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# Framework for tolerance analysis of over-constrained mechanisms with form defects

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## Abstract

The tolerance analysis aims to check if the tolerances specified on mechanism components allow to respect the assembly and functional requirements. Several papers illustrate that the form defects have an impact on the mechanism behavior, but most tolerance analysis techniques neglected them. To perform the tolerance analysis, three issues must be addressed: the geometrical deviations modeling, the geometrical behavior modeling and the technique of tolerance analysis. Therefore, this paper presents one contribution for each issue to perform tolerance analysis of over-constrained mechanism with form defect:

- Modal representation of the form defect
- Behavior modeling technique of the contact with form defect
- Tolerance analysis technique with the integration of the different contact types.

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*Keywords:* Tolerance analysis, form defects, contact types

## 1 Introduction

The aim of tolerance analysis is to analyse the influence of geometric variations of the features on the behavior of a mechanical system. It consists to check that geometric tolerances of the components ensure the compliance of a mechanical system in terms of functional requirements. It is thus necessary to integrate the tolerance analysis process in all stages of product lifecycle in order to improve product quality, to reduce its cost and to ensure its reliability. Moreover, tolerance analysis aims to check if the tolerances specified on the mechanism components allow respecting the assembly and functional requirements. Generally, to perform a tolerance analysis, three main issues should be addressed: geometrical deviations modeling, geometrical behavior modeling of a system and mathematical formulation of a tolerance analysis technique.

In any mechanical system, parts may admit form

defects; these form defects could impact its behavior [1, 2, 3], the non-consideration of form errors could lead to noncompliant assemblies even if all parts respect the geometrical specifications. Hence, there is a need to manage all deviations when dealing with tolerance analysis approaches. The geometrical behavior of systems that are generally considered as over-constrained should be modelled differently. The geometrical behavior modeling consists in the description of the interactions between assembled parts or the contacts. Several works have been developed to deal with contact/assembly modeling. Giordano et al. [4] introduced the clearance and deviation domains which are 3D spaces of relative displacements in an assembly. Morse and Zou proposed the GapSpace Model for two-dimensional assemblies [5]. The model used gaps to describe the possible mating clearance conditions between

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features within the assemblies. Davidson and Shah [6] proposed the T-Maps® model to give a 3D simulation of all potential variations of the features in a system. Homri et al. [7] then Teissandier et al. [8] developed polytopes approach to deal with the tolerance analysis of an over-constrained assembly simulation. Lê et al. [9] proposed the concept of gap hull to study the behavior of a planar joint when mating surfaces are with form defects. Schleich et al. [10] proposed to use the Skin Model representation for the assembly simulation of over-constrained systems. This concept of Skin Model was introduced to provide a global description of parts' surfaces [11]. Most of the existing approaches for contact modeling and assembly simulation when parts are without form defects [10, 12, 13] then with form defects [14, 15] are based on optimization. The main objective is to assess the distance between the mating surfaces potentially in contact in order to avoid interferences. Different techniques have been proposed to geometrically characterize the form defects. Form defects can be characterized using Discrete Fourier Transform [16], morphing mesh-based approach [17], Skin Model [10, 11], Discrete-Cosine-Transformation [18], Discrete Modal Decomposition [19] and Metric Modal Decomposition [14].

This paper presents a global framework for the tolerance analysis of over-constrained systems with form defects. One contribution is developed for each issue of tolerance analysis. A modal representation of a form defect is presented in section 1, the section 2 focuses on the geometrical behavior modeling of a contact with form defects and a mathematical definition of a tolerance analysis technique with the integration of different contact types is proposed in section 3. One application of the framework is given in section 4, followed by a conclusion at the end.

