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Correlations on weakly time-dependent transcritical white-hole flows

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We report on observations made on a run of transcritical flows over an obstacle in a narrow channel. Downstream from the obstacle, the flows decelerate from supercritical to subcritical, typically with an undulation on the subcritical side (known in hydrodynamics as an undular hydraulic jump). In the Analogue Gravity context, this transition corresponds to a white-hole horizon. Free surface deformations are analyzed via the two-point correlation function, which shows the presence of a checkerboard pattern in the vicinity of the undulation. In non-gated flows where the white-hole horizon is far downstream from the obstacle, this checkerboard pattern is shown to be due to low-frequency fluctuations associated with slow longitudinal movement of the undulation. In gated flows, however, the undulation is “attached” to the obstacle, and the correlation pattern is likely due to the scattering of a stochastic ensemble of surface waves.

I. INTRODUCTION

Correlations provide valuable insight into the behavior of stochastic fluctuations. In fluid mechanics they have found utility in the theory of turbulence [1], though they are also useful in the analysis of wave-current interaction when the waves present are due to random noise. That is, if we decompose the flow into a “background” state (in some sense an average value, typically a time-average) and fluctuations around this background, the fluctuations can be considered as a statistical ensemble of waves which interact with each other, or (if their amplitudes are sufficiently small) with the background mean flow alone.

The latter case corresponds to the regime of Analogue Gravity [2–4], which aims to simulate gravitational phenomena using condensed matter experiments [5, 6]. To this end, the background is identified with an effective spacetime metric for the fluctuations, which play the role of test waves propagating in the effective spacetime. Correlations are a useful tool in both theoretical and experimental works in Analogue Gravity [7, 8], primarily as they can capture the pair-wise nature of the analogue of Hawking radiation from an effective horizon. These correlations are particularly involved near an analogue white-hole¹ horizon, where the flow passes from supercritical to subcritical: the expected two-point correlation function exhibits a checkerboard pattern [9], due in that case to correlations between short-wavelength dispersive modes. (We shall have more to say about analogue Hawk-

ing radiation in the discussion section at the end of this paper.)

That being said, there can be subtleties in how the background (or, in the context of Analogue Gravity, the effective metric) is to be defined, which in turn affects the identification of the fluctuations themselves. For example, in a quantum system where averages are taken over an ensemble of experimental realizations, one might like to be able to isolate quantum fluctuations but will necessarily have to include classical ones as well, and the latter might be dominant (see [10–12] for such an example involving density fluctuations in a Bose-Einstein condensate). In a classical context, for a steady water flow which is expected to be statistically stationary, the background can be identified as the constant component of the field (*i.e.*, its time-average) while the fluctuations capture the entirety of the time-dependence of the field. On the other hand, we can relax the assumption of strict stationarity by allowing the background to vary slowly, averaging instead over subintervals of the full duration so that the background is determined separately within each subinterval. In this case, we will inevitably encounter a degree of time-dependence in the background due to fluctuations from one subinterval to the next. Note that these two approaches are rather complementary; indeed, the second effectively divides all the time-dependence into “slow” fluctuations (which can be associated with some movement of the background) and “fast” fluctuations (which occur on top of the background). In this way, the motion of the background can to some extent be separated from the field evolution due to the presence of stochastic fluctuations.

In this paper, we report on a checkerboard-like contribution to the two-point function of free surface deformations downstream from an analogue white-hole horizon

¹ A *white hole*, or sometimes *white fountain*, is the time-reversed version of a black hole, into which nothing can enter and from which nothing can avoid being ejected.

in a 1D water flow over an obstacle. The flow is transcritical (in contrast to previous experimental Analogue Gravity works in water flows which have tended to be purely subcritical [3, 4]), and the effective metric thus contains a white-hole horizon. The flow can be regulated by the partial lowering of a gate at the downstream end of the channel. For flows that are unobstructed at the downstream end, the white-hole horizon is generated by an undular hydraulic jump [13] some distance downstream from the obstacle, and the origin of the correlation pattern is shown to be a slow drift in the longitudinal position of the jump. The associated frequencies are so low that it makes sense to separate this drift from the familiar surface waves and to treat it instead as a slow movement of the background; when this is done, the remaining “fast” fluctuations have a correlation pattern in which the checkerboard is strongly suppressed. Contrastingly, in flows with a partially closed gate at the downstream end (inducing a “backwater effect” [14] that affects the upstream part of the flow), the undulation is typically seen to be “attached” to the obstacle, and we observe no clear contribution associated with a slow drift of the background. We show that, in this case, the low-frequency contribution to the full two-point function is much less significant than in non-gated flows.

The paper is organized as follows. Section II provides a description of the experiment: the details of the water channel, the cameras used to record the free surface, and the numerical post-processing performed on the data. In Section III, we describe our observations, paying particular attention to slow movement on long time scales and the associated patterns in the two-point correlation function. This falls into two parts, with subsections III A and III B dealing with non-gated and gated flows, respectively. We summarize our findings in Section IV and discuss some of the implications. More information and technical details are given in the “Materials and Methods” section.

II. EXPERIMENTAL SETUP

We used an open flow channel (reference H23 from Prodidac), of which a photograph is shown in Figure 1. The channel walls are made of transparent plexiglass with anodized aluminum support, with dimensions ($L = 2.5$ m) \times ($Z = 12$ cm) \times ($W = 5.3$ cm). As mentioned above, an upper gate, which can be lowered into the upper part of the flow, is present at the downstream end of the channel. The maximum flow rate is 35 L/min, provided by a volumetric hydraulic power bench. The flow rate is measured with a flowmeter Vortex F 20 (DN20) from Bamo Mesures, with a range from 5 to 85 L/min. Obstacles to be placed on the channel floor are made by an Ultimaker 5S 3D printing machine with the Cura software, using either PLA (black) or ABS (blue) filaments of diameter 2.85 mm. These obstacles have notches measuring 6 mm in width and 3.5 mm in depth, with which

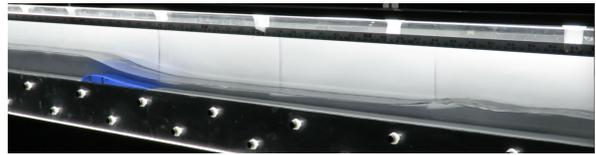


Figure 1. Photograph of the channel, with an ABS obstacle (in blue) placed on the bottom. The variation of the water height can be clearly seen, including the undular hydraulic jump on the far downstream (right) side of the flow.

they are fixed on both sides of the channel. An overhead LED lighting system illuminates the free surface of the flow and allows the side visualization of the meniscus, which is recorded by two grayscale (256) Point Grey cameras with CMOS technology. The images from the two cameras are combined by a Matlab algorithm. They record a total length of 2.05 m with spatial resolution $\delta x = 0.5$ mm, and (for the experiments here considered) a total duration of around 5 minutes at an acquisition rate of $f_{ac} = 32$ fps. No wave maker is used; instead, the fluctuations of the free surface are provided by the noise inherent to the system (turbulence, mechanical vibrations, *etc.*).

