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An improved Hyperstatic Reaction Method for tunnels 1 under seismic loading 2 3 Qiangqiang Sun^a, Dianchun Du^{a,b}, Daniel Dias^{a,c} 4 ^a Laboratory 3SR, CNRS UMR 5521, Grenoble Alpes University, Grenoble, 38000 France 5 ^b School of Civil Engineering, Southeast University, Nanjing, 211189 China 6 ^cAntea Group, Antony 92160, France 7 8 Abstract: Hyperstatic Reaction Method (HRM) is a simple but effective method with 9 attractive features in calculating internal forces and deformations of the tunnel lining. The 10 superior ability of the HRM method under static conditions inspires the further development of this method for tunnel seismic design. This paper implements an improved HRM method 11 12 for estimating seismic-induced internal forces in the tunnel lining in a quasi-static condition. 13 To this end, a modification parameter (a) for approximating the applied active load as well as 14 a varying spring stiffness factor (β) for characterizing the soil-tunnel interaction is 15 incorporated into the existing HRM method. The values of two dimensionless parameters are 16 calibrated through an analytical solution for a wide range of the soil and tunnel lining 17 properties. The efficiency of the improved HRM method is verified against the analytical 18 solution and numerical analysis in several benchmark cases. The comparisons show that the 19 improved HRM method can be used for the preliminary seismic design of tunnels. 20 21 Keywords: Circular tunnel; Hyperstatic Reaction Method; Seismic design; Analytical 22 solution 23 24 25 26 27 28

29 1. Introduction

30 Tunnels are an important part of the infrastructure of modern society and their applications continue to expand in high earthquake risk regions. The severe damages to 31 tunnels in strong earthquakes have demonstrated that the seismic design demand of tunnels 32 33 should be adequately considered (Hashash et al., 2001; Sun et al., 2016). To guide the tunnel 34 seismic design, many efforts have been made by a series of analytical solutions (Bobet 2010; 35 Park et al., 2009; Penzien and Wu, 1998; Zou et al., 2017), numerical analyses (Amorosi and 36 Boldini, 2009; Fabozzi et al., 2018; Patil et al., 2018; Shahrour et al., 2010; Sun and Dias, 37 2019a, 2019b; Sun et al., 2019, 2020; Yu et al., 2016) and experimental studies (Cilingir and 38 Madabhushi, 2011; Bilotta et al., 2014; Chen et al., 2017, 2020).

39 Analytical solutions are very attractive tools for the preliminary seismic design, as 40 they provide easy and fast calculations of internal forces and deformations of the tunnel lining. 41 The analytical procedures generally consider a homogeneous isotropic soil medium subjected 42 to a seismically induced uniform shear strain field, but they disregard any inertial interaction. 43 The soil-tunnel interaction is often assumed as the no-slip and full-slip conditions except Park 44 et al. (2009). Oppositely, numerical analysis is a higher-level and more complex approach for 45 calculating seismic design loads of tunnels. It generally needs careful consideration of many 46 inherent or man-made uncertainties in the numerical simulations (Hashash et al., 2005; Lu 47 and Hwang, 2017; Sun and Dias, 2018), including the model boundary, the mesh size, the soil constitutive models, and the selection of input ground motions, etc. Most of all, it is difficult 48 49 to, a-priori, quantify the degree of the slippage between the soil and the tunnel in a numerical 50 model.

The analysis of a tunnel lining should consider the soil-tunnel interaction, which is a fundamental component to calculate the internal forces develop in the tunnel lining. A simple but effective way of achieving this is to use the Hyperstatic Reaction Method (HRM) (CDTA 2005; Do and Dias, 2018; Duddeck and Erdmann, 1985; Einstein and Schwart, 1979; ITA 2000; Leca and Clough, 1992). This method requires defining the active loads that are directly applied to the tunnel lining by the soil mass; further passive loads are due to the 57 displacement of the tunnel lining because of the soil-tunnel interaction. The HRM method is 58 developed in the Matlab software within the FEM framework, to calculate the internal forces 59 of tunnels in a homogeneous medium. In comparison with traditional numerical analyses, the 60 HRM method provides a fast and accurate prediction with less computational cost (Oreste 61 2005, 2007). These attractive features greatly promote its application to calculate the lining 62 forces of various tunnels, e.g., segmental tunnel (Do et al., 2014, 2015, 2018), U-shaped 63 tunnel (Du et al., 2018a, 2018b), sub-rectangular tunnel (Do et al., 2020), and shotcrete lining 64 (Oreste et al., 2018, 2019). The HRM method allows a great deal of analyses to be performed 65 in a very short time, thus it is an appropriate method for the tunnel probabilistic analysis (Kroetz et al., 2018; Oreste, 2005) and optimization design (Du et al., 2020). 66

67 This paper aims at presenting an improved HRM method for estimating seismic-68 induced internal forces of a deep circular tunnel in homogeneous isotropic soil medium. The 69 paper begins with a brief presentation of the mathematical formulation of this method. Then, 70 the numerical procedure to implement this method under seismic conditions is presented. In 71 this work, two dimensionless parameters are introduced respectively to modify the applied active loads and the soil-tunnel interaction. Next, the formulas of the two parameters are 72 73 presented by calibrating the HRM solution with an analytical solution. Finally, extensive 74 validations of the improved HRM method are conducted in several benchmark cases.

