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A novel approach based on meta-modeling technique and time transformation function for reliability analysis of upgraded automotive components

Rodrigue Sohoin^{a,b,*}, Abdelkhalak El Hami^a, Fabrice Guerin^b, Hassen Riahi^b, Djelali Attaf^c

^aLaboratoire de Mécanique de Normandie, INSA Rouen, Avenue de l'Université BP 0876801, Saint-Etienne-Du-Rouvray, France

^bLaboratoire Angevin de Recherche en Ingénierie des Systèmes - ISTIA, 62 Avenue Notre Dame du Lac, 49000 Angers, France

^cFAURECIA Automotive Seating, Le Pont de Vère, 61100 Caligny, France

Abstract

Early reliability estimation is still a challenging task. The paper presents a novel approach to deal with early reliability estimation of upgraded automotive components. The key idea is to combine reliability analysis based on efficient surrogate models and time transformation function principle. The surrogate model, built using Dimensional Decomposition Method and projection throughout a Lagrange polynomial basis, is used to substitute a time consuming implicit model initially used to compute the fatigue lifetime. The time transformation function is represented by a parametric power law model where the corresponding parameters are obtained through statistical analysis based on both numerical and experimental reliability results of a reference design. The reliability of an upgraded design is easily obtained by applying the time transformation function to the reliability estimation given by performing Monte-Carlo simulations on the surrogate model corresponding to the upgraded design. An application to a mechanical component, used in car seats, clearly illustrates the efficiency and the accuracy of the proposed approach.

Keywords: Early reliability analysis, Dimensional Decomposition Method, Monte-Carlo simulations, Time transformation function, Accelerated lifetime testing, Automotive components.

List of Symbols

The following list describes some symbols that will be later used within the body of the document

ΔR Small step size used to discretize reliability domain

\mathcal{D}_{ref} Reference design

\mathcal{D}_{up} Upgraded design

exp Experimental

$h(t)$ Time transformation function

N_f Fatigue lifetime

num Numerical

$R(t)$ Reliability function

$R_{\mathcal{D}_{ref}}^{exp}(t)$ Experimental reliability function of \mathcal{D}_{ref}

$R_{\mathcal{D}_{ref}}^{num}(t)$ Numerical reliability function of \mathcal{D}_{ref}

$R_{S_i}(t)$ Reliability function under accelerated loading

$R_{S_{nom}}(t)$ Reliability function under operating loading

$S_i, i \in \{1, n_s\}$ Severe stress levels

S_{nom} Operating stress level

$t_{\mathcal{D}_{ref}}^{exp}$ Experimental Lifetime of reference design

$t_{\mathcal{D}_{ref}}^{num}$ Numerical Lifetime of reference design

$t_{\mathcal{D}_{up}}^{exp}$ Experimental Lifetime of upgraded design

$t_{\mathcal{D}_{up}}^{num}$ Numerical Lifetime of upgraded design

1. Introduction

It is widely recognized now, particularly in automotive industry, that reliability is one of the most important parameter which characterizes the performance of the products. Indeed, the reliability can be defined as a measure of the failure ratio of a product over a given time span. Hence, products having the higher reliability level are consequently the most safe. For these reasons, customers are increasingly interested in reliable products, and manufacturers are constrained to provide the associated reliability level of their products in the same way as their prices. In this context, reliability estimate of new products, at early steps of their life-cycle is of a great interest. It helps manufacturers to enhance the reliability of their products before reaching the market, and consequently to cope effectively with the concurrency. In fact, at that stages, engineers still have a large leeway to review the design. Otherwise, modifications at

*Corresponding author at : Laboratoire Angevin de Recherche en Ingénierie des Systèmes - ISTIA, 62 Avenue Notre Dame du Lac, 49000 Angers, France. Tel.: +33-2-32-95-97-53; fax: +33-2-32-95-97-04

Email address: rodrigue.sohoin@univ-angers.fr (Rodrigue Sohoin)

later stages of the products life-cycle could be time consuming and generate extra costs.

Product qualification aims to verify whether an entity fulfill the reliability and quality requirements. In the case of automotive components, the mechanical reliability assessment is generally based on statistical analysis of data, which are provided from testing in the case of a new product, such as accelerated tests [1] for mechanical components subjected to fatigue, or collected during the life-cycle of former releases in the case of upgraded products. In the two cases, early reliability prediction is still a challenging problem, mainly due to the lack of the available data and the expensive cost of the testing process. In the last three decades, numerical-based methods have been developed [2], which could be a serious alternative to address the reliability assessment problem. Briefly, these methods require first, a model of the mechanical component to describe its behavior under various loading conditions, and then the reliability estimate is obtained using a mechano-probabilistic coupling strategy, which aims to perform uncertainties propagation throughout the previous mechanical model.

Unfortunately, most of the numerical reliability approaches are inefficient when dealing with engineering problems, mainly because, on the one hand the simulation of the mechanical models are time consuming since they are often available under implicit form (e.g. finite elements model), and on the other hand the number of the uncertain parameters could be very high, which induce the problem of the *curse of dimensionality* [3], that is the exponential growth of the number of the mechanical model calls with the probabilistic dimension (i.e. the number of the uncertain parameters). In addition, the computation time spent by the numerical reliability method is sensitive to the target reliability level. Thus, higher is the target reliability level (conversely lower is the target failure probability), which is the case of automotive components, higher is the computation time spent to achieve accurate results. Using numerical reliability approaches often needs to take some assumptions, for instance, the distribution type (i.e. normal, uniform, . . .) or/and the statistical parameters (i.e. mean, standard deviation, . . .) of the uncertain parameters, since prior information is not available. This can explain the significant discrepancy often observed between the estimates given by the numerical methods and the *true* reliability. Consequently, the only use of *crude* numerical approaches to perform reliability analysis during products qualification process can lead to erroneous results. At the same time, performing reliability analysis based on limited experimental data is not a trivial task [4–6], and carrying out additional tests, each time one have new products, would not be the best solution. Fortunately, products are mainly evolutionary and not revolutionary, especially for automotive industry. Indeed, for a given product, the upgraded releases have often the same backbone as the reference design, and only a few minor modifications are carried out. Hence, reliability estimation at the beginning of the product development process can be well addressed, if we are able to integrate available data on similar products or former releases, as prior information. In this context, Bayesian-based approaches, which combine testing data and reliability computations, have been found a great interest in the literature. One type of Bayesian

approaches consists first in assigning a prior distribution for the failure probability or the lifetime distribution parameters, based on existing knowledge on the product, and then updating this distribution when new information is available [7]. Such an approach can be only used when reliable information is available to construct the prior distribution. For example, to improve the accuracy of lifetime estimation of an automobile component (Boot seal) while reducing the amount of tests, Guerin et al. [8] have proposed a Bayesian estimation of the lifetime distribution (Log-normal), by incorporating as prior knowledge, results obtained by finite element computation (mean fatigue lifetime) and expert opinion (lifetime variance). The second type of approaches [9], consists firstly in characterizing the responses (e.g. stress level) of the constitutive model (i.e. mechanical model) through the propagation of the uncertainty associated to the input parameters. Secondly, data provided by means of tests are used to update the distributions of the input parameters and consequently update finally the prior reliability estimate. Unfortunately, this later approach suffers from a lack of consistency, since do not use data from reliability testing. For that reason, Zhang et al. [10] developed an original approach which uses reliability testing data in conjunction with results given by computer-based analysis. Despite, the authors have demonstrated its efficiency only on academic examples; this method can be easily extended to practical engineering problems. Guida et al. [11, 12] proposed a Bayesian procedure for making inference on the reliability of upgraded automotive components, by using both failure data relative to a previous design releases and prior information about the technical process used in the modification of the product design itself, instead of additional reliability testing data. Indeed, when dealing with product enhancement, the main goal of designers is to avoid critical failure modes observed in the former releases, through modifications. If such modifications are effective, we can be able to eliminate most of the defects of the previous design and consequently reduce the associated probability of failure of the new design. The effectiveness of the design modifications is measured through an improvement factor defined as the ratio between the new average probability of failure of the enhanced design release and the past probability of failure of the reference design. This factor is defined as a function of random variables to represent the uncertainties related to the effectiveness of the design modifications. This approach, also take into account the effect of working conditions and cost reduction. Based on enhanced subsystem used in already commercialized car model, the authors demonstrated that the approach provides more accurate estimate of the probability of failure. In addition, they clearly showed that informative prior can be very useful to a better control of the uncertainty on the reliability of the enhanced products, by reducing their probability of failure, and the numbers of items that will fail in the new population.

In the literature, the joint use of the Bayesian computation and fuzzy inference is also investigated to perform early reliability analysis. In this context, Yadav et al. [13] have developed an approach able to take into account qualitative information as engineering judgment, expert's opinion, operating and environmental conditions. As a little or no quantitative information is

available at early development stage of new products, especially for reliability analysis, these authors proposed to use fuzzy inference system to address different kinds of qualitative information. Then a Bayesian approach is used to enhance its capability to deal with qualitative information. The applicability of this approach to engineering problems has been illustrated, through reliability analysis of a being enhanced car steering system, for which both qualitative information and reliability testing data are available.

In this paper, a new approach combining, numerical reliability computation, based on high-dimensional representation model approach [14, 15], and Time Transformation Function (TTF), usually used in accelerated test data analysis [16, 17], is developed to deal with early reliability prediction. Indeed, for a reference design which experimental reliability function is known, numerical computations are performed through Monte-Carlo simulations applied to a surrogate model, to obtain the numerical reliability function. Then, a TTF associated to the reference design is derived using both the experimental and the numerical reliability functions. Now, for an upgraded or new release of the reference design, the associated *true* reliability function is simply obtained by applying the TTF to the numerical reliability function of this new design, without performing additional tests.

The paper is organized as follows. In section 2, a brief recall of the mathematical formulation of the structural reliability problem, and the numerical methods commonly used to make estimate of the probability of failure. In section 3 we describe the mathematical setup of the new approach allowing to perform reliability prediction of upgraded products. Section 4 illustrates the effectiveness and the accuracy of the proposed approach through an application to automotive components. Finally, the main findings and conclusions are summarized in section 5.

2. Mathematical formulation of the structural reliability problem

Reliability analysis aims to evaluate the safety level of an engineering system or structure, with respect to a prescribed failure criterion. Commonly, the failure criterion concept can be defined as the gap between two fundamental quantities named the Demand and the Capacity, respectively. In mechanical engineering, as well as in civil engineering, the Demand can be defined as the response of the system, such as a mechanical stress or displacements fields, induced by the loading conditions. The Capacity, on the other hand, represents a threshold scenario, such as an ultimate stress or displacement, beyond which the system collapses.

Mathematically speaking, the failure criterion is characterized by the limit state function, or the performance function, denoted G . It is defined as a mapping of the random variables (X_1, X_2, \dots, X_n) , which represent the uncertain parameters (x_1, x_2, \dots, x_n) , often gathered in the random vector $X = \{X_1, X_2, \dots, X_n\}^T$ for the sake of simplicity. Thus, the random space, can be split up into two regions: the failure domain $\Omega_F = \{x \in \mathcal{D}_X | G(x) < 0\}$ and the safety domain $\Omega_S = \{x \in \mathcal{D}_X | G(x) > 0\}$. The set of points $\Gamma = \{x \in \mathcal{D}_X | G(x) = 0\}$

represents the limit state surface which is the frontier between the failure and the safety domains. Fig. 1 (a), schematically illustrates these concepts in the case of a two-dimensional random space.

Accordingly, the failure probability P_f , which is by definition the complementary event of the reliability R , that is $R = 1 - P_f$, reads :

$$P_f = Prob[G(x) \leq 0] = \int_{G(x) \leq 0} p_X(x) dx = \int_{\mathbb{R}^n} \mathbb{I}_{\Omega_F}(x) p_X(x) dx \quad (1)$$

where p_X is the joint probability density of the random vector X , and \mathbb{I}_{Ω_F} is the indicator function on Ω_F , which is equal to 1 if $G(x) \leq 0$ and 0 otherwise. As can be seen, the estimation of the probability of failure P_f is none other than a computation of a multi-dimensional integral. As the limit state function G is often not available in an explicit form, especially when dealing with engineering problems, the integral in above equation cannot be analytically computed. Instead, numerical methods are frequently employed. The efficiency of the numerical scheme mainly depends on the complexity of the limit state function and the problem dimensionality n . During the last few decades, various reliability analysis methods have been developed [18], such as simulations and moments based method.

