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Lajos Horváth[a], Bo Li[b], Hemei Li[c], Zhenya Liu[c,d*]

[a] Department of Mathematics, University of Utah, Salt Lake City, UT 841120090, USA
[b] Bussiness School, Beijing International Studies University, Beijing, 100024, China
[c] School of Finance, Renmin University of China, Beijing, 100872, China
[d] CERGAM, Aix-Marseille University, 13090 Aix-en-Provence Cedex 02, France

[*] Corresponding author at: No.59, Zhongguancun Street, Haidian District, Beijing, 100872, China.
Email address: zhenya.liu@ruc.edu.cn

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Abstract

In this paper, we introduce the functional analysis method to investigate how betas change over time in the factor models. Based on the China A-share data, we drop the constant beta assumption in the CAPM and multi-factor models to estimate the time-varying betas directly from the functional data regression. The empirical results show that exposures to all risk factors have certain time-varying patterns in the Chinese A-share stock market.

Keywords: Functional factor models, Time-varying beta, Functional regression, Risk factors, Basic functions

JEL classification: C31, C32, C51, G12

1. Introduction

Based on the mean-variance model (Markowitz, 1952), Sharpe (1964), Lintner (1965), and Mossin (1966) introduce the capital asset pricing model (CAPM) and use the regression coefficient beta to measure exposure to market risk. Later, many scholars added firm-specific (unsystematic) risk factors to the CAPM, leading to the multi-factor models (Fama & French, 1992, 1993, 2015; Carhart, 1997; Acharya & Pedersen, 2005). However, most factor models are restricted by the constant beta assumption, and existing studies on time-varying beta are mainly based on the Kalman filter model (Hameed, 1997; Zhou, 2013). In this study, we propose a functional approach (Ramsay & Silverman, 2005; Horváth & Kokoszka, 2012) to dynamic asset pricing models based on the CAPM, the Fama-French three-factor model, and the Fama-French five-factor model to explore the time variation of exposure to risk factors.

There is a long-standing discussion of the validity of these models and statistical testing in these models. As beta is unobservable, the linear regression is used to estimate it. Jayasinghe et al. (2014) and Bu et al. (2019) argue that an ordinary linear regression for constant betas may not be compatible with financial theory. Besides, it should be noted that the constant beta assumption assumes the stationarity of the data. However, betas might change during the observation period. Levy (1971) studies the time dependence of betas for different portfolios and finds that betas are time-dependent for smaller portfolios. Bos & Newbold (1984) and Bollerslev et al. (1988) also show that beta is time-varying. Many other scholars argue that asset returns are not constant (Campbell & Hentschel, 1992; Glosten et al., 1993). In statistical testing, Fama & French (2016) point out that the constant beta assumption may be a potential problem. Bodurtha Jr & Mark (1991) propose the GMM test for the CAPM with time-varying returns and risks, and Velu & Zhou (1999) extend the GMM test to multi-factor asset pricing models.

Because of the considerable evidence that estimated betas are time-varying, some studies on asset pricing investigate the time variation of betas. Merton (1973) and Breeden (1979) extend the one-period assets pricing model to multiple periods, and introduce the intertemporal CAPM and the consumption CAPM, respectively. Many authors advocate conditional assets pricing models, such as the conditional CAPM (Ferson & Harvey, 1991; Jagannathan & Wang, 1996) and the dynamic conditional betas model (Engle, 2016), which they apply to equities, bonds, and portfolios. In empirical research, ARMA, GARCH, and Kalman filter models are often used to study betas' time variations. Blume (1975) and Sunder (1980) confirm the time-dependence of betas in the AR (1) model. Ohlson & Rosenberg (1982) and Collins et al. (1987) obtain the similar results in the ARMA model. Ng (1991) and Lee et al. (2001) analyze time-varying volatility using various GARCH models. Kim et al. (2009) and Borup (2019) combine the time-varying coefficient approach with the Kalman filter technique. Furthermore, Kim & Kim (2016) reject the constant-volatility assumption and employ the nonparametric kernel method to examine time-varying volatility.

In conventional asset pricing models, we usually apply the regression to the scalar data, such as the excess returns at the end of each month. However, this method only relies on the information of discrete-time series data at t and completely ignores the information throughout $t-1$ to t. Functional data analysis aims to

construct functions, such as excess returns curves, to extract additional information between time $t-1$ and t. Ramsay & Silverman (2005) and Ramsay & Silverman (2007) provide introduction to functional data analysis. Functional data analysis has been widely used in quantum mechanics, bioengineering, and other fields, but so far, it is rarely used in finance. Horváth & Kokoszka (2012) develop functional data analysis for time-series data. Based on this theory, Kokoszka et al. (2015) introduce a functional regression to model asset pricing by transforming daily asset returns and the market factor into functions and constructing a functional factor model with both scalar and functional factors. However, Kokoszka et al. (2015) employ the functional factor model with the constant beta assumption. Besides, Kokoszka et al. (2018) propose a statistical significance test for risk factors in a functional regression with functional cross-section returns and scalar risk factors. Horváth & Kokoszka (2012) and Cao et al. (2019) decompose the cross-section returns using functional principal component analysis and find momentum and disposition effects in Chinas A-share market.

Motivated by the functional factor model in Kokoszka et al. (2015, 2018), we introduce functional asset pricing models that can investigate how betas change over time. The main contribution of this study is that it allow time-varying betas in the functional asset pricing models. One advantage of a functional regression is that it will enable researchers to estimate functional coefficients. In contrast to Kokoszka et al. (2018), we use time-series data, and both excess returns and risk factors are functions of time t. Based on our proposed functional factor models, we can test the validity of factor models without the restriction of constant betas.

The rest of this paper is organized as follows: Section 2 introduces functional factor models. Section 3 discusses the estimation of the time-varying beta and the construction of confidence bands. We discuss our data in Section 4 and investigate the time variation of betas in Section 5. Section 6 summarises the application of functional factor models to the Chinese stock market.

2. Conventional and Functional Factor Models

2.1. Conventional factor models

For any security or portfolio, the conventional CAPM [1] is given by the linear regression

$$R_t = \alpha + \beta_1 R_{M,t} + \varepsilon_t, \tag{1}$$

where R_t is the excess returns on any security or portfolio at time t[2]; $R_{M,t}$ is the excess returns on market portfolio according to the modern portfolio theory.

The CAPM has been extended to the three-factor model (Fama & French, 1993)

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \varepsilon_t, \tag{2}$$

[1] For any security or portfolio i, the conventional CAPM can be expressed as $R_t^i = \alpha^i + \beta^i R_{M,t} + \varepsilon_t^i$, for the sake of simplicity, we omit the superscript i.

[2] In this study, we choose monthly data from January 1997 to December 2018 (264 months), so $t = 1, 2, \ldots, 264$.

where SMB_t is the difference between returns on a value-weighted portfolio of small stocks and that of big stocks. HML_t is the difference of returns for high and low book-to-market ratio.

The five-factor model (Fama & French, 2015)

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \varepsilon_t, \tag{3}$$

where RMW_t is the difference of returns for robust and weak operating profit and CMA_t is the difference of returns for conservative and aggressive investment.

2.2. Functional factor models

Now we introduce functional factor models. Along the lines of the conventional factor model, the functional CAPM [3] is defined as

$$R_i(t) = \alpha(t) + \beta(t) R_M(t) + \varepsilon_i(t), \tag{4}$$

where $R_i(t)$ is the functional time-series excess returns on portfolio over time t [4]; $R_M(t)$ is the functional time-series market factor over time t.

The functional three-factor model is

$$R_i(t) = \alpha(t) + \beta_1(t) R_M(t) + \beta_2(t) SMB_i(t) + \beta_3(t) HML_i(t) + \varepsilon_i(t), \tag{5}$$

where $SMB_i(t)$ is the functional time-series size factor, and $HML_i(t)$ is the functional time-series value factor.

Similarly, the functional five-factor model is

$$R_i(t) = \alpha(t) + \beta_1(t) R_M(t) + \beta_2(t) SMB_i(t) + \beta_3(t) HML_i(t) + \beta_4(t) RMW_i(t) + \beta_5(t) CMA_i(t) + \varepsilon_i(t), \tag{6}$$

where $RMW_i(t)$ is the functional time-series profitability factor, and $CMA_i(t)$ is the functional time-series investment factor.

3. Methodology

In this section, we discuss statistical inference for time-varying beta. The general functional factor regression can be expressed as

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t) F_{i,j}(t) + \varepsilon_i(t),$$

or

$$\boldsymbol{R}(t) = \boldsymbol{\beta}(t) \boldsymbol{F}(t) + \boldsymbol{\varepsilon}(t), \tag{7}$$

[3] Similar to the conventional factor models, for any security or portfolio i, the functional CAPM can be expressed as $R_i(t) = \alpha(t) + \beta(t) R_M(t) + \varepsilon_i(t)$.

[4] We consider the excess returns of stock or portfolio i as a function of time t ($t = 1, 2, \ldots, 264$), i.e., $R_i(t)$.

where $\boldsymbol{\beta}(t) = (\beta_1(t), \ldots, \beta_m(t))$ and $\boldsymbol{F}(t) = (\boldsymbol{F}_1(t), \ldots, \boldsymbol{F}_m(t))$ contains all functional time-series factors. Equation (7) is a standard general linear model, we can estimate $\boldsymbol{\beta}(t)$ by the standard least squares criterion. We extend the method to minimize the residual sum of squares in the functional regression case. The least squares function, as a function of $\boldsymbol{\beta}(t)$, in our model is

$$\text{LMSSE}(\boldsymbol{\beta}) = \int [\boldsymbol{R}(t) - \boldsymbol{\beta}(t)\boldsymbol{F}(t)]'[\boldsymbol{R}(t) - \boldsymbol{\beta}(t)\boldsymbol{F}(t)]dt.$$

The functional excess returns $R_i(t)$ is expressed as

$$R_i(t) = \sum_{k=1}^{K} c_{i,k}\phi_k(t) = \boldsymbol{C}_i'\boldsymbol{\Phi}(t).$$

where $\Phi(t)$ are the basis function vector.

Similarly, we assume that

$$\beta_j(t) = \sum_{k=1}^{K_j} b_{j,k}\psi_{j,k}(t) = \boldsymbol{b}_j'\boldsymbol{\psi}_j(t), \tag{8}$$

where $\boldsymbol{b}_j$ is the coefficient vector for functional beta $\beta_j(t)$ and $\boldsymbol{\psi}_j(t)$ is the vector of Fourier basis functions. Hence, the estimation of $\beta_j(t)$ can be based on the estimation of coefficient vector $\boldsymbol{b}_j$.

