in section II.2, must be treated in two steps: 1) first, we have to decide the question of existence (or not) of a direct, causal “interaction” between physiological and mental processes and 2) if the response to the latter question is “no” (and only in this case) it is meaningful to ask whether the psychosomatic correlations are of quantum-like (non-local) or classical (local) nature –or, in other words, to ask whether the psychosomatic state of an individual must be represented or not by an entangled state. The question of the practical realisation of this experimental test will also be briefly discussed in section V.

## **II. The question of the nature of the psychophysical correlations.**

### ***II.1. A brief review of some propositions.***

Let us first remark that the main stream of neurobiology unfortunately tends to *avoid* this debate about the nature of the psychophysical correlations by merely claiming that mental processes are *nothing but* brain processes. This reductionist, materialist position is clearly expressed in Minsky’s famous assertion that “Minds are simply what brains do” (Minsky 1988) and has been upheld by Crick who has claimed that (1994, p. 3):

“ "You," your joys and your sorrows, your memories and your ambitions, your sense of personal identity and free will, are in fact no more than the behavior of a vast assembly of nerve cells and their associated molecules.”

More recently, this position has been reformulated by Tononi within a computational approach of consciousness wherein subjective experience is defined in terms of “integrated information” (Tononi, 2007) and it has been pushed to the extreme by Dennett (1991) who denies the existence of qualia, or by Churchland (1996) who asserts that folk psychology will eventually be eliminated and replaced by neurosciences. However, this reductionist (or even eliminativist) approach of the mind-body problem cannot bring any light on the question of the nature of the psychophysical correlations since mental life is merely (and mysteriously) “identified” to biological or computational processes. Consequently, it will not be considered any more in the rest of this paper.

Let us consider now the main propositions that really aim to explain the nature of the observed psychophysical correlations. In spite of their great diversity, they can indeed be classified according to whether or not they appeal to the existence of a concept of “causal interaction” between the brain (or the body) and the mental, whatever the particular form it takes. The causal explanation is caricatured in Descartes’ dualistic philosophy that asserts the existence of a causal interaction between matter and mind, which are considered as independent substances -respectively called “res extensa” by referring to the property of material things to be extended in space and “res cogitans”, the thinking thing which is not extended in space. This causal interaction between mind and matter was supposed to take place in the pineal gland, an organ situated at the centre of the brain, between the two hemispheres. If current neurobiology has undermined Descartes’ assertions about the pineal gland and its alleged role in the mind-body interaction, Descartes’ dualism of substances has nevertheless been re-interpreted in terms of dualism of processes and has been recently upheld by von Neumann, London and Bauer or Wigner by appealing to a subjectivist interpretation of the wave function collapse. According to



this interpretation, the subject's "consciousness", "mind" or "faculty of introspection" would play an essential role in the collapse of the wave function by selecting one of the possible results of a measurement. Along this line of thought, Beck and Eccles have suggested that mind could subtly change the probability distribution associated with exocytosis (the release of neurotransmitters) and, consequently, influence the information transfer between neurons –a phenomenon which would be amplified to the macroscopic level due to the large number of synaptic clefts of the pyramidal neurons of the cortex (Beck and Eccles 1992).

However, if it is more tenable to focus on the exocytosis phenomenon which takes place in all chemical synapses of the brain rather than on the role of some particular organ, the nature of this "interaction" between the two inhomogeneous, mental and material substances or processes still remains unclear. If physics accounts for *physical* causality and psychology for *mental* causality, what could the laws governing *psychophysical causation* be ? Models of such psychophysical interactions have been provided but up to now they are not very convincing. For example, Walker has suggested to evaluate the action of mind on matter by asserting that the amount of information processed by consciousness in a physical (quantum) transition is equal to Shannon's statistical information associated with the probability of this transition (Walker 1975). For increasing the maximal psychokinetic "effect", Walker's suggestion has been refined by Mattuck (1980) who has suggested that mind would be capable of performing a quantum transition in several steps. However, the subjectivist interpretation of the wave function collapse on which these propositions are based is far from being accepted by the scientific community for its unclarity and its lack of coherence (Uzan 2013, chap. VII). Even more, this interpretation seems rather useless in regard to other attempts to explain the wave function collapse *within the framework of quantum mechanics*, for example in the decoherence approach proposed by Zurek

(1981-1982) -that is, without appealing to any particular role of “consciousness” or of the “faculty of introspection” of the experimentator.

On evolutionary grounds, the existence of such a causal interaction between the brain processes and the mental processes is implicitly assumed for explaining the genesis of mental life and consciousness and, in turn, its possible “effects” on the brain and the body. According to the so-called “emergentist” approach, brain processes would “cause” or “give rise to” mental processes and, in particular, to subjective experience due to its high level of complexity acquired during evolution. In this case, psychic life could be either thought of as a by-product of brain processes, without any function or any possible “effect” on body (Ribot 1899), like the noise produced by a car is useless for its functioning (epiphenomenalism position), or as a higher-level property of the brain which is different from its physical properties and is governed by its own laws that transcend those governing the brain processes –which would ensure that the mental could have a causal efficacy on material processes (Searle 1997) (Sperry 1983) (Lestienne 2012). For example, Searle upholds the thesis of “biological naturalism” of consciousness: on the one hand, consciousness would be a biological phenomenon “caused” by the brain processes of the most evolved animals; on the other hand, this phenomenon would be “natural” like digestion, photosynthesis or mitosis. Moreover, as suggested by Sperry and generally assumed by contemporary emergentist proposals, the subjective properties would be supposed, in turn, to be capable of exerting “control over the biophysical and chemical activities at subordinate levels”. Thus, the emergentist position seems to clearly assume the existence of causal interactions between the mental processes and their somatic (and, in particular, neurophysiological) correlates.

However, all causal explanations of the psychophysical correlations seem to be faced with

the question of the *explanatory gap* (Chalmers 1996) (Nagel 1974): as complex as they could be, how neural processes, which are governed by material (biophysical or biochemical) laws “give rise” or “cause” *subjective experience*, that is, a sensible quality which can only be experimented in the first person ? How could an objective *description* of physiological processes, in the third person, explain the private *experience* of a particular subject? Assuming the primacy of matter can certainly allow a description of *neural correlates* of subjective experience; however, can such an objective description account for *subjective experience* ?

Moreover, a well-known logical argument, called “exclusion problem of mental causation”, has been mentioned by several prominent philosophers, like Kim (Kim 2006, chap. VII and X) or Esfeld (2012), to show how problematic it is to maintain, within a (non reductionist) materialist position, the efficiency of mental causation, from the mental to the physical. This argument shows that, in virtue of the principle of causal closure of the physical domain which underlies materialism, a principle according to which the cause of a physical event (if any) is necessary of physical order, the very notion of mental causation is made illusory or superfluous -which consequently leads to the epiphenomenalist position presented above and then makes problematic any concept of psychophysical causation.

In contrast, neutral monism philosophy is not based on a concept of psychophysical causation and could then be susceptible of dissolving the questions raised by the previous approaches. According to this approach, both mind and matter would be *intelligible aspects* of a deeper level of reality, a “substance” which is neither mental nor physical. Neutral monism philosophy has been developed by Spinoza very rigorously (in an axiomatic-like presentation) in the Ethics (1677) and has been later re-interpreted in purely epistemic terms by Russell, by James and by Mach on slightly different grounds. Neutral monism has also been explored by Jung and

Pauli (Jung 1970; 1988) from the point of view of the deep psychoanalysis (theory of archetypes) and, more recently, within the framework of quantum theory, by Bohm and Hiley (1993) and by Atmanspacher (2003) and Primas (2003; 2009). Let us briefly explore this philosophical system and its recent extensions or re-interpretations:

According to Spinoza, there exists only one infinite, eternal, substance (also called God or Nature) which is neither mind nor matter. Everything in reality is considered as a mode or a modification of this “neutral” substance. Mind and matter are understood as two “attributes” of this single substance, that is, as two intelligible aspects of it, as what we can perceive of it. In Spinoza’s system, one can thus already find the idea of an *epistemic distinction* between the two, mental and physical aspects of an only substance. This idea has been developed by Russell in his “Analysis of Mind” (1921) where he asserts, for example, that we apprehend the neutral substance with different “logical constructions”: physics is used to apprehend its material aspect and psychology its mental aspect. Furthermore, one can also find in the Ethics the idea of a *psychophysical parallelism* regarding the two, mental and physical, *aspects* of the substance since, for Spinoza:

“the order and connection of ideas are the same as those of things”.