The Matlab script processes the interface with a sub-pixel detection method applied to the side meniscus (see Refs. [3, 4, 15–18] for details). The meniscus shows a maximum intensity on the reconstituted image. A first calculation makes it possible to detect this maximum for each position x and each time t . After a first detection of maxima, the aberrant points (due to image problems like drops, blurs, *etc.*) are replaced by an average value of their neighbors, and the maximum brightness is sought again around the positions previously found. This two-step detection has a precision of one pixel. It is then followed by a subpixel approach: around its maximum value, the luminosity is assumed to decrease according to a Gaussian. By fitting the observed luminosity to a Gaussian over five neighboring points, it is possible to find the position of the meniscus to within a fraction of the pixel size δx .

III. OBSERVATIONS

A series of transcritical flows were realised, and recorded for a total duration of ~ 340 seconds. We present here two main cases, distinguished by the presence or absence of a gate at the downstream end of the channel. We observe that this feature seems to determine whether the passage from supercritical to subcritical flow in the downstream region occurs close to the obstacle, or some distance downstream from it. As already noted, we also observe that this property has implications for the content of the fluctuations of the free surface around its mean profile.

Here we describe the most important observations, for

non-gated flows in §III A and for gated flows in §III B. Technical details on how the key quantities are defined and extracted, as well as some supplementary observations, can be found in the “Materials and Methods” section at the end of the paper.

A. Non-gated flows

In this case, the flow typically remains supercritical for a significant distance downstream from the obstacle, then returns to subcriticality via an undular hydraulic jump: the water height increases quite abruptly, and is followed by an undulation of gradually decreasing amplitude [13]. The jump occurs far in the downstream region, and is thus unlikely to be driven by the geometry of the obstacle. This makes it quite different from the flows that are typically considered in theoretical Analogue Gravity works. Instead, the form of the jump (and the white-hole horizon associated with it) is likely to be controlled by dissipative processes induced by turbulence.² Such a hydraulic jump was not observed in our previous experimental work involving transcritical flows, in which a wider channel was used [17]. We thus conjecture that the narrowing of the channel increases the effective dissipation, leading generically to the presence of a jump whenever the flow becomes supercritical, and whenever the downstream end of the flow is free so that there is no backwater effect [14] due to the gate.

In Figure 2 are shown three plots related to a non-gated flow (over an obstacle which is a scaled-down version of that used in Ref. [3]). In the first panel, the mean water height $h_0(x)$, averaged over the entire duration of the recording, is shown, with the expected positions of the black- and white-hole horizons indicated by dashed lines. The second shows the background profiles $h_0(x, t)$ averaged separately in each of 32 equal subintervals of ~ 10 s duration each, having zoomed in on the region of interest where the non-stationarity of the background is most prominent. The various $h_0(x, t)$ are shown simultaneously, so the thickness of the curve illustrates the degree of variation of the background over the entire recording. Notice that, while the upstream side of the flow is relatively stable³, there is a significant degree of variation of the background on the downstream side, and the beginning of the growth of this variation appears to coincide with the position of the undular hydraulic jump. To get an idea of how the background actually varies in time

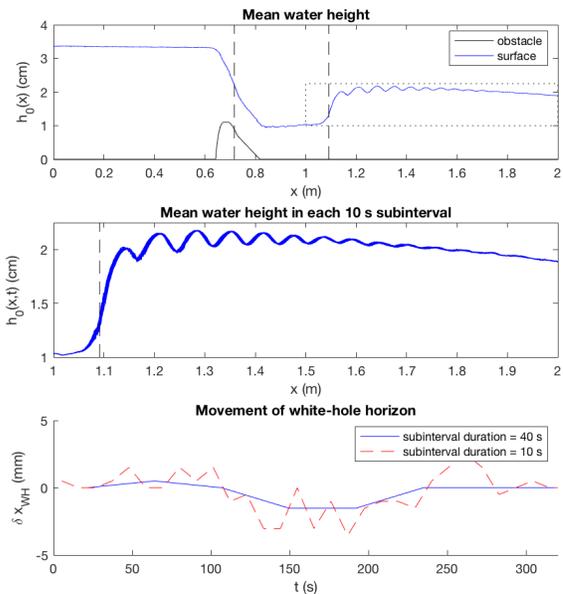


Figure 2. We show here a series of plots characterizing a flow which is free (non-gated) at the downstream end. These are: (1) The obstacle height profile and the mean water height profile, averaged over the entire recording. In vertical dashed lines are shown the positions of the black- and white-hole horizons. (2) A zoom on the region shown by a dotted rectangle in panel (1), but now with the mean water height calculated separately in a series of ~ 10 s subintervals, all shown simultaneously. The thickness of the curve thus indicates the degree of variation in time. (3) The position of the white-hole horizon, calculated in subintervals of duration ~ 40 s (in solid blue) and ~ 10 s (in dashed red). The flow rate is $Q = 15$ L/min, or (reducing to two dimensions) $q = Q/W = 4.7 \times 10^{-3}$ m²/s (where $W = 5.3$ cm is the channel width).

from one subinterval to the next, the measured position of the white-hole horizon is shown in the third plot, suggesting a slow oscillatory nature.⁴

In Figure 3 we turn our attention to the two-point correlation function for free surface deformations: $C(x, x') = \langle \delta h(x) \delta h(x') \rangle$. We focus on the region of interest in the vicinity of the undular hydraulic jump. In the first panel, the background to be subtracted is taken as the time-average over the entire duration of the recording, so that all time-dependence of the free surface is contained in the fluctuations. In the second and third panels, the background is defined and subtracted separately in 32 equal subintervals of duration ~ 10 s. As already mentioned, this procedure effectively defines a time-dependent background and divides the data into “slow” and “fast” fluctuations: those associated with

² Note that a dissipative term associated with turbulence is a required ingredient in theoretical treatments of the undular hydraulic jump; see, *e.g.*, Refs. [19–21].

³ The stability of the upstream part of the flow, around the analogue black-hole horizon, allows us to conclude that the time-dependence we see further downstream is not due to the pump – and therefore the flow – having yet to stabilize. It seems instead to be an inherent feature of the system.

