



**HAL**  
open science

# The role of the leverage effect in the price discovery process of credit markets

Paul Zimmermann

► **To cite this version:**

Paul Zimmermann. The role of the leverage effect in the price discovery process of credit markets. Journal of Economic Dynamics and Control, 2021, 122, pp.104033 -. 10.1016/j.jedc.2020.104033 . hal-03493542

**HAL Id: hal-03493542**

**<https://hal.science/hal-03493542>**

Submitted on 15 Dec 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License

# The Role of the Leverage Effect in the Price Discovery Process of Credit Markets<sup>\*</sup>

Paul Zimmermann<sup>\*,a,b</sup>

<sup>a</sup>*IESEG School of Management, 1 Parvis de la Défense, 92044 Paris, France.*

<sup>b</sup>*LEM-CNRS 9221, 3 rue de la Digue, 59000 Lille, France.*

October 8, 2020

## Abstract

Starting from the analysis of a levered firm's capital structure, I show that corporate default risk becomes measurable through the leverage effect, i.e., the negative correlation observed between stock returns and changes in stock volatility. In this model, the firm's debt-to-asset ratio governs the elasticity of default probabilities relative to equity prices. I use a large dataset of S&P 500 firms and an extended timeframe (2008-2019) to examine the model's empirical implications. First, the firm's corporate leverage enhances the transmission of pricing information from the stock to the credit default swap (CDS) market. Second, equity and credit markets are more likely to be co-integrated when firms employ a higher debt-to-asset ratio. Although the stock market generally dominates the price discovery process, a small cluster of highly-leveraged firms exhibits a dominant CDS market share. Under the effect of corporate leverage, the credit market attracts informed trading and arbitrage resources.

**JEL classification:** G12; G13; G14

**Keywords:** Credit markets; Capital structure arbitrage; Leverage effect; Co-integration; Price discovery.

---

<sup>\*</sup>The author thanks Yann Braouezec, Philippe Raimbourg, Jean-Paul Laurent, Olivier Le Courtois, Franck Moraux, Nicolas Leboisne, Marco Ghitti, Riccardo De Blasis, Ryan Williams, Saeid Hoseinzade, Sarah Wang, seminar participants at EM-Lyon, and conference participants at the 2019 FMA European Conference, the 2019 INFINITY Conference on International Finance, the 2019 EFMA Annual Meeting, the 12<sup>th</sup> IRMC Conference, the 2019 SFA Annual Meeting, and the 2020 FMA Consortium on Asset Management for helpful comments and suggestions.

<sup>\*</sup>*E-mail address:* p.zimmermann@ieseg.fr

This research did not receive any grant from funding agencies in the public, commercial, or not-for-profit sector.

## 1. Introduction

Merton's (1974) structural model of credit risk implies that individual stock and corporate debt securities are reasonably close substitutes. The integration of credit and equity markets suggested by the standard financial theory incites rational investors to bet on the convergence of these two markets by arbitraging stocks against credit derivatives (Kapadia and Pu, 2012). The risk limits to capital structure arbitrage strategies being low,<sup>1</sup> neither the equity market nor the credit market should dominate the price discovery of credit risk. However, recent studies have shown that insider trading may occur in the credit derivatives market and impound the price discovery process (e.g., Acharya and Johnson, 2007; Qiu and Yu, 2012; Kryzanowski et al., 2017). Such breaches to market efficiency raise the question as to which market attracts informed trading and arbitrage resources.

In this paper, I put forward a structural model of the leverage effect to interpret the dominance of credit markets in the price discovery process. In this model, the corporate leverage governs the transmission of price information between the credit and the stock market. When the firm's financial leverage is low, the informational content of the credit market is low and produces low-intensity signals. As a result, credit traders are mostly noise or liquidity traders, and the bulk of the price discovery process primarily occurs in the stock market, in line with multiple empirical studies (e.g., Hilscher et al., 2015). Conversely, when the firm's financial leverage gradually increases, rational and sophisticated credit investors acquire a gradual advantage in the gathering and the processing of information related to the firm's credit quality. As credit traders tend to monopolize the incorporation of private information into prices, the transmission to the stock market intensifies due to the effect of the corporate leverage. The shift in intensity then pushes stock traders to chase the trend and morph into noise traders without their knowing. Everything happens as if the corporate leverage made informed trading migrate to the credit market.

My structural model for price transmission draws on the following insight. Relying on the

---

<sup>1</sup>For the limits to arbitrage, see, for example, Shleifer and Summers (1990) and Shleifer and Vishny (1997).

economic concept of elasticity,<sup>2</sup> I focus on the elasticity of default probabilities relative to stock prices. The intuition behind the credit-equity elasticity is that it conveys the joint correlation between changes in the firm’s market value and changes in the firm’s credit quality. Additionally, it delivers the optimal hedge ratio sought by capital structure arbitrageurs.<sup>3</sup> The paper’s key result is then as follows. The structural framework proves to be rich enough to link the credit-equity elasticity to the elasticity of equity variance relative to stock prices. Simultaneously, the latter captures the so-called “leverage effect”<sup>4</sup> as a linear function of the debt-to-asset ratio, one of the critical indicators of the firm’s financial health. Consequently, a simple function of the firm’s financial leverage turns out to encapsulate the signal for informed trading of the capital structure.

The model put forward offers far-reaching empirical implications. First, the credit-equity elasticity hypothesis brings support to the presence of a long-run equilibrium relationship between a firm’s credit spreads and equity prices. The intuition is that CDS and stock time series cannot drift too far apart from the equilibrium because capital structure arbitrageurs will act to restore the equilibrium relationship. By capturing the non-linear effect of the firm’s leverage, this co-integrating vector is distinct from the linear combination of the credit spread and the stock price already investigated in the literature (e.g., Narayan et al., 2014; Kryzanowski et al., 2017).

Second, the model provides new testable hypotheses concerning the price discovery process at work in credit markets. If equity and credit prices are co-integrated, the permanent-transitory decomposition of Gonzalo and Granger (1995) ensures that they must track a common long-memory component, or efficient price (Hasbrouck, 1995). Meanwhile, an error-correction mechanism must absorb transitory shocks to reflect arbitrage across equity and credit markets. By specifying the co-integrated credit-equity system, my model allows computing the implicit efficient price of credit. Each market’s contribution to the price discovery process then becomes accessible.

Third, as already underscored in the literature (Kapadia and Pu, 2012; Choi and Kim, 2018),

---

<sup>2</sup>The term *elasticity* refers to situations where a change of  $\delta\%$  in a dimensionless financial quantity  $x$  generates a change of  $e\delta\%$  in quantity  $x^e$  for a  $\delta$  close to 0.

<sup>3</sup>See Schaefer and Strebulaev (2008) for a study of the hedge ratios produced by structural models of credit risk.

<sup>4</sup>Coined by Black (1976), the term conventionally designates the negative correlation empirically observed between stock price returns and changes in volatility.

exogenous barriers to arbitrage such as funding constraints, liquidity risks, or short-sale impediments interfere with the co-movements in the equity and credit markets. This paper hypothesizes that the non-linear impact of the leverage effect may be one of the endogenous sources for the lack of integration between the credit and equity markets.

This paper uses a large dataset of S&P 500 firms and an extended timeframe (2008-2019) to examine the transmission of pricing information from the stock market to the credit default swap (CDS) market. By identifying the genuine price innovations arising in the stock market, I offer an empirical methodology to identify the non-linear impact of the financial leverage on the information flow transiting to the credit market. The paper's main finding is the high statistical significance of this leveraged transmission mechanism to the CDS market. Uniform across firms, this newly-identified channel of transmission appears more intense than the direct unleveraged channel already investigated in previous literature.

Most firms in the sample reject the null hypothesis of no (leveraged-)co-integration between their equity and CDS markets. The empirical analysis shows that entities are more likely to be co-integrated when (i) they belong to a business sector perceived as more leveraged; (ii) they employ a higher debt-to-asset ratio; (iii) their CDS price is more volatile. These findings provide evidence that an increase in financial leverage ramps up market activity in capital structure arbitrage. One of the indirect market effects of corporate leverage is thus to intensify the integration between credit and equity markets.

For those firms which are significantly co-integrated, this study draws on the vector error-correction (VECM) approach of Gonzalo and Granger (1995) to identify the respective contributions of each market to the price discovery process. The CDS market share appears to be low and below 30% for the vast majority of firms, consistent with the CDS "sideshow" hypothesis (Hilscher et al., 2015). However, a small cluster of highly-leveraged firms exhibits an extremely dominant CDS market share close to 100%. This new finding provides reliable evidence for the role of the leverage effect in the price discovery process.

This paper relates to the vast empirical literature that investigates the price discovery process in

credit markets. The conventional view states that credit pricing information primarily flows from stock markets to credit markets due to lower transaction costs (e.g., Hilscher et al., 2015). The alternative view underscores the role of private information in the flow of pricing information from credit markets to stock markets (e.g., Acharya and Johnson, 2007; Qiu and Yu, 2012). The most recent literature suggests that both credit and equity markets should potentially lead and lag the other market (Forte and Peña, 2009; Marsh and Wagner, 2016; Lee et al., 2018). By studying the endogenous, non-linear impact of the firm's capital structure, this paper departs from a work of literature mainly focused on exogenous and linear transmission effects.

The article proceeds as follows. Section 2 contains the main theoretical contribution, while Section 3 discusses the economic implications of the theory. Section 4 describes the data used in the empirical analysis developed in Section 5. Finally, Section 6 concludes the article.

## 2. The Model

I now introduce a simple structural framework to build a new approach to the leverage effect.

### 2.1 Structural framework

I start with a basic structural model of the firm in the vein of Black and Cox (1976), Leland (1994, 1998), Leland and Toft (1996). The firm's unlevered asset value  $V = (V_t)_{t \geq 0}$  evolves according to a geometric Brownian motion which is defined on a complete probability space  $(\Omega, \mathcal{F}, P)$ :

$$dV_t = \mu_v V_t dt + \sigma_v V_t dB_t, \quad (1)$$

where  $\mu_v$  is the asset growth rate,  $\sigma_v^2$  is the instantaneous variance of the return on the firm, and  $B_t$  is a standard Brownian motion. Fully-informed managers operate the firm and have access to the complete information filtration  $\mathcal{F}_t := \sigma\{V_s : s \leq t\}$ .

The firm has issued two types of financial claims: equity and an amount of debt interest and principal. The firm pays a net cash outflow rate  $\delta > 0$  to equityholders. Debt is issued to benefit

from the tax shield offered at the constant tax rate  $\theta \in [0; 1]$ . I model it as a consol bond, which is paying interests indefinitely at some constant coupon rate,  $c > 0$ . The optimal amount of debt and its coupon rate may be chosen at time 0 by the structural planners, depending on the initial firm valuation. The considered company is subject to default risk, and the stopping time of the firm's default is  $\tau_b := \inf\{t > 0 : V_t \leq V_b\}$ , where  $V_b > 0$  is a default boundary to be endogenously determined later by having the shareholders optimally liquidate the firm. The rule of absolute priority governs the distribution of assets to bondholders in case of liquidation. Liquidation costs are assumed to be a proportion of the remaining asset value  $V_b$ .

Fully-informed managers of the firm are the agents of equity shareholders. They are entrusted with the strategic choice of optimally liquidating the firm for the benefit of stockholders. As I do not consider agency costs of equity, the managers choose a liquidation policy modeled as an  $(\mathcal{F}_t)$ -stopping time  $\tau_b$  to maximize the residual asset value of the firm at time  $t$ :

$$S_t = \sup_{\tau_b \in \mathcal{T}} \mathbb{E} \left[ \int_t^{\tau_b} e^{-r(s-t)} (\delta V_s + (\theta - 1)c) ds \middle| \mathcal{F}_t \right], \quad (2)$$

where  $\mathcal{T}$  is the set of  $(\mathcal{F}_t)$ -stopping times. Under the technical assumption that the expected asset growth rate is below the risk-free rate (i.e.,  $r > \mu_V + \sigma_V^2/2$ ), Duffie and Lando (2001) solve the optimal control program (2) to find explicit functions for the endogenous default boundary, the optimal equity value, and the value of debt.

To obtain a first passage default model consistent with a reduced-form representation, I follow Duffie and Lando (2001) by assuming noisy accounting information. I assume that debtholders, contrary to the firm's managers, have only access to incomplete accounting information on the state of the firm value  $V$ . More precisely, the investors in the secondary debt market have access to the following pieces of information:

- *Noisy accounting information.* At selected reporting dates  $t_1, t_2, \dots$ , the bondholders have access to a noisy accounting report of assets,  $\hat{V}_t$ .
- *Default state of the firm.* At each time  $t$ , the debtholders know whether the firm managers

have liquidated the firm and bankruptcy has taken place. They observe and use rationally the default indicator process  $1_{\{\tau_B \leq t\}}$ .

At each time  $t$ , the information filtration available to the bond market is thus given by the  $\sigma$ -algebra  $\mathcal{G}_t := \sigma \left\{ (\hat{V}_{t_1}, \hat{V}_{t_2}, \dots, \hat{V}_{t_n}), 1_{\{\tau_B < s\}} \mid 0 \leq s \leq t \right\} \subset \mathcal{F}_t$ , where  $t_n$  is the latest noisy reporting date before  $t$ . As shown by Giesecke (2006) in a more general setting, the imperfect observation of the firm's assets naturally yields a  $\mathcal{G}_t$ -intensity process  $\lambda$  of the default stopping time  $\tau_B$ .

I now turn to the structural analysis of the leverage effect. The next formulation of the leverage effect is a simple consequence of our structural framework. It provides a preliminary link between the logarithmic slope of the equity local volatility surface and an adjusted value of the firm's corporate leverage.

**Lemma 1.** *The logarithmic slope of the equity local volatility surface,  $\sigma(S_t, t)$ , is linked to the firm's financial leverage by the relationship:*

$$\frac{\partial \sigma}{\partial \ln(S)} = -\sigma \cdot (\ell - \varepsilon_\ell), \quad (3)$$

where  $\ell$  is the debt-to-asset ratio,<sup>5</sup> and the adjustment to the financial leverage is given by:

$$\varepsilon_\ell := \frac{SS_{VV}}{S_V^2}, \quad (4)$$

where subscripts denote partial derivatives with respect to the firm's asset value  $V$ . Moreover:

- (a)  $\lim_{V \rightarrow \infty} \varepsilon_\ell = 0$ ;
- (b)  $\lim_{V \rightarrow V_B} \varepsilon_\ell = \frac{1}{2}$ ;
- (c)  $\varepsilon_\ell$  is continuous and bounded over  $[V_B; \infty)$ ;
- (d) Except for states of the firm close to default ( $V \approx V_B$ ), numerical simulations indicate that  $\varepsilon_\ell \ll \ell$ . In these conditions, the slope of the equity instantaneous volatility surface is determined by  $\ell$  at first order in the firm's leverage.