Consequently, neutral monism philosophy clearly suggests an explanation of the psychophysical correlations by referring to the synchronisation of the two intelligible, mental and physical aspects of an only psychophysical unity, that is, *without appealing to any direct causal relation between them* –an appeal which seems rather problematic, as briefly explained above.<sup>2</sup>

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<sup>2</sup> Leibniz has also proposed a parallelist conception of the psychophysical relation which is thus capable, like Spinoza’s proposal, of dissolving the problem of mental causality. For Leibniz, the correlations between the mental and the physical are conceived of in terms of a divin “pre-established harmony”, which means that the chain of physical events, governed by efficient causality, has been synchronized at the beginning, by God, with the chain of mental events,

Some authors have proposed to explore this direction of research within the framework of quantum theory. For example, Bohm and Hiley have described the psychophysical entanglement within the framework of their implicate order theory, as a dynamical process of enfoldment-unfoldment of the mental and material aspects of the individual in a same underlying level of reality -a process illustrated by their famous hologram metaphor (Bohm and Hiley 1993). Atmanspacher and Primas have suggested to regard the (epistemic) co-emergence of the material and mental aspects from the psychophysical neutral level of reality as a symmetry breaking process (Primas 2003) (Atmanspacher 2003). According to this approach, the state of the psychophysical level of reality is mathematically represented as a maximally *entangled state* with respect to a tensorization of its space of representation (Hilbert space) into the material and the mental domain. Furthermore, along the line of thought explored by Jung and Pauli, Primas has shown that the mind-matter entanglement could be parametrized by time, which thus appears as their common ordering principle (Primas 2003).

However, as attractive as the approach of neutral monism and its mathematical representation within the framework of quantum theory could be, it seems that no decisive argument has been provided up to now in order to discard the main stream approaches based on a (rather elusive) concept of psychophysical causation. Can the question about the nature of psychophysical correlations be decided within a quantum-like framework? Namely, can psychophysical correlations be rigorously and consistently explained or not in terms of a generalised form of quantum “entanglement”?

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which is supposed to be governed by final causality. However, and in contrast with Spinoza’s philosophy, Leibniz appeals to infinitely many substances (the monads) from which everything is built, which then leads to a substantialization of mind and matter.

## ***II.2. Can this question be decided on the basis of Bell-type tests ?***

Atmanspacher and Primas have suggested to appeal to Landau's theorem (Landau 1987; 2001) as a possible way of explaining the non-classical nature of the psychophysical correlations from the existence of couples of complementary properties in each of the material and the mental domain (Atmanspacher 2003). Landau's theorem sets out, in the classical case where all observables commute and also in the quantum case where non-commuting observables can exist, the conditions fulfilled by the correlation factor  $R$  between two couples of binary observables  $(A, A')$  and  $(B, B')$ , whose values can be  $+1$  or  $-1$ , respectively measured on two *causally separated* regions  $S1$  and  $S2$  of a composite system, which will be denoted as  $S$ . Landau employs the terms “causally separated” for  $S1$  and  $S2$  in order to justify the fact that all the observables of  $S1$  commute with all the observables of  $S2$  and that joint measurements of these observables can then be performed. This “separability” condition is generally interpreted in a pure physical way, by saying that  $S1$  and  $S2$  *do not interact* or that *no signal* that satisfies relativistic causality can be sent from one of them to the other one. However, in spite of the physical connotation of the terms “interact” and “signal”, which make them rather inappropriate for dealing with the mind-body problem (see section II.1), we will nevertheless use these terms in the following to denote any possible direct causal connection between mind and body.

In Landau's paper (2001), the correlation factor  $R$  for some state of the global system is defined as follows<sup>3</sup>:

$$R = \langle a b \rangle + \langle a' b \rangle + \langle a b' \rangle - \langle a' b' \rangle,$$

where  $a, a', b, b'$  are, respectively, the values of  $A, A', B, B'$  and  $\langle a b \rangle$  denotes the expectation

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<sup>3</sup> Indeed, by changing the definition of one or several of these observables by their “opposite” (which changes their eigenvalues by their opposite), all possible definitions of  $R$  that satisfy Bell

value, for this state, of the product of the values of A and B *in a joint measurement*, the other expectation values being defined in a similar way.

As expected, this theorem shows that Bell's inequalities must be satisfied in the classical case of commuting observables, which characterizes classical (local) correlations:

(L1) For non-interacting sub-systems S1 and S2, if (A, A') and (B, B') are couples of *commuting* observables respectively defined on S1 and S2 then  $|R| \leq 2$  (for all possible states of S).

Moreover, in the case of complementary observables<sup>4</sup>, this theorem shows that  $|R|$  satisfies Tsirelson bound (Tsirelson 1980) which characterizes the maximal degree of violation of Bell inequalities for quantum correlations:

(L2) For non-interacting sub-systems S1 and S2, if (A, A') and (B, B') are couples of *complementary* observables respectively defined on S1 and S2 then  $|R| \leq 2\sqrt{2}$  (for all possible states of S).

The latter assertion provides us with a very powerful means to capture the link between the complementarity of the two couples of observables (respectively defined in each of the two sub-systems) and the nature of their correlations: this theorem shows that *the existence of complementary couples of observables in each of the two parts of the system could be a sufficient condition for existence of non local correlations between these two subsystems*.

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inequalities can be provided. For example, by replacing B' by its opposite, R is defined by another expression which has also been involved in CHSH inequalities:  $R = \langle ab + a'b - ab' + a'b' \rangle$ .

<sup>4</sup> The general notion of "complementarity", which basically refers to the idea of "complement" of a part of an object regarding its totality, is applied here to the observables that are used to represent the properties of an object within the quantum formalism. The latter, specific application of the general notion of "complementarity" to quantum observables, which are mathematically represented by hermitean operators (and not functions of points, like in classical physics), is expressed by their non-commutativity and then, by the fact that the order in which they are measured is not indifferent. This is due to the fact that their respective eigenstates, which are relative to exclusive, complementary descriptions, cannot form a common basis. It is this

Taking now into account the fact that this theorem is established within the framework of the  $C^*$ -algebra of observables that underlies the standard quantum theory but *without any a priori reference to the physical properties of the systems considered*, Atmanspacher and Primas have suggested that the latter result could be used to justify the non-locality of the psychophysical correlations: if the two sub-systems are conceived of as, respectively, the mental domain and the material domain, the existence of couples of complementary properties in each of these sub-systems could be a sufficient condition for the existence of non-local correlations between them. In other words, the psychophysical correlations that characterize the very notion of individual could be explained and explored from the complementarity of some properties defined in each domain, mental and somatic.

However, this idea gives rise to four questions:

(1) *The question about the existence of psychophysical “interactions”*. Landau’s theorem has been shown under the assumption that the two sub-systems  $S_1$  and  $S_2$  are separated (see above). Consequently, this theorem can be used to show that the correlations between two sub-systems are non-local *if any causal interaction between them has been discarded*. If this hypothesis is not satisfied, this theorem *cannot be used* in its present form and nothing can be decided about the quantum (non-local) or the classical (local) nature of the psychophysical correlations. Regarding now the psychophysical correlations, the previous remark means that the question of their non-locality can be decided on the basis of the violation of Bell inequalities only *if the possible existence of causal interactions between the somatic and the mental domains (and, in particular, between the somatic and the mental observables involved in the expression of  $R$ ) has been discarded*. Consequently, according to a strategy of decision based on Landau theorem,

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specific application of the notion of “complementarity” to quantum observables that will be



one has *first* to tackle the question of existence of psychophysical, causal interactions and *then* (in case of a negative answer) the question about non-locality of the psychophysical correlations.