75 2. Hyperstatic reaction method

76 2.1 Mathematical formulation

The calculation scheme of support structures with the HRM method is illustrated in Fig. 1. In the HRM method, the structure is subdivided into a finite number of linear elements. Those elements interact with each other through nodes and they are connected to the soil through normal springs. Therefore, the interaction between the soil and the tunnel support is simulated through the normal springs and active loads. The normal springs allow the reaction produced by the soil to be simulated when the support, which deforms under the applied active loads, moves towards the soil. The tangential spring at the soil-tunnel interface is not 84 considered herein due to the difficulty in determining the maximum shear reaction pressure.

85 The no-slip condition permits to consider the most unfavorable internal force scenario86 (Hashash et al., 2001).

87



88

93

Fig. 1. Calculation scheme of support structures with the HRM method under static conditions. With: σ_v : the vertical loads; σ_h : the horizontal loads; k_n : the stiffness of springs; *R*: the tunnel radius; α_i : the angle of inclination of the element (*i*) with respect to the horizontal; *EJ* and *EA*: bending and normal stiffness of the support; *X* and *Y* are the global Cartesian coordinates.

Fig. 2 illustrates the scheme of a beam-type element, which can develop internal forces.
Since the structure interacts with the soil by the normal springs distributed over the nodes,
one can obtain the stresses on each element once the displacements of the nodes are known.
The unknown displacements can be achieved by defining the global stiffness matrix of the
whole structure elements as well as the connections to the surrounding soil.



99

100 101

Fig. 2. A finite element under the local Cartesian coordinates: *h*: initial node; *j*: final node; θ : rotation; *x* and *y*: local Cartesian coordinates; *v*: transversal displacement; *u*: axial displacement; *L*_i: element length.

102

103 The global stiffness matrix K is assembled by the local stiffness matrix k_i of the i^{th} 104 element in the global Cartesian reference system. In the static analysis, only half of the

structure is considered because of the tunnel center symmetry with respect to the vertical axis. 105

106 The global stiffness matrix *K* is given as follows:

where the terms $k_{(n-1)(n-1)}^{(n-1)}$, $k_{(n-1)n}^{(n-1)}$, $k_{n(n-1)}^{(n-1)}$ and $k_{nn}^{(n-1)}$ represent the 3×3 sub-matrices of 108 the local stiffness matrix k_{n-1} of the $(n-1)^{th}$ element under the global Cartesian coordinates. 109 The local stiffness matrix k_i of the i^{th} element under the global Cartesian reference system can 110 111 be obtained as follows:

112

$$[k]_i = \lambda_i^T \cdot \overline{[k]_i} \cdot \lambda_i \tag{2}$$

where \bar{k}_i is the local stiffness matrix under local Cartesian reference system and λ_i is the 113 transformation matrix respectively: 114

$$115 \qquad \qquad \overline{[k]_{l}} = \begin{bmatrix} \frac{EA}{L_{i}} & 0 & 0 & -\frac{EA}{L_{i}} & 0 & 0 \\ 0 & \frac{12EI}{L_{i}^{3}} & \frac{6EI}{L_{i}^{2}} & 0 & -\frac{12EI}{L_{i}^{3}} & \frac{6EI}{L_{i}^{2}} \\ 0 & \frac{6EI}{L_{i}^{2}} & \frac{4EI}{L_{i}} & 0 & -\frac{6EI}{L_{i}^{2}} & \frac{2EI}{L_{i}} \\ -\frac{EA}{L_{i}} & 0 & 0 & \frac{EA}{L_{i}} & 0 & 0 \\ 0 & -\frac{12EI}{L_{i}^{3}} & -\frac{6EI}{L_{i}^{2}} & 0 & \frac{12EI}{L_{i}^{3}} & -\frac{6EI}{L_{i}^{2}} \\ 0 & \frac{6EI}{L_{i}^{2}} & \frac{2EI}{L_{i}} & 0 & -\frac{6EI}{L_{i}^{2}} & \frac{4EI}{L_{i}} \end{bmatrix}$$

$$116 \qquad \qquad \lambda_{i} = \begin{bmatrix} \cos \alpha_{i} & \sin \alpha_{i} & 0 & 0 & 0 & 0 \\ -\sin \alpha_{i} & \cos \alpha_{i} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos \alpha_{i} & \sin \alpha_{i} & 0 \\ 0 & 0 & 0 & -\sin \alpha_{i} & \cos \alpha_{i} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(4)$$

