Electromagnetic pulse generator: An analytical and numerical study of the Lorentz force in tube crimping processes
Khadija Sofi, Mohammed Hamzaoui, Hassan El Idrissi, Ahmed Nait Sidi Moh, Denis Jouaffre, Abdelkrim Hamzaoui

To cite this version:

HAL Id: hal-03493536
https://hal.science/hal-03493536
Submitted on 21 Nov 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution - NonCommercial 4.0 International License
Electromagnetic pulse generator: an analytical and numerical study of the Lorentz force in tube crimping processes

Khadija SOFIa,b,*, Mohammed HAMZAOUTib, Hassan EL IDRISSIa, Ahmed NAIT SIDI MOHib, Denis JOUAFFREC, Abdelkrim HAMZAOUTId

aLEEA-TI, Department of electrical engineering, FSTM, Hassan II University of Casablanca, Mohammedia, Morocco
bLTI, Intelligent Systems team, EDSTS 585, University of Picardie Jules Verne, Saint-Quentin, France
cPFT Innovaltech - Lycée Condorcet, Saint-Quentin, France
dLGECOS, Department of electrical engineering, Cadi Ayyad University, ENSA, Marrakesh, Morocco

Abstract

In this paper, we address the application of Lorentz force in the electromagnetic pulse crimping process. To do so, we used an analytical method based on mutual inductance and mutual force between two circular coils. This crimping process is employed on conductive materials considered as work-pieces. For different voltages, we developed an analytical model for two circular coils corresponding to the inductance of the coil attached to RLC circuit, and the tube inductance. The magnetic density into the work-piece, the Lorentz force and the magnetic pressure on the tube are evaluated. Obtained results from experimental measurements and simulation are promising.

Keywords: Magnetic pulse generator, single turn coil, dissimilar materials welding, electrical scheme, current pulse

1. Introduction

In the new technology of magnetic pulse joining metal work-pieces, estimating the exact mechanical or magnetic force is essential for a wide range

*Corresponding author
Email address: khadija.sofi@etud.u-picardie.fr (Khadija SOFI)

of scientific and industrial processes. The magnetic pulse crimping process is a high-velocity crimping method of metallic tubes using a magnetic field generated by an EMPG (ElectroMagnetic Pulse Generator). In the field of metallurgy, the EMPG has been studied by many researchers using massive coils and field shapers. The field shaper is a metallic piece used to adapt the massive coil to the deformation area of the metal work-piece [1]. It generally leads to a concentration of the magnetic field that acts on work-pieces in a specific area. As demonstrated in many studies over the last two years, due to the eco-environmental benefits of the magnetic forming process, the development of this technology has great potential for industrial purposes. In terms of the generated magnetic field and negative effect on electronic components such as sensors, many previous studies brought into question the safety of the EMPG [2]. Some of the studies were focused on reducing the adverse effects of the magnetic field on human health [3]. On the other hand, in terms of the force generated by EMPG, many researchers have consistently focused on studying the Lorentz force applied to work-pieces using finite element methods.

The third important parameter concerning EMPG is the temperature. Indeed, the temperature distribution is studied experimentally during the forming process and the results are that with a current pulse with a peak of 150 kA, the active area in the massive coil reaches a maximum temperature of 34°C. This process is called cold forming [4]. In the literature, there has been considerable work addressed to the magnetic force produced in the crimping process of a tube. These studies are performed analytically, for example, using a method of splitting the massive coil into areas that correspond to uniform current. The calculation process depends on magnetic vector potential [5]. Other research works are performed by numerical simulations using FEM (Finite Element Method) and BEM (Boundary Element Method). Many studies about EMPG have focused on identifying and evaluating semi-analytical methods using, either the vector potential $A$ or the injected current and the total energy stored in the EMPG [5].

In the electromagnetic forming process, a new investigation gives us an in-
sight into the geometry of a pressure coil. It presents an analytical method to determine the optimum geometry of the coil. This method is based on two parameters, the produced magnetic pressure on the work-piece and the magnetic energy generated by the EMPG. In this paper, the expression "forming" will designate all magnetic pulse deforming processes. The present research work predominately focuses on the analysis of the crimping process of an aluminum tube. As illustrated in figure 1, high power bank capacitors are charged throughout a transformer, then discharged in the massive coil instantaneously throughout spark-gaps. The discharged current creates an intense magnetic field in the forming area and generates an eddy current in the work-piece (tube). The Lorentz force is generated radially in the active area due to the magnetic field produced by the coil and acts mechanically on the work-piece, which leads to the crimping process. This process is performed without physical contact between the tube and the massive coil. The active area or the working zone defines the gap between the coil and the work-piece.

![Electromagnetic pulse generator: tube crimping setup](image)

Nowadays, several studies have also investigated electromagnetic welding processes. In this research field, [6] explains the formation mechanism of two kinds of wave interfaces in a Fe/Al electromagnetic welding process. Another study [7] proposed a model to predict the critical wall thickness of the inner tube in the same process. The thickness of the tube has been verified experimentally with an error of less than 2.5%. A recent study of the same team of researchers
has investigated the magnetic pulse welding of tubular carbon-fiber-reinforced plastic / aluminum [8]. In an analysis of crimped connections realized by the same process, [9] gives an analytic model to predict the joining zone parameters that was verified experimentally. In a recent study, the magnetic pressure is predicted analytically, by numerical simulations and verified experimentally [10]. This study is very interesting as it takes into account all the work-pieces, the materials properties and the EMPG parameters used in the process. Authors of [11] stated in their research work the advantages of the electromagnetic crimping process over conventional ones. Authors established an Al to copper wire crimping process, the average gap in this case, was minimized by 70% and the pullout strength reaches 1.958kN rather than 0.98kN in the conventional method. As noted in a recent study [12], the trapezoidal shape of the coil increases the velocity of the wire’s crimping by 10% compared to circular or rectangular coils. In order to find the right coil dimensions and the discharge voltage, the same authors conducted numerical simulation research about the copper wire crimping process [13]. The joint properties in the electromagnetic crimping process have also been studied earlier in [14] using metallic filler materials. Authors of the study proved that the filler materials increase the load-bearing capacity. The velocity in the electromagnetic joining process is also studied in the literature. An example of the joining of Al tube to steel rods have been established in [15], it shows that the velocity of the joining reaches a maximum value of 500m/s. The same research team studied the velocity in the joining of steel to steel tubes [16]. In this case, the speed reaches 360m/s.