(2) *The question of defining complementary observables* in each of the mental and somatic domains. Indeed, complementary *processes* in both somatic and mental domains can easily be found. For example, in the somatic domain, actions of some couples of hormones are complementary since they exclude each other by performing opposite tasks, and yet are both necessary for body equilibrium, namely for maintaining minimal fluctuation in physiological variables (like blood pressure, blood glucose concentration, temperature or pH). It is the case for the couple of hormones insulin/glucagon which have opposite effects on blood glucose concentration: the latter tends to increase it whereas the former tends to lower it. Similarly, in the mental field, one can find couples of complementary patterns, like, for example, as emphasized by Bohr, the passive and active attitudes which exclude each other (we cannot at the same moment act and passively observe, these attitudes being alternative) but are both necessary in everyday life. However, as mentioned by Filk (2011), it is an open question to use these complementary processes to define complementary observables in the sense of (generalised) quantum theory, that is, in the sense that *the order of measurements* of these observables is significant. Nevertheless, in section III we will mention some interesting attempts to address this question and a general method to define complementary observables in the psychosomatic field will be provided.

(3) *The question of the possible continuity of the somatic and the mental quantities.* Somatic and mental observables can *a priori* take *continuous* values, as is the case for the previous examples of observables (for example, the concentration of some substance in the

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exclusively used all along this paper (like in Landau's theorem).

blood). One should then work within a continuous representation of the psychosomatic states and show statistical conditions for determining the nature of correlations between two sub-systems for continuous variables observables. Note that the use of continuous variables do not at all prevent the possibility of dealing with discrete variables, like those which are often defined and evaluated in the field of psychology, on the basis of questionnaires. The latter only appear as particular specifications of the more general continuous variables that will be introduced in section III.

(4) *The question of the practical realisation.* In order to propose an experimental test capable to decide the nature of the psychophysical correlations, one has to *specify* a concrete model wherein couples of seemingly *correlated*, measurable quantities can be defined (“correlated” according to the data of psycho-neuro-endocrino-immunology) and to perform series of measurements of these observables on systems (namely, individuals) “prepared” in the same psychophysical state –which seems to lead to several practical difficulties.

All these questions will be debated bellow and precise answers will be proposed.

### **III. A theoretical framework to tackle this question.**

#### ***III.1. Quantum-like representation of the psychosomatic states.***

The theoretical framework where the psychophysical correlations can be represented is a generalised form of quantum theory like the one developed by Atmanspacher, Römer and Walach (2002), where the notion of “system” is defined as *any* part of reality which is an object of attention and investigation and that of “observable” can refer to any meaningful property of it which can be measured in a reproducible way. As mentioned in the introduction, *any a priori reference to material systems and physical quantities has been relaxed* -for example, no Planck

constant and no Schrödinger equation will be involved in this approach. We are here only interested by the mathematical structure of quantum theory and by the possibility it can offer to tackle the mind-body problem. To be precise, and in contrast with the weak version of quantum theory proposed by Atmanspacher, Römer and Walach, we will keep here the *full* mathematical background of standard quantum theory for its representative power: the full (non commutative)  $C^*$  algebra of observables that govern the properties of generalized systems and the corresponding formalism of Hilbert spaces to represent their state.<sup>5</sup>

In the present situation, the “system” is defined as the individual and the notion of “observable” can refer to any mental or somatic quantity which can be measured, the psychosomatic states being represented by vectors in a Hilbert space, noted as  $H$ . Now, *if* we adopt the psychophysical parallelism proposed by Spinoza or by Leibniz as an alternative philosophical position susceptible of dissolving the difficulties inherent to the concept of psychophysical causality (see section II.1), we *a priori* suppose that no direct causal interaction can take place between the physiological and the mental properties. This assumption then allows us to write  $H$  as the tensor product of a Hilbert space  $M$  of purely material states and a Hilbert space  $N$  of purely mental states:

$$(h) \quad H = M \otimes N,$$

where  $M$  is spanned by the eigenvectors of the somatic observables and  $N$  is spanned by the eigenvectors of the mental observables that describe the individual. The reason for which the assumption (h) can be considered as the mathematical expression of Spinoza’s (or Leibniz’) parallelism is that the tensor splitting of the space of psychosomatic states of the individual in a

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<sup>5</sup> It can be shown that any  $C^*$ -algebra is isomorphic to a closed  $*$ -algebra of the bounded operators  $B(H)$  of a Hilbert space  $H$ , the involution  $*$  being in correspondence with the operation

space of purely somatic states and a space of purely mental states is equivalent to the claim that all the somatic observables commute with all the mental observables. In other words, adopting the parallelist position would entail that any possible complementarity relation between somatic observables (acting only on M) and mental observables (acting only on N) would be ruled out. However, *this assumption will not be used a priori* since this paper aims precisely to discuss this question -and other ones which are relevant to the mind-body problem- and to propose an experimental procedure for deciding them on experimental grounds.

A “natural” orthonormal basis of  $H$  can be defined from the collection of possible values of the quantities that are usually considered in both mental and somatic domains. For the somatic domain these quantities are, for example, physico-chemical parameters like the temperature of the body, the blood pressure or the blood concentration of different cells, platelets and other substances (hormones, nutrients, ...); more complex measurable quantities, like the velocity of metabolism or the degree to which an organ dysfunctions, can of course be defined from the latter that are taken as primitive. For the mental domain, the primitive quantities are those identified by psychology and measured by psychological diagnostics and tests, like the types of personality or temperaments and the emotional sensibility –and they can give rise to more complex quantities, like the degree of depression or social phobia. It will be assumed that a *finite* number of such mental and somatic primitive properties is sufficient to define the psychosomatic state on an individual<sup>6</sup> though their possible values, which exclude each other, can *a priori* vary *continuously*. This “natural” orthonormal basis of  $H$  can then be written as:

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of taking the adjoint sub-space for sub-spaces of  $B(H)$ . Thus, any system can be associated with a Hilbert space defined from the algebra of its observables and the set of its states.

<sup>6</sup> This hypothesis about the finite number of elementary patterns from which the state of the individual can be defined has always been adopted and has proved rather fertile, like in the four humours theory of ancient Greece, in that of existence of a finite number of morphological types

$$\Pi = \{ |a^k, b^{k'}\rangle \},$$

where the indexes  $k$  and  $k'$  refer, respectively, to these somatic and mental primitive quantities (which are in finite number). Note again that it is not supposed a priori that the somatic and the mental domain are separated (hypothesis (h)) and then the previous notation  $|a^k, b^{k'}\rangle$  must not be interpreted a priori as a tensor product of purely somatic and mental vectors. It only reminds us that a psychosomatic state is a function of somatic *and* mental variables. For simplicity of writing, one will drop the indexes of the observables and then write the continuous orthonormal basis  $\Pi$  as :

$$\Pi = \{ |a, b\rangle \},$$

where  $a$  and  $b$  run, respectively, on the continuous spectrum of the somatic and mental observables taken as “primitive” for the description of the individual.

It can be supposed that *the spectra of all somatic and mental observables are bounded* due to the phenomenon of homeostasis, which imposes to the possible values of these observables to be regulated within a short interval in order to maintain the stability of the body and of its normal functioning. For example, the (tympanic) temperature of the human body oscillates around  $37^\circ\text{C}$ , and the pH of the blood remains very close to 7,3. Similarly, psychological studies show that mental observables, which evaluate, for example, the intellectual performance or some emotional features of the individual, are also bounded –which defines the notion of “normality” to which refer psychologists. In section IV, we will make the assumption that the spectrum of any observable is bounded by 1, which can be justified by the fact that any

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and associated temperaments (Reich, Sheldon) and, more recently, in the psychoanalytic classification given from a finite number of structural “items” and mental “data” by Marty (1990).

bounded interval can be injected into the continuous interval  $[-1,+1]$  by using an appropriate function.

A vector of  $H$  can be decomposed as a linear combination of vectors of the “natural” basis  $\Pi$ :

$$|\Xi\rangle = \iint_{a,b} c(a,b) |a, b\rangle da db,$$

where the  $c(a,b) = \langle a, b | \Xi \rangle$  are complex coefficients which can always be conceived of as *continuous* and *derivable* functions of the continuous variables  $a$  and  $b$  (which range in a bounded interval).