---

<sup>5</sup>The debt-to-asset ratio is specified with market values instead of book values. This modeling choice reflects not only the firm's tangible assets and working capital but also its intangible assets and growth opportunities.

*Proof.* See Appendix A. □

## 2.2 Structural approach to the “leverage effect”

To substantiate the role of the firm’s financial leverage in the so-called leverage effect, limiting oneself to Equation (3) presents serious shortcomings. Indeed, as it depends on the model of the stock price dynamics, the local volatility surface is not observable in the market. It is thus preferable to rely on a model-free formulation of the volatility surface. Let introduce the Black-Scholes implied volatility  $\widehat{\sigma}_T(K)$  for a given strike price  $K$  and given time to maturity  $T$ .<sup>6</sup> The local volatility slope appearing in Equation (3) is known to be a good predictor of the asymmetry of the implied volatility surface observed in options markets. More precisely, the local volatility skew is twice as steep as the implied volatility skew for short times to expiration (see, for example, Gatheral, 2006). However, as the following technical result derived by Hagan and Woodward (1999) via singular perturbation theory shows, the assumption of short times to expiration may safely be relaxed at a small technical cost.

**Lemma 2 (Hagan and Woodward, 1999).** *Assume a time-separable instantaneous volatility surface:  $\sigma(S_t, t) \equiv \alpha(t)\sigma(S_t)$ . The instantaneous volatility surface  $\sigma(\cdot)$  may be inferred from the volatility surface  $\widehat{\sigma}(\cdot)$  implied by the options market through the following affine transformation:*

$$\widehat{\sigma}_T(K) = \sigma\left(\frac{S+K}{2}\right) \sqrt{\frac{1}{T} \int_0^T \alpha^2(s) ds}, \quad \text{for all } K, S > 0. \quad (5)$$

*Proof.* See Appendix B. □

Let define the slope of the implied volatility surface in the log-strike space as:

$$\widehat{\Sigma}_T := \frac{\partial \widehat{\sigma}_T(K)}{\partial \ln(K)}. \quad (6)$$

An immediate consequence of Lemma 2 is to provide a structural interpretation based on leverage

---

<sup>6</sup> $\widehat{\sigma}_T(K)$  is the volatility number to be input in the Black-Scholes-Merton model (Black and Scholes, 1973) in order to match the European-style call price  $C(K, T)$  observed in the options market.

of the implied volatility “skew”  $\widehat{\Sigma}$ , that is, the negative relationship between implied volatility and strike price.

**Proposition 1 (Structural leverage effect).** *Assume a time-separable local volatility surface:  $\sigma(S_t, t) \equiv \alpha(t)\sigma(S_t)$ . The slope of the implied volatility surface is linked to the firm’s debt-to-asset ratio through:*

$$\widehat{\Sigma}_T = -\frac{\sigma}{2}(\ell - \varepsilon_\ell)\bar{\alpha}_T, \quad (7)$$

where  $\ell$  is the debt-to-asset ratio,  $\varepsilon_\ell$  is given by Equation (4), and  $\bar{\alpha}_T := \sqrt{\frac{1}{T} \int_0^T \alpha^2(s) ds}$ . The variance-equity elasticity is then given by:

$$e_v := \frac{d\sigma^2/\sigma^2}{dS/S} = -2(\ell - \varepsilon_\ell). \quad (8)$$

*Proof.* See Appendix C. □

The alternate hypothesis traditionally advanced for the asymmetry of the implied volatility surface (e.g., Bekaert and Wu, 2000; Wu, 2001) is the volatility feedback effect.<sup>7</sup> Notice how Equation (7) captures the volatility feedback effect (increases in volatility imply increases in  $|\widehat{\Sigma}|$ ) on top of the leverage effect (increases in leverage imply increases in  $|\widehat{\Sigma}|$ ). In our current structural framework, it is thus the role of the firm’s financial leverage to dampen or magnify a possible volatility feedback effect.

In view of Equation (8), it comes as no surprise that some authors call the variance-equity elasticity a “leverage coefficient” (Das and Sundaram, 2007), although the authors recognize that “there is no direct interpretation of this parameter within the Merton framework.” The academic literature devoted to the constant-elasticity-variance (CEV) stock price process has rarely addressed the practical problem of parameter estimation.<sup>8</sup> To the best of my knowledge, Equation (8) is

<sup>7</sup>The economic mechanism goes as follows. As an increase in stock market volatility raises expected stock returns (Campbell and Hentschel, 1992), current stock prices then decline to adjust to these revised expectations. As a result, an increase in volatility ( $\sigma \uparrow$ ) is correlated with negative stock returns, thus raising the value of out-of-the-money stock options and the implied volatility skew ( $|\widehat{\Sigma}| \uparrow$ ).

<sup>8</sup>Beckers (1980) initiates an econometric approach to the variance-equity elasticity estimation. Schroder (1989) is the first to outline the influence of the firm’s debt-equity ratio. More recently, De Spiegeleer et al. (2014) further elaborate upon it in the context of the modeling of hybrid securities.

the first theoretical result to provide an unambiguous structural estimate for the variance-equity elasticity.

### 2.3 The credit-equity elasticity

To exploit the approach to the variance-equity elasticity developed in Section 2.2, I now introduce a simple economic model to link default probabilities with equity volatilities. I reinforce the assumption of financial market completeness by assuming that equity options are continuously tradeable within a significant range of exercise prices before the default event. A stock option trading continuum is needed to exhibit optional equity structures liable to replicate the main features of a conventional CDS instrument. The ultimate purpose is to match the higher moments of the implied volatility surface, such as the volatility “skew.”

If the positive correlation between CDS spreads and the levels and slopes of the implied volatility surface is well known from empiricists (e.g., Cremers et al., 2008), theoretical models that account for this close empirical relationship are still lacking. It is possible, however, to rely on sensitivity-matching analysis to get a better understanding of the links between default probabilities and the dynamics of the implied volatility surface. Grounded in the replication of a CDS instrument by an equity option structure, the following result provides a workable relationship between the CDS spread and the implied volatility skew.

**Lemma 3 (Zimmermann, 2015).** *Under the assumption of an stock option trading continuum, the firm’s default probability on its debt at a given maturity  $T$  is linked at first order to the at-the-money implied volatility,  $\widehat{\sigma}_{ATM}$ , and the implied volatility skew,  $\widehat{\Sigma}$ , as follows:*

$$\lambda_T = k \cdot \widehat{\sigma}_{ATM} \cdot |\widehat{\Sigma}_T|, \quad (9)$$

where the constant normalizing factor  $k$  is typically independent of the equity volatility and reflects the expected recovery rate on the debt.

*Proof.* See Appendix D. □

I now combine the insights from Proposition 1 and Lemma 3 to derive the main result of the paper. I introduce the credit-equity elasticity defined as:

$$e_\lambda := \frac{d\lambda/\lambda}{dS/S}. \quad (10)$$

The next result provides a structural estimate for the credit-equity based on the firm's financial leverage.

**Proposition 2 (Credit-equity elasticity).** *Under the assumptions of a separable local volatility surface and a stock option trading continuum, the credit-equity elasticity is equal to the variance-equity elasticity and amounts to twice the firm's adjusted financial leverage:*

$$e_\lambda = -2(\ell - \varepsilon_\ell), \quad (11)$$

where  $\ell$  is the debt-to-asset ratio and  $\varepsilon_\ell$  is given by Equation (4). The case of an un-levered firm ( $\ell \equiv 0$ ) is consistent with the Black-Scholes paradigm ( $e_\sigma \equiv 0$ ) in which the perfect de-correlation between credit spreads and stock prices ( $e_\lambda \equiv 0$ ) means that the stock price process cannot reach zero and that the default probability reduces to zero.

*Proof.* See Appendix E. □

### 3. Model Implications

In this section, I show that the model's main result, Proposition 2, provides new refutable hypotheses for future empirical research.

#### 3.1 The credit-equity power relationship

The main implication of Proposition 2 is that over a small period  $\Delta t$ , the firm's default probability follows a power relationship relative to the stock price. Taking CDS spreads quoted in the

credit market as a natural proxy for default probabilities yields the relationship:

$$\text{CDS}_{t+\Delta t} = \text{CDS}_t \cdot \left( \frac{S_t}{S_{t+\Delta t}} \right)^{2(\ell-\varepsilon_\ell)}. \quad (12)$$

Consider now a reference credit-equity point  $(\text{CDS}_0; S_0)$  at time 0, and divide the period  $[0; t]$  into  $n$  small periods of length  $\Delta t$ . Repeated use of Equation (12) yields:

$$\frac{\text{CDS}_t}{\text{CDS}_{(n-1)\Delta t}} \times \cdots \times \frac{\text{CDS}_{\Delta t}}{\text{CDS}_0} = \left( \frac{S_{(n-1)\Delta t}}{S_t} \times \cdots \times \frac{S_0}{S_{\Delta t}} \right)^{2(\ell-\varepsilon_\ell)}. \quad (13)$$

Equation (13) leads to the following parameterization of the current CDS spread by a power function of the current stock price:

$$\text{CDS}_t = a \times S_t^{-2(\ell-\varepsilon_\ell)}, \quad (14)$$

where the normalization factor  $a = \text{CDS}_0 \times S_0^{2(\ell-\varepsilon_\ell)}$  reflects the credit-equity reference point. Notice how the power function captures the loose credit-equity de-correlation when stock prices increase, and the sharp credit-equity re-correlation when stock prices fall. In contrast with alternative parameterizations of credit spreads by stock prices based on logarithmic or exponential functions, the power function also ensures sound boundary conditions when stock prices fall close to zero or tend to infinity.<sup>9</sup> Another comparative advantage lies in the scalability of inputs, which can be multiplied by any factor without altering relevant empirical aspects.<sup>10</sup>

Figure 1 provides an empirical illustration of the credit-equity power relationship (14). The scatter plots display 5-year CDS par spreads against common stock prices for Microsoft and General Motors over 2011-2018. We can recognize the monotonicity and the convexity predicted by Equation (14) on both plots. When the equity market value rises significantly, the firm's improved financial health is expected to enhance its creditworthiness. As a result, default probabilities tend smoothly toward a floor, as illustrated by AAA-rated firms such as Microsoft (Figure 1.a). The

<sup>9</sup>The superior capability of power parametric functions for data fitting is not an isolated case in the financial domain. Also known as the family of constant relative risk aversion (CRRA) in the economic literature, the power family is widely used in economics and other social sciences (e.g., Wakker, 2008; Gabaix, 2009).

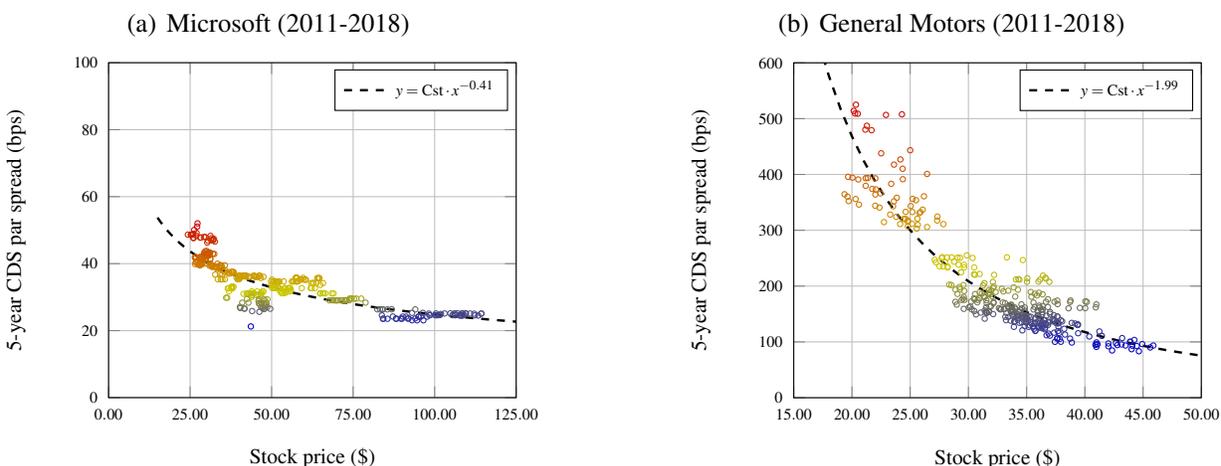
<sup>10</sup>For example, credit risk should be an invariant across the different quoting currencies of the firm's common stock.

credit-equity elasticity fitted over the whole period is  $-0.41$ , thereby suggesting an implicit value of 0.20 for the corporate leverage. Without surprise, the realized debt-to-asset of Microsoft never exceeded 0.19 over 2011-2018 and was as low as 0.10 on average. In a nutshell, harmless corporate leverage will result in a low flow of transmission between credit and equity.

Conversely, a significant fall in the equity market value is expected to signal higher odds of financial distress. As a consequence, default probabilities sharply increase, as illustrated by firms that are close to financial distress, such as General Motors (Figure 1.b). The credit-equity elasticity fitted over the whole period is  $-1.99$ , thereby suggesting a highly leveraged firm on the brink of financial distress. Unsurprisingly, the debt-to-asset of General Motors averaged at 0.60 over the whole period 2011-2018, oscillating between 0.34 and 0.79. In summary, a high flow of credit-equity transmission will need a tight credit-equity profile to transit.

**Figure 1. Credit-equity elasticity**

This figure plots weekly CDS par spreads (5-year, senior unsecured contract) against weekly closing prices for the common equity. Time period: 2011:10 to 2018:09. Data source: Thomson Reuters.



### 3.2 Information transmission between stock and credit markets

The credit-equity power relationship (12) provides new testable hypotheses concerning the transmission mechanisms between stock and credit markets. Taking the logarithm of both sides of Equation (12) and conditioning upon the information filtration  $\mathcal{G}_t$  available at time  $t$  to the credit

market participants yields the CDS return expected by capital structure arbitrageurs:

$$E \left[ (\text{CDS return})_t \mid \mathcal{G}_t \right] = -E \left[ 2(\ell - \varepsilon_\ell)_t (\text{Stock return})_t \mid \mathcal{G}_t \right]. \quad (15)$$

Equation (15) suggests that the stock market channels pricing information to the credit market. However, market participants have to rescale this information by the firm's corporate leverage. Notice how the true value of the firm's corporate leverage is still unknown to the participants in the secondary debt market and must remain within the conditional expectation operator  $E[\cdot | \mathcal{G}_t]$ .