To express the model in a consistent form, we define $\boldsymbol{B} = (\boldsymbol{b}_1', \boldsymbol{b}_2', \ldots, \boldsymbol{b}_m')$ and

$$\boldsymbol{\Psi}(t) = \begin{pmatrix} \boldsymbol{\psi}_1(t) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \boldsymbol{\psi}_m(t) \end{pmatrix}.$$

Thus the functional factor model can be rewritten as

$$\boldsymbol{R}(t) = \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t) + \boldsymbol{\varepsilon}(t),$$

we minimize

$$\begin{aligned} \text{LMSSE}(\boldsymbol{\beta}) &= \int [\boldsymbol{R}(t) - \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)]'[\boldsymbol{R}(t) - \boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)]dt \\ &= \int \boldsymbol{R}'(t)\boldsymbol{R}(t) - 2\boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{B}'\boldsymbol{R}(t) + \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{B}'\boldsymbol{B}\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt. \end{aligned}$$

Minimizing LMSSE($\boldsymbol{\beta}$) is equivalent, according to equation (8), to minimize with respect of the coefficient matrix. We find that the coefficient matrix $\boldsymbol{B}$ is the solution of

$$\boldsymbol{B}\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt = \int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{R}(t)dt.$$

Hence, we need to estimate the coefficient matrix $\boldsymbol{B}$, and get estimate for $\boldsymbol{\beta}(t)$.

Next, we construct confidence bands. The construction of the confidence bands requires the variance and covariance of the estimated coefficient matrix $\hat{\boldsymbol{B}}$.

The estimated coefficient matrix is

$$\begin{aligned} \hat{\boldsymbol{B}} &= \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1} \int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{R}(t)dt \\ &= \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt\right)^{-1} \int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{C}\boldsymbol{\Phi}(t)dt. \end{aligned}$$

Hence we get[5]

$$\text{vec}(\hat{\boldsymbol{B}}) = \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt \right)^{-1} \int \boldsymbol{\Phi}'(t) \otimes \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)dt\text{vec}(\boldsymbol{C}).$$

Let $\boldsymbol{S}_\psi$ be the linear mapping that maps the excess returns $\boldsymbol{R}$ into the coefficient matrix $\boldsymbol{C}$, i.e. $\boldsymbol{C} = \boldsymbol{R}\boldsymbol{S}_\psi$. Then

$$\text{vec}(\boldsymbol{C}) = (\boldsymbol{S}'_\psi(t) \otimes \boldsymbol{I})\text{vec}(\boldsymbol{R}).$$

The variance of $\boldsymbol{R}$ is given by

$$\text{var}(\text{vec}(\boldsymbol{R})) = \Sigma_\varepsilon \otimes \boldsymbol{I},$$

where Σ_ε is the variance-covariance matrix of the error vectors. Then the variance of estimated coefficient matrix $\hat{\boldsymbol{B}}$ is

$$\text{var}(\text{vec}(\hat{\boldsymbol{B}})) = \boldsymbol{A}(\boldsymbol{S}'_\psi(t) \otimes \boldsymbol{I})\Sigma_\varepsilon \otimes \boldsymbol{I}(\boldsymbol{S}'_\psi(t) \otimes \boldsymbol{I})'\boldsymbol{A}',$$

where $\boldsymbol{A} = \left(\int \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)\boldsymbol{\Psi}(t)\boldsymbol{F}(t)dt \right)^{-1} \int \boldsymbol{\Phi}'(t) \otimes \boldsymbol{F}'(t)\boldsymbol{\Psi}'(t)dt$. Hence, we have the variance of the estimated coefficient matrix $\hat{\boldsymbol{B}}$, and therefore we can construct confidence bands for the time-varying betas.

4. Data

In this study, we investigate functional factor models in the Chinese stock market. We select all A shares (except stocks denoted ST and *ST [6]) returns with monthly frequency from the China Stock Market & Accounting Research Database (CSMAR) and all risk factors data with monthly frequency from the China Asset Management Research Center. The data period is from January 1997 to December 2018 (264 months).

We construct 25 (or 100) value-weighted portfolios as observations, and the different observations correspond to different excess returns (dependent variable) and the same risk factors (independent variables).

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[5]The Vec-operator: $\text{vec}(\boldsymbol{A}) = (a_{11}, a_{21}, \ldots, a_{m1}, \ldots, a_{1n}, a_{2n}, \ldots, a_{mn})'$, where $\boldsymbol{A} = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$.

The Kronecker product $\otimes$ of $\boldsymbol{A}_{m\times n}$ and $\boldsymbol{B}_{p\times q}$ is the $mn \times pq$ matrix and defined as follows:

$$\boldsymbol{A} \otimes \boldsymbol{B} = \begin{pmatrix} a_{11}\boldsymbol{B} & \cdots & a_{1n}\boldsymbol{B} \\ a_{21}\boldsymbol{B} & \cdots & a_{2n}\boldsymbol{B} \\ \vdots & \ddots & \vdots \\ a_{m1}\boldsymbol{B} & \cdots & a_{mn}\boldsymbol{B} \end{pmatrix}.$$

[6]In the Chinese stock market, stocks indicated with ST and *ST represent listed companies in deficit for two consecutive fiscal years, respectively. Both types of companies face the risk of delisting.

4.1. Dependent variable: excess returns on 25 (or 100) Size-B/M portfolios

We sort all A-shares by market value (size factor) and divide them into five (or ten) groups. Then, we sort the stocks in each group by the book-to-market ratio and divide them into five (or ten) subgroups. Finally, we get 25 Size-B/M portfolios (or 100 Size-B/M portfolios). The excess returns on value-weighted portfolios are defined by

$$R_{i,t} = \sum_k \frac{mv^i_{k,t}}{\sum_k mv^i_{k,t}} r^i_{k,t}, \quad i = 1, 2, \ldots, 25 \; t = 1, 2, \ldots, 264$$

or

$$R_{i,t} = \sum_k \frac{mv^i_{k,t}}{\sum_k mv^i_{k,t}} r^i_{k,t}, \quad i = 1, 2, \ldots, 100 \; t = 1, 2, \ldots, 264,$$

where $R_{i,t}$ is the excess returns on portfolio i at time t, $r^i_{k,t}$ is the excess returns on stock k at time t in the ith portfolio and $mv^i_{k,t}$ is the market value of stock k.

4.2. Independent variables: risk factors

For each group (25 or 100 portfolios), we apply the same (25 or 100) risk factors (market factor, size factor, value factor, profitability factor and investment factor) constructed following Fama & French (2015).

4.3. Transform the discrete data to functional data

Next, we build functional data for time-series excess returns and risk factors. We map the time-series data $\{R_{i,t}\}_{t=1}^{264}$ into functions to get the functional data $\{R_i(t), 0 \leq t \leq 1\}$, and $R_i(t)$ can be expressed by the basis expansion

$$R_i(t) = \sum_{k=1}^{K} c_{i,k}\phi_k(t) = \boldsymbol{C}_i'\boldsymbol{\Phi}(t),$$

where $\boldsymbol{\Phi}(t)$ is the vector of basis functions, and $\boldsymbol{C}_i$ is a coefficient vector in the basis function expansion. The general functional factor model is

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t) F_{i,j}(t) + \varepsilon_i(t), \quad i = 1, 2, \ldots, 25$$

or

$$R_i(t) = \sum_{j=1}^{m} \beta_j(t) F_{i,j}(t) + \varepsilon_i(t), \quad i = 1, 2, \ldots, 100,$$

where $F_{i,j}(t)$ denotes the functional risk factor vector.

In practice, two types of basis functions are used. The Fourier basis functions are suitable for periodic observations. The spline basis functions can be used for non-periodic observations. We choose the Fourier basis functions because we argue that both excess returns and risk factors show periodicity in the long run. Figure 1 - 5 illustrate the raw data for the factors and functional factors.

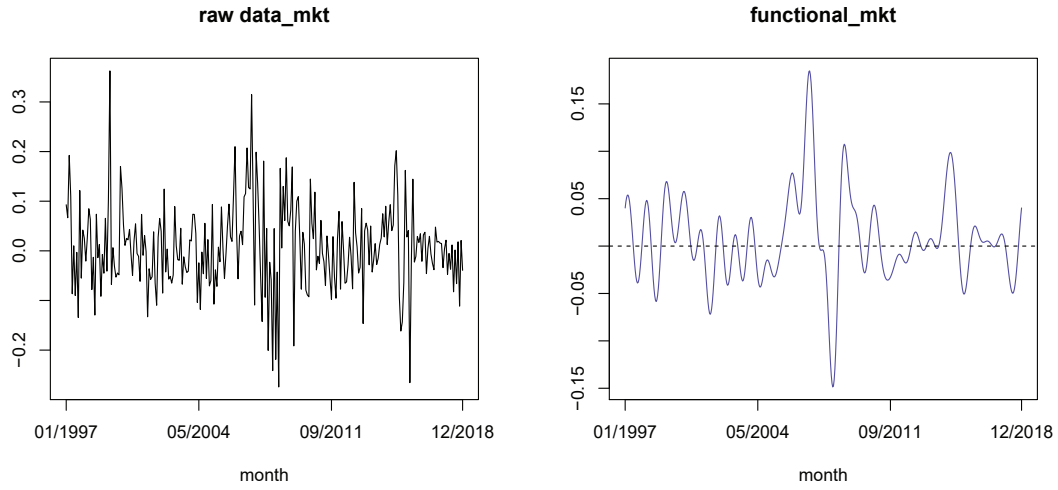

Figure 1: The market factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the market factor, and the right panel shows the functional market factor.

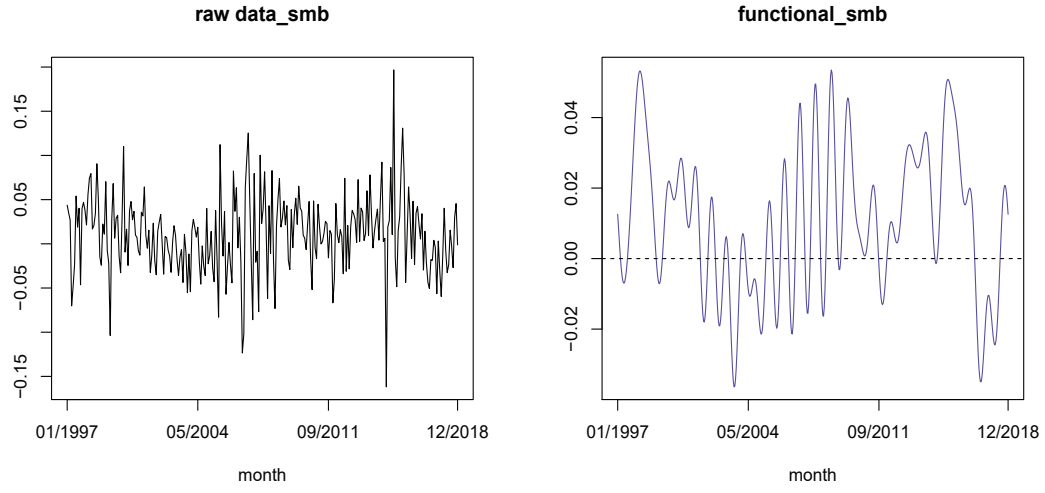

Figure 2: The size factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the size factor, and the right panel shows the functional size factor.