## 2 Modal representation of a form defect

In our previous works [14, 15], a new technique to characterize the form defects has been proposed. For each nominal surface, several elementary defects could be defined. Each elementary defect is mathematically modelled by a modal vector. Elementary form defects are obtained thanks to the translation of all points of a nominal surface. The amplitude of this translation is associated with the modal vector. The final surface is a weighted sum of all elementary defects by their respective amplitudes. These amplitudes are generally measured and expressed in millimetre. The elementary modes are defined from the manufacturing process. They are defined by a designer based on his/her manufacturing knowledge. One modal vector can be expressed as (1):

$$\mathbf{f}_{i,M} = \sum_{k=0}^n \lambda_k \mathbf{f}_{k,M,i,j} \quad (1)$$

Where  $\mathbf{f}_{i,M}$  is a form defect vector that is defined in the point M,  $\mathbf{f}_{k,M,i,j}$  is the  $k^{\text{th}}$  elementary modal vector at the point M of surface j and  $\lambda_k$  denotes the associated amplitude. In this modal representation technique and in order to characterize classical form defects when dimensioning parts, the modal vectors are normalized, as expressed in equation (2).

$$\max \left( \left\| \mathbf{f}_k(M) \right\| \right) = 1 [mm], \quad \forall M \quad (2)$$

Figure 2 depicts the generation of form defects with regard to the modal representation. For each nominal surface in a part  $i$ , a substituted model is associated. This model considers all position and orientation deviations. Then, for the definition of a non-ideal surface, the relation (1) is applied.

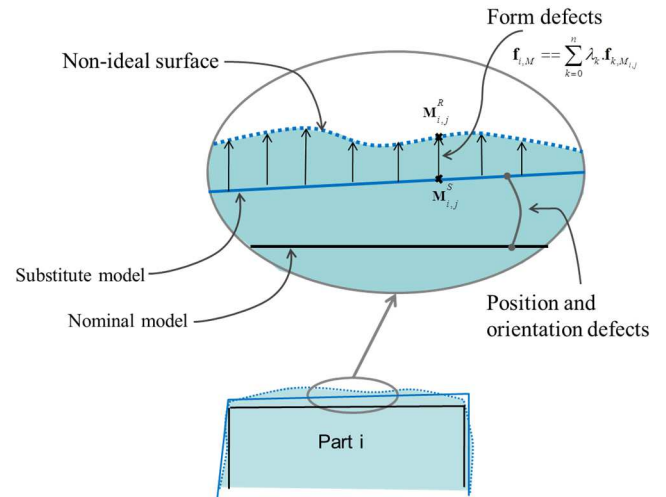


Figure 1. Non-ideal surface representation

## 3 Geometrical behavior modeling of contacts with form defects

The geometrical behavior model considers the surface deviations in each component and the relative displacements between parts with form defects according to the gap. These deviations of the parts' components, the form defects and the gaps are respectively described by the vectors  $\mathbf{x} = \{x_1, \dots, x_n\}$ ,  $\mathbf{f} = \{f_1, \dots, f_n\}$  and  $\mathbf{g} = \{g_1, \dots, g_n\}$ . The behavior modeling involves the definition of the mathematical models that characterize the system behavior. These mathematical models describe the features, the interaction between them and the functional requirements. Globally, there main formulations are defined [13, 14, 15].

- Compatibility equations: constraint parts deviations and gaps. The equalities derive from the different combinations of the admissible displacements of the surfaces in the parts. Globally, the compatibility constraints are denoted by  $C_c(\mathbf{x}, \mathbf{g}) = 0$ .
- Interface constraints: inequalities that characterize the non-interferences between substituted surfaces which are nominally in contact. They limit the gaps between two non-ideal surfaces. Generally, the interface constraints are expressed as  $C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}) \leq 0$ . The interface constraints should be linearized if they are not linear.
- Functional requirements: limit the orientation and location displacements between surfaces, which are in a functional relation. These constraints are generally denoted by  $C_f(\mathbf{x}, \mathbf{f}, \mathbf{g}) \leq 0$ .

In general case, over-constrained mechanisms present different types of contact: fixed, sliding and floating contacts; see Figure 2. These different contacts have to be studied separately when dealing with assembly simulation. Floating contacts have mostly been taken into account. In this section, the three types of contacts are analyzed and modelled. The form defects could impact differently the study of the three contacts. The gap parameters (translations and rotations) characterizing each contact should thus be determined.

