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Reverse Blending: an economically efficient answer to the challenge of fertilizer mass customization

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Abstract. Reasoned fertilization, which is a major concern for sustainable and efficient agriculture, consists of applying customized fertilizers which requires a drastic increase in the number of fertilizer formulae, involving increasing costs due to the multiplication of production batch, of storage areas and of transportation constraints. An alternative solution is given by the Reverse blending, which is a new Blending Problem where inputs are non-pre-existing composite materials that need to be defined in both number and composition, simultaneously with the quantities to be used in the blending process, such as to meet the specifications of a wide variety of outputs, while keeping their number as small as possible. This would replace the production of a large variety of small batches of fertilizers by few large batches of new composite materials whose blending may be performed close to end-users (delayed differentiation), delivering substantial production and logistics cost savings, well in excess of remote blending costs. Reverse Blending presents some analogies with the Pooling Problem which is a two-stage Blending Problem where primary inputs are existing raw materials. An adapted version of this problem may be used to facilitate the design of new composite materials used by Reverse Blending. This paper presents the Reverse Blending approach, whose modeling is based on a quadratic programming formulation, and a large case study to demonstrate its feasibility. Reverse Blending, therefore, may be a disruptive approach to successfully reengineer not only the fertilizer supply chain but any other industry operating in blending contexts to meet a great diversity.

Keywords. mass customization, continuous production, blending, composite material design, fertilizer, pooling problem.

1. Introduction

The fertilizer industry faces a major dilemma which is unable to resolve under the current product/process organization. On the one hand, sustainable and efficient agriculture would require hundreds of different customized fertilizers, amounting to a much greater variety than that currently on offer; and on the other hand, such required diversity cannot be produced at an acceptable cost in the fertilizer plants as they exist today, with continuous production processes geared to batch production. In discrete production, however, mass customization can easily be performed in assembly lines, enabling successive production of different products. The proposed solution can be considered both original and disruptive: fertilizer plants need not produce all the fertilizers required by end users but only a few intermediate products, which are later combined to produce the required fertilizers in small blending units located in the countryside, close to actual fertilizer users. The proposed solution, called Reverse Blending (RB) uses an original quadratic model to formulate a new blending problem where the number of inputs and their component structures are treated as new decision variables. These are combined with the traditional variables concerning input quantities to be mixed so as to produce a given quantity of an output, whose components must comply with a set of constraints. This paper is organized as described below.

Section 2 describes the stakes of RB through a real-life case study of a sample of 700 different fertilizer needs, isolated from the large fertilizer diversity that would be required to support sustainable and efficient agriculture. Additionally, this section shows why such diversity cannot be achieved under the current production system. Section 3 is dedicated to a brief literature review covering the different topics relevant to the multi-disciplinary problem at hand. Section 4 contains a description of RB model, which presents some similarities with the Blending Problem (BP) and the Pooling Problem (PP), to point out their differences with RB. In section 5, a solution of the case study introduced in section 2 is provided, before giving a short conclusion in section 6.

2. The stakes of Reverse Blending

To illustrate the difficulty of the problem at hand, a real-life sample of several hundreds of required customized fertilizers was selected (§2.1). After showing why such mass customization cannot be achieved under the current continuous production system (§2.2), the main features of RB are introduced, explaining why this approach may re-engineer the supply chain so as to overcome all the production problems confronting the

fertilizer industry (§2.3). Understanding those stakes is a pre-condition to understanding the structure of the literature review.

2.1 The need for large fertilizer diversity to support sustainable and efficient agriculture

As explained in section §3.1, in the years to come, a very large range of highly customized fertilizers must be available to provide each farmer with customized and affordable fertilizers capable of increasing agricultural yield while preserving soil fertility to achieve sustainable agriculture. Such fertilizers must comply with specifically adapted formulae whose nutrients and proportions differ according to the pedological characteristics and the crops involved. The three most important nutrients are Nitrogen (N), Phosphorus (P) and Potassium (K). The varying needs for all three nutrients translate into hundreds or even thousands of formulae. This diversity is illustrated through a real-life example of 700 different fertilizer requirements.

This sample was obtained from Fertimap (<http://194.204.220.89/fertimap/map.html>), a fertilization advice application developed by agronomists under the umbrella of the Moroccan Ministry of Agriculture. The application enables selecting a geographical area, a crop and a target yield to calculate the exact need, expressed in kg/ha, of N, P and K. The customized fertilizer solution recommends the adequate fertilizer formula with the right quantities. The sample, an extract of which is provided in Table 1, can be reviewed in the file named `Data_set.xlsx` included in the Mendeley data (<http://dx.doi.org/10.17632/z3sbn5j9z7.1>).

The crop chosen for this study is that for which Morocco recorded the largest shortfall in agricultural production in the year 2017-2018, namely wheat (Ghouibi, 2019) and, more precisely the "bour" variety that grows in rain-fed areas. To select the geographical area, we referred to the land vocation maps for wheat published under Fertimap, by focusing on areas with a high wheat-growing aptitude. This led to a sample of 700 parcels of land (see columns from B to F of the workbook "Data_set"). Note that the best sustainable target yield for "bour" wheat is 4,000 kg/ha (Alaoui, 2005).

For each parcel, Fertimap proposes what is deemed the best possible operational solution to be obtained by blending available fertilizers (Fertimap calculation rules are not disclosed). One or two solutions are given depending on whether both a regional and generic fertilizer formula are available or not. The NPK formula "15-30-10" means that 100 kg of this fertilizer includes 15 kg of N, 30 kg of P and 10 kg of K. The balance as in $100-15-30-10=45$ kg corresponds to the filler material that is added for chemical stabilization purposes and to avoid excessive fertilizer spreading that "burns" the soil. Fillers have no impact on nutritional structure. We retained the proposed operational solutions for 700 NPK requirements and showed their deviations from the

exact NPK requirements: for instance, for parcel 381, the optimal requirement for Phosphorus is 82.24 Kg/ha, whereas the solution that mixes regional fertilizers yields $16.31 \times 30 = 489.3$ Kg/ha, which amounts to a difference of +407.06 Kg/ha. Note that this deviation vastly exceeds the tolerance under forthcoming fertilizer regulation (EU PE-CONS 76/18, 2019).

No. NPK Need	Geographical data			Calculated Nutritional Requirement (Kg/Ha)	Fertimap Recommendations	Deviations from the calculated requirements (Kg/Ha)	
	Region/ Prefecture or province/District	Longitude	Latitude				
1	Grand Casablanca-Settat/Berrechid/Sidi Rahal Chatai (Mun.)	-7.9447	33.4608	N 72.83 P 2.17 K 0	No regional fertilizer formula exists	N - P - K -	
2	Grand Casablanca-Settat/Berrechid/Sidi Rahal Chatai (Mun.)	-7.9369	33.4561	N 140.83 P 0 K 0	No regional fertilizer formula exists	N - P - K -	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
362	Rabat-Salé-Kénitra/Khémisset/Ait Ouribel	-6.168	33.7713	N 138.33 P 26.06 K 112	3.73 q/Ha of 9-23-30 and 3.17 q/Ha of 33-0-0	N -0.15 P 59.73 K -0.1	
363	Rabat-Salé-Kénitra/Khémisset/EL Ganzra	-5.9531	34.0845	N 197.33 P 111.29 K 161	5.37 q/Ha of 9-23-30 and 4.52 q/Ha of 33-0-0	N 0.16 P 12.22 K 0.1	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
381	Tanger-Tetouan-Al Hoceima/Ouezzane/ Kalaat Bouqorra	-5.1789	34.7444	N 79.33 P 82.24 K 163.02	16.3 q/Ha of 15-30-10	N 165.17 P 406.76 K -0.02	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
699	Fès-Meknès/ Taza/ Zarda	-4.4452	33.9143	N 113.83 P 0 K 0	3.45 q/Ha of 33-0-0	N 0 P 0 K 0	
700	Fès-Meknès/ Taza/ Zarda	-4.3941	33.9137	N 125.33 P 0 K 44.24	2.21 q/Ha of 10-20-20 and 3.13 q/Ha of 33-0-0	N 0.06 P 44.2 K -0.04	

Table 1: Extract of the Fertimap sample for 700 NPK requirements for wheat cultivation in a number of Moroccan regions.