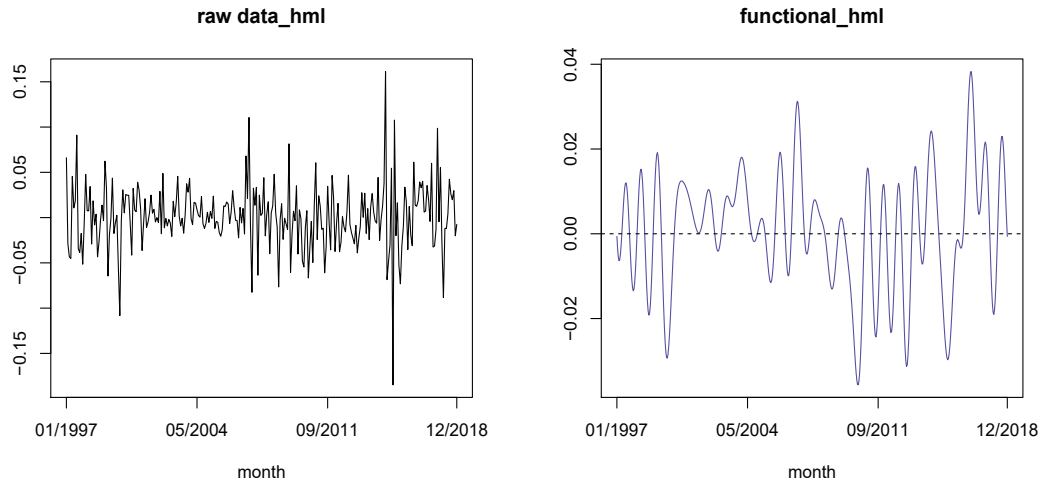

Figure 3: The value factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the value factor, and the right panel shows the functional value factor.

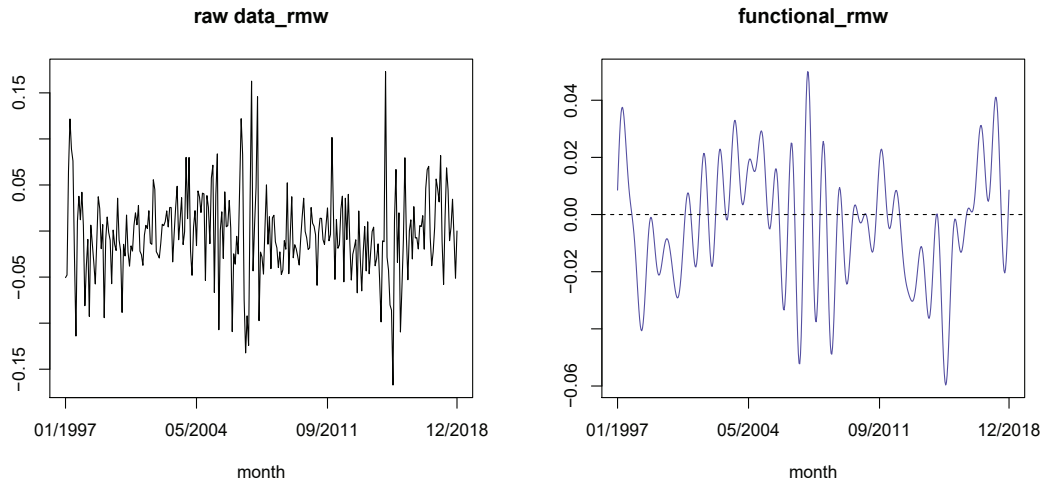

Figure 4: The profitability factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the profitability factor, and the right panel shows the functional profitability factor.

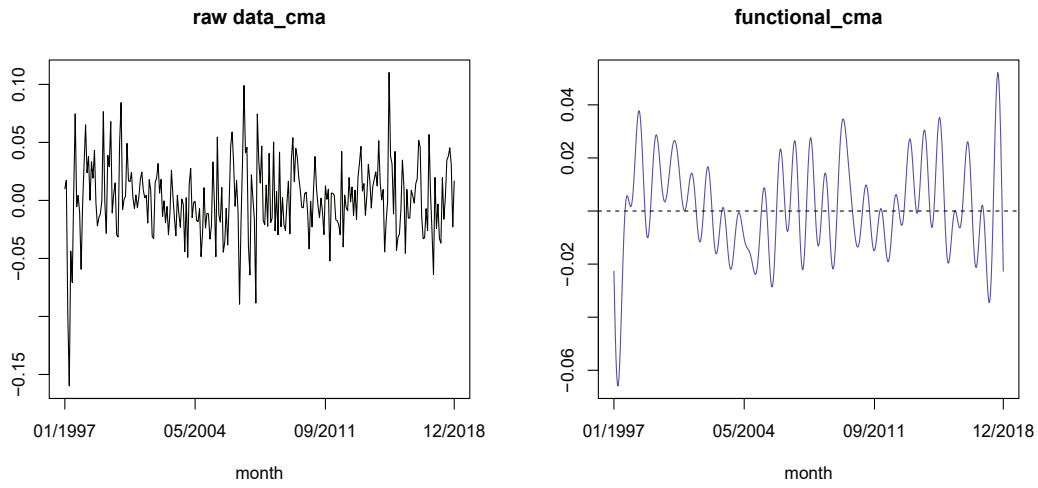

Figure 5: The investment factor in the Chinese stock market (January 1997 - December 2018). The left panel shows raw data of the investment factor, and the right panel shows the functional investment factor.

5. Functional Factor Models

In this section, we investigate the time-varying betas in the Chinese stock market. First, we construct 25 (or 100) Size-B/M portfolios for all A-shares and obtain the value-weighted excess returns.[7] Then, we employ the functional factor models with 25 (or 100) Size-B/M portfolios with time-varying betas (see equations (9) and (10)). As mentioned earlier, one advantage of functional factor models is that we can relax the constant beta assumption. Another advantage is that for every risk factor, in functional factor models, we obtain the conjoint $\beta_j(t)$ of all portfolios, which exhibits common exposure of all A-shares to each risk factor. In Section 5.2, we show that using different portfolio constructions has little influence on the time-dependent betas.

The least squares fitting criterion with 25 (or 100) portfolios is:

$$\min_{\{\beta_j(t)\}_{j=1}^m} \quad \sum_{i=1}^{25} \int_0^1 \left(R_i(t) - \sum_{j=1}^m \beta_j(t) F_{i,j}(t) \right)^2 dt \tag{9}$$

or

$$\min_{\{\beta_j(t)\}_{j=1}^m} \quad \sum_{i=1}^{100} \int_0^1 \left(R_i(t) - \sum_{j=1}^m \beta_j(t) F_{i,j}(t) \right)^2 dt. \tag{10}$$

5.1. Time-varying betas

With the functional excess returns on 25 (or 100) Size-B/M portfolios and functional time-series risk factors, we can employ the functional CAPM, functional three-factor model, and functional five-factor model. Figure 6 - 11 show the time-varying betas, where the solid line displays the estimated $\beta_j(t)$ and the dashed lines

[7] We use excess returns on portfolios instead of individual stocks because there is a lack of data on individual stocks on some trading days due to suspension and other reasons. This does not affect much on the excess returns of portfolios.

indicate the 95% confidence bands for $\beta_j(t)$. Table 1 and 2 report the regression details for the functional factor models, specifically the coefficient vectors $\boldsymbol{b}_j$ for each $\beta_j(t)$). To compare the empirical results of functional factor models with benchmark, we implement conventional asset pricing models with constant parameters (see equations (1)-(3)) for average excess returns of 25 (or 100) Size-B/M portfolios, and the results are shown in Table 3.

Figure 6 (Figure 7) reveals the time dependence of risk exposure to the market factor. We can see that our portfolios have positive exposure to the market factor in the Chinese stock market, which means that the market factor always has a positive effect on the returns of A-shares. From 1997 to 2003, exposure to the market factor is increasing. The Chinese stock market was volatile during the 1990s. In January 1996, the Shanghai Composite Index closed at 537.35, the index went up rapidly and closed at 1393.75 in April 1997. After the enactment of the securities law in 1997, the index dropped nearly 22% to close at 1090.09 in February 1999. In the two years since then, the index went to 2218.03 in June 2001 and then dropped again. The turbulence of China's stock market raises the market risk in the 1990s. During 2005-2006, the Chinese government implemented the non-tradable share reform to improve the governance of listed state-owned enterprises and solve the conflict of interest problem of shareholders in the A-share market. As the Chinese stock market matures, exposure to the market factor will gradually decline. This situation continued until the financial crisis broke out. As Schlueter & Sievers (2014) point out, business risks have an impact on the market beta, and the market beta increased again after the financial crisis broke out. After several ups and downs in the stock market, an increasing number of investors are choosing value investing, and exposure to market risk is no longer as high as it was earlier.