To the best of our knowledge today, very little research has been done on the exact Lorentz force needed for the whole process. Most of this research has been performed experimentally [17] or by numerical simulations using FEM [18]. For the analytical studies, the method has been based on the calculation of the inductance of both the primary coil and the work-piece to determine magnetic pressure is performed in the following studies [19, 20]. A significant part of our research in this paper is focused on the calculation of self and mutual inductance of both the coil and the work-piece. This leads us to determine analytically the
radial Lorentz force needed to achieve the desired deformation. Thus, using the stress/strain modeling of the material of a work-piece, specifically a model that involves a high strain rate deformation of metallic parts, we can predict the velocity of the deformation, the displacement and finally the final shape of this work-piece. This part of the study is not concluded in the paper, however, it is considered as part of future research work. Previous research worked on defining self and mutual inductance of complicated geometries using three dimensional configurations which are presented in [21, 22]. Another critical study has been developed in [23] using Bessel functions to calculate the inductance of non-coaxial coils. Regarding the Lorentz force, the authors of [24] identify a three dimensional analytical case where they have investigated the mutual Lorentz force of two thick coils using Biot & Savart’s law. This study found that the force decreases when the distance between the two coils increases, while the maximum force is reached when the distance is the minimum possible. Since there is no contact between the coil and the tube, we consider this process as an energy transfer system. In the same vein, a thesis based on the same principle presented a wireless energy transfer in a system which rotates on two axes using air transformers [25]. In this thesis both mutual inductance of coaxial and non-coaxial rectangular or circular coils are investigated. In the same field of electromagnetic forming, a case study is based on 1D analytical modelling of the coil and determines the electro-mechanical parameters by defining the magnetic vector potential [5]. Together these studies provide valuable insights into the electromagnetic crimping process and the parameters needed to be defined before using experimental setup applications. Compared with studies that have been already developed in the literature, the main advantage of our study is that the analytical method is based only on the geometry of the coil to measure the current in the coil and other electro-mechanical parameters of the electromagnetic crimping process. There are several important areas where this study can make an original contribution to the electromagnetic forming processes. A specific objective of this study is to predict preliminary the voltage load and the geometry of the work-piece before performing experimental measurement.
and starting to produce the final product industrially. It can also predict the velocity of the crimping process, which will help to automate the process.

The remainder of this paper is as follows: Analytical modelling theory is given in section 2, where we calculate the mutual inductance between the coil and the tube. In addition, we compute the magnetic field generated in the working zone and the Lorentz force applied on the tube. Section 3 presents the numerical method using FEM and BEM. This method predicts the magnetic pressure required to produce the final shape of the work-piece after the crimping process. Numerical simulations are performed using coupled mechanical, thermal and electromagnetic solvers in LS-DYNA software [26, 27]. Results of both studies are detailed in section 5, where we focus mainly on comparing Lorentz force both analytically and numerically. Section 6 is the conclusion and gives some perspectives and future directions for this work.

2. System analytical modelling

In the electromagnetic pulse generator, the interaction between the coil and work-pieces is performed by the mutual inductance and mutual Lorentz force. Furthermore, the eddy current generated in the tube is based on the magnetic field generated itself by the coil. The Lorentz force acting on the tube is due to both the magnetic field produced by the coil and the eddy current in the tube. Using the electrical scheme of the EMPG represented in figure 2, we develop an analytical model based on the Neumann formula of mutual inductance between two circular coils. This model will be used to specify the Lorentz force and its impact on the tube deformation. This method enables the calculation of the self and mutual inductance using only the geometry of the main coil, and it determines the current using these parameters. The calculation method expresses precisely the Lorentz force needed for a specific deformation of the work-piece. To establish the current pulse shape in the coil, we begin by giving an overall equation of the current based on an RLC circuit study using an experimental measurement of these parameters. Then, we develop an analytical method for
the mutual inductance and self-inductance inside the massive coil. The purpose here is to give the exact current shape using only the coil geometry. This result will allow us to express the magnetic field and Lorentz force analytically in a short period of time compared to numerical analysis.

2.1. Electrical scheme of the EMPG

The main electrical scheme that explains the circuit of the process is illustrated in figure 2. The capacitor $C_1$ represents the bank capacitors. The coil is expressed by an $RL$ series circuit ($R$ is the resistance of the coil, and $L$ is its inductance) conducts a current $I_c$, while the tube, conducts the current $I_t$, is expressed by an $RL$ series circuit ($L_t$ is the inductance of the tube and $R_t$ is its resistance). The current is conducted by coaxial cables (24 cables) which connect the bank of capacitors to the main coil. The electrical scheme of these conductors is also an $RLC$ circuit, presented by $C_2$, $R_c$ and $L_c$ and represent, respectively, the capacity, the resistance, and the inductance of the cables. In figure 2, we represent the total inductance and resistance of the primary circuit, which are $L_p = L_c + L$ and $R_p = R_c + R$. The resistance, self and mutual inductance of the scheme is calculated in the next section. In figure 2, the coil and the tube acts as an air transformer, which is governed by Eq. (1). The voltage $V_c$ is defined using the mutual inductance $M_{ct}$ between the coil and the tube. $V_c$ and $V_t$ are respectively, the voltages in the capacitor $C_1$ and in the tube. Using Laplace transfer, we get the resulting expressions in Eq. (1).