When nutritional requirements concern a single component, they can easily be obtained from a composite material which is rich in this nutriment. For instance, the requirements for N can be met using urea (which includes 46% of N). The requisite amount of this type of requirements is obtained by mixing the composite material with the filler in adequate proportions to obtain the required nutritional intake (e.g. to meet the 2nd NPK need, 306.15 kg/ha of formula 46-0-0 should be applied: $306.15 \times 46\% = 140.38$ which exactly matches the 2nd need, plus the filler ($306.15 - 140.38 = 165.32$)). Out of the 700 parcels, 219 required only N, and a single parcel required only P fertilizers.

2.2 Mass customization in continuous production

Mass customization, a major trend in modern economy (Pine II, 1993; Anderson and Pine II, 1997), is the most economically efficient solution to deal with huge diversity. Usually, mass customization is driven by marketing departments that seek to offer products to better meet customers' expectations. In this case, mass customization is an agronomical requirement to achieve sustainable and efficient agriculture. In discrete production, mass customization is essentially performed by assembling alternative components. In continuous production, such as fertilizer production, it consists in producing a sequence of small batches of different products. This involves major cost hikes due to *i*) the massive increase in the number of setup operations, each one of which takes around 2 hours and *ii*) in warehousing requirements (since a limited number of different fertilizers can be stored in the same shed in order to preserve fertilizer integrity), and due to *iii*) the problem of transporting multiple small batches to areas where specific fertilizers are needed.

2.3 The basis of the Reverse Blending solution

RB is based on finding the chemical specifications (in terms of component proportion) of the smallest set, called "canonical basis (CB)", of blending inputs, named "canonical basis inputs (CBIs)". These are composite materials that may not already exist and whose blends enable meeting the specifications (in terms of component proportion) of any fertilizer belonging to a set of customized fertilizers. These blending combinations form a Bill of Materials (BOM) used to produce any quantity of a required fertilizer.

The massive flow concentration delivered by RB would allow: *i*) CBIs to be cost-effectively produced-to-stock, in large batches, at existing chemical plants, since their production process is similar to fertilizer production one; *ii*) few CBIs to be cost-effectively shipped to small blending units located close to end-use areas, such as those currently employed in some countries where fertilizers are mixed to obtain a formula closely matching their specific nutritional requirements; *iii*) the needed fertilizers to be produced-to-order locally, by applying the appropriate BOM formula so as to support efficient and sustainable agriculture. The proposed solution relies on delayed differentiation (see § 3.3) which, rather than taking place at original production sites (usual solution in discrete production), is conducted near customers.

Obviously, market studies of potential demand for a vast variety of fertilizers that depends on crops, soils, superficies and farmer interests, is impracticable. This, however, would not be of real concern since market studies are not a prerequisite in the case of local delayed differentiation of few CBIs particularly as their production cost (other than input and setup costs), are bound to be similar to those of existing fertilizers.

Accordingly, all kinds of demand are treated as potential, focusing on different representative parcels of 1 ha, thus ignoring their overall surface area and whether farmers would be ready to buy the customized fertilizer. This is consistent with the goal of this paper which is to present the RB analytical solution to obtain the required fertilizer diversity with very few CBIs and illustrate its application. If this research is conclusive, further research avenues will need to be explored including the fields of chemistry, agronomy and network design (plants and local blenders location). At that later stage, economic studies based on incremental cash flow reports while considering alternative sets of CBIs, alternative plant locations and alternative networks of the fertilizer supply chain, will of course make sense.

3. Literature review

This literature review covers the three topics relevant to the problem at hand. Section §3.1 discusses the need for a very large variety of fertilizers to support cost-effective, sustainable and efficient agriculture, hitherto incompatible with the current fertilizer production model based on successive production of different fertilizer batches. Section 3.2 discusses the originality of the RB approach compared to one-stage fertilizer blending solutions, particularly at distribution network level. Section 3.3 addresses the concept of delayed differentiation and reviews its possible application to non-discrete production. Section 3.4 analyzes the Pooling Problem (PP) by focusing on its differences with the RB problem.

3.1 Mass-customization of fertilizers: a major issue to address looming global food production crises

In order to feed a global population of 9.1 billion people by 2050, food production will have to increase by about 70% versus its 2005 level (Conforti and Sarris, 2012). Rising to this challenge demands rational fertilization so as to provide plants with needed nutrients in the best way and support efficient and sustainable agriculture. However, due to low fertilizer utilization and high nutrient extraction rates, soils are often unable to yield these nutrients without recourse to supplementary ingredients (Fixen *et al.*, 2015). The latter must contain the right proportion of a number of nutrients. Below the three most important of these nutrients are presented.

- Nitrogen plays a key role in plant growth and crop yield (Hirel *et al.*, 2011). Its availability and internal concentration affect the distribution of biomass between roots and shoots (Bown *et al.*, 2010) as well as metabolism, physiology and plant development (O'Brien *et al.*, 2016).

- Phosphorus is an essential nutrient for root development and nutrient availability (Jin *et al.*, 2015). It is essential for cell division, reproduction and metabolism of plants and allows to store energy and regulate its use (Epstein and Bloom, 2005).
- Potassium plays a major role in regulating the opening and closing of stomata, which is necessary for photosynthesis, the transport of water and nutrients and the cooling of plants (Prajapati, 2012).

Recommending fertilizer blends of these three nutrients requires in-depth knowledge of the different aspects of fertilization including target yield, crop nutrient need and soil nutrient content (Cottenie, 1980). To quantify this content, farmers must perform soil tests so as to be able to manage nutrients and avoid long-term nutritional and health problems (Watson *et al.*, 2002). These tests should be carried out at least every three years (Warncke, 2000) as soil properties change as a result of land management practices and inherent soil characteristics (Jenny, 1941). This translates into a wide range of fertilizer needs, hence the need to produce a wide variety of customized fertilizers which, according to Rakshit *et al.* (2012), are critical to help feed the world's growing population, provide sustainable global food security and protect the environment.

3.2 The Blending Problem

RB may be regarded as an extension of the blending problem (BP) (whose model is detailed in section 4.1) where, unlike in standard BP, inputs are not pre-existing, such that their composition must be determined. Many papers dealing with one-stage blending problems were consulted in the agri-food sector (Paredes-Belmar *et al.*, 2016; E. Steuer, 1984; Karmarkar and Rajaram, 2001; Bilgen and Ozkarahan, 2007; Yoon *et al.*, 1997), mining sector (Kumral, 2003; Williams and Haley, 1959), petroleum sector (Oddsdottir *et al.*, 2013; Bengtsson *et al.*, 2013) and chemical sector (Montante *et al.*, 2016; Jonuzaj and Adjiman, 2017 and many others). In all these papers, all the inputs are existing raw materials whose specifications are input data of the blending model.