⁴ In a different context (namely a 1D Bose gas on which phonons propagate), the randomness in the position of the white-hole horizon from realization to realization might lead to a similar checkerboard-like correlation pattern due to the varying phase of the undulation [11, 12].

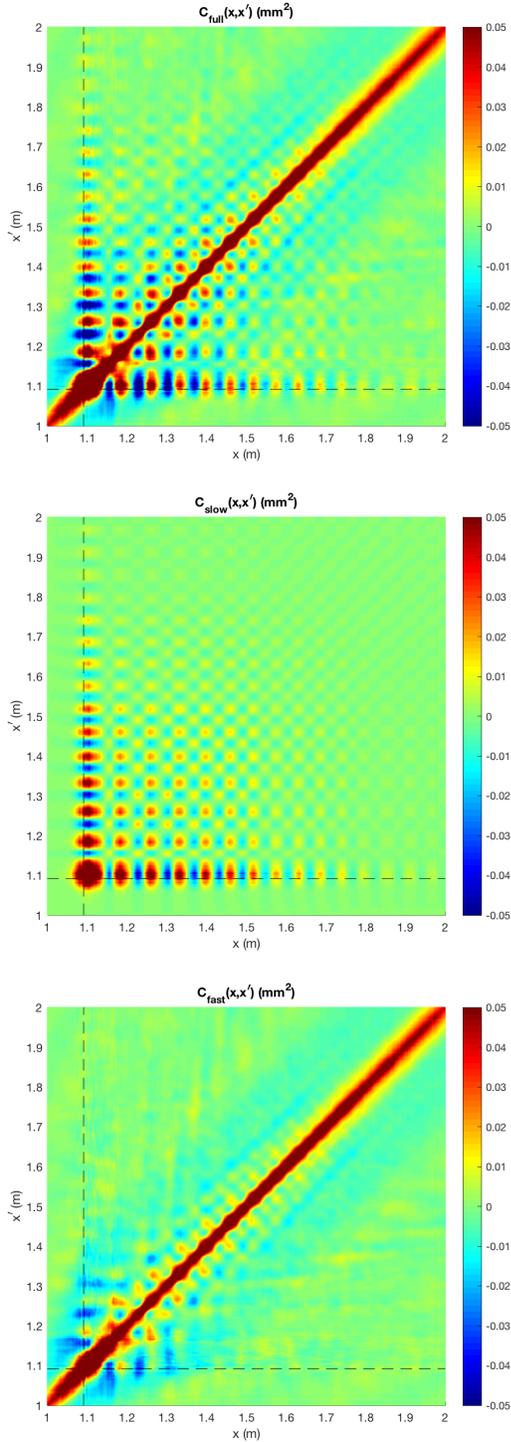


Figure 3. The two-point correlation function of free surface fluctuations in the downstream region, for the same non-gated flow of Fig. 2. The dashed lines indicate the average position of the white-hole horizon. The three correlation functions correspond to different definitions of the fluctuation δh : the full fluctuations (where the background is the time-average over the entire recording), the “slow” fluctuations associated with movement of the background (found by averaging over 10-second subintervals), and the “fast” fluctuations on top of this slowly-varying background.

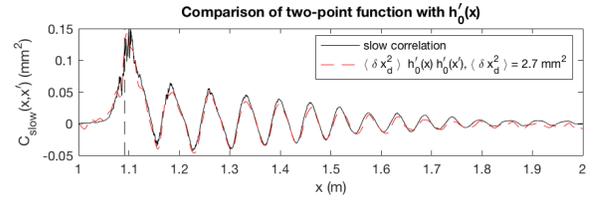


Figure 4. In black is shown a cross-section of the “slow” correlation of Fig. 3, with $x' \approx 1.1$ m fixed close to the point where $C_{\text{slow}}(x, x')$ reaches its maximum value. In red is shown the spatial derivative of the mean water height $h_0(x)$ (averaged over the entire recording), multiplied by a prefactor found by fitting it to the black curve. Through prediction (1), this prefactor gives the variance $\langle \delta x_d^2 \rangle = 2.7 \text{ mm}^2$ of the longitudinal position of the undular jump.

movement of the background from one subinterval to the next, and those with movement of the instantaneous flow within each subinterval. The most prominent feature, clearly visible in both the full and “slow” correlation functions, is a checkerboard pattern in the vicinity of the undulation. The “fast” correlation function is essentially just the autocorrelation along $x = x'$ due solely to the presence of fluctuations.

The observations described above lead us to make the following postulate on the nature of the “slow” fluctuations giving rise to the checkerboard pattern of Fig. 3: namely, that they are mainly due to small longitudinal shifts in the position of the jump and the undulation. To this end, we adopt the following ansatz for the profile of the undular hydraulic jump at any given time: $h_0(x, t) \approx h_0(x - \delta x_d(t))$, where $\delta x_d(t)$ is the instantaneous shift in the position of the jump with respect to its mean position. Assuming that this shift is always sufficiently small – in particular, that it remains at all times much smaller than the wavelength of the undulation – we can make a first-order Taylor expansion of this expression to get the instantaneous fluctuation of the background profile: $\delta h_0(x, t) \approx -\delta x_d(t) h'_0(x)$. Then the equal-time two-point correlation function associated with these fluctuations is

$$\langle \delta h_0(x, t) \delta h_0(x', t) \rangle = \langle \delta x_d^2(t) \rangle h'_0(x) h'_0(x'). \quad (1)$$

This prediction is borne out by Fig. 4, where the two-point correlation function associated with “slow” background fluctuations is shown along a line of fixed x' (chosen to be close to the maximum of the two-point function). This is plotted alongside the derivative of the mean water height, $h'_0(x)$, after multiplication by a fitting parameter. The two curves agree reasonably well, corroborating our claim that the checkerboard pattern is due to a slow longitudinal drift of the undulation. According to prediction (1), the value of the fitting parameter multiplying $h'_0(x)$ is to be equated with $\langle \delta x_d^2 \rangle h'_0(x')$; the fitting procedure thus yields an estimate for the variance of δx_d , which is given in the legend of each plot and whose associated r.m.s value is 1.6 mm. This in turn

seems to be consistent with the spread of the position of the white-hole horizon shown in the third plot of each panel of Fig. 2.

B. Gated flows

In Fig. 5 is shown a series of plots associated with a gated flow, where the upper gate at the end of the water channel has been partially closed, extending 0.8 cm below the surface. The obstacle here has the same shape as that used in Figs. 2-4, but its dimensions are twice as large. The plots are analogous to those shown in Fig. 2, showing the mean water depth over all time, the mean water depth within each of 32 subintervals, and the position of the white-hole horizon as a function of time. As before, there is a noticeable degree of variation of the background in the downstream region, while the upstream region is relatively stable.