The price formation of credit risk can be analyzed in terms of the variation in the CDS price. Equation (15) suggests decomposing the change in CDS spread into several components. The channel of transmission from the stock market provides its first component. On top of this informed trading channel, a second component must capture the firm-specific information about credit risk produced within the credit market. Finally, a third component must reflect the market price of credit risk, that is, the premium demanded by investors for the uncertainty related to default.

Later in Section 5.1, this paper tests the corresponding transmission mechanism from stock to credit markets:

$$(\Delta \text{CDS})_t = \alpha + \beta \times (\ell - \varepsilon_\ell)_t (\text{Stock return})_t + \varepsilon_t, \quad (16)$$

where the intercept  $\alpha$  captures a risk premium that cannot be explained by the integration of credit and equity markets, the coefficient  $\beta$  measures the rate of transmission between markets, and  $\varepsilon_t$  is the pure credit innovation arising in the credit market independently from the stock market. In the mechanism, the interaction between the firm's leverage and the stock returns constitutes the signal which primarily matters to informed traders when filtering stock returns.

### 3.3 Co-integration and price discovery

Letting the time interval  $\Delta t$  grow in Equation (12) and taking the natural logarithm of both sides suggests that the log-price of credit and the log-price of leveraged equity should be co-integrated.

The long-run equilibrium is:

$$\ln(\text{CDS})_t = \beta_0 + \beta_1 \times (\ell - \varepsilon_\ell)_t \ln(\text{Stock})_t + \eta_t, \quad (17)$$

where  $\beta_0$  captures the equilibrium's inception point,  $(1, -\beta_1)'$  is the co-integrating vector, and  $\eta_t$  is a stationary random variable representing the dynamic behavior of the log-CDS-equity price ratio (disequilibrium error). The economic intuition behind Equation (17) is that the log-prices of CDS and leveraged equity should form a co-integrated system because both relate to the fundamental value of credit risk. If the series drift too far apart because of market frictions, capital structure arbitrageurs will act to restore the long-run equilibrium.

The co-integration of credit and equity markets has important implications for the price discovery process of credit risk. The permanent-transitory decomposition of Gonzalo and Granger (1995) suggests that credit and equity prices share a common long-memory component, also referred to as the *efficient price* (Hasbrouck, 1995). Additionally, if equity and credit prices converge in the long run, there must be an error-correction mechanism reflecting arbitrage across both markets. The error-correction mechanism should absorb the transitory shocks that have no permanent effect on CDS and equity prices. As a result, either the CDS price or the stock price must be the first to move permanently, that is, to reflect innovations in the efficient price of credit.

Two price discovery metrics have been proposed in the literature to assess the relative speed at which a price series is the first to impound new information. The first approach initiated by Hasbrouck (1995) focuses on the variance of the innovations in the efficient price. The information share (IS) measures each market's contribution to this variance. The IS metric attributes price discovery to the market that first reflects innovations in the common factor. By contrast, the second approach builds on Gonzalo and Granger's (1995) insight that the common stochastic trend must be a linear combination of the original price series. The component share (CS) measures each market's contribution to this implicit common factor. The CS metric attributes price discovery to the market with the most substantial weight in the common factor (e.g., Baillie et al., 2002;

Hasbrouck, 2002; Yan and Zivot, 2010; Putnins, 2013), i.e., the market that adjusts least to the other.

#### **4. The Data**

For this study, I consider daily closing CDS quotes for the most widely traded, North American reference entities. To build as much as possible a large and representative CDS universe, I impose three requirements. The first constraint is for bid-ask CDS quotes to be available in Thomson Reuters over an extended 12-year sample period running from January 01, 2008, to January 01, 2020.<sup>11</sup> The second constraint is for the corresponding common stocks to continuously trade on the S&P 500 stock index over the full sampling period. Finally, the historical leverage ratio has to be available in Thomson Reuters over the full sampling period. For all the reference entities satisfying the previous three requirements, all the CDS quotes, stock market data, and leverage data are then consistently retrieved from the Thomson Reuters database.<sup>12</sup>

The final single-name CDS list comprises a total of 220 corporate credits from the S&P 500 stock index. For consistency, I consider only CDS par spreads corresponding to U.S.-dollar denominated contracts on the most liquid tenor (5 years), the lowest seniority (senior unsecured debt), and the same restructuring clause. Thomson Reuters provides end-of-day prices by collecting daily single-name CDS quotes from over 30 contributors around the world and applying a rigorous screening procedure to eliminate outliers or doubtful data. Final CDS quotes are thus composite mid spreads calculated by Thomson Reuters and expressed in basis points. The timing for the end-of-day composite calculation is in T+1 (5:00 am GMT). As this last update takes place after the end of trading for U.S. stocks, there is no bias in detecting information flows from stock markets to credit markets.

To measure the firm's financial leverage, I use the ratio of total debt book value to enterprise

---

<sup>11</sup>Firms in the sample may undergo a major credit event leading to an early exit from the S&P stock index over the sample period such as a corporate default, a merger, or an acquisition. The dataset contains at least two S&P 500 constituents that were delisted from the stock index over the period 2008-2019 before filing for bankruptcy at a later stage (Eastman Kodak, Dean Foods).

<sup>12</sup>Mayordomo et al. (2010) offer an in-depth comparative study of the Thomson Reuters database and five other public sources of corporate CDS prices.

**Table 1. Descriptive statistics**

The table reports summary statistics for firm characteristics (Panel A), equity and CDS returns (Panel B), and business sector-level statistics (Panel C). The sample consists of 220 U.S. firms over the period 2008:01 to 2020:01, including only trading days with available CDS spread observations and equity returns. Sample statistics are computed across all observations. Data source: Thomson Reuters.

	5 <sup>th</sup> perc.	25 <sup>th</sup> perc.	Median	Mean	75 <sup>th</sup> perc.	95 <sup>th</sup> perc.	SD	Observations	
<i>Panel A: firm-level statistics</i>									
Firm CDS level (mid-price, bps)	25	44	70	119	121	340	220	548,394	100.0%
Firm leverage (debt to assets)	0.07	0.15	0.23	0.29	0.38	0.71	0.20	548,394	100.0%
Firm size (mkt. cap., \$bn)	3.54	10.43	21.15	43.83	44.90	176.80	68.57	548,394	100.0%
Firm debt (book value, \$bn)	1.20	3.39	6.77	27.70	13.94	98.51	87.62	548,394	100.0%
Daily observations by firm	1,113	2,492	2,758	2,493	2,804	2,901	603	548,394	100.0%
<i>Panel B: equity and CDS returns</i>									
Equity daily return (%)	-2.88	-0.79	0.05	0.02	0.89	2.77	2.06	548,394	100.0%
CDS daily return (%)	-3.39	-0.14	0.00	-0.01	0.07	3.47	3.95	548,394	100.0%
<i>Bidirectional co-movements (<math>\Delta\text{CDS} \cdot \Delta S &lt; 0</math>)</i>									
Equity daily return (%)	-3.27	-0.92	0.04	-0.04	0.96	2.93	2.28	245,698	44.8%
CDS daily return (%)	-4.28	-0.82	-0.01	-0.01	0.32	4.63	3.89	245,698	44.8%
<i>Unidirectional co-movements (<math>\Delta\text{CDS} \cdot \Delta S \geq 0</math>)</i>									
Equity daily return (%)	-2.54	-0.70	0.06	0.07	0.84	2.63	1.86	302,696	55.2%
CDS daily return (%)	-2.41	-0.02	0.00	0.00	0.03	2.32	4.00	302,696	55.2%
	# Firms	Mean CDS (bps)	Mean leverage	Mean debt (\$bn)	Mean size (\$bn)				Observations
<i>Panel C: business sector-level statistics</i>									
Basic Materials	15	117	0.25	4.75	15.82			38,713	7.1%
Consumer Cyclical	36	174	0.27	9.26	25.71			96,288	17.6%
Consumer Non-Cyclical	24	79	0.24	11.10	49.84			65,165	11.9%
Energy	20	141	0.25	11.05	57.94			46,854	8.5%
Financials	36	130	0.44	109.82	41.45			82,174	15.0%
Healthcare	23	67	0.21	12.86	59.71			56,613	10.3%
Industrials	30	83	0.25	19.06	38.30			77,859	14.2%
Technology	18	148	0.21	14.72	89.80			41,445	7.6%
Telecommunications	3	149	0.42	71.37	123.37			6,658	1.2%
Utilities	15	115	0.47	15.12	18.41			36,625	6.7%

value:

$$\frac{\text{Short-term Debt} + \text{Long-term Debt}}{\text{Market Capitalization} + \text{Total Debt} + \text{Minority Interest} + \text{Preferred Stock} - \text{Cash}}. \quad (18)$$

A conservative approach to the financial leverage of financial institutions is in order. For banks, customer deposits do not appear in total debt, while cash on hand includes due from other banks. For insurance companies, policyholders' liabilities do not appear in total debt. The sample thus comprises estimates of the debt-to-asset ratio over the period 2008-2019. Notice that the fluctuations of the firm's market capitalization on top of the changes in total debt book value entail daily variations in the leverage dataset.

Table 1 describes the dataset. Panel A provides summary statistics for CDS levels, leverage data, and characteristics of the firms in the sample. Panel B summarizes the distribution of equity and CDS returns over the entire sample. Only 44.8% of equity-credit observations see stock prices and CDS spreads co-move as financial theory would expect, that is,  $\Delta\text{CDS} \cdot \Delta S < 0$ . This proportion is similar to the one reported by Kapadia and Pu (2012), who document that "short-term Merton model pricing discrepancies are common across firms, frequent, and economically large." Panel C breaks down the sample into business sectors. We notice that the two most leveraged industries are the Utilities (0.47) and Financials (0.44) business sectors.

## 5. Empirical Analysis

In this section, I first provide empirical evidence for the effect of the firm's financial leverage in the transmission of price information from the stock market to the credit market.

### 5.1 Information transmission

#### 5.1.1 Identifying pure stock innovations

I first describe the methodology for identifying true innovations in the stock market due to information revelation. For each firm, I run a regression of stock percentage changes on a constant, four lags of CDS percentage changes to absorb any transmission of delayed information from the credit market, and four stock return lags to capture any autocorrelation in the stock market. To take the elasticity of CDS returns relative to stock returns into account as predicted by the model and Equation (15), the specification also includes interactions of the CDS returns (both contemporaneous and lagged) with the firm's leverage. This approach starts from the conventional view that credit pricing information primarily flows from stocks to CDS (e.g., Hilscher et al., 2015).

Specifically, I estimate the following specification for each firm  $i$ :

$$\begin{aligned}
 (\text{Stock return})_t = & \alpha_i + \sum_{k=0}^4 \left[ \beta_{i,k} + \frac{\beta_{i,k}^\ell}{(\text{Leverage})_t} \right] (\text{CDS return})_{t-k} \\
 & + \sum_{k=1}^4 \gamma_{i,k} (\text{Stock return})_{t-k} + \epsilon_t.
 \end{aligned} \tag{19}$$

I view the residuals  $\epsilon_{i,t}$  from each of these regressions as independent innovations arriving in the stock market. These innovations are either not relevant or just not appreciated by the credit market at the time. By contrast, the coefficients  $\beta_{i,0}$  and  $\beta_{i,0}^\ell$  are akin to linear and “leveraged” measures of the feedback information flowing from the CDS market to the stock market. This approach is similar to the one by Acharya and Johnson (2007) and Qiu and Yu (2012), who isolate CDS market innovations at time  $t$  by controlling for both stock and CDS returns between  $t$  and  $t - k$ .

The contemporaneous linear response  $\beta_{i,0}$  is statistically significant at the 5% level for 22% of the firms. The contemporaneous leveraged response,  $\beta_{i,0}^\ell$ , is even more significant at 33%. For the sake of robustness, I then consider the following aggregated measures:

$$\beta_i := \sum_{k=0}^4 \beta_{i,k}, \quad \beta_i^\ell := \sum_{k=0}^4 \beta_{i,k}^\ell. \tag{20}$$

These measures capture the overall feedback information flowing from the CDS market to the stock

**Table 2. Feedback information from CDS to stock markets**

In the first stage, I run the time-series regression for each firm  $i$ :

$$(\text{Stock return})_t = \alpha_i + \sum_{k=0}^4 \left[ \beta_{i,k} + \frac{\beta_{i,k}^\ell}{(\text{Leverage})_t} \right] (\text{CDS return})_{t-k} + \sum_{k=1}^4 \gamma_{i,k} (\text{Stock return})_{t-k} + \epsilon_t \quad (19)$$

In the second stage, I sort firms into quintile portfolios based on the first-stage estimates of  $\beta_i = \sum_{k=0}^4 \beta_{i,k}$  (resp.  $\beta_i^\ell = \sum_{k=0}^4 \beta_{i,k}^\ell$ ), Q1 being the quintile with the smallest (most negative) estimates, and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the averages (across firms) of the time-series means of the characteristics for each firm. Within each quintile, robust  $p$ -values calculated via standard errors corrected for heteroscedasticity and serial correlation (Newey-West, 1987) are combined across firms via Fisher's sum of logarithms method (e.g., Heard and Rubin-Delanchy, 2018). \*\*\*, \*\* and \* denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. The sample period is 2008:01 to 2020:01. Data source: Thomson Reuters.

	Q1	Q2	Q3	Q4	Q5
<i>Panel A: Properties of firms in different <math>\beta</math>-quintiles</i>					
Average $\beta_i$	-5.341	-0.094	0.006	0.129	0.649
$t$ -statistic	(-1.14)	(-18.28)	(1.53)	(15.10)	(4.59)
Combined $p$ -value	0.000***	0.275	1.000	0.079	0.000***
Avg. firm leverage	0.30	0.24	0.24	0.29	0.38
Avg. CDS level (bps)	167	88	94	114	148
<i>Panel B: Properties of firms in different <math>\beta^\ell</math>-quintiles</i>					
Average $\beta_i^\ell$	-3.292	-0.970	-0.522	-0.151	9.253
$t$ -statistic	(-3.95)	(-50.44)	(-25.03)	(-9.35)	(1.19)
Combined $p$ -value	0.000***	0.000***	0.000***	1.000	0.001***
Avg. firm leverage	0.26	0.30	0.31	0.29	0.29
Avg. CDS level (bps)	102	119	104	131	156
‡ Firms	44	44	44	44	44

market at the firm level. The aggregated linear response  $\beta_i$  remains significant at the 5% level for only 18% of the firms. However, the level of statistical significance of the aggregated leveraged response  $\beta_i^\ell$  now rises to at least 49% of the firms. This gap in the level of statistical significance stands a first hint as to the role of the leverage effect in the feedback price transmission from credit markets to stock markets.