To support the specificities of our field of application (fertilizer sector), particular focus was placed on the papers dealing with fertilizer blending problems. Indeed, fertilizer blending problems have drawn a lot of attention, as demonstrated in the papers by Babcock *et al.* (1984), Mínguez *et al.* (1988), Ashayeri *et al.* (1994), Traoré *et al.* (2007), Lima *et al.* (2011), Aldeseit (2014), Srichaipanya *et al.* (2014), M. Cole *et al.* (2015), Loh *et al.* (2015) and their respective references. These problems consist in determining the optimal quantities to be taken from a subset of inputs in order to produce one or more fertilizer formulae. The inputs used can either refer to composite materials, conventionally considered as nutritious raw materials (Cole *et al.*, 2015; Srichaipanya *et al.*, 2014) or to fertilizers, obtained by chemical reaction or by blending, for which the

proportion of each nutrient is known (Mínguez *et al.*, 1988). In both cases, these inputs (composite materials), either supplied or produced, are already available. As far as we know, all the inputs associated with the one-stage blending problems addressed in the literature, already exist.

3.3 Delayed differentiation

To benefit from the advantages of both mass production and customization, the concept of Mass Customization (MC) was first coined by Stanley Davis in 1987. MC is “the low-cost production of high variety, even individually customized goods and services” (Pine, 1993). One of its key techniques is delayed differentiation (McIntosh *et al.*, 2010). It refers to combining some common processes and delaying other differentiation processes as late as possible until the supply chain achieves cost effectiveness (Boone *et al.*, 2007). To do so, the design of product structure and supply chain processes must be performed simultaneously (Shao and Ji, 2008; B. Lavigne *et al.*, 2012). The Product Differentiation Point (PDP) in MC can be delayed to increase flexibility and responsiveness to customer demand. This concept, that has attracted much attention from researchers (Skipworthy and Harrison, 2004; Shao and Ji, 2008; Algeddawy and Elmaraghy, 2010; Wong *et al.*, 2011; Daaboul and Da Cunha, 2014; and others), is defined as an operation that transforms one or more common products into customized products based on customers’ needs. Where to locate it is a crucial decision in MC and has given rise to much discussion in the literature: while delaying PDP is generally considered to reduce investment in the value-added component inventory and to enable responsiveness to customer orders (Shao and Ji, 2008), several optimization models, all involving discrete production, have been developed for optimal location purposes (e.g. Algeddawy and Elmaraghy, 2010; Hanafy and Elmaraghy, 2015).

It is also admitted that the PDP’s optimal location depends on the production strategy adopted. In Assemble-to-order (ATO) operations (e.g. automobile), PDP is usually carried out at assembly lines, within the production plant. In Make-to-stock (MTS) operations, PDP can be either located at the production site or at distribution centers but customization is usually a simple matter of labeling and packaging (e.g. Nestle). Although delayed differentiation has been proven to work well in discrete production (e.g. Hewlett-Packard Products), this analysis indicates that it has not been employed in continuous flow production, except to customize packaging.

With RB, delayed differentiation concerns both products and packaging and takes place in small capacity blending units located near end users (PDP); the product differentiation, made-to-order, is based on blending ready-made inputs (CBIs), by using a predetermined BOM.

3.4 The Pooling Problem

According to chemical experts, CBIs may be produced like the blended fertilizers that are obtained by mixing chemically compatible composites. This production mode presents a number of similarities with the pooling problem (PP) which refers to multi-stage blending problems (C. Chang *et al.*, 2019) starting from raw materials. Given the availabilities of a number of raw materials and the aim of achieving optimum income from final blend sales, this problem aims to define the composition of “intermediate blends” as well as the quantities to be mixed in order to meet the needs of various final blends whose composition must satisfy known requirements (Audet *et al.*, 2004). Owing to the specificities of non-convex quadratic programming, the majority of PP studies focus on proposing improved resolution methods (C. Chang *et al.*, 2019). There are very few papers that have applied pooling in a real industrial context: Petrochemical sector (Misener *et al.*, 2010; Zheng and Zhang, 2016; Cheng *et al.*, 2018), solid materials recycling sector (C. Chang *et al.*, 2019) and wastewater treatment sector (Misener and Floudas, 2010). To the best of our knowledge, the pooling problem has not yet been applied to fertilizers, but if one considers the production of CBIs, from existing composite materials, as a possible extension of RB, PP may be an interesting alternative to RB, assuming it can be adapted to address specific constraints. With this in mind, several important differences were pointed between RB and PP as usually dealt with in the literature.

- a) **Nature of products.** In PP, the primary inputs used to produce the intermediate blends are raw materials that correspond either to crude elements (e.g. petrochemical: hydrocarbon for oil (Misener *et al.*, 2010)/ agri-food: wheat for flour (Akkerman *et al.*, 2010)), wastewater streams (e.g. wastewater treatment (Ting *et al.*, 2016)) or recycled metals (e.g. aluminum recycling (Chang and Jiyoun, 2015)). Whereas in RB, the primary inputs to be used to produce CBIs are composite materials that may be new chemical products to be designed and produced. For example, Ammonium sulfate and Urea, components used in fertilizer production, are composite materials that cannot be extracted from earth or obtained by agriculture. On the other hand, phosphate ore, petroleum or wheat are raw materials whose common main components list is known. This is not the case for composite materials, such as Ammonium sulfate and Urea which do not share the same components. Thus, their combination in producing a CBI may be precluded and this constraint may also prevent the free combination of CBIs to produce a fertilizer.
- b) **Blend feasibility constraints.** When dealing with composite materials as opposed to raw materials, PP must be tailored to address new constraints: *i*) those related to composite materials incompatibilities: the composite materials to be used to produce a CBI must be chemically and physically compatible (hazard-free)

(e.g. an urea composite cannot be mixed with a TSP composite (a mono-component containing 46% of phosphorous)); *ii*) those related to the inheritance of these incompatibilities: in the previous example, in producing a fertilizer, a CBI using urea cannot be mixed with another using TSP. Constraints related to the minimum quantities *i*) of composite materials required to produce a CBI and *ii*) of CBIs to be used to produce a fertilizer must also be taken into account.

- c) **Number of end-products.** In the context of reasoned fertilization, which is increasingly being adopted, the diversity of finished products to be delivered is much greater than that of the industries that have been studied (the maximum diversity found in the body of research amounts to around thirty items (Akkerman *et al.*, 2010), whereas the diversity enabled by RB is to the tune of several hundreds).
- d) **Number of intermediate blends.** As PP is used to model systems that have intermediate mixing tanks (pools) in the blending process (Audet *et al.*, 2004), the number 'I' of intermediate blends is generally modeled by a fixed parameter which is equal to the number of pools installed in the manufacturing plant (Adhya *et al.*, 1999; Alfaki, 2012; Audet *et al.*, 2004; Floudas and Aggarwal, 1990; Meyer & Floudas, 2006; Misener and Floudas, 2010; Visweswaran, 2009). On the other hand, given the on-floor storage of granular fertilizers, the size 'I' of the canonical basis is not constrained by the number of available pools. Rather, it refers to the minimum number of CBIs needed to produce any fertilizer.
- e) **Network configuration.** In traditional pooling problems, the network structures defining the connections between sources and pools; pools and final products; and sources and final products is fixed and only the flows must be optimized (Meyer and Floudas, 2006). This assumes that the locations of the blending processes are fixed within the production sites, which is not the case in the RB approach where small blending units are to be installed outside the production sites (near the final consumer) and their optimal location is to be determined in future research. Indeed, unlike the pooling problem, which is generally solved in an operational context where it is adapted to the current supply chain structure, the RB model must be developed at a strategic level rather than an operational one, since it should be designed with a long-term view. It also should be noted that most PPs introduce capacity constraints on the nodes (tanks) and on the arcs (transportation), which are irrelevant to the problem of fertilizer mass customization.
- f) **Composition constraints of intermediate products.** In PPs, no composition constraints exist in the definition of intermediate blends, as they result from the composition of existing raw materials which are of the same kind. In RB, on the other hand, one must remember that, except for filler, any CBI, a composite material to be created, will combine existing composite materials, since no single component material (like

Nitrogen) can exist in a pure and stable state in an industrial environment. This led us to introduce in RB a number of composition constraints to enforce the weight proportion of a given component (e.g. N) to be below an established ceiling (e.g. 46%).