A very noticeable difference with respect to Fig. 2 is that, while there is still an undulation as the flow decelerates on the downstream side of the obstacle, it occurs on top of the obstacle, and not some distance downstream after an abrupt change in the flow. The undular hydraulic jump is suppressed by the presence of the gate at the downstream end of the flume: there is a “backwater effect” [14] that prevents its appearance far downstream from the obstacle. Consequently, since the flow decelerates rather quickly, the supercritical region between the two horizons is rather short. Less intuitively, the undulation itself is also significantly shorter than its counterpart in non-gated flows. Numerical simulations of the flow are able to reproduce this observation (see “Materials and Methods”).

With the undulation occurring on top of the obstacle, we might intuitively expect that it has no room to move around longitudinally, and that the checkerboard pattern observed in non-gated flows to be associated with the movement of the undular jump is strongly suppressed. This indeed seems to be the case, as illustrated by the two-point correlation functions for the gated flow, shown in Fig. 6. As before, we show first the full correlation function, followed by its division into “slow” and “fast” contributions. The obvious difference with respect to the previous case is that the “slow” contribution to the correlations is much less significant; indeed, the full correlation function is practically indistinguishable from the “fast” contribution alone. Moreover, while the “slow” fluctuations generate a checkerboard pattern reminiscent of that in the non-gated flow (albeit much weaker), for the gated flow they are not straightforwardly related to shifts in the longitudinal position of the undulation. The most telling sign of this appears in Fig. 7, where a cross-section of the “slow” correlation function is plotted alongside the spatial derivative of the water height (multiplied by a fitted prefactor) in order to check the validity of prediction (1). Unlike what was observed in Fig. 4, there is a clear discrepancy here, and the two profiles cannot be

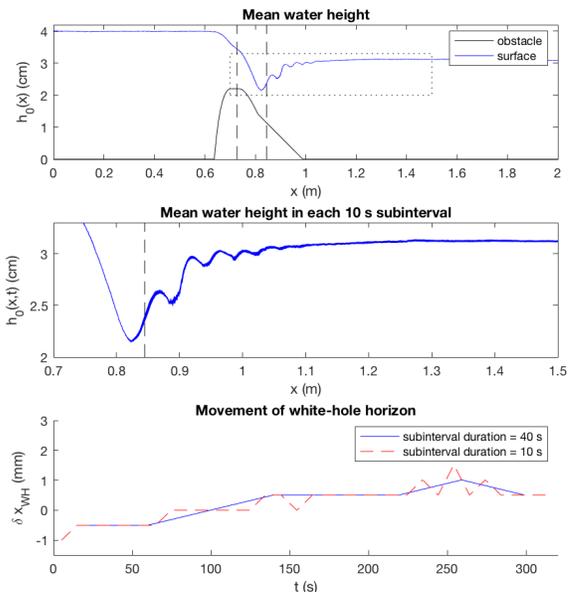


Figure 5. We show here a series of plots characterizing a flow which is gated at the downstream end. These are: (1) The obstacle height profile and the mean water height profile, averaged over the entire recording. In vertical dashed lines are shown the positions of the black- and white-hole horizons. (2) A zoom on the region shown by a dotted rectangle in panel (1), but now with the mean water height calculated separately in a series of ~ 10 s subintervals, all shown simultaneously. The thickness of the curve thus indicates the degree of variation in time. (3) The position of the white-hole horizon, calculated in subintervals of duration ~ 40 s (in solid blue) and ~ 10 s (in dashed red). The flow rate is $Q = 9.8$ L/min, or (reducing to two dimensions) $q = Q/W = 3.1 \times 10^{-3}$ m²/s (where $W = 5.3$ cm is the channel width).

said to match.

Notice also that, in the “slow” contribution to the two-point correlation function, there is a clear offset in the far downstream region where the water surface is flat. This indicates a significant *vertical* shift in the water height as opposed to a horizontal one, since horizontal motion of a flat surface generates no fluctuation. We do not know the cause of this vertical motion, though it could be related to the same kind of back-reaction effects observed in [22], due to the transport of mass by surface waves. In any case, since the value of this offset is about an order of magnitude smaller than the amplitude of the oscillations seen on the right of Fig. 4, we cannot rule out its presence in Fig. 4 and are thus unable to pin down this feature as being unique to gated flows.

Finally, we note that, while there is a clear but weak checkerboard pattern associated with “slow” fluctuations, there is a much stronger checkerboard-like pattern associated with “fast” fluctuations (in addition to the strong autocorrelation along the diagonal $x = x'$). Since we cannot link it straightforwardly to the motion of the background flow, this suggests that this checkerboard-like feature is really due to the scattering of surface waves,

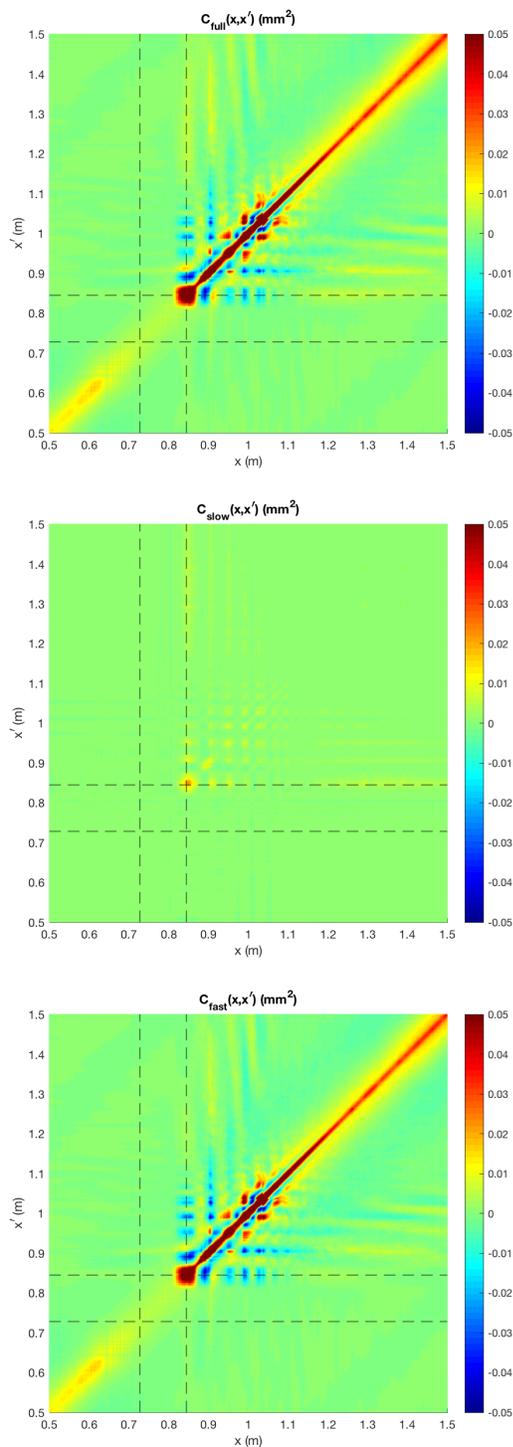


Figure 6. The two-point correlation function of free surface fluctuations in the downstream region, for the same gated flow of Fig. 5. The three correlation functions correspond to different definitions of the fluctuation δh : the full fluctuations (where the background is the time-average over the entire recording), the “slow” fluctuations associated with movement of the background (found by averaging over 10-second subintervals), and the “fast” fluctuations on top of this slowly-varying background.

