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Asteroid satellite ephemeride service. Mutual occultations and eclipses.

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ABSTRACT

A tool has been developed for computing the ephemerides of asteroid satellites, which are needed to plan and verify new observations, as well as for space missions.

The proposed ephemeris server is available online at <http://www.sai.msu.ru/neb/nss/html/multisat/nssAsthe.htm> and <http://nsdb.imcce.fr/multisat/nssAsthe.htm>

Motion models for 62 asteroid satellites are included, which are computed based on all available observations. The proposed ephemeris server can provide relative satellite positions for a series of time instants specified by the user. The server displays the time intervals of mutual occultations and eclipses in asteroid systems with satellites, as well as times of satellite elongations, when the observing conditions are most favorable.

The circumstances of all mutual occultations and eclipses in the satellite systems are determined. During the years 2021-2025, events are to occur in 21 of the 62 systems considered. The plots show the time intervals of the phenomena.

The reliability of the computed ephemerides is verified by comparing their predictions with the results published by other authors.

We use our original method to estimate the accuracy of asteroid satellite ephemerides. Thus the asteroid satellite ephemeris server provides not only ephemerides but also the uncertainties of the predicted satellite positions. The low accuracy of the ephemerides of some satellites is due to the fact that their orbits are determined from a small number of observations spanning a short time interval.

Our tool can be used with more accurate orbital parameters and for new satellites to be discovered in the future. We hope to work on this initial release and to consider adding updates including refined astrometry and additional systems.

1. Introduction


The knowledge about the dynamics of asteroid satellites is in demand for space missions. It is also needed for the theory of the formation and evolution of the solar system. This knowledge is obtained by building satellite motion models based on observations. The models have the form of satellite ephemerides, which allow the relative position of objects to be computed for whatever times in the future and past; they are also needed to plan new observations and validate the results obtained. Furthermore, ephemerides allow the time intervals and circumstances of mutual occultations and eclipses in asteroid systems with satellites to be calculated in advance. Photometric observations of these events are needed to provide additional data on mutual satellite positions.

In this work we develop a tool for providing ephemerides of asteroid satellites to any user online via the Internet. The tool computes Keplerian orbits of satellites with the parameters of the ephemerides inferred from all available observations (Emelyanov, Drozdov, 2020).

We explain the choice of satellites for ephemerides in the next section.

We compute the ephemerides of 62 asteroid satellites and provide them online via the ephemeris server. The server offers remarkable features including the detection of mutual occultations and eclipses of a satellite with their asteroid.

Our ephemeris is based on observations that have errors, which results in that the ephemeris have errors too. We pay special attention to obtaining estimates of the ephemeris errors. These estimates are obtained by generating 1000

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samples of the orbital parameters by the Monte Carlo method using the covariance matrix of parameters as described in the paper (Emelyanov, 2010).

This paper has the following layout. In the first part we explain the selection of the satellites with motion models incorporated in our ephemeris server. In the second part we describe the tools that we provide for deriving the ephemerides of asteroid satellites. The third part is dedicated to the ephemerides of mutual occultations and eclipses in asteroid-satellite systems. The following sections describes the estimates of the uncertainties of the predicted satellite positions and how they can be obtained via the ephemeris server.

2. Asteroid satellites included in the ephemeride service

We selected the satellites for which suitable motion models are available and perturbing factors can be neglected.

In our previous study (Emelyanov, Drozdov, 2020), we analyzed perturbing factors and evaluated the suitability of the Keplerian model. We reproduce here some of the results. The perturbing factors considered are : the non-sphericity of the primary, the gravitational influence of the Sun and other satellites. Our estimates show that short-period perturbations due to these factors are completely small and negligible. Perturbations from the Sun may be significant only for several satellites. We published the values of the parameter characterizing these perturbations in (Emelyanov, Drozdov, 2020). Long-period perturbations due to the oblateness of bodies are absent. Only secular perturbations remain, which appear as the precession of the orbit around some fixed axis. The position of this axis is determined from observations. To determine the coordinates of this axis the period of observations has to be comparable with the precession period. These perturbations are significant only in the systems of 22 Kalliope, 45 Eugenia, and 87 Sylvia. Our analysis shows the orbital precession for the last two satellites to be too small to allow the determination of the coordinates of the symmetry axis of the primary. An analysis of 22 Kalliope (Emelyanov et al., 2019) shows that the axis of the precession cone lies in the range of 1.5 degrees and our attempts to determine the precession from observations were unsuccessful. Further details can be found in (Emelyanov et al., 2019).

Asteroids with two satellites having noticeable mutual attraction are excluded from consideration.

Our program for computing the orbits in the ephemeris server can adopt any model of precessing elliptical satellite motion with no special action required for this.

We ultimately selected only those satellites for which the adopted Keplerian model of motion is suitable and disturbing factors can be neglected. We actually selected the orbits of 62 asteroid satellites, 13 of which belong to the main belt, two asteroids are Trojans moving in the orbit of Jupiter, and the remaining ones are trans-Neptunian objects.

We calculate the ephemerides using the orbital parameters determined in our previous study and published by Emelyanov, Drozdov (2020). These parameters are available in electronic form and can be taken from the file attached to this cited paper as Supplementary material.

3. Features of the proposed asteroid satellite ephemeride server

The asteroid satellite ephemeris server is a computer program that can be launched by a user from the Internet at <http://www.sai.msu.ru/neb/nss/html/multisat/nssAsthe.htm> or <http://nsdb.imcce.fr/multisat/nssAsthe.htm>

The Cartesian coordinates of the satellite relative to the asteroid at given times are calculated using the known parameters of the Keplerian orbit that we determined earlier from astrometric observations (Emelyanov, Drozdov, 2020).