\[
\begin{align*}
V_c(p) &= V_2(p) + R_p I_c(p) + pL_p I_c(p) + pM_{ct} I_t(p) \\
V_t(p) &= R_t I_t(p) + pL_t I_t(p) + pM_{ct} I_c(p) = 0
\end{align*}
\]
From \( V_2(p) \) and \( V_t(p) \) expressed in Eq. (1), we get the following expressions for currents \( I_c \) and \( I_t \).

\[
\begin{align*}
I_t(p) &= -\frac{pM_{ct}}{R_t + pL_t} I_c(p) \\
I_c(p) &= -p C_1 V_c(p) = p C_2 V_2(p)
\end{align*}
\]  
\((2)\)

During the procedure that we followed, the parasitic capacitance in the coil is considered very small and does not have a significant effect on the current pulse. Therefore, we ignore the parasitic capacitance during the calculation of the current. \( I_c \) symbolizes the current pulse flowing through the coil, and \( I_t \) is the current generated in the tube.

From Eq. (1) and Eq. (2), we obtain the differential equation (3). It expresses the electrical behavior of the EMPG.

\[
R_t(1 + \frac{C_1}{C_2})v_c(t) + (L_t(1 + \frac{C_1}{C_2}) + R_p R_t C_1) \frac{dv_c(t)}{dt} +
(R_p L_t + L_p R_t) C_1 \frac{d^2 v_c(t)}{dt^2} + (L_t L_p - M^2_{ct}) C_1 \frac{d^3 v_c(t)}{dt^3} = 0
\]  
\[(3)\]

In what follows, we begin the numerical simulation by injecting the computed current pulse into the coil in order to deform the tube. It is worth noting that in the analytical section, the current settings calculated in the numerical analysis do not serve for the results. Only the peak of the current is computed using the self and the mutual inductance and depending only on the coil dimensions and the input voltage.

### 2.2. Self and Mutual Inductance

Mutual inductance is considered as one of the most useful parameters in numerous applications such as generators and transformers [28]. Therefore, since we have noticed an air core transformer system in our EMPG system, we have developed the following method; by identifying the mutual inductance between the coil and the tube. The massive coil and the tube’s geometry are illustrated in figure 3. It is important to clarify that the geometric structure of the pieces is based on the experimental research in our previous studies [3, 4]. The current in the massive coil and the field shaper, represented in these studies,
circulates only in on the interior surface of the tube. Consequently, this massive coil can be replaced by a one-turn coil, as illustrated in figure 3. \( R_1 \) and \( R_2 \) represents the inner and outer radius of the coil respectively while \( R_3 \) and \( R_4 \) are respectively the inner and outer radius of the tube. The current is injected into two bars located at the gap of the coil represented by C in figure 3. To begin this

![Figure 3: Geometric representation of the coil and the tube](image)

process, we represent the coil or the tube in figure 4. First, we consider a hollow cylinder with \( R_{i1} \) as an inner radius and \( R_{i2} \) as an outer radius while its length is \( Z_{i2} - Z_{i1} \). The cylinder generates a resistance \( R_{si} \) and a self-inductance \( L_i \). Before computing the mutual inductance, we begin by using the expression of the self-inductance \( L_i \). The method adopted in the calculation of the self-inductance

![Figure 4: Representation of a hollow cylinder](image)

\( L_i \) has been developed in [29] for a rectangular coil, and expressed with the Cartesian coordinates \( x, y, z \) using a sextuplet definite integral. However, since we are using a cylinder coil in our study, the calculus will be developed with cylindrical coordinates. Therefore we give the following expression of \( L_i \) as
is calculated using the following expression

\[ L_i = \frac{\mu_0}{4\pi S_i^2} Q_i \]  

(4)

where \( S_i^2 = (R_{i2} - R_{i1})^2 (Z_{i2} - Z_{i1})^2 \) is the rectangular cross section surface. \( Q_i \) is expressed using the following definite integrals as shown in Eq. (5).

\[
Q_i = \int_{\alpha}^{2\pi - \alpha} \int_{Z_{i1}}^{Z_{i2}} \int_{R_{i1}}^{R_{i2}} \int_{R_{i1}}^{R_{i2}} \frac{r R \cos(\theta) dR d\theta dZ dz d\theta}{\sqrt{r^2 + R^2 - 2r R \cos(\theta) + (z - Z)^2}}
\]

(5)

In the analysis method, the integral of \( Q_i \) on \( \theta \) is performed between \( \alpha \) and \( 2\pi - \alpha \). Since the coil has a gap (C) for current input and output, \( \alpha \neq 0 \) represents the angle at the beginning of the coil at the gap while in this case we perform the inductance of the tube with \( \alpha = 0 \). As a result, the primitive of \( Q_i \) is given in Eq. (6), and its integral is expressed by Eq. (7).

\[
Q_i = Q_i[r(2), t(2), z(2)] - Q_i[r(1), t(2), z(2)] - (Q_i[r(2), t(1), z(2)] - Q_i[r(1), t(1), z(2)])
\]

\[
- (Q_i[r(2), t(2), z(1)] - Q_i[r(1), t(2), z(1)]) - (Q_i[r(2), t(1), z(1)] - Q_i[r(1), t(1), z(1)])
\]

(6)

where, \( r(1) = R_{i1}, r(2) = R_{i2}, \theta(1) = \alpha, \theta(2) = 2\pi - \alpha, z(1) = Z_{i1} \) and \( z(2) = Z_{i2} \).