Among these methods, Monte-Carlo Simulations (MCS) [19] can be used to compute the integral in Eq. (1). The main principle consists in interpreting the probability of failure P_f as the mathematical expectation $\mathbb{E}[\cdot]$ of the indicator function \mathbb{I}_{Ω_F} , that is to say $P_f = \mathbb{E}[\mathbb{I}_{\Omega_F}]$. The numerical implementation of the Monte-Carlo Simulations is quite simple. Let us consider M realizations $\{x^1, x^2, \dots, x^M\}$ of the of the random vector X , drawn from its associated joint probability density p_X . The estimator \hat{P}_f of the probability of failure P_f can be obtained by :

$$\hat{P}_f = \frac{1}{M} \sum_{k=1}^M \mathbb{I}_{\Omega_F}(x^k) \quad (2)$$

Unfortunately, experience has shown that Monte-Carlo Simulations scheme becomes unpractical to compute high order probability of failure, mainly due to its slow convergence. Hence, when the limit state function is represented through time consuming implicit model, the computation cost becomes unaffordable. To overcome the inefficiency of Monte-Carlo Simulations, the First Order Reliability Method (FORM) has been developed [20].

To apply FORM, the reliability problem is rewritten in the standard random space, where the random vector $X = \{X_1, X_2, \dots, X_n\}^T$ is transformed into standard normal random vector $U = \{U_1, U_2, \dots, U_n\}^T$ (i.e. zeros mean, unit standard deviation and independent components), by means of isoprobabilistic transform $X = T(U)$. In the general case where the components of the random vector X are dependents, the Rosenblatt transform or the Nataf transform [21] are used. As a consequence, the probability of failure in Eq. (1) rewrites :

$$P_f = \int_{G(x) \leq 0} p_X(x) dx = \int_{H(u) \leq 0} \varphi_U(u) du \quad (3)$$

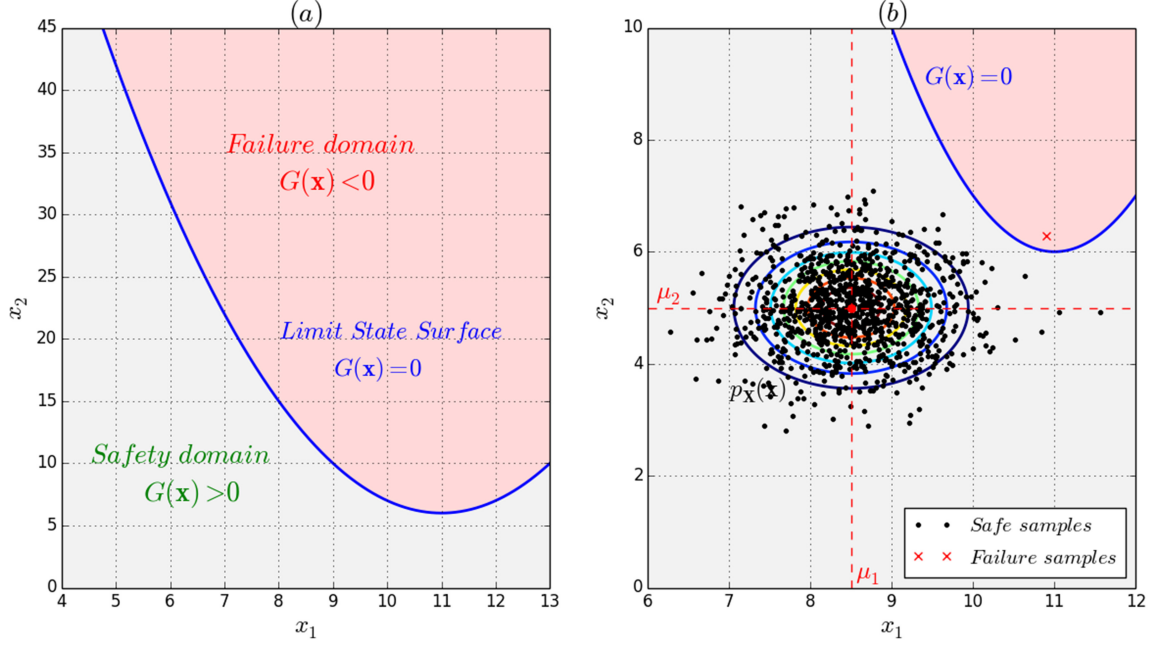


Fig. 1. (a) Limit state surface, failure domain and safety domain concepts, (b) Monte-Carlo Simulations

where $H(U) \equiv G \circ T(U) = G(X)$ is the limit state function in the standard random space and φ_U is the standard multivariate normal probability density function.

Then, FORM substitutes, in the standard random space, the limit state function H by an hyperplane tangent to the true failure domain at the Most Probable Failure Point P^* , also called the design point, which is defined as the nearest point of the limit state surface to the origin of the standard random space (see Fig. 2 (b)).

Once the coordinates $u^* = \{u_1, u_2, \dots, u_n\}^T$ of the most probable failure point P^* are found, the Hasofer-Lind reliability index $\beta_{HL} = \|u^*\|$ is computed, and the first order approximation of the probability of failure reads:

$$P_f \approx P_{f,FORM} = \Phi(-\beta_{HL}) \quad (4)$$

The FORM approximation is often satisfactory, especially for high values of the reliability index, provided that the most probable failure point P^* is well identified. It is clear from Fig. 2 (b) that FORM approximation is exact when the true limit state function is linear. Unfortunately, this situation is rarely encountered in real life problems. For this reason, the Second Order Reliability Method has been developed. As depicted in Fig. 2 (b) it uses a quadratic surface in order to better approximate the true failure domain. Based on Breitung [22] the SORM estimation of the probability of failure writes :

$$\begin{aligned} P_f \approx P_{f,SORM} &= \Phi(-\beta_{HL}) \times \prod_{i=1}^{n-1} \frac{1}{\sqrt{1 - \beta_{HL} \cdot \kappa_i}} \\ &= P_{f,FORM} \times \prod_{i=1}^{n-1} \frac{1}{\sqrt{1 - \beta_{HL} \cdot \kappa_i}} \end{aligned} \quad (5)$$

where $\kappa_i, i \in 1, \dots, (n-1)$ are the main curvatures of the limit state function at the most probable failure point P^* .

As can be seen from Eq. (5), SORM simply improves the estimation of the probability of failure given by FORM through a ponderation by a correction factor $\prod_{i=1}^{n-1} \frac{1}{\sqrt{1 - \beta_{HL} \cdot \kappa_i}}$ including information about the curvature of the limit state function. SORM becomes inefficient when the reliability problem dimensionality n is high. This is mainly due to the computation of the $(n-1)$ curvatures, which requires the evaluation of the second order derivatives of the limit state function with respect to the random variables. This problem can be artfully avoided by the use of meta-models, also known in probabilistic analysis as surrogate models or response surface. They aim to substitute the true limit state function, obviously if it has an implicit form, by an analytical formulation often constructed by a projection onto a polynomial basis. Quadratic Response Surface (QRS) method [23–26] is the widely used meta-modeling technique to perform reliability analysis based on FORM approximation, mainly due to its simplicity and efficiency for moderate dimensionality problems. Based on QRS method, the first order derivatives of the limit state function, required in the optimization procedure, are computed on quadratic polynomial approximation, with or without cross terms, rather than the direct use of the true limit state function. Although, the quadratic polynomial approximation is built in each iteration of the optimization scheme, the QRS method remains efficient compared to Monte-Carlo Simulations or crude FORM. In last years, meta-modeling techniques have known a large breakthrough, and others approaches have been developed, such as those based on, Polynomial Chaos Expansion (PCE) [27], Collocation method [28–30], Support Vector Machine (SVM) [31–33], kriging [33, 34], neural network [35] and Dimension Decomposition Method (DDM) [36]. This later is used in this paper to perform reliability analysis, since it provides the better balance between efficiency and accuracy.

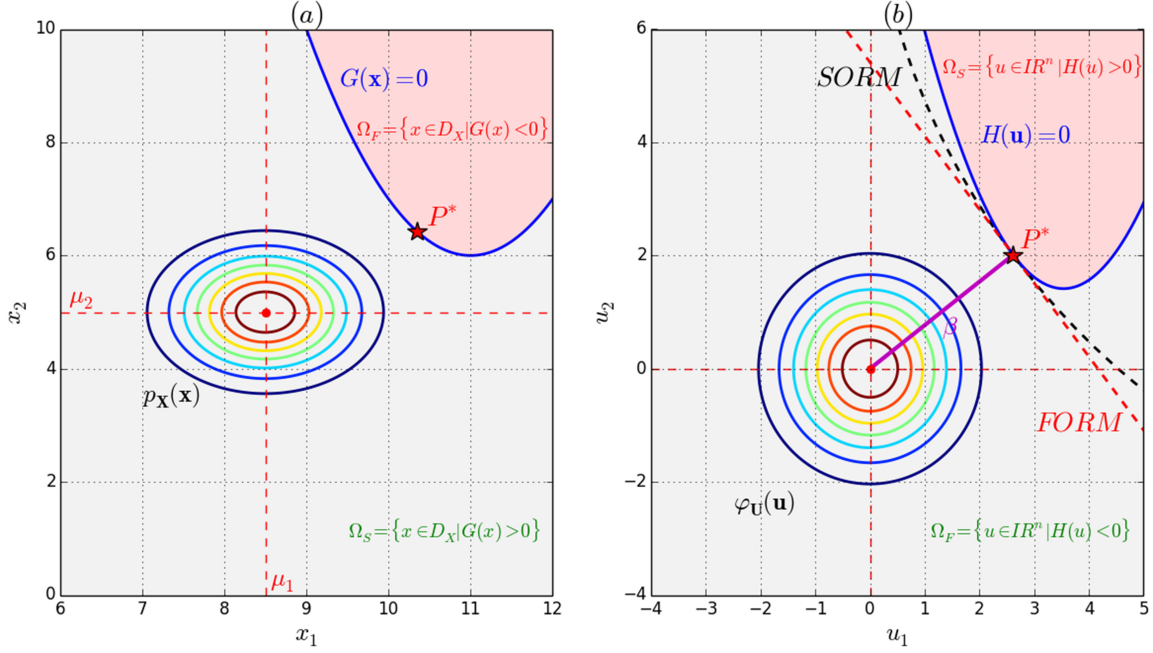


Fig. 2. (a) Reliability concepts in the physical random space, (b) FORM and SORM approximations in the standard random space

3. The proposed approach

3.1. Dimensional Decomposition Method for reliability analysis

The key idea of the DDM is to construct a response surface to approximate an input-output relation. This approach consists in the discretization of a multidimensional function into a set of finite and hierarchic sums of simple functions of the basic random variables. In the sequel, the developed mathematical formulations are derived in the standard random space. Thus, let us consider a physical model \mathcal{M} defined by the mapping $y = \mathcal{M}(u)$. Here, y is a scalar representing the response of the model \mathcal{M} which depends on a set of uncertain parameters gathered in the n -dimensional vector $u = \{u_1, u_2, \dots, u_n\}^T$ in the standard random space. According to the DDM, the response y may be expanded onto elementary functions, as follows:

$$y = \mathcal{M}(u) = y_0 + \sum_{k=1}^n y_k(u_k) + \sum_{k_1=1}^n \sum_{k_2 > k_1}^n y_{k_1, k_2}(u_{k_1}, u_{k_2}) + \dots + y_{k_1, \dots, k_n}(u_{k_1}, \dots, u_{k_n}) \quad (6)$$

where y_0 is a real constant, which is obtained as a response of the model \mathcal{M} at a specific reference point $u^r = \{u_1^r, u_2^r, \dots, u_n^r\}^T$ (e.g. the mean point), y_k , y_{k_1, k_2} and y_{k_1, \dots, k_n} are univariate, bivariate and n -variate functions, respectively. Note that this expansion is only valid for independent uncertain parameters, and can be easily derived for non-scalar response $y = \{y_1, y_2, \dots, y_m\}^T$, $m > 1$.

For practical implementation, the above expansion should be truncated and only lower order components are to be considered. Therefore, the s -dimensional approximation \hat{y}_s of the response y

reads:

$$y \cong \hat{y}_s = \sum_{i=0}^s (-1)^i C_{n-s+i-1}^i \sum_{k_1=1}^{n-s+i-1} \dots \sum_{k_{s-i}=k_{s-i-1}-1}^n y_{k_1, \dots, k_{s-i}}(\tilde{u}) \quad (7)$$

where $y_{k_1, \dots, k_{s-i}}$ are $(s-i)$ -variate functions representing the $(s-i)^{th}$ order contribution among the parameters u_{k_1}, \dots, u_{s-i} gathered in the vector \tilde{u} .