The heliocentric coordinates of the asteroid are calculated with our special program EMEASTER from the geocentric angular coordinates of the asteroid obtained with the Miriade ephemeris server (IMCCE, France) available at <http://vo.imcce.fr/webservices/miriade/?> forms.

The user can choose one of the 62 satellites on the request page, set the necessary parameters, send the request and receive the satellite ephemeris on a separate page. The request is made by filling the appropriate data entry fields in several menus.

The user manual with a detailed description of the features of the server is available via the link given on the main page of the server. This paper is not a proper place for the user manual, but it would be good to learn some of the important features of the server before starting to use it.

The ephemeris is produced on a separate page that appears when the request is submitted. At the top of that page some data about the system is displayed, including the satellite orbit parameters followed by the ephemeris table.

The user selects one of four ephemeris types:

- ephemeris with a constant time step,
- ephemeris in 1 day step only for the times when the inclination of the orbital plane (the tilt) to the direction toward the Sun or observer is such that mutual occultations and eclipses are possible,
- ephemeris with the time step selected by the program only for the times when mutual occultations and eclipse occur,
- ephemeris for the times of maximum satellite elongation.

The user can choose a point on the Earth (IAU observatory code) for which ephemeris is displayed. The output table gives the right ascension and declination of the asteroid and various satellite coordinates (topocentric and heliocentric) relative to the primary component for the time instants of the ephemeris.

Two special features of the ephemeris server are most important. The first is that the server can output the times of the beginning, the maximum phase, and the end of each mutual occultation and eclipse in the asteroid system with the satellite. These times are necessary for organizing and performing photometric observations of the events.

The second is the output of the times of maximum separation between the satellite and asteroid (maximum elongation) when the conditions are most favorable for astrometric observations.

The purpose of obtaining asteroid satellite ephemeris can be to compare it with the results of new observations. Inevitable deviations of the relative satellite coordinates from its ephemeris provide very valuable information, which should be used to further adjust the satellite motion model.

4. Circumstances of mutual occultations and eclipses of asteroid and satellite

Photometry of asteroids with satellites during their mutual occultations and eclipses is an efficient source of positional data about the satellites. However, these events are rare and observers and researchers look for each such occasion.

It is clear that the geocentric and heliocentric directions pass through the plane of the satellite orbit twice per revolution of the asteroid around the Sun, and these configurations lead to the events considered. However, such cases are rare for trans-Neptunian objects (TNO) with long orbital periods.

Our newly developed ephemeris server provides the circumstances of all mutual occultations and eclipses in asteroid-satellite systems for the time interval 2021–2025. According to our calculations, mutual occultations and eclipses are to occur in 21 of the 62 systems included in the ephemeris server.

For observations of mutual occultations and eclipses of the asteroid with satellite it is important to know the times of the beginning and end of each phenomenon. It is this data that the ephemeride server provides for each phenomenon. The time intervals of a mutual occultation and mutual eclipse may overlap. The duration of each phenomenon can range from several minutes to 3 hours and no phenomena are to occur for many satellites in the time interval from 2021 to 2025 inclusive.

To determine the times of the phenomena our program calculates the topocentric or heliocentric angular distance between the centers of the bodies and compares this distance with the sum of the radii of the disks of the bodies, which are assumed to be spherical.

It would be interesting to see an event schedule for each satellite as illustrated in Fig. 4.

During some events the asteroid may be located in the sky close the sun making observations difficult to perform. The thick and thin lines in Fig. shows the graphs of the time intervals, when phenomena occur without and with interference from the sun, respectively. The adopted criterion of the interference from the sun is that the topocentric angular distance between the asteroid and the sun is less than 50 degrees.

The graphs show that for some satellites mutual occultations and eclipses occur almost every year during 10 months. For other satellites, these phenomena are rare. Graphs allow you to select an object and plan observations.

The ephemeris tool used for predicting the events gives us the most probable satellite positions corresponding to the observations used to derive the ephemeris. However, the prediction of the events may be unreliable because of the limited precision of the ephemerides. We estimate the precision of our ephemerides as described in the special sections 6 and 7 below. Based on these estimates we attribute a quality index Q for the resulting event predictions and provide it in Table 1. See the explanation of the meaning of the index Q in the caption to the table.

More detailed estimates of the accuracy of phenomena prediction can be obtained using the proposed server as described at the end of the Section 7.

Note that we give overestimates predicted circumstances of mutual events for the reasons described in the Section 6 below.