117 After knowing the global stiffness matrix K, the unknown vector of nodal displacement $S = [S_1, S_2, ..., S_n]^T$ under the global Cartesian reference system can be determined by the 118 119 following relation:

120
$$[K] \cdot [S] = [F] \tag{5}$$

121 where $F = [F_1, F_2, ..., F_n]^T$ is the vector of the nodal forces applied to the numerical model, 122 such as the external active loads applied to the lining. Note that $S_1, S_2, ..., S_n$ are the sub-123 vectors composed of three displacements of each node, respectively; $F_1, F_2, ..., F_n$ are the 124 sub-vectors composed of three external forces applied to each node, respectively. Once the 125 vector *S* is calculated, a conversion of nodal displacements at the local reference system of 126 the element is easily calculated. The characteristic of nodal stresses can then be determined 127 through the local stiffness matrix.

A pressure-displacement relation is adopted to describe the interaction between the soil and the tunnel. Unlike other methods, in which the normal stiffness is set as a constant value (i.e., a Mohr-Coulomb type), Oreste (2007) introduced a nonlinear relationship between the reaction pressure and the support normal displacement (Fig. 3):

132
$$p = p_{lim} \cdot \left(1 - \frac{p_{lim}}{p_{lim} + \eta_0 \cdot \delta}\right) \tag{6}$$

where p_{lim} is the maximum reaction pressure that the ground can offer, and η_0 is the initial stiffness of the ground. This relation represents the simplest way to describe the behavior of the ground when the initial stiffness and the limit pressure are known with a certain confidence. By performing a plate load test in the ground, it is possible to note a curve loaddisplacement that is very similar to the hyperbolic one (AFTES, 1997; Do et al., 2014).





Fig. 3. Nonlinear relationship between the reaction pressure *p* and the support normal displacement δ : η_0 : initial spring stiffness; p_{lim} : maximum reaction pressure.

142

143 The apparent stiffness η^* of the ground is given by the p/δ ratio that can be calculated 144 at each node of the support structure:

145
$$\eta^* = \frac{p_{lim}}{\delta} \cdot \left(1 - \frac{p_{lim}}{p_{lim} + \eta_0 \cdot \delta}\right) \tag{7}$$

The ground reaction generally depends on the ground elasticity parameters and tunnel
radius (AFTES, 1997; Oreste, 2007). In this study, the following formula is applied to
determinate the initial normal stiffness of the ground (Do et al., 2014; Möller, 2006):

149
$$\eta_0 = \beta \frac{1}{1+\nu_s} \cdot \frac{E_s}{R}$$
(8)

150 where v_s and E_s are respectively the Poisson's ratio and Young's modulus of soil; R is the 151 tunnel radius; β is a dimensionless factor.

152 For non-cohesion soil, the maximum reaction pressure p_{lim} can be calculated based on 153 the friction angle φ , Poisson's ratio v_{s} , and active loads (Do et al., 2014; Oreste, 2007; Vu et 154 al., 2017):

155
$$p_{lim} = \frac{1+sin\varphi}{1-sin\varphi} \cdot \Delta\sigma_{conf}$$
(9)

where $\Delta \sigma_{conf}$ is the confining pressure that acts on the tunnel perimeter. It can be defined by the following equation:

$$\Delta\sigma_{conf} = \frac{\sigma_h + \sigma_v}{2} \cdot \frac{v_s}{1 - v_s} \tag{10}$$

159 The normal stiffness of each spring can then be given by the formula:

160
$$k_{i} = \eta_{i}^{*} \cdot \left[\frac{L_{i-1} + L_{i}}{2}\right] = \frac{p_{lim}}{\delta_{i}} \cdot \left(1 - \frac{p_{lim}}{p_{lim} + \eta_{0} \cdot \delta_{i}}\right) \frac{L_{i-1} + L_{i}}{2}$$
(11)

Using Eqs. 1 to 11, the internal forces in the tunnel lining are therefore calculated through the nodal displacements and the local stiffness matrices of each element. It should be emphasized that the normal springs will disappear in zones in which the support structure moves towards the tunnel. Therefore, only compressive loads are possible in the normal direction, in which the tunnel support moves towards the ground. More details of the HRM method can be found in the literature (Do et al., 2014, 2015; Du et al., 2018a, 2018b, 2020; Duddeck and Erdmann, 1985; Leca and Clough, 1992; Oreste 2005, 2007; Vu et al., 2017).