Table 2 sorts the firms into quintiles based on their aggregated response and examines the average characteristics for firms within each quintile. Panel A of Table 2 shows that the linear aggregated response  $\beta_i$  can be positive, in stark contrast with structural models of default risk. Neither the CDS level nor the leverage appears to vary much across quintiles. When combining  $p$ -values within quintiles, only the lowest and the highest quintiles display statistical significance. By contrast, Panel B of Table 2 shows that the aggregated leveraged response  $\beta_i^\ell$  is negative for

most firms, in line with structural models of credit risk. Moreover, the high degree of combined statistical significance is almost uniform across quintiles. In this sense,  $\beta_i$  and  $\beta_i^\ell$  appear as complementary measures of the feedback transmission channel existing from the CDS market to the stock market.

### 5.1.2 Panel regression evidence

I can now exploit the stock price innovations identified in the previous section to study the information flow from stock markets to credit markets. To select a proxy for the CDS market activity, I follow the literature on CDS determinants, which traditionally relies on CDS spread changes (e.g., Alexander and Kaeck, 2008; Ericsson et al., 2009; Zhang et al., 2009; Friewald et al., 2014; Meine et al., 2016; Han et al., 2017).<sup>13</sup> To bring to light the leverage effect predicted by the model and Equation (16), I regress spread CDS changes against the interaction of the stock innovations (both contemporaneous and lagged) with the firm's financial leverage. To entirely capture the marginal effect of stock innovations on CDS changes, I include its constitutive terms on top of the interaction term. The specification also contains four lags of CDS changes to purge the credit market of any residual autocorrelation.

More specifically, I estimate the following panel specification by pooled regression:

$$\begin{aligned}
(\Delta \text{CDS})_{i,t} = & \sum_{k=0}^4 b_k (\text{Stock innovation})_{i,t-k} + \beta_0 (\text{Leverage})_{i,t} + \beta_1 (\Delta \text{Leverage})_{i,t} \\
& + \sum_{k=0}^4 b_k^\ell (\text{Leverage})_{i,t} \times (\text{Stock innovation})_{i,t-k} + \sum_{k=1}^4 c_k (\Delta \text{CDS})_{i,t-k} \\
& + \beta_2 (\Delta r_{10})_t + \beta_3 (\Delta(r_{10} - r_2))_t + \beta_4 (\text{VIX})_t + \beta_5 (\Delta \text{VIX})_t + \beta_6 (\text{Ret}_{\text{S\&P}})_t + e_{i,t},
\end{aligned} \tag{21}$$

where the first-stage residuals  $\hat{e}_{i,t}$  provide a proxy for the real stock innovations of firm  $i$ ,  $r_2$  (resp.  $r_{10}$ ) measures the 2-year (resp. 10-year) Treasury bill rate, VIX measures the CBOE volatility index, and  $\text{Ret}_{\text{S\&P}}$  is the contemporaneous daily log-return on the S&P 500 stock index. **Interpret**

<sup>13</sup>CDS changes are more appropriate than CDS percentage changes as a proxy for CDS market activity because the latter are purged of any level-dependent effects by construction.

the linear combination  $\sum_{k=0}^4 b_k$  as a measure of the unconditional information flow from the stock market to the credit market. The linear combination  $\sum_{k=0}^4 b_k^\ell$  offers a measure of the interaction with the firm's leverage or "leveraged" information flow.

As control variables, the regression also includes the common macroeconomic CDS predictors highlighted in the literature on credit spread determinants (e.g., Collin-Dufresne et al., 2001; Ericsson et al., 2009). Several empirical studies have documented the negative relation between changes in credit spreads and changes in the level of interest rates (e.g., Longstaff and Schwartz, 1995; Duffee, 1998). Other empirical studies have also documented the slope of the yield curve's predictive power on future economic activity (e.g., Estrella and Hardouvelis, 1991). Many asset pricing studies have highlighted the impact of idiosyncratic equity volatility on corporate bond yields and CDS spreads (e.g., Campbell and Taksler, 2003; Zhang et al., 2009). Finally, several authors use the return of the aggregate stock market, usually proxied by the S&P 500 index, as an additional determinant of credit spreads (Collin-Dufresne et al., 2001; Ericsson et al., 2009).

To assess the statistical significance of the interaction effect, I partition the full sample into the two sub-samples of bidirectional co-movements ( $\Delta\text{CDS} \cdot \Delta S < 0$ ) and unidirectional co-movements ( $\Delta\text{CDS} \cdot \Delta S \geq 0$ ). Indeed, we cannot anticipate the sign of the interaction coefficient over the complete sample since more than 55% of the observations occur when CDS and stock prices co-move in the same direction due to market frictions, microstructure noise, delays in information transmission, and other limits to market integration.<sup>14</sup> As structural models of the firm (Merton, 1974) predict a negative credit-risk elasticity, we thus expect a negative interaction term only when the stocks and CDS prices co-move in opposite directions and reinforce market integration.

Table 3 reports estimates for the specification (21) with firm fixed effects. Columns (D) and (G) confirm that, when taken alone, the un-leveraged information flow  $\sum_{k=0}^4 \hat{b}_k$  is highly significant on each of the two sub-samples, with signs as financial theory would expect. Over the complete sample, however, these two effects cancel out, and the flow estimate appears to be no longer significant

---

<sup>14</sup>Equity-credit co-movements in the same direction may also have firm-specific informative content, as in the case of asset substitution (Jensen and Meckling, 1976). When stockholders of levered firms attempt to transfer wealth from the bondholders by substituting projects that increase the firm's riskiness, the value of the stockholders' equity rises, and so does the credit spread on bondholders' claims.

**Table 3. Information flow from stock market to CDS market**

This table reports OLS panel estimates and  $t$ -statistics for the coefficients of the following pooled regression:

$$(\Delta \text{CDS})_{i,t} = \sum_{k=0}^4 b_k (\text{Stock innovation})_{i,t-k} + \sum_{k=0}^4 b_k^\ell (\text{Leverage})_{i,t} (\text{Stock innovation})_{i,t-k} + \sum_{k=1}^4 c_k (\Delta \text{CDS})_{i,t-k} + \beta_0 (\text{Leverage})_{i,t} + \beta_1 (\Delta \text{Leverage})_{i,t} + \beta_2 (\Delta r_{10})_t + \beta_3 (\Delta (r_{10} - r_2))_t + \beta_4 (\text{VIX})_t + \beta_5 (\Delta \text{VIX})_t + \beta_6 (\text{Returns}_{\text{S\&P}})_t + e_{i,t} \quad (21)$$

The interest rate level (resp. slope) is measured by the 10-year Treasury bill rate  $r_{10}$  (resp. the difference  $r_{10} - r_2$  between the 10-year and 2-year Treasury bill rates). VIX measures the volatility index provided by the CBOE.  $\text{Returns}_{\text{S\&P}}$  is the contemporaneous daily log-return on the S&P 500 stock index.  $t$ -statistics in parentheses are calculated via standard errors clustered by date to correct for heteroscedasticity and cross-sectional correlation. \*\*\*, \*\* and \* denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. The sample period is 2008:01 to 2020:01. Data source: Thomson Reuters.

	Full sample			$\Delta \text{CDS} \cdot \Delta S < 0$			$\Delta \text{CDS} \cdot \Delta S \geq 0$		
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)
$\sum_{k=0}^4 b_k$	-14.827 (-0.82)	139.333*** (4.21)	138.993*** (5.22)	-127.272*** (-3.84)	181.975** (3.24)	181.118*** (5.00)	95.465*** (8.80)	30.434 (1.58)	30.255 (1.87)
$\beta_0$		-0.176 (-0.22)	0.217 (0.66)		-1.515 (-1.03)	-0.489 (-0.98)		-0.229 (-0.34)	-0.044 (-0.16)
$\sum_{k=0}^4 b_k^\ell$		-373.372** (-3.28)	-372.326*** (-3.83)		-690.615*** (-3.82)	-688.404*** (-5.15)		173.848* (2.36)	174.224* (2.81)
$\sum_{k=0}^4 c_k$	0.074 (0.43)	0.071 (0.41)	0.072 (0.57)	0.192 (0.89)	0.185 (0.86)	0.186 (2.14)	0.184** (2.84)	0.183** (3.08)	0.183* (2.79)
$\beta_1$	276.880*** (8.81)	280.299*** (7.83)	280.182*** (6.97)	310.121*** (6.44)	280.185*** (5.14)	279.597*** (5.66)	143.770*** (8.40)	193.117*** (9.35)	193.290*** (6.96)
$\beta_2$	-5.487*** (-3.42)	-5.072** (-3.08)	-5.066*** (-5.38)	-10.739*** (-3.69)	-10.86*** (-3.71)	-10.849*** (-4.73)	0.792 (1.20)	0.695 (1.07)	0.691 (1.09)
$\beta_3$	0.566 (0.25)	0.563 (0.25)	0.554 (0.53)	10.889** (2.73)	10.565** (2.83)	10.549*** (3.21)	-2.722* (-2.41)	-1.617 (-1.47)	-1.618 (-1.74)
$\beta_4$	0.011 (1.24)	0.013 (1.53)	0.012 (1.90)	-0.015 (-0.78)	-0.014 (-0.77)	-0.018 (-1.68)	0.006 (1.13)	0.007 (1.56)	0.007 (1.86)
$\beta_5$	0.109 (1.82)	0.088 (1.55)	0.088* (2.04)	-0.068 (-0.68)	-0.035 (-0.40)	-0.033 (-0.52)	0.138*** (3.31)	0.068 (1.71)	0.069* (2.10)
$\beta_6$	-102.697*** (-7.51)	-102.752*** (-9.08)	-102.832*** (-9.32)	-144.070*** (-6.10)	-167.028*** (-8.34)	-167.174*** (-8.68)	-71.176*** (-8.39)	-45.323*** (-5.95)	-45.305*** (-6.27)
Firm F.E.	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No
Industry F.E.	No	No	Yes	No	No	Yes	No	No	Yes
Clustered SE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted $R^2$	0.100	0.110	0.110	0.200	0.214	0.215	0.094	0.154	0.155
Observations	548,394	548,394	548,394	245,698	245,698	245,698	302,696	302,696	302,696

in column (A). The main finding of Table 3 is located in column (E), where the interaction term  $\sum_{k=0}^4 \widehat{b}_k^\ell$  is economically substantial and statistically significant at the 0.1% threshold. The estimate has the awaited negative sign predicted by structural models of credit risk. Its magnitude ( $-690.6$ ) is more than five times that of the flow of direct, un-leveraged information ( $-127.3$ ) taken alone. Even though roughly 55% of overall equity-credit observations lack firm-specific informative content, this newly-identified channel of transmission is strong enough for the coefficient to remain negative and significant at the 1% threshold when we estimate the pooled regression on the full sample in column (B). ~~More importantly, as soon as we include the interaction term in regressions (B) and (E), the un-leveraged estimate  $\sum_{k=0}^4 \widehat{b}_k$  becomes significantly positive in contrast with financial theory.~~ Everything happens as if the interaction with the corporate leverage impounds the main channel of information flowing from stocks to credit markets.

Table 3 provides an additional robustness test to ensure that the industries with high leverage, such as the Utilities and Financials business sectors, do not drive the previous results entirely. Columns (C), (F), and (I) show that including industry fixed effects instead of firm fixed effects generates no systematic departure from our main results. At the very least, including industry fixed effects improves the statistical significance of the interaction term marginally.

An alternative portfolio approach is also useful to study how information transmission varies with the extent of firm leverage. In the first stage, every year of the sample period, I sort all the firms by leverage into five groups. In the second stage, I run the following panel regression within each of the five leverage-sorted portfolios:

$$(\Delta \text{CDS})_{i,t} = a + \sum_{k=0}^4 b_k (\text{Stock innovation})_{i,t-k} + \sum_{k=1}^4 c_k (\Delta \text{CDS})_{i,t-k} + e_{i,t}, \quad (22)$$

where the first-stage residuals  $\widehat{\varepsilon}_{i,t}$  provide a proxy for the real stock innovations of firm  $i$ . Notice that no specific conditioning based on leverage is imposed on the variables. The goal is then to compare the magnitude of the information flow from stocks to CDS,  $\sum_{k=0}^4 b_k$ , across the leverage-sorted groups of firms.

**Table 4. Information flow as a function of the leverage**

In the first stage, I sort firms every year into quintile portfolios based on their average annual leverage, Q1 being the quintile with the smallest leverage and Q5 being the quintile with the largest leverage. In the second stage, I run the following panel regression within each quintile portfolio of firms:

$$(\Delta \text{CDS})_{i,t} = \sum_{k=0}^4 b_k (\text{Stock innovation})_{i,t-k} + \sum_{k=1}^4 c_k (\Delta \text{CDS})_{i,t-k} + e_{i,t}. \quad (22)$$

This table reports OLS panel estimates and  $t$ -statistics for the regression coefficients. The summary statistics reported for each quintile are the averages (across firms) of the time-series means of the characteristics for each firm.  $t$ -statistics in parentheses are calculated via standard errors clustered by date to correct for heteroscedasticity and cross-sectional correlation. \*\*\*, \*\* and \* denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. The sample period is 2008:01 to 2020:01. Data source: Thomson Reuters.

	Q1	Q2	Q3	Q4	Q5
<i>Sort variable: firm leverage</i>					
Mean	0.09	0.17	0.24	0.35	0.60
<i>Portfolio characteristics</i>					
$\sum_{k=0}^4 b_k$	-22.416*** (-11.80)	-32.945*** (-11.18)	-41.881*** (-8.79)	-81.542*** (-12.64)	-205.293*** (-4.48)
$\sum_{k=1}^4 c_k$	0.078 (2.17)	0.141*** (5.38)	0.215*** (4.90)	0.138*** (3.99)	0.066 (0.37)
Avg. CDS level (bps)	54	67	88	133	253
Firm F.E.	Yes	Yes	Yes	Yes	Yes
Adjusted $R^2$	0.012	0.019	0.039	0.031	0.101
Observations	109,580	109,542	109,535	109,548	109,577

Table 4 reports estimates for the specification (22). For each leverage-sorted portfolio, the information flow estimate,  $\sum_{k=0}^4 \hat{b}_k$ , appears to be negative and statistically significant at the 0.1% threshold. The main finding here is the monotonic relationship between the information flow's estimate and firm leverage and CDS level. The flow estimate's absolute value increases from the portfolio of firms with the lowest leverage (Q1) to the portfolio of firms with the highest leverage (Q5). Between Q1 and Q5, a ninefold increase in the flow estimate corresponds to a sixfold increase in leverage and a fivefold increase in the CDS level. As a robustness test, I also estimate specification (22) with industry fixed effects (unreported). These tests show quasi-identical results to the estimation with firm fixed effects and ward off the possibility of an industry-induced bias in our pooled-regression findings.