Asteroid satellite ephemeride

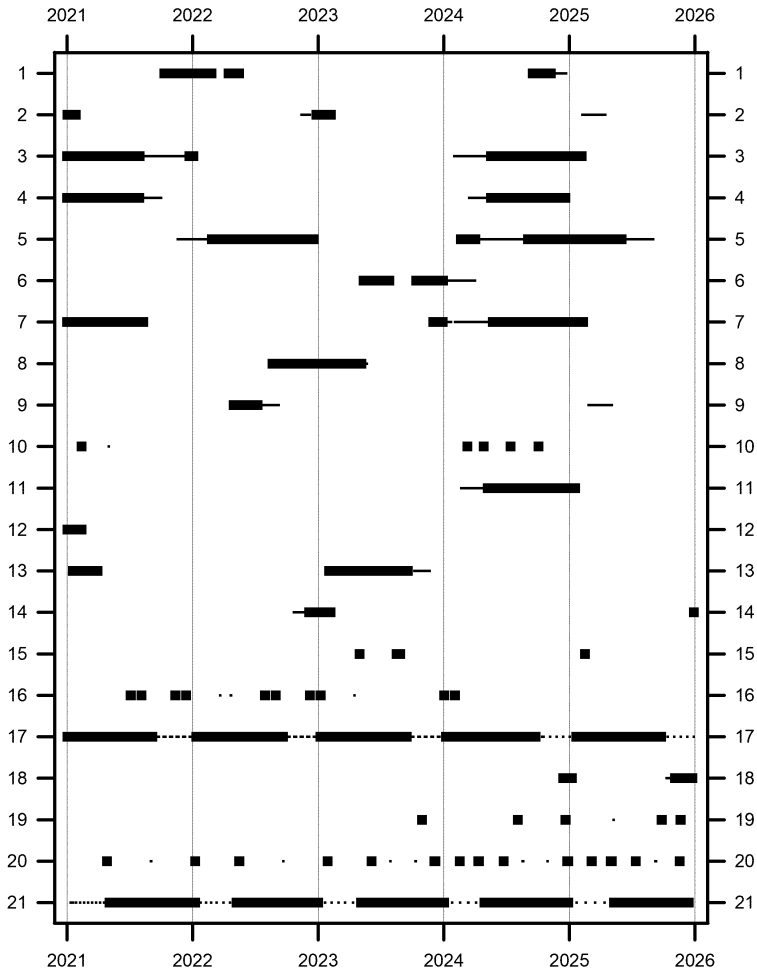


Figure 1: Periods of the mutual occultations and eclipses in the systems of asreroids with satellites. Here N is the number of the system in Table 1.

5. Comparison with the results of Grundy et al. (2018)

Grundy et al. (2018) calculated the circumstances and ephemerides of mutual occultations and eclipses in the system of asteroid (617) Patroclus, a Trojan in the orbit of Jupiter, and its satellite Menoetius. We also calculated the ephemerides of the phenomena in this system for the same dates as in (Grundy et al., 2018) and can now compare results and check our calculations.

The parameters of our solution for the orbit of the satellite Menoetius includes eccentricity, which we find to be 0.0051 ± 0.0068 (1 sigma) based on observations. This is a formal solution with the range including zero value. Grundy et al. (2018) computed the ephemerides of the phenomena for the case of circular orbit. For our comparison to be correct we had to compute a special solution inferring the parameters of the circular orbit of the satellite from observations and calculating the ephemerides for this case.

We use our tool to calculate the times of beginning and end of the phenomena, as well as their duration, for all the cases given in (Grundy et al., 2018). We then determine the differences between our results and those of Grundy et al. (2018) including the durations of the phenomena. Figs. 2 and 3 shows the results of our comparison for mutual occultations and mutual eclipses, respectively.

As is evident from the figures, the two sets of calculations compared yield ephemerides that are very close to each other. Small differences can be explained by the fact that in our algorithm ephemeris times are in increments of 7 minutes and do not coincide with the exact times of the beginning and end of the phenomena. The comparison leads us to conclude that both our satellite motion model and the algorithm of detection of mutual occultations and eclipses

Table 1

Asteroids and satellites sorted by number N in the Fig. 1. Semi-major axis a in astronomical units is indicative of the type of the asteroid. Q is the ephemeris quality established in the following way: if the errors measured along the apparent relative track of satellite exceed the value 0.5 of the semi-major axis of the orbit on the periode 2022–2025, then the ephemeris is considered unreliable and the sign "-" is assigned to the index Q . The sign "!" is assigned to the index Q in opposite case. The values r_m , r_s , σ_{mut} are the average radius of the asteroid, radius of the satellite and the uncertainty of the sum of these radii correspondingly adopted in this work (see the Section 8).

N	Asteroid	Satellite	a, au	Q	r_m	r_s	σ_{mut}
1	(22) Kalliope	Linus	2.91	!	91.414	14.000	14.986
2	(45) Eugenia	Petit-Prince	2.72	!	99.427	3.500	22.885
3	(87) Sylvia	Remus	3.48	!	147.309	5.300	26.432
4	(87) Sylvia	Romulus	3.48	!	147.315	5.400	26.569
5	(90) Antiope	S/2000(90)1	3.15	-	44.123	41.900	1.738
6	(107) Camilla	S/2001(107)1	3.49	!	102.331	8.000	30.883
7	(107) Camilla	S/2016(107)1	3.49	-	101.393	1.750	30.423
8	(121) Hermione	S/2002(121)1	3.44	!	93.500	16.000	3.400
9	(283) Emma	S/2003(283)1	3.04	-	67.350	4.500	2.762
10	(379) Huenna	S/2003(379)1	3.13	!	43.735	2.900	1.324
11	(617) Patroclus	Menoetius	5.22	!	53.000	49.000	7.433
12	(624) Hektor	Skamandrios	5.25	-	92.000	6.000	5.220
13	(702) Alauda	Pichi unem	3.20	-	100.980	1.755	2.363
14	(762) Pulcova	S/2001(762)1	2.16	-	70.860	9.500	3.584
15	(3749) Balam	S/2002(3749)1	2.24	-	1.975	0.920	0.215
16	(26308) 1998 SM165	S/2001(26308)1	47.41	!	134.000	40.500	15.652
17	(42355) Typhon	Echidna	38.27	-	81.000	44.500	4.610
18	(80806) 2000 CM105	companion	42.36	-	80.000	60.500	10.030
19	(148780) Altjira	companion	44.11	!	123.000	110.500	25.495
20	(469705) 2005 EF298	companion	44.31	!	69.000	52.500	8.670
21	2003 QR91	companion	46.48	-	103.500	94.500	17.903

are correct and very close to those of Grundy et al. (2018).