168 2.2 HRM method under seismic conditions

169 Analytical solutions assume the ovaling deformation of the cross-section is the most 170 critical deformation of circular tunnels during a seismic event (Hashash et al., 2001; Lu and 171 Hwang, 2017; Sun and Dias, 2019), as illustrated in Fig. 4a. Since the inertia effect of a deep 172 tunnel is relatively small (i.e., dynamic interaction can be ignored), the ovaling deformation can further be simplified as a pseudo-static case in a two-dimensional plane strain condition 173 174 (Bobet, 2010; Park et al., 2009). Hence, the seismic-induced stress and deformations can easily be predicted when the external shear stress is applied to the far-field, as shown in Fig. 175 4b. This stress state is equivalent to a compressive and tensile free-field stress at 45° with the 176 direction of pure shear, as presented in Fig. 4c. 177



The acting shear stress τ , can be calculated by (Penzien and Wu, 1998):

179
$$\tau = \frac{E_s \gamma_{max}}{2(1+\nu_s)} \tag{12}$$

180 where γ_{max} is the maximum shear strain at the position of tunnel axis.

181



182

183 Fig. 4. Transversal response in 2D plane strain conditions (a) ovaling deformation; (b) corresponding seismic shear

184 loading; (c) equivalent static loading; (d) equivalent static loading with the HRM method.

185

186 As previously mentioned, the internal forces in the tunnel lining are influenced by the

187 exact evaluation of the active loads (σ_h and σ_v) and the definition of the initial stiffness of the

188 normal springs (η_0). In the HRM method, the normal springs can only work in compression 189 Compressive active loads applied to the tunnel in one direction will automatically produce 190 tensile loads in a perpendicular direction. This leads to the shear stress that acts on the lining 191 is not identical to the one in a quasi-static condition. Therefore, a parameter (*a*) is necessary 192 to be adopted to modify the active loads that are applied to the tunnel lining under seismic 193 conditions, as shown in Fig. 4d.

In previous static analyses, the value of β was usually set to 1 (Mashimo and Ishimura, 2005; Molins and Arnau 2011) or 2 (Do et al., 2014; Vu et al., 2017). For a given case, using a constant value of β is reasonable. However, this hypothesis is questioned when a wide range of properties of the soil and tunnel lining is considered, since the value of β is related to the properties of the soil and tunnel lining. In this work, a varying dimensionless factor (β) is thus utilized to realistically describe the soil-tunnel interaction.

To determine the values of two dimensionless parameters, a thorough parametric analysis is then required. To this end, the results obtained from the HRM method are verified against the ones calculated by an analytical solution (Bobet, 2010), considering a wide range of practical properties of the soil and tunnel lining. The quasi-static numerical model is not adopted in this study since there is no reference to calibrate the model. This consideration allows the accuracy of the improved HRM method is comparable to the analytical solution.

3. Numerical implementation

In this section, the assumptions of the improved HRM method are first summarized.
Following that, the numerical procedure to implement the improved HRM method with the
help of parametric analysis is presented.

210 *3.1 Underlying assumptions*

Since the analytical solution is adopted to calibrate two dimensionless parameters, the
improved HRM method inevitably shares the same underlying assumptions or limitations of
analytical solutions (Bobet, 2010; Park et al., 2009; Penzien and Wu, 1998):

214	•	Two-dimensional plane strain models condition is assumed, considering the
215		ovaling deformation of the tunnel's cross-section is the most critical mode.
216	•	A homogenous, isotropic and linear elastic half-space is considered.
217	•	The tunnel lining is represented as a cylindrical shell with thin walls.
218	•	The stress history in the medium and the lining is not considered and the tunnel
219		excavation is assumed to take place simultaneously with the installation of the
220		lining.
221	•	The external forces are applied under fully drained conditions, ignoring any
222		inertial interaction.
223		

224 3.2 Numerical procedure

To implement the HRM method under seismic conditions, the formulas of the two dimensionless parameters need to be determined. Extensive comparisons between the seismic-induced internal force increments predicted by the HRM method and the ones calculated by the Bobet's solution are performed.

