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Trade-in for remanufactured products: Pricing with double reference effects

Peng Ma ^a, Yeming Gong ^b, Prakash Mirchandani ^c

^a School of Management Science and Engineering, Nanjing University of Information Science and Technology, Nanjing 210044, P.R. China

^b EMLYON Business School, 23 avenue Guy de Collongue, Ecully Cedex 69134, France

^c Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260, United States

Abstract

When organizations introduce programs to trade in old products for remanufactured ones to promote remanufacturing, they offer two options to consumers: buying new products or trading in for remanufactured products, where the latter is a new practice. While most of the extant literature considers only the single reference price effect, there are quality differences in the problem of trading in for remanufactured products. Thus, the reference quality effect cannot be overlooked as before. To handle this new setting, we consider consumers' double reference effects to examine a manufacturer selling both new and remanufactured products. We also consider the remanufacturing subsidy and the consumer rebate ratio. We analyze five models and develop equilibrium solutions thereof to understand the impacts of double reference parameters and government incentives on pricing strategies, the manufacturer's profits, and the consumer surplus. Computational examples reveal that (i) both the manufacturer's profits and the consumer surplus benefit from the double reference effects when the reference price parameter is relatively larger and the reference quality parameter is relatively smaller; (ii) the remanufacturing subsidy is beneficial to the manufacturer, and the consumer rebate ratio only impacts and improves the retail prices of remanufactured products, but does not change the profits of the manufacturer; (iii) when the customers only consider reference price effect, lower unit remanufacturing cost, higher remanufacturing rates, and lower consumers' discount rates for remanufactured products benefit the manufacturer. Surprisingly, when only the reference quality effect behavior is considered, higher unit remanufacturing cost, lower remanufacturing rates, and higher customers' discount rates can offset some of the negative impacts of

Email addresses: mapeng88@126.com (Peng Ma ^a), GONG@em-lyon.com (Yeming Gong ^b), pmirchan@katz.pitt.edu (Prakash Mirchandani ^c)

reference quality effects.

Keywords: Remanufacturing, Double Reference Effects, Remanufacturing Subsidy, Consumer Rebate, Pricing Strategies

1. Introduction

While trading in old products for new ones is an industry-wide practice in automobiles, technology, and electronics (Ray et al., 2005; Agrawal et al., 2016), our research is partially motivated by a new business practice: trading in old products for remanufactured products. This initiative has been recently established by the National Development and Reform Commission (chinairn.com). The products that qualify for the trade-in program for remanufactured products include automobile parts (i.e., automobile gearbox, automobile engine, engineering machinery, and machine tool). In the program for trading in old products for remanufactured products, the government subsidizes remanufactured products of firms, while firms offer consumers a rebate if they return an old product and purchase a remanufactured one of the same type. Consumers can choose to buy new products or trade in old products for remanufactured products, and participating in the trade-in program for remanufactured products allows consumers to benefit from the lower retail prices of remanufactured products compared with the new products. However, consumers may have lower quality valuations for the remanufactured products, thus they will feel a loss in quality when using remanufactured products compared with new ones. The customers' feeling of gain from lower prices of remanufactured products and feeling of loss from lower quality perception of remanufactured products can be understood as reference effects behaviors, which will significantly influence a manufacturer's pricing decisions, sales quantities, and profits if the manufacturer participates in the trade-in program for remanufactured products.

Reference prices are certain anchoring levels of prices that consumers have in mind and to which they compare the shelf prices of specific products. Researchers introduce and study the impacts of reference price effects from the following two perspectives. Some of researchers consider that the consumers' purchase behavior is impacted not only by the utilities derived from products, but also by the gain-loss utility in comparison with the reference products (Kahneman and Tversky, 1979; Tversky and Kahneman, 1991; Li and Jain, 2016; Amaldoss and He, 2018; Hong et al., 2018; Hong et al., 2019). Other researchers consider reference price effects in which the differences between the

reference prices and the shelf prices have a direct impact on the demands of products (Greenleaf, 1995; Kopalle et al., 1996; Fibich et al., 2003; Popescu and Wu, 2007). Several researchers consider reference quality effects (Gavious and Lowengart, 2012; Liu et al., 2016). Reference quality is the perceived product quality formed over time through some information, such as consumers' experience of purchasing this product, and past product quality levels. In practice, consumers usually use this quality information to directly compare with the current product quality before making purchase decisions. Similar to the reference price effects, reference quality can also influence the market demands or the utilities of customers. The literature above focuses on reference price effects or reference quality effects independently, which are acceptable and appropriate in the existing settings stated in their papers.

Double reference effects will significantly influence consumers' purchase decisions, which then affect the quantities of products sold and their prices. Moreover, double reference effects are non-trivial and concurrently present in the setting of trade-in program for remanufactured products. However, no researchers hitherto have addressed the impacts of double reference effects in optimizing operations of the trade-in program for remanufactured products. To fill this gap, we address the following research questions: (i) Considering the double reference effects, how should the manufacturer make pricing decisions in the trade-in program for remanufactured products? (ii) How do double reference effects influence pricing decisions, sales quantities, the profits of the manufacturer, and the consumer surplus?(iii) How do government incentives impact pricing decisions, sales quantities, and manufacturer's profits, considering double reference effects?

To answer these questions, we consider a two-period model with limited remanufactured product supply in the second period. In the first period, the manufacturer only sells new products to the consumers. In the second period, the manufacturer participates in the trade-in program for remanufactured products, and sells both new and remanufactured products to consumers. We consider the following five scenarios: consumers do not consider any reference effects (Model I); consumers only consider reference price effects (Model II); consumers only consider reference quality effects (Model III); consumers consider double reference effects with no subsidies (Model IV); and consumers consider double reference effects with subsidies (Model V). In Model V, we consider two types of government incentives: a remanufacturing subsidy given to the manufacturer and a rebate given to the consumers who participate in the trade-in program for remanufactured products. As

government incentives aim to boost remanufacturing, we also study the impacts of such incentives on pricing and profits with special double reference effects.

Our findings below offer decision-makers interesting management insights when designing incentive policies to stimulate remanufacturing.

(1) The consumers' reference price effects are beneficial to the manufacturer in the case that customers only consider reference price effects (Model II). This is to say, it is beneficial to the manufacturer for selling remanufactured products in Model II. The reference quality effects are always non-beneficial to the manufacturer when consumers only consider reference quality effects (Model III). Thus, it is not suitable for selling remanufactured products to the consumers in Model III.

(2) When double reference effects are considered, we find that the retail prices of new products in Period 1 are higher than those of new products in Period 2 when the reference quality parameter is relatively larger and the reference price parameter is relatively small. The manufacturer can sell more new products in Period 2 than remanufactured products in Period 2 in Model IV. We also find that double reference effects are beneficial to the manufacturer under certain conditions. The smaller reference quality parameter and larger reference price parameter will benefit the manufacturer. Our study reveals that the impacts of double reference effects on the consumer surplus are similar to the impacts of these effects on the manufacturer's profits.

(3) We investigate the impacts of remanufacturing subsidies and consumer rebate ratios on the pricing decisions and manufacturer's profits. The remanufacturing subsidy thus benefits the manufacturer and helps promote remanufactured products.

(4) We do the sensitivity analysis to investigate the influences of reference price parameter and reference quality parameter on the profitability of double reference effects with different unit remanufacturing costs, different remanufacturing rates, and different customers' discount rates for remanufactured products. Our study shows that when the customers only consider reference price effects, lower unit remanufacturing cost, higher remanufacturing rates, and lower consumers' discount rates for remanufactured products benefit the manufacturer. Surprisingly, when only the reference quality effects behavior is considered, higher unit remanufacturing cost, lower remanufacturing rates, and higher customers' discount rates benefit the manufacturer.

The main contributions of our study are threefold: (1) We first introduce both the reference

price effects and reference quality effects into study of the remanufacturing practices considering the new practice of trade-in program for remanufactured products. (2) We study the impacts of double reference effects on the pricing decisions, sales quantities, the profits of the manufacturer, and the consumer surplus. (3) We examine the impacts of remanufacturing subsidies and consumer rebate ratios on the pricing decisions and manufacturer's performance.

2. Literature review

In this section, we review the reference price and reference quality effects, summarize the results of recent studies on optimal pricing strategies for remanufactured products, and review the extant literature on trade-in programs in industries.

2.1. Reference price effects and reference quality effects

A substantial corpus of literature has focused on dynamic pricing problems with reference price effects (Kopalle et al., 1996; Fibich et al., 2003; Popescu and Wu, 2007; Miraldo, 2009; Nasiry and Popescu, 2011; Azar, 2013; Zhang et al., 2014; Dye and Yang, 2016; Li and Jain, 2016; Chen et al., 2017; Hong et al., 2018; Hong et al., 2019). We summarize them as follows. Kopalle et al. (1996) assume that the market demand is a function of actual prices and reference prices. They derive optimal dynamic pricing policies under monopoly model and duopoly model respectively. Fibich et al. (2003) obtain the explicit expression for the optimal pricing strategy, open-loop equilibria, and closed-loop equilibria of the nonsmooth problems with asymmetric reference price effects in marketing models. Furthermore, Popescu and Wu (2007) address the dynamic pricing problem of a monopolist in a market with a frequently purchased product or service, where the demand is sensitive to the pricing history of the firm. Considering heterogeneous qualities, Miraldo (2009) studies the impacts of reference pricing reimbursement on firms' pricing strategies in a horizontal differentiation model. Nasiry and Popescu (2011) introduce the peak-end rule into the reference price mechanism to study dynamic pricing problems. Azar (2013) investigates the optimal strategy of multi-product firms, while Zhang et al. (2014) study the optimal strategic pricing in a supply chain with a retailer and a manufacturer. Dye and Yang (2016) introduce the reference price effects into optimal pricing for deteriorating products. Li and Jain (2016) assume the consumers' first-period price represented as a reference price in the extended model to study the impacts of

fairness concerns and historical reference price. Chen et al. (2017) use a reference model to study the computational side of a dynamic pricing problem. Hong et al. (2018) study a green-product pricing problem by considering consumers' reference point. Lastly, Hong et al. (2019) study the green-product design strategies for a two-stage supply chain by considering consumers' reference behaviors. However, these works do not analyze the impacts of reference quality effects.

On the other hand, some scholars have introduced reference price effects in other areas. For example, Zhang et al. (2013) study the supply chain coordination based on the assumption that the reference price not only depends on the consumers' prior purchase experiences, but also on the local and national advertising level. Becker-Peth and Thonemann (2016) introduce reference-dependent valuation into a decision model to study the behavioral aspects of revenue-sharing contracts. Denoyel et al. (2017) address a structured framework for practitioners to conduct an experiment of reference pricing for healthcare.

Still, other scholars have also considered reference quality effects in their investigations (Gavious and Lowengart, 2012; Liu et al., 2016). Gavious and Lowengart (2012) focus on the relationship between price and reference quality to study their combined effects on the profits. Liu et al. (2016) consider the consumers' reference quality effects and study the pricing strategies in a dynamic supply chain. However, these works do not consider reference price effects.

2.2. Optimal pricing strategies for remanufactured products

Researchers have considered the reverse channel choice and pricing decisions in a remanufacturing setting (Gong, 2013). The problem of choosing the appropriate reverse channel structure for collecting used products from consumers is addressed by Savaskan et al. (2004). Thereafter, Savaskan et al. (2006) consider competing retailers; they study the interaction between the reverse channel choice and strategic product pricing decisions in the forward channel. In a later study, Wu and Zhou (2017) extend Savaskan et al.'s (2004) conclusion to address the impacts of supply chain competition on the optimal reverse channel choice of the manufacturers' remanufacturing. Similarly, Ma et al. (2017) also extend Savaskan et al.'s (2004) work to a research setting wherein the market demand depends on the marketing effort. They consider the retailer's distributional fairness concerns behavior and potential recycle cost advantages of the retailer and the third party. Recently, Wang et al. (2019) consider three competitive recycling-market: the manufacturer does not participate in recycling; the manufacturer competes with the remanufacturer for recycling; and

the manufacturer outsources collecting to the retailer. Different from above papers, we focus on pricing decisions in trade-in program for remanufactured products.

Some of researchers have investigated the remanufacturing from the perspective of consumer behavior. The problem of integrating market segmentation and production technology choice is addressed in a remanufacturing setting by Debo et al. (2005). Mitra and Webster (2008) then investigate a two-period model wherein a manufacturer makes and sells a new product, while a remanufacturer competes with the manufacturer in the second period. However, they do not consider consumer rebate, while we do consider the trade-in program for remanufactured products as well as consumers' reference effects. Ferguson and Toktay (2006) build models to support the manufacturer's recovery strategy in a competitive remanufacturing market. Ferrer and Swaminathan (2006) treat both new and remanufactured products as indistinguishable, while Ferrer and Swaminathan (2010) contend that they are differentiated. Atasu et al. (2008) provide manufacturers with guidelines for remanufacturing and product design. Then, two types of reuse-simple (remanufacturing only) and full reuse (remanufacturing and upgrading) models are analyzed by Galbreth et al. (2013). At the same time, Xiong et al. (2013) examine a decentralized closed-loop supply chain with a manufacturer and a key component supplier. He (2015) studies the acquisition pricing and remanufacturing decisions with both the demand and supply uncertainties in a closed-loop supply chain. Then, the optimal emissions taxation policy of remanufacturing is characterized by Yenipazarli (2016). Gan et al. (2017) address the pricing decision for short life-cycle products in a closed-loop supply chain wherein new products are sold by traditional retail stores while the remanufactured products are sold through the manufacturer's direct channel. Similar to above papers, we also consider the consumer behavior for buying new and remanufactured products. However, the above works neither consider the trade-in programs, nor the reference effects.

Several researchers have focused on the implementations of take-back legislation in the remanufacturing industry (Toyasaki et al., 2011; Atasu and Van Wassenhove, 2012; Atasu et al., 2013; Esenduran and Kemahlioglu-Ziya, 2015; Esenduran et al., 2016, 2017; Zhou et al., 2017; Chen et al., 2019; Mazahir et al., 2019). Specially, Toyasaki et al. (2011) study monopolistic and competitive take-back schemes for recycling end-of-life electrical and electronic equipment. Atasu and Van Wassenhove (2012) find that there is a strong need for research on the implementations of environmental legislation from an operations perspective. Atasu et al. (2013) compare two practical

forms of take-back legislation application, i.e., manufacturer-operated systems, and state-operated systems. Esenduran and Kemahlioglu-Ziya (2015) compare two schemes (i.e., individual scheme and collective scheme) with regard to the costs, which are imposed on firms and environmental benefits. Esenduran et al. (2016) consider three levels of legislation (i.e., no take-back legislation, legislation with collection targets, and legislation with collection and reuse targets) respectively, and study how various levels of legislation influence manufacturing, remanufacturing, and collection decisions. Esenduran et al. (2017) study the influences of regulation on remanufacturing levels, consumer surplus, and the original equipment manufacturer's profits. Zhou et al. (2017) study the competition between recyclers with take-back legislations and address the government's decision on the subsidy model. Recently, Chen et al. (2019) study how a regulator develops efficient collection targets to provide desirable economic and environmental outcomes for a given application of take-back legislation. Mazahir et al. (2019) address an analytical framework of the product take-back legislation with product reuse. Esenduran et al. (2019) study the impacts of competition for collection and recycling of valuable waste on the implications of take-back regulation. Different from above literature, we do not consider the implications of take-back legislation. Moreover, we focus on a new practice (i.e., trading in old products for remanufactured ones) and study the impacts of double reference effects.

Some scholars have examined remanufacturing from other perspectives too. Especially, Heese et al. (2005) investigate the impacts of used product take-back on companies and consumers. Yoo et al. (2015) analyze pricing and return policies in a closed-loop supply chain, where the supplier is the leader. Govindan et al. (2015) review reverse logistics and closed-loop supply chains. Agrawal et al. (2015) make use of behavioral experiments to investigate how remanufactured products and the remanufacturer's identity affect the perceived values of new products. Abbey et al. (2015) conduct experiments on consumer preferences to identify optimal pricing strategies for new and remanufactured products. De Giovannia et al. (2016) consider a dynamic closed-loop supply chain with one manufacturer and one retailer, where both members invest in a product recovery program to improve the rate of return of old products. More recently, Genc and De Giovann (2020) tackle the impacts of some innovation-led lean programs under a closed-loop supply chain setting.

2.3. Trade-in programs in practice

Several scholars have investigated the application of the trade-in old products programs. Ray et al. (2005) study the optimal pricing and trade-in strategies for durable and remanufactured products, by especially focusing on a case wherein replacement consumers are only interested in trade-ins. Li et al. (2011) concentrate on trade-in programs in business-to-business markets. Ma et al. (2013) study a dual-channel supply chain with a manufacturer, a retailer, and an e-tailer by considering government subsidy. While Ma et al. (2013) focus on trade-in for new products, we consider the application of trade-in program for remanufactured products. Agrawal et al. (2016) examine when and how an original equipment manufacturer should offer a trade-in rebate to collect used products to obtain better price discrimination and weaken competition with third-party remanufacturers. Miao et al. (2017) examine three closed-loop supply chain models: centralized collection, retailer collecting, and manufacturer collecting. Moreover, Miao et al. (2017) also focus on the trade-in for new products. At the same time, Han et al. (2017) research the trade-old-for-remanufactured program. Herein, they consider consumers with three options: using the old product, participating in the trade-old-for-remanufactured program, and buying the new product. However, neither the consumer rebate, nor the impact of double reference effects is considered in these studies.

3. Problems description

From the perspective of consumers, remanufactured products typically have the same or similar performance characteristics as new products do. There is considerable evidence to indicate that consumers do not consider them to be perfect substitutes. In fact, they perceive remanufactured products to have lower quality in comparison (Guide and Li, 2010; Subramanian and Subramanyam, 2012). Given the main notations and assumptions in following Table 1, we will demonstrate the models later.