An Adaption of PP was designed, which we called ‘Adapted Pooling Problem’ (APP), that considers all the specificities of RB with the aim of creating as many CBIs as possible from existing composite materials, with the remaining CBIs having to be created as new composite materials.

4. Modelling

To clearly account for the differences and complementarities between the Blending Problem (BP), the Reverse Blending problem (RB) and the Pooling Problem (PP), their mathematical formulation (§4.1 to §4.3) must be examined. This led to an Adapted Pooling Problem (APP) (§4.4), which induced a general approach, named Extended Reverse Blending (ERB), combining APP with RB (§4.5).

4.1 Blending Problem

Indexes and parameters

i	Existing inputs (raw material or composite material) ($i = 1, \dots, I$).
j	Output index ($j = 1, \dots, J$).
c	Component index ($c = 1, \dots, C$) / common to all models.
β_{cj}^{\min}	Minimal proportion weight of component c in the weight of output j .
β_{cj}^{\max}	Maximal proportion weight of component c in the weight of output j .
D_j	Demand for output j , expressed in weight.
A_i	Availability of input i , expressed in weight.
α_{ci}	Proportion of component c in the total weight of input i (raw material or composite material).
ω_i	Unit cost of input i .

Variables

x_{ij} Quantity of input i used in the production of output j .

Problem definition

$$\text{Min}(\sum_j \sum_i \omega_i x_{ij}) \quad (1)$$

Subject to

$$\sum_i x_{ij} = D_j, \forall j \quad (2)$$

$$\beta_{cj}^{\min} \leq \sum_c \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj}^{\max}, \forall c, \forall j \quad (3)$$

$$\sum_j x_{ij} \leq A_i, \forall i \quad (4)$$

Comments

- Relation (1): cost minimization.
- Relation (2): satisfaction of demand.
- Relation (3): satisfaction of composition constraints of an output.
- Relation (4): respect of input availabilities.
- Implicitly, the formulation is a mono-period one (steady state).

4.2 Reverse Blending Problem (RBP)

As RBP aims to meet a potential demand for any fertilizer (without a priori market study), in RBP formulation, fertilizer demand must be conventional. In addition, specifications of inputs, whose blends produce the required fertilizers, are to be defined. It follows that input costs are unknown at this stage, such that a formulation based on economic optimization would be pointless as neither demand nor costs are known. RBP only aims to find a feasible blending solution in a steady state for a set of fertilizers as well as the composition of the CBIs used, whose number must be as low as possible to reduce production, storage and transportation problems.

Indexes and parameters

- j Becomes the index of a fertilizer.
- i Becomes the index of a CBI, which is now a new composite material to be created, ($i = 2, \dots, I$) where parameter ' I ' is the acceptable upper limit for the number of CBIs. Composite $i = 1$ is the filler.
- c Component index ($c = 1, \dots, C$); only $C=4$ components are considered here; the filler is component $c=4$.
- D_j Becomes conventional demand for fertilizer j , expressed in Kg/ha, needed to cover the nutrient requirements of a 1 ha plot of land.
- β_{cj} Target proportion of component c in the total weight of output (fertilizer) j .
- η_{cj} Maximal absolute deviation of component c in the total weight of output (fertilizer) j .
- τ_{ci} Maximal proportion weight of component $c < 4$ in the total weight of CBI $i > 1$ (see feature f in §3.4).
- τ_{4i} Minimal proportion weight of component $c=4$ (filler) in the total weight of CBI $i > 1$.
- α_{ci} The proportion of component c in the total weight of CBI $i=1$. For this CBI, which is filler, $\alpha_{c1} = 0, \forall c < 4$ and $\alpha_{41} = 1$ (as the CBI filler is 100% made of filler).
- M Upper boundary of any CBI quantity used to produce an output (e.g., $M=100000$).
- κ Minimal weight percentage of a CBI used to produce an output (a fertilizer), if that CBI is used (e.g., $\kappa = 0.01$).

Variables

- x_{ij} Quantity of CBI i used in the production of fertilizer j .
- α_{ci} Represents the weight proportion of component c in the total weight of CBI $i > 1$; note that in the blending problem, α_{ci} is a parameter and not a variable for $i > 1$, as CBI $i = 1$ only comprises the filler (implying that α_{ci} are parameters and not decision variables).
- w_{ij} Binary variable which is equal to 1 if CBI i is used to produce fertilizer j and 0 otherwise.
- y_i Binary variable which is equal to 1 if CBI i is used and 0 otherwise. The set of $I = \sum y_i$ inputs constitutes a Canonical Basis enabling the production of any fertilizer.

Problem definition

$$\text{Min} \left[\sum_i y_i \right] \quad (5)$$

Subject to

$$\sum_c x_{ij} = D_j, \forall j \quad (2)$$

$$\sum_j x_{ij} \leq M \cdot y_i, \forall i \quad (6)$$

$$\left| \beta_{cj} - \eta_{cj} \leq \sum_i \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj} + \eta_{cj}, \forall c, j \mid \beta_{cj} > 0 \right. \\ \left. \sum_i \alpha_{ci} \cdot x_{ij} / D_j = 0, \forall c, j \mid \beta_{cj} = 0 \right. \quad (3')$$

$$\sum_c \alpha_{ci} = 1, \forall i \quad (7)$$

$$\left| \alpha_{ci} \leq \tau_{ci}, \forall i, c \mid c < 4 \right. \quad (8)$$

$$\left| \alpha_{4i} \geq \tau_{4i}, \forall i \right. \\ \left. \begin{array}{l} x_{ij} \leq M \cdot w_{ij} \\ x_{ij} \geq \kappa \cdot w_{ij} \cdot D_j \end{array}, \forall i, j \right. \quad (9)$$

Comments

- Relation (5) defines the optimization criteria designed to obtain a feasible solution (i.e., respecting relation 2, 3' and 7 to 9) where $I = \sum_i y_i$ is as low as possible.
- Relation (6) enforces y_i to be equal to 1 only if CBI i is used; that said, the objective function (5) aims to maximize the number of CBIs unused (the set of CBIs i where $y_i = 0$) so as to keep I as small as possible.
- Relations (3') replace relation (3) while tolerating deviation η_{cj} from the target values β_{cj} when $\beta_{cj} > 0$. Clearly, (3') is equivalent to (3) when $\eta_{cj} = (\beta_{cj}^{\max} - \beta_{cj}^{\min}) / 2$ and $\beta_{cj} = \beta_{cj}^{\min} + \eta_{cj}$, but this formulation is chosen on account of fertilizer regulation (EU PE-CONS 76/18, 2019) which sets target values and tolerances unlike in the classical blending problems where the specifications are expressed as a min-max bracket. Due to relations (3'), the problem becomes a quadratic one.
- Relation (7) constrains the CBI composition to ensure the aggregate of component proportion weights equals 100%.

- Relations (8) constrain the CBI nutrient proportion not to exceed its maximal allowed value while having a minimum proportion of filler (see feature f in §3.4).
- Relations (9), which connect the continuous variables to the binary ones, ensure that if CBI i is to be used to produce fertilizer j , it must represent at least $\kappa\%$ of this fertilizer's weight.