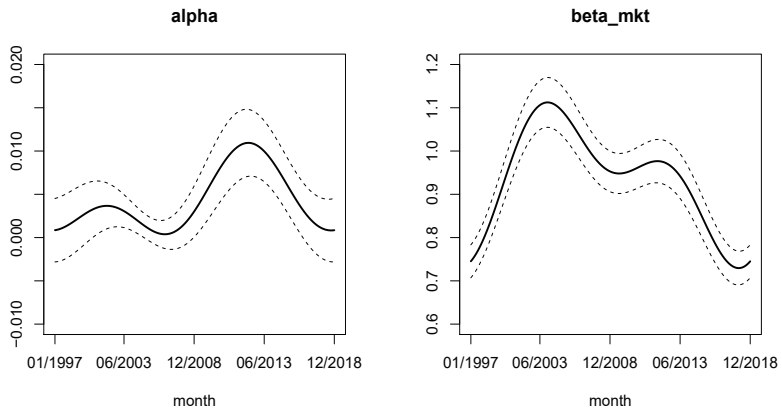

Figure 6: Time-varying beta of the functional CAPM with 25 Size-B/M portfolios

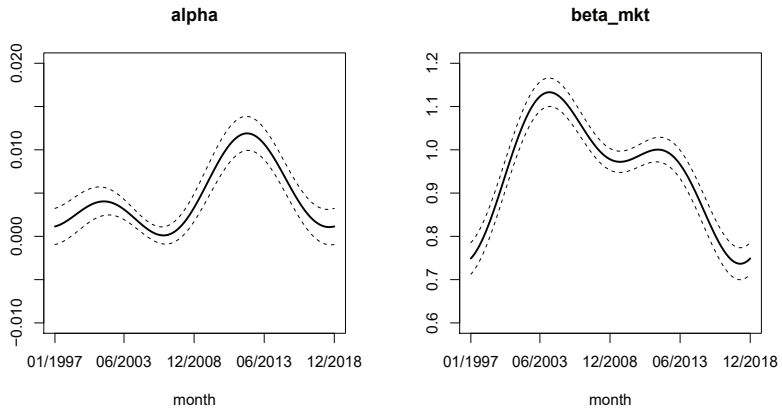

Figure 7: Time-varying beta of the functional CAPM with 100 Size-B/M portfolios

Figure 8 (Figure 9) shows the time dependence of risk exposure to the market, size, and value factors. We conclude: (a) Exposure to the market factor is always positive, which is the same as in the functional CAPM. (b) Exposure to the size factor is positive in two periods (before June 1999 and after December 2006). From 2000 to 2006, exposure to the size factor is negative, possibly due to the non-tradable share structure in the Chinese stock market. Li (2012) shows that China's stock market reforms, such as the non-tradable reform, will influence the stock market performance. Before 2005, there were two kinds of shares in Chinese listed companies: tradable and non-tradable shares. Non-tradable shares were dominant, which seriously hindered the circulation of listed companies' shares, so the size effect of the listed company was hardly reflected in the price. After the non-tradable share reform, the securities market has become more market-oriented, and small-scale companies have gradually shown higher excess returns. (c) Exposure to the value factor is positive only from 2011 to 2016, so the value factor had a positive effect on A shares only for a short period (from 2011 to 2016). Looking back to the history of the Chinese stock market, exposure to the value factor is related to the performance of the stock market (Fama & French, 1996). The Chinese stock market is much more complicated and turbulent before 2011, as it has a large or small bear and bull markets. In the two years after the non-tradable share reform, the Shanghai Composite Index multiplied from 1258.05 in January 2006 to 5261.56 in December 2007 and then collapsed abruptly due to the global financial crisis. During the financial crisis, exposure to the value factor reached its lowest point. Since then, exposure to the value factor has gradually increased and become positive in the period from 2011 to 2016, during which the stock market is relatively stable. Therefore, the value factor's positive impact may be related to stock market stability, and investors often pursue value investing when the stock market is stable.

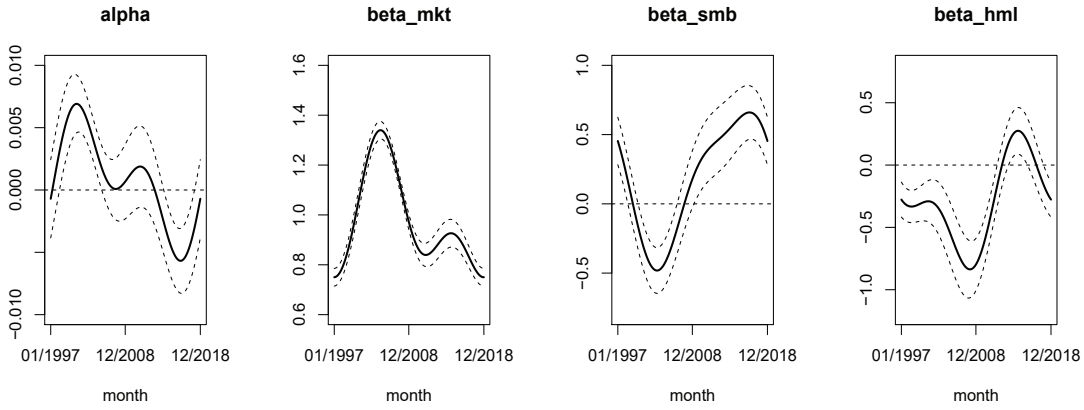

Figure 8: Time-varying beta of the functional 3-factor model with 25 Size-B/M portfolios

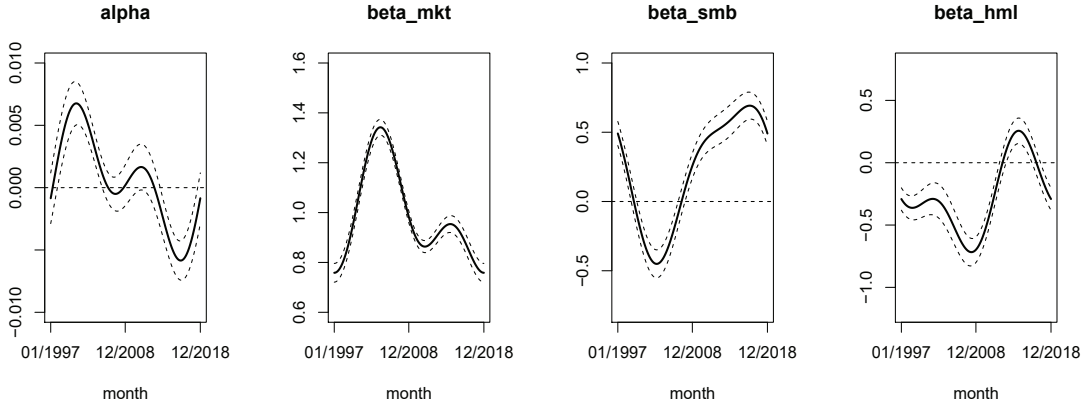

Figure 9: Time-varying beta of the functional 3-factor model with 100 Size-B/M portfolios

Figure 10 (Figure 11) shows the time dependence of risk exposure to the market, size, value, profitability, and investment factors. Generally, exposure to the market, size, and value factors in the functional five-factor model are similar to that in the functional three-factor model, and both the profitability factor and investment factor have negative effects on A shares most of the time. We observe: (a) The market factor has a positive effect on A shares at all times. (b) Comparing to the functional three-factor model, both models show that the size factor has a positive effect from 2012 to 2016, possibly because the Chinese stock market remained in a relatively stable period after 2011 and became a bear market after June 2015. The size factor has a positive effect during a stable period. (c) Both the functional three-factor and five-factor models show that the value factor has a positive effect from 2011 to 2016. As with the size factor, the value factor has a positive effect in a stable period. (d) Exposure to the profitability factor is almost always negative in the whole sample period, and reached its minimum after the financial crisis. Valuation theory says that the company's profitability is usually positively related to expected returns (Haugen et al., 1996; Fama & French, 2006). Nevertheless, portfolios have negative exposure to the profitability factor in the Chinese stock market; that is, most portfolios do not have high excess returns compared with a robust-profitability portfolio. (e) Exposure to the investment factor gradually changed from negative to positive after the non-tradable share reform in China. However, during the financial crisis, exposure to the investment factor reached its maximum and began to decline due to stock market turbulence.

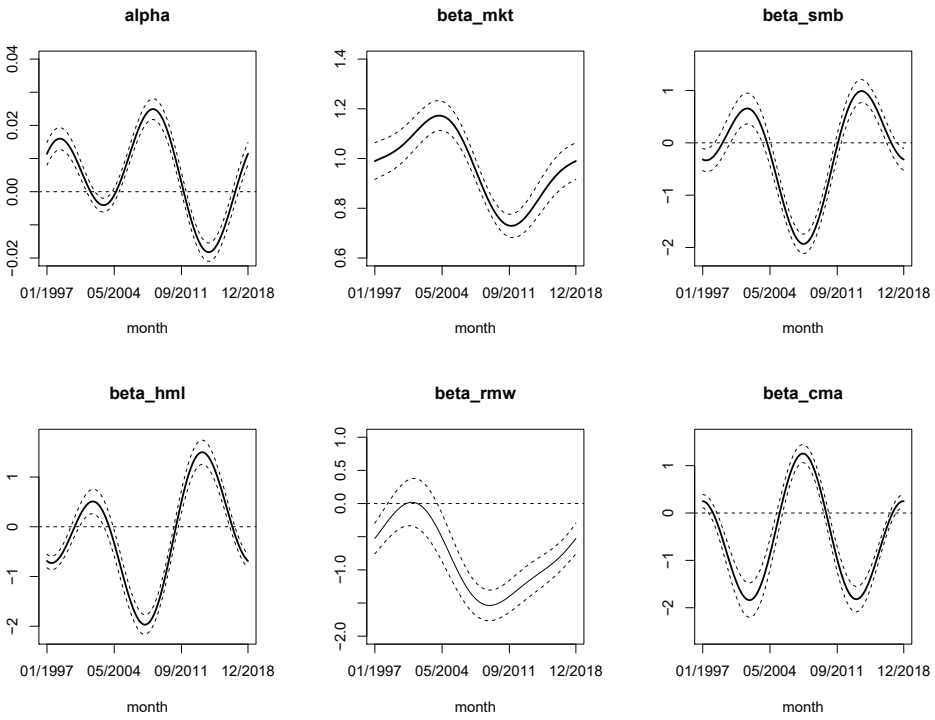

Figure 10: Time-varying beta of the functional 5-factor model with 25 Size-B/M portfolios

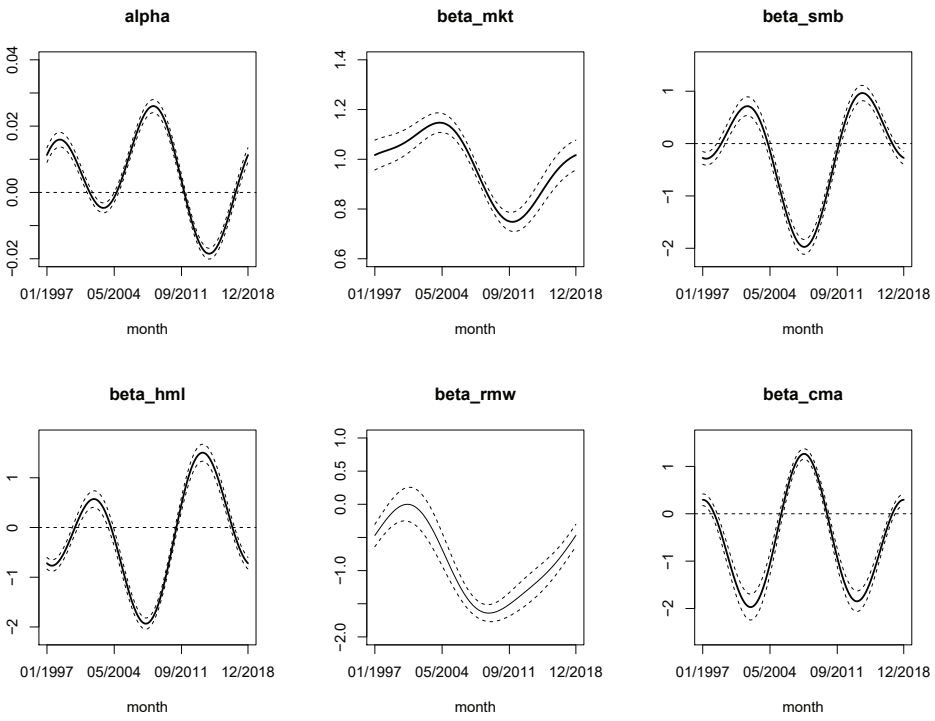

Figure 11: Time-varying beta of the functional 5-factor model with 100 Size-B/M portfolios