Table 1. The main model parameters

Parameter	Definition
Δ	Total number of the consumers in the market
θ	Consumers' quality valuation of new product
δ	Consumers' value discount for the remanufactured product
ρ	The proportion of used products from the first period can be collected
λ^i	A Lagrange multiplier used in Lagrange multiplier method
p_n^i	Retail price for the new product in the first period
p_{2n}^i, p_{2r}^i	Retail prices of the new and remanufactured product in the second period, respectively
c_n, c_r	Marginal costs of making the new and remanufactured products, respectively
q_n^i	New product sales in the first period
q_{2n}^i, q_{2r}^i	New and remanufactured product sales in the second period, respectively
τ	The trade-in value of the old product given by the manufacturer
Π_M^i	The profit of the manufacturer
U_n	Consumer utility function for the new product in the first period
U_{2n}^i	Consumer utility function for the new product in the second period
U_{2r}^i	Consumer utility function for the remanufactured product in the second period
φ	Consumer rebate ratio, which equals consumer rebate/the remanufactured product's price
s	The subsidy received by the manufacturer for making one unit of the remanufactured product
β	The reference price parameter of consumers
γ	The reference quality parameter of consumers
$()^*$	Optimal results

Note: $i = \text{I, II, III, IV, and V}$, which represent models I, II, III, IV and V, respectively.

In the following study, the size of potential consumers in the market is Δ . Consumers are heterogeneous in their valuation, θ (i.e., represented as the consumers' quality valuation of new products), uniformly distributed on $[0,1]$. In the first period, the condition for a consumer to buy a new product is that the net utility from buying a new product is nonnegative, that is, $U_n^i = \theta - p_n^i \geq 0$, where $i = \text{I, II, III, IV, and V}$ represent models I, II, III, IV, and V, respectively. Then, the sales quantity of new products is $q_n^i = \Delta(1 - p_n^i)$. In the second period, the consumers decide on choosing new products or trading in for remanufactured ones. Those who choose to trade in for remanufactured products will exhibit the reference quality, reference price, or double reference effects.

Without loss of generality, we normalize the quality of the new product to 1 and denote it by $0 < \delta < 1$, the relative quality of the remanufactured product (Atasu et al., 2008). That is to say, the consumer values the quality of the new product at θ , while he or she values the remanufactured product lower, that is, $\delta\theta$. Now, we demonstrate the consumer utilities in the second period.

We demonstrate Model I (without reference effects) as follows. In the second period, the consumer can choose to buy a new product or trade in for a remanufactured product. If a consumer who bought a new product in the first period chooses the trade-in in the second period, the utility he or she obtains in the second period is:

$$U_{2r}^{\text{I}} = \delta\theta - p_{2r}^{\text{I}} + \tau, \quad (1)$$

where τ is the trade-in value of the old product given by the manufacturer. Otherwise, if the consumer buys a new product in the second period, the utility he or she obtains is:

$$U_{2n}^{\text{I}} = \theta - p_{2n}^{\text{I}} + \tau, \quad (2)$$

where the consumer can sell the old product in the secondary market if he or she chooses to buy a new product in the second period. Here, the unit salvage value that the consumer obtains in the secondary market for the old product is assumed equal to τ .

Similar to Model I, we build utility functions of customers in the following four models respectively, i.e., consumers only consider reference price effects (Model II), consumers only consider reference quality effects (Model III), consumers consider double reference effects with no subsidies (Model IV), and consumers consider double reference effects with subsidies (Model V). In Model

II (with only reference price effects), the consumer who bought a new product in the first period chooses the trade-in in the second period and will only consider the retail price of a new product in the first period as a reference price. In Model III (with only reference quality effects), the consumer who bought a new product in the first period chooses the trade-in in the second period and will only consider the quality of a new product in the first period as a reference quality. In Model IV (with double reference effects and without subsidies), the consumer who bought a new product in the first period chooses the trade-in in the second period and will consider both the retail price and quality of a new product in the first period as the reference price and the reference quality, respectively. In Model V (with double reference effect and subsidies), we not only consider the consumers' double reference effects, but also the impacts of remanufacturing subsidies and the consumer rebate ratio.

4. Models

4.1. Benchmark: Model I, without reference effects

In the second period, if $U_{2r}^I > U_{2n}^I$ and $U_{2r}^I > 0$, the consumer chooses to trade in for a remanufactured product. Following the analytical approach of Chiang et al. (2003), we obtain the sales quantity of the remanufactured product as follows:

$$q_{2r}^I = \Delta \left(\frac{p_{2n}^I - p_{2r}^I}{1 - \delta} - \frac{p_{2r}^I - \tau}{\delta} \right). \quad (3)$$

Similarly, if $U_{2n}^I \geq U_{2r}^I$, the number of consumers who buy the new product is:

$$q_{2n}^I = \Delta \left(1 - \frac{p_{2n}^I - p_{2r}^I}{1 - \delta} \right). \quad (4)$$

We assume that only proportion ρ of used products from the first period can be recycled and remanufactured in the second period (Debo et al., 2005; Geyer et al., 2007; Atasu et al., 2008). The manufacturer sells both new and remanufactured products to the consumers. Then, the manufacturer's objective function during two periods is as follows:

$$\max_{p_n^I, p_{2n}^I, p_{2r}^I} \Pi_M^I = q_n^I (p_n^I - c_n) + (q_{2n}^I (p_{2n}^I - c_n) + q_{2r}^I (p_{2r}^I - c_r - \tau)), \quad (5)$$

$$s.t. \quad q_{2r}^I \leq \rho q_n^I. \quad (6)$$

where q_n^I is the sales quantity of the new product in Period 1, q_{2n}^I is the sales quantity of the new product in Period 2, and q_{2r}^I is the sales quantity of the remanufactured product in Period 2.

The term $q_n^I(p_n^I - c_n)$ in the objective function denotes the manufacturer's profits in Period 1, while $q_{2n}^I(p_{2n}^I - c_n) + q_{2r}^I(p_{2r}^I - c_r - \tau)$ denotes the manufacturer's profits in Period 2. The constraint reveals that proportion ρ of used products from Period 1 can be collected and remanufactured in Period 2.

When both new and remanufactured products are sold, the manufacturer has two pricing options:

1. Retain the low-price strategy of p_{2r}^I (i.e., $p_{2r}^I \leq \delta p_{2n}^I + (1-\delta)\tau$) to sell remanufactured products to consumers. We can derive the respective sales quantities of the new and remanufactured products through Eqs. (3) and (4).

2. Retain the high-price strategy of p_{2r}^I (i.e., $p_{2r}^I > \delta p_{2n}^I + (1-\delta)\tau$) to maximize profits from the new product only. In this case, $U_{2n}^I > U_{2r}^I$ always holds, that is, consumers do not buy the remanufactured product. However, both the government and the manufacturer want to sell the remanufactured product to consumers. Hence, we do not consider this case in our study.

Then, we can obtain the following proposition.

Proposition 1. (i) *When the constraint ($q_{2r}^I \leq \rho q_n^I$) is not binding, the optimal retail prices of new products in Periods 1 and 2, and the optimal retail price of remanufactured product in Period 2 are as follows:*

$$p_n^{I*} = \frac{1}{2}(c_n + 1), p_{2n}^{I*} = \frac{1}{2}(\tau + c_n + 1), p_{2r}^{I*} = \frac{1}{2}\delta + \tau + \frac{1}{2}c_r,$$

(ii) *The constraint ($q_{2r}^I \leq \rho q_n^I$) is binding if $\rho \leq \frac{(c_n - \tau)\delta - c_r}{\delta(1 - c_n)(1 - \delta)}$. The optimal retail prices of new products in Periods 1 and 2, and the optimal retail price of the remanufactured product in Period 2 are as follows:*

$$p_n^{I*} = \frac{(2\delta^2 - 2\delta)\rho^2 + ((-\tau + c_n)\delta - c_r)\rho - c_n - 1}{-2 + (2\delta^2 - 2\delta)\rho^2}, p_{2n}^{I*} = \frac{1}{2}(c_n + 1 + \tau),$$

$$p_{2r}^{I*} = \frac{1}{2}(\delta + 2\tau + c_r) + \frac{\delta(1 - c_n)(1 - \delta)\rho + (\tau - c_n)\delta + c_r}{-2 + (2\delta^2 - 2\delta)\rho^2}.$$

For detailed results and proof, see the Appendix.

Proposition 1 provides the optimal retail prices and sales quantities of new and remanufactured products in two periods when the constraint is binding or not, respectively. Besides, Proposition 1 also provides the manufacturer's optimal profits when the constraint ($q_{2r}^I \leq \rho q_n^I$) is binding or not.

When the constraint ($q_{2r}^I \leq \rho q_n^I$) is not binding, the retail prices of new products in Period 1 only depend on the marginal costs of manufacturing new products. The trade-in value of the old product given by the manufacturer (τ) increases the retail prices of new and remanufactured products in Period 2. Moreover, the higher the customers value for the remanufactured products, the higher the retail prices of remanufactured products. When the constraint ($q_{2r}^I \leq \rho q_n^I$) is binding, the retail prices of new products in Period 2 are the same as those under the case that constraint ($q_{2r}^I \leq \rho q_n^I$) is not binding.

When the constraint ($q_{2r}^I \leq \rho q_n^I$) is binding, we also study the impacts of remanufacturability rate on the optimal profits of the manufacturer (see Figure 1). Figure 1 shows that the manufacturer's profits increases with the remanufacturability rate (ρ) firstly and then decrease with it. This is because that: the sales quantities of remanufactured products increase with the parameter ρ , which results in the increase of the manufacturer's profits. However, when the parameter ρ become relatively large (i.e., $\rho > 0.6$), the competition intensity between new and remanufactured products in Period 2 become higher, which results in the decrease of the manufacturer's profits.

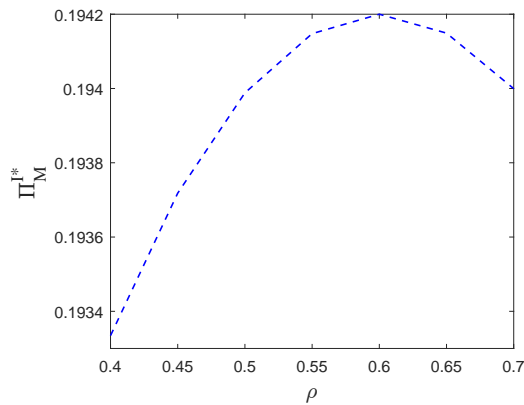


Figure 1: The impacts of ρ on manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02$)

4.2. Model II, with only reference price effects

In Model II, consumers consider the retail price of the new product in Period 1 as the reference price. Thus, if a consumer who bought a new product in the first period chooses to trade in for remanufactured products in the second period, the utility he or she obtains in the second period is:

$$U_{2r}^{\text{II}} = \delta\theta - p_{2r}^{\text{II}} + \tau + \beta(p_n^{\text{II}} - p_{2r}^{\text{II}}), \quad (7)$$

where β is a reference price parameter that shows how consumers view a reference price (i.e., the retail price of the new product in the first period) to be larger than the observed price (i.e., the retail price of the remanufactured product in Period 2). Moreover, parameter β creates an asymmetric effect on consumer utility, and then influences the market demand.

Otherwise, if the consumer chooses to buy a new product in the second period, he or she can sell the old products in the secondary market, and obtain unit salvage value τ . Then, the utility obtained is $U_{2n}^{\text{II}} = \theta - p_{2n}^{\text{II}} + \tau$.

Similar to Model I, we only consider the manufacturer's low-price strategy (i.e., $p_{2r}^{\text{II}} \leq \frac{\beta p_n^{\text{II}} + (-\tau + p_{2n}^{\text{II}})\delta + \tau}{\beta + 1}$) to sell remanufactured products to consumers. If $U_{2r}^{\text{II}} > U_{2n}^{\text{II}}$ and $U_{2r}^{\text{II}} > 0$, consumers choose to trade in for remanufactured products, and we can thus obtain the sales quantity of the remanufactured product as follows:

$$q_{2r}^{\text{II}} = \Delta \left(\frac{\beta(p_n^{\text{II}} - p_{2r}^{\text{II}}) + p_{2n}^{\text{II}} - p_{2r}^{\text{II}}}{1 - \delta} - \frac{-\beta p_n^{\text{II}} + \beta p_{2r}^{\text{II}} - \tau + p_{2r}^{\text{II}}}{\delta} \right). \quad (8)$$

Similarly, if $U_{2r}^{\text{II}} \leq U_{2n}^{\text{II}}$ and $U_{2n}^{\text{II}} \geq 0$, the number of consumers who buy the new product is:

$$q_{2n}^{\text{II}} = \Delta \left(1 - \frac{\beta(p_n^{\text{II}} - p_{2r}^{\text{II}}) + p_{2n}^{\text{II}} - p_{2r}^{\text{II}}}{1 - \delta} \right). \quad (9)$$

The manufacturer's objective function is:

$$\max_{p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}}} \Pi_M^{\text{II}} = q_n^{\text{II}}(p_n^{\text{II}} - c_n) + (q_{2n}^{\text{II}}(p_{2n}^{\text{II}} - c_n) + q_{2r}^{\text{II}}(p_{2r}^{\text{II}} - c_r - \tau)), \quad (10)$$

$$s.t. \quad q_{2r}^{\text{II}} \leq \rho q_n^{\text{II}}. \quad (11)$$

The term $q_n^{\text{II}}(p_n^{\text{II}} - c_n)$ in the objective function denotes the manufacturer's profits in Period 1, while $q_{2n}^{\text{II}}(p_{2n}^{\text{II}} - c_n) + q_{2r}^{\text{II}}(p_{2r}^{\text{II}} - c_r - \tau)$ denotes the manufacturer's profits in Period 2. The constraint reveals that proportion ρ of used products from the first period can be collected and remanufactured in Period 2.

Using the Lagrange multiplier, the optimization problem in Eqs. (10)–(11) can also be represented by Eq. (12):

$$L(p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}}) = q_n^{\text{II}}(p_n^{\text{II}} - c_n) + (q_{2n}^{\text{II}}(p_{2n}^{\text{II}} - c_n) + q_{2r}^{\text{II}}(p_{2r}^{\text{II}} - c_r - \tau)) - \lambda^{\text{II}}(q_{2r}^{\text{II}} - \rho q_n^{\text{II}}). \quad (12)$$

Then, we can obtain the following proposition.

Proposition 2. (i) When the constraint $(q_{2r}^{\text{II}} \leq \rho q_n^{\text{II}})$ is not binding, we can derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.

(ii) The constraint $(q_{2r}^{\text{II}} \leq \rho q_n^{\text{II}})$ is binding if

$\rho \leq \frac{-2(((\tau - c_n + 1)\delta + 2\tau - c_n + 2c_r - 1)\beta + (2\tau - 2c_n)\delta + 2c_r)(\beta + 1)}{(((c_n - 1)\delta^2 + (\tau - c_n + 1)\delta + 2\tau + 2c_r - 2)\beta^2 + ((4c_n - 4)\delta^2 + (2\tau - 6c_n + 4)\delta + 2c_r)\beta + 4\delta(c_n - 1)(\delta - 1))}$. We can also derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.

The detailed results are provided in the Appendix.

Proposition 2 provides the optimal retail prices and sales quantities of new and remanufactured products in two periods and provides the optimal profits of the manufacturer under the model wherein consumers behave with respect to reference price effects. We not only do numerical studies in Section 5 to show business insights, but also conduct the sensitivity analysis in Section 6.

When the constraint $(q_{2r}^{\text{II}} \leq \rho q_n^{\text{II}})$ is binding, we also study the impacts of remanufacturability rate (ρ) and reference price parameter (β) on the optimal profits of the manufacturer (see Figure 2). Figure 2 shows that the manufacturer's profits always increase with the remanufacturability rate in a given range (i.e., $\rho \in [0.4, 0.7]$). Moreover, the larger reference price parameter will benefit the manufacturer.

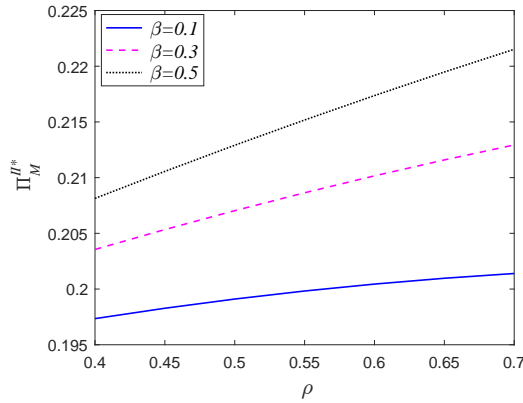


Figure 2: The impacts of ρ and β on manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02$)

4.3. Model III, with only reference quality effects

In Model III, in the second period, the consumer can choose to buy a new product or trade in for a remanufactured product. For the consumer who chooses to purchase the new product in the second period, the utility obtained is $U_{2n}^{\text{III}} = \theta - p_{2n}^{\text{III}} + \tau$. Otherwise, when we consider the consumers' reference quality effects behavior (i.e., the consumer considers the quality valuation of the new product in the first period as a reference quality), if he or she trades in for remanufactured products in the second period. Then, the utility obtained is:

$$U_{2r}^{\text{III}} = \delta\theta - p_{2r}^{\text{III}} + \tau - \gamma(\theta - \delta\theta), \quad (13)$$

where γ is the reference quality parameter that creates an asymmetric effect on consumer utility, and then influences the market demand. Moreover, γ is a parameter corresponding to how consumers view a reference quality that is higher than the observed quality (i.e., the quality valuation of the remanufactured product in the second period).