6. Some considerations for estimates of ephemeris precision

To simulate mutual phenomena, we made assumption that both bodies are spheres and their visible images are uniform disks. We know, however, that the real bodies are non-spherical. The circumstances of mutual phenomena might change if this fact is taken into account. However, for most of the considered asteroids with satellites, there is no information about the orientation of these elongated bodies at the moments of the ephemeris. In the cases when there are data on asteroid rotation, they are not reliable. Thus, we have no choice but to follow the assumptions made.

Observation errors are random values. This results in that the ephemeris based on these observations are also random. The least squares method gives us the most likely ephemeris. However, because of ephemeris uncertainties, real coordinates may significantly differ from those obtained from the ephemeris. It is inevitable that predictions of mutual phenomena have limited certainty. If, during the observation, the phenomenon did not take place, it does not mean that the observers wasted their time as such information is also valuable because it can be used to refine the orbit. On the contrary, if we do not observe the phenomenon that has taken place, we will lose valuable information. Thus, we prefer to give overestimated predicted circumstances of mutual events. Due to random observation errors, there is a possibility that real events could not be predicted. Such situations are inevitable. When some mutual phenomena are not predicted but observed this carries additional valuable data that serve to refine the ephemeris.

It is evident that the results of observations have errors. Each measured coordinate is a random value. Hence, the parameters of the orbit found from observations are random too. The ephemeris obtained from such an orbit are also random. The least-squares method gives us the most likely ephemeris. But there are other possible random realizations of ephemeris. We give variations to the parameters by the Monte Carlo method. Using the parameter covariance matrix, we get the ephemeris variations. Such variations provide us with the error estimates for the ephemeris. For this end,

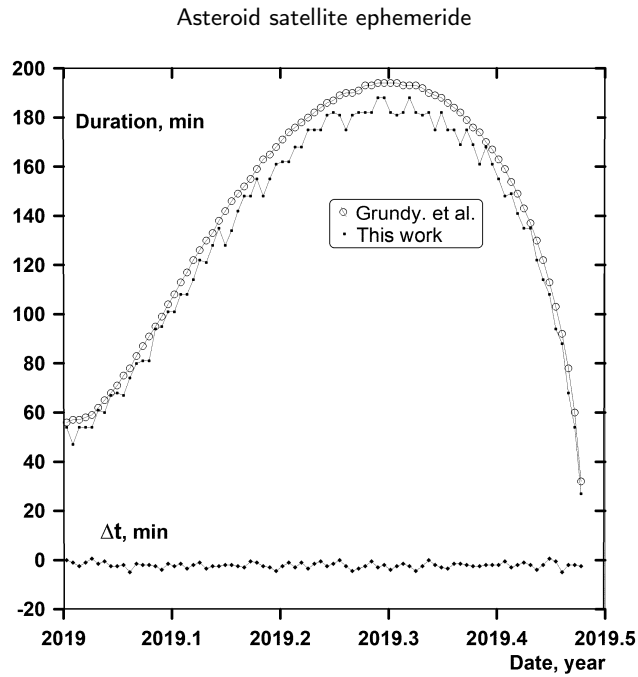


Figure 2: Comparison of the ephemerides of the mutual occultations in the system of asteroid (617) Patroclus with the results of Grundy et al. (2018).

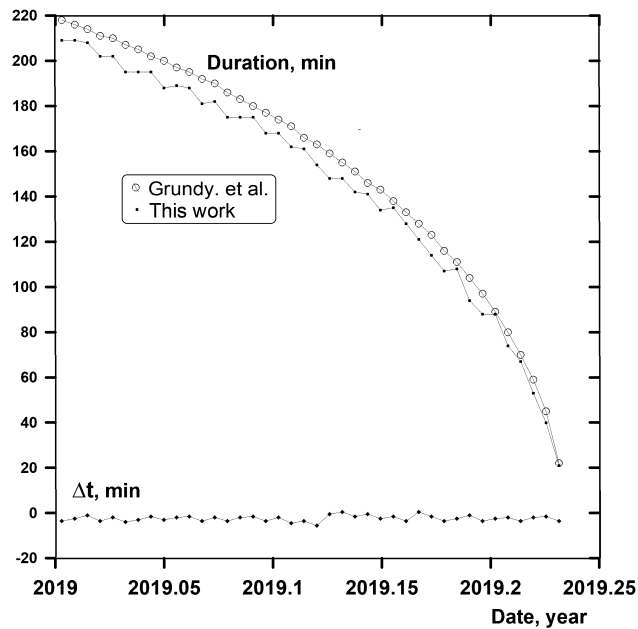


Figure 3: Comparison of the ephemerides of the mutual eclipses in the system of asteroid (617) Patroclus with the results of Grundy et al. (2018).

we follow the method described by Emelyanov (2010).

We do not take into account the solar phase that also leads to some little overestimation of the event prediction.

We give an ephemeris based on the observations used. See the table given in the supplementary material to our article (Emelyanov, Drozdov, 2020). For 35 of the 62 satellites, we have by 12 or less measured positions for each satellite. This is with seven refined parameters. For most satellites, observations cover only a small part of their orbits. Under these conditions, we cannot count on reliable ephemeris. New observations are necessary which could be used

to refine our ephemeris. Our ephemeris server could be appropriate for this purpose.