Table 1: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0685	-0.0401	-0.0119	0.0229	-0.0269	
Market factor	15.1877	0.9243	-1.2103	0.1693	-1.0153	0.2010
Functional Three-factor Model						
α	0.0123	0.0439	-0.0091	0.0363	-0.0105	
Market factor	15.9079	2.0642	-1.3010	-1.0733	-1.3455	
Size factor	2.8540	-5.8313	1.6908	-0.2215	1.6446	
Value factor	-4.7360	-3.3843	3.0480	-0.0041	-2.7960	0.2048
Functional Five-factor Model						
α	0.0734	0.0624	-0.0734	0.1012	0.1463	
Market factor	15.6311	2.1801	0.1384	-0.7365	0.1415	
Size factor	-3.1744	-1.2970	9.2743	-1.5089	-10.5884	
Value factor	-2.9267	-5.4290	7.3809	-1.0726	-13.0348	
Profitability factor	-13.1955	6.4993	5.0407	0.3500	-1.9246	
Investment factor	-8.5420	-0.1847	-5.7555	0.3540	14.6527	0.2165

Table 2: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0740	-0.0438	-0.0120	0.0277	-0.0274	
Market factor	15.4739	0.8829	-1.3317	0.1187	-1.0494	0.1928
Functional Three-factor Model						
α	0.0082	0.0426	-0.0072	0.0376	-0.0112	
Market factor	16.1191	1.9201	-1.3112	-1.0403	-1.3865	
Size factor	3.6129	-5.8935	1.4474	-0.1743	1.7797	
Value factor	-4.3223	-3.0301	2.3910	-0.4082	-2.5844	0.2003
Functional Five-factor Model						
α	0.0750	0.0580	-0.0796	0.1058	0.1497	
Market factor	15.7536	1.9901	0.2493	-0.6252	-0.2562	
Size factor	-3.1156	-0.6659	9.7549	-1.7854	-10.5950	
Value factor	-2.6380	-4.9560	7.0438	-1.6655	-13.2370	
Profitability factor	-14.3465	6.5660	6.0725	0.7583	-1.5171	
Investment factor	-8.9860	-0.9261	-5.5226	1.2188	15.2399	0.2084

Table 3: Conventional factor models for average excess returns of 25 or 100 Size-B/M portfolios

	α	β_1	β_2	β_3	β_4	β_5	R^2
CAPM							
25 portfolios	0.0098	0.1393					0.0173
	(1.75)	(2.15)					
100 portfolios	0.0103	0.1464					0.0183
	(1.79)	(2.21)					
Three-factor Model							
25 portfoios	0.0100	0.1429	0.0037	-0.2304			0.0254
	(1.73)	(2.16)	(0.03)	(-1.42)			
100 portfolios	0.0104	0.1496	0.0080	-0.2293			0.0260
	(1.76)	(2.21)	(0.06)	(-1.38)			
Five-factor Model							
25 portfoios	0.0123	0.1347	-0.2868	-0.4182	-0.1303	0.3743	0.0386
	(2.05)	(1.87)	(-1.16)	(-2.14)	(-0.49)	(1.36)	
100 portfolio	0.0129	0.1399	-0.3055	-0.4269	-0.1481	0.3899	0.0403
	(2.10)	(1.90)	(-1.21)	(-2.14)	(-0.54)	(1.39)	

Note: The number in parenthesis is the t-statistic.

5.2. The robustness of the results

5.2.1. Different portfolio constructions

To investigate whether different portfolio constructions with all A shares affect the time variation of betas, we perform functional factor models with 25 (or 100) Size-OP portfolios (OP represents operating profitability), 25 (or 100) Size-Inv portfolios (Inv represents investment) respectively. We show that different portfolio constructions have little influence on the time variation of betas (see Figure 12 - 17). The results with 25 (or 100) Size-Inv portfolios are shown in Appendix A.

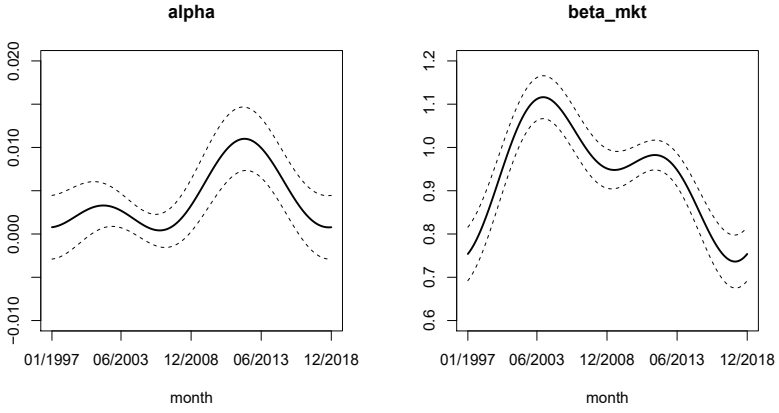

Figure 12: Time-varying beta of the functional CAPM with 25 Size-OP portfolios

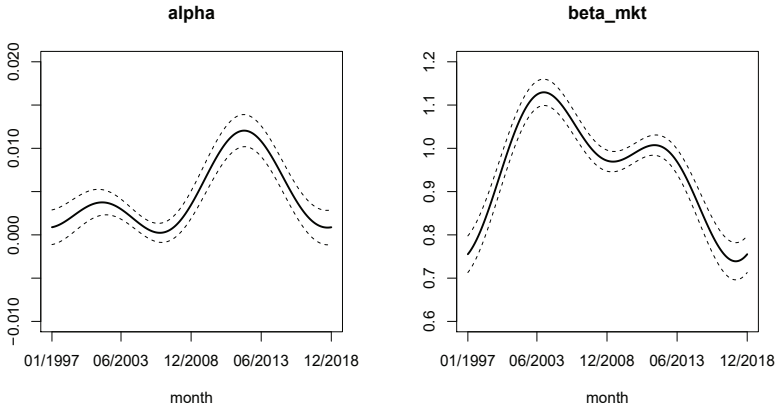

Figure 13: Time-varying beta of the functional CAPM with 100 Size-OP portfolios

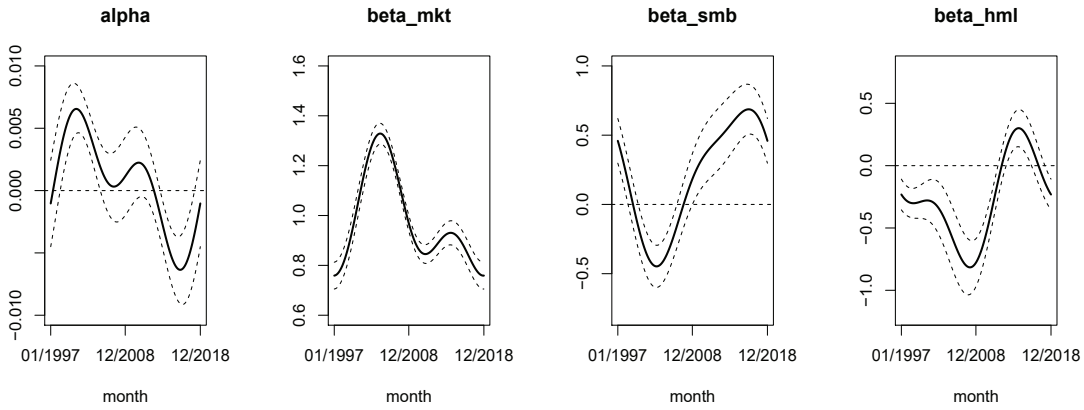

Figure 14: Time-varying beta of the functional 3-factor model with 25 Size-OP portfolios

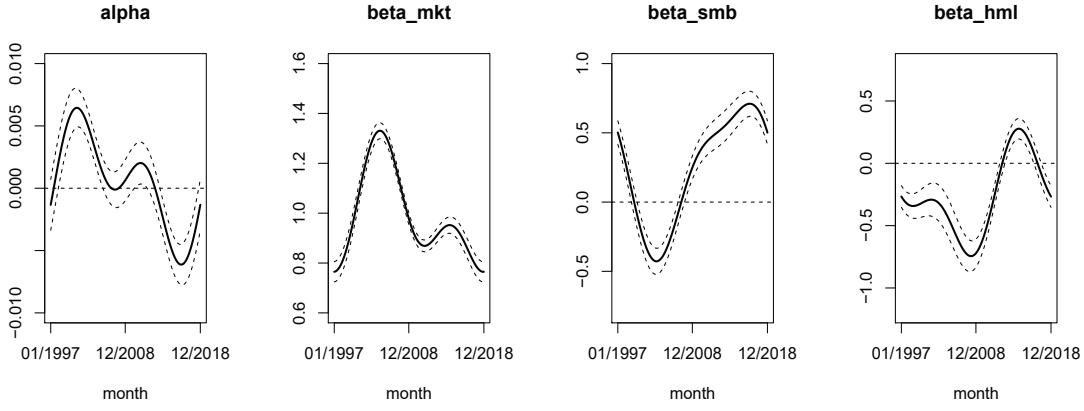

Figure 15: Time-varying beta of the functional 3-factor model with 100 Size-OP portfolios