Similar to Model I, we only consider the manufacturer's low-price strategy (i.e., $p_{2r}^{\text{III}} \leq (p_{2n}^{\text{III}} - \tau)(\delta(\gamma + 1) - \gamma) + \tau$) to sell remanufactured products to consumers. If $U_{2r}^{\text{III}} > U_{2n}^{\text{III}}$ and $U_{2r}^{\text{III}} > 0$, consumers choose to trade in for remanufactured products. From $U_{2r}^{\text{III}} = U_{2n}^{\text{III}}$, we can derive that the consumer whose valuation $\theta = \frac{p_{2n}^{\text{III}} - p_{2r}^{\text{III}}}{(1+\gamma)(1-\delta)}$ is indifferent between the new and remanufactured products. From $U_{2r}^{\text{III}} > 0$, we can derive $\theta > \frac{p_{2r}^{\text{III}} - \tau}{\delta(1+\gamma) - \gamma}$. Hence, the demand of the remanufactured product is:

$$q_{2r}^{\text{III}} = \Delta \left(\frac{p_{2n}^{\text{III}} - p_{2r}^{\text{III}}}{(1+\gamma)(1-\delta)} - \frac{p_{2r}^{\text{III}} - \tau}{\delta(1+\gamma) - \gamma} \right). \quad (14)$$

Similarly, if $U_{2r}^{\text{III}} \leq U_{2n}^{\text{III}}$, the number of consumers who buy the new product is:

$$q_{2n}^{\text{III}} = \Delta \left(1 - \frac{p_{2n}^{\text{III}} - p_{2r}^{\text{III}}}{(1+\gamma)(1-\delta)} \right). \quad (15)$$

The manufacturer starts the second period with the opportunity to recycle ρq_n^{III} cores from product that was sold in Period 1. In addition to the sales quantity of the new product (q_{2n}^{III}), the manufacturer also chooses the number of units to remanufacture, q_{2r}^{III} . With new products in Period 1 and two products on the market in Period 2, the demand functions are given by $q_n^{\text{III}} = \Delta(1 - p_n^{\text{III}})$, Eqs. (14) and (15). Then, the manufacturer's optimization problem during two periods is:

$$\max_{p_n^{\text{III}}, p_{2n}^{\text{III}}, p_{2r}^{\text{III}}} \Pi_M^{\text{III}} = q_n^{\text{III}}(p_n^{\text{III}} - c_n) + (q_{2n}^{\text{III}}(p_{2n}^{\text{III}} - c_n) + q_{2r}^{\text{III}}(p_{2r}^{\text{III}} - c_r - \tau)), \quad (16)$$

$$s.t. \quad q_{2r}^{\text{III}} \leq \rho q_n^{\text{III}}. \quad (17)$$

After solving the manufacturer's optimization problem, we can derive the following proposition:

Proposition 3. (i) When $\lambda^{III} = 0$, constraint (17) is not binding. The optimal retail prices of new products in Periods 1 and 2 as well as the optimal retail price of the remanufactured product in Period 2 are as follows:

$$p_n^{III*} = \frac{1}{2}(c_n + 1), p_{2n}^{III*} = \frac{1}{2}(\tau + c_n + 1), p_{2r}^{III*} = \frac{1}{2}(\gamma\delta - \gamma + \delta + 2\tau + c_r),$$

(ii) Constraint (17) is binding if $\rho \leq \frac{-(\gamma+1)(\tau-c_n)\delta+(\tau-c_n)\gamma-c_r}{(\gamma+1)((\gamma+1)\delta-\gamma)(-1+c_n)(\delta-1)}$. The optimal retail prices of new products in Periods 1 and 2 as well as the optimal retail price of the remanufactured product in Period 2 are as follows:

$$\begin{aligned} p_n^{III*} &= \frac{1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} (2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1) \\ &\quad + (-\gamma + 1)(\tau - c_n)\delta + (\tau - c_n)\gamma - c_r)\rho - c_n - 1), \\ p_{2n}^{III*} &= \frac{1}{2}(\tau + c_n + 1), \\ p_{2r}^{III*} &= \frac{1}{2}(\delta\gamma - \gamma + \delta + 2\tau + c_r) + \frac{1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} \\ &\quad \times ((\delta(\gamma + 1) - \gamma)(c_n - 1)(\gamma + 1)(\delta - 1)\rho - (-\tau + c_n)(\gamma + 1)\delta + (-\tau + c_n)\gamma + c_r). \end{aligned}$$

For detailed results and proof, see Appendix.

Proposition 3 provides the optimal retail prices and sales quantities of new and remanufactured products in two periods and provides the optimal profits of the manufacturer under the model wherein consumers behave with respect to reference quality effects. When the constraint ($q_{2r}^{III} \leq \rho q_n^{III}$) is not binding, the retail prices of new products in Period 1 only depend on the marginal costs of making new products. The trade-in value of the old product given by the manufacturer (τ) also increases the retail prices of new and remanufactured products in Period 2. Moreover, the higher the customers value for the remanufactured products, the higher the retail prices of remanufactured products. When the constraint ($q_{2r}^{III} \leq \rho q_n^{III}$) is binding, the retail prices of new products in Period 2 are the same as those under the case that constraint ($q_{2r}^{III} \leq \rho q_n^{III}$) is not binding.

We now study the impacts of the reference quality parameter on the optimal values of the retail prices and sales quantities if the constraint ($q_{2r}^{III} \leq \rho q_n^{III}$) is not binding. After taking the first derivative of $p_n^{III}, p_{2n}^{III}, p_{2r}^{III}, q_n^{III}, q_{2n}^{III}, q_{2r}^{III}$ with respect to γ , we can derive following proposition.

Proposition 4. (i) The reference quality parameter (γ) does not impact the optimal retail prices of new products in Period 1 and Period 2, and also does not impact the sales quantity of the new product in Period 1; (ii) The optimal retail price of the remanufactured product decreases with the reference quality parameter; (iii) The optimal sales quantity of the new product in Period 2 always increases with the reference quality parameter if $c_n > c_r + \tau$, while the optimal sales quantity of the remanufactured product in Period 2 always decreases with the reference quality parameter.

Proposition 4 shows that consumers' reference quality effects will benefit the sales of the new product in Period 2, but hurt the sales of the remanufactured product in Period 2.

When the constraint ($q_{2r}^{III} \leq \rho q_n^{III}$) is binding, we also study the impacts of remanufacturability rate (ρ) and reference quality parameter (γ) on the optimal profits of the manufacturer (see Figure 3). Figure 3 shows that the manufacturer's profits always decrease with the remanufacturability rate in a given range (i.e., $\rho \in [0.4, 0.7]$). Moreover, the lower reference price parameter will benefit the manufacturer.

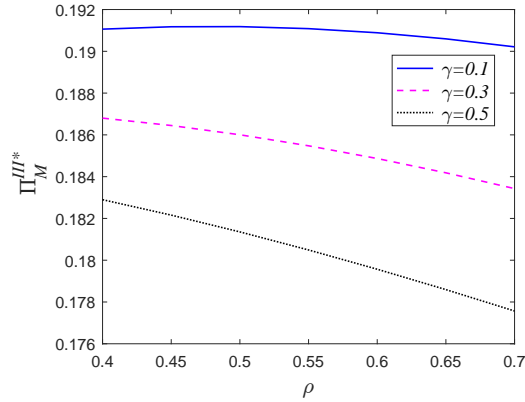


Figure 3: The impacts of ρ and γ on manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02$)

4.4. With double reference effects

4.4.1. Model IV, with double reference effects and without subsidies

In Model IV, we consider the impacts of double reference effects on the pricing decisions of the manufacturer. In Period 2, consumers who bought a new product in Period 1 will choose to buy the new product or participate in the trade-in for remanufactured products program. Thus, if a consumer who bought a new product in the first period chooses the trade-in in the second period,

the utility obtained in the second period is:

$$U_{2r}^{IV} = \delta\theta - p_{2r}^{IV} + \tau + \beta(p_n^{IV} - p_{2r}^{IV}) - \gamma(\theta - \delta\theta). \quad (18)$$

However, if the consumer buys a new product in the second period, the utility obtained is $U_{2n}^{IV} = \theta - p_{2n}^{IV} + \tau$.

Similar to Model I, we only consider the manufacturer's low-price strategy (i.e., $p_{2r}^{IV} \leq \frac{(p_{2n}^{IV} - \tau)(\delta(\gamma+1) - \gamma) + \beta p_n^{IV} + \tau}{\beta + 1}$) to sell remanufactured products to consumers. If $U_{2r}^{IV} > U_{2n}^{IV}$ and $U_{2r}^{IV} > 0$, consumers choose to trade in for remanufactured products. Then, we can obtain the demand of remanufactured product as follows:

$$q_{2r}^{IV} = \Delta \left(\frac{(-p_n^{IV} + p_{2r}^{IV})\beta - p_{2n}^{IV} + p_{2r}^{IV}}{(\gamma + 1)(\delta - 1)} - \frac{-\beta p_n^{IV} + \beta p_{2r}^{IV} - \tau + p_{2r}^{IV}}{\delta\gamma + \delta - \gamma} \right). \quad (19)$$

Similarly, if $U_{2r}^{IV} \leq U_{2n}^{IV}$ and $U_{2n}^{IV} \geq 0$, the number of consumers who buy the new product is:

$$q_{2n}^{IV} = \Delta \left(1 - \frac{(-p_n^{IV} + p_{2r}^{IV})\beta - p_{2n}^{IV} + p_{2r}^{IV}}{(\gamma + 1)(\delta - 1)} \right). \quad (20)$$

The manufacturer's optimization problem is:

$$\max_{p_n^{IV}, p_{2n}^{IV}, p_{2r}^{IV}} \Pi_M^{IV} = q_n^{IV}(p_n^{IV} - c_n) + q_{2n}^{IV}(p_{2n}^{IV} - c_n) + q_{2r}^{IV}(p_{2r}^{IV} - c_r - \tau), \quad (21)$$

$$s.t. \quad q_{2r}^{IV} \leq \rho q_n^{IV}. \quad (22)$$

After solving the manufacturer's optimization problem, we can derive following proposition:

Proposition 5. (i) When the constraint ($q_{2r}^{IV} \leq \rho q_n^{IV}$) is not binding, we can derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.

(ii) The constraint ($q_{2r}^{IV} \leq \rho q_n^{IV}$) is binding if $\rho \leq -\frac{1}{\Phi}(2(((\gamma + 1)(\tau - c_n + 1)\delta + (-\tau + c_n - 1)\gamma + 2\tau - c_n + 2c_r - 1)\beta + 2(\gamma + 1)(\tau - c_n)\delta + (-2\tau + 2c_n)\gamma + 2c_r)(\beta + 1))$, we can also derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.

For parameter Φ , the optimal solutions, and proof, see the Appendix.

Proposition 5 provides the optimal retail prices and sales quantities of new and remanufactured products in two periods and provides the optimal profits of the manufacturer under the model wherein consumers behave with respect to double reference effects.

4.4.2. Model V (with double reference effects and subsidies)

In Model V, the government subsidizes firms to remanufacture products and the firms offer consumers a rebate if they return an old product and buy a remanufactured product. Consumers who bought a new product in Period 1 will choose to buy the new product or participate in the trade-in program for remanufactured products in Period 2. Thus, if a consumer who bought a new product in the first period chooses the trade-in in the second period, the utility obtained in the second period is:

$$U_{2r}^V = \delta\theta - p_{2r}^V + \tau + \varphi(p_{2r}^V - \tau) + \beta(p_n^V - (p_{2r}^V - \varphi(p_{2r}^V - \tau))) - \gamma(\theta - \delta\theta), \quad (23)$$

where φ is the consumer rebate ratio, which equals the consumer rebate/the remanufactured product's unit retail price. If the consumer buys a new product in the second period, the utility obtained is: $U_{2n}^V = \theta - p_{2n}^V + \tau$.

Similar to Model I, we only consider the manufacturer's low-price strategy (i.e., $p_{2r}^V \leq \frac{((\beta+1)\varphi+(\gamma+1)(-1+\delta))\tau - \beta p_n^V - p_{2n}^V(\delta(\gamma+1) - \gamma)}{(\varphi-1)(\beta+1)}$) to sell remanufactured products to consumers. If $U_{2r}^V > U_{2n}^V$ and $U_{2r}^V > 0$, the consumers choose to trade in remanufactured products. Then, we can derive the demand of the remanufactured product as follows:

$$q_{2r}^V = \Delta \left(\frac{-(\varphi-1)(\beta+1)p_{2r}^V + \tau(\beta+1)\varphi - \beta p_n^V - p_{2n}^V}{(\gamma+1)(\delta-1)} - \frac{-(\varphi-1)(\beta+1)p_{2r}^V + \tau(\beta+1)\varphi - \beta p_n^V - \tau}{\delta(\gamma+1) - \gamma} \right).$$

Similarly, if $U_{2r}^V \leq U_{2n}^V$ and $U_{2n}^V \geq 0$, the number of consumers who buy the new product is:

$$q_{2n}^V = \Delta \left(1 - \frac{-(\varphi-1)(\beta+1)p_{2r}^V + \tau(\beta+1)\varphi - \beta p_n^V - p_{2n}^V}{(\gamma+1)(\delta-1)} \right).$$

The manufacturer's optimization problem is:

$$\max_{p_n^V, p_{2n}^V, p_{2r}^V} \Pi_M^V = q_n^V(p_n^V - c_n) + q_{2n}^V(p_{2n}^V - c_n) + q_{2r}^V(p_{2r}^V - c_r - \tau - \varphi(p_{2r}^V - \tau) + s), \quad (24)$$

$$s.t. \quad q_{2r}^V \leq \rho q_n^V. \quad (25)$$

Using the Lagrange multiplier, the optimization problem in Eqs. (24)–(25) can also be represented by following problem:

$$L(p_n^V, p_{2n}^V, p_{2r}^V) = q_n^V(p_n^V - c_n) + q_{2n}^V(p_{2n}^V - c_n) + q_{2r}^V(p_{2r}^V - c_r - \tau - \varphi(p_{2r}^V - \tau) + s) - \lambda^V(q_{2r}^V - \rho q_n^V).$$

where s is the subsidy received by the manufacturer for making one unit of a remanufactured product. Here, we assume s is an exogenous variable in our model (Miao et al., 2017; Han et al., 2017).

After solving the manufacturer's optimization problem, we can derive the following proposition:

Proposition 6. (i) *When the constraint ($q_{2r}^V \leq \rho q_n^V$) is not binding, we can derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.*

(ii) *The constraint ($q_{2r}^V \leq \rho q_n^V$) is binding if $\rho \leq -\frac{1}{\Psi}(4(\beta + 1)((-0.5(\gamma + 1)(\tau - c_n + 1)\delta + (0.5\tau - 0.5c_n + 0.5)\gamma + s - \tau + 0.5c_n - c_r + 0.5)\beta - (\gamma + 1)(\tau - c_n)\delta + (\tau - c_n)\gamma + s - c_r))$. Then, we can also derive the optimal retail prices and sales quantities of new products in Periods 1 and 2, the optimal retail price and sales quantity of the remanufactured product in Period 2, and the manufacturer's optimal profits.*

For the parameter Ψ , the optimal solutions, and proof, see the Appendix.

Proposition 6 provides the optimal retail prices and sales quantities of new and remanufactured products in two periods and provides the optimal profits of the manufacturer under the model wherein consumers behave with respect to double reference effects. Two types of government incentives are considered: the remanufacturing subsidy and the consumer rebate.

The optimal values of Propositions 2, 3, 5 and 6 are very complex, but their expressions do exist, thus we will use numerical studies to investigate the impacts of model parameters and compare different models in the next section.

5. Analysis

In this section, we only consider the constraint of the remanufactured product's quantity to be binding. We first compare Model I with Models II, III, and IV, and then study the impacts of reference price parameter (β) and reference quality parameter (γ) on the retail prices, sales quantities, and manufacturer's profits, respectively. Then, we investigate the impacts of reference effects parameters on the consumer surplus. Finally, we study the impacts of the remanufacturing subsidy (s) and the consumer rebate ratio (φ) on the retail prices, sales quantities, and the manufacturer's profits under Model V.

5.1. Model comparison

5.1.1. Impacts of β on prices, sales quantities, and profits (Model I vs. Model II)

We now investigate the impacts of the reference price parameter (β) on the optimal retail prices, sales quantities, and the manufacturer's profits.

Both the retail prices of new products in Periods 1 and 2 decrease with the reference price parameter, while the retail prices of remanufactured products in Period 2 increase with the reference price parameter (See Figure 4). Correspondingly, both the sales quantities of new products in Periods 1 and 2 increase with the reference price parameter, and the sales quantities of remanufactured products in Period 2 also increase with the reference price parameter (See Figure 5).

Figure 6 illustrates the impacts of the reference price parameter (β) on the manufacturer's profits. The manufacturer's profits always increase with the reference price parameter. In other words, the reference price effects are beneficial to the manufacturer.

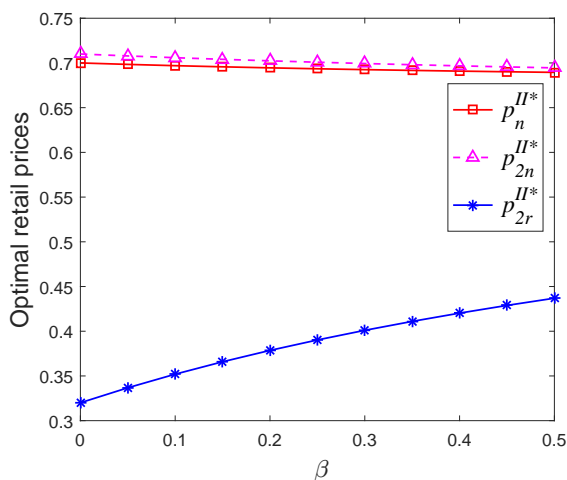


Figure 4: The impacts of β on retail prices

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$

$\tau = 0.02, \rho = 0.6.$ (Atasu et al., 2008)

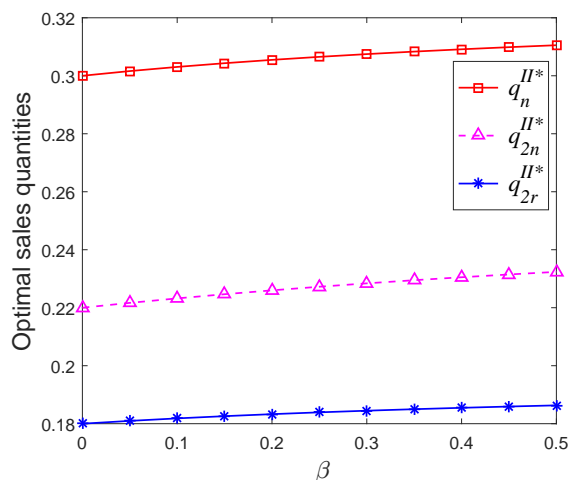


Figure 5: The impacts of β on sales quantities

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$

$\tau = 0.02, \rho = 0.6$

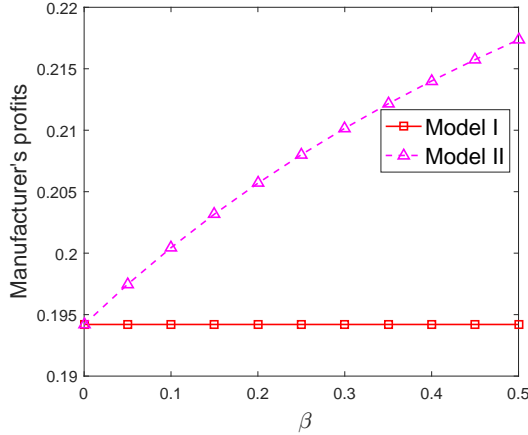


Figure 6: The impacts of β on manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02, \rho = 0.6$)

5.1.2. Impacts of γ on prices, sales quantities, and profits (Model I vs. Model III)

We now investigate the impacts of reference quality parameter (γ) on the retail prices, sales quantities, and the manufacturer's profits.