229 In the initial calibration phase, the parameters of the tunnel lining are as follows: 230 Young's modulus $E_l = 24.8$ GPa and Poisson's ratio $v_l = 0.2$. The tunnel radius (R) varies from 231 3 m to 5 m according to engineering practice. A larger tunnel radius is not adopted since the 232 analytical solution accuracy is not acceptable (Hashash et al., 2001, 2005). Concerning the 233 lining thickness (t), the relative thickness t/R ratio varies from 1/15 to 1/10 to consider the 234 commonly used values in practice. The relative thickness utilized satisfies the assumption of 235 the analytical solutions (i.e., the thickness should be relatively small in comparison with the 236 tunnel diameter). For the soil parameter, Poisson's ratio $v_3=0.25$ is adopted, while Young's 237 modulus (E_s) varies parametrically from 1 to 500 MPa to capture a wide range of soil 238 conditions (i.e., from very soft to hard soils). It should be noted that for the HRM method, the 239 friction angle of soil should be defined to calculate the maximum reaction pressure (Eq. 9). A low p_{lim} will prevent the development of the normal stress at the soil-tunnel interface, whereas 240 the analytical solution allows normal stress increases continuously with tunnel deformation 241

242 (i.e., linear relation). Thus, the upper limit value of the friction angle of soil (i.e., about 50°) is 243 adopted to produce sufficient reaction pressure. Although this work is academic research, the 244 soil and tunnel parameters are taken from the literature (Bobet, 2010; Du et al., 2020; 245 Hashash et al., 2001, 2005; Park et al., 2009). A value of γ_{max} =0.00252 is adopted in the initial calibration phase, which is identical to the one used in Hashash et al. (2001). Once the input 246 247 soil and tunnel lining parameters are defined, the calibration work of the two parameters (i.e., 248 a and β) can then be performed. The main schemes to calibrate two parameters are presented 249 in Table 1 and Fig. 5.



250

251

Fig. 5. Calibration flowchart of the two parameters.

253 Table 1 Overview of the calibration process. Step Description 1 Generate the input soil and lining parameters $\{t_i, R_i, E_{si}\}$ from the defined parameter ranges. 2 Select a pair of parameters $\{t_l, R_l, E_{sl}\}$, then respectively compute the corresponding seismic-induced axial force and bending moment $\{T_B, M_B\}$ using Bobet's solution, and the initial values of $\{T_H, M_H\}$ using HRM method on the basis of $a=\beta=1$. 3 Calculate the relative error between the two methods. 4 If $e_T \leq 0.02$ and $e_M \leq 0.02$, output a and β , otherwise, update these two parameters until the target accuracy is satisfied. 5 Repeat steps 2 to 4 until all cases are considered. 6 Obtain the optimal formulas of a and β as the functions of t_i , R_i , E_{si} using program SigmaPlot, with the maximum coefficient of determination (i.e., close to 1). 254

255 Figs. 6 and 7 present the calibration results of parameters a and β for various values 256 of E_s . In Fig. 6, the representative results of t/R equals to 1/10 and 1/15 and tunnel radius 257 equals to 3 m and 5 m are shown, while Fig. 7 presents the results for different ratios of t/R258 and E_s . It shows that both the values of the parameter β and the parameter *a* increase with 259 increasing value of soil Young's modulus, until peak amplitude is reached. A further increase 260 in the value of E_s leads to a decrease in the values of parameters a and β . Besides, the same 261 t/R ratio predicts identical values of a and β . Fig. 7 further shows that the maximum values of 262 the parameter a and β increase with the decrease of the t/R ratio. Regardless of the individual impact of thickness or radius, the values of two parameters are only influenced by the relative 263 264 thickness t/R and soil Young's modulus E_s .









272 After all the calibration works are completed, the relationships between the two 273 parameters with the soil and tunnel lining properties can be established, with the help of the 274 data analysis software SigmaPlot. The optimal functions are selected based on the goodness 275 of fit (i.e., the coefficient of determination is close to 1). The parameter β can be given by:

276
$$\beta = a_1 + b_1 \cdot c_1^{E_s} + d_1 \cdot E_s$$
(13)

277 while:

278
$$a_1 = 8.4 + 2.1 \cdot \cos(55.7 \cdot \frac{t}{R} - 2.8)$$
 (14)

279
$$b_1 = -6.9 + 2.0 \cdot \cos(56.4 \cdot \frac{t}{R} + 0.3)$$
 (15)

280
$$c_1 = -79.7 + 80.7 \cdot \left[1 - e^{\left(-281 \cdot \left(\frac{t}{R}\right) + 3079 \cdot \left(\frac{t}{R}\right)^2 - 11764 \cdot \left(\frac{t}{R}\right)^3 \right)} \right]$$
(16)

281
$$d_1 = -0.014 + 0.01 \cdot \cos(40.3 \cdot \frac{t}{R} + 2.1)$$
(17)

The coefficients a_1 , b_1 , c_1 , and d_1 are expressed as the functions of the *t/R* ratio, as shown in Figs. 8a~d and their best fitting solutions are presented in Eqs. 14~17. As for the parameter *a*, it is also a function of the relative thickness and Young's modulus of soil. It is further demonstrated that the parameter *a* can be normalized by the parameter β , as follows:

286
$$a = \beta / (a_2 + b_2 \cdot c_2^{E_s} + d_2 \cdot E_s)$$
(18)