### 5.1.3 Firm-level regression evidence

The pooled regression described above forces all firms to have the same dynamic properties. I now estimate separate dynamics for each firm. I also include five lags of unleveraged stock innovations to compare the intensity of the leveraged, non-linear information flow with the direct transmission of information. This alternative specification allows testing for differences among nested models by running a likelihood ratio (LR) test. The LR test statistic then measures whether the inclusion of leveraged regressors significantly improves the goodness of fit of the regression model.

Specifically, I estimate the following specification for each firm  $i$ :

$$\begin{aligned}
 (\Delta \text{CDS})_t = & a_i + \sum_{k=0}^4 \left[ b_{i,k} + b_{i,k}^\ell (\text{Leverage})_t \right] \times (\text{Stock innovation})_{t-k} \\
 & + \sum_{k=1}^4 c_{i,k} (\Delta \text{CDS})_{t-k} + e_t
 \end{aligned} \tag{23}$$

where the first-stage residuals  $\hat{\varepsilon}_{i,t}$  provide a proxy for the real stock innovations of firm  $i$ . I then consider the following aggregated measures for each firm  $i$ :

$$b_i := \sum_{k=0}^4 b_{i,k}, \quad b_i^\ell := \sum_{k=0}^4 b_{i,k}^\ell. \tag{24}$$

Table 5 reports estimates for the specification (23). Panel A shows the summary statistics for the estimated linear responses  $b_i$ . We observe a positive and statistically insignificant mean at 61.3. By contrast, the mean of the leveraged response  $b_i^\ell$  is  $-207.7$  and significantly different from zero, thereby validating the non-linear role of the leverage. As a robustness check, I also run for each firm an LR test of the null hypothesis  $H_0 : b_{i,0}^\ell = \dots = b_{i,4}^\ell = 0$ . This procedure provides a collection of independent LR test statistics and  $p$ -values. I then use Fisher's sum of the logarithms method (e.g., Heard and Rubin-Delanchy, 2018) to combine these  $p$ -values and to assess whether the inclusion of leveraged predictor variables improves the model's goodness of fit. Panel A reveals that this combined LR  $p$ -value is highly significant at the 0.1% level.

**Table 5. Firm-level information flow from stocks to CDS**

In the first stage, I run the time-series regression for each firm  $i$ :

$$(\Delta \text{CDS})_t = a_i + \sum_{k=0}^4 [b_{i,k} + b_{i,k}^\ell (\text{Leverage})_t] (\text{Stock innovation})_{t-k} + \sum_{k=1}^4 c_{i,k} (\Delta \text{CDS})_{t-k} + e_t. \quad (23)$$

For each firm, a likelihood ratio test measures rejection of the null hypothesis  $H_0 : b_{i,0}^\ell = \dots = b_{i,4}^\ell = 0$ .  $LR$  is the combination across firms of  $p$ -values via Fisher's sum of the logarithms method (e.g., Heard and Rubin-Delanchy, 2018). In the second stage, I sort firms into quintile portfolios based on the first-stage estimates of  $b_i^\ell := \sum_{k=0}^4 b_{i,k}^\ell$ , Q1 being the quintile with the smallest (most negative) estimates, and Q5 being the quintile with the largest estimates. The summary statistics reported for each quintile are the averages (across firms) of the time-series means of the characteristics for each firm. Within each quintile,  $LR$  is the combination across firms of  $p$ -values from likelihood ratio tests via Fisher's sum of the logarithms method. \*\*\*, \*\* and \* denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. The sample period is 2008:01 to 2020:01. Data source: Thomson Reuters.

<i>Panel A: Univariate properties of <math>b_i</math> and <math>b_i^\ell</math></i>					
Average $b_i$	61.3		Average $b_i^\ell$		-207.7
$t$ -statistic	(1.42)		$t$ -statistic		(-2.78)
			$LR$ (combined $p$ -val.)		0.000***
Min	-1,551.2		Min		-10,873.8
Max	7,201.1		Max		1,917.6
	Q1	Q2	Q3	Q4	Q5
<i>Panel B: Properties of firms in different <math>b_i^\ell</math>-quintiles</i>					
Average $b_i^\ell$	-1,222.9	-212.0	-55.2	55.6	396.0
$t$ -statistic	(-3.80)	(-21.32)	(-9.90)	(12.06)	(6.89)
$LR$ (combined $p$ -val.)	0.000***	0.993	0.999	0.084	0.000***
Avg. firm leverage	0.37	0.31	0.25	0.22	0.30
Avg. CDS level (bps)	225	113	85	78	111
# Firms	44	44	44	44	44
Obs.	108,479	113,255	113,255	101,930	107,978

Panel B sorts the firms into quintile portfolios of firms based on their aggregated interaction coefficient  $b_i^\ell$ . These average estimates are significantly different from zero in every quintile. The combined  $LR$   $p$ -value turns out to be highly significant in the most leveraged portfolio (Q1). More importantly, Panel B reports the average firm leverage and credit risk (CDS level) within each quintile. One observes a decreasing pattern of firm leverage as well as CDS level across the portfolios with a negative interaction estimate (Q1-Q3). The interaction term's absolute value decreases from the quintile with the highest leverage (Q1) to the quintile with the lowest leverage (Q4). In summary, the information transmission at the firm level is more pronounced when firms' leverage is high.

## 5.2 Co-integration and price discovery

### 5.2.1 Co-integration

In this section, I now investigate the role of the leverage effect in the co-integration of credit and equity markets. Equation (17) suggests that the log-CDS price and the leveraged log-stock price should form a co-integrated system. The intuition is that CDS spread and stock price time series cannot drift too far apart from the equilibrium because capital structure arbitrageurs will act to restore the long-run equilibrium relationship.

The testing procedure draws on the VECM approach to co-integration modeling suggested by Johansen (1988, 1991). Let introduce for each company  $i$  the credit-equity log-price process:

$$\mathbf{X}_{i,t} := \begin{bmatrix} \ln(\text{CDS})_{i,t} \\ \ln(\text{Stock})_{i,t} \end{bmatrix} \quad (25)$$

A preliminary step consists in checking that the two components of the log-price process are indeed integrated to the same order. For this purpose, I run systematic unit root tests for nonstationarity.

In the first stage, I then select each firm's optimal lag length by fitting a VAR model for the leveraged log-price process:

$$\mathbf{X}_{i,t}^\ell := \begin{bmatrix} \ln(\text{CDS})_{i,t} \\ (\text{Leverage})_{i,t} \cdot \ln(\text{Stock})_{i,t} \end{bmatrix}. \quad (26)$$

The order of the VAR,  $p_i$ , is selected by the Schwarz Bayesian information criterion. In the second stage, I estimate the following fully specified VECM by maximum likelihood:

$$\Delta \mathbf{X}_{i,t}^\ell = \Pi_i \mathbf{X}_{i,t-1}^\ell + \sum_{k=1}^{p_i} \Gamma_{i,k} \Delta \mathbf{X}_{i,t-k}^\ell + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}, \quad (27)$$

where  $\Pi_i$  is the long-run impact matrix,  $\Gamma_{i,k}$  are short-run impact matrices,  $\boldsymbol{\mu}_i$  is a drift vector, and  $\mathbf{u}_{i,t} = (u_{i,t}^1, u_{i,t}^2)'$  are independent 2-dimensional Gaussian disturbances. Assuming  $\mathbf{X}_{i,t}^\ell$  is co-

**Table 6. Co-integration tests results**

This table reports co-integrated firms for which the Johansen trace statistic rejects the null hypothesis of non-co-integration  $H_0 : \text{rank}(\Pi_i) = 0$  when estimating the VECM  $\Delta \mathbf{X}_{i,t}^\ell = \Pi_i \mathbf{X}_{i,t-1}^\ell + \sum_{k=1}^{p_i} \Gamma_{i,k} \Delta \mathbf{X}_{i,t-k}^\ell + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}$  (resp.  $\Delta \mathbf{Y}_{i,t}^\ell = \Pi_i \mathbf{Y}_{i,t-1}^\ell + \sum_{k=1}^{p_i} \Gamma_{i,k} \Delta \mathbf{Y}_{i,t-k}^\ell + \boldsymbol{\mu}_i + \mathbf{u}_{i,t}$ ) by maximum likelihood. A time trend is included in the long-run relation. Panel A reports the number and proportion of co-integrated entities by threshold of co-integration. \*\*\*, \*\* and \* denote statistical significance at the 0.1%, 1%, and 5% levels, respectively. Panel B reports firm-level sample statistics computed across all observations. Data source: Thomson Reuters.

	$p < 0.001^{***}$		$p < 0.01^{**}$		$p < 0.05^*$		Total
	# Firms	Prop. (%)	# Firms	Prop. (%)	# Firms	Prop. (%)	
<i>Panel A: number of co-integrated firms</i>							
Full sample	51	23.7	80	37.2	106	49.3	215
Basic Materials	4	26.7	4	26.7	5	33.3	15
Consumer Cyclical	6	17.1	8	22.9	14	40.0	35
Consumer Non-Cyc.	3	13.0	6	26.1	9	39.1	23
Energy	3	15.8	7	36.8	8	42.1	19
Financials	19	52.8	23	63.9	26	72.2	36
Healthcare	2	8.7	9	39.1	13	56.5	23
Industrials	9	30.0	12	40.0	12	40.0	30
Technology	1	6.2	3	18.8	6	37.5	16
Telecommunications	0	0.0	1	33.3	3	100.0	3
Utilities	4	26.7	7	46.7	10	66.7	15
	5 <sup>th</sup> perc.	Median	Mean	75 <sup>th</sup> perc.	95 <sup>th</sup> perc.	SD	Obs.
<i>Panel B: firm-level statistics of co-integrated firms</i>							
Firm CDS level (bps)	26	75	117	130	309	204	241,054
Firm leverage	0.08	0.25	0.32	0.42	0.80	0.22	241,054
Firm size (\$bn)	4.50	20.08	49.20	58.64	187.60	68.63	241,054
Firm debt (\$bn)	1.24	6.91	42.05	16.86	358.51	117.00	241,054
<i>Panel C: equity and CDS returns of co-integrated firms</i>							
Equity daily return (%)	-2.81	0.05	0.02	0.86	2.73	2.14	241,054
CDS daily return (%)	-3.44	0.00	-0.02	0.06	3.41	5.47	241,054

integrated implies that the rank of  $\Pi_i$  can neither be null due to the error correction mechanism nor equal to 2 due to nonstationarity. As a result,  $\Pi_i$  must be of rank 1, and there must exist two vectors  $\boldsymbol{\alpha}_i = (\alpha_{i1}, \alpha_{i2})'$  and  $\boldsymbol{\beta}_i = (1, -\beta_i)'$  such that  $\Pi_i = \boldsymbol{\alpha}_i \boldsymbol{\beta}_i'$ . In this case, even though  $\mathbf{X}_{i,t}^\ell$  is not stationary, the stochastic deviations from the long-run equilibrium must be stationary around a potential deterministic trend:

$$\eta_{i,t} := \boldsymbol{\beta}_i' \mathbf{X}_{i,t}^\ell + \rho_i t = \ln(\text{CDS})_{i,t} - \beta_i (\text{Leverage})_{i,t} \ln(\text{Stock})_{i,t} + \rho_i t \sim I(0). \quad (28)$$

I allow for a linear time trend in the co-integrating relationship to accommodate the trending nature of data like stock prices.

I use Johansen's (1988, 1991) likelihood ratio statistic to test for the rank of the long-run impact matrix. Rejecting the null hypothesis that  $\text{rank}(\Pi_i) = 0$  validates the existence of a co-integrating vector  $\beta_i$ . I repeat the testing procedure with the symmetric process:

$$\mathbf{Y}_{i,t}^\ell := \begin{bmatrix} \ln(\text{CDS})_{i,t}/(\text{Leverage})_{i,t} \\ \ln(\text{Stock})_{i,t} \end{bmatrix}. \quad (29)$$

Finally, I consider an entity  $i$  as co-integrated if either  $\mathbf{X}_{i,t}^\ell$ , or  $\mathbf{Y}_{i,t}^\ell$ , or both processes reject the null of non-co-integration at the statistical level of 5%.

Table 6 reports co-integration results. In a preliminary step, I run Augmented Dickey-Fuller tests to verify that the stock price and CDS spread time series share the same order of integration. To conserve space, I do not present the results of these tests, which provide unambiguous evidence that all the series are  $I(1)$  and their first differences  $I(0)$ . These results are typical for asset price time series.

Panel A reports the number of significantly co-integrated firms that qualify for the VECM stage. We find support for co-integration at the 5% level for 106 entities out of 215, which represents 50% of the firms. A third of the sample does not hint at co-integration at all, while 20 firms barely miss the 5% rejection threshold. The Financials, Telecommunications, and Utilities business sectors appear the most co-integrated, with co-integration ratios close or above 70%. Without surprise, Table 1 reveals that these three sectors are also the most dependent on external debt financing, with average debt-to-asset ratios well over 40%. This finding is the first hint of the role of the leverage effect in the integration of credit and equity markets.

Panel B reports summary statistics for co-integrated entities at the 5% threshold. The average debt-to-asset ratio (resp. CDS level) in this sub-sample is 3 points (resp. 4 basis points) higher. In the case of deeply co-integrated entities at the 0.1% level, unreported statistics show that this leverage differential is even more acute with an excess 4.5 points (resp. 14 basis points). With an average market capitalization of 44.5\$bn versus 49.2\$bn, the size of co-integrated firms seems on par with the overall sample. Their indebtedness, however, is much more pronounced with an

average debt size of 42.1\$bn versus 27.9\$bn. Everything happens as if an increase in corporate leverage ramps up market activity in capital structure arbitrage. These additional findings provide reliable evidence that one of the leading market effects of corporate leverage is to intensify the integration between the credit and equity markets.

Panel C reports summary statistics for daily stock and CDS returns of co-integrated firms. While stock returns statistics are similar to the full sample, the standard deviation of CDS returns is much more pronounced, with 5.47% versus 4.53%. This finding is even more spectacular for highly co-integrated entities with an (untabulated) standard deviation at 6.85%. Without surprise, some excess volatility in CDS prices appears necessary to stimulate capital structure arbitrage activity.

### 5.2.2 *Decomposition of price discovery*

I now study the contribution of each market to the price discovery process. The co-integrating feature suggests adopting the classical vector error-correction model (VECM) approach to price discovery formalized by Gonzalo and Granger (1995). The intuition is that error-correcting adjustments to transitory shocks must occur in either the stock market or the CDS market to maintain the long-run equilibrium relationship between both time series.