If the effect of the high order interactions is assumed to be weak, it is clear from Eq. (7) that the DDM can be very efficient to approximate implicit time consuming models. Indeed, two uses of this approach can be distinguished: the dimensionality reduction of multidimensional integrals such as those encountered in statistical moments computation of a random quantity, or the construction of surface response of explicitly-unknown models.

By definition the l^{th} order statistical moment of the random variable Y representing the variability of the response y induced by uncertain parameters of the model \mathcal{M} , can be written as:

$$m_Y^l \cong \mathbb{E}[Y^l] = \int_{\Omega_Y} y^l p_Y(y) dy = \int_{\mathbb{R}^n} \mathcal{M}^l(u) \varphi_U(u) du \quad (8)$$

where $\mathbb{E}[\cdot]$ denotes the mathematical expectation and φ_U is the standard multivariate normal probability density function.

The evaluation of the integral in Eq. (8), using traditional computation schemes such as cubature rules, is not a trivial task, especially in the case of implicit integrand and high dimensional integration. Replacing, the integrand in Eq. (8) by its approximation given by the DDM, based on Eq. (7), the l^{th} order statistical

moment m_Y^l is rewritten as:

$$m_Y^l \cong \sum_{i=0}^s (-1)^i C_{n-s+i-1}^i \sum_{k_1=1}^{n-s+i-1} \cdots \sum_{k_{s-i}=k_{s-i-1}-1}^n \mathbb{E} \left[y_{k_1, \dots, k_{s-i}}^l(\tilde{u}) \right] \quad (9)$$

The expectation term $\mathbb{E} \left[y_{k_1, \dots, k_{s-i}}^l(\tilde{u}) \right]$ in the above equation, reads:

$$\mathbb{E} \left[y_{k_1, \dots, k_{s-i}}^l(\tilde{u}) \right] = \int_{\mathbb{R}^{(s-i)}} y_{k_1, \dots, k_{s-i}}^l(\tilde{u}) \varphi_{\tilde{U}}(\tilde{u}) d\tilde{u} \quad (10)$$

where $\varphi_{\tilde{u}}$ is the probability density function of the $(s-i)$ -dimensional standard normal random variable $\tilde{U} = \{U_{k_1}, \dots, U_{k_{s-i}}\}^T$.

Now, based on Eqs. 9 and 10, it is clear that the computation of the statistical moment m_Y^l , is reduced to the evaluation of a set of $(s-i)$ -dimensional at most integrals, than a tedious n -dimensional one. These elementary integrals can be accurately and efficiently computed using Gauss cubature formula. In order to have a complete picture of the variability of the response y , its probability density function p_Y can be easily constructed using a statistical moments based-technique [37]. Note that, the use of such technique requires a high accuracy level on the estimates of the four first statistical moments of the random quantity, namely the mean, the standard deviation, the skewness and the kurtosis.

In the previous development, the approximation of the response of a given model provided by the DDM can be used implicitly to reduce the dimensionality of integrals, which means without really constructing the response surface. It is clear from Eq. 7 that an analytical formulation of the response of the model can be obtained if and only if the component functions $y_{k_1, \dots, k_{s-i}}$, derived through the decomposition process, are known. This issue can be easily addressed by considering the later functions as shape functions. Indeed, the component functions can be simply constructed by interpolation throughout a polynomial basis.

In this work, the polynomial interpolation is cast in the Lagrange basis, as follow:

$$y_{k_1, \dots, k_{s-i}}(u_{k_1}, \dots, u_{k_{s-i}}) = \sum_{j_1=1}^N \cdots \sum_{j_{s-i}=1}^N \mathcal{M}(u_{k_1}^{j_1}, \dots, u_{k_{s-i}}^{j_{s-i}}) \times l_{j_1}(u_{k_1}) \times \cdots \times l_{j_{s-i}}(u_{k_{s-i}}) \quad (11)$$

where $l_{j_{s-i}}(u_{k_{s-i}})$ are univariate Lagrange polynomials constructed around a set of experimental points $u_{k_{s-i}}^{j_{s-i}}$, $1 \leq j_{s-i} \leq N$, and $\mathcal{M}(u_{k_1}^{j_1}, \dots, u_{k_{s-i}}^{j_{s-i}})$ are real coefficients.

One more step needed before the construction of the polynomial approximation, is to build a suitable Experimental Design (ED). The components of the ED can be chosen as the Gauss-Hermite integration points used previously in the computation of the expectation in Eq. 10, or as uniformly distributed points around a reference point $u^r = \{u_1^r, u_2^r, \dots, u_n^r\}^T$, obtained

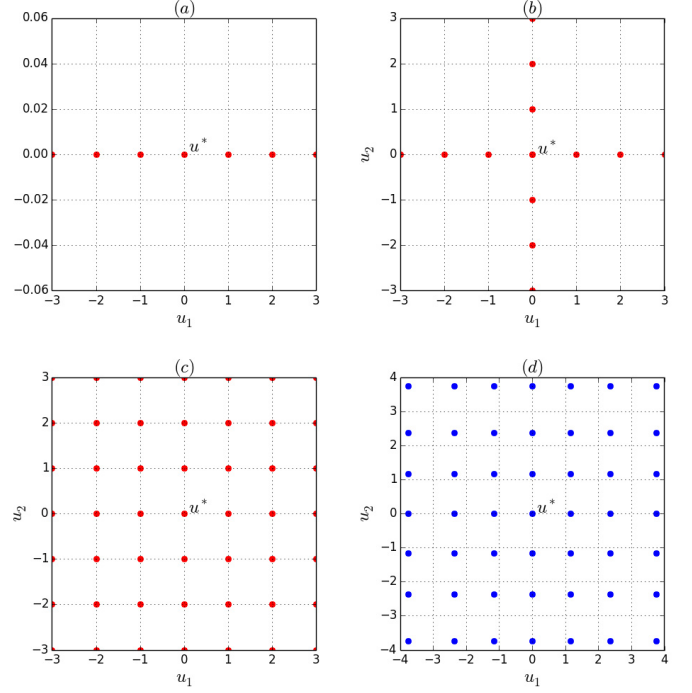


Fig. 3. Experimental Designs (a) $n=1, s=1$, (b) $n=2, s=1$, (c) $n=2, s=2$ (d) Gauss-Hermite integration points

through the following recursive formula [36]:

$$\left\{ u_k^j \right\}_{j=1}^N = \left\{ u_k^r - \frac{N-1}{2\delta_k}, u_k^r - \frac{N-3}{2\delta_k}, \dots, u_k^r, \dots, u_k^r + \frac{N-3}{2\delta_k}, u_k^r + \frac{N-1}{2\delta_k} \right\} \quad (12)$$

where u_k^r is the coordinate of the reference point in the k^{th} direction of the random space and δ_k is a perturbation which control the distance between the points of the ED.

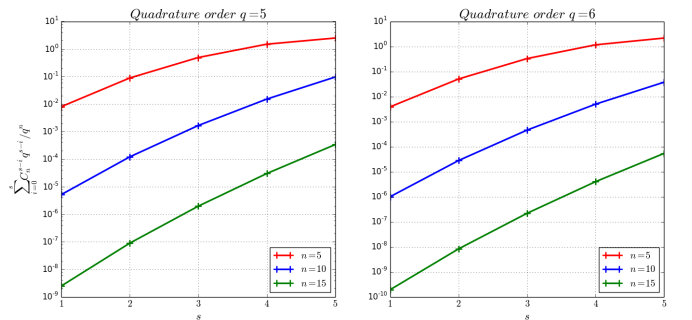


Fig. 4. Effectiveness of the DDM compared to Quadrature method

A suitable choice of the reference point is of a great importance to have an accurate interpolation. It has been shown that the mean point gives good results when dealing with statistical moments analysis. In the case of reliability analysis, the most probable failure point P^* is the best candidate. However, FORM analysis should be performed to compute its coordinates

which probably increases the computation time. Fig. 3 shows a comparison of experimental designs obtained through the recursive formula (Eq. 12) for different dimensionality n and decomposition order s , and Gauss-Hermite integration points.

In order to demonstrate the effectiveness of the meta-modeling method, the number of calls to the implicit model, required by the DDM is compared to that of the quadrature method (figure 4). For example, with decomposition order $s = 1$, $s = 2$ and $s = 3$ the DDM makes it possible to reduce the number of calls to the implicit model compared to the direct integration method respectively by a factor of 1969, 136, 17 when the quadrature order $q = 3$, and respectively by a factor of 26215, 1380, 125 when the quadrature order $q = 4$.

Note that the DDM does not directly give the reliability function. This is an intermediate step that allows to obtain a model easier to compute the output of the mechanical model that will be in the context of this work, a lifetime of the component. DDM will be coupled with Monte-carlo simulations to construct the lifetime distribution, thus the numerical reliability function.

3.2. Construction of the Time Transformation Function (TTF)

During product qualification process, an Accelerated Life Testing (ALT) based approach [16] is often used to estimate the reliability of the underlying product. Briefly, this technique aims to reduce the testing time, required to obtain an estimation of the reliability, by applying a much more severe stress levels S_i , $i \in \{1, n\}$, than the one observed under normal operating conditions (S_{nom}). Fig. 5 (a) schematically summarizes the ALT procedure. The failure events observed during testing are then used to construct the reliability function $R_{S_i}(t)$ associated to the accelerated loading conditions. Finally, by means of a Time Transformation Function (TTF), also called acceleration law in the ALT context, the reliability function $R_{S_i}(t)$ is extrapolated to obtain the *true reliability* function $R_{S_{nom}}(t)$, associated to the operating conditions (see Fig. 5 (b)). For practical purposes, it is assumed [38] that the failure times observed at different stress levels are linearly dependent, and the timescale transformation is constant, which means we have a true linear acceleration. Therefore, the reliability function under accelerated loading conditions $R_{S_i}(t)$ can be related to the reliability function under operating loading conditions $R_{S_{nom}}(t)$ as follows:

$$R_{S_i}(t) = R_{S_{nom}}(TTF \times t) \quad (13)$$

where TTF is the so called time transformation function.

The principle of TTF is used in this paper to construct the reliability function of upgraded automotive components. Let us consider a reference design \mathcal{D}_{ref} and $h(t)$ the associated TTF, which measures the gap between the experimental reliability function $R_{\mathcal{D}_{ref}}^{exp}(t)$, obtained through experimental testing, and the numerical reliability function $R_{\mathcal{D}_{ref}}^{num}(t)$, obtained through structural reliability analysis as presented in section 3.1. Note that, both reliability functions $R_{\mathcal{D}_{ref}}^{exp}(t)$ and $R_{\mathcal{D}_{ref}}^{num}(t)$ correspond to the operating loading conditions of the component.

Now, let us consider an upgraded design \mathcal{D}_{up} , which is a new release of the reference design \mathcal{D}_{ref} , and for which only the

numerical reliability function is available $R_{\mathcal{D}_{up}}^{num}(t)$. Since the upgraded design \mathcal{D}_{up} has the same backbone as the reference design \mathcal{D}_{ref} , that only few modifications are performed to adapt the design to new customers requirements, the time transformation function $h(t)$ should be conserved. Indeed, on the one hand the reference and the upgraded releases are very similar since they are made from the same constitutive material, went through same manufacturing process, subjected to similar operating conditions and failure modes. On the other hand, the same assumptions and hypothesis are made for the finite elements models representing the reference and the upgraded releases. Therefore, the time transformation function can also be reasonably assumed to be the same for both product versions. Consequently, the true reliability function of the upgraded design \mathcal{D}_{up} , also referred to as experimental reliability function $R_{\mathcal{D}_{up}}^{exp}(t)$, reads as follows:

$$R_{\mathcal{D}_{up}}^{exp}(t) = (R_{\mathcal{D}_{up}}^{num} \circ h)(t) = R_{\mathcal{D}_{up}}^{num}(h(t)) \quad (14)$$

Where \circ is the function composition symbol.

The following steps describe in details the proposed approach to construct the time transformation function $h(t)$ and an estimate of the true reliability function of an upgraded design:

- **Step 1: Discretization of the reliability domain**

Let $[0, 1]$ be the set of possible values of the reliability function and ΔR a small step size used to discretize this domain. Here, ΔR is fixed to 0.0125 in order to obtain a sufficiently large sample size (i.e. greater than 30). It is assumed that the lifetime t is strictly positive variable and modeled as a lognormal distribution. Thus, to avoid a null or infinite lifetime values using the inverse distribution function, the bounds $R = 0$ and $R = 1$ are excluded. The sample points in $]0, 1[$ are obtained by the following recursive formula:

$$D_R = \{\Delta R, 2 \times \Delta R, \dots, 1 - \Delta R\} \quad (15)$$

It is important to note that other distributions than the lognormal one can be used without any particular modification of the the formulation of the proposed approach.