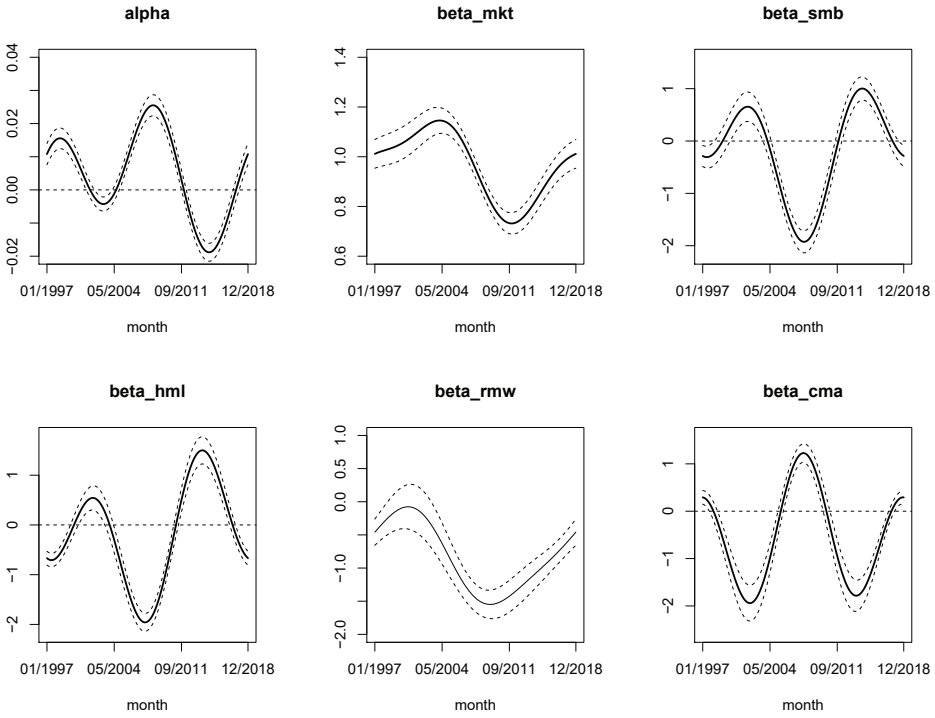

Figure 16: Time-varying beta of the functional 5-factor model with 25 Size-OP portfolios

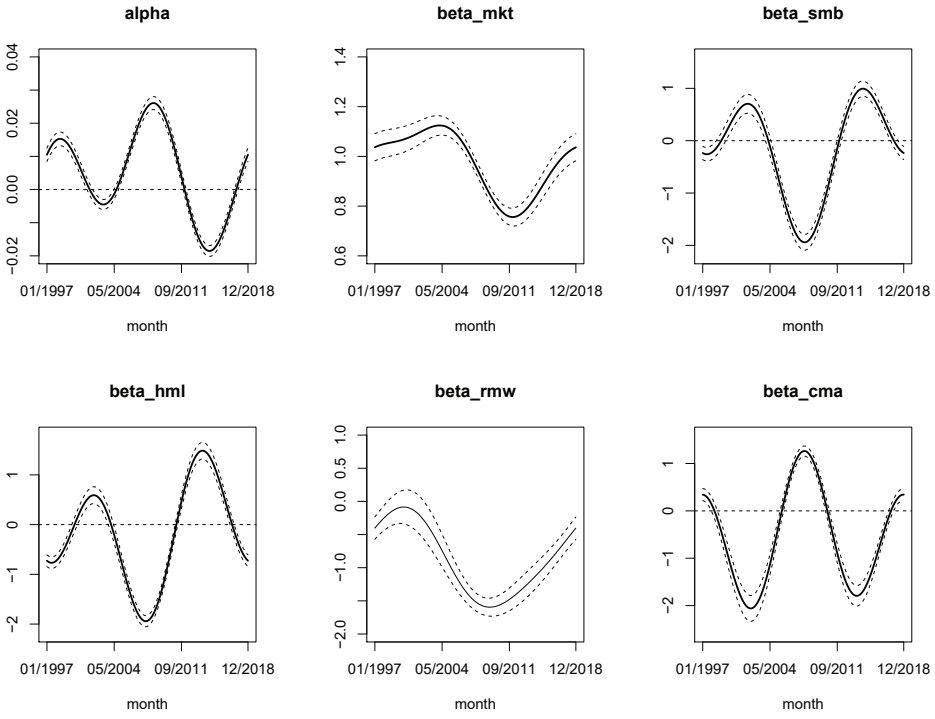

Figure 17: Time-varying beta of the functional 5-factor model with 100 Size-OP portfolios

Table 4: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-OP portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0678	-0.0420	-0.0140	0.0228	-0.0252	
Market factor	15.2627	0.9248	-1.1516	0.2109	-1.0291	0.2042
Functional Three-factor Model						
α	0.0092	0.0444	-0.0138	0.0382	-0.0075	
Market factor	15.9265	2.0043	-1.2286	-1.0055	-1.3308	
Size factor	3.1283	-5.7862	1.6948	-0.3718	1.5159	
Value factor	-4.3647	-3.4515	3.1996	-0.0853	-2.6831	0.2118
Functional Five-factor Model						
α	0.0711	0.0624	-0.0801	0.1040	0.1474	
Market factor	15.6219	2.0435	0.2878	-0.6781	0.2499	
Size factor	-3.0152	-1.2114	9.4331	-1.8720	-10.4542	
Value factor	-2.6679	-5.2267	7.4475	-1.1088	-13.0541	
Profitability factor	-13.5796	5.8189	5.5413	0.2759	-1.3543	
Investment factor	-8.7690	-0.9937	-5.3378	0.5203	14.9002	0.2198

Table 5: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-OP portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0736	-0.0455	-0.0144	0.0267	-0.0277	
Market factor	15.4866	0.8723	-1.2587	0.2236	-1.0637	0.1937
Functional Three-factor Model						
α	0.0078	0.0418	-0.0125	0.0374	-0.0111	
Market factor	16.1095	1.8975	-1.2486	-0.9398	-1.3731	
Size factor	3.7256	-5.7859	1.5571	-0.3005	1.7158	
Value factor	-4.2848	-3.1898	2.6713	-0.3640	-2.6040	0.2014
Functional Five-factor Model						
α	0.0730	0.0576	-0.0842	0.1062	0.1467	
Market factor	15.7804	1.8600	0.3746	-0.5502	0.3354	
Size factor	-2.7861	-0.7330	9.7748	-2.1016	-10.3958	
Value factor	-2.6966	-4.7765	7.0515	-1.5304	-13.2967	
Profitability factor	-14.1769	5.6757	6.2191	0.5276	-0.9772	
Investment factor	-8.9708	-1.7240	-5.2188	1.1588	15.4871	0.2092

5.2.2. Different basis functions

To check whether the choice of basis functions will affect the time-varying betas, we apply the spline basis functions to our functional factor models with 25 (or 100) Size-B/M portfolios and get the similar time-variations in betas over time (see Figure 18 - 23).

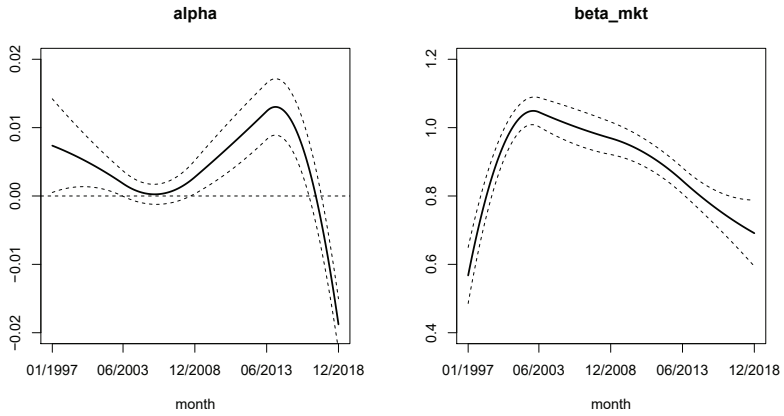

Figure 18: Time-varying beta (spline basis functions) of the functional CAPM with 25 Size-B/M portfolios

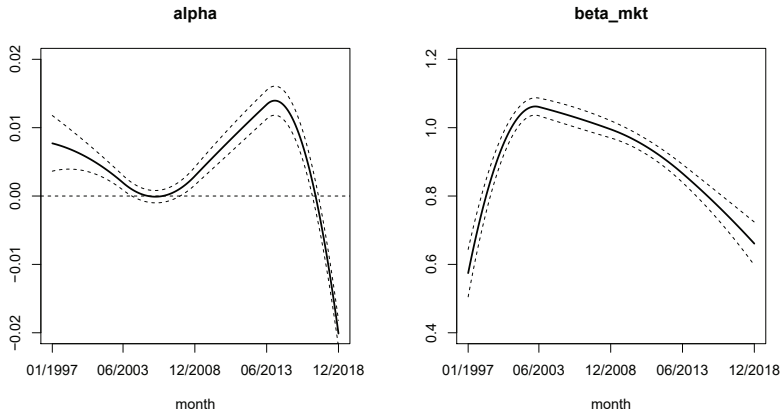

Figure 19: Time-varying beta (spline basis functions) of the functional CAPM with 100 Size-B/M portfolios

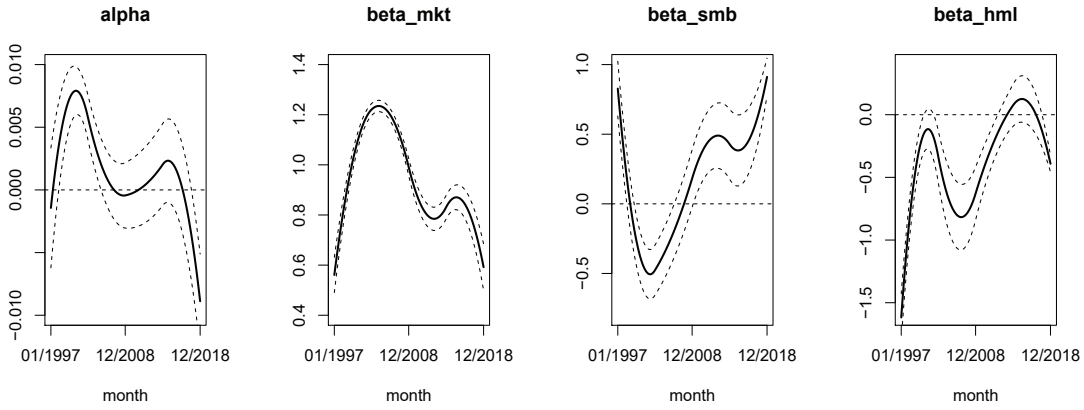

Figure 20: Time-varying beta (spline basis functions) of the functional 3-factor model with 25 Size-B/M portfolios

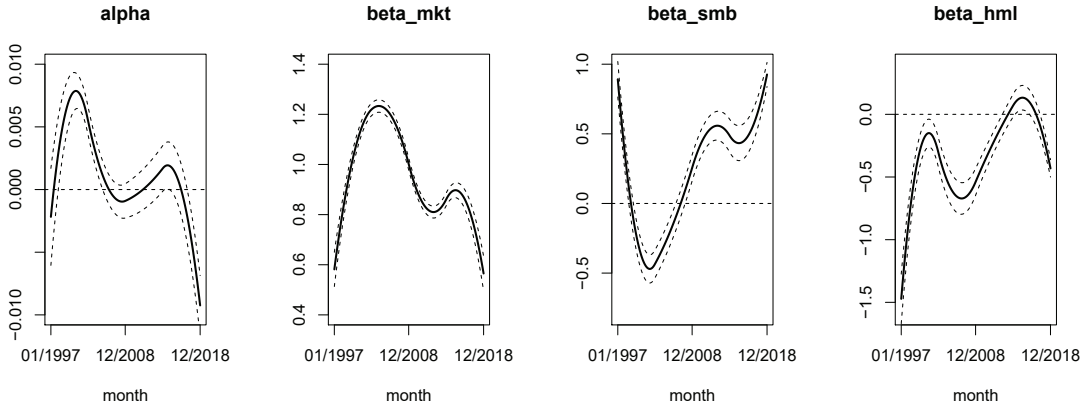

Figure 21: Time-varying beta (spline basis functions) of the functional 3-factor model with 100 Size-B/M portfolios