\[
Q_i = \frac{1}{72} \left[ \frac{6}{5} \left( r^2 \left( 1 - \frac{\sin^2(2\theta)}{2} - 3 \cos^2(\theta) \sin^2(\theta) \right) + z^2(z^2 - 3r^2) \right) \sqrt{r^2 + z^2} 
\]

\[
- 12r^4 \sin(\theta) \cos(\theta) z^2 \arctan \left( \frac{\sin(\theta) \cos(\theta)}{z \sqrt{r^2 + z^2}} \right) 
\]

\[
+ \sin^2(\theta) \arctan \left( \frac{\cos(\theta) z}{\sin(\theta) \sqrt{r^2 + z^2}} \right) + \cos^2(\theta) \arctan \left( \frac{\sin(\theta) z}{\cos(\theta) \sqrt{r^2 + z^2}} \right) 
\]

\[
- 3r \cos(\theta) (r^2 \sin^2(\theta)(r^2 \sin^2(\theta) - 6z + z^4)) \ln(r \cos(\theta) + \sqrt{r^2 + z^2}) 
\]

\[
- 3r \sin(\theta) (r^2 \cos^2(\theta)(r^2 \cos^2(\theta) - 6z + z^4)) \ln(r \sin(\theta) + \sqrt{r^2 + z^2}) 
\]

\[
- 3z r^4 \left( 1 - \frac{\sin^2(2\theta)}{2} - 6 \cos^2(\theta) \sin^2(\theta) \right) \ln(z + \sqrt{r^2 + z^2}) \right] 
\]

(7)

In the second step we calculate the resistance \( R_{si} \). To do so, \( R_{si} \) of each piece is calculated using the following expression \( R_{si} = \rho \frac{l}{S} \), where \( l \) represents the length of the wire of each conductor, \( S \) is the cross-section area, and \( \rho \) represents...
the electrical resistivity of the material. Based on this formula, the resistance is expressed by Eq. (8).

$$R_{S_i} = \rho \frac{\pi (R_2 + R_1)}{(R_2 - R_1)(Z_2 - Z_1)}$$ (8)

After calculating the resistance, the next step is the calculation of the mutual inductance $M_{ct}$ between two coaxial hollow cylinder coils represented by the tube and the coil using Neumann formula. Figure 5 shows a 3D configuration of the coil in situ with the tube. The blue hollow cylinder (external piece) represents the coil of length $Z_2 - Z_1$ while the red one (internal piece) represents the tube of length $Z_4 - Z_3$.

$$M_{ct} = \mu_0 \frac{(R_2 - R_1)^2(Z_2 - Z_1)^2(R_4 - R_3)^2(Z_4 - Z_3)^2}{Q}$$ (9)

$$Q = \int_0^\pi \int_{Z_1}^{Z_2} \int_{Z_3}^{Z_4} \int_{R_1}^{R_2} \int_{R_3}^{R_4} \frac{r R \cos(\theta) dr dz d\theta}{\sqrt{r^2 + R^2 - 2r R \cos(\theta) + (z - Z)^2}}$$ (10)

After integrating along $r, R, z$ and $Z$, the expression of $Q$ becomes:

$$Q = \int_0^\pi G[R_1, R_2, R_3, R_4, Z_1, Z_2, Z_3, Z_4, \theta] \cos(\theta) d\theta$$ (11)

where

$$G = \left[ |F(r, R, z, Z, \theta)|^{R_2, R_3, Z_2, Z_4} \right]^{R_1, Z_1, Z_3}$$ (12)
Since $F(r, R, z, Z, \theta) \cos(\theta)$ presents singularities at $\theta = 0$ and $\pi$, the integral along $\theta$ is solved using Newton cotes method of 1st order (Trapezoidal Rule) and L’Hospital’s rule as expressed in Eq. (13) and in Eq. (14).

\[
\int_{a}^{b} F(r, R, z, Z, \theta) \cos(\theta) \, d\theta = \frac{b - a}{2} \left[ F(r, R, z, Z, a) \cos(a) + F(r, R, z, Z, b) \cos(b) \right]
\]

\[
\begin{aligned}
\lim_{\theta \to 0} F(r, R, z, Z, \theta) \cos(\theta) &= \frac{(z - Z)(r^2 - R^2)^2}{8} \ln[z - Z + Y] + \frac{(z - Z)^2}{6} r^3 \ln[r - R + Y] + \\
&\quad \frac{(z - Z)^2}{6} R^3 \ln[R - r + Y] - \frac{(r - R)^2}{30} (r^2 + 3rR + R^2) Y - \\
&\quad \frac{Y(z - Z)^2}{20} \left( \frac{Y^2}{9} + \frac{rR}{3} + \frac{3(r^2 + R^2)}{2} \right)
\end{aligned}
\]

\[
\begin{aligned}
\lim_{\theta \to \pi} F(r, R, z, Z, \theta) \cos(\theta) &= -\frac{(z - Z)(r^2 - R^2)^2}{8} \ln[z - Z + X] \\
&\quad - \frac{(z - Z)^2}{6} (r^3 + R^3) \ln[r + R + X] \\
&\quad + \frac{(r + R)^2}{30} (r^2 - 3rR + R^2) X + \\
&\quad \frac{X(z - Z)^2}{20} \left( \frac{X^2}{9} - \frac{rR}{3} + \frac{3(r^2 + R^2)}{2} \right)
\end{aligned}
\]

where: $X = \sqrt{(r + R)^2 + (z - Z)^2}$ and $Y = \sqrt{(r - R)^2 + (z - Z)^2}$.