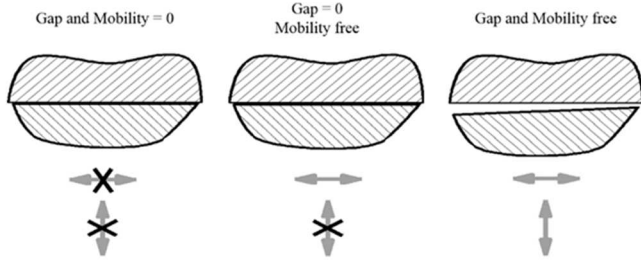


Figure 2. Fixed, sliding and floating contacts.

a) Fixed and sliding contacts

When parts' form defects are considered in fixed or sliding contact analysis, the gap components have generally to be determined. The determination could be based on optimization. The components which are set to zero when parts are rigid without form defects have to be computed in case of non-ideal mating surfaces. The determination of the gap components allows knowing the real position of the mating surfaces permitting the mechanism to be physically functional. To determine the admissible contact configuration of fixed and sliding contacts, the Signed Distance Approach (SDA) [14] has been used. The mathematical definition of the SDA concept is given by (3):

$$\begin{cases} \forall (\mathbf{M}_{k,j}^R, \mathbf{M}_{i,j}^R) \in (S_k, S_i), \mathbf{M}_{k,j}^R \mathbf{M}_{i,j}^R \cdot \mathbf{n} \geq 0 \\ \text{with} \\ \|\mathbf{M}_{k,j}^R \mathbf{M}_{i,j}^R \wedge \mathbf{n}\| = 0 \end{cases} \quad (3)$$

With  $\mathbf{n}$  the normal vector of the nominal surface of the part  $k$ ,  $S_k$  and  $S_i$  represent respectively the non-ideal surfaces of parts  $k$  and  $i$ ,  $\mathbf{M}_{k,j}^R$  and  $\mathbf{M}_{i,j}^R$  are two corresponding points belonging respectively to surfaces  $S_k$  and  $S_i$  as shown in Figure 3.

The determination of the admissible configuration of the fixed and sliding contacts, as discussed, is mathematically achieved based on optimization resolution. The considered optimization problem can be expressed as (4):

$$\begin{cases} \text{Min}_{\mathbf{g}_{slid-fix}} f(\mathbf{g}_{slid-fix}) \\ \text{subject to } \mathbf{M}_{k,j}^R \mathbf{M}_{i,j}^R \cdot \mathbf{n} \geq 0, \forall \mathbf{M}_{k,j}^R \in S_k, \forall \mathbf{M}_{i,j}^R \in S_i \end{cases} \quad (4)$$

Where  $\mathbf{g}_{slid-fix}$  denotes the gap in the fixed or sliding contacts. The contact analysis results from the computation of the clearance torsor modeling the gap [14, 15, 21] and that represents the relative displacements between the mating surfaces. This analysis depends globally on the surface class.

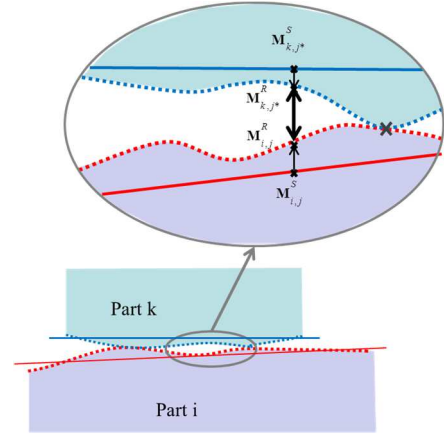


Figure 3. Fixed or sliding contact behavior modeling- illustration

b) Floating contacts

This type of contacts is mostly considered in the existing approaches for assembly simulation [4, 7, 10, 12, 13, 14]. Based on the geometrical behavior modeling by sets of constraints and particularly the interface constraints, non-interferences between mating surfaces could be assessed. These interfaces constraints are sensitive to the consideration of parts' form defects. The figure 4 illustrates a floating contact modeling. According to this figure, the set of the interface constraints can mathematically be expressed as (5).

$$\mathbf{g}_{float-S_k/S_i} \cdot \mathbf{n} \leq \Delta g + f_1 + f_2 \quad (5)$$

Where  $\mathbf{g}_{float-S_k/S_i}$  defines the gap parameters between the non-ideal surfaces  $S_k$  and  $S_i$ ,  $\Delta g$  is the nominal gap between the nominal surfaces and  $f_1$  and  $f_2$  denotes the form defect amplitudes respectively, to the nominal surface of part  $k$  and part  $i$ .