The relationships between the coefficients a_2 , b_2 , c_2 , and d_2 and t/R ratios are respectively shown in Figs. 8e~h and the corresponding solutions are presented in Eqs. 19~22:

290
$$a_2 = 1.4 - 0.4 \cdot \left[1 - e^{\left(98 \cdot \left(\frac{t}{R}\right) - 1159 \cdot \left(\frac{t}{R}\right)^2 + 3615 \cdot \left(\frac{t}{R}\right)^3\right)} \right]$$
(19)

291
$$b_2 = 0.3 + 0.7 \cdot \left[1 - e^{\left(88\left(\frac{t}{R}\right) - 1091 \cdot \left(\frac{t}{R}\right)^2 + 3751 \cdot \left(\frac{t}{R}\right)^3\right)} \right]$$
(20)

292
$$c_2 = -37.7 + 38.7 \cdot \left[1 - e^{\left(-250\left(\frac{t}{R}\right) + 2444 \cdot \left(\frac{t}{R}\right)^2 - 8135 \cdot \left(\frac{t}{R}\right)^3\right)} \right]$$
(21)

293
$$d_2 = 0.005 + 1 \cdot \left[1 - e^{\left(\frac{t}{R} - 16 \cdot \left(\frac{t}{R}\right)^2 + 75 \cdot \left(\frac{t}{R}\right)^3\right)} \right]$$
(22)
294





297

Fig. 8. Coefficients fitting curves for the formulas of the parameters a and β .

298 Once the formulas of two parameters are determined, it is necessary to examine their 299 efficiencies. Thus, the values of parameters β and β/a computed by the proposed formulas are 300 compared respectively with the true values (i.e., obtained during the calibration process), as 301 shown in Fig. 9. These analyses correspond to t/R=1/10, 1/12, 1/14, and 1/15, R=3 m, and 302 various E_s values. The comparison results indicate that the proposed formulas can represent 303 the values of parameters β and β/a . The comparisons for other cases also show good 304 agreement, and the results are not presented for the sake of brevity. Furthermore, Fig. 10 305 compares the seismic internal force increments that are calculated respectively by the 306 analytical solution and the HRM method (using the proposed formulas). These analyses correspond to t/R=1/10, 1/12, and 1/15, E_s varies from 1 to 500 MPa, and R=3 and 5 m. It 307 308 demonstrates that the proposed formulas of two parameters have the capability for predicting 309 the true tunnel lining forces.







 $(t/R=1/15\sim1/10).$





Fig. 10. Comparison of the axial forces and bending moments calculated by the proposed formulas and the

ones calculated by the analytical solution (R=3 and 5 m).

319 4. Method validation

In this section, extensive validations are performed to demonstrate the effectiveness of the improved HRM method. The first validation attempts to examine the accuracy of the improved HRM method using different properties of the soil and tunnel lining, which are not considered in section 3. Then, two academic cases that are taken from the literature are respectively used in validations 2 and 3. In each validation, the improved HRM method is compared with the analytical solution or/and the numerical analysis.







328

329

Fig. 11. Effect of the maximum shear strain on the calculated lining forces (R=3 and 5 m).

330 The formulas of parameters *a* and β are proposed in the case of $\gamma_{max}=0.00252$. The 331 comparison between the improved HRM method and the analytical solution in terms of the 332 maximum axial force and bending moment is performed for different values of γ_{max} . The 333 amplitude of γ_{max} varies parametrically from 0.001 to 0.005, in the case of t/R=1/10, E_s varies 334 from 1 to 500 MPa, and R=3 and 5 m. The other soil and tunnel lining properties are identical to the ones used in section 3. As shown in Fig. 11, the maximum axial force and the bending 335 336 moment calculated by the improved HRM method coincide well with the analytical solution. Two methods capture the same trend: axial forces and bending moments increase with 337 increasing applied shear strains. It indicates that the improved HRM method can be 338 339 successfully applied in a larger strain range, although the proposed formulas correspond to 340 γ_{max}=0.00252.

341





Fig. 12. Effect of the lining Young's modulus on the calculated lining forces (R=3 m).

345 The validation is also performed for the varying Young's modulus of tunnel lining 346 (E_l) . The value of E_l varies from 20 to 40 GPa, which corresponds to the commonly used 347 concrete strength. The analysis corresponds to $\gamma_{max}=0.00252$, t/R=1/10, and R=3 m. The other 348 soil and lining properties are the same as the ones used in section 3. Fig. 12 shows the 349 comparison for different E_l values in terms of maximum axial force and bending moment. The 350 maximum axial forces from the HRM method result in an almost perfect match with the 351 Bobet's solution, whereby the differences are within 5% for all three cases. Compared to the 352 analytical solution, the improved HRM method also accurately predicts the maximum 353 bending moments, particularly for the cases of $E_l=20$ and 24.8 GPa. For the value of E_s varies from 100 to 500 MPa, the improved HRM method tends to slightly underestimate the 354

calculated bending moment in the case of E_l =40 GPa, with the maximum difference is about 9%.