In the first stage, I test at the level of each firm  $i$  the co-integration of the CDS and stock price series following the methodology of Section 5.2.1. For those entities which are significantly co-integrated at the 5% threshold, I retrieve the co-integrating residuals  $\hat{\eta}_{i,t}$  and the lag order  $p_i$  providing the optimal fit for the underlying VAR process.

In the second stage, I measure the contribution of each market to the price discovery process

by estimating the following VECM by OLS:

$$\begin{aligned}
(\text{CDS return})_{i,t} &= \alpha_1 \hat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{1,k} (\text{CDS return})_{i,t-k} + u_{i,t}^1, \\
(\text{Stock return})_{i,t} &= \alpha_2 \hat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + u_{i,t}^2,
\end{aligned} \tag{30}$$

where the first-stage co-integrating residuals  $\hat{\eta}_{i,t}$  provide error-correcting terms, the lagged stock and CDS returns capture market imperfections, and  $u_{i,t}^{1,2}$  are i.i.d. disturbances. If the stock market is contributing significantly to the price discovery process, then  $\alpha_1$  should be negative and statistically significant as the CDS market continuously adjusts to absorb transitory noise frictions. Conversely, if the CDS market dominates the price discovery process,  $\alpha_2$  should be positive and statistically significant as the stock market continuously responds to transitory shocks.

Comparing the adjustment coefficients  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  allows estimating the market that least adjusts to transitory deviations. This market will stand as the closest to the fundamental value of credit risk. Following the literature on credit price discovery (e.g., Narayan et al., 2014), I use the component share (CS) metric to measure the relative shares of each market in the permanent component:

$$\text{CS}_{\text{CDS}} := \frac{\alpha_2}{\alpha_2 - \alpha_1}, \quad \text{and} \quad \text{CS}_{\text{Stock}} := \frac{\alpha_1}{\alpha_1 - \alpha_2}, \tag{31}$$

provided that  $\hat{\alpha}_1 \neq \hat{\alpha}_2$ .<sup>15</sup> I also rely on the information share (IS) metric to measure the relative adjustment speed of each market to innovations in the permanent component. The lower bound and upper bound of information share are given as follows (Hasbrouck, 1995):

$$\text{IS}^{\text{Low}} := \frac{\alpha_2^2 (\sigma_1^2 - \sigma_{12}^2 / \sigma_2^2)}{\alpha_2^2 \sigma_1^2 - 2\alpha_1 \alpha_2 \sigma_{12} + \alpha_1^2 \sigma_2^2}, \quad \text{IS}^{\text{Up}} := \frac{(\alpha_2 \sigma_1 - \alpha_1 \sigma_{12} / \sigma_1)^2}{\alpha_2^2 \sigma_1^2 - 2\alpha_1 \alpha_2 \sigma_{12} + \alpha_1^2 \sigma_2^2}, \tag{32}$$

where  $\text{Var}(u_t^1) := \sigma_1^2$ ,  $\text{Var}(u_t^2) := \sigma_2^2$ , and  $\text{cov}(u_t^1, u_t^2) := \sigma_{12}^2$ . I follow the standard practice in the

---

<sup>15</sup>Notice that  $0 \leq \text{CS}_{\text{CDS}} \leq 1$  as soon as  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  have the expected negative and positive sign, respectively. If  $\hat{\alpha}_1 = 0$ , there is no evidence of price discovery in the stock market ( $\text{CS}_{\text{CDS}} \equiv 1$ ). If  $\hat{\alpha}_2 = 0$  there is no evidence of price discovery in the CDS market ( $\text{CS}_{\text{CDS}} \equiv 0$ ).

price discovery literature (e.g., Baillie et al., 2002) to average the lower and upper bounds:

$$IS_{CDS} := \frac{IS_{CDS}^{Low} + IS_{CDS}^{Up}}{2}, \quad \text{and} \quad IS_{Stock} := \frac{IS_{Stock}^{Low} + IS_{Stock}^{Up}}{2}. \quad (33)$$

Table 7 reports price discovery metrics for the stock and CDS markets. We restrict the analysis to the sub-sample of 106 firms that reject the null hypothesis of non-co-integration at the 5% threshold.

Panel A reports the stock market’s price discovery metrics. For the stock market to significantly impact the efficient price of credit, the adjustment coefficient  $\hat{\alpha}_1$  must be negative and statistically significant. Co-integrated firms are thus ranked into deciles based first on their increasing component share, and second on their decreasing leverage. For more than half of the firms, a component share close or equal to 100% indicates a high degree of proximity of the stock market with the permanent component of the credit-equity price system. As a result, the stock market monopolizes the price discovery process with a market share of 74.8% on average. To a lesser extent, the information share metric confirms the informational leadership of the stock market, with an average market share of 55.5%. This result is consistent with the CDS “sideshow” hypothesis (Hilsher et al. 2015) for which informed traders globally favor the stock market to the CDS market because of transaction costs. We notice that the last three deciles (D8–D10) concentrate firms with average leverage well below the overall sample mean (0.29).

Panel B reports the smaller set of firms whose CDS market heavily weighs on the permanent component of the credit-equity price system. In this case, the adjustment coefficient  $\hat{\alpha}_2$  must be positive and statistically significant. We thus focus on the 49 firms whose  $\hat{\alpha}_2$  turns out to be significant at the 5% level. Confined to the last three deciles (D8–D10), Panel C reveals strong evidence for the role of the leverage effect. With a component share near 100%, CDS prices are closely aligned with the fundamental value of credit as they preempt the bulk of informed trading. The top decile (D10) contains highly-leveraged firms with leverage above the 95<sup>th</sup> percentile (0.58)

**Table 7. CDS and equity market shares of the price discovery process**

This table reports price discovery metrics for the stock and CDS markets. In the first stage, I select the co-integrated firms for which the Johansen trace statistic rejects the null of non-co-integration at the 5% threshold and retrieve error-correcting terms  $\hat{\eta}_{i,t}$ . In the second stage, I estimate the following VECM by OLS:

$$\begin{aligned}
 (\text{CDS return})_{i,t} &= \alpha_1 \hat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{1,k} (\text{Leverage})_{i,t} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{1,k} (\text{CDS return})_{i,t-k} + u_{i,t}^1, \\
 (\text{Stock return})_{i,t} &= \alpha_2 \hat{\eta}_{i,t-1} + \sum_{k=1}^{p_i} b_{2,k} (\text{Stock return})_{i,t-k} + \sum_{k=1}^{p_i} c_{2,k} \frac{(\text{CDS return})_{i,t-k}}{(\text{Leverage})_{i,t}} + u_{i,t}^2,
 \end{aligned} \tag{30}$$

I assess the statistical significance of the adjustment coefficients  $\hat{\alpha}_1$  and  $\hat{\alpha}_2$  via robust standard errors corrected for heteroscedasticity and serial correlation (Newey-West, 1987). Panel A reports co-integrated firms for which  $\hat{\alpha}_1$  is significant at the 5% level, ranked into deciles based on the second-stage estimate of the stock market's component share,  $CS_{\text{Stock}}$ . Panel B reports firms for which  $\hat{\alpha}_2$  is significant at the 5% level, ranked into deciles based on the second-stage estimate of the CDS market's component share,  $CS_{\text{CDS}}$ . Panel C reports firms for which  $\hat{\alpha}_2$  is significant at the 5% level, ranked into deciles based on the second-stage estimate of the CDS market's information share,  $IS_{\text{Stock}}$ . The summary statistics reported for each decile are the averages (across firms) of the time-series means of the characteristics for each firm. Data source: Thomson Reuters.

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Total
<i>Panel A: stock market</i>											
Average component share, $CS_{\text{Stock}}$ (%)	0.0	26.8	53.5	80.3	96.8	100.0	100.0	100.0	100.0	100.0	74.8
Average information share, $IS_{\text{Stock}}$ (%)	7.4	25.7	49.2	61.4	76.4	65.8	67.9	66.0	65.3	78.6	55.5
Average firm leverage	0.41	0.30	0.21	0.30	0.58	0.45	0.31	0.23	0.17	0.08	0.31
Average CDS level (bps)	142	108	87	121	192	131	141	88	62	51	115
Number of firms	11	11	11	11	11	11	11	11	11	7	106
<i>Panel B: CDS market's component share</i>											
Average component share, $CS_{\text{CDS}}$ (%)	7.0	13.8	23.6	38.3	50.0	50.0	71.0	100.0	100.0	100.0	54.5
Average firm leverage	0.32	0.17	0.44	0.18	0.20	0.41	0.29	0.17	0.34	0.60	0.31
Average CDS level (bps)	82	78	174	58	94	155	115	51	134	184	111
Number of firms	5	5	5	5	5	5	5	5	5	4	49
<i>Panel C: CDS market's information share</i>											
Average information share, $IS_{\text{CDS}}$ (%)	18.6	30.9	40.6	47.2	53.4	59.2	75.6	91.2	96.5	99.1	60.5
Average firm leverage	0.20	0.21	0.27	0.39	0.35	0.40	0.43	0.38	0.18	0.24	0.31
Average CDS level (bps)	63	98	133	105	102	195	84	154	66	108	111
Number of firms	5	5	5	5	5	5	5	5	5	4	49

of the average debt-to-asset distribution by firms. In this decile, the average CDS level rises above 200 basis points, a level corresponding to a credit rating lower than Baa2/BBB.

Panel C examines the same set of firms as Panel C from a slightly different standpoint. Whereas Panel C focuses on the CDS market's relative avoidance of noise, the focus now shifts to the relative speed of adjustment to innovations in the permanent component of credit. The average information share of the CDS market (60.5%) is in line with market shares reported in the recent study by Kryzanowski et al. (2017).

## **6. Conclusions**

A parsimonious structural framework is sufficient to build a theoretical model connecting the firm's financial leverage and the variance-equity elasticity. This elasticity amounts to twice the debt-to-assets ratio—a standard measure of the corporate leverage. This key feature enables putting the so-called “leverage effect” into a credit risk perspective, thus giving its full meaning to a four-decade-old term (Black, 1976). It provides a non-linear mechanism of information transmission between the equity and credit markets.

An empirical analysis over a large dataset of S&P 500 firms and an extended timeframe (2008-2019) highlights the non-linear role of the corporate leverage in the transmission of price information between stock markets and credit markets. Based on the dynamic interaction between firm leverage and stock price innovations, this newly-identified channel of transmission impounds the flow of information transiting from stocks to CDS markets. It affects all firms uniformly, irrespective of their level of indebtedness, their CDS spread quoted in the market, or their business sector. As the corporate leverage increases, it stimulates market activity in capital structure arbitrage and strengthens the co-integration of credit and equity markets. The strength of the long-run equilibrium uniting CDS spreads with stock prices correlates with the debt-to-asset ratio for co-integrated firms.

In line with previous studies, two-thirds of the firms in the sample see their price discovery process widely dominated by the equity market, with stocks impounding more than 70% of the

process. However, I find a significant portion of highly-leveraged firms for which half of the discovery process or more is occurring in the CDS market. The leverage effect could explain some of the pricing discrepancies observed between stock and CDS markets. The recent literature usually attributes these mispricings to various CDS market inefficiencies such as illiquidity or opaqueness. By contrast, the leverage effect provides an economic rationale for the limits to capital structure arbitrage and the lack of integration between equity and credit markets.

### Appendix A. Proof of Lemma 1

A standard application of Ito's lemma shows that the firm's asset volatility is linked to the equity instantaneous volatility by the relationship:

$$\sigma S = \frac{\partial S}{\partial V} \sigma_v V. \quad (\text{A.1})$$

This can be differentiated with respect to the equity market value to get:

$$\frac{\partial \sigma}{\partial S} = \frac{\partial}{\partial S} \left( S_v \sigma_v \frac{V}{S} \right) = \frac{\partial S_v}{\partial S} \times \sigma_v \frac{V}{S} + S_v \sigma_v \times \frac{S - V}{S^2}, \quad (\text{A.2})$$

where I have used the fact that the firm's business risk,  $\sigma_v$ , is a constant independent from  $S$  and the firm's capital structure. I can apply the chain rule

$$\frac{\partial S_v}{\partial S} = \frac{\partial S_v}{\partial V} \frac{\partial V}{\partial S} = \frac{S_{VV}}{S_v}, \quad (\text{A.3})$$

which yields after substitution into Equation (A.2):

$$S \frac{\partial \sigma}{\partial S} = \frac{S_{VV}}{S_v} \sigma_v V - S_v \sigma_v \frac{V - S}{S}. \quad (\text{A.4})$$

Equation (A.1) also enables to express the unknown asset volatility  $\sigma_v = \sigma S / (S_v V)$  which can now be eliminated from Equation (A.4). Noticing that the debt-to-asset ratio  $\ell = (V - S) / V$ , I find:

$$S \frac{\partial \sigma}{\partial S} = \sigma \frac{SS_{VV}}{S_v^2} - \sigma \ell. \quad (\text{A.5})$$

which yields Equation (3) in a straightforward way.

It remains to be checked that the adjusting term  $\varepsilon_\ell = (SS_{VV}) / S_v^2$  to the leverage is bounded on  $[V_B; \infty)$ . In the structural setting of Duffie and Lando (2001), the bankruptcy boundary is given by:

$$V_B = -\frac{C\gamma}{A(1+\gamma)}, \quad (\text{A.6})$$

while the optimal equity market value is available as an explicit function of the asset value:

$$S = AV - AV_B \left( \frac{V}{V_B} \right)^{-\gamma} + C \left[ 1 - \left( \frac{V}{V_B} \right)^{-\gamma} \right], \quad (\text{A.7})$$

where:

$$A := \frac{\delta}{r - \mu_v - \sigma_v^2 / 2}, \quad C := (\theta - 1) \frac{c}{r}, \quad \gamma := \frac{\mu_v + \sqrt{\mu_v^2 + 2r\sigma_v^2}}{\sigma_v^2}. \quad (\text{A.8})$$

To prove (a), simple calculations show that when  $V$  goes to  $\infty$ , we have  $S \sim AV$ ,  $S_v \sim A$ , and  $S_{VV} \sim 1/V^{\gamma+2}$  so that:

$$\frac{SS_{VV}}{S_v^2} \sim \frac{1}{AV^{\gamma+1}} \longrightarrow 0. \quad (\text{A.9})$$

To prove (b), notice that when  $V$  tends to  $V_B$ , there is an indeterminate form  $\frac{0}{0}$ . Applying l'Hôpital's rule yields:

$$\lim_{V \rightarrow V_B} \varepsilon_\ell = \lim_{V \rightarrow V_B} \frac{S}{S_v^2} \cdot \lim_{V \rightarrow V_B} S_{VV} = \lim_{V \rightarrow V_B} \frac{1}{2S_{VV}} \cdot \lim_{V \rightarrow V_B} S_{VV} = \frac{1}{2}. \quad (\text{A.10})$$

Table A.1 reports numerical simulations for  $\varepsilon_\ell$  computed with the numerical assumptions from Duffie and Lando (2001). In accordance with the previous asymptotic results (A.9) and (A.10),  $\varepsilon_\ell$  is always less than half the debt-to-asset ratio. It takes low asset values and deep states of financial

distress to produce values of the same magnitude as the firm's leverage.