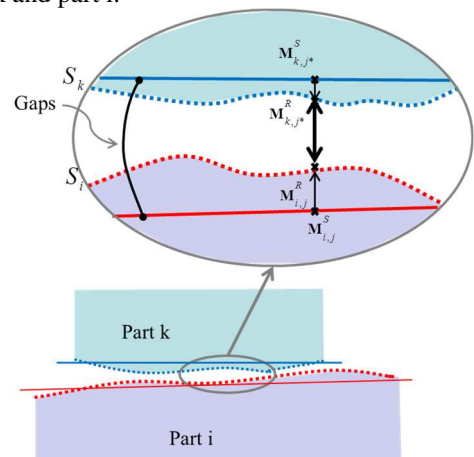


Figure 4. Floating contact behavior modeling - illustration

c) Synthesis

This section gives a brief synthesis on the geometrical behavior of contact modeling. When form defects are considered in a tolerance analysis framework, contacts should be studied according to their types. Three different

types of contacts are analysed. The behavior of fixed and sliding contacts is modelled by optimization and thanks to the signed distance concept. For the floating contacts, a set of constraints characterizing the non-interferences between mating surfaces should be defined. These linearized constraints (in general case) should be respected. The behavior modeling of different contacts is integrated in a global framework for tolerance analysis of over-constrained systems with form defects. Two probabilities of failure, relative to the assembly and relative to the functionality of the system are thus computed.

#### 4 Tolerance analysis framework

This section focuses on the proposition of a tolerance analyse framework considering the previous two issues: form defect modeling by modal representation and contact behavior modeling considering different types of contacts in an over-constrained mechanical systems. The proposed approach aims to assess the mechanism assembly and its geometrical functioning and that by computing two different failure probabilities by optimization and Monte Carlo simulation:

- Functionality assessment: “for all admissible gap configurations of the mechanism and for all acceptable deviations, there exists a functional characteristic such that the geometrical behavior and the functional requirements are respected” [13, 15].

$$\forall \mathbf{g} \in \left\{ \begin{array}{l} \mathbf{g} \in \mathbf{R}^n / C_c(\mathbf{x}, \mathbf{g}) = 0 \cap C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{float}) \leq 0 \\ \cap C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{fix-slid}) = 0 \end{array} \right\}; C_f(\mathbf{x}, \mathbf{f}, \mathbf{g}) \geq 0$$

- Assembly assessment: “there exists a gap configuration such that the assembly requirement modelled by interface constraints and compatibility constraints are verified” [13, 15].

$$\exists \mathbf{g} \in \left\{ \begin{array}{l} \mathbf{g} \in \mathbf{R}^n / C_c(\mathbf{x}, \mathbf{f}, \mathbf{g}) = 0 \cap C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{float}) \leq 0 \\ \cap C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{fix-slid}) = 0 \end{array} \right\}$$

Monte Carlo simulation is used in order to determine if the functional and the assembly conditions are respected.

The objective of the calculation of the probability of assembly failure is not the identification of the values of the gap parameters but their existence. That ensures the feasibility of the problem. A counter is added to compute the number of times that the optimization is not achieved. Based on the resulted value of the counter at the end of the large number N of Monte Carlo simulation, the probability of assembly failure is performed.

In the case of functionality assessment of the system, the gap parameters associated to the functional requirements are computed by optimization. A counter is created in order to quantify the number of times that the functional requirements are not respected. The resulted value of this counter at the end of the simulation and the number N of the MCS are used to assess the probability of

the functionality failure.