4.3 Pooling Problem (PP)

The formulation of the standard pooling problem proposed by Gupte *et al.* (2016) is presented below. This standard version applies to network flow models consisting of arcs and nodes with capacity limit constraints for both the nodes (tanks) and arcs (devices connecting different units). However, to make the distinction between PP and RB problem easier, this version was adapted to only retain the constraints representing similarities with a RB problem. That said, the limiting capacity constraints are eliminated and the variable decisions now concern quantities and proportions instead of arc flows. Additionally, in PP problems, unlike in RBP, in addition to arcs connecting inputs to pools (intermediate blends) and pools to outputs, there are flows from inputs to outputs.

Indexes and parameters

- I Number of pools (fixed value).
- k Index of an existing raw material ($k = 1, \dots, K$).
- i Becomes the index of an intermediate input to create ($i = 1, \dots, I$) by blending raw materials.
- j As in the BP, j is the index for an output (final blend) ($j = 1, \dots, J$).
- φ_{kj} The cost of sending a unit flow on the arc connecting raw material k to final blend j .
- φ_{ki} The cost of sending a unit flow on the arc connecting raw material k to intermediate input i .
- φ_{ij} The cost of sending a unit flow on the arc connecting intermediate input i to final blend j .
- γ_{ck} Proportion of component c in the total weight of raw material k .
- A_k Availability of raw material k , expressed in weight (replaces A_i in the BP formulation).
- D_j Becomes the maximum demand for j .
- β_{cj}^{\min} Minimal proportion weight of component c in the weight of output j .
- β_{cj}^{\max} Maximal proportion weight of component c in the weight of output j .

Variables

- x_{ij} Becomes the quantity of intermediate input i used in the production of output j .
- α_{ci} Weight proportion of component c in the total weight of intermediate input i .
- y_{ki} Quantity of raw material k used in the production of intermediate input i .
- z_{kj} Quantity of raw material k sent directly to be used in the production of output j .

Problem definition

$$\text{Min}(\sum_{j,k} \varphi_{kj} \cdot z_{kj} + \sum_{k,i} \varphi_{ki} \cdot y_{ki} + \sum_{i,j} \varphi_{ij} \cdot x_{ij}) \quad (10)$$

Subject to

$$\sum_i x_{ij} + \sum_k z_{kj} \leq D_j, \forall j \quad (11)$$

$$\sum_j x_{ij} = \sum_k y_{ki}, \forall i \quad (12)$$

$$\sum_j z_{kj} + \sum_i y_{ki} \leq A_k, \forall k \quad (13)$$

$$\sum_k \gamma_{ck} \cdot y_{ki} = \alpha_{ci} \cdot \sum_j x_{ij}, \forall c, i \quad (14)$$

$$\beta_{jc}^{\min} \leq \frac{\sum_k \gamma_{ck} z_{kj} + \sum_i \alpha_{ci} x_{ij}}{(\sum_k z_{kj} + \sum_i x_{ij})} \leq \beta_{jc}^{\max}, \forall j, c \quad (3'')$$

Comments

- Relation (10): cost minimization.
- Relation (11) maximum withdrawal constraint (the sum of the quantities of raw materials to be withdrawn z_{kj} and/or intermediate inputs x_{ij} must not exceed demand D_j).
- Relation (12) flow conservation constraint of intermediate inputs.
- Relation (13) raw material availability constraint (the outgoing flows from the raw material tanks, whether those going to the intermediate tanks or the outputs tanks, must not exceed A_k).
- Relation (14) flow conservation constraint of components.
- Relation (3'') variant of constraint (3) of output specifications taking into account outflows from the pools and the raw material tanks.

4.4 Adapted Pooling Problem (APP)

After presenting the APP model (§4.4.1), an explanation of its resolution approach is given in §4.4.2.

4.4.1 APP formulation

In APP, one tries to use PP, as far as possible, to produce the maximum number J_1 of fertilizers obtained by blending I_1 CBIs made with existing composite materials that are assumed to be available. I_1 is a parameter that is gradually increased until reaching the possible highest value of J_1 (see §4.4.2).

Adaptation of the pooling formulation

- k Becomes the index of an available composite material (instead of a raw material).
- i Becomes the index of a CBI made by blending composite materials ($i = 1, \dots, I_1$) where I_1 is a parameter to be adjusted (and not a fixed parameter, as in PP).

New parameters

- $\zeta_{kk'}$ Boolean parameter equal to 1 if composite materials k and k' are incompatible, and 0 otherwise.
- κ' Minimal weight percentage of a composite material k used to produce a CBI, if that composite material is used.
- ε Very low value (e.g., $\varepsilon = 0.00001$).

New variables

- v_j Binary variable equal to 1 if fertilizer j is produced and 0 otherwise.
- z_{ki} Binary variable equal to 1 if composite material k is used to produce CBI i and 0 otherwise.

Problem definition

$$\text{Max} \left[\sum_j v_j \right] \quad (15)$$

Subject to

$$\sum_i x_{ij} = D_j, \forall j \quad (2)$$

$$\left| \beta_{cj} - \eta_{cj} \leq \sum_i \alpha_{ci} \cdot x_{ij} / D_j \leq \beta_{cj} + \eta_{cj}, \forall c, j \mid \beta_{cj} > 0 \right. \quad (3')$$

$$\left. \sum_i \alpha_{ci} \cdot x_{ij} / D_j = 0, \forall c, j \mid \beta_{cj} = 0 \right. \quad (3')$$

$$\left| \begin{array}{l} x_{ij} \leq M \cdot w_{ij} \\ x_{ij} \geq \kappa \cdot w_{ij} \cdot D_j \end{array} \right., \forall i, j \quad (9)$$

$$\left| \begin{array}{l} \sum_i x_{ij} \leq M \cdot v_j \\ \sum_i x_{ij} \geq \varepsilon \cdot v_j \end{array} \right., \forall j \quad (16)$$

$$\sum_j x_{ij} = \sum_k y_{ki}, \forall i \quad (12)$$

$$\left| \begin{array}{l} y_{ki} \leq M \cdot z_{ki} \\ y_{ki} \geq \kappa' \cdot z_{ki} \cdot \sum_j x_{ij} \end{array} \right., \forall k, i \quad (17)$$

$$z_{ki} + z_{k'i} \leq 1, \forall i, k, k' \mid k \neq k' \wedge \zeta_{kk'} = 1 \quad (18)$$

$$w_{ij} + w_{i'j} \leq 3 - \zeta_{kk'} \cdot (z_{ki} + z_{k'i}), \forall j, \forall i, i' \mid i' \neq i, \forall k, k' \mid k \neq k' \wedge \zeta_{kk'} = 1 \quad (19)$$