We will also keep the probability interpretation of quantum theory, which is indeed *the only possible form of probability measure that can be defined on a Hilbert space according to Gleason theorem* (Gleason 1957). We will then assert that:

(Pr) if  $A$  and  $B$  are, respectively, a somatic and a mental observable, then  $|\langle a, b | \Xi \rangle|^2$

is the probability density to measure  $A = a$  and  $B = b$  on an individual in state  $|\Xi\rangle$ .

The adoption of this rule, which implies the normalisation condition:

$$\iint_{a,b} |\langle a, b | \Xi \rangle|^2 da db = 1,$$

means that the psychosomatic states form a sub-ensemble of  $L^2$ , the space of square-summable functions, and that, consequently, by using Plancherel theorem (Plancherel 1910), any psychosomatic state in the  $\Pi$ -representation can be decomposed as a weighted sum of elementary (“plane-wave”) functions as:

$$(1) \quad \langle a, b | \Xi \rangle = \iint_{\lambda_a, \lambda_b} c(\lambda_a, \lambda_b) \exp i(\lambda_a a + \lambda_b b) d\lambda_a d\lambda_b,$$

where  $\lambda_a$  and  $\lambda_b$ , the rates of change of  $\langle a, b | \Xi \rangle$  regarding, respectively,  $a$  and  $b$ , are real numbers, and  $c(\lambda_a, \lambda_b)$  is the Fourier transform of  $\langle a, b | \Xi \rangle$  :

$$c(\lambda_a, \lambda_b) \equiv \langle \lambda_a, \lambda_b | \Xi \rangle = \iint_{a,b} \langle a, b | \Xi \rangle \exp -i (\lambda_a a + \lambda_b b) da db,$$

the basis  $|\lambda_a, \lambda_b\rangle$  being defined by :

$$(2) \quad \langle a, b | \lambda_a \lambda_b \rangle =_{\text{df}} \exp i (\lambda_a a + \lambda_b b) .$$

### ***Temporal Evolution of Psychosomatic States.***

Chronobiology and chronopsychology have shown that the change of (most of) the physiological and psychological features of the individual is periodic. These changes are governed by internal clocks, which are roughly synchronised with astronomical cycles (Reinberg 2000; Lestienne 1990; Finger 1982). In the somatic domain, it is for example the case for the circadian change (a 24-hours change) of the body temperature or that of the release of hormones, like cortisol or melatonin, by the endocrine glands, which gives rise to a corresponding change in their concentration in the blood. In the mental domain we can mention the circadian rhythm of learning or that of the emotional sensibility (Minors and Waterhouse 1981) (Beugnet-Lambert et al. 1988). The periodic change of all the physiological and mental variables involved in the menstrual cycle, roughly synchronized with the lunar rhythm, is also well known.

By Fourier theorem, any periodic change can be decomposed as a weighted sum of harmonic functions. Consequently, the periodic change of the physiological and the mental variables involved in the psychosomatic state described, in the  $\Pi$ -representation, by equation (1) can be mathematically represented as follows:

$$\langle a, b | \Xi(t) \rangle = \iint_{\lambda_a, \lambda_b} c(\lambda_a, \lambda_b) \exp i [(\lambda_a a + \lambda_b b) - (\omega_a(\lambda_a) + (\omega_b(\lambda_b)) t] d\lambda_a d\lambda_b ,$$

where  $\omega_a (\lambda_a)$  and  $\omega_b (\lambda_b)$  are the pulsations associated with the harmonic changes of a and b in the Fourier decomposition of the global state  $\langle a, b | \Xi (t) \rangle$ . Now, as shown by chronobiological and chronopsychological studies (Reinberg 2000), it can be supposed that the rhythmic change of all the properties involved the description of the psychosomatic state is dominated by a same circadian rhythm (with period of 24 hours). Consequently, the previous state can be written as a function of only one pulsation  $\omega_0$ :

$$\omega_0 = \omega_a (\lambda_a^0) = \omega_b (\lambda_b^0) = 2\pi / 24 \text{ hours},$$

modulated by a weighted superposition of harmonic functions of the dispersion of the values of  $\lambda_a$  and  $\lambda_b$  around, respectively,  $\lambda_a^0$  and  $\lambda_b^0$ . Moreover, in the limiting case where the amplitudes of the harmonic contributions of the modulation are negligible near that of the main harmonic function with pulsation  $\omega_0$ , the psychosomatic state can simply be written as follows:

$$\langle a, b | \Xi (t) \rangle = \iint_{\lambda_a, \lambda_b} c(\lambda_a, \lambda_b) \exp i [(\lambda_a a + \lambda_b b) - \omega_0 t] d\lambda_a d\lambda_b,$$

or, in a more compact form, as:

$$(3) \quad \langle a, b | \Xi (t) \rangle = \langle a, b | \Xi (0) \rangle \exp - i \omega_0 t .$$

The latter form will be useful to understand intuitively which quantity the “derivative” observable that will be defined in the next section refer to.

### ***III.2. Complementary observables for the psychosomatic domain.***

In view of the difficulty, mentioned in section II, for defining complementary observables from the existing complementary processes in somatic and mental domains, different approaches have been proposed. For example, Atmanspacher and Filk have worked out a model of bistable perception of an ambiguous stimulus (the Necker cube) where they consider the complementarity of the measurements at two different times of *the same observable* in order to show the



entanglement of perceptual states from the violation of *temporal* Bell inequalities that involve measurements of the same observable at different times (Atmanspacher and Filk 2010) (Filk 2011). Let us also mention Wallach's and Römer's suggestion to define a notion of complementarity between mental and physiological observables within the framework of the weak version of quantum theory that has been mentioned in section III.1 (Walach 2005) (Walach and Römer 2011)<sup>7</sup>. These researchers appeal to the concept of complementarity between *global* observables, which are relative to the description of the whole system (and, in particular, the whole human organism), and *local* observables, which are relative to the description of some of its parts, like its organs (Walach 2005, p. 385-386). For, within this theoretical framework, a system can be said in an *entangled state* if this system is in an eigenstate of the global observable but *not* of the local observables<sup>8</sup> (Atmanspacher, Filk and Römer 2009a).

However, none of the previous interesting approaches will be explored here since, as mentioned in section II from a proposition formulated by Atmanspacher and Primas, we aim to define complementary observables in *each* of the somatic and the mental domain and to use some Bell-type condition(s) in order to decide the question of the nature of psychophysical correlations. Let us then follow the latter idea.

The method to construct complementary observables that will be suggested hereafter is very general and can be used in both somatic and mental domains. It relies on the idea of complementarity between what Römer (2006) calls "*substance*" observables and "*process*"

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<sup>7</sup> Walach (2005) has also suggested to deal with this notion of complementarity by referring to the mental and physiological *aspects* of the individual. For example, studying pain leads to the possibility of defining complementary observables which are, respectively, relative to the experience of pain, that is, to the mental aspect of the individual, and relative to its neurological substrates, that is, to the somatic aspect of the individual (2005, p. 384).

<sup>8</sup> This condition is indeed a weakening of the non-factorability condition in standard quantum theory.

observables within the framework of the weak version of generalized quantum theory (Amanspacher H, Römer H, Walach H 2002). For this author, substance observables refer to timeless features of a system, like the description of its state (for example, its position, its shape or its colour), and are compatible with an inner time observable that could be defined according to Mc Taggart's concept of A-time. In contrast, *process* observables are relative to its change, to the transition between its possible, intermediate states and are complementary to an inner A-time observable. The energy observable of quantum physics is a typical example of process observable since it is related to the temporal evolution of a system, which is expressed by the fact that a precise value of energy and a precise location in time cannot be provided together with arbitrary accuracy.