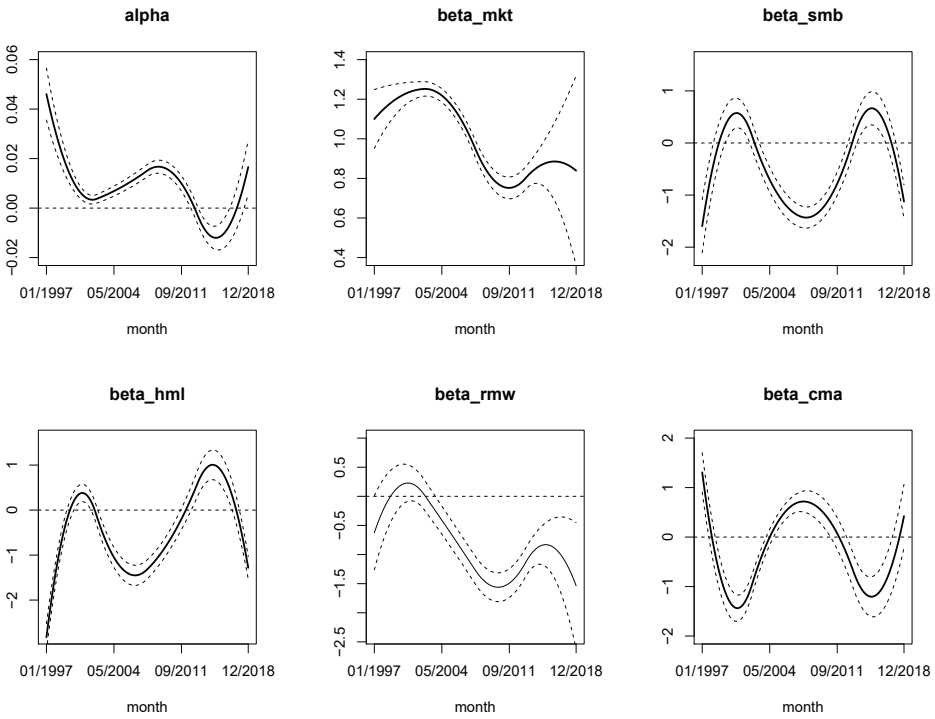

Figure 22: Time-varying beta (spline basis functions) of the functional 5-factor model with 25 Size-B/M portfolios

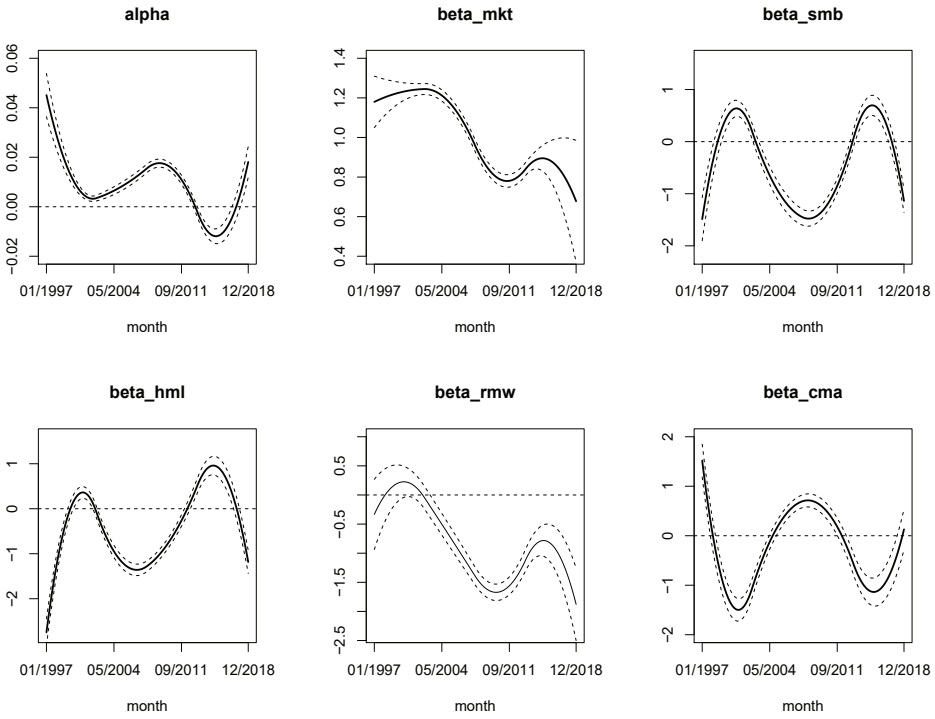

Figure 23: Time-varying beta (spline basis functions) of the functional 5-factor model with 100 Size-B/M portfolios

Table 6: Coefficient matrix $\hat{\boldsymbol{B}}$ (spline basis functions) with 25 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	$\boldsymbol{b}_6$	R^2
Functional CAPM							
α	0.0074	0.0053	-0.0017	0.0073	0.0175	-0.0188	
Market factor	0.5677	1.0909	0.9992	0.9388	0.7503	0.6912	0.2026
Functional Three-factor Model							
α	-0.0014	0.0123	-0.0007	-0.0002	0.0042	-0.0089	
Market factor	0.5608	1.1433	1.2964	0.6867	0.9753	0.5919	
Size factor	0.8273	-0.6921	-0.2639	0.6278	0.2292	0.9116	
Value factor	-1.6176	0.4515	-1.1202	-0.1381	0.2876	-0.3911	0.2091
Functional Five-factor Model							
α	0.0461	-0.0002	0.0077	0.0224	-0.0282	0.0166	
Market factor	1.1001	1.2460	1.2580	0.6529	0.9480	0.8391	
Size factor	-1.5973	1.5512	-1.3016	-1.5454	1.6946	-1.1246	
Value factor	-2.8261	1.6284	-1.8645	-0.7818	1.9748	-1.2830	
Profitability factor	-0.6250	0.6402	-0.5931	-1.9200	-0.3618	-1.5356	
Investment factor	1.3075	-2.5841	0.6985	0.7380	-2.1196	0.4238	0.2170

Table 7: Coefficient matrix $\hat{\boldsymbol{B}}$ (spline basis functions) with 100 value-weight Size-B/M portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	$\boldsymbol{b}_6$	R^2
Functional CAPM							
α	0.0077	0.0064	-0.0026	0.0084	0.0183	-0.0201	
Market factor	0.5745	1.0878	1.0332	0.9571	0.7752	0.6608	0.1945
Functional Three-factor Model							
α	-0.0022	0.0126	-0.0016	-0.0003	0.0036	-0.0092	
Market factor	0.5815	1.1453	1.2923	0.7120	1.0085	0.5651	
Size factor	0.8920	-0.6781	-0.1944	0.7049	0.2698	0.9260	
Value factor	-1.4723	0.2911	-0.8936	-0.1848	0.3226	-0.4313	0.2009
Functional Five-factor Model							
α	0.0451	0.0001	0.0069	0.0241	-0.0288	0.0181	
Market factor	1.1796	1.2406	1.2485	0.6757	1.0042	0.6777	
Size factor	-1.4882	1.6048	-1.2293	-1.6750	1.7826	-1.1386	
Value factor	-2.7466	1.5203	-1.6826	-0.8719	1.9236	-1.1834	
Profitability factor	-0.3337	0.6114	-0.6997	-2.0699	-0.1731	-1.8778	
Investment factor	1.5188	-2.6415	0.5369	0.8588	-1.9869	0.1285	0.2090

6. Conclusion

Our paper introduces functional factor models with time-varying betas (the functional CAPM, functional three-factor model, and functional five-factor model). It investigates the time variations of exposure to risk factors (market, size, value, profitability, and investment). With functional factor models, we can drop the constant beta assumption in the CAPM and multi-factor models to estimate the time-varying betas. The empirical results for the Chinese stock market show: (a) The market factor always has a positive effect on the returns of A-shares over the whole period. In the early years of the stock markets establishment, exposure to the market factors increased dramatically; after 2003, exposure to the market factor began to decrease. (b) The size factor had a positive effect on the returns of A-shares from 2012 to 2016, a relatively long period of stability in China. (c) Similar to the size factor, the value factor had a positive effect from 2011 to 2016. (d) The profit factor had a negative impact on the returns of A-shares in most of the time; exposure to the profitability factor reached its minimum after the financial crises. (e) The investment factor has a solid cyclical pattern.

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Appendix A. Empirical Results with Size-Inv Portfolios

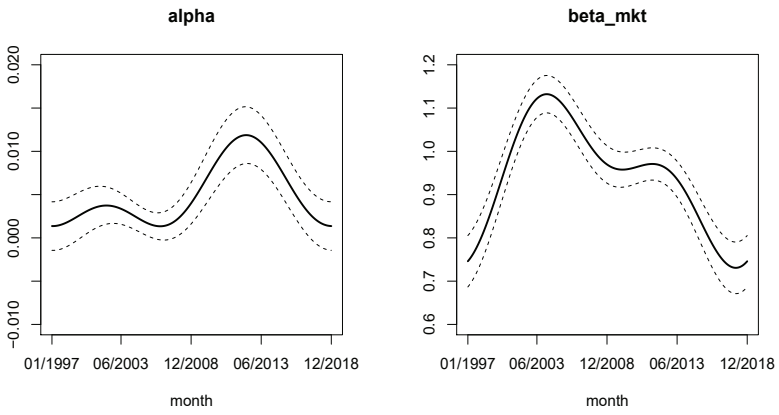

Figure A.24: Time-varying beta of the functional CAPM with 25 Size-Inv portfolios

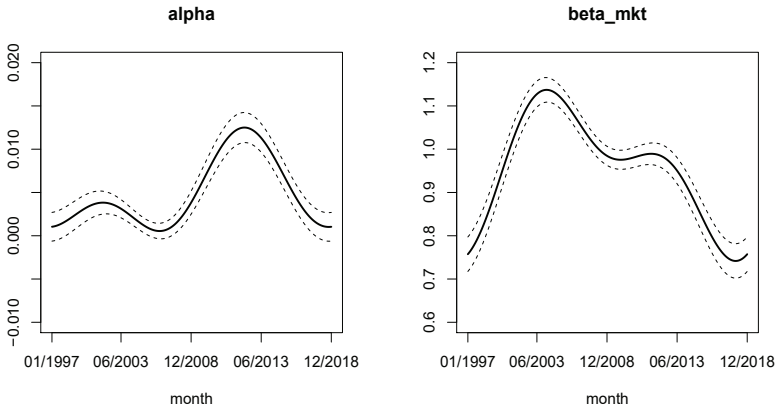

Figure A.25: Time-varying beta of the functional CAPM with 100 Size-Inv portfolios