**Table A.1. Numerical magnitude of the financial leverage adjustment  $\varepsilon_\ell$**

This table reports the numerical magnitude of  $\varepsilon_\ell$  as a function of the firm's asset value  $V$  for five different bankruptcy boundaries corresponding to five different levels of asset volatility. The corresponding debt-to-asset ratio  $\ell$  is also reported. Pricing assumptions are those of Duffie and Lando (2001): annual debt interest charge  $c = 8.00$ , corporate tax rate  $\theta = 0.35$ , bankruptcy costs 0.3, payout rate  $\delta = 0.05$ , risk-free rate  $r = 0.06$  per annum, asset growth rate  $\mu_V \equiv 0.01$  per annum.

$V$	Bankruptcy boundary									
	$\sigma_V = 0.25$		$\sigma_V = 0.20$		$\sigma_V = 0.15$		$\sigma_V = 0.10$		$\sigma_V = 0.05$	
	$V_B = 45.34$		$V_B = 49.30$		$V_B = 54.88$		$V_B = 63.37$		$V_B = 78.01$	
	$\varepsilon_\ell$	$\ell$	$\varepsilon_\ell$	$\ell$	$\varepsilon_\ell$	$\ell$	$\varepsilon_\ell$	$\ell$	$\varepsilon_\ell$	$\ell$
$V_B$	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00	0.50	1.00
50	0.44	0.99	0.49	1.00	-	-	-	-	-	-
60	0.35	0.93	0.38	0.96	0.43	0.99	-	-	-	-
80	0.22	0.82	0.23	0.84	0.24	0.88	0.27	0.92	0.44	1.00
100	0.15	0.73	0.15	0.75	0.14	0.77	0.13	0.82	0.09	0.89
120	0.11	0.65	0.10	0.67	0.09	0.69	0.06	0.72	0.01	0.78
140	0.08	0.59	0.07	0.60	0.05	0.62	0.03	0.65	0.00	0.70
160	0.06	0.53	0.05	0.55	0.04	0.56	0.02	0.59	0.00	0.63
180	0.05	0.49	0.04	0.50	0.03	0.51	0.01	0.53	0.00	0.58
200	0.04	0.45	0.03	0.46	0.02	0.47	0.01	0.49	0.00	0.53

## Appendix B. Proof of Lemma 2

In the sequel, I simplify the argument of Hagan and Woodward (1999) to prove Equation (5) by singular perturbation theory. For the sake of notational simplicity, I will assume zero interest rates. The stock pays no dividends, which implies a zero drift under the risk-neutral probability measure associated with the money market account. The stock price diffuses according to the dynamics:

$$dS_t = \alpha(t)\sigma(S_t)S_t dW_t. \quad (\text{B.1})$$

The undiscounted risk-neutral value  $C(S, t) = \mathbb{E}\{(S_T - K)^+ | S\}$  of a European-style call option with strike  $K$  and time to maturity  $T$  evolves according to the Black-Scholes-Merton partial differential equation (PDE):

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2(S)S^2 \frac{\partial^2 C}{\partial S^2} = 0, \quad (\text{B.2})$$

subject to appropriate boundary and terminal conditions.

- *Re-scaling procedure.* Denoting  $f : x \mapsto x\sigma(x)$  and  $\varepsilon := f(K) \ll 1$ , I introduce the following change of variables:

$$\tau := \int_t^T \alpha^2(u) du, \quad x := \frac{S-K}{\varepsilon}, \quad (\text{B.3})$$

in order to re-scale the call value as  $\tilde{C}(\tau, x) := C(t, S)/\varepsilon$ . The new PDE in the variables  $(\tau, x)$  verified by the re-scaled call value is as follows:

$$-\frac{\partial \tilde{C}}{\partial \tau} + \frac{1}{2} \frac{f^2(K + \varepsilon x)}{f^2(K)} \frac{\partial^2 \tilde{C}}{\partial x^2} = 0. \quad (\text{B.4})$$

Expanding in power series of  $\varepsilon$ , we note that:

$$f^2(K + x\varepsilon) = f^2(K) \left( 1 + 2 \frac{f'(K)}{f(K)} x\varepsilon \right) + O(\varepsilon^2). \quad (\text{B.5})$$

Substituting in Equation (B.4), the PDE can now be written at first order in  $\varepsilon$ :

$$\frac{\partial \tilde{C}}{\partial \tau} - \frac{1}{2} \frac{\partial^2 \tilde{C}}{\partial x^2} = v x \varepsilon \frac{\partial^2 \tilde{C}}{\partial x^2} + O(\varepsilon^2), \quad (\text{B.6})$$

where  $v := f'(K)/f(K)$ . Expanding the re-scaled price  $\tilde{C}$  in power series of  $\varepsilon$  as  $\tilde{C}^0 + \varepsilon \tilde{C}^1 + O(\varepsilon^2)$ , we are led to solve the following system of PDEs at first order in  $\varepsilon$ :

$$\begin{cases} \frac{\partial \tilde{C}^0}{\partial \tau} - \frac{1}{2} \frac{\partial^2 \tilde{C}^0}{\partial x^2} = 0, & \tilde{C}^0(0, x) = x^+, \\ \frac{\partial \tilde{C}^1}{\partial \tau} - \frac{1}{2} \frac{\partial^2 \tilde{C}^1}{\partial x^2} = v x \frac{\partial^2 \tilde{C}^0}{\partial x^2}, & \tilde{C}^1(0, x) = 0. \end{cases} \quad (\text{B.7})$$

- *Solving the re-scaled problem.* Standard techniques apply to solve the first heat-like PDE.

The solution for  $\tilde{C}^0$  is given by:

$$\tilde{C}^0(\tau, x) = xN\left(\frac{x}{\sqrt{\tau}}\right) + \sqrt{\frac{\tau}{2\pi}}e^{-x^2/2\tau}, \quad (\text{B.8})$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \tilde{C}^0}{\partial x} = N\left(\frac{x}{\sqrt{\tau}}\right), \quad \frac{\partial^2 \tilde{C}^0}{\partial x^2} = \frac{e^{-x^2/2\tau}}{\sqrt{2\pi\tau}}, \quad \frac{\partial \tilde{C}^0}{\partial \tau} = \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}. \quad (\text{B.9})$$

In the same way, the solution for  $\tilde{C}^1$  is given by:

$$\tilde{C}^1(\tau, x) = vx\tau \frac{e^{-x^2/2\tau}}{2\sqrt{2\pi\tau}}, \quad (\text{B.10})$$

as it can be checked by means of the following elementary calculations:

$$\frac{\partial \tilde{C}^1}{\partial \tau} = \frac{(vx\tau + vx^3)e^{-x^2/2\tau}}{4\tau\sqrt{2\pi\tau}}, \quad \frac{\partial^2 \tilde{C}^1}{\partial x^2} = \frac{(-3vx\tau + vx^3)e^{-x^2/2\tau}}{2\tau\sqrt{2\pi\tau}}. \quad (\text{B.11})$$

Moreover, we notice that:

$$\tilde{C}^1(\tau, x) = \tau vx \frac{\partial \tilde{C}^0}{\partial \tau}. \quad (\text{B.12})$$

Substituting Equation (B.12) in the re-scaled price expansion of  $\tilde{C}$ , we obtain the solution for the re-scaled price at first order in  $\varepsilon$ :

$$\tilde{C}(\tau, x) = \tilde{C}^0(\tau, x) + \varepsilon \tau vx \frac{\partial \tilde{C}^0}{\partial \tau} + O(\varepsilon^2) = \tilde{C}^0(\tau + \varepsilon \tau vx + O(\varepsilon^2), x). \quad (\text{B.13})$$

- *Solving for the option price in the physical space.* The unscaled call price may then be deduced as follows:

$$C(t, S) = \varepsilon \tilde{C}(\tau, x) = \varepsilon \tilde{C}^0(\tau(1 + \varepsilon vx) + O(\varepsilon^2), x) = \tilde{C}^0(\varepsilon^2 \tau(1 + \varepsilon vx) + O(\varepsilon^4), \varepsilon x). \quad (\text{B.14})$$

Noting that  $\varepsilon x = S - K$ , we obtain the option price with respect to physical variables:

$$C(t, S) \approx \tilde{C}^0(\tau^*, S - K), \quad (\text{B.15})$$

where  $\tau^* \simeq \varepsilon^2 \tau (1 + \nu(S - K))$ . We also note that  $\varepsilon = f(K)$  may be developed around the midpoint  $(K + S)/2$  for spot prices close to the call strike  $K$ :

$$f^2(K) = f^2\left(\frac{K+S}{2}\right) \left(1 + \frac{f'\left(\frac{K+S}{2}\right)}{f\left(\frac{K+S}{2}\right)}(K-S)\right) + o(K-S), \quad (\text{B.16})$$

which gives at leading order:

$$\tau^* = \tau f^2\left(\frac{K+S}{2}\right) + O(K-S). \quad (\text{B.17})$$

- *The Black-Scholes-Merton case.* The preceding whole line of reasoning may be applied to the pure Black-Scholes model, which means performing the same calculations for the following stock price dynamics:

$$dS_t = \hat{\sigma}_K S_t dW_t, \quad (\text{B.18})$$

where  $\hat{\sigma}_K$  is the constant Black-Scholes implied volatility at strike  $K$  and expiry  $T$ . In this specific case we note that  $f$  is the identity function while  $\tau = \hat{\sigma}_K^2(T - t)$ ,  $\nu = 1/K$  and  $\varepsilon = K$ . Applying Equation (B.15) with the previous parameters, the Black-Scholes price is then given by  $\tilde{C}^0(\tau_{BS}^*, S - K)$  where we have at leading order:

$$\tau_{BS}^* \simeq \hat{\sigma}_K^2(T - t) \left(\frac{K+S}{2}\right)^2 + O(K-S). \quad (\text{B.19})$$

- *Linking local volatility with implied volatility.* As the option price observed in the market is both given by the local volatility model (B.15) and the Black-Scholes model, we can write:

$$\tilde{C}^0(\tau^*, S - K) = \tilde{C}^0(\tau_{BS}^*, S - K). \quad (\text{B.20})$$

As  $\tilde{C}^0$  is strictly increasing in the re-scaled time to maturity variable  $\tau$ , we thus obtain  $\tau^* = \tau_{BS}^*$ . Substituting Equations (B.17) and (B.19) in this last relationship, we get at leading order the following relationship which is valid for stock prices in the vicinity of the strike price:

$$\hat{\sigma}_K^2(T-t) \simeq \sigma^2 \left( \frac{K+S}{2} \right) \int_t^T \alpha^2(u) du. \quad (\text{B.21})$$

This is Equation (5).

### Appendix C. Proof of Proposition 1

By definition of the variance-equity elasticity:

$$e_v = \frac{d\sigma^2/\sigma^2}{dS/S} = \frac{2}{\sigma} \cdot \frac{d\sigma}{d \ln S}. \quad (\text{C.1})$$

But Lemma 2 ensures that the slope of the local volatility  $\sigma(\cdot)$  at  $S$  is twice the slope of the implied volatility  $\hat{\sigma}_T(\cdot)$  at  $K$ , ignoring the factor  $\bar{\alpha}_T$ . As a consequence, we have:

$$\frac{d\sigma}{d \ln S} = \frac{2}{\bar{\alpha}_T} \cdot \frac{d\hat{\sigma}}{d \ln K}, \quad (\text{C.2})$$

which yields after substitution of the local volatility slope in (C.1):

$$e_v = \frac{4}{\bar{\alpha}_T \sigma} \cdot \frac{d\hat{\sigma}}{d \ln K}. \quad (\text{C.3})$$

Substituting the structural formulation (7) of the implied volatility skew into (C.3) yields:

$$e_v = \frac{4}{\bar{\alpha}_T \sigma} \left( -\frac{\sigma}{2} (\ell - \varepsilon_\ell) \bar{\alpha}_T \right) = -2(\ell - \varepsilon_\ell), \quad (\text{C.4})$$

which is Equation (8).

## Appendix D. Proof of Lemma 3

A natural option structure matching the moments of a default swap instrument is the *credit risk reversal* (Ilinski, 2003). This optional structure combines long out-of-the-money put options with short at-the-money call options. The out-of-the-money put option is intended to replicate the default swap payoff on the occurrence of a credit event, that is, upon a jump to zero of the stock price. Simultaneously, the at-the-money call option is intended to provide exposure to the third moment of the implied volatility surface. It turns out that a specific choice for the geometry of the credit risk reversal structure offers no entry cost and as little convexity as possible between the two option exercise prices.<sup>16</sup> This last feature ensures an approximate static replication of the default swap instrument.

To match the first two moments of a *binary* default swap,<sup>17</sup> let us show that once the put strike,  $K_p$ , has been chosen arbitrarily, the call strike,  $K_c$ , and the put (resp. call) quantity  $n_p$  (resp.  $n_c$ ) should be chosen as follows:

$$\begin{cases} K_c = F_T^2 / K_p, \\ n_p = F_T / K_p, \\ n_c = -1. \end{cases} \quad (\text{D.1})$$

Hedging the structure with forward contracts, we can assume no dividends, no carrying costs as well as no implied volatility skew for the sake of simplicity. Let define  $P := n_p p - c$  as the upfront premium for the credit risk reversal, where  $p$  (resp.  $c$ ) is the put (resp. call) theoretical price. The usual Black-Scholes formulae can be used to calculate this upfront cost:

$$\begin{aligned} P &= [n_p K_p N(d_1) - n_p F_T N(d_2) - F_T N(d_1) - K_c N(d_2)] e^{-rT} \\ &= [(n_p K_p - F_T) N(d_1) - (n_p F_T - K_c) N(d_2)] e^{-rT} \\ &= 0, \end{aligned} \quad (\text{D.2})$$

---

<sup>16</sup>It is still possible to use more complex structures, such as combinations of risk reversals or put spreads, to match the higher-order sensitivities of the default swap instrument more closely.