Based on the resistance, self-inductance and mutual inductance equations of the coil and the tube, we calculate the current in the next section. In addition, we define the magnetic field, Lorentz force and magnetic pressure, operating on the tube during the crimping process.

2.3. Governing equations of magnetic parameters

Dynamic deformation of metal pieces during welding or crimping is due to the magnetic loading condition. This deformation is obtained from the magnetic pressure which is, itself, generated by the Lorentz force [30]. To predict the required Lorentz force for the tube’s crimping, we used a theoretical approach based on mutual forces and Biot & Savart’s law in the working zone and the
current in the coil \[31\].

The current in the coil is calculated as stated in Eq. (2). From this equation, we extract the magnetic field strength using Biot & Savart’s law at the point \(M(r, z)\) referring to figure 5. The magnetic field is expressed in Eq. (15). This allow us to establish the expression of the magnetic field at a point \(M(r, z)\) in the working zone. This means that the red tube represented in figure 5 is not taken into account while calculating the magnetic field.

The current density \(J\) flowing through the coil is expressed by: \(J = \frac{I_c}{(Z_2 - Z_1)(R_2 - R_1)}\)

and the magnetic field is expressed by:

\[
\vec{H}(M) = \frac{1}{4\pi} \int \int \int_V \vec{J}d\tilde{\nu} \land (-\nabla G(\vec{r}|\vec{r'}))
\]  

(15)

\(G\) represents the Green’s function in 3 dimensions. Supposing a point \(M(\tilde{r}, \tilde{z}, \tilde{\theta})\) that belongs to the coil, \(G\) will be given by the following equation:

\[
G(\vec{r}|\vec{r'}) = \frac{1}{|\vec{r} - \vec{r'}|} = \frac{1}{\sqrt{r^2 + \tilde{r}^2 - 2r\tilde{r}\cos(\tilde{\theta}) + (z - \tilde{z})^2}}
\]

Therefore, the gradient of \(G\) is expressed as follows: \(-\nabla G(\vec{r}|\vec{r'}) = \frac{\vec{r} - \tilde{r'}}{|\vec{r} - \tilde{r'}|^3}\)

The radial and axial components of the magnetic field, represented respectively by \(H_r\) and \(H_z\) are expressed in Eq. (16).

\[
\begin{align*}
H_r(r, z) &= \frac{J}{4\pi} \sum_{i,k=1}^{2\pi - \alpha} (-1)^{i+k} \left\{ \phi((\alpha + b)K(k) - aE(k)) + \int_{\alpha}^{2\pi - \alpha} \Psi_r d\theta \right\} \\
H_z(r, z) &= \frac{J}{4\pi} \sum_{i,k=1}^{2\pi - \alpha} (-1)^{i+k} \left\{ \int_{\alpha}^{2\pi - \alpha} \left( \Psi_z + f \ln[d + c] \right) d\theta \right\}
\end{align*}
\]  

(16)

After integrating along \(\tilde{r}\) and \(\tilde{z}\), final expressions of \(H_r\) and \(H_z\) are given in Eq. (17). The integral along \(\tilde{\theta}\) is done such as \(\alpha < \theta < 2\pi - \alpha\).

\[
\begin{align*}
H_r(r, z) &= \frac{J}{4\pi} \sum_{i,k=1}^{2\pi - \alpha} (-1)^{i+k} \left\{ \phi((\alpha + b)K(k) - aE(k)) + \int_{\alpha}^{2\pi - \alpha} \Psi_r d\theta \right\} \\
H_z(r, z) &= \frac{J}{4\pi} \sum_{i,k=1}^{2\pi - \alpha} (-1)^{i+k} \left\{ \int_{\alpha}^{2\pi - \alpha} \left( \Psi_z + f \ln[d + c] \right) d\theta \right\}
\end{align*}
\]  

(17)
with:

$$\begin{align*}
\Psi_r &= r \cos^2(\theta) \ln[d + c] \\
\Psi_z &= r \cos(\theta) \ln[f + c] + \nu \arctan\left(\frac{df \csc(\theta)}{rc}\right) - z_j + \nu \arctan\left(\frac{\nu}{f}\right)
\end{align*}$$

(18)

Other parameters are defined as follows in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>$a$</td>
</tr>
<tr>
<td>b</td>
<td>$b$</td>
</tr>
<tr>
<td>c</td>
<td>$c$</td>
</tr>
<tr>
<td>d</td>
<td>$d$</td>
</tr>
<tr>
<td>f</td>
<td>$f$</td>
</tr>
<tr>
<td>k</td>
<td>$k$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\nu$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi$</td>
</tr>
</tbody>
</table>

Table 1: Electrical parameters

The expressions of elliptical integrals $K(k)$ and $E(k)$ are given in Eq. (19).

$$\begin{align*}
K(k) &= \int_0^{\pi/2} \frac{1}{\sqrt{1 - k \sin^2(\theta)}} d\theta \\
E(k) &= \int_0^{\pi/2} \frac{1}{\sqrt{1 - k \sin^2(\theta)}} d\theta
\end{align*}$$

(19)

The known formula of the magnetic induction $\vec{B}$ is expressed in the general following Eq. (20).

$$\vec{B} = \mu \vec{H}$$

(20)

where: $\vec{H} = H_r \vec{u}_r + H_z \vec{u}_z$

In the work-piece, the current density $J_t$ is a function of $B$ in Eq. (21). The parameter $r$ represents the variable radius inside the tube. For simplification purposes, we suppose that the current density is uniform at each circle of radius $r$ inside the tube.

$$J_t = -\frac{\partial \vec{B}}{\mu \partial r}$$

(21)

where $\mu$ represents the magnetic permeability of the material of the tube. The radial Lorentz force acting on the work-piece is given by the Eq. (22).