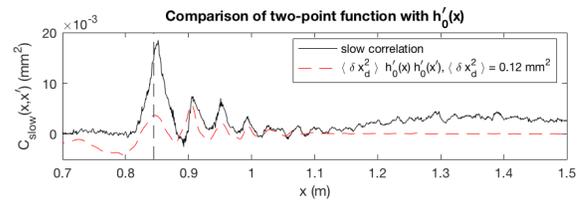


Figure 7. In black is shown a cross-section of the “slow” correlation of Fig. 6, with $x' \approx 0.85$ m fixed close to the point where $C_{\text{slow}}(x, x')$ reaches its maximum value. In red is shown the spatial derivative of the mean water height $h_0(x)$ (averaged over the entire recording), multiplied by a prefactor found by fitting it to the black curve. If the two curves were in good agreement, this would have been associated with a value of $\langle \delta x_d^2 \rangle$ through prediction (1); however, given the clear discrepancies between the two curves, prediction (1) is not valid here and the quoted value of $\langle \delta x_d^2 \rangle$ is not physically meaningful.

and in particular to a scattering process that efficiently produces short-wavelength dispersive modes just downstream of the white-hole horizon. It is known in Analogue Gravity that a long-wavelength counter-current wave incident on the white-hole horizon will scatter primarily into one of the two short-wavelength dispersive modes, and that the analogue Hawking effect is in this case precisely the coupling with the other dispersive mode [9, 23]. It is thus tempting to conclude that the rather complex correlation pattern in the first and third panels of Fig. 6 is to some degree a signal of the analogue Hawking process.⁵ This could be verified by a careful analysis of the corresponding scattering coefficients, which is beyond the scope of this paper but is an intriguing direction for further research.

IV. SUMMARY AND DISCUSSION

We have performed a series of experimental runs in which a stationary transcritical flow is realised in a narrow 1D channel. Broadly speaking, such flows can be categorized into two groups according to the behavior of the white-hole horizon and the associated undulation, which is linked to the presence or absence of a gate restricting the upper part of the flow at the downstream end of the channel. In the absence of such a gate, the white-hole

⁵ While this might seem in conflict with the expected low-frequency thermal behavior $\sim \sqrt{\omega_H/\omega}$ of the associated scattering coefficients, this behavior is recovered only in a finite frequency range $[\omega_c, \omega_H]$, where ω_H is the Hawking temperature and ω_c is a low-frequency cutoff associated with the finite length of the supercritical region [24]. The shortness of the supercritical region compared with the non-gated case may help to explain the relative absence of a strong signal for “slow” frequencies. We estimate $\omega_H \sim 1$ Hz, so there is room for a Hawking-type signal in the “fast” frequency range $\omega > 1/(10\text{ s})$.

horizon occurs some distance downstream from the obstacle, being generated spontaneously by the occurrence of an undular hydraulic jump. In the two-point function of free surface deformations, we observe a checkerboard pattern in this downstream region that is associated with slow fluctuations, and can be reasonably well-described by a degree of randomness in the longitudinal position of the jump. On the other hand, when a gate is present, the white-hole horizon and its associated undulation tend to occur on top of the obstacle. In this case, while there is still a checkerboard-like pattern in the vicinity of the undulation, it is not associated with slow fluctuations nor with a straightforward longitudinal shift of the undulation.

Although we are not yet able to give precise physical reasons for the observed behavior, it seems to be physically intuitive, at least in part. That the undulation is longitudinally mobile in the free flow seems reasonable, given that the flow is not restricted at the downstream end, nor much at the white-hole horizon which occurs some distance from the obstacle. Indeed, being far from the obstacle, the undulation in this case occurs in a background which is approximately translation-invariant, so it makes sense that there is some translational freedom that manifests through a degree of variability in the longitudinal position.

By contrast, the “attachment” of the undulation to the obstacle in the case of gated flows removes this translational freedom, and the checkerboard pattern associated with slow fluctuations is strongly suppressed. Nevertheless, there remains a checkerboard-like pattern associated with fast fluctuations. As pointed out above, it is tempting to view this pattern as an indication of the production of correlated pairs of short-wavelength dispersive modes due to the scattering of counter-current waves at the horizon. These correlated pairs are precisely the (stimulated, classical) analogue of Hawking radiation at the white-hole horizon, and should in principle be associated with a scattering coefficients that are approximately given by a Planck spectrum whose temperature is determined by the flow properties at the horizon [9, 23]. This could be verified by a careful analysis of the corresponding scattering coefficients, which we hope to achieve in the near-future.

That being said, as far as potential for observation of the Hawking effect is concerned, the non-gated flow still seems to have certain important advantages over the gated flow. The supercritical region in between the two horizons is relatively long and smooth, so we can expect a straightforward scattering of the incident co-current waves at the black-hole horizon and for the products of this scattering to impinge on the white-hole horizon, with no significant complications in between. One of these products is a negative-energy counter-current wave, which will be incident on the white-hole horizon and subject to the Hawking effect there in the same manner described above. The length and flatness of the supercritical region provides room for Fourier analysis of the incident modes. Finally, the extended length of the undu-

lation itself indicates that the short-wavelength dispersive modes (which are the products of the Hawking process) exist over a large region, also allowing greater room for Fourier analysis and their possible detection. This is in contrast to the very limited undulations typically observed in gated flows, where it seems the dispersive modes cease to be solutions at a relatively short distance from the horizon. We thus believe that the regimes we have observed provide a promising scenario in which to detect the thermal analogue Hawking process in a transcritical white-hole flow.