We might think that this idea could be implemented in the psychosomatic field by merely considering couples of properties that characterize, respectively, the state of an individual and its rate of change (which defines a process). For example, we could consider the couple of quantities that, respectively, refer to the physiological parameters of the body (state description) and to those that determine the velocity of metabolism (process description). In the mental domain, we could consider, for example, the couple of quantities that respectively measure the degree of emotional sensibility, which contributes to the description of the mental *state*, and its rate of change, which describes an evolution *process*. However, it is not clear that these couples of property could really implement Römer's idea since, for example, a degree of emotional sensibility, which is supposed to play the role of substance observable, cannot be easily defined independently of a change of attitude and it then relies, in some way, on a notion of process. Moreover, it is not clear that these couple of properties could be mathematically interpreted in terms of non-commuting observables in the strict sense of (generalized) quantum theory –which

is a basic requirement in Römer's paper quoted previously. This idea is reconsidered hereafter within the framework of the generalised version of quantum theory presented here wherein Römer's substance/process complementarity will be interpreted in terms of canonical, "conjugate" observables.

Let us suppose that A is a somatic observable whose action on a psychosomatic state  $|\Xi\rangle$  is to compute the value a of the associated quantity, which means that in the  $\Pi$ -representation its action can be written as:

$$\langle a, b | A | \Xi(t) \rangle = a \langle a, b | \Xi(t) \rangle.$$

Let us now define an observable A' that computes, in the  $\Pi$ -representation, the rate of change of the state  $|\Xi(t)\rangle$  associated with a variation of "a". A' can be defined (to within a constant) as the derivative of  $|\Xi(t)\rangle$  regarding to a: <sup>9</sup>

$$(4) \quad \langle a, b | A' | \Xi(t) \rangle =_{\text{df}} -i \partial/\partial a \langle a, b | \Xi(t) \rangle.$$

The observables A and A' are complementary since (in the  $\Pi$ -representation):

$$\langle a, b | A A' | \Xi(t) \rangle = -i a \partial/\partial a \langle a, b | \Xi(t) \rangle, \text{ whereas}$$

$$\langle a, b | A' A | \Xi(t) \rangle = -i \partial/\partial a (a \langle a, b | \Xi(t) \rangle) = -i [a \partial/\partial a \langle a | \Xi \rangle + \langle a | \Xi(t) \rangle].$$

Their commutator is thus equal (within a constant) to the identity operator Id (in H):

$$[A, A'] = i \text{Id},$$

which is different from the null operator 0.

If the (formal) action of A on a psychophysical state  $|\Xi\rangle$  provides the value "a" of a definite quantity (like the concentration of some particular substance in the blood), the quantity  $\lambda_a$  that corresponds to the action of A', the "derivative" operator, refers (within a constant) to its

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<sup>9</sup> It has been supposed that the states of H are, in the  $\Pi$ -representation, complex functions differentiable in a and b.

“frequency” –in a general sense, like in physics the wave number  $k$  can be called a “spatial frequency”. This can be seen by computing (in the  $\Pi$ -representation ) the action of  $A'$  on the vectors of the basis  $\{|\lambda_a \lambda_b \rangle\}$  defined by equation (2) and by using the definition (4) of the action of  $A'$ :

$$\langle a, b | A' | \lambda_a \lambda_b \rangle = -i \partial/\partial a \langle a, b | \lambda_a \lambda_b \rangle = \lambda_a \langle a, b | \lambda_a \lambda_b \rangle.$$

The intuitive meaning of the coefficient  $\lambda_a$  measured by the observable  $A'$  can be seen as follows. As suggested by the experimental data of chronobiology (Reinberg 2000), all the periodic changes of the psychosomatic state seem to be synchronized and they give rise to an only circadian (24 hours) rhythm for the global state of the individual, which can indeed be approximated by a harmonic function of pulsation  $\omega_0 = 2\pi / 24$  hours since the circadian rhythm is dominant in most of the rhythmic changes of the features of the individual (see section III.1). Then, the global, time rate of change of the psychosomatic state can be computed as:

$$\partial/\partial t \langle a, b | \Xi(t) \rangle = -i \omega_0 \langle a, b | \Xi(t) \rangle,$$

where  $\langle a, b | \Xi(t) \rangle$  is defined by equation (3) and according to the aforementioned approximation.

On the other hand, taking into account the (local) time rate of change of the variable “a”, the same (global) time rate of change of the psychosomatic state  $\langle a, b | \Xi(t) \rangle$  can be calculated as:

$$\partial/\partial t \langle a, b | \Xi(t) \rangle = \partial/\partial a \langle a, b | \Xi(t) \rangle \cdot \partial a/\partial t = -i \lambda_a \langle a, b | \Xi(t) \rangle \cdot \partial a/\partial t.$$

Then, by identification of the two previous results, and under the aforementioned approximation that the global change of the psychosomatic state is a circadian harmonic change, the parameter  $\lambda_a$  measured by the “derivative” observable  $A'$  can be computed as:

$$\lambda_a = \omega_0 / (\partial a/\partial t).$$

This expression of  $\lambda_a$  shows that the conjugate observable  $A'$  does not exactly evaluate the time rate of change  $\partial a/\partial t$  of the variable “a” but, as briefly mentioned above, its associated

general “frequency”, which denotes the rate of the “natural”, global pulsation  $\omega_0$  of the psychosomatic state to its local rate of change ( $\partial a/\partial t$ ).

The case of mental observables can of course be treated in a similar way, by considering the observables that evaluate a mental property B and the associated “generalized frequency”  $\lambda_b$  which, as shown above, compares its time rate of change to the global pulsation of the psychosomatic state. Finally, the previous suggestion to define couples of complementary observables in both somatic and mental domains gives rise to a general method for defining an infinite number of couples of complementary observables by linear combinations of these observables. For example, for any couple of complementary (somatic or mental) observables X and X' defined as above (that is, such that  $[X, X'] = i \text{Id}$ ), the observables defined by:

$$Y = X + X' \quad \text{and} \quad Y' = X - X'$$

are complementary since  $[Y, Y'] = -2 [X, X'] = -2 i \text{Id}$ , which is different from 0.

#### **IV. Deciding the question about the nature of psychophysical correlations.**

The question of the nature of psychophysical correlations can be decided *experimentally* on the basis of a (double) Bell-type condition capable of determining 1) whether the somatic and the mental domains “interact” or not, and 2) in case of negative response to this first question, whether psychosomatic states are entangled in the strict sense of (generalized) quantum theory. In order to realize this program, two conditions will be shown for generalized systems and continuous variables observables with spectrum bounded by 1: a “signalling” condition for characterizing the existence of any form of causal “interaction” between two sub-systems, and a condition of entanglement for the states of a composite system -which, indeed, extends the usual Bell-type condition.

#### ***IV.1. Signalling condition for generalized systems with continuous observables.***

For two couples of continuous variables observables  $(A, A')$  and  $(B, B')$  with spectrum bounded by 1 and respectively defined on two parts  $S1$  and  $S2$  of a generalized system  $S$ , let us denote the CHSH correlation factor calculated for the set of observables  $J = \{(A, A'); (B, B')\}$  with continuous spectrum bounded by 1 and for the state  $|\Xi\rangle$  as  $R(J, |\Xi\rangle)$ . As in section II.1, the latter is defined by the following expectation value:

$$R(J, |\Xi\rangle) = \langle a b + a' b + a b' - a' b' \rangle_{|\Xi\rangle}$$

where  $a, a', b$  and  $b'$  denote the possible values of the observables of  $J$ .

*Definition:* We will say that two systems  $S1$  and  $S2$  are  $J$ -separated if the observables  $A$  and  $A'$  of  $S1$  do not interact with the observables  $B$  and  $B'$  of  $S2$ , whatever is the particular form of this “interaction”.

We will now show the following signalling condition *for all set  $J$  of observables*:

$$(C) \quad S1 \text{ and } S2 \text{ are } J\text{-separated if and only if for all state } |\Xi\rangle, |R(J, |\Xi\rangle)| \leq \beta_J$$

where  $\beta_J$ , which will be computed hereafter, is the maximum value of  $R(J, |\Xi\rangle)$  *under the condition that  $S1$  and  $S2$  are  $J$ -separated*.