Comments

- Relation (15) maximizes the number of fertilizers that may be produced with I_1 CBIs obtainable by blending existing composite materials.; as I_1 is a parameter that must be as low as possible, APP is a parametric quadratic problem.
- Relation (2) is taken from RB while relations (3') and (9) are taken from RBP.
- Relations (16) enforce v_j to be equal to 1 if fertilizer j is produced by a blend of CBIs, otherwise to 0.
- Relation (12) is a flow conservation constraint taken from PP. In a two-stage blending perspective, the previous relations deal with the first stage and the following ones, with the second stage.
- Relations (17) similar to relations (9) ensure $z_{ki} = 1 \Leftrightarrow y_{ki} > 0$ (meaning composite material k is used to produce CBI i) and enforce the proportion of composite material k in the weight of CBI i to exceed a proportion \mathcal{K}^1 (e.g., $\mathcal{K}^1=1\%$), if composite material k is used to produce CBI i .
- According to feature b of §3.4, relation (18), aiming to prevent blending incompatible composite materials in the CBI BOM, forbid the use of chemically incompatible mixtures when creating CBI granules. So, if for example the two composite materials k and k' are incompatible (i.e. $\zeta_{kk'} = 1$), then these composites cannot be combined in the blend producing CBI i (i.e. $z_{ki} + z_{k'i} \leq 1$, z_{ki} being a binary variable which is equal to 1 when composite k is used to produce CBI i). These incompatibilities are discussed in the dataset section (§5.1) and illustrated in Table 3.
- According to feature b of §3.4, relation (19), aiming to prevent blending incompatible CBIs in the fertilizer BOM, ensures compliance with the chemical incompatibilities inherited from the first blending stage when the CBIs are mixed to produce a fertilizer. Let CBIs i and i' respectively use composites k and k' (involving $z_{ki} + z_{k'i} = 2$), both CBIs being obtained by mixing compatible material composites. Where these two composites are not compatible ($\zeta_{kk'} = 1$), then CBIs i and i' must not be combined in the production of any fertilizer j (involving: $w_{ij} + w_{i'j} \leq 3 - (z_{ki} + z_{k'i}) \Leftrightarrow w_{ij} + w_{i'j} \leq 1$, w_{ij} being a binary variable which is equal to 1 when CBI i is used to produce fertilizer j).

4.4.2 APP resolution

APP being a parametric quadratic problem where one searches the highest value of the fertilizers produced with a canonical basis of I_1 CBIs (quadratic feature) with the lowest possible value of I_1 (parametric feature), the following algorithm must be used to find the optimal solution (optimal size I_1 of the canonical basis and optimal fertilizer blends, i.e. $x_{ij} / D_j, \forall j \in \Omega_1, i \in \mathcal{I}_1$).

- 1 $I_1 = 0$
- 2 $J_1^* = 0$
- 3 $I_1 \leftarrow I_1 + 1$
- 4 Solve the APP
- 5 If $J_1 > J_1^*$ then

6 $J_1^* \leftarrow J_1$
 7 Go to 3
 8 Else go to 9
 9 Stop

4.5 Extended Reverse Blending (ERB)

This approach, summarized in Figure 1, combines APP, whose solution is a set \mathcal{S}_1 of I_1 CBIs, made of existing composite materials, the blending of which can produce the greatest set Ω_1 of J_1 fertilizers. Then, a RBP is run to find the smallest set \mathcal{S}_2 of new composite materials that corresponds to I_2 new CBIs which, combined with the initial set of CBIs, gives a blending solution for the J_2 remaining fertilizer requirements. In the end, the specifications for all fertilizers $\Omega = \Omega_1 \cup \Omega_2$ are satisfied.

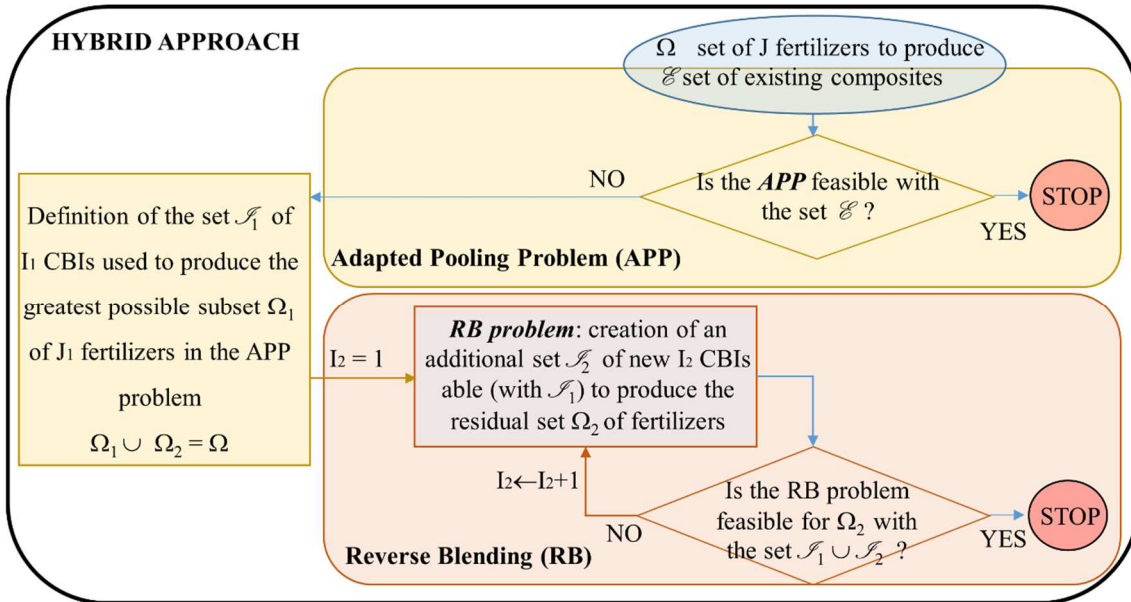


Figure 1: Extended Reverse Blending Approach.

5. The Application

As explained in the introduction, the RB was tested on a sample of 700 fertilizer solutions. The aim was to find the minimum number of CBIs whose physical blending would successfully meet all 700 NPK requirements. The results of the RB models are presented in §5.2 before being discussed in §5.3.

5.1. Dataset

The 700 NPK requirements are given by Fertimap in Kg/ha. But as fertilizers are marketed in the form of percentage formulae, it is necessary to convert the quantitative NPK requirements into needs expressed in fertilizer formulae. Indeed, since any fertilizer contains a proportion of filler, this conversion was operated by setting the filler proportion. The idea was to do so in such a way as to obtain formulae that are similar to those of currently marketed fertilizers as this would facilitate their production using currently available composite materials. Accordingly, some 70 NPK fertilizers, listed in the catalogs of world leading fertilizer manufacturers, were analyzed. As a result, a set of 8 filler proportions was defined, resulting in 482 different formulae capable of matching Fertimap's 700 NPK needs. These needs may be satisfied by other formulae of varying filler weight. Be that as it may, changing fertilizer formulae will not alter the approach. These formulae as well as demand D_j for them, are presented in Table 2 for the NPK requirements listed in Table 1. For example, to meet NPK requirement No. 362, 531.52 Kg/ha of formula 26.03–4.90–21.07 must be applied. By multiplying these % by the required quantity (531.52 Kg/ha), we obtain respectively 138.33, 26.06 and 112 Kg/ha of N, P and K which corresponds exactly to the 362th NPK need. As explained at the end of §2.1, by excluding one-component fertilizers, the models are targeted on the remaining $J=700-220+2=482$ fertilizers. Using all 482 formulae, the RB and ERB solutions meeting all 700 NPK needs are listed.