The main objective of the optimization technique is to determine the gap components between surfaces potentially in floating contacts, especially those involved in the assessment of the functional requirements. Therefore, the objective function is a function of the gap parameters of the functional contact and it is optimized such that all the interface constraints, the compatibility equations, and the functional requirements are satisfied.

In our study, the optimization problem is defined by the following equation [15]:

$$\begin{aligned} \min_{\mathbf{g}} \quad & f(\mathbf{g}) \\ \text{subject to} \quad & C_c(\mathbf{x}, \mathbf{g}) = 0 \\ & C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{float}) \leq 0 \\ & C_i(\mathbf{x}, \mathbf{f}, \mathbf{g}_{fix-slid}) = 0 \end{aligned} \quad (6)$$

Where  $\mathbf{g}_{float}$  identifies the gap parameters of floating contacts,  $\mathbf{g}_{fix-slid}$  defines the gap parameters associated to fixed and sliding contacts and  $\mathbf{g}$  represents the gap parameters combing all contacts in any mechanical system.

As synthesis of the tolerance analysis framework, different sets of constraints are firstly defined. The geometrical deviations and the form defects are generated to be integrated in the constraints definition. The geometrical behavior of fixed and sliding contacts is subsequently modelled and that based on signed distance concept. The gap parameters identified in this step are integrated in the global optimization in the tolerance analysis procedure. In parallel, floating contacts are analyzed and associated gap parameters are identified based on the definition of a set of interface constraints. The probabilities of assembly and functionality failure are computed at the end. Figure 5 presents the global framework of tolerance analysis of over-constrained assemblies, main objective of the paper.

#### 5 Discussion and conclusion

In the first part of this section, we consider to apply the proposed framework on a simplified mechanical clutch of pump. The system is composed of 7 parts two contact types, different sliding contacts and one floating contact, see Figure 6. One assembly condition is considered in this example. The aim of the study is to assess probability of assembly failure.

In order to apply the previous framework and as a first step, the different constraints are mathematically formalized, the involved deviations are defined based on the Small Displacements Torsor concept [20]; the different gaps characterizing the relative position between different mating surfaces of the system parts potentially in contact are represented by torsors. The form defects associated to the mating surfaces are generated using the modal representation as presented in section 2.