Finally, it should be pointed out that the precise origins of the movement of the undulation in the free flows are not known, despite the possibility of such movement being rather intuitive. In particular, we cannot rule out the possibility that these slow fluctuations are themselves the result of an analogue Hawking effect. For the Hawking process, being described by a thermal spectrum, is (in 1D) most prevalent for very low frequencies. Moreover, since the downstream side of the flow lies far from the obstacle in a regime where the system is almost translation-invariant, it makes sense that one of the zero-frequency solutions be characteristic of this symmetry, and thus that $\delta h \propto h'_0(x)$ be a solution of the linearized wave equation on top of such a flow. A more definitive answer to this question requires a more precise treatment of the linearized wave equation on top of a background flow where the undulation is already at its saturated level. This is beyond the scope of the present work, but would be an interesting direction for further research.

SUPPLEMENTAL MATERIAL

Background and fluctuations

In this section, we give some theoretical details concerning the decomposition of the full water height $h(x, t)$ into a “background” and a fluctuation propagating on top of that background.

There is inevitably some ambiguity in the decomposition of the full water depth into a background plus a fluctuation:

$$h(x, t) = h_0(x, t) + \delta h(x, t). \quad (2)$$

Only the full water depth, $h(x, t)$, is measured experimentally. The decomposition into the background, $h_0(x, t)$, and the fluctuation, $\delta h(x, t)$, is imposed in the data analysis, and is inherently ambiguous, for there are many ways to define $h_0(x, t)$ (at least when averaging over time, rather than taking an ensemble average). The simplest way is simply to define a time-independent $h_0(x)$ as being the average of $h(x, t)$ over the entire duration of the recording. If the background profile drifts over this duration, then that drift will be included in the fluctuation $\delta h(x, t)$. We may, however, split the full duration into shorter subintervals, defining $h_0(x, t)$ as the local average over each segment. Crucially, we should see a clear

dependence on the duration of the segments if there is a separation of scales between the typical time associated with the passage of surface waves and the typical time associated with the evolution of the background. If the segment duration is much shorter than the latter, then the background will be approximately constant over each segment, and the movement of the background will be included in $h_0(x, t)$ rather than in $\delta h(x, t)$, the latter then capturing the “true” surface waves. On the other hand, if the segment duration is large compared to the time scale associated with the movement of the background, then this movement will be included in $\delta h(x, t)$ and will be treated as a fluctuation on an equal footing with the surface waves.

This, of course, affects all quantities used to characterise the statistical properties of the fluctuations. In particular, the (time-averaged) two-point correlation function $\langle \delta h(x, t) \delta h(x', t') \rangle_t$ is affected. To see this explicitly, let us write the fully time-dependent water depth as:

$$h(x, t) = h_0(x) + \delta h_{\text{slow}}(x, t) + \delta h_{\text{fast}}(x, t). \quad (3)$$

We identify $h_0(x) + \delta h_{\text{slow}}(x, t)$ with the time-dependent background, $h_0(x, t)$. $h_0(x)$ is just the average of $h(x, t)$ over the entire duration of the recording. The division of the remainder into $\delta h_{\text{slow}}(x, t)$ and $\delta h_{\text{fast}}(x, t)$ is then a matter of choice, depending on the duration of the sub-intervals averaged over in order to define $h_0(x, t)$. The equal-time two-point function for the total height is

$$\begin{aligned} \langle h(x, t) h(x', t) \rangle_t &= h_0(x) h_0(x') + \langle \delta h_{\text{slow}}(x, t) \delta h_{\text{slow}}(x', t) \rangle_t \\ &\quad + \langle \delta h_{\text{fast}}(x, t) \delta h_{\text{fast}}(x', t) \rangle_t. \end{aligned} \quad (4)$$

The cross-terms must vanish because $h_0(x)$ and $\delta h_{\text{slow}}(x, t)$ are themselves defined as averages over time, with the average of the perturbation over each sub-interval necessarily vanishing. Now, $\langle h(x, t) h(x', t) \rangle_t$ and $h_0(x) h_0(x')$ are unambiguously defined for any single recording. The only ambiguity rests in the division into the two-point functions for δh_{slow} and δh_{fast} . So we cannot lose any information when exploring different partitions of the whole duration into sub-intervals; we only divide the information differently into “slow” and “fast” perturbations.

Extraction of relevant quantities

The mean water height $h_0(x)$ is calculated by averaging $h(x, t)$ over all time, with the typical recording duration being 340 seconds. Once this is known, the (depth-averaged) flow velocity profile $v(x)$ and the wave speed $c(x)$ are calculated using

$$v(x) = \frac{q}{h_0(x)}, \quad c(x) = \sqrt{g h_0(x)}, \quad (5)$$

where $q = Q/b$ is the flow rate per unit width ($b = 5.3$ cm is the width of the channel) and $g = 9.8$ m/s² is the acceleration due to gravity. These expressions hold in the

absence of significant vorticity and when the surface is relatively flat (*i.e.*, $|dz/dx| \ll 1$). They are expected to be accurate in the upstream region, though the presence of vorticity downstream from the obstacle is expected to make them less accurate there. The horizons are simply defined as the points where $v(x) = c(x)$, the black-hole horizon in the upstream region where the flow passes from sub- to supercritical, and the white-hole horizon in the downstream region where it returns to subcritical. It is interesting to note that, even though the white-hole horizon occurs in the downstream region where the validity of Eqs. (5) could be questioned, it typically coincides well with the beginning of the region where significant movement of the background is observed.

The instantaneous free surface deformation is simply defined as $\delta h(x, t) = h(x, t) - h_0(x)$. The full two-point correlation function is then just the average over all time of the product of $\delta h(x, t)$ observed simultaneously at two points:

$$\begin{aligned} C_{\text{full}}(x, x') &= \langle \delta h(x, t) \delta h(x', t) \rangle_t \\ &= \frac{1}{N_t} \sum_{j=1}^{N_t} \delta h(x, t_j) \delta h(x', t_j), \end{aligned} \quad (6)$$

where N_t is the total number of discrete measurement times (typically $N_t = 2^{13} = 8192$).