7. Estimates of ephemeris precision

The observations from which the asteroid satellite motion models were derived have errors and span short time intervals for some satellites. These circumstances lead to errors in the resulting ephemeris. Estimating the errors of an ephemeris is a difficult task. The least-squares method used to determine the parameters of satellite orbits also yields estimates of parameter errors, which, however, do not allow determining the errors of the ephemeris.

Emelyanov (2010) offers three methods for evaluating ephemeris errors arising due to the errors of the underlying observations. In any case, one needs to know either the accuracy of the observations or the covariance matrix of parameter errors. The required matrix is computed in the process of determining parameters from observations. When applying the proposed methods it is assumed that observation errors are random in nature. We have no choice but to adopt this hypothesis. We also assume that the Keplerian motion model corresponds to the accuracy of observations. Any actual violation of the assumptions made can only increase the errors of the ephemerides.

Given these circumstances, we evaluate the accuracy of the ephemerides by the method proposed in (Emelyanov, 2010) using the covariance matrix of parameter errors. The method gives us estimates of the accuracy of satellite coordinates at any given time instants. It is very important how the accuracy estimates are presented. First, the ephemeris errors along the satellite trajectory are more important. Secondly, absolute coordinate errors should be related to the size of the satellite orbit.

The current asteroid satellite ephemeris server provides accuracy estimates in the form of a table that contains errors of the satellite's apparent positions projected along and across the path of apparent relative motion. A special column of the table gives the angular value of the apparent semimajor axis of the satellite's orbit. Finally, the server provides errors along and across the trajectory, expressed in the units of the semi-major axis.

In order to have a clear idea of the accuracy of the ephemeris, the server provides a graph of the errors along the trajectory, expressed in the units of the semi-major axis. A link to this graph appears as soon as any request is made for the ephemeris of a satellite.

The server uses a special sequence of time instants. It takes N_{rev} points on one revolution of the satellite. Then it makes a roll for P_{jet} days, again tracks the satellite's rotation, and so on. This is necessary to track the behavior of the error on one revolution and show the ephemeris errors over a long time interval. The server selects N_{rev} and P_{jet} individually for each satellite. The time interval spans from the time of the first observation to January 1, 2025.

Examples of graphs for assessing the accuracy of ephemerides are shown in Figs 4, 5, 6. As is evident from Figs. 4 and 5, the ephemeris errors for these satellites are much smaller than the sizes of their orbits. These ephemerides can be used at least over the time interval until 2025. For satellite Pichi unem of the asteroid (702) Alauda the ephemeris becomes useless after 2015, as the satellite position errors become as large as the size of its orbit. This is so because the observations cover a very short time interval.

We summarize the uncertainties of the ephemerides for all 62 asteroid in a table, where for each of the 62 satellites, the maximum uncertainties of the predicted satellite positions are given for each of three time intervals: Jan 1, 2022 - Dec 31, 2023, Jan 1, 2023 - Dec 31, 2024, Jan 1, 2024 - Dec 31, 2025. The table is attached to this paper as Supplementary material and is available online. Excerpts from this table are given here in Table 2. Satellites with the best ephemerides are listed at the top of the table, and those with the poorest ephemerides are listed at the bottom. As is evident from the table, the accuracy of the ephemeris depends significantly on the length of the time interval and the number of observations used to determine the orbit.

Estimating ephemeris uncertainties for predicted mutual phenomena requires special treatment. We call here an apparent position and apparent velocity of the satellite always counting them relative to the main body. The error of the moment of the phenomenon is determined by the error of the relative position of the satellite calculated along the apparent trajectory. The position error across the trajectory determines the depth of the phenomenon, that is, the drop of the light flux from the system. With a certain level of error, the phenomenon may not occur at all. We estimate the uncertainty for mutual phenomena in the following way. First, the uncertainty for the moment of the phenomenon σ_t is calculated using the formula

$$\sigma_t = \sigma_a / V_a$$

where σ_a is the uncertainty for the relative position along the apparent trajectory, V_a is the apparent velocity of the satellite along the apparent trajectory. Due to an error in the relative position of the satellite at the time of the predicted maximum of the phenomenon, the phenomenon may not occur. We calculate the relative distance of the satellite for

Asteroid satellite ephemeride

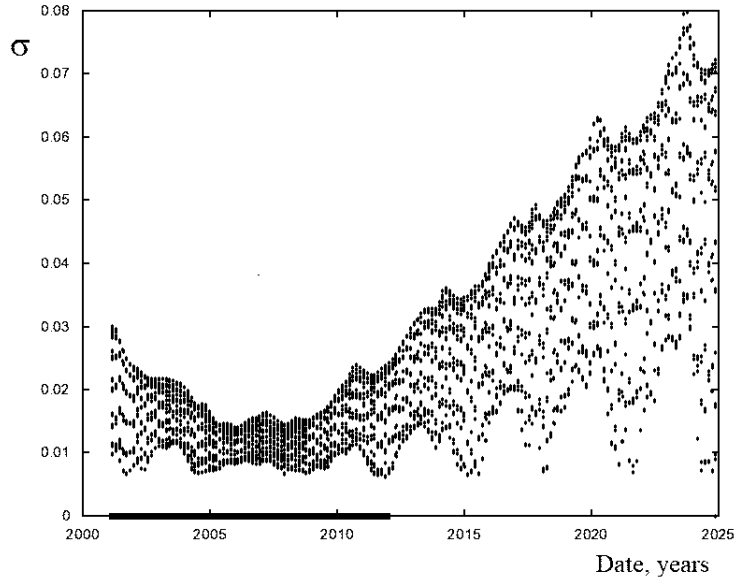


Figure 4: Uncertainty - standard deviation (σ) - of the ephemeris of asteroid (87) Sylvia's satellite Remus, calculated along the apparent relative trajectory of the satellite and expressed in the units of the apparent semi-major axis of the orbit. The bold line on the axis shows the observation time interval.