*Proof of the condition (C):*

$\Rightarrow$ ) Suppose that  $S1$  and  $S2$  are  $J$ -separated. In his derivation of conditions (L1) and (L2) presented in section II.2, Landau (2001) has shown that for separated systems (and then for  $J$ -separated systems) and *for binary (+1/-1) observables* the quantity  $\langle c^2 \rangle_{|\Xi\rangle}$  such that  $R = \langle c \rangle_{|\Xi\rangle}$  can be computed for state  $|\Xi\rangle$  as (Landau 2001, section 8):

$$\langle c^2 \rangle_{|\Xi\rangle} = 4 + \langle [A, A'] [B', B] \rangle_{|\Xi\rangle}.$$

However, for *continuous* variables observables with spectrum bounded by 1, that is, with:

$$|a| \leq 1; |b| \leq 1; |a'| \leq 1; |b'| \leq 1$$

it is easy to check that the previous equality must be replaced by:

$$\langle c^2 \rangle_{|\Xi\rangle} \leq 4 + \langle [A, A'] [B', B] \rangle_{|\Xi\rangle}$$

and, since  $\langle c \rangle_{|\Xi\rangle}^2 \leq \langle c^2 \rangle_{|\Xi\rangle}$  :

$$(5) \quad |R(J, |\Xi\rangle)|^2 \leq 4 + \langle [A, A'] [B', B] \rangle_{|\Xi\rangle}$$

By the well-known property that the absolute expectation value of any observable in a state which is the eigenvector associated to its *maximum* (supposed non degenerated) eigenvalue is equal to its norm, the maximum of the expectation value involved in the right member of inequality (5) is equal to the norm  $|| [A, A'] [B', B] ||$ . Consequently, *under the condition that S1 and S2 are J-separated*, the maximum value of the right member of inequality (5) for the set of observables J is  $(4 + || [A, A'] [B', B] ||)$ , which means that:

$$(6) \quad \beta_J = (4 + || [A, A'] [B', B] ||)^{1/2}.$$

and then for any state  $|\Xi\rangle$  of H:

$$(7) \quad |R(J, |\Xi\rangle)| \leq \beta_J,$$

where  $\beta_J$  is determined by equation (6).

Note that this result is more general than the conditions (L1) and (L2) since the latter are only particular cases of it:

- if  $[A, A'] [B', B] = 0$ , that is, (at least) one of the couples (A,A') and (B,B') commute. In this

case  $\beta_J = 2$  and then (7) reduces to  $|R(J, |\Xi\rangle)| \leq 2$ , that is, Bell inequality shown in (L1).

- if  $[A, A'] [B', B] \neq 0$ , that is, both couples of observables are complementary. In this case,

$\beta_J = 2\sqrt{2}$  (since the maximum value of the norm  $|| [A, A'] [B', B] ||$  is 4) and then (7)

reduces to  $|R(J, |\Xi\rangle)| \leq 2\sqrt{2}$ , Tsirelson bound, which is shown in (L2).

⇐ ) We will indeed show the contrapositive assertion on the converse:

If S1 and S2 are not J-separated, then there is a state  $|\Xi\rangle$  such that  $|R_J(|\Xi\rangle)| > \beta_J$ .

If S1 and S2 are not J-separated, the observables A and A' of S1 can “interact” in some way with those, B and B', of S2. Now, many forms of “interaction” between these observables can exist and, without specifying it, a precise characterization of the statistical dependence of, say, the values of A (defined on S1) on the values of B or B' (defined on S2) cannot be provided. However, as remarked in Introduction, the *consequences* of any form of “interaction” can be characterized by some general conditions, like Lewis' counterfactual condition or some probabilistic condition. For continuous variables observables, it seems indeed more convenient to refer to the change in the correlation factor between these observables defined, respectively, on S1 and on S2.

The fact that S1 and S2 “interact” (regarding J) can be expressed by an increase of the correlation factor  $R(J,|\Xi\rangle)$ , for any state  $|\Xi\rangle$ , with regard to its value when they are separated. Without loss of generality, we can suppose that this increase of the correlation factor can be written as an increase in its absolute value, that is, we can suppose that  $|R(J,|\Xi\rangle)|$  is greater than  $|R_{\text{sep}}(J, |\Xi\rangle)|$ , which is computed when S1 and S2 are J-separated<sup>10</sup>:

$$|R(J, |\Xi\rangle)| > |R_{\text{sep}}(J, |\Xi\rangle)|.$$

Now, as shown above (see the previous demonstration), there exists a state, say  $|\Xi_0\rangle$ , for which  $|R_{\text{sep}}(J, |\Xi\rangle)|$  is maximized, this maximal value being equal to  $\beta_J$ , which is given by

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<sup>10</sup> If it is not the case (for example, if both correlation factors are negative or if  $R(J, |\Xi\rangle)$  is positive and  $R_{\text{sep}}(J,|\Xi\rangle)$  is negative), we can always define another set of observables J' for which  $R_{\text{sep}}(J',|\Xi\rangle)$  and then  $R(J',|\Xi\rangle)$  are positive –for example, by changing A and A' by their opposite.



equation (6). Consequently, when S1 and S2 are not J-separated there exists a state  $|\Xi_0\rangle$  such that:

$$|R(J, |\Xi_0\rangle)| > \beta_J (= |R_{\text{sep}}(J, |\Xi_0\rangle)|).$$

#### ***IV.2. Bell-type entanglement condition for generalized systems with continuous variables observables.***

Let us consider a set of observables  $J = \{(A, A'); (B, B')\}$  with *continuous* spectrum  $[-1; +1]$ , where  $(A, A')$  and  $(B, B')$  are respectively defined on two *non-interacting* sub-systems S1 and S2 of a generalized system S. Due to this no signalling assumption, the state space of S can be written as the tensor product of  $H_1 \otimes H_2$  –which entails that all observables of  $H_1$  commute with all observables of  $H_2$ .

It is then possible, *under this assumption*, to show the following condition, where  $R(J, |\Xi\rangle)$  denotes the CHSH correlation factor calculated for the state  $|\Xi\rangle$  and the set J of observables:

(D)  $|\Xi\rangle$  is separable (or factorable) if and only if for any set J of observables  $|R(J, |\Xi\rangle)| \leq 2$

*Proof of the condition (D):*

$\Rightarrow$ ) The fact that  $|\Xi\rangle$  is separable means that it can be written as the product of a state of S1 and a state of S2:  $|\Xi\rangle = |\Xi_1\rangle \otimes |\Xi_2\rangle$ . Since all observables of S1 commute with all observables of S2, due to the tensor product form of H, the observables A and B commute, which means that they exclusively act, respectively, on S1 and S2, and that, consequently, the expectation value  $\langle ab \rangle_{|\Xi\rangle} = \langle \Xi | A B | \Xi \rangle$  involved in  $R(J, |\Xi\rangle)$  can also be factored as the

product of the expectation value of A for the state  $|\Xi_1\rangle$  of H1 and of the expectation value of B for the state  $|\Xi_2\rangle$  of H2:

$$\langle \Xi | A B | \Xi \rangle = \langle \Xi_1 | A | \Xi_1 \rangle \langle \Xi_2 | B | \Xi_2 \rangle .$$

A similar factorisation can of course be done for the expectation values that involve the other couples of observables (A,B'), (A',B) and (A', B'). The correlation factor can then be written as follows (the writing has been simplified, the index 1 referring to the state  $|\Xi_1\rangle$  of S1 and the index 2 referring to the state  $|\Xi_2\rangle$  of S2):

$$\begin{aligned} R(J, |\Xi\rangle) &= \langle A \rangle_1 \langle B \rangle_2 + \langle A \rangle_1 \langle B' \rangle_2 + \langle A' \rangle_1 \langle B \rangle_2 - \langle A' \rangle_1 \langle B' \rangle_2 \\ &= \langle A \rangle_1 (\langle B \rangle_2 + \langle B' \rangle_2) + \langle A' \rangle_1 (\langle B \rangle_2 - \langle B' \rangle_2). \end{aligned}$$

Consequently, the absolute value of  $R_{|\Xi\rangle}$  is such that:

$$\begin{aligned} |R(J, |\Xi\rangle)| &= |\langle A \rangle_1 (\langle B \rangle_2 + \langle B' \rangle_2) + \langle A' \rangle_1 (\langle B \rangle_2 - \langle B' \rangle_2)| \\ &\leq |\langle A \rangle_1| |\langle B \rangle_2 + \langle B' \rangle_2| + |\langle A' \rangle_1| |\langle B \rangle_2 - \langle B' \rangle_2| \\ &\leq |\langle B \rangle_2 + \langle B' \rangle_2| + |\langle B \rangle_2 - \langle B' \rangle_2| \leq 2, \end{aligned}$$

the two last inequalities (of the last line) being established by taking into account the fact that the spectrum of any observable is bounded by 1.