The division into subintervals is achieved by factorizing $N_t = n_{\text{sub}} \times N_t^{\text{sub}}$, representing a total of n_{sub} subintervals each containing N_t^{sub} discrete measurement times. Then profile of the mean water height $h_0^{\text{sub}}(x, t)$ can be calculated within each subinterval. The “slow” fluctuation is defined as the difference between this and the overall mean $h_0(x)$, while the “fast” fluctuation is defined as the remainder:

$$\begin{aligned} \delta h_{\text{slow}}(x, t) &= h_0^{\text{sub}}(x, t) - h_0(x) \\ \delta h_{\text{fast}}(x, t) &= \delta h(x, t) - \delta h_{\text{slow}}(x, t) = h(x, t) - h_0^{\text{sub}}(x, t). \end{aligned} \quad (7)$$

The “slow” and “fast” contributions to the two-point correlation function are then straightforwardly defined:

$$\begin{aligned} C_{\text{slow}}(x, x') &= \langle \delta h_{\text{slow}}(x, t) \delta h_{\text{slow}}(x', t) \rangle_t, \\ C_{\text{fast}}(x, x') &= \langle \delta h_{\text{fast}}(x, t) \delta h_{\text{fast}}(x', t) \rangle_t. \end{aligned} \quad (8)$$

The derivative $h_0'(x)$, needed to compare the observed “slow” two-point correlation function with the prediction (1), is calculated as follows. The height profile $h_0(x)$ is smoothed by applying a window in the Fourier transform so as to remove noise of high spatial frequency. The window takes the form

$$W(k) = \frac{1}{2} \left(\tanh \left(\frac{k + k_{\text{cut}}}{\sigma_k} \right) - \tanh \left(\frac{k - k_{\text{cut}}}{\sigma_k} \right) \right), \quad (9)$$

where we take $k_{\text{cut}} = 400$ m⁻¹ and $\sigma_k = 50$ m⁻¹. We then multiply by $i k$ and take the inverse Fourier transform,

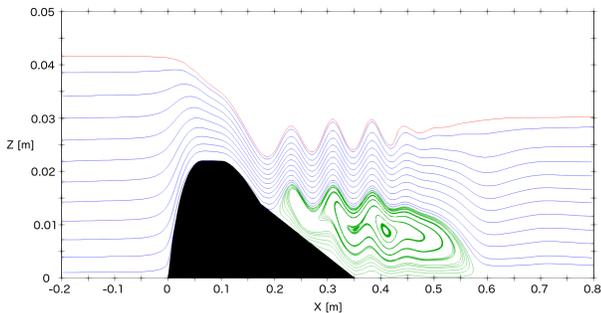


Figure 8. Numerical simulation of background flow. Here are shown the streamlines of the flow over the obstacle as produced by the numerical simulation (see details in text), for a flow with a gate at the downstream end. The short undulation on top of the obstacle is clearly seen. It is also observed to be associated with a circulation layer on the bottom of the flow, shown here in green. This makes the upper part of the flow effectively flat, allowing the undulation to extend further downstream than it otherwise would have.

which yields $h'_0(x)$ for the relevant values of x . (It generates rather large deviations at the edges of the spatial window, but these are not relevant for our purposes.)

In comparing the two-point function with the prediction (1), we fix x' to be close to the first maximum of $C_{\text{slow}}(x, x')$, and then simply perform a linear fit to the amplitude needed to match $h'_0(x)h'_0(x')$ with the observed two-point function. We only include those points within the displayed spatial window (corresponding to the dotted rectangle on the plots of the mean water height).

Numerical simulation of background flow

An original two-dimensional free-surface flow code was used to simulate numerically the transcritical flows studied experimentally in this paper. A projection method is applied to the incompressible variable density Navier-Stokes equations to decouple velocity and pressure unknowns. Away from the interfaces (water-air and obstacle-water), partial differential operators (divergence, gradient, Laplacian operator) and nonlinear terms are discretized on a fixed Cartesian grid using standard second-order finite difference approximations. Several techniques are used to account for the presence of the two interfaces while avoiding the generation of conformal meshes. An Immersed Boundary Method [25] enforces the no-slip boundary condition on the rigid obstacle and the free surface evolution is tackled with the Level-Set technique [26].

Results for a gated flow, with the obstacle of Figs. 5-7, are given in Figure 8. This shows the streamlines of the flow once a steady state has been reached, starting from an initial water height of 3.3 cm (so that the obstacle is always completely submerged). The simulation

neglects the transverse direction, so in order to mimic the increased effective dissipation due to the narrowness of the channel and friction at the walls, the viscosity of water has been increased by a factor of 10. The results are in qualitative agreement with what is seen experimentally in Fig. 5: after an initial acceleration on top of the obstacle, the flow immediately decelerates and induces an undulation, which exists over a short region and vanishes in the asymptotic downstream region.

Notice that the undulation appears to be coupled with a circulation layer on the bottom of the channel, and that the non-circulatory layer on top has a depth which is essentially constant in the vicinity of the undulation, effectively cancelling out the downstream slope of the obstacle. This helps to explain why the undulation extends as far as it does and why it disappears rather abruptly. The undulation is essentially a (rather large-amplitude) zero-frequency free surface deformation, satisfying $\omega(k) = 0$ where k is the wave vector of the undulation. This means that the phase velocity of the undulation, ω/k , vanishes, but by addition of velocities this is just equal to $v_{\text{ph}}(k) + v_{\text{flow}}$, where $v_{\text{ph}}(k)$ is the phase velocity in the rest frame of the fluid and v_{flow} is the flow speed. Due to capillary effects, $v_{\text{ph}}(k)$ has a minimum at ~ 23 cm/s, so v_{flow} needs to be larger than this in order for a non-trivial zero-frequency solution to exist. A straightforward application of the formula $\bar{v}(x) = q/h(x)$ (for the depth-averaged flow velocity \bar{v}) indicates that $\bar{v}(x)$ dips below 23 cm/s significantly before the end of the undulation. However, in the numerical results of Fig. 8 we see that the non-circulating flow near the surface has a relatively shallow constant depth some distance from the obstacle, so that the surface flow dips below 23 cm/s only towards the end of the circulation layer. We indeed see that the undulation ends in rough coincidence with the end of the circulation layer.

Dependence on subinterval duration

In Fig. 9 are plotted the “slow” correlation functions of Figs. 4 and 7 for varying durations of subintervals in which the background is defined by time-averaging. What is most notable is the relative stability of the curves with respect to the subinterval duration. We do see a significant change in the non-gated case when changing from a duration of 43 seconds to 21 seconds. This indicates that we cross the time scale associated with the slow fluctuations: a duration of 43 seconds is long enough that the background has already undergone a noticeable change, and this change gets labelled as “fast” rather than “slow”.

Correlations in Fourier space

Given that scattering on a stationary background proceeds independently for different frequencies, it can be

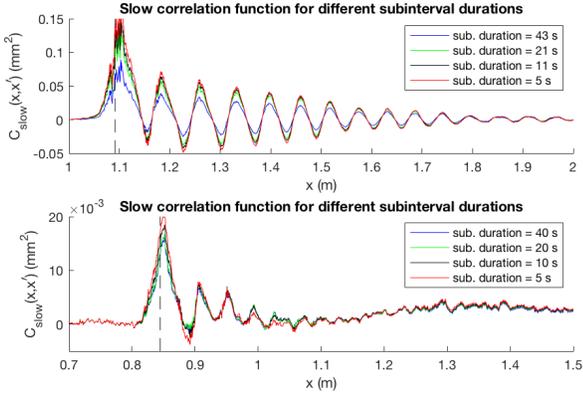


Figure 9. $C_{\text{slow}}(x, x')$ of Figs. 2 (upper, non-gated flow) and 5 (lower, for gated flow), for different values of the subinterval duration. In both cases the profile is quite stable.