The comparison of the improved HRM method with the Bobet's solution for a wide 357 358 range of soil Poisson's ratios is performed, as shown in Fig. 13. The analysis corresponds to the value of v_s varies from 0.1 to 0.4, in the case of $\gamma_{max}=0.00252$, t/R=1/10, and R=3 m. The 359 360 other soil and lining properties are the same as the ones used in section 3. The axial forces 361 calculated by the improved HRM method and the Bobet's solution both decrease with the 362 increase of ν_s value. The maximum error of the axial force is around 7%. Concerning the 363 bending moment, the improved HRM method matches well with the analytical solution when 364 the soil Young's modulus is less than 300 MPa. Further increases in the value of E_s , a 365 relatively larger discrepancy with a maximum error up to 15% occurs. However, the soil 366 Poisson's ratio has an unimportant effect on the calculated bending moments as compared to the applied shear strain and Young's modulus of the tunnel lining, due to the relatively low 367 368 computed bending moments.

369





Fig. 13. Effect of Poisson's ratio of soil on the calculated lining forces (*R*=3 m).

372 *4.2 Validation 2*

The numerical examples in Hashash et al. (2001) are employed to further verify the accuracy of the improved HRM method. The lining forces calculated by the improved HRM method are compared with the quasi-static numerical model and Bobet's solution respectively.

The properties of soil and lining are presented in Table 2.

377

2	7	a
3	1	Õ

 Table 2 Soil and tunnel lining properties (Hashash et al., 2001).

	Properties	Value
	Soil parameter	
Case 1	Young's modulus, <i>Es</i>	312 MPa
	Poisson's ratio, Vs	0.3
Case 2	Young's modulus, Es	312 MPa
	Poisson's ratio, Vs	0.49
Case 3	Young's modulus, Es	185.4 MPa
	Poisson's ratio, Vs	0.49
	Lining parameter	
	Young's modulus, E_l	24.8 GPa
	Tunnel thickness, t	0.3 m
	Tunnel radius, R	3 m
	Poisson's ratio, v_l	0.2

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380 Before going into the detailed comparison, more details about the numerical model 381 should be presented for clarity. A two-dimensional quasi-static numerical model is built using 382 the finite difference code FLAC (Itasca, 2011). The numerical model is 50 m wide and 50 m 383 high while the circular tunnel is placed at the numerical model center, as illustrated in Fig. 14. 384 The model dimensions and tunnel burial depth adopted here are enough to get rid of the 385 boundary effects (Lu and Hwang, 2017; Zou et al., 2017). The mesh size is 1×1 m and the 386 mesh around the tunnel is refined. Before the quasi-static numerical analysis, a static analysis is conducted to determine the initial stress in the lining. The displacements in both directions 387 388 along the model bottom are restricted and the horizontal displacements are also fixed for the 389 lateral boundaries.

A deformation based quasi-static numerical model is used in this study, in which the seismic-induced ovaling deformation is applied to the model considering inverted triangular displacements along the two lateral boundaries of the model and a uniform lateral displacement at the model's top boundary (Lu and Hwang, 2017; Sun and Dias, 2019a). For the quasi-static analysis, the horizontal and vertical displacements are fixed along the bottom and two lateral boundaries. The vertical displacement along the top boundary is also fixed. The displacement applied method and boundary conditions used here are consistent with the assumptions of analytical solutions (Bobet, 2010; Park et al., 2009). The shear strain γ_{max} =0.00252 is adopted, which corresponds to a maximum displacement applied along the model boundary equal to 0.126 m.



Fig. 14. Quasi-static numerical model.

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404 Spring-type interface elements of zero thickness are used to model the soil-tunnel 405 interaction condition. Based on the sensitivity analysis, the normal stiffness of spring with a value of $K_n = 10^{10}$ N/m³ is used in this study (Itasca, 2011). The difficulty is to select a proper 406 407 shear stiffness value (K_s) which associates with the soil-tunnel slippage. Thus, a parametric 408 analysis is performed considers a range of values of shear spring stiffness. Fig. 15 compares 409 the axial forces and bending moments calculated by the quasi-static numerical model with 410 different values of K_s , corresponding to soil and tunnel properties shown in Table 2 (case 1). 411 The lining forces predicted by the improved HRM method and the Bobet's solution are also 412 presented. The shear spring stiffness has a pronounced influence on the calculated internal 413 forces, particularly for the axial forces. The results of numerical analyses are in good 414 agreement with the Bobet's solution and the improved HRM method only when the value of K_s equals to 5×10⁸ N/m³. This value is also adopted to simulate the soil-tunnel condition for 415 416 cases 2 and 3, as listed in Table 2. Fig. 15 again indicates that the soil-tunnel interface 417 parameters should be determined with caution in the numerical analysis.