<sup>17</sup>A binary default swap instrument is an instrument making a single payment of 1\$ in case of a default event.

where  $d_1 = \ln(F_T/K_c)/(\sigma\sqrt{T}) + \sigma\sqrt{T}/2$  and  $d_2 = d_1 - \sigma\sqrt{T}$ . Similarly, the convexity  $\gamma$  of the credit risk reversal is zero since:<sup>18</sup>

$$\frac{\gamma_c}{\gamma_p} = \frac{N'(d_1)}{N'(d_2)} = \exp\left(-\frac{\ln(F_T/K_c) + \ln(K_p/F_T)}{2}\right) = \exp\left(\frac{2\ln n_p}{2}\right) = n_p. \quad (\text{D.3})$$

Let us now consider the expected payoff of the delta-hedged credit risk reversal upon a default event, denoted  $L$ . This expected loss appears to be tightly constrained by the credit risk reversal geometry. Indeed, in case of a jump to zero of the stock price, the delta-hedged credit risk reversal pays off the put notional  $n_p K_p$  minus its initial delta  $\delta F_T$  in cash:

$$L = F_T - \delta F_T = (1 - n_p \delta_p + \delta_c) F_T, \quad (\text{D.4})$$

where  $\delta_p$  (resp.  $\delta_c$ ) is the initial hedge ratio of the put (resp. call). Denoting  $\delta_p^0$  (resp.  $\delta_c^0$ ) the delta of the put (resp. call) option struck at  $F_T$ , the call-put parity yields  $\delta_c^0 - \delta_p^0 = 1$ . With a strike  $K_p$  sufficiently close to  $F_T$ , we have  $n_p \delta_c^0 - \delta_c^0 \simeq 1$ . Substituting into Equation (D.5) yields:

$$L \simeq [n_p(\delta_p^0 - \delta_p) - (\delta_c^0 - \delta_c)] F_T. \quad (\text{D.5})$$

Recall now that  $\partial\delta/\partial K = -\gamma F_T/K$  then gives the sensitivity of the delta in the Black-Scholes model. Applying this general result for  $K_p$  and  $K_c$  sufficiently close to  $F_T$ , a first-order Taylor expansion yields:

$$\begin{aligned} \delta_{0,p} - \delta_p &\approx -F_T \bar{\gamma}_p \ln(F_T/K_p), \\ \delta_{0,c} - \delta_c &\approx -F_T \bar{\gamma}_c \ln(F_T/K_c), \end{aligned} \quad (\text{D.6})$$

where  $\bar{\gamma}_p$  (resp.  $\bar{\gamma}_c$ ) is the average convexity between  $F_T$  and  $K_p$  (resp.  $K_c$ ). Substituting into Equation (D.5), the expected payoff upon default turns out to depend explicitly on the log-distance

---

<sup>18</sup>The Black-Scholes convexity is  $\gamma_p = \gamma_c = N'(d_1)/(S_0\sigma\sqrt{T})$ , where  $N'(x) = \exp(-x^2/2)/\sqrt{2\pi}$ .

between the strikes:

$$L \simeq \bar{\gamma} F_T^2 \ln(K_c/K_p), \quad (\text{D.7})$$

where  $\bar{\gamma} = n_p \bar{\gamma}_p = \bar{\gamma}_c$  is the average convexity between the strikes.

In the presence of an implied volatility skew  $\hat{\sigma}_p > \hat{\sigma}_{ATM} > \hat{\sigma}_c$ , the upfront premium of the credit risk reversal has to be locally adjusted for the put (resp. call) implied volatility  $\hat{\sigma}_p$  (resp.  $\hat{\sigma}_c$ ). At first order, the adjustment cost to the premium is:

$$(\hat{\sigma}_p - \hat{\sigma}_{ATM}) \times n_p v_p - (\hat{\sigma}_{ATM} - \hat{\sigma}_c) \times v_c, \quad (\text{D.8})$$

where  $v_p$  (resp.  $v_c$ ) is the put (resp. call) sensitivity to volatility<sup>19</sup> calculated at  $\hat{\sigma}_{ATM}$ . Using the fact that  $n_p v_p = n_p \hat{\sigma}_{ATM} \gamma_p F_T^2 T = \hat{\sigma}_{ATM} \gamma_c F_T^2 T = v_c$ , the upfront premium becomes:

$$P \simeq (\hat{\sigma}_p - \hat{\sigma}_c) \hat{\sigma}_{ATM} \bar{\gamma} F_T^2 T. \quad (\text{D.9})$$

Finally, the fair spread  $s_T$  of a binary default swap instrument of maturity  $T$  may be assimilated to the annualized premium to be paid for protection,  $P/T$ , against the expected payoff upon default,  $L$ . Dividing Equations (D.7) and (D.9), the fair spread is given by:

$$s_T \simeq \hat{\sigma}_{ATM} \cdot \frac{\hat{\sigma}_p - \hat{\sigma}_c}{\ln(K_c/K_p)}. \quad (\text{D.10})$$

Substituting Equation (D.10) into the standard credit identity  $s_T = \lambda_T \times (1 - R)$ , where  $R$  is the expected recovery rate on debt, yields Equation (9).

## Appendix E. Proof of Proposition 2

Letting  $K$  converge to  $S$  in Equation (5) shows that the implied volatility and the local volatility coincide at the money. Inserting  $\hat{\sigma}_{ATM} = \bar{\alpha}_T \sigma$  and substituting Equation (7) for  $|\Sigma_T|$  into Equ-

<sup>19</sup>The Black-Scholes sensitivity to the implied volatility  $\hat{\sigma}$  is given by  $\hat{\sigma} \gamma F_T^2 T$ .

tion (9) yields:

$$\lambda_T = \frac{k\bar{\alpha}_T^2 (\ell - \varepsilon_\ell)}{2} \sigma^2, \quad (\text{E.1})$$

which leads to:

$$\frac{d\lambda_T}{\lambda_T} = \frac{d\sigma^2}{\sigma^2}. \quad (\text{E.2})$$

An immediate consequence in terms of elasticity is  $e_\lambda = e_\sigma$ .

## References

- Acharya, V.V. and Johnson, T.C. (2007) Insider Trading in Credit Derivatives, *Journal of Financial Economics* **84**, 110–141.
- Alexander, C., and Kaeck, A. (2008) Regime dependent determinants of credit default swap spreads, *Journal of Banking and Finance* **32**, 1008–1021.
- Baillie, R.T., Booth, G.G., Tse, Y. and Zobotina, T. (2002) Price discovery and common factor models, *Journal of Financial Markets* **5**, 309–321.
- Beckers, S. (1980) The Constant Elasticity of Variance Model and Its Implications For Option Pricing, *Journal of Finance* **35**, 661–673.
- Bekaert, G. and Wu, G. (2000) Asymmetric Volatility and Risk in Equity Markets, *Review of Financial Studies* **13**, 1–42.
- Black, F. (1976) Studies of Stock Price Volatility Changes. In: Proceedings of the Meetings of the American Statistical Association, Business and Economics Statistics Section, pp. 177–181.
- Black, F. and Cox, J.C. (1976) Valuing Corporate Securities: Some Effects of Bonds Indenture Provisions, *Journal of Finance* **31**, 351–367.
- Black, F. and Scholes, M. (1973) The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* **3**, 637–654.
- Campbell, J. and Hentschel, L. (1992) No News is Good News: An Asymmetric Model of Changing Volatility in Stock Returns, *Journal of Financial Economics* **31**, 281–318.
- Campbell, J. and Taksler, G. (2003) Equity Volatility and Corporate Bond Yields, *Journal of Finance* **58**, 2321–2349.
- Collin-Dufresne P., Goldstein, R.S., and Martin, J.S. (2001) The Determinants of Credit Spread Changes, *Journal of Finance* **56**, 2177–2207.
- Choi, J. and Kim, Y. (2018) Anomalies and market (dis)integration, *Journal of Monetary Economics* **100**, 16–34.
- Cremers, M., Driessen, J., Maenhout, P. and Weinbaum, D. (2008) Individual Stock-Option Prices and Credit Spreads, *Journal of Banking and Finance* **32**, 2706–2715.

- Das, S.R. and Sundaram, R.K. (2007) An Integrated Model for Hybrid Securities, *Management Science* **53**, 1439–1451.
- De Spiegeleer, J., Schoutens, W. and Van Hulle, C. (2014) *The Handbook of Hybrid Securities*. John Wiley & Sons Ltd, Chichester.
- Duffee, G.R. (1998) The Relation Between Treasury Yields and Corporate Bond Yield Spreads, *Journal of Finance* **53**, 2225–2241.
- Duffie, D. and Lando, D. (2001) Term Structures of Credit Spreads with Incomplete Accounting Information, *Econometrica* **69**, 633–664.
- Ericsson J., Jacobs, K., and Oviedo, R. (2009) The Determinants of Credit Default Swap Premia, *Journal of Financial and Quantitative Analysis* **44**, 109–132.
- Estrella A., and Hardouvelis, G.A. (1991) The Term Structure as a Predictor of Real Economic Activity, *Journal of Finance* **46**, 555–576.
- Forte, D. and Peña, J.I.. (2009) Credit spreads: An empirical analysis on the informational content of stocks, bonds, and CDS, *Journal of Banking and Finance* **33**, 2013–2025.
- Friewald, N., Wagner, C., and Zechner, J. (2014). The Cross-Section of Credit Risk Premia and Equity Returns, *Journal of Finance* **69**, 2419–2469.
- Gabaix, X. (2009) Power Laws in Economics and Finance, *Annual Review of Economics* **1**, 255–293.
- Gatheral, J. (2006) *The Volatility Surface: A Practitioner’s Guide*. John Wiley & Sons Ltd, Hoboken.
- Giesecke, K. (2006) Default and information, *Journal of Economic Dynamics & Control* **30**, 2281–2303.
- Gonzalo, J., and Granger, C. (1995) Estimation of Common Long-memory Components in Cointegrated Systems, *Journal of Business and Economic Statistics* **13**, 27—35.
- Hagan, P.S., and Woodward, D.E. (1999) Equivalent Black Volatilities, *Applied Mathematical Finance* **6**, 147–157.
- Han, B., Subrahmanyam, M., and Zhou, Y. (2017). The term structure of credit spreads, firm fundamentals, and expected stock returns, *Journal of Financial Economics* **124**, 147–171.
- Hasbrouck, J. (1995) One Security, Many Markets: Determining the Contributions to Price Discovery, *Journal of Finance* **50**, 1175–1199.
- Hasbrouck, J. (2002) Stalking the “efficient price” in market microstructure specifications: an overview, *Journal of Financial Markets* **5**, 329—339.
- Heard, N.A., and Rubin-Delanchy, P. (2018). Choosing between methods of combining  $p$ -values. *Biometrika* **105**, 239–246.
- Hilscher, J., Pollet, J.M., and Wilson, M. (2015). Are Credit Default Swaps a Sideshow? Evidence that Information Flows from Equity to CDS Markets. *Journal of Financial and Quantitative Analysis* **50**, 543–567.

- Ilinski, K. (2003) Pricing Credit from Equity Options, Working paper, JPMorgan Chase.
- Jensen, M., and Meckling, M. (1976). Theory of the Firm: Managerial Behavior, Agency Costs and Ownership Structure, *Journal of Financial Economics* **3**, 305–360.
- Johansen, S. (1988) Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control* **12**, 231–254.
- Johansen, S. (1991) Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica* **59**, 1551–1580.
- Kapadia, N. and Pu, X. (2012) Limited arbitrage between equity and credit markets, *Journal of Financial Economics* **105**, 542–564.
- Kryzanowski, L., Perrakis, S., and Zhong, R., (2017) Price discovery in equity and CDS markets, *Journal of Financial Markets* **35**, 21–46.
- Lee J., Naranjo, A. and Velioglu, V. (2018) When do CDS Spreads Lead? Rating Events, Private Entities, and Firm-specific Information Flows, *Journal of Financial Economics* **130**, 556–578.
- Leland, H.E. (1994) Corporate Debt Value, Bond Covenants, and Optimal Capital Structure, *Journal of Finance* **49**, 1213–1252.
- Leland, H.E. (1998) Agency Costs, Risk Management, and Capital Structure, *Journal of Finance* **53**, 1213–1252.
- Leland, H.E. and Toft, K.B. (1996) Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads, *Journal of Finance* **51**, 987–1019.
- Longstaff, F.A. and Schwartz, E. (1995). A Simple Approach to Valuing Risky Fixed and Floating Rate Debt, *Journal of Finance* **50**, 789–819.
- Marsh, I.W. and Wagner, W. (2016) News-Specific Price Discovery in Credit Default Swap Markets, *Financial Management* **45**, 315–340.
- Mayordomo, S., Peña, J.I. and Schwartz E. (2010) Are all Credit Default Swap Databases Equal?, *NBER Working Paper 16590*.
- Meine, C., Supper, H., and Weiss, G.N.F. (2016). Is Tail Risk Priced in Credit Default Swap Premia? *Review of Finance* **20**, 287–336.
- Merton, R.C. (1974) On the Pricing of Corporate Debt: The Risk Structure of Interest Rates, *Journal of Finance* **29**, 449–470.
- Narayan, P.K., Sharma, S.S., and K.S. Thuraiamy. (2014). An analysis of price discovery from panel data models of CDS and equity returns. *Journal of Banking & Finance* **41**, 167–177.
- Newey, W.K., and K.D. West. (1987). A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* **55**, 703–708.
- Putnins, T.J. (2013) What do price discovery metrics really measure? *Journal of Empirical Finance* **23**, 68–83.
- Qiu, J., and Yu, F. (2012) Endogenous liquidity in credit derivatives, *Journal of Financial Economics* **103**, 611–631.

- Schroder, S. (1989) Computing the Constant Elasticity of Variance Option Pricing Formula, *Journal of Finance* **44**, 211–219.
- Schaefer, S.M., and Strebulaev, I.A. (2008) Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds, *Journal of Financial Economics* **90**, 1–19.
- Shleifer, A., and Summers, L. (1990) The Noise Trader Approach to Finance, *Journal of Economic Perspectives* **4**, 19–33.
- Shleifer, A., and Vishny, R.W. (1997) The Limits of Arbitrage, *Journal of Finance* **52**, 35–55.
- Wakker, P.P. (2008) Explaining the Characteristics of the Power (CRRA) Utility Family, *Health Economics* **17**, 1329–1344.
- Wu, G. (2001) The Determinants of Asymmetric Volatility, *Review of Financial Studies* **14**, 837–859.
- Yan, B. and Zivot, E. (2010) A structural analysis of price discovery measures, *Journal of Financial Markets* **13**, 1–19.
- Zimmermann, P. (2015) Revisiting the Credit-Equity Power Relationship, *Journal of Fixed Income* **24**, 77–87.
- Zhang, B.Y., Zhou, H. and Zhu, H. (2009). Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms, *Review of Financial Studies* **22**, 5099–5131.