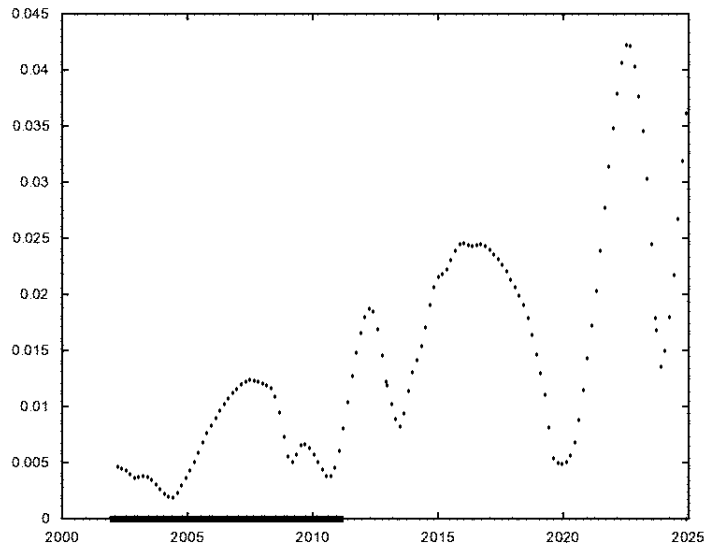


Figure 5: Uncertainty - standard deviation (σ) - of the ephemeris of asteroid 2000 CF105 satellite, calculated along the apparent relative trajectory of the satellite and expressed in the units of the apparent semi-major axis of the orbit. The bold line on the axis shows the observation time interval, which spans approximately 9 years.

each random ephemeris sample. Each time we record whether an event has occurred or not. For all the samples, the probability P_{ev} of whether the event takes place is calculated. This is one more estimate for the uncertainty in predicting the phenomenon. The values σ_t and P_{ev} are given as part of the output of the ephemeris. To get this, it is necessary to choose the first year of the mutual event ephemeris 2021, since the uncertainties were calculated in advance beginning from 2021.

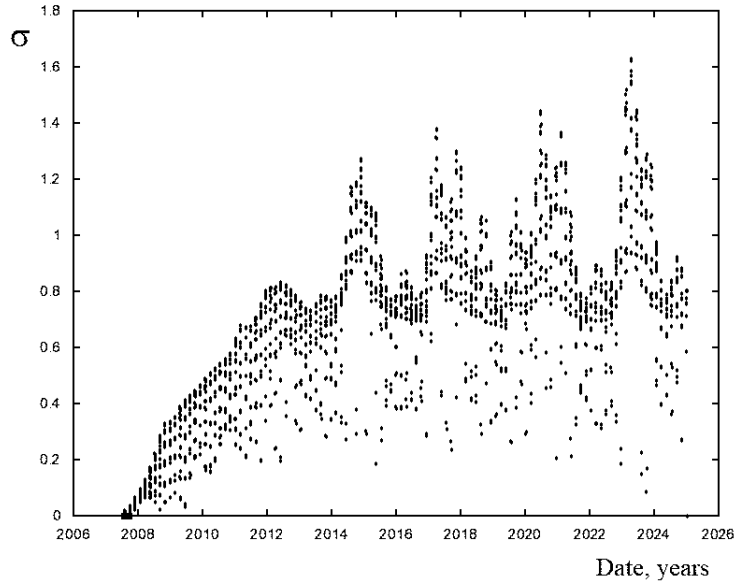


Figure 6: Uncertainty - standard deviation (σ) - of the ephemeris of asteroid (702) Alauda's satellite Pichi unem, calculated along the apparent relative trajectory of the satellite and expressed in the units of the apparent semi-major axis of the orbit. The bold line on the axis shows the observation time interval, which spans approximately 50 days.

8. Taking into account the shape and size of bodies when calculating the ephemeris of mutual occultations and eclipses

First of all, it should be noted that by now there are very few data on the shapes of asteroids with satellites and about rotation of these bodies. Thus, it is hard to evaluate their influence upon ephemeris. Additional difficulties arise from complex dependence of moments of the phenomena on both the orbital and physical parameters of the bodies.

The primary of an asteroid-satellite system can have non-spherical shape. It can be approximated by the triaxial ellipsoid with semiaxes a_m , b_m , c_m . At the current level of observations, shapes of the satellite cannot be surely determined so we consider it to be spherical.

Suppose that the orientation of the main body in space is known at the time of the ephemeris. Then the primary's shape can be projected on image plane, that is on a plane perpendicular to the line of sight. Projection of the triaxial ellipsoid (its shadow) will always be an ellipse. We denote the semiaxes of this ellipse on the image plane as a and b .

Let the X axis be directed from the center of the primary to the East, with Y axis pointing to the North. We denote the angle between the axis of the ellipse and the direction from the primary to the satellite as φ . The configuration is shown in Fig 7.