- **Step 2: Experimental and numerical lifetime distribution for the reference design**

Let $(\mu_{\mathcal{D}_{ref}}^{exp}, \sigma_{\mathcal{D}_{ref}}^{exp})$ and $(\mu_{\mathcal{D}_{ref}}^{num}, \sigma_{\mathcal{D}_{ref}}^{num})$ be the parameters of the experimental and the numerical lifetime distributions of the reference design \mathcal{D}_{ref} , respectively. For a given value $D_R(i) \in D_R, i = 1, \dots, n_R$ of the reliability function, the corresponding experimental and numerical lifetimes $t_{\mathcal{D}_{ref}}^{exp}$ and $t_{\mathcal{D}_{ref}}^{num}$ read, respectively:

$$t_{\mathcal{D}_{ref}}^{exp}(i) = \exp \left[\Phi^{-1}(1 - D_R(i)) \times \sigma_{\mathcal{D}_{ref}}^{exp} + \mu_{\mathcal{D}_{ref}}^{exp} \right] \quad (16)$$

$$t_{\mathcal{D}_{ref}}^{num}(i) = \exp \left[\Phi^{-1}(1 - D_R(i)) \times \sigma_{\mathcal{D}_{ref}}^{num} + \mu_{\mathcal{D}_{ref}}^{num} \right] \quad (17)$$

Where Φ denotes the standard normal cumulative distribution function.

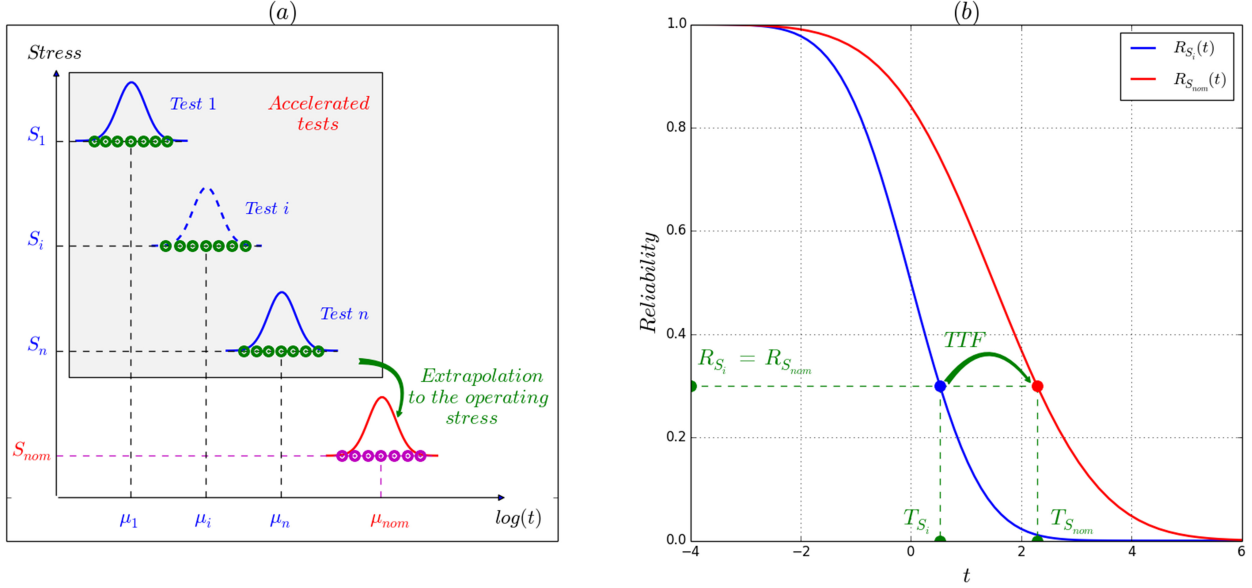


Fig. 5. (a) Accelerated Life Testing principle, (b) Definition of the Time Transformation Function (TTF)

- **Step 3 : Time transformation function of the reference design**

As so far mentioned, the time transformation function $h(t)$ represents the relationship between the experimental and the numerical reliability functions. In the framework of this paper, the well-known parametric power law model is used as time transformation function. This model is especially used when the damage process is sensitive to a particular loading conditions, such as fatigue. It reads:

$$h(t) = A \times t^{\beta_1}, \forall t > 0 \quad (18)$$

where A and β_1 are two unknown parameters.

Assuming that the numerical lifetime t^{num} is known, applying the logarithm to Eq. 18, and let $\beta_0 = \ln(A)$, the associated experimental lifetime t^{exp} is derived as follows:

$$\ln(t^{exp}) = \ln(h(t^{num})) = \beta_0 + \beta_1 \times \ln(t^{num}) \quad (19)$$

The parameters β_0 and β_1 are then obtained by performing regression analysis based on the set of points $\{t_{\mathcal{D}_{ref}}^{exp}(i), t_{\mathcal{D}_{ref}}^{num}(i)\}_{i=1}^{n_R}$ associated to the reliability sample $D_R = \{\Delta R, 2 \times \Delta R, \dots, i \times \Delta R, \dots, 1 - \Delta R\}$ previously defined in step 2:

$$\beta_1 = \frac{\sum_{i=1}^{n_R} (\ln(t_{\mathcal{D}_{ref}}^{num}(i)) - \mu_{\mathcal{D}_{ref}}^{num}) \times (\ln(t_{\mathcal{D}_{ref}}^{exp}(i)) - \mu_{\mathcal{D}_{ref}}^{exp})}{\sum_{i=1}^{n_R} (\ln(t_{\mathcal{D}_{ref}}^{num}(i)) - \mu_{\mathcal{D}_{ref}}^{num})^2} \quad (20)$$

$$\beta_0 = \mu_{\mathcal{D}_{ref}}^{exp} - \beta_1 \times \mu_{\mathcal{D}_{ref}}^{num} \quad (21)$$

where $\mu_{\mathcal{D}_{ref}}^{exp} = \frac{1}{n_R} \sum_{i=1}^{n_R} \ln(t_{\mathcal{D}_{ref}}^{exp}(i))$ and $\mu_{\mathcal{D}_{ref}}^{num} = \frac{1}{n_R} \sum_{i=1}^{n_R} \ln(t_{\mathcal{D}_{ref}}^{num}(i))$ are the means values of the experimental and numerical lifetime distributions, respectively.

- **Step 4 : Experimental lifetime of the upgraded design**

Structural reliability analysis is performed on the upgraded design to construct first, the corresponding numerical lifetime distribution. A set of numerical lifetime points are obtained by performing inverse reliability analysis based on the reliability sample $D_R = \{\Delta R, 2 \times \Delta R, \dots, i \times \Delta R, \dots, 1 - \Delta R\}$:

$$t_{\mathcal{D}_{up}}^{num}(i) = \exp \left[\Phi^{-1}(1 - D_R(i)) \times \sigma_{\mathcal{D}_{ref}}^{num} + \mu_{\mathcal{D}_{up}}^{num} \right] \quad (22)$$

Now, since the time transformation function remains the same for the two designs, the experimental lifetime distribution of the upgraded design can be obtained through:

$$t_{\mathcal{D}_{up}}^{exp}(i) = \exp \left[\Phi^{-1}(1 - D_R(i)) \times \sigma_{\mathcal{D}_{up}}^{exp} + \mu_{\mathcal{D}_{up}}^{exp} \right] \quad (23)$$

- **Step 5 : Experimental reliability function of the upgraded design**

Once a sample of experimental lifetime points of the upgraded design is obtained, a lognormal model can be fitted on this sample in order to estimate the lifetime distribution parameters.

The so-called bootstrap [39] approach is used here for this purpose. This numerical method, based on re-sampling technique from a prior population, is widely used for many statistical problems such as hypothesis testing and confidence intervals estimation. In our case, it provides the

estimates of the lifetime distribution parameters and the associated confidence intervals. In addition, confidence bounds of the reliability function can be also derived.

First, we randomly generate \mathcal{B} *sub-samples*, also called *bootstrap samples*, based on sampling with replacement technique from a *training sample*, which corresponds in our case to the experimental lifetime sample $\{t_{\mathcal{D}_{up}}^{exp}(i)\}_{i=1}^{n_R}$ associated to the upgraded design. Consequently, it is clear that the bootstrap has the same size as the training sample, that is to say n_R points. Some of them are redundant and others are not selected since the sampling process is perfectly random.

Then, for each bootstrap sample, the parameters (μ, σ) of the lifetime distribution are obtained through a Maximum-Likelihood-Estimation method. For a lognormal distribution, the likelihood function reads:

$$\begin{aligned} L(t|\mu, \sigma) &= \prod f(t_i|\mu, \sigma) \\ &= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^{n_R} \left(\prod_{i=1}^{n_R} \frac{1}{t_i}\right) \exp\left[\sum_{i=1}^{n_R} \frac{-(\ln(t_i) - \mu)^2}{2\sigma^2}\right] \end{aligned} \quad (24)$$

The estimators $\hat{\mu}$ and $\hat{\sigma}^2$ of the lognormal parameters are obtained by :

$$\hat{\mu} = \frac{1}{n_R} \sum_{i=1}^{n_R} \ln(t_i) \quad (25)$$

$$\hat{\sigma}^2 = \frac{1}{n_R} \sum_{i=1}^{n_R} (\ln(t_i) - \hat{\mu})^2 \quad (26)$$

When all the \mathcal{B} *bootstrap samples* are evaluated, a set of \mathcal{B} values of each of the lifetime distribution parameters $\hat{\mu}$ and $\hat{\sigma}$ are obtained, and gathered in the samples $\Xi_{\mathcal{B}} = \{\hat{\mu}_k\}_{k=1}^{\mathcal{B}}$ and $\Sigma_{\mathcal{B}} = \{\hat{\sigma}_k\}_{k=1}^{\mathcal{B}}$, respectively. After sorting the components of these samples in ascending order, the mean values associated to $\hat{\mu}$ and $\hat{\sigma}$ corresponding to the 50% fractile, are obtained :

$$\hat{\mu}_{\mathcal{D}_{up}}^{exp} = \Xi_{\mathcal{B}}(0.5 \times \mathcal{B}) \quad (27)$$

$$\hat{\sigma}_{\mathcal{D}_{up}}^{exp} = \Sigma_{\mathcal{B}}(0.5 \times \mathcal{B}) \quad (28)$$

Let us consider a target confidence interval $\delta \times 100\%$. In the case of symmetric distribution of the parameters, the corresponding lower and upper bounds are respectively $(1 - \delta)/2$ and $(1 + \delta)/2$ fractiles. For instance, if we are interested in the parameters $\hat{\mu}_{\mathcal{D}_{up}}^{exp}$, the associated lower and upper bounds are given by:

$$\hat{\mu}_{\mathcal{D}_{up}}^{exp}(lower) = \Xi_{\mathcal{B}}\left(\frac{1 - \delta}{2} \times \mathcal{B}\right) \quad (29)$$

$$\hat{\mu}_{\mathcal{D}_{up}}^{exp}(upper) = \Xi_{\mathcal{B}}\left(\frac{1 + \delta}{2} \times \mathcal{B}\right) \quad (30)$$

Finally, the confidence bounds for the reliability distribution are constructed in the same manner. Indeed, for a lifetime t_i , the reliability is computed for each value $\Xi_{\mathcal{B}}(k)$ and $\Sigma_{\mathcal{B}}(k)$, as follows:

$$R_i^k = 1 - \Phi\left(\frac{\ln(t_i) - \Xi_{\mathcal{B}}(k)}{\Sigma_{\mathcal{B}}(k)}\right) \quad (31)$$

The obtained \mathcal{B} values $\{R_i^k\}_{k=1}^{\mathcal{B}}$ are sorted in ascending order and the $(1 - \delta)/2$ and $(1 + \delta)/2$ fractiles are retained as the minimum and the maximum values of the reliability at the lifetime t_i . The lower and upper bounds curves of the experimental reliability function of the upgraded design are obtained by repeating the above process for all lifetimes t_i .

4. Application to automotive components

In this section, two releases of an automotive mechanical component are studied to assess the advantages of the proposed approach. The first release, for which experimental data are available, is taken as the reference design \mathcal{D}_{ref} to construct the associated time transformation function of the automotive component based on both, statistical analysis of the experimental data and numerical assessment of the structural reliability. The second one is a modified release of the reference design to better respond to car constructors' requirements. It is considered as the upgraded design \mathcal{D}_{up} , where the corresponding *true reliability* function (i.e. experimental reliability) will be derived by applying the time transformation function to the numerical reliability function obtained through Monte-Carlo simulations on the DDM-based surrogate model. For the upgraded design only few tests are performed in order to assess the accuracy of the reliability function.