Figure 7 illustrates how the retail prices of new products in Period 1 increase smoothly with the reference quality parameter, while the retail prices of remanufactured products in Period 2 decrease quickly with the reference quality parameter. The intuition is that, with the quality valuation of new products in Period 1 as a reference quality, consumers feel more loss when the reference quality parameter becomes larger. Moreover, the differences between the retail prices of new products in Periods 1 and 2 are small, and the retail prices of new products in Period 1 are higher than those in Period 2 when $\gamma > \underline{\gamma} \approx 0.21$. Figure 8 illustrates how the sales quantities of new products in Period 2 increase with the reference quality parameter, while both the sales quantities of new products in Period 1 and remanufactured products in Period 2 decrease with the consumers' reference quality effects.

In Figures 7 and 8, the manufacturer can retain the retail prices of new products in Period 2 at a constant, but the sales quantities of new products in Period 2 can be increased. This result is different from our general understanding. Surprisingly, both the retail prices and sales quantities of remanufactured products in Period 2 decrease with the reference quality parameter. Thus, the manufacturer should raise consumer awareness of the remanufactured products' quality to reduce reference quality effects.

We also compare the manufacturer's profits between Models I and III in Figure 9, and find that

the manufacturer's profits always decrease with the reference quality parameter.

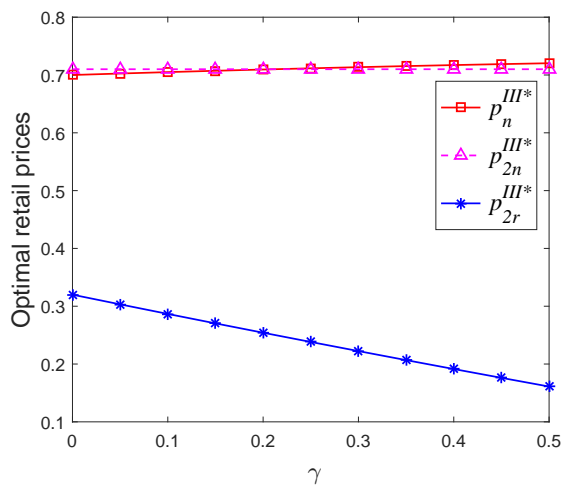


Figure 7: The impacts of γ on retail prices

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$
 $\tau = 0.02, \rho = 0.6$

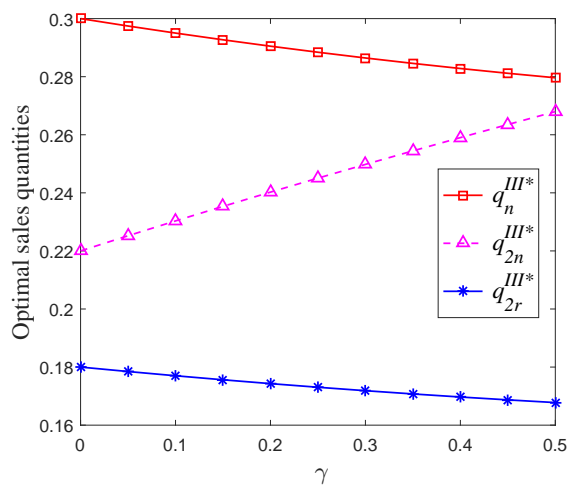


Figure 8: The impacts of γ on sales quantities

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$
 $\tau = 0.02, \rho = 0.6$

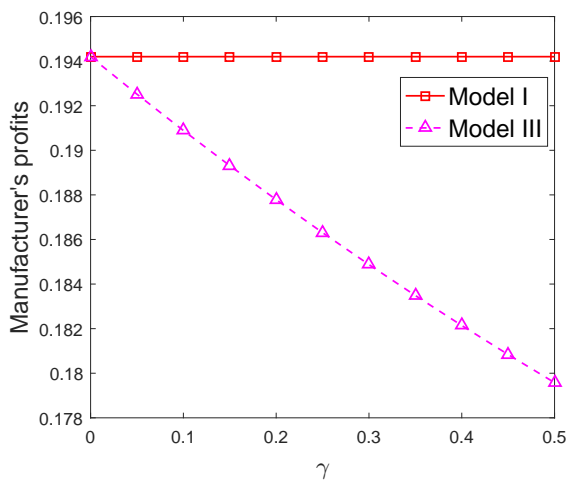


Figure 9: The impacts of γ on the manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02, \rho = 0.6$)

5.1.3. Impacts of β and γ on prices, sales quantities, and profits (Model I vs. Model IV)

In this section, we investigate the impacts of reference effects parameters β and γ on the retail prices, sales quantities, and the manufacturer's profits.

In Figure 10, both the reference effects parameters will impact the retail prices of new and remanufactured products. Especially, the impacts of the reference quality parameter (γ) on the retail prices of remanufactured products are larger than those on the retail prices of new products in both periods (Figure 10). Figure 11 illustrates how the sales quantities of new products in Period 1 are the largest, while those of remanufactured products in Period 2 are the smallest.

Figure 12 illustrates how the manufacturer's profits under Model IV are the highest when the reference price parameter is relatively large, and the reference quality parameter is relatively small. Thus, the manufacturer should improve consumer awareness of price differentiation between the new and remanufactured products, and reduce their awareness of quality differentiation between the new and remanufactured products.

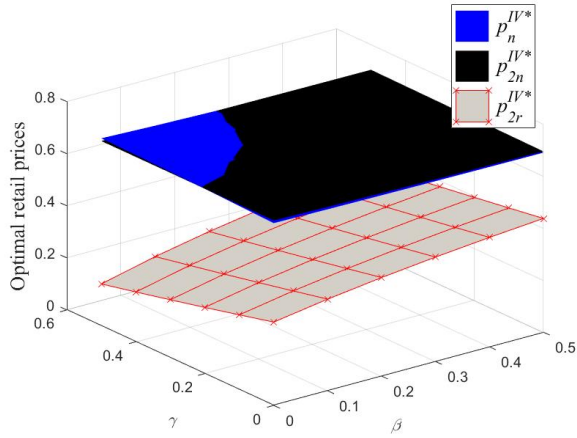


Figure 10: The impacts of β and γ on retail prices

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$

$\tau = 0.02, \rho = 0.6$

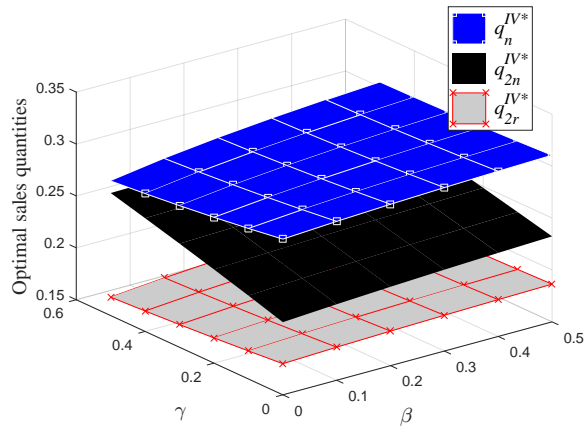


Figure 11: The impacts of β and γ on sales quantities

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$

$\tau = 0.02, \rho = 0.6$

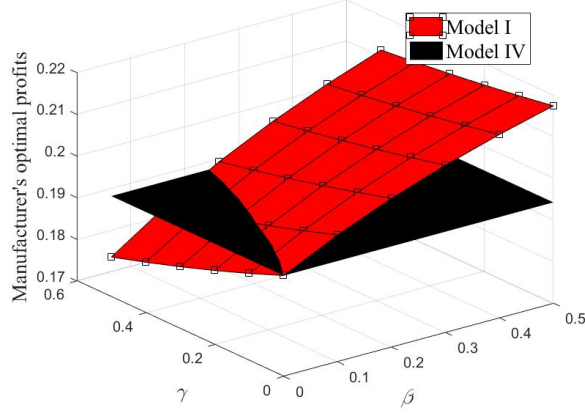


Figure 12: The impacts of β and γ on profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02, \rho = 0.6$)

5.2. Impacts of β and γ on consumer surplus

We now address the impacts of double reference effects on the consumer surplus under Models I, II, III, IV, and V, respectively. After substituting optimal retail prices of Proposition 1 into following equation CS^{I^*} , we can derive the optimal consumer surplus under Model I:

$$CS^{I^*} = \Delta \left(\int_{p_n^{I^*}}^1 (\theta - p_n^{I^*}) d\theta + \int_{\frac{p_{2r}^{I^*} - \tau}{\delta}}^{\frac{p_{2n}^{I^*} - p_{2r}^{I^*}}{1 - \delta}} (\delta\theta - p_{2r}^{I^*} + \tau) d\theta + \int_{\frac{p_{2n}^{I^*} - p_{2r}^{I^*}}{1 - \delta}}^1 (\theta - p_{2n}^{I^*} + \tau) d\theta \right),$$

Similarly, we can substitute the optimal retail prices under Models II, III, IV, and V into equations $CS^{II^*}, CS^{III^*}, CS^{IV^*}$ and CS^{V^*} , respectively, and derive the corresponding optimal consumer

surplus:

$$\begin{aligned}
CS^{\text{II}^*} &= \Delta \left(\int_{p_n^{\text{II}^*}}^1 (\theta - p_n^{\text{II}^*}) d\theta + \int_{\frac{-\beta p_n^{\text{II}^*} + \beta p_{2r}^{\text{II}^*} - \tau + p_{2r}^{\text{II}^*}}{\delta}}^{\frac{\beta(p_n^{\text{II}^*} - p_{2r}^{\text{II}^*}) + p_{2n}^{\text{II}^*} - p_{2r}^{\text{II}^*}}{1-\delta}} (\delta\theta - p_{2r}^{\text{II}^*} + \tau + \beta(p_n^{\text{II}^*} - p_{2r}^{\text{II}^*})) d\theta \right. \\
&\quad \left. + \int_{\frac{\beta(p_n^{\text{II}^*} - p_{2r}^{\text{II}^*}) + p_{2n}^{\text{II}^*} - p_{2r}^{\text{II}^*}}{1-\delta}}^1 (\theta - p_{2n}^{\text{II}^*} + \tau) d\theta \right), \\
CS^{\text{III}^*} &= \Delta \left(\int_{p_n^{\text{III}^*}}^1 (\theta - p_n^{\text{III}^*}) d\theta + \int_{\frac{p_{2n}^{\text{III}^*} - p_{2r}^{\text{III}^*}}{(1+\gamma)(1-\delta)}}^{\frac{p_{2n}^{\text{III}^*} - p_{2r}^{\text{III}^*}}{\delta(1+\gamma) - \gamma}} (\delta\theta - p_{2r}^{\text{III}^*} + \tau - \gamma(\theta - \delta\theta)) d\theta \right. \\
&\quad \left. + \int_{\frac{p_{2n}^{\text{III}^*} - p_{2r}^{\text{III}^*}}{(1+\gamma)(1-\delta)}}^1 (\theta - p_{2n}^{\text{III}^*} + \tau) d\theta \right), \\
CS^{\text{IV}^*} &= \Delta \left(\int_{p_n^{\text{IV}^*}}^1 (\theta - p_n^{\text{IV}^*}) d\theta + \int_{\frac{-\beta p_n^{\text{IV}^*} + \beta p_{2r}^{\text{IV}^*} - \tau + p_{2r}^{\text{IV}^*}}{\delta\gamma + \delta - \gamma}}^{\frac{(-p_n^{\text{IV}^*} + p_{2r}^{\text{IV}^*})\beta - p_{2n}^{\text{IV}^*} + p_{2r}^{\text{IV}^*}}{(\gamma+1)(\delta-1)}} (\delta\theta - p_{2r}^{\text{IV}^*} + \tau + \beta(p_n^{\text{IV}^*} - p_{2r}^{\text{IV}^*}) - \gamma(\theta - \delta\theta)) d\theta \right. \\
&\quad \left. + \int_{\frac{(-p_n^{\text{IV}^*} + p_{2r}^{\text{IV}^*})\beta - p_{2n}^{\text{IV}^*} + p_{2r}^{\text{IV}^*}}{(\gamma+1)(\delta-1)}}^1 (\theta - p_{2n}^{\text{IV}^*} + \tau) d\theta \right), \\
CS^{\text{V}^*} &= \Delta \left(\int_{p_n^{\text{V}^*}}^1 (\theta - p_n^{\text{V}^*}) d\theta + \int_{\frac{-\beta p_n^{\text{V}^*} + \beta p_{2r}^{\text{V}^*} - \tau + p_{2r}^{\text{V}^*}}{\delta(\gamma+1) - \gamma}}^{\frac{-(\varphi-1)(\beta+1)p_{2r}^{\text{V}^*} + \tau(\beta+1)\varphi - \beta p_n^{\text{V}^*} - p_{2n}^{\text{V}^*}}{(\gamma+1)(\delta-1)}} (\delta\theta - p_{2r}^{\text{V}^*} + \tau + \varphi(p_{2r}^{\text{V}^*} - \tau) \right. \\
&\quad \left. + \beta(p_n^{\text{V}^*} - (p_{2r}^{\text{V}^*} - \varphi(p_{2r}^{\text{V}^*} - \tau))) - \gamma(\theta - \delta\theta)) d\theta + \int_{\frac{-(\varphi-1)(\beta+1)p_{2r}^{\text{V}^*} + \tau(\beta+1)\varphi - \beta p_n^{\text{V}^*} - p_{2n}^{\text{V}^*}}{(\gamma+1)(\delta-1)}}^1 (\theta - p_{2n}^{\text{V}^*} + \tau) d\theta \right).
\end{aligned}$$

We now study the impacts of the reference price parameter (β) and reference quality parameter (γ) on the consumer surplus under Models I, II, III, and IV. The difference between Model IV and Model V is that the impacts of the remanufacturing subsidy and consumer rebate ratio are considered in Model V. However, the optimal solution structures of Models IV and V are similar. Thus, the analysis of Model V is omitted here.

Figure 13 illustrates that the consumer surplus increases quickly with the reference price parameter (β) when β is relatively small, and increases smoothly with β when β is relatively large. Further, the consumer surplus decreases in the reference quality parameter (γ) with almost the same decreasing rate.

Figure 14 illustrates that the consumer surplus under Model IV is the lowest when the reference price parameter is relatively small and the reference quality parameter is relatively large. On the other hand, the consumer surplus under Model IV obtains the highest when the reference price parameter is relatively larger and the reference quality parameter is relatively smaller. Compared with

Figure 12, the impacts of the double reference effects on the manufacturer's profits and consumer surplus are similar.

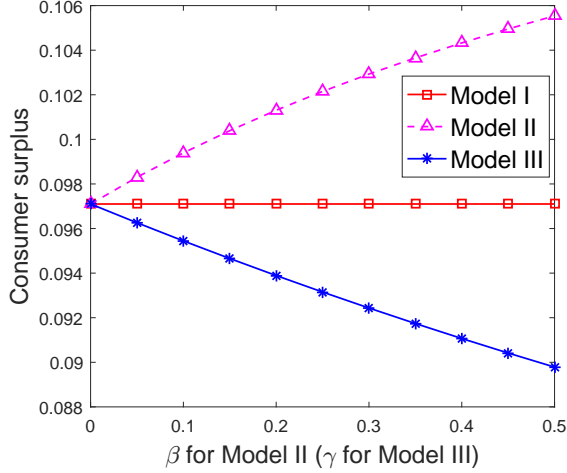


Figure 13: The impacts of β and γ on consumer surplus

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$
 $\tau = 0.02, \rho = 0.6$

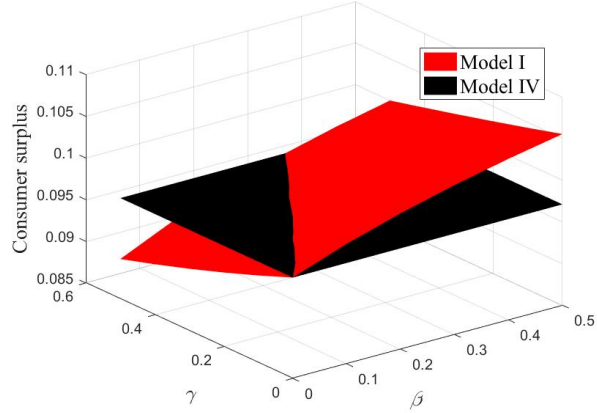


Figure 14: The impacts of β and γ on consumer surplus under Model IV

Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1,$
 $\tau = 0.02, \rho = 0.6$

5.3. Impacts of subsidies on retail prices, sales quantities and profits under Model V

5.3.1. Impacts of the remanufacturing subsidy (s)

We now study the impact of the remanufacturing subsidy on the optimal retail prices, sales quantities, and the manufacturer's profits. Figure 15 shows that the retail prices of new products in both periods and the retail prices of remanufactured products in Period 2 will always decrease with s . This is because a larger s induces the manufacturer to lower the remanufactured products' retail prices, which, in turn, puts a downward pressure on the new products' retail prices in Period 2 and influences the pricing decisions of new products in Period 1. Figure 16 illustrates that the sales quantities of remanufactured products increase with s , while those of new products in Period 2 decrease with s . In Figure 17, the manufacturer's profits increase with s . Further, the manufacturer obtains higher profits under Model V than under Model I when $s = 0$. The consumer rebate ratio does not affect the manufacturer's profits (see section 5.3.2), and hence when $s = 0$, the manufacturer's profits under Model V are similar to those under Model IV. This is because the

point (β, γ) is in the decision space wherein the manufacturer benefits from the reference effects, which further validates Figure 12.