We introduce the coordinate system x, y with the origin in the ellipse's center, the x axis being directed along the semiaxis a . At the beginning or end of the phenomenon, the projection of the satellite will look like a circle touching the ellipse at some point with the coordinates x, y . Let x_s, y_s be the coordinates of the satellite center. With given a, b, x_s, y_s , the coordinate x of the touch point is determined from the algebraic equation of the fourth degree

$$x \left[\left(1 - \frac{b^2}{a^2}\right) a \sqrt{a^2 - x^2} + by_s \right] = a \sqrt{a^2 - x^2} x_s. \quad (1)$$

The value $0 \leq x \leq a$ can also be calculated by using the dichotomy method to solve the equation

$$f(x) = x \left[\left(1 - \frac{b^2}{a^2}\right) a \sqrt{a^2 - x^2} + by_s \right] - a \sqrt{a^2 - x^2} x_s = 0.$$

with $f(0) < 0$, $f(a) > 0$.

Table 2

The uncertainties of the ephemeris predictions (Excerpt of the full table available online). Maximum values are selected during each year. The errors are measured along the apparent relative track of the satellite and normalised to the current value of the apparent semi-major axis. Here T is the time interval of observations (in years) and N is the number of satellite positions used to fit the orbital parameters.

Asteroid	Satellite	2022	2023	2024	T	N
2001 QW322	companion	0.00715	0.00790	0.00782	16.137	89
(22) Kalliope	Linus	0.00966	0.01011	0.01008	16.740	114
(87) Sylvia	Romulus	0.01043	0.01154	0.00979	10.816	113
(107) Camilla	S/2001(107)1	0.01181	0.01116	0.01211	15.444	106
(525462) 2005 EO304	companion	0.01501	0.01427	0.01142	12.191	53
(88611) Teharonhiawako	Sawiskera	0.01883	0.01294	0.01317	9.096	18
(136199) Eris	Dysnomia	0.01944	0.01975	0.02007	9.392	14
2006 BR284	companion	0.01545	0.01979	0.02093	10.975	68
(508788) 2000 CQ114	companion	0.01376	0.01428	0.01526	11.828	8
...
(624) Hektor	Skamandrios	0.89811	0.87476	0.78748	5.325	13
2003 QR91	companion	0.83758	0.83207	0.80410	0.687	5
(283) Emma	S/2003(283)1	1.01608	0.79035	0.79413	1.454	26
(702) Alauda	Pichi unem	1.20614	1.63145	0.97547	0.133	9
(762) Pulcova	S/2001(762)1	1.23618	1.04861	0.95327	2.938	10
(3749) Balam	S/2002(3749)1	1.28227	1.45400	1.25356	1.430	11

The ephemeris provide us with the functions $x_s(t)$, $y_s(t)$. For the moments of the beginning or end of the phenomenon we have

$$(x - x_s)^2 + (y - y_s)^2 = r_s^2, \quad (2)$$

where r_s is the satellite image radius. At these moments, the distance between the centers of the bodies, d , is determined by the relation

$$d^2 = x_s^2 + y_s^2.$$

In fact, the angle of rotation of the main body is unknown. We could not find in publications the models where the rotation angle could be obtained for any moment of time. The only thing that can be found in publications is the coordinates of the rotation axis of the main body for some asteroids with satellites. The reliability of these data is low.

The shape, that is the semiaxes of the approximating ellipsoid, are known only for some of the objects in consideration. For the objects listed in the Table 1, the semiaxes a_m , b_m , c_m are known only for the first six ones. Therefore, we have to consider the angle φ in Fig 7 as a random value varying in the range $0 \leq \varphi < 2\pi$, considering the direction from the primary to the satellite to be constant.

We have calculated the average value of d and the corresponding rms for those six asteroids with satellites for which the semiaxes a_m , b_m , c_m are known. Since the orientation of the bodies is unknown, to overestimate the values for $d(\varphi)$, we took a to be equal to the maximum of the semiaxes a_m , b_m , c_m , while b is equal to the minimum of them. With the given value for φ , the value for d was calculated by iterations using the relations (1) and (2), assuming in the zero approximation $d = \sqrt{x^2 + y^2} + r2$ and taking into account the relations $x_s = d \cos(\varphi)$, $y_s = d \sin(\varphi)$. Then, the averaged value \bar{d} and its uncertainty σ_d were calculated. In fact, we assumed that the main body is a sphere with the radius $r_m = \bar{d} - r2$ determined with the uncertainty $\sigma_m = \sigma_d$.

The initial data for asteroids with satellites were taken from (Johnston, 2020). This database gives also the values for the satellite radii, r_s , and the corresponding uncertainties, σ_s . For the systems for which the sizes of the primary a_m , b_m , c_m are given in (Johnston, 2020), we have defined r_m and σ_m in the way described above. For other systems, r_m and σ_m were taken directly from (Johnston, 2020). Then we used the sum $r_{mut} = r_m + r_s$ as a random variable with uncertainty $\sigma_{mut} = \sqrt{\sigma_m^2 + \sigma_s^2}$. The values r_m , r_s , σ_{mut} adopted in our work are given in the Table 1.