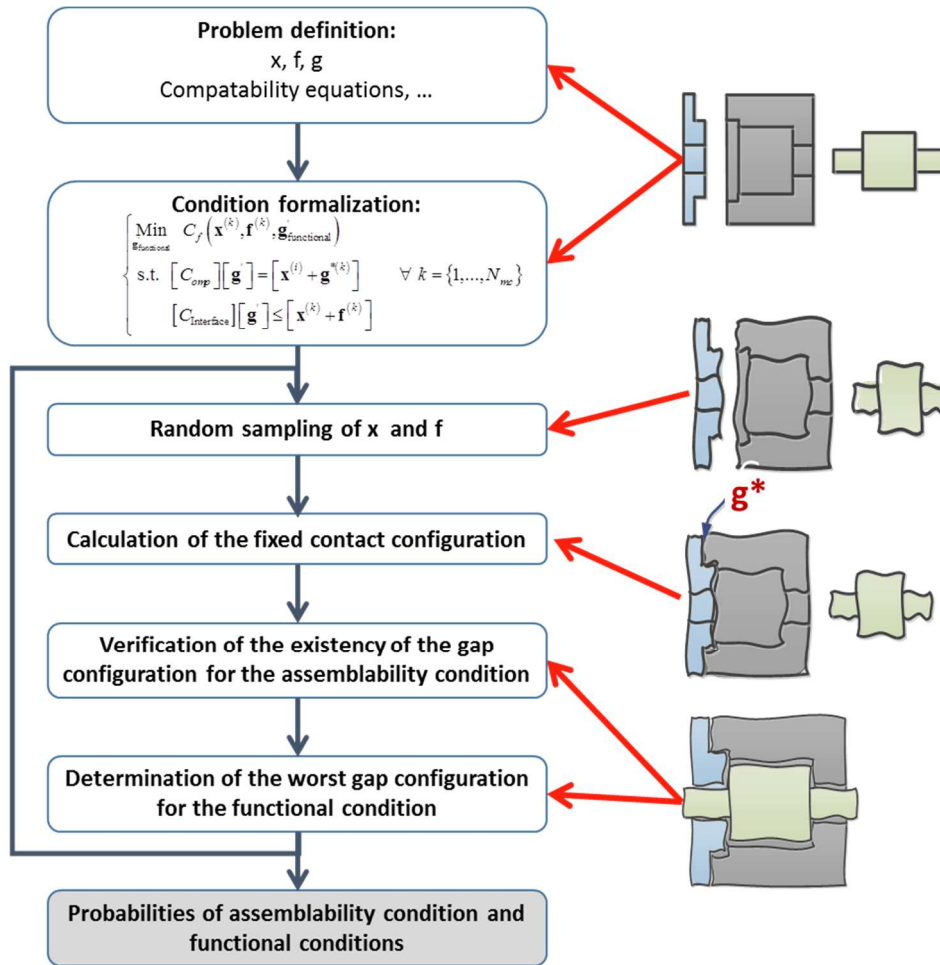


Figure 5. Tolerance analysis framework

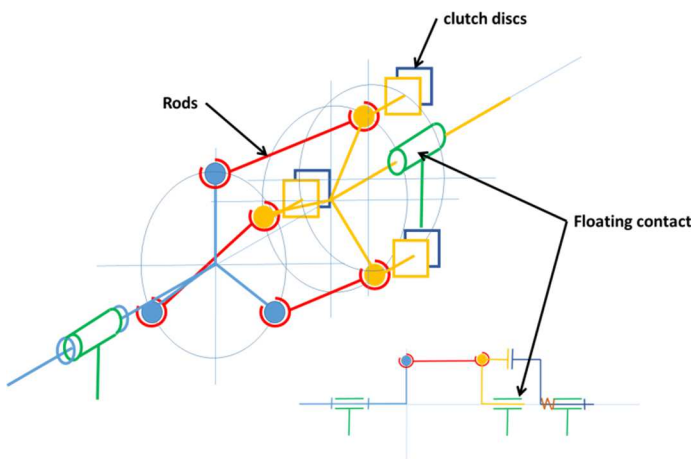


Figure 6. Kinematic graph representation of the studied mechanism

In the case of a sliding contact, an optimization problem based on the signed distance approach is defined. The objective function is written in terms of gap parameters. The main objective is to determine the admissible configuration of the gap parameters. Once found, these gap parameters allow the definition of the

interface constraints associated to the concerned contacts. These constraints permit the definition of a global optimization modeling the assembly condition. All involved constraints are linearized in order to make resolvable the optimization problem, a linear resolution technique such a simplex algorithm can be used. The following table gives the numeral results of the application of the tolerance analysis framework on the mechanical clutch of pump. The assembly failure probability is thus computed. Two different values for the nominal gap associated to the floating contact are considered.

Table 1. Assembly assessment

<i>Tolerance analysis framework</i>		
	<b>Example 1</b>	<b>Example 2</b>
<i>Nominal gap</i>	0.01 mm	0.05 mm
<i>Assembly failure (%)</i>	2.10 e-03	9.90 e-04
<i>Computing time</i>	3h22min	3h20min

As conclusion, the paper deals with a statistical tolerance analysis framework of over-constrained mechanisms considering two issues: parts are with form defects and three types (fixed, sliding, and floating) of contacts are differently modeled. Firstly, the paper gives a comprehensive review on the modal representation of the

form defects. Then, the geometrical behavior modeling is performed for the three types of contacts. To deal with a statistical tolerance analysis technique, the probability of assembly failure and the probability of functionality failure of a mechanical system are computed. In the context of fixed or sliding contacts behavior modeling, Monte Carlo simulation has been used, this technique is too consuming in terms of computation time. To overcome this issue, further resolution should be adapted. As future work and to overcome this issue, a probabilistic model for the gap modeling when dealing with fixed and sliding contacts will be developed.

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