Generally speaking, the remanufacturing subsidy not only impacts the decisions of the current period, but also those of the earlier period. It also changes the competitive intensity of new and remanufactured products.

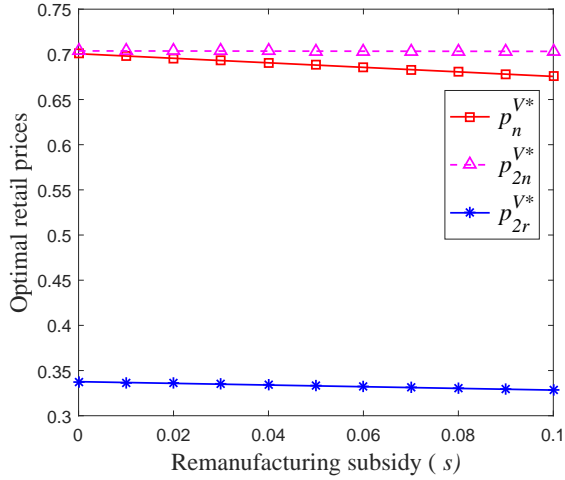


Figure 15: The impacts of s on retail prices
 Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02,$
 $\rho = 0.6, \varphi = 0.1, \beta = 0.3$ and $\gamma = 0.4$

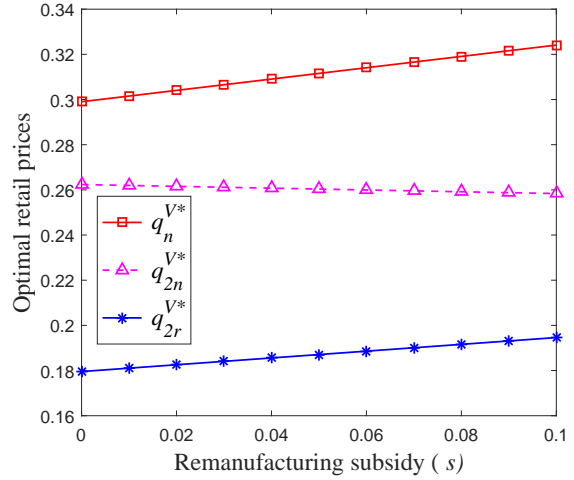


Figure 16: The impacts of s on sales quantities
 Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02,$
 $\rho = 0.6, \varphi = 0.1, \beta = 0.3$ and $\gamma = 0.4$

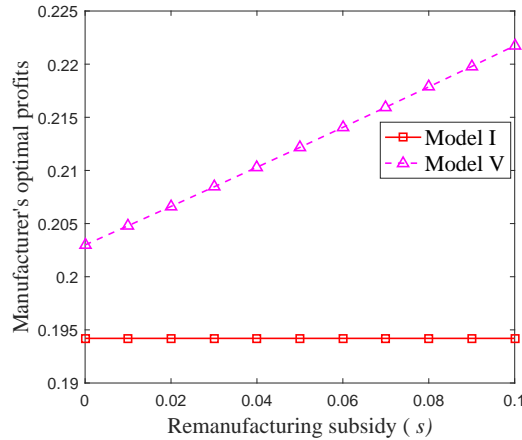


Figure 17: The impacts of s on manufacturer's profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02, \rho = 0.6, \varphi = 0.1, \beta = 0.3$ and $\gamma = 0.4$)

5.3.2. Impacts of the consumer rebate ratio (φ)

We now study the impacts of the consumer rebate ratio (φ). Figure 18 also shows that the retail prices of remanufactured products always increase with the consumer rebate ratio. However, higher retail prices and higher costs offset each other. Moreover, we can also derive that: Improving the consumer rebate ratio cannot change the sales quantities of the new and remanufactured products in Period 2, which results in no change in the manufacturer's profits. Here, we omit related Figures about later result.

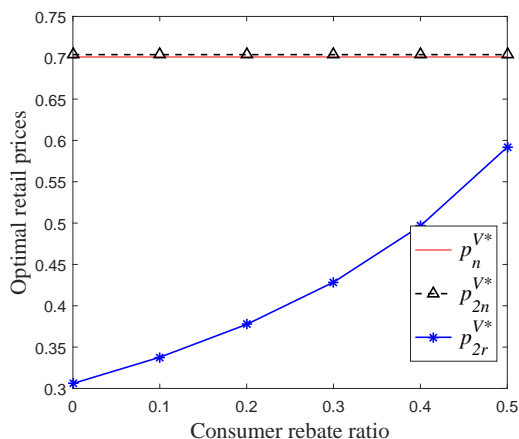


Figure 18: The impacts of φ on retail prices (Note: $\delta = 0.5$, $\Delta = 1$, $c_n = 0.4$, $c_r = 0.1$, $\tau = 0.02$, $\rho = 0.6$, $s = 0$, $\beta = 0.3$ and $\gamma = 0.4$)

In the next section, we will do sensitivity analysis to study the impacts of reference price parameter on the extra profits considering reference price effects. We also study the impacts of reference quality parameter on the extra profits considering reference price effects.

6. Sensitivity Analysis

Compared with Model I, this section focuses on Model II and Model III to investigate the impacts of reference price parameter (β) and reference quality parameter (γ) on the profitability of double reference effects with different unit remanufacturing costs, different remanufacturability rates, and different customers' discount rates for remanufactured products, respectively.

6.1. The impacts of β and γ on the extra profits considering reference effects with different unit remanufacturing costs

Figure 19 addresses the impacts of reference price parameter (β) on the manufacturer's extra profits considering reference price effects under different unit remanufacturing costs. Figure 19 shows that the manufacturer can obtain more extra profits when the unit remanufacturing cost is relatively low.

On the other hand, Figure 20 addresses the impacts of reference quality parameter (γ) on the manufacturer's extra profits considering reference quality effects under different unit remanufacturing costs. Figure 20 shows that the manufacturer can lose less profits when the unit remanufacturing cost is relatively high, which is a surprising and interesting result. Higher unit remanufacturing cost can offset some of the negative impacts of reference quality effects.

Now, we explain why the result above is suitable and correct. Higher unit remanufacturing cost will make the manufacturer improve the retail prices of remanufactured products, which results in the decrease of sales quantities of remanufactured products. Recall that the reference quality effects have a negative impact on the performance of selling remanufactured products (Figures 7 and 8), and the manufacturer's profits decrease with the reference quality effects (See Figure 9), thus decrease of sales quantities of remanufactured products will benefit the manufacturer.

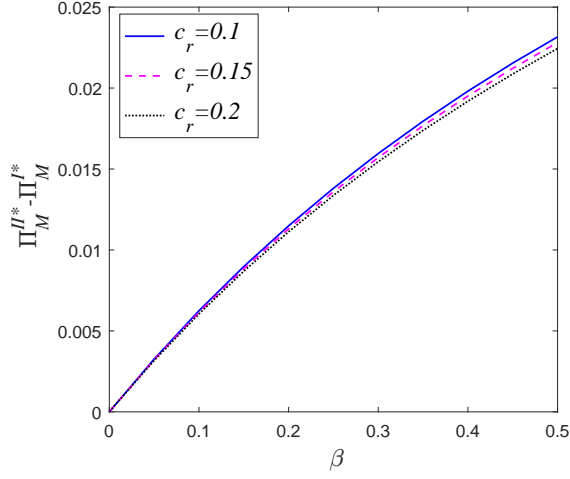


Figure 19: The impacts of β on the manufacturer's extra profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, \tau = 0.02$, and $\rho = 0.6$)

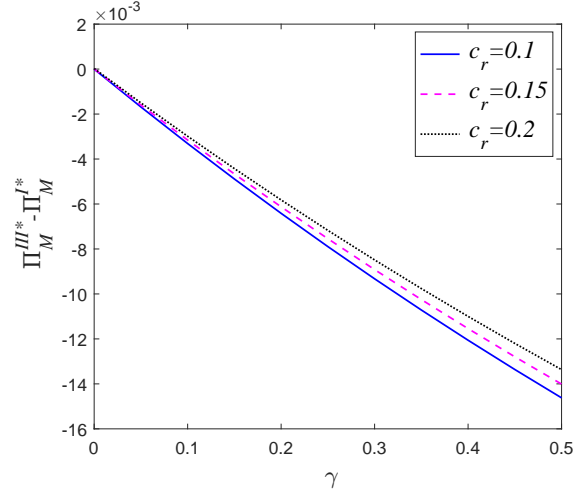


Figure 20: The impacts of γ on the manufacturer's extra profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, \tau = 0.02$, and $\rho = 0.6$)

6.2. The impacts of β and γ on the extra profits considering reference effects with different remanufacturability rates

Figure 21 investigates the impacts of reference price parameter (β) on the manufacturer's extra profits considering reference price effects under different remanufacturability rates. Figure 21 shows that the manufacturer can obtain more extra profits considering reference price effects when the remanufacturability rate is relatively high.

On the other hand, Figure 22 investigates the impacts of reference quality parameter (γ) on the manufacturer's extra profits considering reference quality effects under different remanufacturability rates. Figure 22 shows that the manufacturer can lose less profits when the remanufacturability rate is relatively low, which is surprising and interesting. Lower remanufacturability rate can offset some of the negative impacts of reference quality effects.

Now, we explain why the result above is suitable and correct. Lower remanufacturability rate means that the sales quantities of remanufactured products are small. Recall that the reference quality effects have a negative impact on the performance of selling remanufactured products (Figures 7 and 8), and the manufacturer's profits decrease with the reference quality effects (See Figure 9), thus lower remanufacturing rate will benefit the manufacturer.

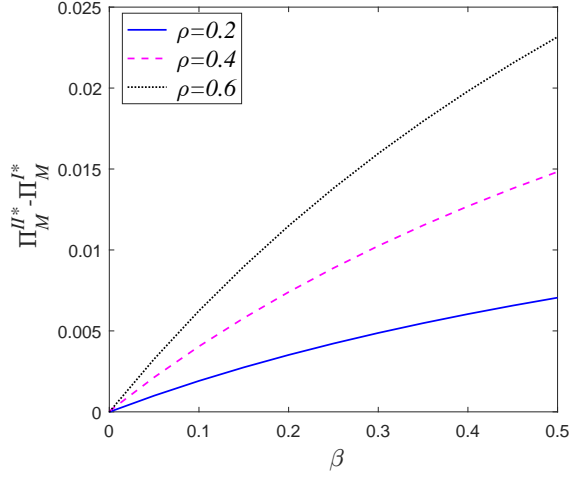


Figure 21: The impacts of β on the manufacturer's extra profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1$ and $\tau = 0.02$)

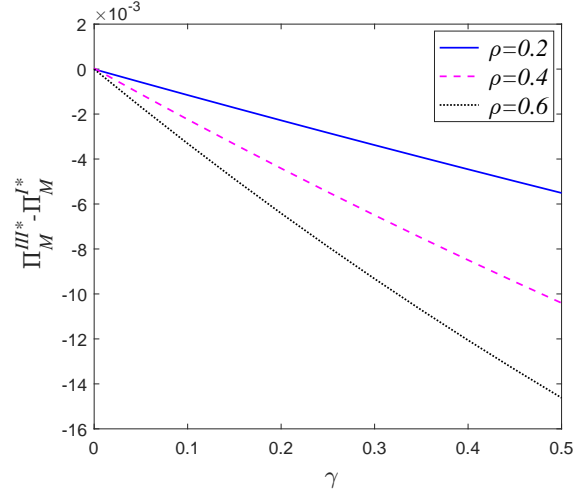


Figure 22: The impacts of γ on the manufacturer's extra profits (Note: $\delta = 0.5, \Delta = 1, c_n = 0.4, c_r = 0.1$ and $\tau = 0.02$)

6.3. The impacts of β and γ on the extra profits considering reference effects with different discount rates for the remanufactured products

Considering customers' different discount rates for the remanufactured products, Figure 23 investigates the impacts of reference price parameter on the manufacturer's extra profits with reference price effects. Figure 23 shows that the manufacturer can obtain more extra profits from reference price effects when customers' discount rate for the remanufactured products is relatively low.

Similarly, Figure 24 investigates the impacts of reference quality parameter on the manufacturer's extra profits considering reference quality effects. Figure 24 shows that the manufacturer will lose less profits when the discount rate is relatively high, which is a surprising and interesting result. High discount rate can offset some of the negative impacts of reference quality effects.

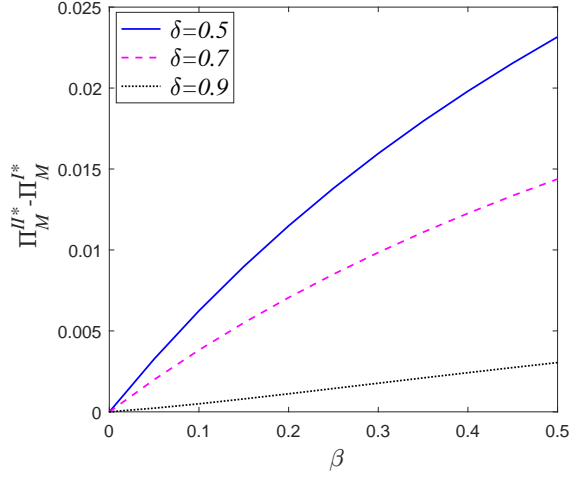


Figure 23: The impacts of β on the manufacturer's extra profits (Note: $\Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02$, and $\rho = 0.6$)

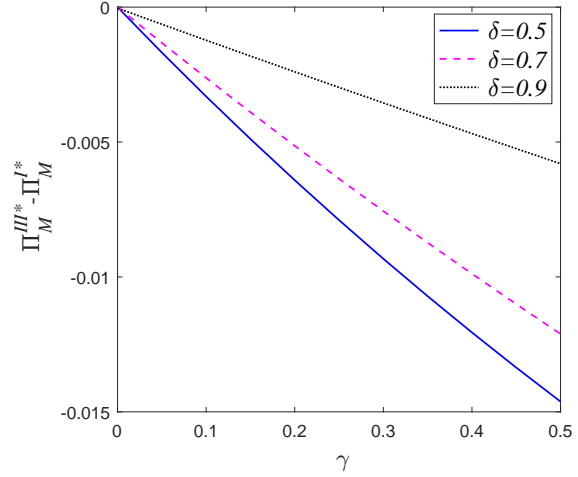


Figure 24: The impacts of γ on the manufacturer's extra profits (Note: $\Delta = 1, c_n = 0.4, c_r = 0.1, \tau = 0.02, \rho = 0.6$)

7. Concluding remarks

The main contribution of this research is that it provides the managerial insights on the impacts of double reference effects with respect to pricing decisions and manufacturer's performance of the trade-in program for remanufactured products. Particularly, we examine how the pricing decisions are affected by only considering the consumers' reference price effects: We find that this effect benefits the manufacturer. When the reference price parameter is relatively small, the manufacturer's profits increase relatively quickly. Second, we examine the impacts of the reference quality effects on the pricing decisions and manufacturer's performance. We find that the manufacturer's profits always decrease with the consumers' reference quality effects. We also derive the threshold value wherein the retail prices of new products in Period 1 are higher than those of new products in Period 2. Third, we consider the double reference effects, and find that they benefit the manufacturer when the reference price parameter is relatively larger and the reference quality parameter is relatively smaller.

Furthermore, upon investigating the impacts of the double reference effects on consumer surplus in five scenarios, we find that consumer surplus increases with the reference price effects in Model II. However, it decreases with the reference quality effects in Model III. Here, the double reference

effects benefit the consumers in Model IV when the reference price parameter is relatively larger and the reference quality parameter is relatively smaller.

Then, we study the impacts of the remanufacturing subsidies and consumer rebate ratios on the pricing decisions and manufacturer's performance. We find that remanufacturing subsidy is beneficial to the manufacturer. The consumer rebate ratio improves the retail prices of remanufactured products, but it does not impact the retail prices of new products, sales quantities, and the manufacturer's profits.

Finally, we focus on Model II and Model III, and study the impacts of double reference parameters on the profitability of double reference effects with different unit remanufacturing cost, different remanufacturability rates, and different customers' discount rates for remanufactured products. We find that lower unit remanufacturing cost, higher remanufacturability rates, and lower consumers' discount rates for remanufactured products benefit the manufacturer if the customers only consider reference price effects. Higher unit remanufacturing cost, lower remanufacturability rates, and higher customers' discount rates benefit the manufacturer if only the reference quality effects behavior is considered.

We admit the limitation of some theoretical analysis due to complex calculation, since we consider more factors in double reference effects, which are nontrivial with new management practice. The results are validated with sensitivity analysis. Future research could consider heterogeneous consumers, who can make different valuations for remanufactured products. We can also extend our model to multiple markets wherein there is a competitor offering alternative remanufactured products or new products to consumers.