In most cases when mutual occultations and eclipses in the systems of asteroids with satellites are observed, in the beginning and in the end of the event the components either approach or move away from one another. Errors in

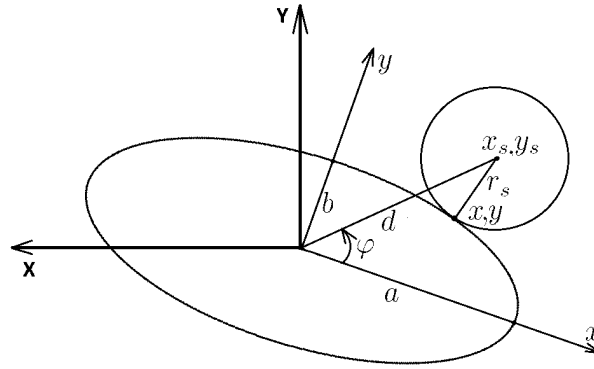


Figure 7: Apparent configuration of asteroid and satellite at the moment of the beginning of mutual occultation.

the moments of phenomena arise due to the errors of orbital parameters, as well as because of the uncertainty of the mutual distance σ_{mut} . That is why the ephemeris server generates the value σ_{dim} which is calculated using the relation $\sigma_{dim} = \sigma_{mut}/V_{along}$, where V_{along} is the relative velocity of the satellite along the apparent relative trajectory. It is the uncertainty σ_{dim} which arises because of uncertainty in the sizes and shapes of the components. The value for σ_{dim} is given separately in the output of the server.

Note that, when calculating the probability P_{ev} of whether the event takes place or not (see above), the mutual distance between the bodies was taken as a set of 300 samples of a random variable with uncertainty σ_{mut} . Thus, this probability takes into account not only errors of the model arising from the errors of observations but also the errors in the sizes and shapes of the objects.

As result, if the year 2021 is chosen as the initial date of the ephemeris of mutual phenomena, in the line with the moment of apparent approach of the satellite to the asteroid, near the “max” code, the ephemeris server will output three values: $\sigma_t, \sigma_{dim}, P_{ev}$.

9. Conclusions

We developed an asteroid satellite ephemeris calculation tool — an ephemeris server, which we propose here for the use by the community. Ephemerides are calculated with motion models based on all available observations. The satellite orbit parameters were found earlier and published in (Emelyanov, Drozdov, 2020).

The server includes motion models of 62 asteroid satellites. When requesting an ephemeris the user can choose from different ways of setting the ephemeris time instants and various types of satellite coordinates. The ephemerides of the satellites include the right ascension and declination of the asteroid. The server also allows the user to determine the conditions of visibility of an asteroid at any observatory from IAU list.

Ephemerides are needed for new observations and inevitable deviations of the results of new observations from the computed ephemeris provide valuable information for further refinements of satellite motion models.

Of particular value are photometric observations of mutual occultations and eclipses in asteroid systems with satellites. These phenomena occur rarely and not in all systems. The ephemeris server gives the beginning and end time instants for each such event. In this study we calculate the time periods of the phenomena and show the corresponding graphs for the time interval from 2021 to 2025. The graphs allow the user select an object and plan observations. We found that mutual occultations and eclipses are to occur for 21 of the 62 satellites considered.

To test the method for determining the ephemeris of asteroid satellites, we compared our computed times of mutual occultations and eclipses for the system of asteroid (617) Patroclus and its satellite Menoetius with the results obtained by Grundy et al. (2018). We conclude from the comparison that our satellite motion model and algorithm of detection of mutual occultations and eclipses are correct and yield results that are very close to those of Grundy et al. (2018).

The ephemerides obtained from the server have unavoidable errors. The main source of ephemeris errors is the inaccuracy of observational data. The accuracy of the ephemeris degrades over time. The shorter the observational arc or time interval, the larger the degradation rate. As a result, the ephemeris provided by the server should be considered as the most probable satellite positions found based on available observations.

Estimating the accuracy of an asteroid satellite ephemeris is a particularly difficult problem. We solved this problem using the method described in (Emelyanov, 2010). Thus, the asteroid satellite ephemeris server provides not only the ephemerides but also the uncertainties of the predicted satellite positions.

Our analysis of the precision of the ephemerides for the entire ensemble of 62 asteroid satellites leads us to the following conclusions.

In the time interval until 2025 the ephemerides for 13 satellites are unreliable. They need to be searched for again because we do not know on which side of the asteroid the satellite will be in the near future. For 28 satellites, the errors of the ephemerides until 2025 amount to 0.1-0.5 of the semimajor axis of the orbit. Only 21 satellites have reliable ephemerides with errors not exceeding 0.1 of the the apparent semi-major axis of the orbit. For nine of them the ephemerides are fairly accurate with errors no greater than 0.02 of the apparent semi-major axis of the orbit.

For 9 satellites from the list in the Table 1, the server shows that the probability of detecting a mutual phenomenon does not exceed 0.2. However, you can count on the predicted moment of the phenomenon as the most likely.

Obviously, the low accuracy of the ephemerides of some satellites is due to the fact that their orbits are determined from a small number of observations spanning a short time interval (Emelyanov, Drozdov, 2020) as is evident from Table 2.

Although the accuracy of the ephemerides of asteroid satellites is currently not high, we plan to maintain the functionality of the ephemeris server and improve the accuracy as new observations appear.

This tool is open for more precise orbital parameters and new satellites to be discovered in the future. We hope to work on this initial release and to consider adding updates of astrometry and additional systems.

The proposed tool can be used with more accurate orbital parameters and applied to new satellites that will be discovered in the future. We hope to work on this initial release and to consider further upgrading it by using refined astrometry and incorporating additional systems.

We also plan to publish a detailed description of the method for assessing the accuracy of ephemerides, which we used for asteroid satellites.

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