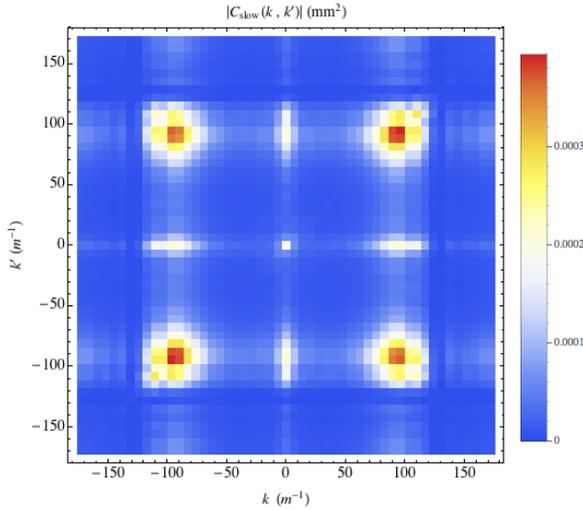


Figure 10. Magnitude of the two-point correlation function in Fourier space, for the “slow” fluctuations in the downstream region of the non-gated flow of Figs. 2-4.

illuminating to consider correlations between different Fourier modes rather than between different points in space. To this end, restricting our attention to the non-gated flow of Fig. 2, we define the Fourier amplitudes

$$\delta h_j(k, t) = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} dx e^{-ikx} \delta h_j(x, t), \quad (10)$$

where $x_1 = 1.15$ m and $x_2 = 2.06$ m are taken as the limits of the downstream region. (In practice, we take a discrete FFT rather than an integral to get $\delta h_j(k, t)$.) The subscript j here stands for the various definitions of $\delta h(x, t)$ defined above, with $j \in \{\text{full}, \text{slow}, \text{fast}\}$. Then we can define the Fourier-space two-point correlation function analogously to that in position-space:

$$C_j(k, k') = \langle \delta h_j(k, t) \delta h_j^*(k', t) \rangle_t. \quad (11)$$

By construction, we have $C_j(k, k') = C_j^*(k', k)$; moreover, since $\delta h(x, t)$ is real and consequently $\delta h(k, t) =$

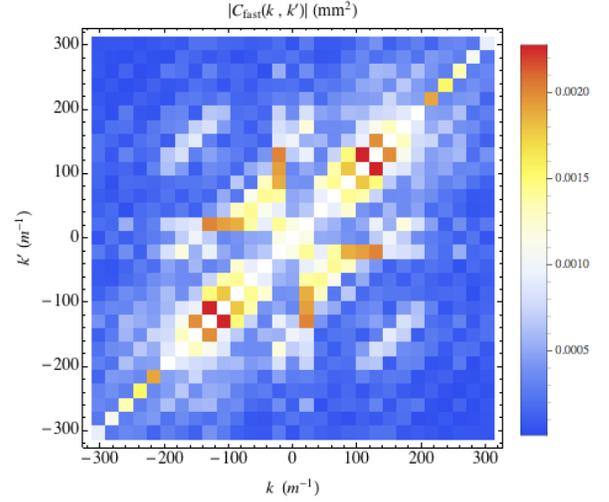


Figure 11. Magnitude of the two-point correlation function in Fourier space, for the “fast” fluctuations in the vicinity of the undulation on top of the obstacle in the gated flow of Figs. 5-7.

$\delta h^*(-k, t)$, we also have $C_j(k, k') = C_j(-k', -k)$. These mean that the magnitude $|C_j(k, k')|$ must be symmetric under reflection in both the diagonal $k = k'$ and the anti-diagonal $k = -k'$.

In Fig. 10 is shown the magnitude of $C_{\text{slow}}(k, k')$ for the Fourier-space correlation function in the downstream region (*i.e.*, in the vicinity of the undulation) of the non-gated flow of Fig. 2. The key observation here is the simple 3×3 structure of the correlations, occurring between three distinct wave vectors: $k = 0$ and $k = \pm k_u$, where k_u is the wave vector associated with the undulation. (We see that $k_u \sim 90 \text{ m}^{-1}$, corresponding to a wavelength of $2\pi/k_u \sim 7$ cm, which is consistent with the undulation pattern seen in Figs. 2-4.) This is consistent with the physical interpretation given in Eq. (1). Fourier transforming $\delta h(t, x) \approx -\delta x_d(t) h'_0(x)$ in space, we have $\delta h(k, t) \approx -\delta x_d(t) [h'_0]_k$, where $[h'_0]_k$ is just the Fourier transform of $h'_0(x)$. Then

$$C_{\text{slow}}(k, k') \approx \langle \delta x_d^2(t) \rangle_t [h'_0]_k [h'_0]_{k'}^*. \quad (12)$$

This particular form of $C_{\text{slow}}(k, k')$ requires that, not only are the required symmetries described above respected, but we also have $|C_{\text{slow}}(k, k')| = |C_{\text{slow}}(k, -k')|$ (and similarly for $k \rightarrow -k$). That is, the magnitude of the correlation function is symmetric under reflection in the horizontal and vertical axes. Although this is only an approximate symmetry that depends on the validity of prediction (1), it is clearly borne out by the correlations in Fig. 10.

Finally, in Fig. 11 we show the magnitude of the Fourier-space correlation function $C_{\text{fast}}(k, k')$ for the region on top of the obstacle in the gated flow of Figs. 5-7. We choose to show this for the “fast” fluctuations since it is these that generate the main checkerboard-like feature in the position-space correlation functions of Fig. 6. The

key feature of interest is a clear correlation between positive and negative k , extending from about $|k| = 100 \text{ m}^{-1}$ to $|k| = 200 \text{ m}^{-1}$. This corresponds to wavelengths on the order of 3-6 cm, consistent with both the undulation of Fig. 5 and the oscillatory pattern seen in Fig. 6. This correlation between short-wavelength dispersive modes of positive and negative k is precisely what is engendered by the Hawking scattering process [9, 23]. Moreover, while the correlations occur over a broad spread in k (unlike what we see in Fig. 10 for the non-gated flow), they do have a clear orientation in the (k, k') -plane that follows the expected locus of the correlation pattern (see Fig. 3 of [4]).

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