No. Need	Geographical coordinates		The specifications of the outputs					Demand (D_j) (kg for 1 ha)
	Longitude	Latitude	Output j	The nutritional specifications (β_{cj})				
				%N	%P	%K	%Filler	
1	-7.9447	33.4608	$j=1$	38.84%	1.16%	0.00%	60.00%	187.50
2	-7.9369	33.4561	$j=2$	46.00%	0.00%	0.00%	54.00%	306.15
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
362	-6.168	33.7713	$j=362$	26.03%	4.90%	21.07%	48.00%	531.52
363	-5.9531	34.0845	$j=363$	29.41%	16.59%	24.00%	30.00%	670.89
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
381	-5.1789	34.7444	$j=381$	11.24%	11.65%	23.10%	54.00%	705.63
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
699	-4.4452	33.9143	$j=699$	46.00%	0.00%	0.00%	54.00%	247.46
700	-4.3941	33.9137	$j=700$	25.13%	0.00%	8.87%	66.00%	498.74

Table 2: The composition (β_{cj}) of the fertilizer formulae (outputs) and the demands (D_j)

Concerning input data of the RB problem relating to CBI composition constraints ensured by relation (8), the maximum proportions of N, P and K in a CBI are set at $\tau_{1i} = 46\%$, $\tau_{2i} = 46\%$ and $\tau_{3i} = 75\%$, $\forall i > 1$ and the minimum proportion of filler at $\tau_{4i} = 5\%$. It is assumed that a CBI must represent at least $\kappa=1\%$ of a fertilizer weight. For the tolerated absolute deviation, a value of $\eta_{cj} = 0.0003, \forall c, j | c < 4$ is used (which complies more than exceeds EU regulation requirements). For the APP input data, only the seven most important NPK material composites in the fertilizer market are considered (see Table 3).

			Composite material k						
			Filler	Urea	TSP	KCl	MAP	DAP	Potas. nitrates
			$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
Composition of the composite materials (γ_{ck})									
Component c	%N	$c=1$	0	46	0	0	11.2	18.3	13.0
	%P	$c=2$	0	0	46	0	55.0	46.4	0
	%K	$c=3$	0	0	0	60	0	0	46.0
	% Filler	$c=4$	100.0	54.0	54.0	40.0	33.8	35.3	41.0
Matrix of the composite materials incompatibilities ($\zeta_{kk'}$)									
Composite material k	Filler	$k=1$	0	0	0	0	0	0	0
	Urea	$k=2$	0	0	1	1	0	0	0
	TSP	$k=3$	0	1	0	0	0	0	0
	KCl	$k=4$	0	1	0	0	0	0	0
	MAP	$k=5$	0	0	0	0	0	0	0
	DAP	$k=6$	0	0	0	0	0	0	0
	Potas. nitrates	$k=7$	0	0	0	0	0	0	0

Table 3: Composition of composite materials (γ_{ck}) and their incompatibilities matrix ($\zeta_{kk'}$).

When $\zeta_{kk'} = 1$, it is considered that composite materials k and k' are strictly incompatible. Yet, in practice, the blending of certain composite materials, forbidden above, can in fact, be tolerated subject to specific ratios depending on the composite material and ambient conditions. Since the exact values of these ratios are currently ignored (to be determined by experimentation), the limited incompatibilities are deemed to be strict. It is assumed that the industrial process used for CBI production requires, for any composite material k , a minimum proportion $\kappa=1\%$ for it to be used in the production of a CBI.

5.2 Results

The results below were produced by the Xpress solver using a PC (Intel® Xeon® CPU E3-1 240 v5 @ 3.50 GHz - 64 Go RAM). Please note that this paper ignores RB computational efficiency aspects as at this stage of this research, the problem can be solved with commercial software. When dealing with much larger problems, computational efficiency will of course be addressed. The results of RB and ERB are successively examined before discussing their results.

5.2.1 RB results

The application of the RBP model (see §4.2) to the above data set (variety of about seven hundred fertilizer solutions using about 482 different formulae) shows that only 8 CBIs are needed, including one acting as filler ($i=1$) (see Table 4 and Reverse_Blending_results.xlsx, included in the Mendeley data).

			CBI i							
			$i=1$ (filler)	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$
Component c	%N	$c=1$	0	46	0	22.53	29.78	12.83	52.20	20.32
	%P	$c=2$	0	0	46	0.15	23.53	0	42.32	0.49
	%K	$c=3$	0	0	0	0	0	47.09	0.49	74.19
	% Filler	$c=4$	100	40	50	77.31	46.69	40.08	5	5

Table 4: Optimal composition of RB CBIs (α_{ci})

Table 5 contains an extract of the optimal quantities x_{ij} of these CBIs to be withdrawn to meet demand for each fertilizer.

		CBI i								Demand (kg for 1 ha)
		$i=1$ (filler)	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$\sum_i x_{ij} = D_j$
Output j	$j=1$	0.00	127.53	0.00	51.05	8.92	0.00	0.00	0.00	187.50
	$j=2$	0.00	306.15	0.00	0.00	0.00	0.00	0.00	0.00	306.15
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=362$	154.75	166.41	0.00	0.00	0.00	0.00	59.80	150.56	531.52
	$j=363$	156.84	38.20	0.00	0.00	0.00	0.00	260.52	215.31	670.89
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=381$	304.62	0.00	115.12	0.00	0.00	0.00	66.60	219.29	705.63
	\vdots	\dots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=699$	0.00	247.46	0.00	0.00	0.00	0.00	0.00	0.00	247.46
	$j=700$	158.54	246.25	0.00	0.00	0.00	93.95	0.00	0.00	498.74

Table 5: The optimal quantities to be withdrawn from the RB CBIs (x_{ij})

5.2.2 ERB results

First, with the APP model, the results determine the optimal CBIs, that can be created from the composite materials of table 3, which maximizes the set of J_1 producible fertilizers ($J_1 = 626$ fertilizers). These 626 can be obtained by compatible blends of just 9 CBIs (see the detail of the APP solution in the file Adapted_Pooling_results.xlsx, included in the Mendeley data). Then, the RBP model is used to define the remaining optimal CBIs set that can be used, combined with the CBIs found with APP, to produce the set Ω_2 of $J_2=74$ remaining fertilizers. To this end, only two additional CBIs must be created ($i=10$ and $i=11$) (see Table 6 and file Extended_RB_results.xlsx included in the Mendeley data).

		Extended Reverse Blending : CBI i to be obtained with											
		The existing composite materials (Result of the APP model)									New composite materials to be created (Result of the RB model)		
		$i=1$ (filler)	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	
Component c	%N	$c=1$	0	46	0	18.30	37.05	20.25	12.64	31.27	12.60	13.31	75
	%P	$c=2$	0	0	46	46.40	15	0	0	22.23	2.64	14.12	0.75
	%K	$c=3$	0	0	0	0.00	0	35.90	46.39	0	42.67	48.58	10.71
	% Filler	$c=4$	100	54	54	35.30	47.95	43.86	40.97	46.50	42.09	23.99	13.54

Table 6: Optimal composition of the hybrid extended RB CBIs (α_{ci})

An extract of the quantities x_{ij} of these CBIs to be withdrawn to produce each output j is presented in Table

7 while the quantities y_{ki} to be withdrawn from each composite k to meet each CBI i are shown in Table 8.

		Extended Reverse Blending CBIs											Demand (kg for 1 ha) $\sum_i x_{ij} = D_j$
		APP CBIs									RB CBIs		
		$i=1$ (filler)	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	$i=11$	
Output j	$j=1$	26.36	156.45	<i>0.00</i>	4.69	0.00	0.00	<i>0.00</i>	0.00	0.00	0.00	0.00	187.50
	$j=2$	0.00	306.15	<i>0.00</i>	0.00	0.00	0.00	<i>0.00</i>	0.00	0.00	0.00	0.00	306.15
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=362$	22.29	141.11	<i>0.00</i>	56.13	0.00	311.99	<i>0.00</i>	0.00	0.00	0.00	0.00	531.52
	$j=363$	22.61	<i>0.00</i>	151.15	0.00	<i>0.00</i>	<i>0.00</i>	0.00	<i>0.00</i>	0.00	284.54	212.59	670.89
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=381$	254.37	<i>0.00</i>	78.22	0.00	<i>0.00</i>	<i>0.00</i>	0.00	<i>0.00</i>	0.00	324.93	48.11	705.63
	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
	$j=699$	0.00	247.46	<i>0.00</i>	0.00	0.00	0.00	<i>0.00</i>	0.00	0.00	0.00	0.00	247.46
	$j=700$	157.28	218.21	<i>0.00</i>	0.00	0.00	123.24	<i>0.00</i>	0.00	0.00	0.00	0.00	498.74

In **bold**: the non-zero quantities that are taken from the used CBIs.