We have then shown that for any (generalised) continuous variables observables A, B, A' and B', if the state  $|\Xi\rangle$  is separable then  $|R(J, |\Xi\rangle)| \leq 2$ .

$\Leftarrow$ ) We will show the contrapositive: if  $|\Xi\rangle$  is entangled then there exists a set J of observables such that  $|R(J, |\Xi\rangle)| > 2$ .

Any state of H can be written, in the continuous  $\Pi$ -representation, as in equation (1):

$$\langle a, b | \Xi \rangle = \iint_{\lambda_a, \lambda_b} c(\lambda_a, \lambda_b) \exp i(\lambda_a a + \lambda_b b) d\lambda_a d\lambda_b .$$

However, as shown by Lamata and Léon (2005), this state (like any continuous variable state), can be discretised *with the desired accuracy* by defining on H1 and H2, respectively, two discrete and complete sets of orthogonal functions of  $L^2$ , noted as  $O_m(a)$  and  $O_n(b)$ . The state  $\langle a, b | \Xi \rangle$  can then be approximated with the desired accuracy as follows:

$$\langle a, b | \Xi \rangle = \sum_{m,n} C_{m,n} O_m(a) O_n(b).$$

By using now the usual Schmidt procedure, it can be decomposed on the Schmidt basis as a diagonal sum of biorthogonal terms:

$$\langle a, b | \Xi \rangle = \sum_m \alpha_m \langle a | \Xi^1_m \rangle \langle b | \Xi^2_m \rangle,$$

where  $|\Xi^1_m\rangle$  and  $|\Xi^2_m\rangle$  are, respectively, states of H1 and H2, indexed by only one index and the coefficients  $\alpha_m$  are real numbers. The state  $\langle a, b | \Xi \rangle$  is an entangled state if (at least) *two of the coefficients  $\alpha_m$  are different from 0*.

The following of this demonstration is similar to those that are given in the literature within the framework of strict quantum physics (see Gisin 1991, for example), but with the difference that it is applicable to generalized systems (that is, there is no a priori reference to physics, like for example to the notion of spin).

The previous entangled state (with two non-zero coefficients) can always be written as:

$$\langle a, b | \Xi \rangle = \langle a, b | \Psi \rangle + \langle a, b | \Psi_\perp \rangle,$$

where  $\langle a, b | \Psi \rangle =_{\text{df}} \alpha_1 \langle a, \Xi^1_1 \rangle \langle b, \Xi^2_1 \rangle + \alpha_2 \langle a, \Xi^1_2 \rangle \langle b, \Xi^2_2 \rangle$ , and

$\langle a, b | \Psi_\perp \rangle =_{\text{df}} \sum_{m>2} \alpha_m \langle a, \Xi^1_m \rangle \langle b, \Xi^2_m \rangle$  are orthogonal functions of H.

That is, focussing on the state vectors for convenience of notation:

$$|\Xi\rangle = |\Psi\rangle + |\Psi_\perp\rangle.$$

By applying an appropriate unitary transformation to  $|\Xi\rangle$ , the entangled state  $|\Psi\rangle$  can always be considered as a vector of the sub-space  $h_1 \otimes h_2$  of  $H_1 \otimes H_2$  where  $h_1$  is spanned by the eigenstates  $|a+\rangle$  and  $|a-\rangle$  of an observable A with eigenvalues +1 and -1 defined on it, and  $h_2$  is

spanned by the eigenstates  $|b+\rangle$  and  $|b-\rangle$  of an observable B with eigenvalues +1 and -1 defined on it. The state  $|\Psi\rangle$  can then be written as<sup>11</sup>:

$$|\Psi\rangle = c_1 |a+\rangle|b-\rangle + c_2 |a-\rangle|b+\rangle,$$

with  $c_1^2 + c_2^2 = 1$  (normalization condition).

It is easy to show that if the action of the observables A, A' (on h1) and , B, B' (on h2) are defined as follows:

$$A |a+\rangle = |a-\rangle \text{ and } A |a-\rangle = |a+\rangle,$$

$$A' |a+\rangle = |a+\rangle \text{ and } A' |a-\rangle = - |a-\rangle$$

$$B |b+\rangle = (c_1 c_2 / 2) |b-\rangle + (|b+\rangle \text{ and } B |b-\rangle = (c_1 c_2 / 2) |b+\rangle - |b-\rangle.$$

$$B' |b+\rangle = (c_1 c_2 / 2) |b-\rangle - |b+\rangle \text{ and } B' |b-\rangle = (c_1 c_2 / 2) |b+\rangle + |b-\rangle,$$

the expectation values, in the state  $|\Psi\rangle$ , of the correlation factor R (J,  $|\Psi\rangle$ ) can be computed as:

$|R(J, |\Psi\rangle)| = 2 (1 + c_1^2 c_2^2) > 2$  for all non-zero  $c_1$  and  $c_2$ , and then for any entangled state  $|\Xi\rangle$  of  $H=H1 \otimes H2$  -since any entangled state  $|\Xi\rangle$  of H determines a state  $|\Psi\rangle$  of  $h1 \otimes h2$  and then a couple  $(c_1, c_2)$  of non-zero real numbers such that  $c_1^2 + c_2^2 = 1$ .

We have then shown that for any entangled, continuous state  $|\Xi\rangle$  of  $H=H1 \otimes H2$ , there exists a set J of observables such that  $|R(J, |\Xi\rangle)| > 2$ .

## V. Strategy of decision.

Taking into account the previous results, which have been shown for generalized systems and continuous variables observables, it is possible to decide the question of the nature of the psychophysical correlations. To do this, the sub-systems S1 and S2 are, respectively, identified

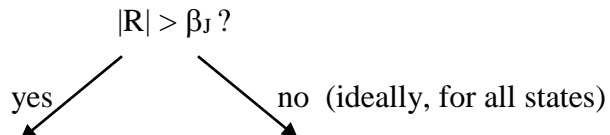
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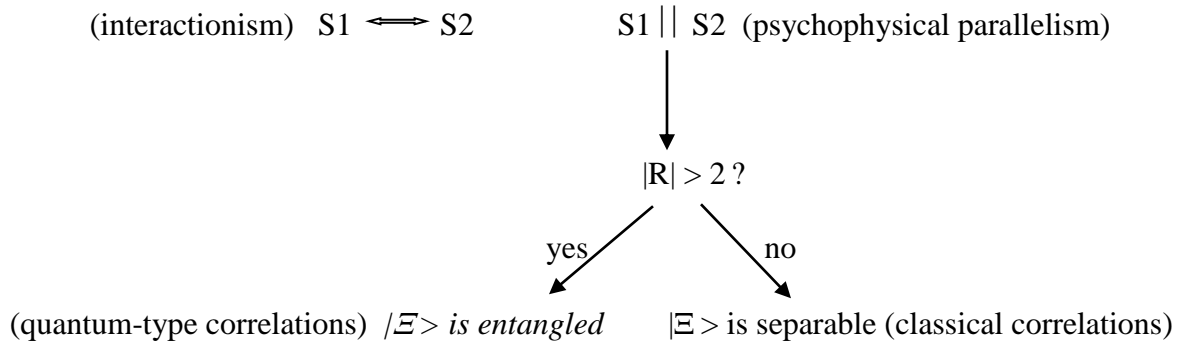
<sup>11</sup> This decomposition is, of course, mathematically similar with that of a singlet state for a pair of particles in physics.

with the somatic and the mental aspects of the individual (which defines the composed system  $S$ ) and we choose two couples of somatic and mental properties, respectively denoted  $(A, A')$  and  $(B, B')$ . On each of the individuals of a large group which are supposed to be “prepared” in the same state  $|\Xi\rangle$ , we have to (randomly) perform the joint measurement of one of the somatic properties  $(A, A')$  and of one of the mental properties  $(B, B')$ . Then the expectations values for all the outcomes of the possible couples of properties  $(A, B)$ ,  $(A, B')$ ,  $(A', B)$ ,  $(A', B')$  and the corresponding correlation factor  $R(J, |\Xi\rangle)$  can be calculated.