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420 421

Fig. 15. Calibration of the value of shear spring stiffness.

	HRM	Bobet	FLAC	HRM vs Bobet (%)	HRM vs FLAC (%)
Casel					
$T_{\rm max}$	1066.54	1046.49	1106.5	1.88	-3.74
$M_{\rm max}$	179.37	159.27	183.7	11.21	-2.41
Case2					
$T_{\rm max}$	924.52	813.57	861.90	12.0	6.77
$M_{\rm max}$	159.55	139.14	145.83	12.79	8.60
Case3					
T_{\max}	570.27	507.20	536.84	11.06	5.86
$M_{\rm max}$	149.25	133.67	140.10	10.44	6.13

 Table 3 Comparison of the results of the improved HRM method, Bobet's solution, and FLAC.

422

The lining forces calculated by the improved HRM method, analytical solution, and 423 numerical model for three cases are listed in Table 3. The axial forces and bending moments 424 425 calculated by the improved HRM method agree very well with the FLAC results. The maximum error is around 7% for the axial force and reaches 9% for the bending moment in 426 case 2. The maximum axial force of the improved HRM method matches perfectly with the 427 Bobet's solution in case 1, whereby the error is less than 2%. A larger error (~12% for axial 428 429 forces, ~13% for bending moments) between the improved HRM method and the Bobet's solution is observed in case 2. As compared to the Bobet's solution, the improved HRM 430 431 method slightly overestimates the lining forces for all the cases considered, providing a 432 conservative seismic design.

This section attempts to validate the effectiveness of the improved HRM method in a wide range of tunnel radii. The soil and tunnel properties are taken from the literature (Do et al., 2015), as shown in Table 4. The comparisons between the Bobet's solution and the improved HRM method are given in Fig. 16, in terms of the maximum axial force and bending moment.

440

441

Table 4 Soil and tunnel lining properties (Do et al., 2015).

Properties	value				
Soil parameter					
Young's modulus, Es	450 MPa				
Poisson's ratio, Vs	0.3				
Input shear strain, ymax	0.0035%				
Lining parameter					
Young's modulus, <i>E</i> _l	35 GPa				
Tunnel thickness, t	0.4 m				
Tunnel radius, R	4.5-8 m				
Poisson's ratio, <i>v</i> _l	0.15				

442

The improved HRM method captures the same trend between the lining force and 443 444 tunnel radius with the Bobet's solution: both the axial force and bending moment increase 445 with increasing tunnel radius. Although the soil and lining properties used here are different 446 with the ones adopted for obtaining formulas of two parameters, a good agreement between 447 the improved HRM method and the Bobet's solution is observed for the axial forces for all 448 the tunnel radii compared here, with the maximum error is less than 7%. For the bending 449 moment, a slightly larger difference with a maximum error of 25% occurs when the tunnel radius equals to 4.5 m. This could be attributed to the obvious underestimation of the bending 450 moment for the rigid tunnel, as also illustrated in Fig. 12b. The accuracy of the improved 451 452 HRM method in computing bending moment is significantly improved as the tunnel radius 453 increases, with an error of about 2% in the case of R=8 m.



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Fig. 16. Comparison of axial forces and bending moments calculated by three methods at various shear strains.

458 **5.** Conclusions

This study proposed an alternative numerical procedure to compute the seismic design loads in a deep circular tunnel lining due to seismic-induced ovaling deformation, using the Hyperstatic Reaction Method. The mathematical formulas of the HRM method were first introduced in brief. Then, the underlying assumptions and algorithms to implement the HRM method under seismic conditions were presented in detail. Considering a wide range of properties of soil and tunnel lining in practice, the formulas of two dimensionless parameters *a* and β were given for the preliminary seismic design, with the help of parametric analysis.

To verify the effectiveness of the improved HRM method, extensive validations were performed in a wide range of numerical examples. The improved HRM method was validated with the analytical solution and the quasi-static numerical model. The comparative studies have demonstrated that the improved HRM method could be an effective tool for the preliminary seismic design of circular tunnels with a certain confidence. Nevertheless, the formulas of two parameters were calibrated through an analytical solution, thus they inevitably shared the same assumptions and limitations with the analytical solution.

The novelty and contribution of this paper, first, lied on providing a new and
alternative approach to fast calculate the seismic design loads. Second, the applicability of the
HRM method was extended from the static to the seismic condition. This was particularly

476 useful for researchers and designers who were familiar with the HRM method for tunnel static

477 design.

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