$$\vec{F} = \vec{J}_t \times \vec{B}$$

(22)

To simplify Lorentz force expression, we try to reduce the size of the magnetic field vector model as much as possible. In order to minimize the computational resources and time needed to resolve the model, we use symmetry and boundary
conditions of the model. After performing the numerical simulation, we found that the radial magnetic field \( H_r \simeq 0 \). Since the magnetic field vector \( \vec{B} \) has only the axial component \( B_z \) and \( \vec{J}_t \) is perpendicular to \( \vec{B} \), the Lorentz force may be presented by Eq. (23) [19].

\[
\vec{F} = -\frac{1}{\mu} \frac{\partial \vec{B}}{\partial r} = -\frac{1}{2\mu} \frac{\partial (\vec{B}^2)}{\partial r}.
\] (23)

The final step in the analytical method focuses on the expression of the magnetic pressure applied to the tube. Since in Eq. (23) we compute the volume magnetic force, then the magnetic pressure will be presented by the integral of this force along with the thickness of the tube as given in Eq. (24).

\[
P = \int_0^d \vec{F} dy
\] (24)

where \( d \) represents the tube’s thickness.

In the following section, we address a numerical method using FEM and BEM approaches and based on the presented geometry of figure 3.

3. Numerical method

After representing the system with the analytical method, a numerical method based on finite element method is performed to verify the analytical results of Lorentz force and magnetic field magnitudes. To do so, we first study the electromagnetic field based on Maxwell equations and the electromagnetic solver developed by LS-DYNA. From Maxwell equations, the current density is given in Eq. (25) and the magnetic field is expressed in Eq. (26).

\[
\vec{J} = \sigma \vec{E} + \vec{J}_s
\] (25)

\[
\vec{B} = \mu_0 \vec{H} = \vec{\nabla} \times \vec{A}
\] (26)

where \( \vec{A} \) is the magnetic vector potential. From Ampère’s law, we obtain the electrical field \( \vec{E} \) as expressed in Eq. 27, where \( \phi \) is the electrical scalar potential.

\[
\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}
\] (27)
As we use Coulomb gauge: \( \nabla (\sigma \vec{A}) = 0 \) and since there is no charge accumulation, the charge density is \( \rho = 0 \). The parameter \( \sigma \) represents the electrical conductivity of the conductor. As a result, the scalar potential verifies the equations (28) and (29).

\[
\nabla (\sigma \nabla \phi) = 0 \tag{28}
\]

\[
\sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times \left( \frac{1}{\mu_0} \nabla \times \vec{A} \right) + \sigma \nabla \phi = \vec{J}_s \tag{29}
\]

In this method, the expressions of the Lorentz force and the magnetic pressure are calculated using the same equations (23) and (24) used in the analytical method.

4. Numerical values of system parameters

4.1. Parameters of the EMPG

To address the current shape, it is crucial to state the EMPG parameters that are given experimentally by the manufacturer of the EMPG as shown in table 2. These parameters are taken into account in Eq. (3) to get the input voltage and the current across the coil, as stated in figure 6. In this figure, the current pulse corresponds to various voltage loads (2.5 kV, 5.5 kV, 8.5 kV). The voltage 8.5 kV corresponds to the maximum load generated by the EMPG.

| Table 2: EMPG parameters

<table>
<thead>
<tr>
<th>( C_1 )</th>
<th>( L_c )</th>
<th>( R_c )</th>
<th>( C_2 )</th>
<th>Maximum Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>690 ( \mu F )</td>
<td>44 ( nH )</td>
<td>2.4 ( m\Omega )</td>
<td>2.112 ( nF )</td>
<td>25 ( kJ )</td>
</tr>
</tbody>
</table>

The current presented in figure 6 is obtained from experimental measurements through the coil during the process. It is measured using a Rogowski coil with a wide bandwidth (100 Hz to 5 MHz). The cutoff frequency has a value of 5 MHz. For each output voltage unit measured, the Rogowski coil has a conversion value of 128kA/V and its sensitivity is 7mV/A. This current
wave is loaded directly by using two bars inserted in the 3D model of the coil of the numerical simulations. The frequency $f$ of the current pulse produced by 8.5 kV load is $f = 20 \text{ kHz}$ as shown in figure 6 and the current peak is about 580 kA. At 5.5 kV, the current peak is 400 kA and at 2.5 kV, we reach 190 kA.

4.2. 3D Modeling parameters

Material parameters used in analytical, simulation and experimental methods are given in table 3. In the finite element analysis approach, the Lorentz force is used as a load on the tubes to model their deformation in 3D. During the process, we used a coil made of a copper alloy called siclanic$^{\circledR}$ and a tube made of aluminium. Aluminum and siclanic$^{\circledR}$ parameters have been addressed in [32].

As for the finite element analysis, the meshing of the coil and the tube is made only of hexahedron elements. The coil is decomposed to 3200 elements while the tube has 8000 elements. In order to minimize the simulation’s duration, we used BEM approach to mesh the air in the system. The boundary conditions are defined only in the elements close to the gap (C) presented in figure 3. These elements are used as the input and output bars of the currents injected the coil.
### Table 3: Materials parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Siclanic ®</th>
<th>Aluminum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass density (kg/m³)</td>
<td>8900</td>
<td>2700</td>
</tr>
<tr>
<td>Electrical resistivity (Ω.cm)</td>
<td>3.95e⁻⁶</td>
<td>2.29e⁻⁶</td>
</tr>
<tr>
<td>Electrical conductivity (S/m)</td>
<td>5.8e⁷</td>
<td>3e⁷</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>130</td>
<td>68.9</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.29</td>
<td>0.33</td>
</tr>
</tbody>
</table>

5. Obtained results

In this section, we report our obtained results during the analytical and the numerical simulation phases and give the most significant parameters and variables that impact the tube crimping process. It should be noted that the study of the impact of the tube’s thickness is not addressed in this paper.