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Appendix

Proof of Proposition 1. To solve the optimization problem in Eqs. (5)–(6), we use the Lagrange multiplier and obtain the Lagrangian and the Karush–Kuhn–Tucker optimality conditions as follows:

$$\begin{aligned}
L(p_n^I, p_{2n}^I, p_{2r}^I) &= q_n^I(p_n^I - c_n) + (q_{2n}^I(p_{2n}^I - c_n) + q_{2r}^I(p_{2r}^I - c_r - \tau)) - \lambda^I(q_{2r}^I - \rho q_n^I), \\
\frac{\partial L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_n^I} &= -\Delta(\rho\lambda^I - c_n + 2p_n^I - 1) = 0, \\
\frac{\partial L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2n}^I} &= \frac{\Delta(2p_{2n}^I - c_n - 1 + \delta - 2p_{2r}^I + c_r + \tau + \lambda^I)}{-1 + \delta} = 0, \\
\frac{\partial L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2r}^I} &= \frac{\Delta((\tau + c_n - 2p_{2n}^I)\delta - 2\tau - c_r + 2p_{2r}^I - \lambda^I)}{(-1 + \delta)\delta} = 0. \\
\lambda^I(q_{2r}^I - \rho q_n^I) &= 0, q_{2r}^I - \rho q_n^I \leq 0, \lambda^I \geq 0.
\end{aligned}$$

We can obtain the second order derivatives as follows:

$$\begin{aligned}
\frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial (p_n^I)^2} &= -2\Delta, \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_n^I \partial p_{2n}^I} = \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_n^I \partial p_{2r}^I} = 0, \\
\frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2n}^I \partial p_n^I} &= 0, \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial (p_{2n}^I)^2} = -\frac{2\Delta}{1 - \delta}, \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2n}^I \partial p_{2r}^I} = \frac{2\Delta}{1 - \delta}, \\
\frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2r}^I \partial p_n^I} &= 0, \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial p_{2r}^I \partial p_{2n}^I} = \frac{2\Delta}{1 - \delta}, \text{ and } \frac{\partial^2 L(p_n^I, p_{2n}^I, p_{2r}^I)}{\partial (p_{2r}^I)^2} = -\frac{2\Delta}{(1 - \delta)\delta}.
\end{aligned}$$

Due to $|H_1| = -2\Delta < 0$, we find that the third-order determinant of the Hessian is $|H_3| = -\frac{8\Delta^3}{(1-\delta)\delta} < 0$. The second-order determinant of the Hessian is $|H_2| = \frac{4\Delta^2}{1-\delta} > 0$. Thus, the first order condition of $L(p_n^I, p_{2n}^I, p_{2r}^I)$ yields the optimal solutions as follows:

$$p_n^I(\lambda^I) = \frac{1}{2}(-\lambda^I \rho + c_n + 1), p_{2n}^I = \frac{1}{2}(c_n + 1 + \tau), p_{2r}^I(\lambda^I) = \frac{1}{2}(\delta + 2\tau + \lambda^I + c_r)$$

We now consider two cases, namely, case 1: $\lambda^I = 0$ and $q_{2r}^I - \rho q_n^I \neq 0$; and case 2: $\lambda^I \neq 0$ and $q_{2r}^I - \rho q_n^I = 0$.

(i) When $\lambda^I = 0$, the constraint (6) is not binding, and we substitute $\lambda^I = 0$ into $p_n^I(\lambda^I)$ and $p_{2r}^I(\lambda^I)$, and then substitute $p_n^I, p_{2n}^I, p_{2r}^I$ into $q_n^I, q_{2n}^I, q_{2r}^I$ and Π_M^I . Thus, we can obtain the optimal solutions as follows:

$$\begin{aligned} p_n^{I*} &= \frac{1}{2}(c_n + 1), p_{2n}^{I*} = \frac{1}{2}(\tau + c_n + 1), p_{2r}^{I*} = \frac{1}{2}\delta + \tau + \frac{1}{2}c_r, \\ q_n^{I*} &= \frac{1}{2}\Delta(1 - c_n), q_{2n}^{I*} = \frac{\Delta(-1 + \delta - \tau + c_n - c_r)}{-2 + 2\delta}, q_{2r}^{I*} = \frac{\Delta((\tau - c_n)\delta + c_r)}{2(-1 + \delta)\delta}, \\ \Pi_M^{I*} &= \frac{\Delta((-c_n^2 - 2\tau + 4c_n - 2)\delta^2 + (2c_n^2 + (-2\tau - 2c_r - 4)c_n + 2 + \tau^2 + (2c_r + 2)\tau)\delta + c_r^2)}{4(1 - \delta)\delta}. \end{aligned}$$

(ii) When $\lambda^I \neq 0$, the constraint (6) is binding if $\rho \leq \frac{(c_n - \tau)\delta - c_r}{\delta(1 - c_n)(1 - \delta)}$, substituting $p_n^I(\lambda^I)$ and $p_{2r}^I(\lambda^I)$ into q_n^I and q_{2r}^I . Then, we can derive $q_n^I(\lambda^I) = \frac{1}{2}\Delta(\rho\lambda^I - c_n + 1)$ and $q_{2r}^I(\lambda^I) = \frac{\Delta((\tau - c_n)\delta + c_r + \lambda^I)}{2(-1 + \delta)\delta}$. We substitute $q_n^I(\lambda^I)$ and $q_{2r}^I(\lambda^I)$ into $q_{2r}^I - \rho q_n^I = 0$, and obtain that $\lambda^{I*} = \frac{\rho(c_n - 1)\delta^2 + ((1 - c_n)\rho + \tau - c_n)\delta + c_r}{\delta^2\rho^2 - \delta\rho^2 - 1}$. After substituting λ^{I*} into $p_n^I, p_{2n}^I, p_{2r}^I, q_n^I, q_{2n}^I, q_{2r}^I$ and Π_M^I , we can obtain the main results as follows:

$$\begin{aligned} p_n^{I*} &= \frac{(2\delta^2 - 2\delta)\rho^2 + ((-\tau + c_n)\delta - c_r)\rho - c_n - 1}{-2 + (2\delta^2 - 2\delta)\rho^2}, p_{2n}^{I*} = \frac{1}{2}(c_n + 1 + \tau), \\ p_{2r}^{I*} &= \frac{1}{2}(\delta + 2\tau + c_r) + \frac{\delta(1 - c_n)(1 - \delta)\rho + (\tau - c_n)\delta + c_r}{-2 + (2\delta^2 - 2\delta)\rho^2}, \\ q_n^{I*} &= \frac{\Delta(((\tau - c_n)\delta + c_r)\rho + c_n - 1)}{-2 + (2\delta^2 - 2\delta)\rho^2}, \\ q_{2n}^{I*} &= -\frac{\Delta(-\delta^2\rho^2 + \rho((\tau - c_n + c_r + 1)\rho + c_n - 1)\delta + \tau - c_n + 1)}{2\delta^2\rho^2 - 2\delta\rho^2 - 2}, \\ q_{2r}^{I*} &= \frac{\Delta\rho(((\tau - c_n)\delta + c_r)\rho + c_n - 1)}{-2 + (2\delta^2 - 2\delta)\rho^2}, \end{aligned}$$

and $\Pi_M^{I*} = \frac{1}{4 - (4\delta^2 - 4\delta)\rho^2} (\Delta(((-2\tau + 2c_n - 1)\delta^2 + (c_n^2 + (-2\tau - 2c_r - 2)c_n + 1 + \tau^2 + (2c_r + 2)\tau)\delta + c_r^2)\rho^2 + 2(c_n - 1)((\tau - c_n)\delta + c_r)\rho + 2c_n^2 + (-2\tau - 4)c_n + \tau^2 + 2\tau + 2))$.

□

Proof of Proposition 2. Similar to Model I, we use the Lagrange multiplier and obtain the Lagrangian and the Karush–Kuhn–Tucker optimality conditions:

$$\begin{aligned}
L(p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}}) &= q_n^{\text{II}}(p_n^{\text{II}} - c_n) + q_{2n}^{\text{II}}(p_{2n}^{\text{II}} - c_n) + q_{2r}^{\text{II}}(p_{2r}^{\text{II}} - c_r - \tau) - \lambda^{\text{II}}(q_{2r}^{\text{II}} - \rho q_n^{\text{II}}), \\
\frac{\partial L(p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}})}{\partial p_n^{\text{II}}} &= \frac{\Delta}{(1 - \delta)\delta} ((\rho\lambda^{\text{II}} - c_n + 2p_n^{\text{II}} - 1)\delta^2 + ((-p_{2n}^{\text{II}} + c_n)\beta - \rho\lambda^{\text{II}} + c_n - 2p_n^{\text{II}} + 1)\delta \\
&\quad + \beta(p_{2r}^{\text{II}} - \lambda^{\text{II}} - \tau - c_r)) = 0, \\
\frac{\partial L(p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}})}{\partial p_{2n}^{\text{II}}} &= \frac{\Delta((- \beta - 2)p_{2r}^{\text{II}} + \beta p_n^{\text{II}} + \tau + \lambda^{\text{II}} + \delta - c_n + c_r + 2p_{2n}^{\text{II}} - 1)}{-1 + \delta} = 0, \\
\frac{\partial L(p_n^{\text{II}}, p_{2n}^{\text{II}}, p_{2r}^{\text{II}})}{\partial p_{2r}^{\text{II}}} &= \frac{\Delta}{(-1 + \delta)\delta} (((-p_{2n}^{\text{II}} + c_n)\delta - \tau - \lambda^{\text{II}} - c_r - p_n^{\text{II}} + 2p_{2r}^{\text{II}})\beta + (\tau + c_n - 2p_{2n}^{\text{II}})\delta \\
&\quad - 2\tau - \lambda^{\text{II}} - c_r + 2p_{2r}^{\text{II}}) = 0, \\
\lambda^{\text{II}}(q_{2r}^{\text{II}} - \rho q_n^{\text{II}}) &= 0, q_{2r}^{\text{II}} - \rho q_n^{\text{II}} \leq 0, \lambda^{\text{II}} \geq 0.
\end{aligned}$$

Following the proof of Proposition 1, we can prove the concavity of the above objective function, since the only constraint is convex. Thus, we can obtain the optimal solutions by solving the first order conditions. After substituting the optimal values of p_n^{II} , p_{2n}^{II} , and p_{2r}^{II} into the sales quantities of new and remanufactured products, and then into the profit of the manufacturer, the detailed results are as follows:

(i) When $\lambda^{\text{II}} = 0$, the constraint (11) is not binding. The optimal solutions can be summarized as follows:

$$\begin{aligned}
p_n^{\text{II}*} &= \frac{1}{2(\beta+2)^2\delta^2+(-8\beta-8)\delta+2\beta^2} ((\beta+2)^2(c_n+1)\delta^2 + ((\tau-c_n+1)\beta^2 + (2\tau-6c_n-4)\beta - 4c_n-4)\delta + \\
&\quad 2\beta((\tau+c_r)\beta+c_r)), \\
p_{2n}^{\text{II}*} &= \frac{1}{2(\beta+2)^2\delta^2+(-8\beta-8)\delta+2\beta^2} ((2\beta^2c_n + (2\tau+6c_n+4)\beta + 4\tau+4c_n+4)\delta^2 + ((-2\tau+c_n-2c_r+1)\beta^2 + \\
&\quad (-4\tau-4c_n-2c_r-4)\beta - 4\tau-4c_n-4)\delta + \beta^2(\tau+c_n+1)), \\
p_{2r}^{\text{II}*} &= \frac{1}{2(\beta+2)^2\delta^2+(-8\beta-8)\delta+2\beta^2} ((2\beta+4)\delta^3 + ((c_n+1)\beta^2 + (2\tau+4c_n+2c_r)\beta + 8\tau+4c_r-4)\delta^2 + ((\tau- \\
&\quad c_n+1)\beta^2 + (-4\tau-2c_n-4c_r-2)\beta - 8\tau-4c_r)\delta + 2\beta^2(\tau+c_r)), \\
q_n^{\text{II}*} &= -\frac{\Delta}{2(\beta+2)^2\delta^2+(-8\beta-8)\delta+2\beta^2} ((\beta+2)^2(c_n-1)\delta^2 + ((\tau-c_n+1)\beta^2 + (2\tau-6c_n+4)\beta - 4c_n+4)\delta + \\
&\quad 2((\tau+c_r-1)\beta+c_r)\beta), \\
q_{2n}^{\text{II}*} &= -\frac{\Delta}{2((\beta+2)^2\delta^2+(-4\beta-4)\delta+\beta^2)} ((2(-\beta-2)\delta^2 + ((2\tau-c_n+2c_r-1)\beta^2 + 2(4\tau-3c_n+3c_r+1)\beta + \\
&\quad 4\tau-4c_n+4c_r+4)\delta - (\tau-c_n+1)\beta^2)), \\
q_{2r}^{\text{II}*} &= \frac{((\tau-c_n+1)\delta+2\tau-c_n+2c_r-1)\beta+(2\tau-2c_n)\delta+2c_r)\Delta(\beta+1)}{(\delta^2+1)\beta^2+(4\delta^2-4\delta)\beta+4\delta^2-4\delta}, \\
\Pi_M^{\text{II}*} &= -\frac{\Delta}{4((\beta+2)^2\delta^2+(-4\beta-4)\delta+\beta^2)} (((-1-c_n)^2\beta^2 + 4(-c_n^2-\tau+3c_n-2)\beta - 4c_n^2-8\tau+16c_n-8)\delta^2 +
\end{aligned}$$

$$((4\tau - 2c_n + 4c_r - 2)(\tau - c_n + 1)\beta^2 + 4(3c_n^2 + (-5\tau - 3c_r - 3)c_n + 3\tau^2 + (3c_r + 1)\tau + c_r + 2)\beta + 8c_n^2 + 4(-2\tau - 2c_r - 4)c_n + 8 + 4\tau^2 + 4(2c_r + 2)\tau)\delta + (-c_n^2 + (6 - 2\tau - 4c_r)c_n + 6\tau^2 + (-6 + 8c_r)\tau - 4c_r + 4c_r^2 - 1)\beta^2 + 2(4\tau - 2c_n + 4c_r - 2)c_r\beta + 4c_r^2).$$

(ii) The constraint (11) is binding if

$$\rho \leq \frac{-2(((\tau - c_n + 1)\delta + 2\tau - c_n + 2c_r - 1)\beta + (2\tau - 2c_n)\delta + 2c_r)(\beta + 1)}{((c_n - 1)\delta^2 + (\tau - c_n + 1)\delta + 2\tau + 2c_r - 2)\beta^2 + ((4c_n - 4)\delta^2 + (2\tau - 6c_n + 4)\delta + 2c_r)\beta + 4\delta(c_n - 1)(\delta - 1)}. \text{ The optimal solutions are as follows:}$$

$$p_{2n}^{\text{II}^*} = \frac{1}{((\delta^2\beta^2 + (4\delta - 4)\beta + 4\delta - 4)\delta\rho^2 + (-4\beta^2 - 4\beta)\rho - 4(\beta + 1)^2)}((\delta\beta^2 + (4\delta - 4)\beta + 4\delta - 4)\delta\rho^2 - (((\tau - c_n + 1)\delta + 2\tau + 2c_r + 2)\beta + (2\tau - 2c_n)\delta + 2c_r)(\beta + 1)\rho - 2(\beta + 1)^2(c_n + 1)),$$

$$p_{2n}^{\text{II}^*} = \frac{1}{((\delta^2\rho^2 - 4\rho - 4)\beta^2 + (-8 + (4\delta^2 - 4\delta)\rho^2 - 4\rho)\beta - 4 + (4\delta^2 - 4\delta)\rho^2)}((- \delta(-\delta c_n + \tau + c_r - 1)\rho^2 + ((-c_n + 1)\delta - 2\tau - 2c_n - 2)\rho - 2\tau - 2c_n - 2)\beta^2 + (((\tau + 3c_n + 2)\delta - 2\tau - 2c_n - c_r - 2)\delta\rho^2 + ((-c_n + 1)\delta - 2\tau - 2c_n - 2)\rho - 4\tau - 4c_n - 4)\beta + 2(-1 + (\delta^2 - \delta)\rho^2)(\tau + c_n + 1)),$$

$$p_{2r}^{\text{II}^*} = \frac{1}{\delta((\beta + 2)^2\delta - 4\beta - 4)\rho^2 + (-4\beta^2 - 4\beta)\rho - 4(\beta + 1)^2}(((\beta + 2)\delta^2 + (\beta^2 + (\tau + c_n + c_r + 1)\beta + 4\tau + 2c_r - 2)\delta + (-2\tau - 2c_r - 2)\beta - 4\tau - 2c_r)\delta\rho^2 + ((c_n - 1)(\beta + 2)\delta^2 + ((-\tau + c_n - 1)\beta^2 - 2\beta c_n - 2c_n + 2)\delta - 2((\tau + c_r + 1)\beta + 2\tau + c_r)\beta)\rho + 2(\beta + 1)((\tau - c_n - 1)\delta + (-c_n - 1)\beta - 2\tau)),$$

$$q_n^{\text{II}^*} = \frac{\Delta}{(\delta^2\rho^2 - 4\rho - 4)\beta^2 + (-8 + (4\delta^2 - 4\delta)\rho^2 - 4\rho)\beta - 4 + (4\delta^2 - 4\delta)\rho^2}(((\tau - c_n + 1)\delta + 2\tau + 2c_r - 2)\rho + 2c_n - 2)\beta + ((2\tau - 2c_n)\delta + 2c_r)\rho + 2c_n - 2)(\beta + 1),$$

$$q_{2n}^{\text{II}^*} = -\frac{\Delta}{\rho^2(\beta + 2)^2\delta^2 - 4\rho^2(\beta + 1)\delta - 4(1 + (\rho + 1)\beta)(\beta + 1)}((- \rho^2(\beta + 2)\delta^2 + (((\tau + c_r - 1)\rho + c_n - 1)\beta^2 + ((4\tau - 2c_n + 3c_r)\rho + 3c_n - 3)\beta + (2\tau - 2c_n + 2c_r + 2)\rho + 2c_n - 2)\rho\delta + 2(1 + (\rho + 1)\beta)(\tau - c_n + 1)(\beta + 1))),$$

$$q_{2r}^{\text{II}^*} = \frac{\Delta}{(\delta\beta^2 + (4\delta - 4)\beta + 4\delta - 4)\delta\rho^2 + (-4\beta^2 - 4\beta)\rho - 4(\beta + 1)^2}(((\tau - c_n + 1)\delta + 2\tau + 2c_r - 2)\beta + (2\tau - 2c_n)\delta + 2c_r)\rho + 2(c_n - 1)(\beta + 1))(\beta + 1)\rho,$$

$$\lambda^{\text{II}^*} = \frac{1}{((\delta^2\rho^2 - 4\rho - 4)\beta^2 + (4\delta^2\rho^2 - 4\delta\rho^2 - 4\rho - 8)\beta + 4\delta^2\rho^2 - 4\delta\rho^2 - 4)}((\rho(c_n - 1)\delta^2 + (\tau - c_n + 1)(\rho + 2)\delta + (2\tau + 2c_r - 2)\rho + 4\tau - 2c_n + 4c_r - 2)\beta^2 + (4\rho(c_n - 1)\delta^2 + ((2\tau - 6c_n + 4)\rho + 6\tau - 6c_n + 2)\delta + 2\rho c_r + 4\tau - 2c_n + 8c_r - 2)\beta + 4\rho(c_n - 1)\delta^2 + ((-4c_n + 4)\rho + 4\tau - 4c_n)\delta + 4c_r),$$

$$\Pi_M^{\text{II}^*} = -\frac{1}{((\delta\beta^2 + (4\delta - 4)\beta + 4\delta - 4)\delta\rho^2 + (-4\beta^2 - 4\beta)\rho - 4(\beta + 1)^2)}((((\tau - c_n + 1)\delta + \tau + c_r - 1)(\tau + c_r - 1)\beta^2 + ((-\tau + c_n - 1)\delta^2 + (c_n^2 + (-4\tau - 3c_r)c_n + 3\tau^2 + 3\tau c_r + c_r + 1)\delta + 2c_r(\tau + c_r - 1))\beta + (-2\tau + 2c_n - 1)\delta^2 + (c_n^2 + (-2\tau - 2c_r - 2)c_n + 1 + \tau^2 + (2c_r + 2)\tau)\delta + c_r^2)\rho^2 + (\beta + 1)((c_n - 1)(\tau - c_n + 1)\delta + c_n^2 + (2c_r - 4)c_n + \tau^2 - 2c_r + 3)\beta + 2(c_n - 1)((\tau - c_n)\delta + c_r))\rho + (2c_n^2 + (-2\tau - 4)c_n + \tau^2 + 2\tau + 2)(\beta + 1)^2)\Delta).$$