4.1. Reliability analysis of the reference design

The studied component is a mechanical component of car seat that allows locking and unlocking the longitudinal sliding of the seat. A schematic sketch of the product is shown in Fig. 6 (a). As a part of the product qualification process, ALT was carried out on the concerned component to evaluate its reliability. A dominant failure mode is observed under cyclic loading (see Fig. 6 (b)), which is the breaking of the component at the upper right side.

In order to reduce the testing process time, which is the main goal of the accelerated lifetime testing, three stress levels $s_3 > s_2 > s_1$ much higher than the operating stress s_{nom} are used. Stress in ALT can be of different types depending on the application case (mechanical stress, displacement, voltage, temperature, etc...). In this application, the stress is relative to displacement stroke. Three levels of displacement $\{\mathbf{u}_k\}_{k=1}^3$

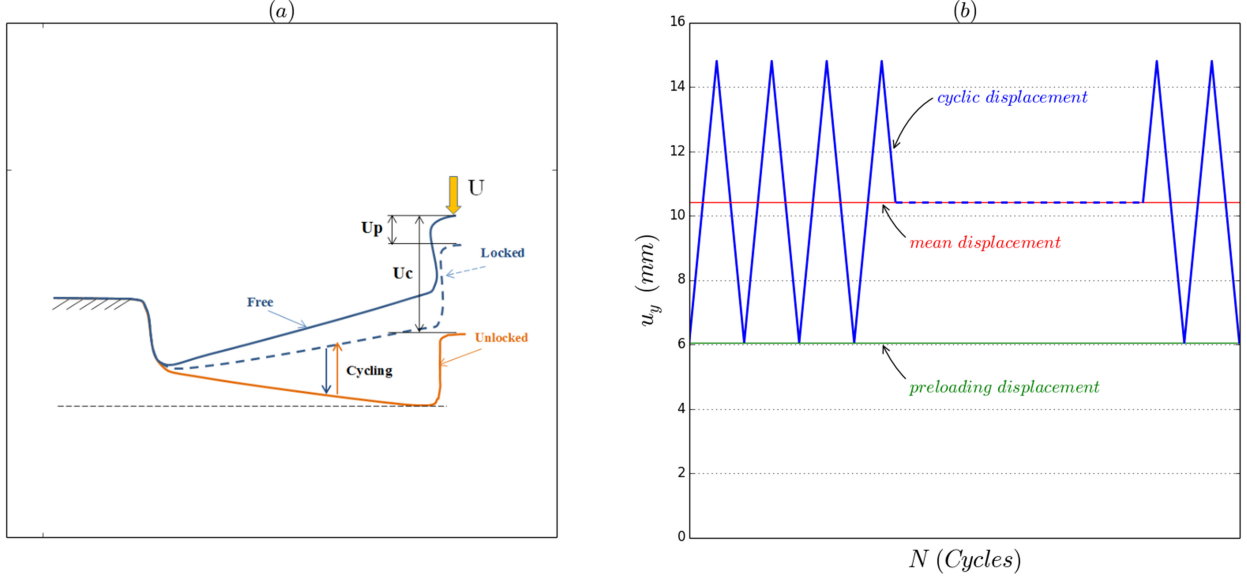


Fig. 6. (a) Sketch of the reference design, (b) Applied load for the reference design

higher than nominal displacement u_{nom} have been applied (Table 1) and the obtained results are given in Table 2. For the displacements u_3 and u_2 five specimens are tested, only four specimens are used for the displacement u_1 . The testing process is stopped when the full collapse of the specimen is observed, and the fatigue lifetime N_f , defined as the number of loading cycles required to reach the failure, is recorded.

Table 1
Accelerated fatigue testing Stress

Accelerated stress levels : displacement strokes			
-high-	-medium-	-low-	-nominal-
u_1	u_2	u_3	u_{nom}
29.3 mm	27.3 mm	24 mm	22.3 mm

Table 2
Accelerated fatigue testing results : Lifetimes (cycles)

Accelerated fatigue lifetimes at each stress levels			
	-high-	-medium-	-low-
$N_f(\text{cycles})$	9088	12797	27305
	9909	16845	31457
	8883	13177	26580
	8358	14252	23759
	8899	24275	

Statistical analysis is performed assuming a lognormal distribution for the fatigue lifetime N_f and an inverse power law as acceleration model, which reads:

$$\bar{N}_f(s) = \frac{1}{K \times s^{\gamma_1}} \quad (32)$$

where s denotes the stress level, \bar{N}_f is the median fatigue lifetime at stress level s , and K and γ_1 are real unknown parameters of the model.

Now applying the logarithm on the both sides of Eq. (32) in order to linearize the acceleration law with respect to the stress level, the median fatigue lifetime may be cast as a function of s as follows:

$$\mu_{N_f}(s) = \ln(\bar{N}_f(s)) = \gamma_0 - \gamma_1 \times \ln(s) \quad (33)$$

where $\gamma_0 = -\ln(K)$

It follows that the lognormal cumulative distribution function, used to represent the variability of the fatigue lifetime N_f , may be written in the following form:

$$F(N_f, \mu_{N_f}(s), \sigma) = \Phi\left(\frac{\ln(N_f) - \gamma_0 + \gamma_1 \times \ln(s)}{\sigma}\right) \quad (34)$$

where $\mu_{N_f}(s)$ is the scale parameter and σ represents the shape parameter.

The shape parameter σ is assumed to be constant for all stress level s . This assumption is a commonly used [40, 41] which allows to simplify Eq.34 but also means that the failure mechanism remains the same for all the applied stress levels. Indeed, in accelerated life testing, when over-stressing the component, one has to ensure that the component will fail in the same manner. This assumption is not always valid [42], thus need to be validated using the obtained results from the ALT.

As previously mentioned, Maximum-Likelihood-Estimation method is used to compute the parameters of the acceleration model. The log-likelihood function reads :

$$\mathcal{L}(\gamma_0, \gamma_1, \sigma) = \sum_{k=1}^3 \left(\sum_{i=1}^{n_k} \ln \left(\frac{1}{N_{f_i} \sigma \sqrt{2\pi}} \exp \left[-\frac{(\ln(N_{f_i}) - \gamma_0 + \gamma_1 \ln(s_k))^2}{2\sigma^2} \right] \right) \right) \quad (35)$$

The estimators of the parameters of the acceleration model, as well as those of the fatigue lifetime distribution are reported in Table 3. $\hat{\mu}_{\mathcal{D}_{ref}}^{exp}$ and $\hat{\sigma}_{\mathcal{D}_{ref}}^{exp}$ correspond respectively to the scale and shape parameter estimators of the lognormal distribution of the experimental lifetime of \mathcal{D}_{ref} .

To validate the assumption that the shape parameter $\hat{\sigma}_{\mathcal{D}_{ref}}^{exp}$ is constant for all the three stress levels, the likelihood ratio test is performed. For each stress level s_i , the log-likelihood L_i is computed as well as the log-likelihood L_a considering all the stress levels. Then the statistic $T = 2 \times (L_1 + L_2 + \dots + L_k - L_a)$ is calculated and compared to $\chi^2(1 - \alpha, k - 1)$, that is the $1 - \alpha$ order quantile of the Chi-square distribution χ^2 , with $k - 1$ degrees of freedom, where k is the number of stress levels and α is the risk of error. According to this test, if $T \leq \chi^2(1 - \alpha, k - 1)$ then constant shape assumption is validated and if $T > \chi^2(1 - \alpha, k - 1)$ the assumption is rejected. In this paper $k = 3$ and a risk $\alpha = 0.1$ is used leading to $T = 2.151$ and $\chi^2(1 - \alpha, k - 1) = 4.605$. Therefore, $T \leq \chi^2(1 - \alpha, k - 1)$ which validated the assumption.

The median fatigue lifetime $\bar{N}_{f,0}$ of the component at operating loading conditions, is also estimated and shows that the failure of the component occurs during Low-Cycles Fatigue (LCF) stage, since the fatigue lifetime do not exceeds 10^5 cycles.

Table 3
Estimates of the acceleration model and fatigue lifetime distribution parameters of the reference design

$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\mu}_{\mathcal{D}_{ref}}^{exp}$	$\hat{\sigma}_{\mathcal{D}_{ref}}^{exp}$	$\bar{N}_{f,0}$
28.281	-5.669	10.681	0.090	43525

In addition, the experimental reliability function of the reference design and the corresponding 95% confidence bounds are plotted in Fig. 7 (a).

In order to construct the time transformation function associated to the studied mechanical component, numerical analysis is also carried out based on the proposed approach presented in section 3. First of all, a 3D finite element model of the reference design has been developed using the software Abaqus [43]. The mechanical behavior of the constitutive material follows the well-known Ramberg-Osgood law [44] and the Coffin-Manson-Morrow's fatigue model [45] was used to compute the fatigue lifetime. The use of this fatigue model is motivated by the experimental results which clearly show that the failure of the mechanical component occurs during low-cycles fatigue stage. The main advantages of the Coffin-Manson-Morrow fatigue model, compared to the original Coffin-Manson fatigue model, is that it takes into account the effect of the mean stress σ_m as

shown in the following equation:

$$\frac{\Delta \varepsilon_t}{2} = \frac{\sigma'_f - \sigma_m}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \quad (36)$$

where $\Delta \varepsilon_t/2 = \Delta \varepsilon_e/2 + \Delta \varepsilon_p/2$ is the strain amplitude which is function of the elastic and plastic strain amplitudes, σ'_f is the fatigue strength coefficient, ε'_f is the fatigue ductility coefficient, b is the fatigue strength exponent, c is the fatigue ductility exponent, E is the elastic modulus and $2N_f$ is the number of reversals to failure. The parameters σ'_f , ε'_f , b and c of the fatigue model are obtained through hardness coefficient of the constitutive material [45]. Note that, the fatigue lifetime N_f is obtained by solving Eq. (36) based on numerical scheme, where the total strain amplitude $\Delta \varepsilon_t/2$ is given by the finite elements model.

Now we are interested in the study the effect of the uncertainties observed on loading parameters and materials properties, on the response of the mechanical model defined as the fatigue lifetime N_f . The uncertain parameters are the elastic modulus E , the fatigue strength coefficient σ'_f , the fatigue strength exponent b and the cyclic loading amplitude u_c . They are modeled as independent lognormal variables, where the statistical characteristics (i.e. the mean value μ and the standard deviation σ) are given in Table 4.

Table 4
Distribution type and statistical characteristics of the uncertain parameters of the reference design

Parameter	Distribution	μ	σ
$E(GPa)$	Lognormal	210	2.1
$\sigma'_f(MPa)$	Lognormal	2339.724	23.39
ε'_f	Constant	0.45	-
b	Lognormal	0.09	9.10^{-4}
c	Constant	0.59	-
$u_p(mm)$	Constant	4.15	-
$u_c(mm)$	Lognormal	14.8	0.158

Since the mechanical model has an implicit form (i.e. finite elements model), which is time consuming to be used directly to carry out uncertainty propagation analysis, a surrogate model based on the DDM has been constructed. The mechanical response, defined as the fatigue lifetime N_f , is then represented through a polynomial approximation $\hat{N}_f(E, \sigma'_f, b, u_c)$, based on univariate dimensional decomposition ($s = 1$) and 7th order experimental design. That is, 7 experimental points are used in each direction of the random space, and obtained through the recursive formula (see Eq. 12), where the reference point is taken as the mean point. Consequently, only 24 finite elements analysis are needed to construct the surrogate model. One should note that the experimental design is firstly constructed in the standard random space and then generated in the physical random space based on isoprobabilistic transformation as explained in section 2.