5.1. Numerical simulation results

The first set of analyses examine the impact of the voltage load on the tube’s deformation and crimping at the current peak. In this section, The results are established using the following geometry configuration: \( R_1 = 11 \text{ mm} \), \( R_2 = 11.5 \text{ mm} \), \( R_3 = 10 \text{ mm} \) and \( R_4 = 10.5 \text{ mm} \). The following numerical simulation gives the results about the tube’s crimping at 8.5 kV voltage load. Figure 7 represents the coil-tube setup and shows the crimping process of the tube.

Figure 8 gives the current density magnitude during this process. The current density distribution is referring to the generated eddy currents due to the magnetic field induced by the massive coil. These figures represent the tube’s radial deformation at 4 µs (8.a), 8 µs (8.b), 14 µs (8.c) and 22 µs (8.d). The results obtained from these figures demonstrate that the current presents a high and an intense magnitude and is mainly concentrated at the center of the coil and the tube. This result is explained by the fact that the current always pursues the shortest path in the conductor. From figures 8-b and 8-c, the current
density shows higher magnitudes in the outside periphery of the coil and the tube which is due to the variation of the current pulse.

The Lorentz force distribution is plotted radially and axially in the pieces. Figure 9 illustrates the distribution of the Lorentz force radially in the coil and the tube. As shown in these figures, the Lorentz force is always increasing from the inside to the outside surface of the tube. This is explained by the fact that the magnetic field magnitude is decreasing from the inner radius of the coil to its center. Figure 10 provides a general scheme of the process development, it represents an axial deformation and Lorentz force distribution in the tube at 4 μs (9.a), 8 μs (9.b), 14 μs (9.c) and 22 μs (9.d). It presents the variation of Lorentz force magnitude during the deformation process. What stands out in both figures 8 and 10 is the concentration of both Lorentz force and current density on the interior surfaces of the coil and the tube, that is to say in the working zone.

A positive correlation is found between them by numerical analysis. Furthermore, an interesting result is the skin effect in the deformation area of the tube and also on the interior surface of the coil. However, the deformation is almost entirely uniform except in the gap between the input and the output bars. This gap impacts the final deformation as it should be minimal in order to produce a uniform magnetic field inside the coil and uniform shape of the tube. It is to be noted that the initial gap between the coil and the tube changes the results.
of the crimping process. The maximum values of Lorentz force and magnetic pressure are reached when this gap is initially set at the minimum possible.

To analyze the impact of the voltage load on the deformation of the tube, three figures are represented in figure 11 and illustrate the tube’s deformation and the Lorentz force distribution for 2.5 kV, 5.5 kV and 8.5 kV voltage loads at 22 μs. These figures proves that the Lorentz force is increasing when the voltage is higher. This leads to produce higher pressure on the tube and as a result a strong crimping.

Afterwards, we can find radial Lorentz force, magnetic pressure as well as the velocity variations in one element of the tube. This element is located at the front edge of the outer cylinder of the tube. The main characteristics and variables of the process are shown in figures 12 and 13 illustrating respectively, the development of Lorentz force magnitude and magnetic pressure over time at the center of the coil and in the tube-coil working zone.

Magnetic pressure, Lorentz force and velocity are obtained at the outer ra-
At this point, both magnitudes are maximum for each voltage load, while we notice the minimum at \( x, y, z = 0, -10.5, 0 \) corresponding to the gap of the input and output bars of the current. This result indicates that the Lorentz force increases if the gap between the coil and the tube decreases. Therefore, in order to obtain the maximum deformation, the gap needs to be very small, which enables us to have the maximum magnetic impact on the work-piece. In the results illustrated in figures 12 and 13, we observe a time difference between the peaks in the graphs. A possible explanation for the magnetic pressure peak might be that it is based on the force variation in the element’s thickness. The pressure is produced due to the Lorentz force applied on the cross-section of the tube, and we cannot say that its peak should correspond exactly to the peak of this force at the same time. Another interesting finding was that the Lorentz force peak is not generated at the same time as the peak of the current.

Figure 9: Representation of the Lorentz force distribution radially during deformation at: a) 4 \( \mu \text{s} \); b) 8 \( \mu \text{s} \); c) 14 \( \mu \text{s} \); d) 22 \( \mu \text{s} \).
Figure 10: Lorentz force distribution in the outer surface of the tube at: a) 4 $\mu$s; b) 8 $\mu$s; c) 14 $\mu$s; d) 22 $\mu$s.

Figure 11: Lorentz force distribution for various voltage loads: a) 2.5 $kV$; b) 5.5 $kV$; c) 8.5 $kV$ pulse. This discrepancy could be attributed to the fact that the Lorentz force is produced due to the magnetic field $B$ generated by the coil and the current density in the tube, this current density is based on the current $I_t$ in the tube expressed by the previous Eq. (2). As mentioned in this formula, $I_t$ expresses a
phase shift of $\pi/2$, and therefore, the Lorentz force also presents a phase shift according to Eq. (22) which may explain the time difference between the current and the force plots.

In figure 14, we represent the velocity variation at various voltage loads. The results show that the deformation velocity is higher when we apply the maximum load.