□

Proof of Proposition 3. Similar to Model I, we use the Lagrange multiplier and obtain the

Lagrangian and the Karush–Kuhn–Tucker optimality conditions:

$$\begin{aligned}
L(p_n^{\text{III}}, p_{2n}^{\text{III}}, p_{2r}^{\text{III}}) &= q_n^{\text{III}}(p_n^{\text{III}} - c_n) + (q_{2n}^{\text{III}}(p_{2n}^{\text{III}} - c_n) + q_{2r}^{\text{III}}(p_{2r}^{\text{III}} - c_r - \tau)) - \lambda^{\text{III}}(q_{2r}^{\text{III}} - \rho q_n^{\text{III}}), \\
\frac{\partial L(p_n^{\text{III}}, p_{2n}^{\text{III}}, p_{2r}^{\text{III}})}{\partial p_n^{\text{III}}} &= -\Delta(\rho\lambda^{\text{III}} - c_n + 2p_n^{\text{III}} - 1) = 0, \\
\frac{\partial L(p_n^{\text{III}}, p_{2n}^{\text{III}}, p_{2r}^{\text{III}})}{\partial p_{2n}^{\text{III}}} &= \frac{\Delta((\gamma + 1)\delta + \tau - \gamma - c_n + c_r + 2p_{2n}^{\text{III}} - 2p_{2r}^{\text{III}} + \lambda^{\text{III}} - 1)}{(\gamma + 1)(\delta - 1)} = 0, \\
\frac{\partial L(p_n^{\text{III}}, p_{2n}^{\text{III}}, p_{2r}^{\text{III}})}{\partial p_{2r}^{\text{III}}} &= \frac{\Delta}{((\gamma + 1)\delta - \gamma)(\gamma + 1)(\delta - 1)}((\gamma + 1)(\tau + c_n - 2p_{2n}^{\text{III}})\delta \\
&+ (-\tau - c_n + 2p_{2n}^{\text{III}})\gamma - 2\tau - c_r + 2p_{2r}^{\text{III}} - \lambda^{\text{III}}) = 0. \\
\lambda^{\text{III}}(q_{2r}^{\text{III}} - \rho q_n^{\text{III}}) &= 0, q_{2r}^{\text{III}} - \rho q_n^{\text{III}} \leq 0, \lambda^{\text{III}} \geq 0.
\end{aligned}$$

Similar to the proof of Proposition 1, we consider following two cases, namely, case 1: $\lambda^{\text{III}} = 0$ and $q_{2r}^{\text{III}} - \rho q_n^{\text{III}} \neq 0$; and case 2: $\lambda^{\text{III}} \neq 0$ and $q_{2r}^{\text{III}} - \rho q_n^{\text{III}} = 0$. Then, we can obtain following results:

(i) When $\lambda^{\text{III}} = 0$, the constraint (17) is not binding. The optimal solutions can be summarized as follows:

$$\begin{aligned}
p_n^{\text{III}*} &= \frac{1}{2}(c_n + 1), p_{2n}^{\text{III}*} = \frac{1}{2}(\tau + c_n + 1), p_{2r}^{\text{III}*} = \frac{1}{2}(\gamma\delta - \gamma + \delta + 2\tau + c_r), \\
q_n^{\text{III}*} &= \frac{1}{2}\Delta(1 - c_n), q_{2n}^{\text{III}*} = \frac{\Delta((-\gamma - 1)\delta + \tau + \gamma - c_n + c_r + 1)}{2(\gamma + 1)(1 - \delta)}, \\
q_{2r}^{\text{III}*} &= \frac{\Delta((\gamma + 1)(\tau - c_n)\delta + (-\tau + c_n)\gamma + c_r)}{2((\gamma + 1)\delta - \gamma)(\delta - 1)(\gamma + 1)}, \\
\Pi_M^{\text{III}*} &= \frac{1}{4}\Delta(1 - c_n)^2 - \frac{\Delta((\delta - 1)\gamma + c_n - c_r - \tau + \delta - 1)(-\tau + c_n - 1)}{4(\gamma + 1)(\delta - 1)} \\
&\quad - \frac{\Delta((\delta - 1)(-\tau + c_n)\gamma + (-\tau + c_n)\delta - c_r)((\delta - 1)\gamma - c_r + \delta)}{4(\gamma + 1)((\delta - 1)\gamma + \delta)(\delta - 1)}.
\end{aligned}$$

(ii) The constraint (17) is binding if $\rho \leq \frac{-(\gamma+1)(\tau-c_n)\delta+(\tau-c_n)\gamma-c_r}{(\gamma+1)((\gamma+1)\delta-\gamma)(-1+c_n)(\delta-1)}$. The optimal solutions are

as follows:

$$\begin{aligned}
p_n^{\text{III}^*} &= \frac{1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} (2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1) \\
&\quad + (-\gamma + 1)(\tau - c_n)\delta + (\tau - c_n)\gamma - c_r)\rho - c_n - 1), \\
p_{2n}^{\text{III}^*} &= \frac{1}{2}(\tau + c_n + 1), \\
p_{2r}^{\text{III}^*} &= \frac{1}{2}(\delta\gamma - \gamma + \delta + 2\tau + c_r) + \frac{1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} \\
&\quad \times ((\delta(\gamma + 1) - \gamma)(c_n - 1)(\gamma + 1)(\delta - 1)\rho - (-\tau + c_n)(\gamma + 1)\delta + (-\tau + c_n)\gamma + c_r), \\
q_n^{\text{III}^*} &= \frac{\Delta((\gamma + 1)(\tau - c_n)\delta + (-\tau + c_n)\gamma + c_r)\rho + c_n - 1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)}, \\
q_{2n}^{\text{III}^*} &= -\frac{\Delta}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} ((\delta(\gamma + 1) - \gamma)((-\gamma - 1)\delta + \tau + \gamma - c_n + c_r + 1)\rho^2 \\
&\quad + (\delta(\gamma + 1) - \gamma)(c_n - 1)\rho + \tau - c_n + 1), \\
q_{2r}^{\text{III}^*} &= \frac{\rho\Delta((\gamma + 1)(\tau - c_n)\delta + (-\tau + c_n)\gamma + c_r)\rho + c_n - 1}{-2 + 2\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)}, \\
\lambda^{\text{III}^*} &= \frac{(\delta(\gamma + 1) - \gamma)(c_n - 1)(\gamma + 1)(\delta - 1)\rho + (\gamma + 1)(\tau - c_n)\delta + (-\tau + c_n)\gamma + c_r}{\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1) - 1},
\end{aligned}$$

and

$$\begin{aligned}
\Pi_M^{\text{III}^*} &= -\frac{1}{-4 + 4\rho^2(\delta(\gamma + 1) - \gamma)(\gamma + 1)(\delta - 1)} (((-2\tau - 2c_n + 1)(\gamma + 1)^2\delta^2 + ((4\tau - 4c_n + 2)\gamma \\
&\quad + c_n^2 + (-2\tau - 2c_r - 2)c_n + 1 + \tau^2 + (2c_r + 2)\tau)(\gamma + 1)\delta + (-2\tau + 2c_n - 1)\gamma^2 \\
&\quad + (-c_n^2 + (2\tau + 2c_r + 2)c_n - 1 - \tau^2 + (-2c_r - 2)\tau)\gamma + c_r^2)\rho^2 + 2((\gamma + 1)(\tau - c_n)\delta \\
&\quad + (-\tau + c_n)\gamma + c_r)(c_n - 1)\rho + 2c_n^2 + (-2\tau - 4)c_n + \tau^2 + 2\tau + 2)\Delta).
\end{aligned}$$

□

Proof of Proposition 4. Taking the first derivative of $p_n^{\text{III}^*}, p_{2n}^{\text{III}^*}, p_{2r}^{\text{III}^*}, q_n^{\text{III}^*}, q_{2n}^{\text{III}^*}, q_{2r}^{\text{III}^*}$ with respect to γ , we can derive the following results: $\frac{\partial p_n^{\text{III}^*}}{\partial \gamma} = \frac{\partial p_{2n}^{\text{III}^*}}{\partial \beta} = \frac{\partial q_n^{\text{III}^*}}{\partial \beta} = 0, \frac{\partial p_{2r}^{\text{III}^*}}{\partial \beta} = -\frac{1}{2}(1 - \delta) < 0; \frac{\partial q_{2n}^{\text{III}^*}}{\partial \gamma} = \frac{(c_n - c_r - \tau)\Delta}{2(\beta + 1)^2(1 - \delta)} > 0$ if $c_n > c_r + \tau; \frac{\partial q_{2r}^{\text{III}^*}}{\partial \gamma} = \frac{((\beta + 1)^2(\tau - c_n)\delta^2 - 2((\tau - c_n)\beta - c_r)(\beta + 1)\delta + (\tau - c_n)\beta^2 - 2\beta c_r - c_r)\Delta}{2(1 - \delta)((\beta + 1)\delta - \beta)^2(\beta + 1)^2} < 0$ for all γ . □

Proof of Proposition 5. Similar to Model I, we use the Lagrange multiplier and obtain the

Lagrangian and the Karush–Kuhn–Tucker optimality conditions:

$$\begin{aligned}
L(p_n^{\text{IV}}, p_{2n}^{\text{IV}}, p_{2r}^{\text{IV}}) &= q_n^{\text{IV}}(p_n^{\text{IV}} - c_n) + q_{2n}^{\text{IV}}(p_{2n}^{\text{IV}} - c_n) + q_{2r}^{\text{IV}}(p_{2r}^{\text{IV}} - c_r - \tau) - \lambda^{\text{IV}}(q_{2r}^{\text{IV}} - \rho q_n^{\text{IV}}), \\
\frac{\partial L(p_n^{\text{IV}}, p_{2n}^{\text{IV}}, p_{2r}^{\text{IV}})}{\partial p_n^{\text{IV}}} &= \Delta(-p_n^{\text{IV}} + c_n) - \Delta(-1 + p_n^{\text{IV}}) + \frac{\Delta\beta(p_{2n}^{\text{IV}} - c_n)}{(\gamma + 1)(\delta - 1)} \\
&+ \frac{\Delta\beta(-p_{2r}^{\text{IV}} + c_r + \tau)}{(\gamma + 1)(\delta - 1)((\gamma + 1)\delta - \gamma)} - \frac{\lambda^{\text{IV}}((\gamma + 1)((\gamma + 1)\delta - \gamma)(\delta - 1)\rho - \beta)\Delta}{(\gamma + 1)((\gamma + 1)\delta - \gamma)(\delta - 1)} = 0, \\
\frac{\partial L(p_n^{\text{IV}}, p_{2n}^{\text{IV}}, p_{2r}^{\text{IV}})}{\partial p_{2n}^{\text{IV}}} &= \frac{((\gamma + 1)\delta - \gamma + (-\beta - 2)p_{2r}^{\text{IV}} + \beta p_n^{\text{IV}} + \tau + \lambda^{\text{IV}} - c_n + c_r + 2p_{2n}^{\text{IV}} - 1)\Delta}{(\gamma + 1)(\delta - 1)} = 0, \\
\frac{\partial L(p_n^{\text{IV}}, p_{2n}^{\text{IV}}, p_{2r}^{\text{IV}})}{\partial p_{2r}^{\text{IV}}} &= \frac{1}{(\gamma + 1)(\delta - 1)((\gamma + 1)\delta - \gamma)} (((\gamma + 1)((-p_{2n}^{\text{IV}} + c_n)\beta + \tau + c_n - 2p_{2n}^{\text{IV}})\delta \\
&+ ((p_{2n}^{\text{IV}} - c_n)\beta - \tau - c_n + 2p_{2n}^{\text{IV}})\gamma + (-\tau + 2p_{2r}^{\text{IV}} - \lambda^{\text{IV}} - c_r - p_n^{\text{IV}})\beta - 2\tau + 2p_{2r}^{\text{IV}} - \lambda^{\text{IV}} - c_r)\Delta) = 0, \\
\lambda^{\text{IV}}(q_{2r}^{\text{IV}} - \rho q_n^{\text{IV}}) &= 0, q_{2r}^{\text{IV}} - \rho q_n^{\text{IV}} \leq 0, \lambda^{\text{IV}} \geq 0.
\end{aligned}$$

Similar to the proof of Proposition 1, we consider following two cases, namely, case 1: $\lambda^{\text{IV}} = 0$ and $q_{2r}^{\text{IV}} - \rho q_n^{\text{IV}} \neq 0$; and case 2: $\lambda^{\text{IV}} \neq 0$ and $q_{2r}^{\text{IV}} - \rho q_n^{\text{IV}} = 0$. Then, we can obtain following results:

(i) When $\lambda^{\text{IV}} = 0$, the constraint (22) is not binding. The optimal solutions can be summarized as follows:

$$\begin{aligned}
p_n^{\text{IV}^*} &= \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2 - 4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)} ((\beta + 2)^2(\gamma + 1)^2(c_n + 1)\delta^2 + \\
&(-2(\beta + 2)^2(c_n + 1)\gamma + (\tau - c_n + 1)\beta^2 + (2\tau - 6c_n - 4)\beta - 4c_n - 4)(\gamma + 1)\delta + (\beta + 2)^2(c_n + 1)\gamma^2 + \\
&((-\tau + c_n - 1)\beta^2 + (-2\tau + 6c_n + 4)\beta + 4c_n + 4)\gamma + 2((\tau + c_r)\beta + c_r)\beta), \\
p_{2n}^{\text{IV}^*} &= \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2 - 4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)} (2(\gamma+1)^2(\beta^2c_n + (\tau+3c_n+2)\beta + \\
&2\tau + 2c_n + 2)\delta^2 - 4((\beta^2c_n + (\tau + 3c_n + 2)\beta + 2\tau + 2c_n + 2)\gamma + (0.5\tau - 0.25c_n + 0.5c_r - 0.25)\beta^2 + \\
&(\tau + c_n + 0.5c_r + 1)\beta + \tau + c_n + 1)(\gamma + 1)\delta + (2\beta^2c_n + (2\tau + 6c_n + 4)\beta + 4\tau + 4c_n + 4)\gamma^2 + ((2\tau - \\
&c_n + 2c_r - 1)\beta^2 + (4\tau + 4c_n + 2c_r + 4)\beta + 4\tau + 4c_n + 4)\gamma + \beta^2(\tau + c_n + 1)), \\
p_{2r}^{\text{IV}^*} &= \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2 - 4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)} (2(\gamma+1)^3(\beta+2)\delta^3 + 2(\gamma+1)^2((-3\beta- \\
&6)\gamma + (0.5c_n + 0.5)\beta^2 + (\tau + 2c_n + c_r)\beta + 4\tau + 2c_r - 2)\delta^2 - 4((-1.5\beta - 3)\gamma^2 + ((0.5c_n + 0.5)\beta^2 + (\tau + \\
&2c_n + c_r)\beta + 4\tau + 2c_r - 2)\gamma + (-0.25\tau + 0.25c_n - 0.25)\beta^2 + (\tau + 0.5c_n + c_r + 0.5)\beta + 2\tau + c_r)(\gamma + \\
&1)\delta + (-2\beta - 4)\gamma^3 + ((c_n + 1)\beta^2 + (2\tau + 4c_n + 2c_r)\beta + 8\tau + 4c_r - 4)\gamma^2 + ((-\tau + c_n - 1)\beta^2 + (4\tau + \\
&2c_n + 4c_r + 2)\beta + 8\tau + 4c_r)\gamma + 2\beta^2(\tau + c_r)), \\
q_n^{\text{IV}^*} &= \frac{\Delta}{(2(\beta+2)^2(\gamma+1)^2\delta^2 - 4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)} (1 - ((\beta+2)^2(\gamma+1)^2(c_n+1)\delta^2 + \\
&(-2(\beta + 2)^2(c_n + 1)\gamma + (\tau - c_n + 1)\beta^2 + (2\tau - 6c_n - 4)\beta - 4c_n - 4)(\gamma + 1)\delta + (\beta + 2)^2(c_n + 1)\gamma^2 + \\
&((-\tau + c_n - 1)\beta^2 + (-2\tau + 6c_n + 4)\beta + 4c_n + 4)\gamma + 2((\tau + c_r)\beta + c_r)\beta),
\end{aligned}$$

$$q_{2n}^{IV*} = -\frac{\Delta}{((\beta+2)^2(\gamma+1)^2\delta^2-2((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+(\beta+2)^2\gamma^2+(4\beta+4)\gamma+\beta^2)}((-(\gamma+1)^2(\beta+2)\delta^2+(\gamma+1)((4+2\beta)\gamma+(\tau-0.5c_n+c_r-0.5)\beta^2+(4\tau-3c_n+3c_r+1)\beta+2\tau-2c_n+2c_r+2)\delta+(-\beta-2)\gamma^2+((0.5-\tau+0.5c_n-c_r)\beta^2+(-4\tau+3c_n-3c_r-1)\beta-2-2\tau-2c_r+2c_n)\gamma-0.5(\tau-c_n+1)\beta^2)),$$

$$q_{2r}^{IV*} = \frac{\Delta}{((\beta+2)^2(\gamma+1)^2\delta^2-2((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+(\beta+2)^2\gamma^2+(4\beta+4)\gamma+\beta^2)}(((\tau-c_n+1)\beta+2\tau-2c_n)(\gamma+1)\delta+((-\tau+c_n-1)\beta-2\tau+2c_n)\gamma+(2\tau-c_n+2c_r-1)\beta+2c_r)(\beta+1)\Delta).$$

(ii) The constraint (22) is binding if $\rho \leq -\frac{1}{\Phi}(2(((\gamma+1)(\tau-c_n+1)\delta+(-\tau+c_n-1)\gamma+2\tau-c_n+2c_r-1)\beta+2(\gamma+1)(\tau-c_n)\delta+(-2\tau+2c_n)\gamma+2c_r)(\beta+1))$, where $\Phi = (((\gamma+1)^2(c_n-1)\delta^2+(\gamma+1)((-2c_n+2)\gamma+\tau-c_n+1)\delta+(c_n-1)\gamma^2+(-\tau+c_n-1)\gamma+2\tau+2c_r-2)\beta^2+(4(\gamma+1)^2(c_n-1)\delta^2+2(\gamma+1)((-4c_n+4)\gamma+\tau-3c_n+2)\delta+(4c_n-4)\gamma^2+(-2\tau+6c_n-4)\gamma+2c_r)\beta+4(\delta-1)(c_n-1)(\delta(\gamma+1)-\gamma)(\gamma+1))$.