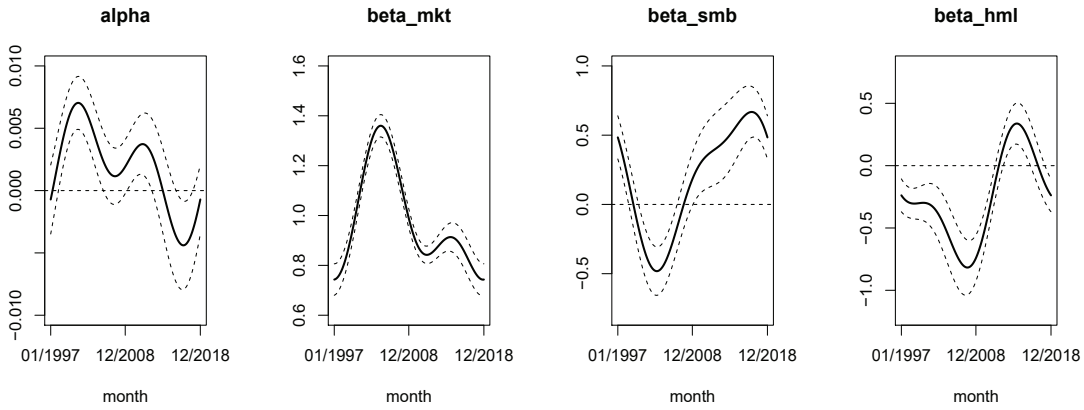

Figure A.26: Time-varying beta of the functional 3-factor model with 25 Size-Inv portfolios

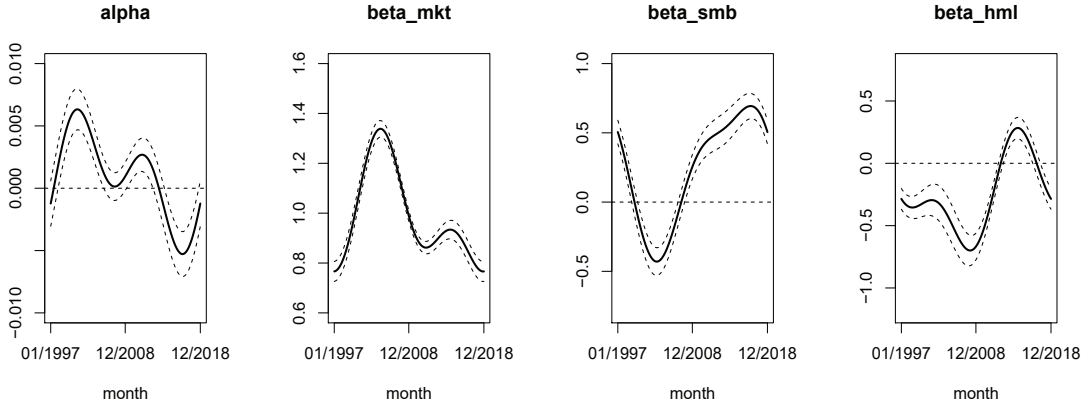

Figure A.27: Time-varying beta of the functional 3-factor model with 100 Size-Inv portfolios

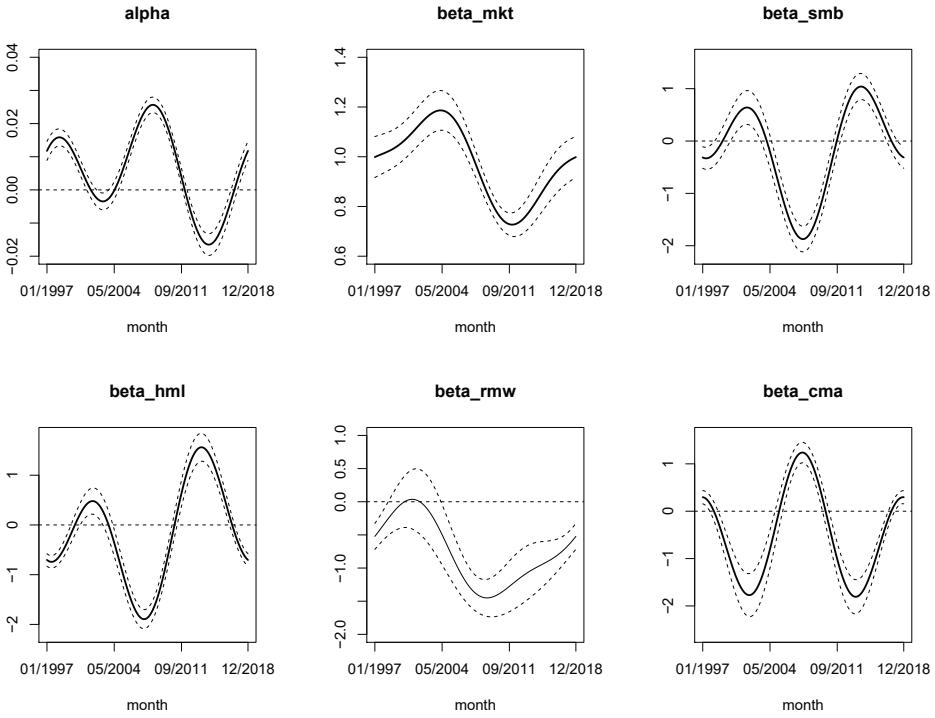

Figure A.28: Time-varying beta of the functional 5-factor model with 25 Size-Inv portfolios

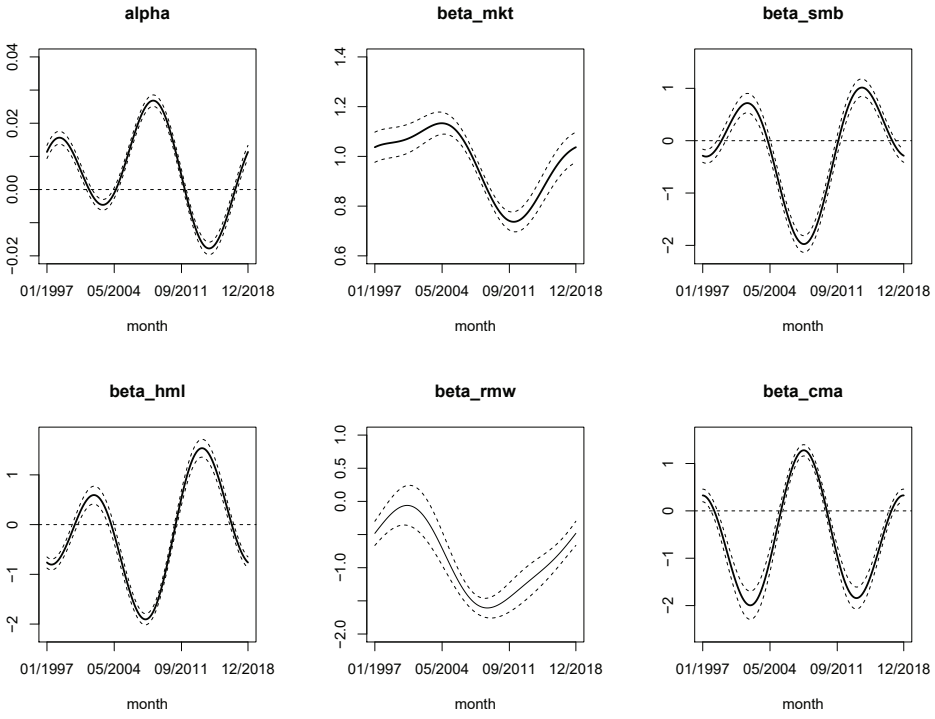

Figure A.29: Time-varying beta of the functional 5-factor model with 100 Size-Inv portfolios

Table A.8: Coefficient matrix $\hat{\boldsymbol{B}}$ with 25 value-weight Size-Inv portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0801	-0.0443	-0.0146	0.0196	-0.0263	
Market factor	15.2965	1.0508	-1.3031	0.0925	-0.9896	0.2080
Functional Three-factor Model						
α	0.0290	0.0338	-0.0161	0.0346	-0.0150	
Market factor	15.9649	2.2212	-1.4356	-1.0749	-1.3346	
Size factor	2.8167	-5.5896	1.8389	-0.3275	1.8716	
Value factor	-4.2286	-4.0609	2.9558	0.3259	-2.6180	0.2154
Functional Five-factor Model						
α	0.0860	0.0554	-0.0758	0.0960	0.1440	
Market factor	15.7517	2.2476	0.0879	-0.7704	0.2078	
Size factor	-2.7254	-1.8044	8.9829	-1.2806	-10.5627	
Value factor	-2.4465	-6.0799	6.8165	-0.6723	-12.9895	
Profitability factor	-12.4034	5.9513	4.7625	0.4820	-2.1523	
Investment factor	-8.1057	0.1966	-5.4027	0.0222	14.5346	0.2231

Table A.9: Coefficient matrix $\hat{\boldsymbol{B}}$ with 100 value-weight Size-Inv portfolios

	$\boldsymbol{b}_1$	$\boldsymbol{b}_2$	$\boldsymbol{b}_3$	$\boldsymbol{b}_4$	$\boldsymbol{b}_5$	R^2
Functional CAPM						
α	0.0782	-0.0474	-0.0155	0.0260	-0.0281	
Market factor	15.4862	0.9980	-1.3236	0.1262	-0.9734	0.1962
Functional Three-factor Model						
α	0.0144	0.0354	-0.0136	0.0358	-0.0133	
Market factor	16.0898	2.0172	-1.3053	-0.9860	-1.2866	
Size factor	3.6786	-5.6800	1.5478	-0.2042	1.7981	
Value factor	-4.0751	-3.3008	2.2550	-0.2346	-2.5596	0.2037
Functional Five-factor Model						
α	0.0806	0.0534	-0.0839	0.1026	0.1509	
Market factor	15.7404	1.9822	0.3208	-0.5988	0.4205	
Size factor	-2.9324	-0.9960	9.6985	-1.6874	-10.8225	
Value factor	-2.4168	-5.1033	6.6233	-1.4078	-13.4735	
Profitability factor	-14.2424	5.8592	5.9251	0.6462	-1.5528	
Investment factor	-8.8781	-1.0287	-5.4331	0.8587	15.4496	0.2117