At 8.5 kV, we reach a high velocity of 420 m/s while at 2.5 kV, we reach 25 m/s which explain the non deformation of the tube at this load. At 5.5 kV, we reach a velocity of 150 m/s. To validate the velocity data in this field, we compared our results to some existing experimental studies that are cited in the introduction section. In [19], the velocity measured using a Photon Doppler
Velocimetry (PDV) system reaches 160 m/s at about 8 kV and a current peak of 300 kA. This result corresponds to the one where the load is 5.5 kV. The difference between the two results may be due to the coil and the tube’s geometry but as well as the EMPG parameters. In the study conducted by [8], the velocity reaches experimentally 300 m/s with a current peak of 300 kA. Furthermore, the velocity reaches about 270 m/s at about 380 kA in [20]. All these results show that this process produces a very high velocity to reach the deformation of the work-piece. However and to the best of our knowledge, we did not find the same coil geometry in previous studies and consequently, we cannot precisely conclude the validation of our results using previous experimental researches.

5.2. Analytical results

This section aims to compare the two results (numerical and analytical) and to examine the electrical parameters of the coil and the tube. It is worth nothing that results provided in this section present only the magnitude of each parameter and do not give their variations at any time during the crimping process. The following results are given for radius $R_1 = 11$ mm, $R_2 = 11.5$ mm, $R_3 = 10$ mm and $R_4 = 10.5$ mm. For the geometry configuration as represented in figure 3, we begin by stating the self-inductance, resistance and mutual inductance of the tube and the coil as stated in the table 4.

<table>
<thead>
<tr>
<th>$L(nH/m)$</th>
<th>$R(m\Omega)$</th>
<th>$L_t(nH/m)$</th>
<th>$R_t(m\Omega)$</th>
<th>$M_{ct}(nH/m)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>1.74</td>
<td>1.75</td>
<td>6.4</td>
<td>1.37</td>
</tr>
</tbody>
</table>

The results obtained from the table above gives an insight into the coupling coefficient $K = \frac{M}{\sqrt{L \cdot L_t}}$ of the system. In this case, it has a value of 0.4. This result is very significant as it specifies how the magnetic flux produced by the coil will interact with the tube.

The second set of results focuses on the analytical method where the load
voltage is 8.5 kV. For this value, the current magnitudes $I_c$ and $I_t$ in both the coil and the tube are given respectively by $I_c = 2.43e3$ kA and $I_t = 1.009$ kA. Using Eq. (26) $\vec{B} = \mu \vec{H}$, the magnetic field magnitude $B$, which is generated in the outer radius of the tube at the deformation area, has a value of $B = 63.76$ T, while at the inner radius of the tube, it is about $B = 69$ T. It is apparent from this data that the magnetic field generated in the coil-tube gap is very intense. These parameters are decreasing when the voltage load is decreasing.

If we now turn to Lorentz force and magnetic pressure acting directly on the tube, we find remarkable results; as follows: the Lorentz force magnitude at the outer radius of the tube is about $F = 625$ GN while the magnetic pressure is about $P = 0.16$ GPa. At 8.5 kV, both magnitudes increase from the outer radius to the inner radius of the tube. Consequently, we find that at the inner radius, Lorentz force is about 639 GN while the magnetic pressure is about 0.193 GPa. However, at 2.5 kV and 5.5 kV, we found that these parameters are decreasing from the inner radius to the outer radius of the tube. These results may be explained by the fact that the magnetic field reaches maximum values at the inner radius of the tube. As for the numerical simulation results, the Lorentz force at the outer radius is about 654 GN while the magnetic pressure is 0.17 GPa which corresponds to the analytic results. The last part of the results is illustrated in figures 15 and 16. It provides a comparison of the Lorentz force (figure 15) and the magnetic pressure (figure 16) analytically and by numerical simulation at the outer radius of the tube.

From these figures, we conclude that the analytic results are almost similar to the numerical ones at 2.5 kV and 8.5 kV. However at 5.5 kV, according to the numerical results, we observe a significant error of 15% precisely in the magnetic pressure. As a conclusion, these results suggest that there is an association between the analytic model and the numerical one. They also provide important insights into the modeling of the process based on the electrical parameters of the coil and the work-piece.
6. Conclusion and perspectives

The main objective of this study is to identify Lorentz force and the magnetic pressure analytically and by numerical simulations in the case of crimping a metallic tube using an electromagnetic pulse generator. The most prominent finding to emerge from this study is that both methods give a maximum value of the force when the voltage load is higher. The first contribution is to determine the analytical Lorentz force applied to a metallic tube. The analytic method has identified the self-inductance and the resistances of the coil and the tube as well as the mutual inductance. The second contribution of this work was dedicated to the numerical method based on Maxwell equations, where the results of the deformation pattern are estimated numerically. The findings reflect those of a similar study of the tube crimping process by a multi-turn solenoid coil [18]. The maximum magnetic pressure $P_{\text{max}}$ found at 8.5 kV is $P_{\text{max}} = 0.17 \text{ GPa}$. This value provides a higher deformation of the tube. In the current research work, the numerical and analytical findings give almost the same Lorentz force magnitude at the coil-tube working zone as well as the magnetic pressure except at 5.5 kV load where we observed a significant error in the magnetic pressure results. These results provide a strong hypothesis that our model presents the parameters of the coil and the work-piece properly. A comparison of the two results shows that both methods give the same results of both magnitudes.
Overall, these results provide valuable insights into the representation of the crimping process analytically since the amount of time for analytical calculations does not exceed 3 seconds. However, the numerical method is very demanding in terms of computing time (about 3 hours). The main weakness of this study is that the analytical model does not consider the variation through time of the magnetic parameters in both the tube and the coil, which could be usefully explored in future research work. Furthermore, instead of the crimping process, an additional study could assess the electromagnetic pulse welding process using the developed methods in this paper.

Acknowledgment

Authors would like to gratefully thank Hauts-de-France region for its financial support and for the Innovaltech platform that served as the experimental platform for our research work.

References


doi:10.1109/ISEF.2017.8090724.


URL http://www.matweb.com/