The optimal solutions are as follows:

$$p_n^{IV*} = \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2-4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)}(-(\beta+2)^2(\gamma+1)^2(\rho\lambda^{IV*}-c_n-1)\delta^2+2(\gamma+1)((\beta+2)^2(\rho\lambda^{IV*}-c_n-1)\gamma+(0.5\tau-0.5c_n+0.5)\beta^2+(2\rho\lambda^{IV*}+\tau-3c_n-2)\beta+2\rho\lambda^{IV*}-2c_n-2)\delta-(\beta+2)^2(\rho\lambda^{IV*}-c_n-1)\gamma^2+((-\tau+c_n-1)\beta^2+(-4\rho\lambda^{IV*}-2\tau+6c_n+4)\beta-4\rho\lambda^{IV*}+4c_n+4)\gamma+2\beta((\tau+c_r+\lambda^{IV*})\beta+c_r+\lambda^{IV*})),$$

$$p_{2n}^{IV*} = \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2-4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)}(2(\gamma+1)^2(\beta^2c_n+(\tau+3c_n+2)\beta+2\tau+2c_n+2)\delta^2-(\gamma+1)((4\beta^2c_n+(4\tau+12c_n+8)\beta+8\tau+8c_n+8)\gamma+(-c_n+2\tau+(\rho+2)\lambda^{IV*}+2c_r-1)\beta^2+(4\tau+4c_n+2c_r+2\lambda^{IV*}+4)\beta+4\tau+4c_n+4)\delta+(2\beta^2c_n+(2\tau+6c_n+4)\beta+4\tau+4c_n+4)\gamma^2+((-c_n+2\tau+(\rho+2)\lambda^{IV*}+2c_r-1)\beta^2+(4\tau+4c_n+2c_r+2\lambda^{IV*}+4)\beta+4\tau+4c_n+4)\gamma+\beta^2(\tau+c_n+1)),$$

$$p_{2r}^{IV*} = \frac{1}{(2(\beta+2)^2(\gamma+1)^2\delta^2-4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)}(2(\gamma+1)^3(\beta+2)\delta^3-(\gamma+1)^2((6\beta+12)\gamma+(\rho\lambda^{IV*}-c_n-1)\beta^2+((2\rho-2)\lambda^{IV*}-2\tau-4c_n-2c_r)\beta-8\tau-4c_r-4\lambda^{IV*}+4)\delta^2+2(\gamma+1)((3\beta+6)\gamma^2+((\rho\lambda^{IV*}-c_n-1)\beta^2+((2\rho-2)\lambda^{IV*}-2\tau-4c_n-2c_r)\beta-8\tau-4c_r-4\lambda^{IV*}+4)\gamma+(0.5\tau-0.5c_n+0.5)\beta^2+((\rho-2)\lambda^{IV*}-2\tau-c_n-2c_r-1)\beta-4\tau-2c_r-2\lambda^{IV*})\delta+(-2\beta-4)\gamma^3+((- \rho\lambda^{IV*}+c_n+1)\beta^2+((-2\rho+2)\lambda^{IV*}+2\tau+4c_n+2c_r)\beta+8\tau+4c_r+4\lambda^{IV*}-4)\gamma^2+((-\tau+c_n-1)\beta^2+((-2\rho+4)\lambda^{IV*}+4\tau+2c_n+4c_r+2)\beta+8\tau+4c_r+4\lambda^{IV*})\gamma+2\beta^2(\tau+c_r+\lambda^{IV*})),$$

$$q_n^{IV*} = \frac{\Delta}{(2(\beta+2)^2(\gamma+1)^2\delta^2-4((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta+2(\beta+2)^2\gamma^2+(8\beta+8)\gamma+2\beta^2)}(((\beta+2)^2(\gamma+1)^2(\rho\lambda^{IV*}-c_n+1)\delta^2-2(\gamma+1)((\beta+2)^2(\rho\lambda^{IV*}-c_n+1)\gamma+(0.5\tau-0.5c_n+0.5)\beta^2+(2\rho\lambda^{IV*}+\tau-3c_n+2)\beta+2\rho\lambda^{IV*}-2c_n+2)\delta+(\beta+2)^2(\rho\lambda^{IV*}-c_n+1)\gamma^2+((4\rho\lambda^{IV*}+2\tau-6c_n+4)\beta+4\rho\lambda^{IV*}-4c_n+4)\gamma-2((\tau+c_r+\lambda^{IV*}-1)\beta+c_r+\lambda^{IV*})\beta)),$$

$$q_{2n}^{IV*} = -\frac{\Delta}{((2(\gamma+1)^2\delta^2+(-4\gamma^2-4\gamma)\delta+2\gamma^2+2)\beta^2+8(\delta-1)(\gamma+1)(\delta(\gamma+1)-\gamma)\beta+8(\delta-1)(\gamma+1)(\delta(\gamma+1)-\gamma))}(((\gamma+1)(-c_n+2\tau+(\rho+2)\lambda^{IV*}+2c_r-1)\delta+((- \rho-2)\lambda^{IV*}-2\tau+c_n-2c_r+1)\gamma-\tau+c_n-1)\beta^2+2((-\gamma-1)\delta+\gamma+(\rho+$$

$$\begin{aligned}
& 3)\lambda^{IV^*} + 4\tau - 3c_n + 3c_r + 1)(\delta(\gamma + 1) - \gamma)\beta + 4((-\gamma - 1)\delta + \tau + \gamma - c_n + c_r + \lambda^{IV^*} + 1)(\delta(\gamma + 1) - \gamma)), \\
q_{2r}^{IV^*} &= \frac{\Delta}{((\beta+2)^2(\gamma+1)^2\delta^2 - 2((\beta+2)^2\gamma+2\beta+2)(\gamma+1)\delta + (\beta+2)^2\gamma^2 + (4\beta+4)\gamma+\beta^2)}((\beta+1)((\gamma+1)((\tau - c_n + 1)\beta + 2\tau - \\
& 2c_n)\delta + ((-\tau + c_n - 1)\beta - 2\tau + 2c_n)\gamma + (-c_n + 2\tau + (\rho + 2)\lambda^{IV^*} + 2c_r - 1)\beta + 2c_r + 2\lambda^{IV^*})), \\
\lambda^{IV^*} &= \frac{1}{((-4+(\delta(\gamma+1)-\gamma)^2\rho^2-4\rho)\beta^2+(-8+4(\delta(\gamma+1)-\gamma)(\delta-1)(\gamma+1)\rho^2-4\rho)\beta-4+4(\delta(\gamma+1)-\gamma)(\delta-1)(\gamma+1)\rho^2)}(((\gamma+1)^2(c_n - \\
& 1)\delta^2 + (\gamma+1)((-2c_n+2)\gamma + \tau - c_n + 1)\delta + (c_n - 1)\gamma^2 + (-\tau + c_n - 1)\gamma + 2\tau + 2c_r - 2)\rho + 2(\gamma+1)(\tau - \\
& c_n + 1)\delta + (-2\tau + 2c_n - 2)\gamma + 4\tau - 2c_n + 4c_r - 2)\beta^2 + ((4(\gamma+1)^2(c_n - 1)\delta^2 + 2(\gamma+1)((-4c_n+4)\gamma + \\
& \tau - 3c_n + 2)\delta + (4c_n - 4)\gamma^2 + (-2\tau + 6c_n - 4)\gamma + 2c_r)\rho + 6(\gamma+1)(\tau - c_n + \frac{1}{3})\delta + (-6\tau + 6c_n - 2)\gamma + \\
& 4\tau - 2c_n + 8c_r - 2)\beta + 4(\delta(\gamma+1) - \gamma)(\delta - 1)(\gamma+1)(c_n - 1)\rho + 4(\gamma+1)(\tau - c_n)\delta + (-4\tau + 4c_n)\gamma + 4c_r).
\end{aligned}$$

□

Proof of Proposition 6. Similar to Model I, we use the Lagrange multiplier and obtain the Lagrangian and the Karush–Kuhn–Tucker optimality conditions:

$$\begin{aligned}
L(p_n^V, p_{2n}^V, p_{2r}^V) &= q_n^V(p_n^V - c_n) + q_{2n}^V(p_{2n}^V - c_n) + q_{2r}^V(p_{2r}^V - c_r - \tau - \varphi(p_{2r}^V - \tau) + s) - \lambda^V(q_{2r}^V - \rho q_n^V), \\
\frac{\partial L(p_n^V, p_{2n}^V, p_{2r}^V)}{\partial p_n^V} &= \Delta(-p_n^V + c_n) - \Delta(-1 + p_n^V) + \frac{\Delta\beta(p_{2n}^V - c_n)}{(\gamma+1)(\delta-1)} \\
& - \frac{\Delta\beta(\varphi(-p_{2r}^V + \tau) + s - \tau - c_r + p_{2r}^V)}{(\delta(\gamma+1) - \gamma)(\gamma+1)(\delta-1)} - \frac{\lambda^V((\delta(\gamma+1) - \gamma)(\gamma+1)(\delta-1)\rho - \beta)\Delta}{(\delta(\gamma+1) - \gamma)(\gamma+1)(\delta-1)} = 0, \\
\frac{\partial L(p_n^V, p_{2n}^V, p_{2r}^V)}{\partial p_{2n}^V} &= -\frac{1}{(\gamma+1)(\delta-1)}(-(\beta+2)(\varphi-1)p_{2r}^V + \tau(\beta+2)\varphi - \beta p_n^V + (-\gamma-1)\delta \\
& + s - 2p_{2n}^V - \lambda^V - \tau + \gamma + c_n - c_r + 1)\Delta = 0, \\
\frac{\partial L(p_n^V, p_{2n}^V, p_{2r}^V)}{\partial p_{2r}^V} &= -\frac{1}{(\delta(\gamma+1) - \gamma)(\delta-1)(\gamma+1)}((\varphi-1)((-\gamma+1)(p_{2n}^V - c_n)\delta + (p_{2n}^V - c_n)\gamma \\
& + (2\varphi-1)\tau - 2\varphi p_{2r}^V + s + 2p_{2r}^V - \lambda^V - c_r - p_n^V)\beta - 2(p_{2n}^V - 0.5\tau - 0.5c_n)(\gamma+1)\delta \\
& + (2p_{2n}^V - \tau - c_n)\gamma + (2\varphi-2)\tau - 2\varphi p_{2r}^V + s + 2p_{2r}^V - \lambda^V - c_r)\Delta) = 0.
\end{aligned}$$

Similar to the proof of Proposition 1, we consider following two cases, namely, case 1: $\lambda^V = 0$ and $q_{2r}^V - \rho q_n^V \neq 0$; and case 2: $\lambda^V \neq 0$ and $q_{2r}^V - \rho q_n^V = 0$. Then, we can obtain following results:

(i) When $\lambda^V = 0$, the constraint (25) is not binding. We only provide the equilibrium outcome of $p_n^{V^*}$, since other outcomes are too complex.

$$\begin{aligned}
p_n^{V^*} &= \frac{1}{((2(\gamma+1)^2\delta^2 + (-4\gamma^2 - 4\gamma)\delta + 2\gamma^2 + 2)\beta^2 + 8(\gamma+1)(\delta-1)(\delta(\gamma+1) - \gamma)\beta + 8(\gamma+1)(\delta-1)(\delta(\gamma+1) - \gamma))}(((\gamma+1)^2(c_n + \\
& 1)\delta^2 + (\gamma+1)((-2c_n - 2)\gamma + \tau - c_n + 1)\delta + (c_n + 1)\gamma^2 + (-\tau + c_n - 1)\gamma - 2s + 2\tau + 2c_r)\beta^2 + (4(\gamma + \\
& 1)^2(c_n + 1)\delta^2 + 2((-4c_n - 4)\gamma + \tau - 3c_n - 2)(\gamma+1)\delta + (4c_n + 4)\gamma^2 + (-2\tau + 6c_n + 4)\gamma - 2s + 2c_r)\beta + \\
& 4(\delta(\gamma+1) - \gamma)(\delta-1)(c_n + 1)(\gamma+1)).
\end{aligned}$$

(ii) The constraint (25) is binding if $\rho \leq -\frac{1}{\Psi}(4(\beta + 1)((-0.5(\gamma + 1)(\tau - c_n + 1)\delta + (0.5\tau - 0.5c_n + 0.5)\gamma + s - \tau + 0.5c_n - c_r + 0.5)\beta - (\gamma + 1)(\tau - c_n)\delta + (\tau - c_n)\gamma + s - c_r)$, where $\Psi = ((-\gamma + 1)^2(c_n - 1)\delta^2 - (\gamma + 1)((-2c_n + 2)\gamma + \tau - c_n + 1)\delta + (-c_n + 1)\gamma^2 + (\tau - c_n + 1)\gamma + 2s - 2\tau - 2c_r + 2)\beta^2 + (-4(\gamma + 1)^2(c_n - 1)\delta^2 - 2(\gamma + 1)((-4c_n + 4)\gamma + \tau - 3c_n + 2)\delta + (-4c_n + 4)\gamma^2 + (2\tau - 6c_n + 4)\gamma + 2s - 2c_r)\beta - 4(\delta(\gamma + 1) - \gamma)(\delta - 1)(c_n - 1)(\gamma + 1)$.

We also only provide the equilibrium outcome of $p_n^{V^*}$, since other outcomes are too complex.

$$p_n^{V^*} = \frac{1}{((2(\gamma+1)^2\delta^2+(-4\gamma^2-4\gamma)\delta+2\gamma^2+2)\beta^2+8(\gamma+1)(\delta-1)(\delta(\gamma+1)-\gamma)\beta+8(\gamma+1)(\delta-1)(\delta(\gamma+1)-\gamma))} (((-\gamma+1)^2(\rho\lambda^{V^*}-c_n-1)\delta^2+2(\gamma+1)((\rho\lambda^{V^*}-c_n-1)\gamma+0.5\tau-0.5c_n+0.5)\delta+(-\rho\lambda^{V^*}+c_n+1)\gamma^2+(-\tau+c_n-1)\gamma-2s+2\tau+2c_r+2\lambda^{V^*})\beta^2+(-4(\gamma+1)^2(\rho\lambda^{V^*}-c_n-1)\delta^2+8(\gamma+1)((\rho\lambda^{V^*}-c_n-1)\gamma+0.5\rho\lambda^{V^*}+0.25\tau-0.75c_n-0.5)\delta+(-4\rho\lambda^{V^*}+4c_n+4)\gamma^2+(-4\rho\lambda^{V^*}-2\tau+6c_n+4)\gamma-2s+2c_r+2\lambda^{V^*})\beta-4(\gamma+1)(\delta-1)(\rho\lambda^{V^*}-c_n-1)(\delta(\gamma+1)-\gamma)),$$

$$\lambda^{V^*} = \frac{1}{((-4+(\delta(\gamma+1)-\gamma)^2\rho^2-4\rho)\beta^2+(-8+4(\gamma+1)(\delta-1)(\delta(\gamma+1)-\gamma)\rho^2-4\rho)\beta-4+4(\gamma+1)(\delta-1)(\delta(\gamma+1)-\gamma)\rho^2)} (((\gamma+1)^2(c_n-1)\delta^2+(\gamma+1)((-2c_n+2)\gamma+\tau-c_n+1)\delta+(c_n-1)\gamma^2+(-\tau+c_n-1)\gamma-2s+2\tau+2c_r-2)\rho+2(\gamma+1)(\tau-c_n+1)\delta+(-2\tau+2c_n-2)\gamma-4s+4\tau-2c_n+4c_r-2)\beta^2+((4(\gamma+1)^2(c_n-1)\delta^2+2(\gamma+1)((-4c_n+4)\gamma+\tau-3c_n+2)\delta+(4c_n-4)\gamma^2+(-2\tau+6c_n-4)\gamma-2s+2c_r)\rho+6(\gamma+1)(\tau-c_n+\frac{1}{3})\delta+(-6\tau+6c_n-2)\gamma-8s+4\tau-2c_n+8c_r-2)\beta+4(\gamma+1)(\delta-1)(c_n-1)(\delta(\gamma+1)-\gamma)\rho+4(\gamma+1)(\tau-c_n)\delta+(-4\tau+4c_n)\gamma-4s+4c_r).$$

Then, we complete the proof of Proposition 6.

□