If  $|R(J, |\Xi\rangle)| > \beta_J$ , we can conclude, by the property (C) shown in section IV.1, that  $S1$  and  $S2$  “interact” in some way. If the latter condition is not satisfied *for a significant number of measurements, which involve the same set  $J$  of observables but different states  $|\Xi\rangle$* , we can suppose to be in presence of no-signalling systems and it becomes meaningful to use condition (D) to check whether the state  $|\Xi\rangle$  is entangled or not. Let us notice that if  $\beta_J$  is not strictly greater than 2, the second question has no meaning since all couples of observables of  $J$  commute and then we are in the classical case where all states are separable. This means that if we want to obtain a more complete response about the question of the nature of psychophysical correlations (that is, about signalling *and*, in case of negative response, about entanglement), this test must be performed with *complementary observables*. For example, we can consider the “conjugate” observables defined in section III.2 and for which  $\beta_J = \sqrt{5}$  (since in this case  $[A, B] [B', A'] = \text{Id}$ ).

This algorithm of decision for the question of the nature of psychophysical correlations *for complementary observables* ( $\beta_J > 2$ ) can be presented as follows, where, for convenience, the correlation factor  $R(J, |\Xi\rangle)$  for couple of complementary observables has been noted as  $R$ :





If it can be shown in a significant manner, that is, for a sufficient number of experiments involving different couples of somatic and mental observables, that the psychosomatic state of the individual must be represented by an entangled state of the state space  $H$ , this would mean that the psychosomatic domain requires to be explored according to a quantum-like approach like the one presented here<sup>12</sup> and that *all possible consequences* of this conclusion must be drawn. In particular, typically quantum effects in the psychosomatic domain, like interference effects, would be expected and detected by appropriate experiments.

Let us now briefly discuss the “practical problem” mentioned in section II.2. In order to decide the question of the nature of psychophysical correlations according to the strategy mentioned above, we have first to specify complementary couples of observables in each of the somatic and the mental domain whose values can be measured with accuracy. Testing the previous condition on couples of observables which seem to be *correlated* (according to existing data) will, of course, have more chance to give significant results. We can, in particular, consider hormonal processes in as much as endocrinology provides us with very accurate data relative to the double functionality (mental and somatic) of these substances produced by the endocrine glands: hormonal processes govern bodily processes, like functioning of organs or immune processes, *and* mental processes, like emotional sensibility or depression (Griffin and Ojeda

2004). For example, it has been observed that thyroid hormones T3 and T4 govern both metabolism and emotional sensibility and that hyperthyroidism gives rise to somatic symptoms, like fast heart rate or muscles weakness, *and* psychological symptoms, like irritability, lack of motivation or depression. These observations suggest that we can take as “good” observables in the somatic domain the concentration of these hormones in the blood, that of the white cells (involved in the immune system) or the muscular tonus and their conjugate observables (defined as in section III.2); and in the mental domain, we can measure the degree of emotional sensibility (Riggio et al. 1985), of irritability -for which a scale has been proposed (Stringaris et al. 2012)- or of depression -for example, by using the questionnaire called Hamilton Rating Scale for Depression (Hedlund and Viewig 1979)- and their conjugate observables (which involve their rate of change but are not identical to them, as shown in section III.2).

Another difficulty of this experimental test is that the previous decision process relies on *statistical* tests, which supposes to “prepare” a large population of individuals in the same psychosomatic states and seems at first glance rather difficult. However, the decision process proposed here has been established *by focussing on specific couples of observables* (those involved in R, the correlation factor), *independently of the other somatic or mental features* of the global psychosomatic state of the individual. Consequently, the proposed decision process will be significant under the only condition that we have “prepared” the individuals on which the statistical test is performed in the same psychosomatic state *regarding these selected observables*. This condition can be fulfilled by selecting, among all possible observables, only those that have approximately the same statistical distribution of their values for all the individuals, which could then be realised by choosing a group of individuals with very close heart rate, blood pressure or

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<sup>12</sup> As mentioned in section I (introduction) and in section III.1 and III.2, other interesting

concentrations of some specific hormones *and*, regarding the mental features, with very close temperaments –which can be evaluated by referring to some known psychosomatic classification (see, for example, Janssen 2006, chap 5).

### **Conclusion.**

It seems that the long-standing and still unresolved problem of the nature of psychophysical correlations, which has already benefited from some rigorous philosophical analysis showing how problematic is the concept of “psychophysical causation”, can be decided within the quantum paradigm. For, the typically quantum concepts (complementarity and entanglement) and the formalism in which they can be rigorously expressed provide us with powerful tools, which can be used to tackle this difficult question. We could say, in some sense, that the proposed strategy of decision for this question is “conditioned” by the quantum-like concepts and the associated quantum formalism that are used in this strategy. However, as shown by history of science, it is always the case that proposing a new theory, deciding a question or discovering a new particle relies on the current paradigm, which indeed makes possible the emergence of this theory, the formulation of this response or the discovering of this new particle. In the present case, we can conjecture that the question about the nature of psychophysical correlations has found an appropriate framework in which it be decided, namely, the quantum paradigm with its powerful formalism wherein, in particular, the concept of “entanglement” and its rigorous formulation can be used to describe the psychophysical correlations without appealing to the problematic concept of psychophysical causality.

More specifically, the proposed strategy of decision relies on a statistical test, namely, an evaluation of the psychophysical correlation factor involved in two Bell-type conditions, which

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quantum-like approaches have been developed according to the same direction of research.



have been shown within the framework of a generalised quantum theory and for continuous variables observables. If this test shows that, for a significant number of different sets of complementary observables  $J$  and for different psychosomatic states, the CHSH correlation factor lies in the interval  $[2, \beta_J]$ , where  $\beta_J$  the maximal value of this correlation factor computed under the no signalling assumption, it can be concluded on experimental basis that *the psychosomatic correlations must be understood in terms of entanglement relation*. This means that it would not be any more possible to think of the individual as the place where physiological processes and mental processes “interact” (like in Descartes’ interactionnist position), neither as a material system from which has “emerged” mental properties capable, in turn, to “act” on it. This result would give reason to the line of thought developed for a long time by ancient Greek physicians like Hippocrate, by great philosophers like Spinoza and, more recently, by psychosomaticians like Alexander and by the contemporary, prominent scientists mentioned in this paper and who can be considered as the precursors of this quantum-like approach. In this case, the individual must be conceived of as an indivisible totality, which parallel, non-“interacting”, physiological and mental processes are nevertheless strongly correlated according to a strictly defined, quantum-like entanglement relation.

The forthcoming result of this statistical test can thus be considered of great importance for deciding the long-standing question of the nature of psychophysical correlations. Meanwhile, let us notice that the mathematical representation of psychosomatic states by entangled states within a generalized version of quantum theory has already given rise to fertile applications, like explicative and predictive models of the placebo effect and of some psychiatric disorders (Uzan 2014). The quantum-like representation of the psychosomatic correlations can also provide very fertile intuitions about their origin and then about that of many (if not all) somatic and mental

dysfunctions, since it clearly suggests that this origin has to be found in the “preparation process” of the system under investigation, namely, the individual -that is, in the ontogenesis and the phylogenies. This idea seems to find a clear confirmation in some authors who have suggested, on the basis of evolutionary considerations, that the individual has been progressively prepared to face with the primordial emotional conflicts by implementing biological “programs” capable to manage them (Denton 2005) (Hamer 2005).

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