In *italic*: the "0" (zero quantities) corresponding to CBIs that are unused because of their incompatibility with the used CBIs (the ones in **bold**).

Table 7: Optimal quantities required for ERB CBIs (x_{ij})

Table 7 shows that the CBIs were used in minimum quantities of $0.01 \times D_j$ Kg/ha and that the inheritance of the incompatibilities, forced by relation (19), was taken into account (e.g., as urea is not compatible with TSP ($\zeta_{2,3} = 1$), when a CBI which is made from TSP is used (case of CBI 3), CBIs made from urea (CBIs 2, 5, 6 and 8) are excluded and vice versa). The bold/italic font is used to emphasize the fact that if a CBI i is used (written in bold), then the quantity to be extracted from the inputs that are incompatible with this CBI i is zero (written in italic).

Table 8 shows that the quantities y_{ki} all correspond to 100% of the total quantity required for each CBI ($\sum_k y_{ki} = \sum_j x_{ij}$) thus satisfying relation (12). Also, thanks to relation (17), the minimum proportion κ set at 1% for all the composites as well as the incompatibilities of Table 3 were taken into account.

		APP								RB	
		<i>filler</i>	Urea	TSP	KCl	MAP	DAP	Potassium nitrates	$\sum_k y_{ki}$		$\sum_j x_{ij}$
		<i>k=1</i>	<i>k=2</i>	<i>k=3</i>	<i>k=4</i>	<i>k=5</i>	<i>k=6</i>	<i>k=7</i>			
APP CBI <i>i</i>	<i>i=1</i> (<i>filler</i>)	40562.06	0.00	0.00	0.00	0.00	0.00	0.00	40562.06	=	40562.06
	<i>i=2</i>	0.00	108230.83	<i>0.00</i>	<i>0.00</i>	0.00	0.00	0.00	108230.83	=	108230.83
	<i>i=3</i>	0.00	<i>0.00</i>	5623.60	0.00	0.00	0.00	0.00	5623.60	=	5623.60
	<i>i=4</i>	0.00	0.00	0.00	0.00	0.00	49659.79	0.00	49659.79	=	49659.79
	<i>i=5</i>	0.00	5044.22	<i>0.00</i>	<i>0.00</i>	0.00	2409.66	0.00	7453.88	=	7453.88
	<i>i=6</i>	0.00	2701.88	<i>0.00</i>	<i>0.00</i>	0.00	0.00	9598.68	12300.55	=	12300.55
	<i>i=7</i>	0.00	<i>0.00</i>	0.00	48.92	0.00	0.00	1726.20	1775.12	=	1775.12
	<i>i=8</i>	17.54	270.32	<i>0.00</i>	<i>0.00</i>	0.00	264.82	0.00	552.68	=	552.68
	<i>i=9</i>	135.46	0.00	0.00	0.00	266.84	0.00	5153.16	5555.45	=	5555.45
RB CBI <i>i</i>	<i>i=10</i>								13703.54	=	13703.54
	<i>i=11</i>								7948.44	=	7948.44

In **bold**: the non-zero quantities that are taken from the used composites.

In *italic*: the "0" (zero quantities) corresponding to composites that are unused because of their incompatibility with the used composites (the ones in **bold**).

Table 8: Optimal quantities y_{ki} required for each composite k

5.3 Discussion

In addition to the filler, RB came up with 7 CBIs whose possible combination fully satisfies the 700 NPK requirements. Besides accurately delivering the exact nutritional requirements, this drives huge flow massification by reducing the flows to be managed from 100% to just 1.57% (subject to laboratory experiments finding chemically stable reactions for the development of these new formulae). Otherwise, if a CBI technical feasibility is to be achieved by blending available compatible composite materials (through the APP), then the number 'I' of CBIs may increase slightly. Indeed, using the ERB hybrid approach, the number of the required CBIs went up from 7 (possibly unfit for manufacturing) CBIs (standard RB) to 10 CBIs (extended RB) (see Table 6) which are known to yield eight feasible formulae. In both cases, 700 fertilizer solutions can be provided with no more than 10 CBIs.

With such a disruptive solution, which as yet to be implemented, no benchmark is possible with any fertilizer company. Nevertheless, if one takes the real life example of 700 fertilizer needs, involving 482 different fertilizers formulas, it is obvious that no fertilizer plant can achieve annual production to stock (or to order) of at least 482 batches of these fertilizers, involving as many setups (each requiring around 2 hours) and solve the associated problems of separate storage and transportation. The results show that these requirements can be met with a canonical basis of 8 new inputs with Reverse Blending (or 11, with Extended Reverse Blending). Clearly, this flow massification drastically reduces the issues confronting fertilizer plants. These are such that currently they are able to cope at best with a diversity of a few dozens of different fertilizers (for example, the world's ten

largest fertilizer companies offer a variety of less than 180 different fertilizers). It can be argued that the proposed solution involves remote blending, and thus additional costs. To this, it can be answered that remote blending of fertilizers is already done in some countries, with very poor results, as it is limited to a mix of two or three existing fertilizers, which hardly satisfies all nutrient requirements.

After showing that a feasible solution ensuring maximum flow massification may be found, we outline in the conclusion the next steps of this research.

6 Conclusion

Reverse blending appears to be an economically efficient way of delivering customized fertilizers that satisfy the needs of sustainable agriculture while reducing production and transportation costs and issues thanks to flow consolidation (as detailed in §2.2). This, however, involves reengineering the production and distribution processes, which means that industries wishing to implement RB must be prepared to redesign their supply chain. RB, therefore, forms part of a long-term strategy with a predictable impact on the supply chain. Indeed, flow consolidation will deliver multiple streamlining opportunities such as increasing production capacity, reducing storage problems and avoiding shipping costs and problems. While the potential interest of the RB approach may be obvious, a number of issues still need to be addressed: implementing the approach requires the expertise of a multidisciplinary team. Bringing scientists from a variety of different backgrounds together is a crucial part of fixing the world's problems especially when it comes to projects that involve multiple scientific disciplines.

- First, agronomists should be asked to define the exact nutrient requirements for land parcels across countries. These requirements depend on the “soil/crop” combinations and their identification requires state-of-the art agronomic expertise. Additionally crops will range from the current ones to those that are targeted by local policy.
- Secondly, chemists shall be called upon to further expand the composite materials set, address the assumptions (mentioned at the end of §5.1) used to simplify the constraints related to composite materials incompatibilities and eventually experiment and study the stability of new chemical reactions that would be essential for the new formulae to be designed.
- Thirdly, it is planned to work on large projects and study the impact of this transformation of production processes on OCP's supply chain (our sponsor) and eventually design new distribution and production

schemes. These will be based on several scenarios describing the impact on transport of massive flow consolidation as well as the nature, location and sizing of post-manufacturing blending facilities to be installed in Africa, one of OCP's largest markets.

To conclude, RB may be used by other industries, like cosmetics, which is well on its way to marketing 100% customized products. Lastly, with reference at least to the fertilizer sector, RB effectiveness does not boil down to achieving corporate goals but, more importantly, appears to be a major lever in solving the looming food supply crises facing mankind and, as such, clearly belongs to the field of corporate social responsibility.

7 Data Set and Results are included in the Mendeley data (<http://dx.doi.org/10.17632/z3sbn5j9z7.1>)

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