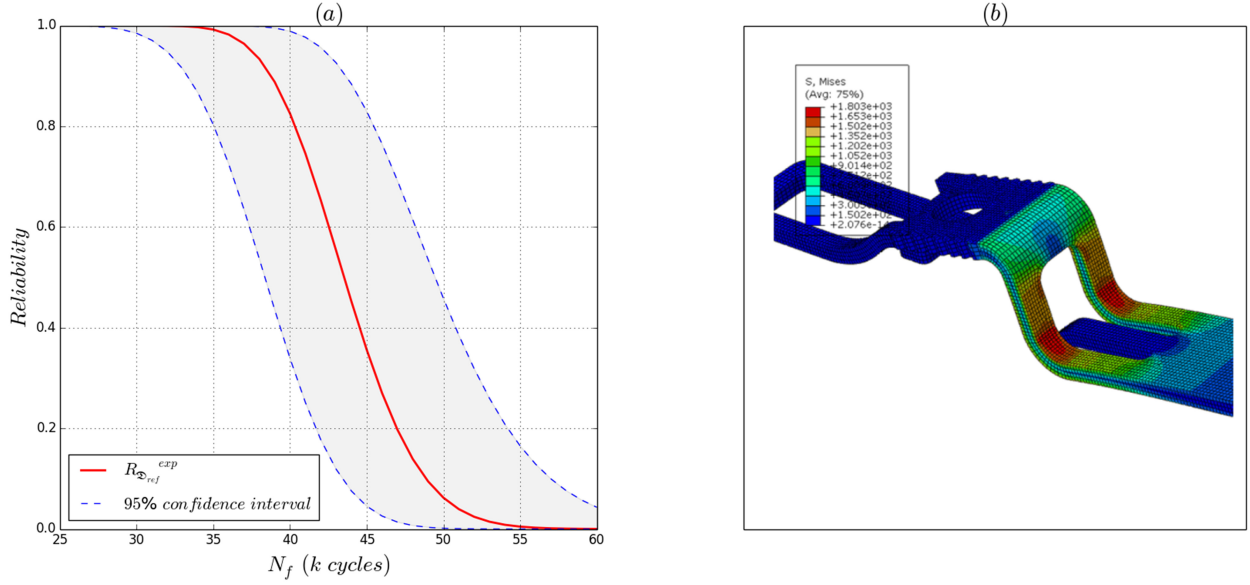


Fig. 7. (a) Experimental reliability of the reference design $R_{\mathcal{D}_{ref}}^{exp}$, (b) Finite elements mesh of the reference design

In order to evaluate the accuracy of the obtained surrogate model, the polynomial approximation is plotted with respect to the uncertain parameters E, σ_f', b and u_c . Every time, only one uncertain parameter is taken as a variable quantity and the others remaining parameters are fixed at their respective mean values.

It is important to note that the experimental points used to assess the accuracy of the polynomial approximation are different from those used in the construction of the surrogate model. They are sampled in the range $[\mu_i \pm 3\sigma_i]$, where μ_i and σ_i are the mean and the standard deviation of the i^{th} uncertain parameter. As shown in Fig. 8, univariate approximation matches the exact mechanical model, which is represented here by a finite element model, in the range $[\mu_i \pm 3\sigma_i]$. Thus, Monte-Carlo simulations can be applied to the surrogate model to efficiently perform uncertainty propagation analysis. This procedure is efficient since it does not require any additional finite elements analysis. Indeed, the four first statistical moments, namely the mean, the standard deviation, the skewness and the kurtosis, of the fatigue lifetime are computed (see Table 5) based on 10^5 Monte-Carlo simulations. The probability density function is also constructed, which gives more information compared to the statistical moment since each possible event is weighted by a probability of occurrence.

The probability density function of the fatigue lifetime is compared to standard distributions. As depicted in Fig. 9 (a), it can be accurately approximated by lognormal distribution, which is more simple to use (i.e. since an analytical formulation is already available) to carry out reliability analysis, for instance. It should be noted that the equivalent scale parameter $\hat{\mu}_{\mathcal{D}_{ref}}^{num}$ and shape parameter $\hat{\sigma}_{\mathcal{D}_{ref}}^{num}$ of the lognormal distribution of the fatigue lifetime are also computed (see Table 5).

The probability density function of the fatigue lifetime is then integrated to obtain the numerical reliability function $R_{\mathcal{D}_{ref}}^{num}$ and compared to the experimental reliability function $R_{\mathcal{D}_{ref}}^{exp}$ obtained

Table 5

Statistical moments of the fatigue lifetime of the reference design from surrogate model based on the DDM

Statistical moments of the fatigue lifetime				
Mean	Standard deviation	Skewness	Kurtosis	Coefficient of variation
65546.68	10826.76	0.178	3.078	16.5%
Lognormal distribution parameters				
	$\hat{\mu}_{\mathcal{D}_{ref}}^{num}$		$\hat{\sigma}_{\mathcal{D}_{ref}}^{num}$	
	11.08		0.164	

previously through experimental test results analysis. The gap observed between the numerical estimations and the *true reliability* (see Fig. 9 (b)), can be explained through, on one hand, the simplifying assumptions made in the finite elements analysis such as the boundary conditions which can be a little bit different from those taken in the experimental testing process, and on the other hand, the lack of information about the uncertain parameters. Indeed, the probabilistic models (i.e. random variables) and the associated statistical characteristics should be obtained through statistical analysis to be able to represent the true variability of the uncertain parameters. In this study, the coefficients of variation of the uncertain parameters are obtained through engineers experiment feedback.

We are interested now on the construction of the time transformation function of the studied mechanical component which is defined as the relationship between the numerical and the experimental reliability function. To do this, the approach proposed in section 3.2 is used. Let $N_{f, \mathcal{D}_{ref}}^{exp}(i)$ and $N_{f, \mathcal{D}_{ref}}^{num}(i)$ respectively the experimental and numerical estimates of the fatigue lifetime associated to a same reliability level R_i . The relationship be-

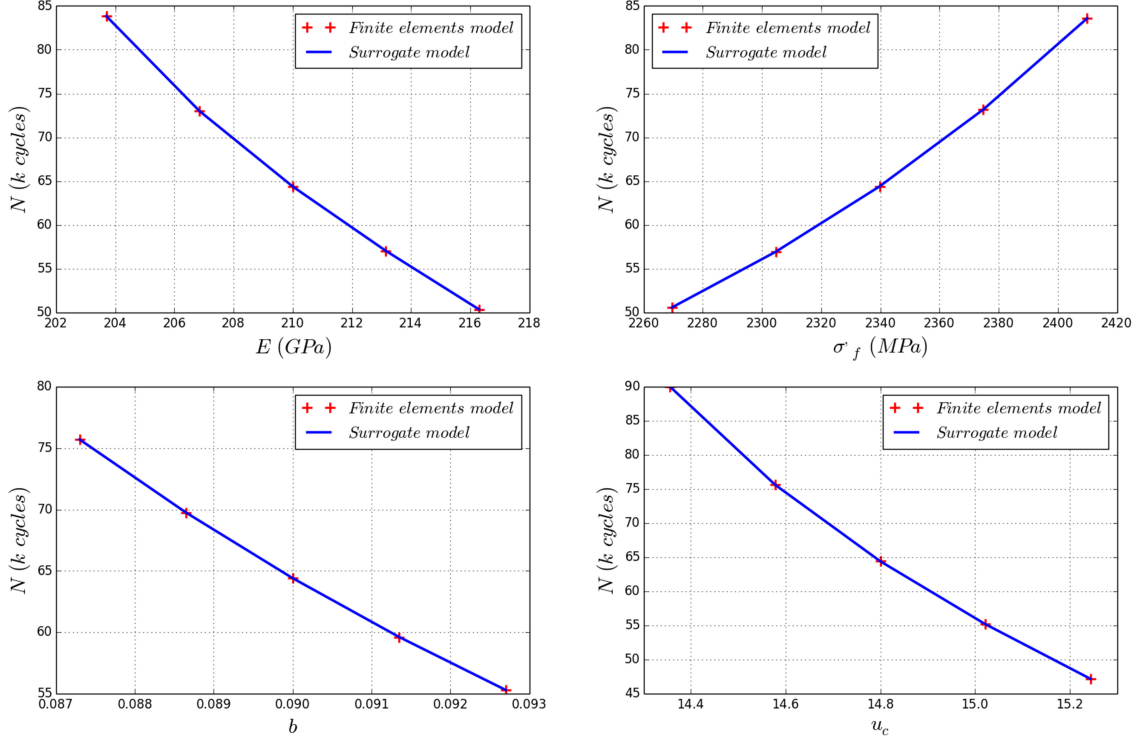


Fig. 8. Evaluation of the accuracy of the surrogate model for the reference design

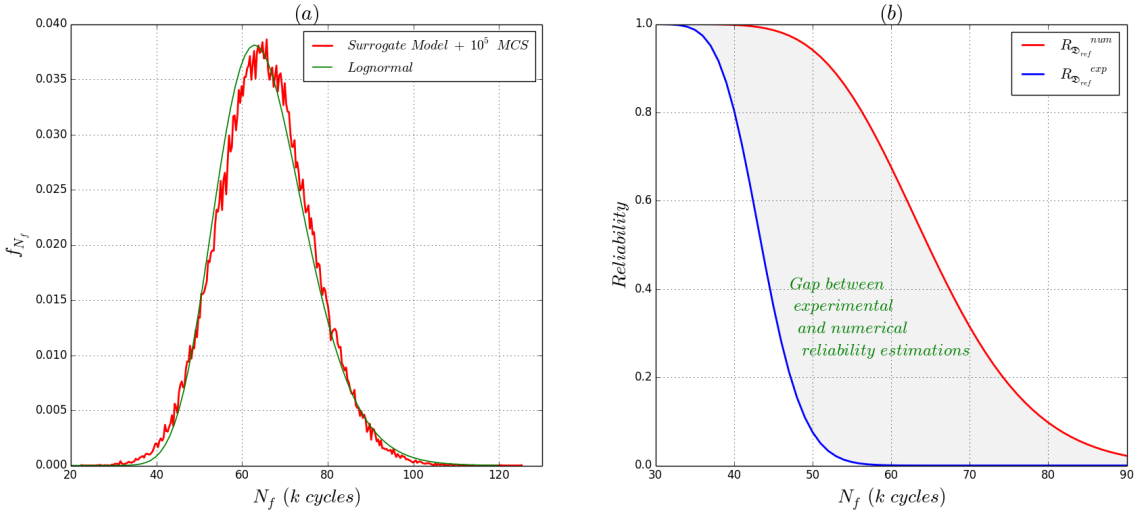


Fig. 9. (a) Probability density function of the fatigue lifetime N_f for the reference design, (b) Comparison of the experimental and numerical estimations of the reliability for the reference design

tween $N_{f,\mathcal{D}_{ref}}^{exp}(i)$ and $N_{f,\mathcal{D}_{ref}}^{num}(i)$ is written as follows using the time transformation function $h(t)$ described by the Eq. (18):

$$N_{f,\mathcal{D}_{ref}}^{exp}(i) = h\left(N_{f,\mathcal{D}_{ref}}^{num}(i)\right) = A \times \left[N_{f,\mathcal{D}_{ref}}^{num}(i)\right]^{\beta_1} \quad (37)$$

Based on the third step of the methodology developed in section 3.2, the parameters A and β_1 of the transformation function have been computed and the corresponding estimated values \hat{A} and $\hat{\beta}_1$ are reported in Table 5. The power law model is compared to linear and 2nd order polynomial approximations, and is

found to be the best to fit the regression points as shown in Fig. 10.

Table 6
Estimates of the parameters of the time transformation function

Linear	2 nd order polynomial	Power law		
R^2	R^2	R^2	\hat{A}	$\hat{\beta}_1$
0.9984	0.9914	1	61.198	0.593

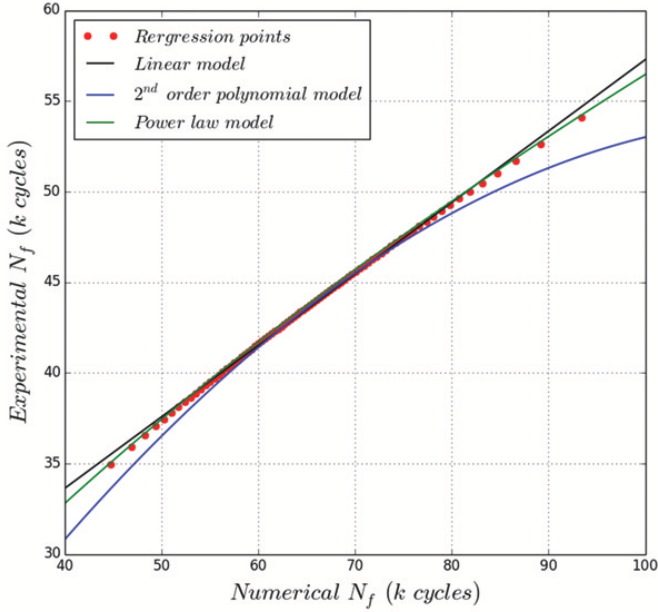


Fig. 10. Comparison of the regression models used to construct the time transformation function

4.2. Reliability analysis of the upgraded design

The reference design \mathcal{D}_{ref} of the mechanical component is now modified to cope with some car constructors requirements. Obviously, it is necessary to assess the effect of these modifications on the reliability of the upgraded design \mathcal{D}_{up} . The problem is that FAURECIA Company, which is in charge of the qualification process, would not want performs experimental tests on the upgraded design to estimate the corresponding reliability, since it is time consuming and expensive. The idea is to use results of reliability analysis carried out on previous designs as prior knowledge to estimate the reliability of the upgraded design. Indeed, this later has the same backbone as the reference design, since the modifications are not significant. Consequently, the time transformation function should be the same for the two designs. In other words, the time transformation function is rather related to the product and invariant against the design. Thus, *the true reliability* (i.e. experimental reliability) can be easily obtained by applying the time transformation function to the results of numerical reliability analysis carried out on the upgraded design, following the method developed in section 3.

First of all, a 3D finite elements model is developed on the software Abaqus to compute the fatigue lifetime of the upgraded design, using the same assumptions on the boundary conditions as for the reference design. It is important to note that the upgraded design is built of the same material as the reference design and Ramberg-Osgood constitutive law and Coffin-Manson-Morrow fatigue model are also used. However, the loading conditions have been a little bit modified (see Fig. 11 (a)) since the preloading displacement u_p and the cyclic displacement u_c are equal now to 4.8 mm and 8 mm, respectively. In addition, as schematically depicted in Fig. 11 (a) a spring model is taken into account in the finite elements analysis to reproduce the real operating conditions of the new design. A sketch of the finite

elements mesh is depicted in Fig. 12.

Then probabilistic analysis is performed combining surrogate model and Monte-Carlo simulations to assess the effect of the variability of the uncertain parameters (see Table 7) on the fatigue lifetime of the upgraded design. The deterministic parameter k in Table 7 refers to the stiffness of the spring model used in the finite elements analysis.

Table 7
Distribution type and statistical characteristics of the uncertain parameters of the upgraded design

Parameter	Distribution	μ	σ
E (GPa)	Lognormal	210	2.1
σ_f' (MPa)	Lognormal	2339.72	23.39
ε_f	Constant	0.45	-
b	Lognormal	0.09	9.10^{-4}
c	Constant	0.59	-
u_p (mm)	Constant	4.8	-
u_c (mm)	Lognormal	8	0.08
k (N/mm)	Constant	3.359	-

Firstly, the accuracy of the surrogate model is evaluated by plotting the obtained analytical formulation with respect to the uncertain parameters as shown in Fig. 13. As can be seen, the polynomial approximation built through DDM reproduce very well the real behavior of the studied mechanical component represented here by a finite elements model. Thus, it can be used to perform reliability analysis. It is important to note that the surrogate model is obtained through univariate dimensional decomposition ($s = 1$) and Lagrange polynomial constructed on the same experimental design as for the reference design.

The four first statistical moments of the fatigue lifetime N_f are computed through 10^5 sample obtained through Monte-Carlo simulations on the surrogate model (see Table 8). The equivalent scale parameter $\hat{\mu}_{\mathcal{D}_{up}}^{num}$ and shape parameter $\hat{\sigma}_{\mathcal{D}_{up}}^{num}$ of the lognormal distribution are also estimated.

Table 8
Statistical moments of the fatigue lifetime of the upgraded design from surrogate model based on the DDM

Statistical moments of the fatigue lifetime				
Mean	Standard deviation	Skewness	Kurtosis	Coefficient of variation
32382.93	5751.87	0.335	3.249	17.8%
Lognormal distribution parameters				
$\hat{\mu}_{\mathcal{D}_{up}}^{num}$		$\hat{\sigma}_{\mathcal{D}_{up}}^{num}$		
10.370		0.176		

Based only on the results of the numerical analysis for both reference and upgraded designs, and comparing the mean values of the fatigue lifetime, it is clear that the reference design better resist to the corresponding operating loading conditions since

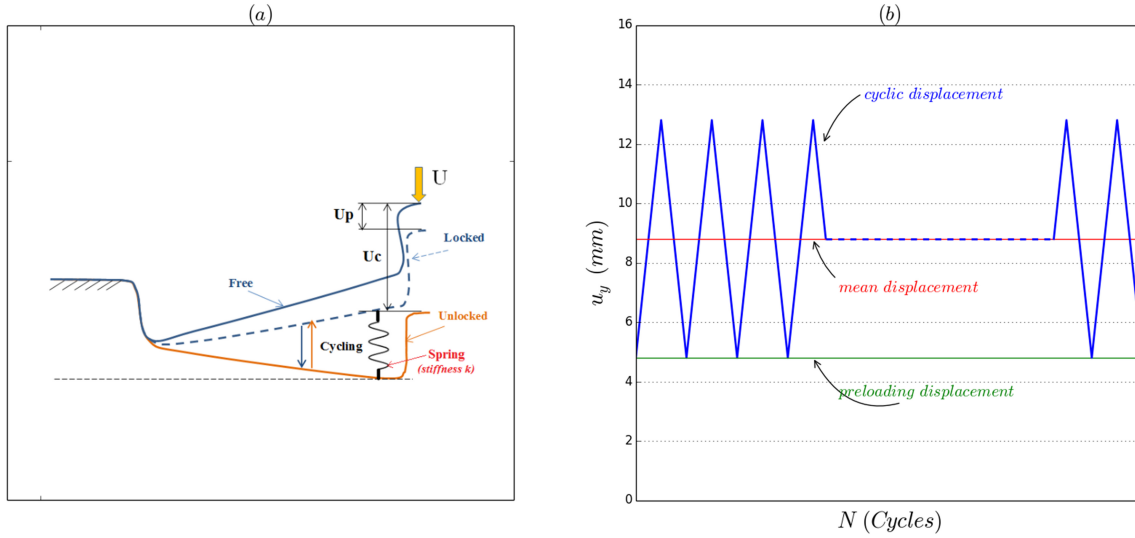


Fig. 11. (a) Sketch of the upgraded design, (b) Applied load for the upgraded design

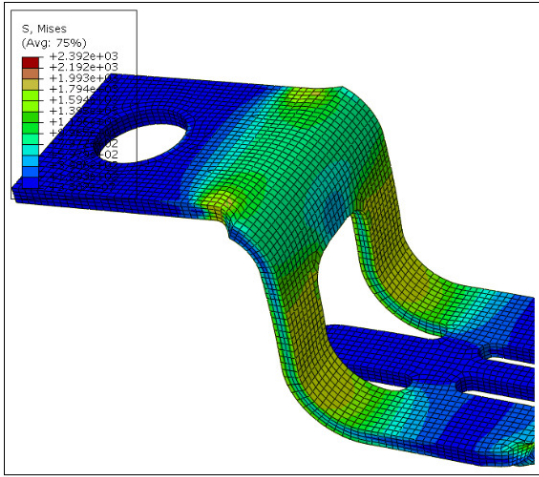


Fig. 12. Finite elements mesh of the upgraded design

the fatigue lifetime is much longer. As a preliminary conclusion, we can say that the modifications carried on the mechanical component will have harmful effect on its reliability. This information is very useful for the engineers since the product is still in the design stage, and has not reached the market. Indeed, they still have a large leeway to enhance the product without expensive efforts.

From the sample set given by Monte-Carlo simulations an histogram of the fatigue lifetime may be built. Then, as depicted in Fig. 14 (a), a graphical representation of the probability density function of the fatigue lifetime is obtained using kernel smoothing technique on the previous histogram. In addition, the probability density function is compared to standard distribution models for which analytical formulation is available. As can be seen, the lognormal model fits very well the probability density function of the numerical fatigue lifetime of the upgraded design.

To validate the assumption of same time transformation function for the reference and upgraded designs, a linearity analysis

is performed using the probabilistic mechanical models of both designs. In Fig. 16, each point $(CV(y^{up}); CV(y^{ref}))$ represents the estimated variabilities (coefficient of variations) of the responses of both mechanical models given same variabilities levels (perturbations) introduced on their input parameters. As can be seen, a linear relationship with a slope $\beta_L \approx 1$ and an y-intercept $\alpha_L \approx 0$ can be derived between the variabilities of the responses of the reference and the upgraded design, which means that they have the same behaviour. Therefore, we can conclude that the time transformation function is conserved for both designs.

Having previously obtained the time transformation function of the mechanical component based on results of both experimental and numerical reliability analysis of the reference design, the *true reliability* (i.e. experimental reliability) of the upgraded design can be easily obtained by applying the time transformation function to the numerical reliability of the upgraded design, as explained in section 3. Firstly, after discretizing the reliability domain, a sample set of expected experimental lifetime for the upgraded design is generated. Then, thanks to the bootstrap technique the statistical characteristics, typically the mean $\hat{\mu}_{\mathcal{D}_{up}}^{exp}$ and the standard deviation $\hat{\sigma}_{\mathcal{D}_{up}}^{exp}$, of the lognormal distribution of the experimental fatigue lifetime, are obtained in conjunction with the associated 95% confidence bounds. The obtained values are given in Table 9.

Table 9
Lognormal distribution parameters of the experimental fatigue lifetime for the upgraded design

$\hat{\mu}_{\mathcal{D}_{up}}^{exp}$			$\hat{\sigma}_{\mathcal{D}_{up}}^{exp}$		
Lower	Median	Upper	Lower	Median	Upper
10.237	10.261	10.283	0.085	0.0986	0.113

It should be noted that a symmetric confidence interval has been chosen here since the estimators $\hat{\mu}_{\mathcal{D}_{up}}^{exp}$ and $\hat{\sigma}_{\mathcal{D}_{up}}^{exp}$ can be well

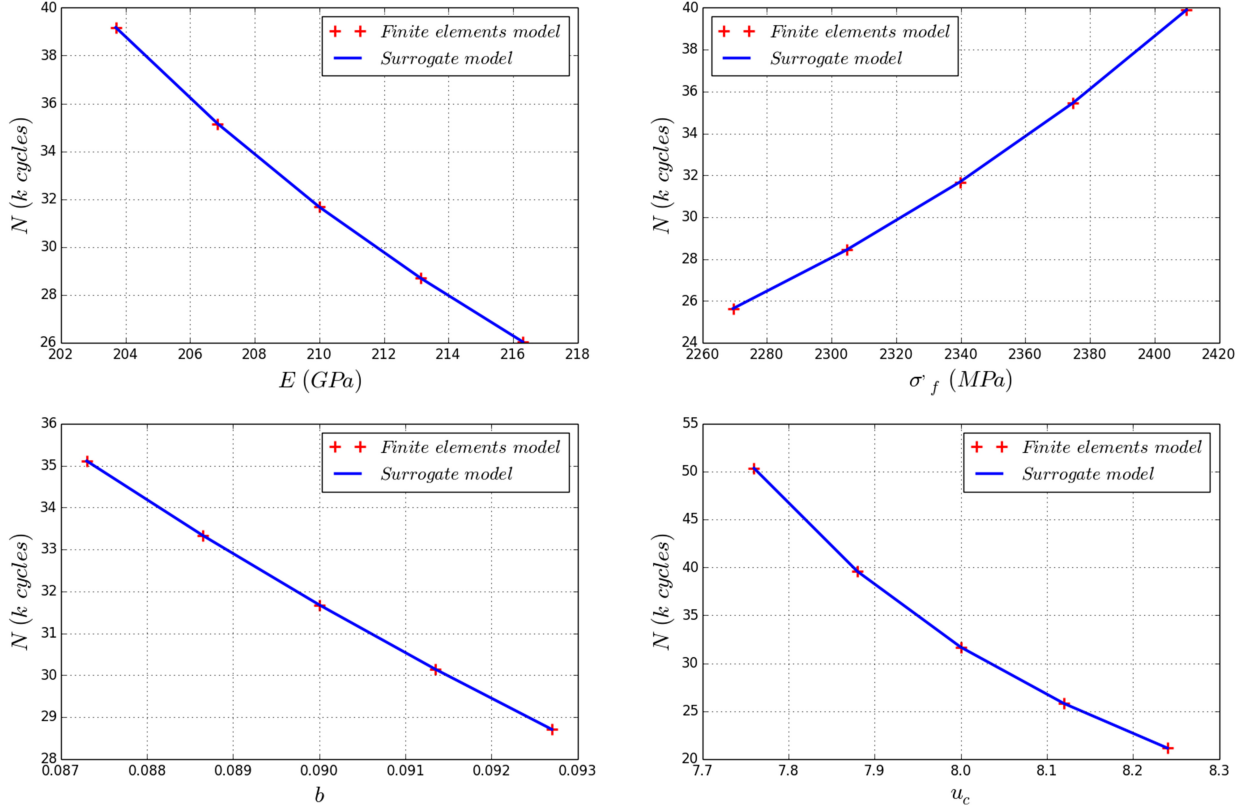


Fig. 13. Evaluation of the accuracy of the surrogate model for the upgraded design

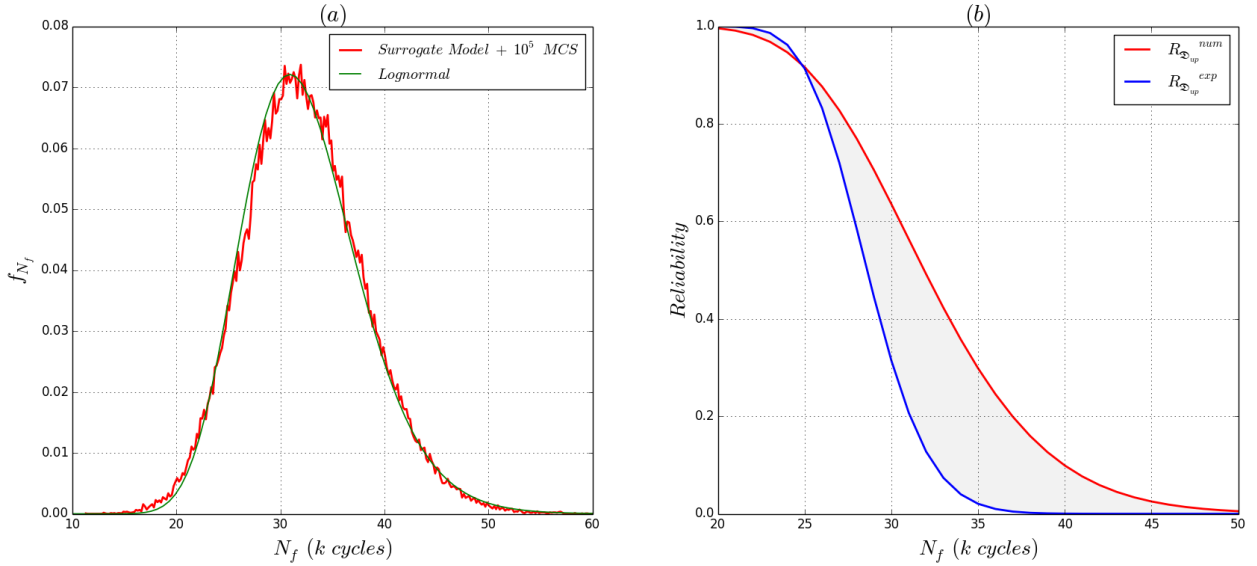


Fig. 14. (a) Probability density function of the fatigue lifetime N_f for the upgraded design, (b) Comparison of the experimental and numerical estimations of the reliability for the upgraded design

fitted by normal distribution as shown in Fig. 15 (a) and Fig. 15 (b).

Finally, the experimental reliability $R_{D_{up}}^{exp}$ of the upgraded design can be easily derived through the integration of the probability density function of the experimental fatigue lifetime $N_{f, D_{up}}^{exp}$. As depicted in Fig. 14 (b), for high reliability levels (i.e. ~ 1) a

small discrepancy is observed between the experimental and the numerical estimations since the relative error does not exceeds 2%. However, this later (i.e. the discrepancy) is more significant when the reliability tends to 0. This discrepancy is mainly due to the assumptions taken in the finite elements analysis and the modeling of the variability of the uncertain parameters. It is

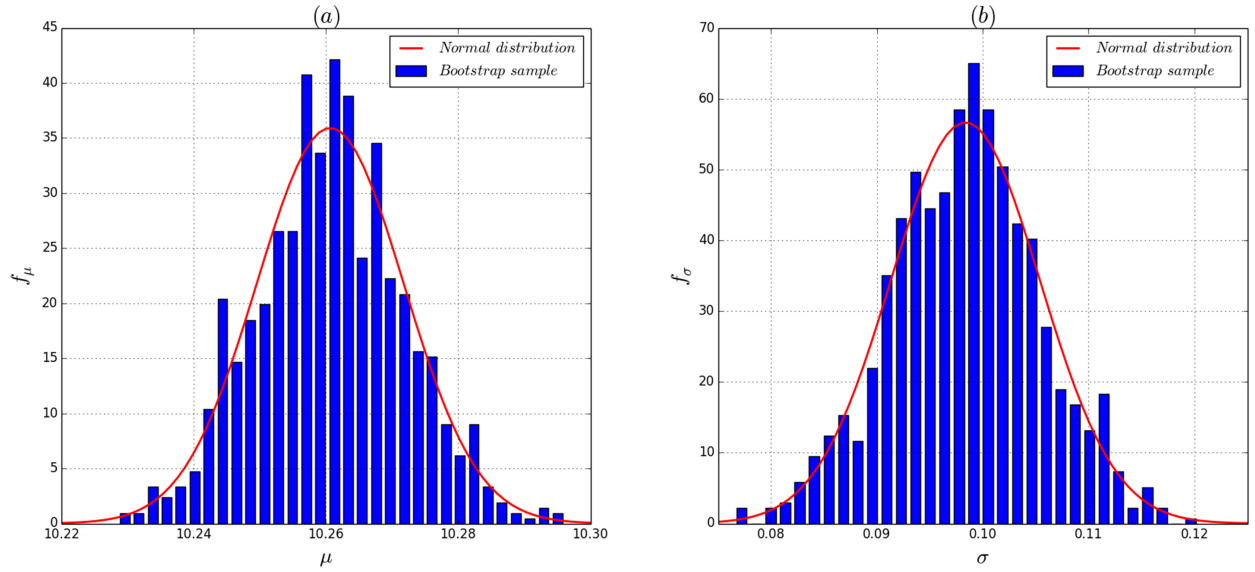


Fig. 15. Scatter on the estimators of statistical parameters of the experimental fatigue lifetime distribution of the upgraded design

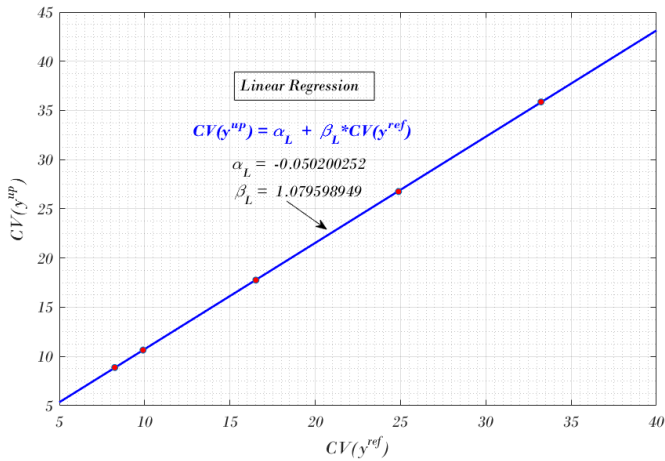


Fig. 16. Linear relationship between the mechanical models representing the reference and the upgraded designs (conservation of the mechanical behavior)

important here to remind that the main subject of the proposed approach is to construct an accurate prior knowledge of the true reliability of the underlying component at the design stage, in order to reduce the number of accelerated tests, required later at the qualification stage. Only for validation purposes, a few tests reproducing nominal operating conditions have been performed on a prototype of the upgraded release of the mechanical component. Tests results consists of 5 failures times corresponding to the number of cycles to fatigue (24983, 26660, 27400, 27970, 32500).

Fig. 17 compares the experimental test results and the experimental reliability function obtained through the proposed approach. For graphical comparison purpose, we have also plotted in the same figure, the numerical reliability curve $R_{\mathcal{D}_{up}}^{num}$ of the upgraded design \mathcal{D}_{up} . As can be seen, regarding the 5 experimental points, the reliability function obtained through the proposed approach is an acceptable estimate of the upgraded

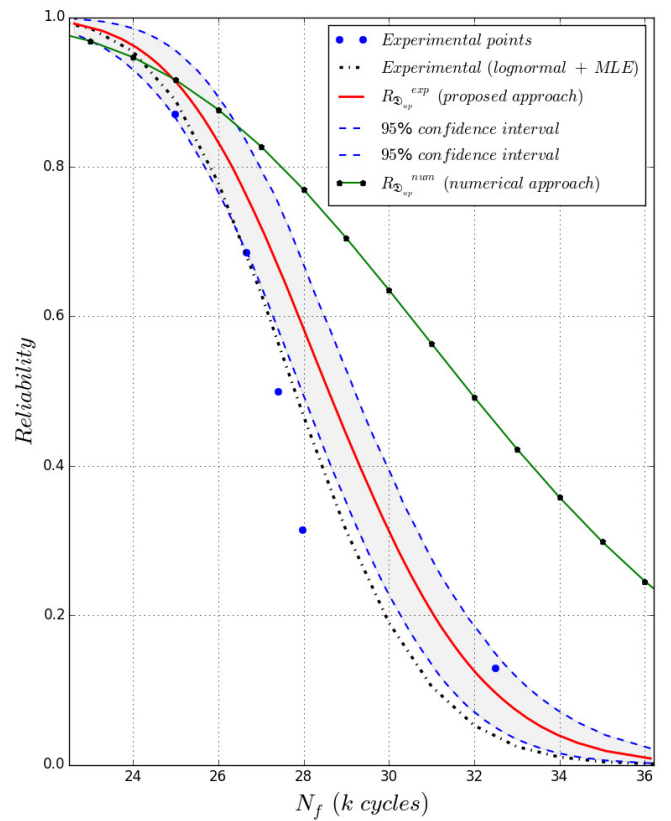


Fig. 17. Comparison of the reliability estimation given by the proposed approach and experimental test results performed on the upgraded design

design reliability, especially for high and low reliability levels since these later are too close to the median curve and belongs in the 95% confidence interval. Also note that, the area of interest on the reliability curves is limited to the reliability values located in the interval $[0.7, 1[$, that generally corresponds to customer

reliability requirements. Therefore, the proposed approach is efficient and sufficiently accurate to carry out early reliability analysis of upgraded automotive components. Indeed, one has to keep in mind that the aim of the proposed approach, from an industrial point of view, is to reduce as possible the number of experimental tests to estimate the true reliability of upgraded design. In this context, the reliability of the upgraded design obtained by the proposed approach can be enhanced further, using for instance, Bayesian updating, based only on few testing.

5. Conclusion

In this paper, we propose a novel approach, combining efficient surrogate models and time transformation concept, to deal with early reliability analysis of upgraded automotive components. The proposed approach is composed of five steps. In summary, first, the TTF associated to the automotive component is derived from reliability analysis based on both experimental data and numerical computation of the fatigue lifetime of a reference design. The fatigue lifetime distribution is built by performing Monte-Carlo simulations on a surrogate model. The surrogate model, which approximates the mechanical behavior of an automotive component, initially derived from an implicit formulation (i.e. finite elements model) is built using the DDM and projection on Lagrange polynomial basis. Secondly, the true reliability of any upgraded design of the same automotive component is easily derived by applying the TTF to the numerical reliability obtained through finite elements analysis and DDM-based surrogate model. In addition, a bootstrap-based sampling technique is used to obtain confidence interval of the reliability estimates.

The efficiency and the accuracy of the proposed approach are evaluated through reliability analysis of two design releases of a mechanical component used to locking and unlocking the longitudinal sliding of car seat. The results have shown that the use of meta-modeling technique to perform uncertainty propagation throughout a mechanical model contribute significantly to the efficiency of the proposed approach since only 2×24 finite elements analysis (i.e. 24 calls of the finite elements per each release of the studied mechanical component) are needed to obtain a surrogate model able to reproduce accurately the real mechanical behavior of the automotive component initially given by a time consuming implicit model. This is mainly due to the structure of the polynomial approximation given by the DDM, where high order interactions of weak effect are automatically omitted. Indeed, a simple univariate dimensional decomposition is sufficient to correctly reproduce the nonlinear behavior of the mechanical component. It has been also shown that a power model is suitable to build the time transformation function since it fits better the evolution of the reliability than linear and 2nd order polynomial models. In addition, the time transformation function is rather related to the product than the design releases. That is, it remains the same for the upgraded and the reference designs. The reliability function of the upgraded design, derived through the proposed approach, is close to the reliability obtained through experimental testing, since the observed

discrepancy is very small. This later is mainly due to the assumption made on the probabilistic models (i.e. the distribution type and the statistical characteristics of the random variables) representing the variability of the uncertain parameters. In summary, the proposed approach combining meta-modeling-based reliability analysis and time transformation function concept provides a good alternative, to carry out early reliability estimation on upgraded automotive components.

Further works should be focused on improving the accuracy of the proposed approach. One way is to perform statistical analysis to determine objectively the statistical characteristics and the distribution type of the random variables representing the uncertain input parameters in order to better represent their real variabilities. Another way is to combine the proposed approach with Bayesian computations that aim to optimizing an experimental qualification plan.

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