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Abstract: While previous studies overlook the collusive behavior of retailers under bidirectional competition, we study a closed-loop supply chain (CLSC) consisting of one manufacturer and two competing retailers whose competition exists in the forward and reverse channels. Considering the retailers' potential collusive behavior and the upstream manufacturer's interactive decisions, we build three two-tier game models: Stackelberg-collusion model, Stackelberg-Nash model, and Stackelberg-Stackelberg model. We first obtain the equilibrium decisions of the manufacturer and the retailers in the three models. Furthermore, we investigate whether collusion is beneficial to the two retailers. We find that the retailers' collusion always brings remarkable profit improvement to them as a whole, while the smaller retailers may suffer severe profit loss. A profit-sharing contract is, therefore, designed to guarantee that each retailer can gain more profit by collusion. We also compare the environmental benefit and social welfare among the three models and find that collusion may do harm to the environment or social welfare compared with other models.
Keywords: supply chain management; recycling competition; demand competition; collusion; closed-loop supply chain

1. Introduction

Remanufacturing is now attracting increasing attention because of the related regulations and legislations, the well-accepted sustainable development concept, as well as its economic benefit. In a wide range of industrial sectors, such as automobiles and consumer electronics, companies have been making efforts to collect used products, which are also referred to as cores (Ferguson and Toktay, 2006; Ferrer and Swaminathan, 2006; Östlin et al., 2008; Bulmus et al., 2014).

This research is partially motivated by the observation of competing firms' cooperation/collusion in the recycling business. For example, in Sweden, Netherlands, Belgium, and Norway, some competing firms build alliances to collect electrical and electronics waste (Atasu and Subramanian, 2012). Panasonic has created recycling systems to handle returned products that are from their competitors in Japan (Modak et al., 2016). In such cases, competing stakeholders may act cooperatively to make corresponding decisions to make themselves better off. This collusive behavior has attracted increasing attention of scholars, including Clark and Houde (2013), Hu et al. (2014), Hosseini-Motlagh et al. (2018), and Zheng et al. (2018).

However, relevant research on retailers' collusive behavior with bidirectional (forward and reverse) competitions in a closed-loop supply chain is rather limited. Most existing literature pay attention to the competition in one direction and usually ignore retailers' collusion. Therefore, it remains unclear how such combinational competitive conditions influence the equilibrium decisions of supply chain members. Further, it is also debatable whether retailers should adopt collusion considering the upstream manufacturer's pricing strategy. In this research, we seek to answer the following question: Should two competing retailers adopt collusion when they face forward and reverse competitions in a closed-loop supply chain? Specifically, we solve three research problems. First, what are the equilibrium decisions of two retailers and the manufacturer when retailers make collusion and do not? How do equilibrium decision variables differ among different models? Second, does collusion always bring more profit to retailers compared with the case of no collusion? Third, from the perspectives of environment impact or social welfare, is collusion still the optimal choice for retailers?

To answer these research questions, we consider two cases with and without collusion. In the former case, the retailers make decisions jointly on ordering quantities and reverse investment to maximize their total profits. In the latter case, the two retailers make decisions, and the objective is to maximize individual profit. Based on the game theory, the decision-making process in the latter case may be simultaneous or sequential. For simultaneous game, the two retailers make decisions at the same time by anticipating the other's strategy. In equilibrium, no one can gain more profit by changing his strategy unilaterally. For sequential game, one retailer (the leader) moves first, and the other retailer (the follower) moves later after observing the leader's decisions. In the three models, the manufacturer moves before the retailers, that is, the manufacturer first determines the wholesale and transfer prices, and then the two retailers make decisions. To summarize, based on the behavior of the manufacturer and two retailers, we study three two-tier models, namely, Stackelberg-collusion model, Stackelberg-Cournot model, and Stackelberg-Stackelberg model.

This study provides several contributions in the closed-loop supply chain. First, different from previous research that usually ignores retailers' collusive behavior in a closed-loop supply chain and only considers one directional competition, we study retailers' competition in two directions (i.e., forward and reverse channels) with three types of behavior modes (i.e., collusion, Nash game, and Stackelberg game). By deriving the equilibrium decisions, we find that the manufacturer's pricing decisions are different from those in the case with only forward competition, as is studied in most literature. Second, by comparing the profits among the three models, we find that collusion always brings two competing retailers noticeable profit improvement, even though the manufacturer may change his pricing decisions when collusion takes place. Numerical experiments reveal the conditions under which the profit improvement is the most remarkable. However, the smaller retailer may suffer severe profit loss because of collusion. Third, we design a Nash arbitration scheme to guarantee a win-win situation. We also compare the environmental benefit and social welfare among three models and find that collusion may not be the optimal choice.

This paper proceeds as follows: In Section 2, we make a relevant literature review and state our contribution. In Section 3, we present three model settings and basic assumptions. In Section 4, we derive the equilibrium decisions and profits in the three models. Section 5 presents a comparative analysis among different models. The concluding remarks are in Section 6. The proof is in the Appendix.

2. Literature Review

Our research is closely related to three streams: (1) competition/cooperation in closed-loop supply chains; (2) interactive behaviors of closed-loop supply chain members; (3) sustainable supply chain management. They are presented in following Sections 2.1-2.3 and the research gap and our contributions is presented in Section 2.4.

2.1 Competition/cooperation in a closed-loop supply chain

This research is related to the stream on competition/cooperation in a closed-loop supply chain. Some research focuses on forward channel competition. The forward channel demand competition between an original equipment manufacturer (OEM) and an independent remanufacturer (IR) has been investigated, taking into account the product quality, social welfare (Mitra and Webster, 2008), or government subsidies (Oersdemir et al., 2014). Fuzzy theory, which can handle the uncertainties in the supply chain, has been combined with the game theory to study the pricing decisions with retail competition (Wei and Zhao, 2011). The demand competition between two supply chains is also investigated, and the equilibrium strategies are obtained (Wu and Zhou, 2017). Other research focuses on reverse channel competition. The competition between two independent stakeholders on collecting cores in the reverse channel has been widely investigated, including the competition between a retailer and a third party (Huang et al., 2013; Yi et al., 2016), a manufacturer and a retailer (He et al., 2019), and a manufacturer and a third party (Liu et al, 2017). In addition, the competition between two recycling alliance is also attracting increasing attention (Ma et al., 2018). To summarize, most of the studies the competition in a closed-loop supply chain in single dimension except very limited studying bidirectional competition including the following. Bulmus et al. (2014) consider the competitions between an original equipment manufacturer and an independently operating remanufacturer. Johari and Hosseini-Motlagh (2019) consider the competitions between two retailers in the forward channel and the reverse competition between two third-party collectors.

Faced with competition, supply chain members have the incentive to cooperate for the sake of higher profits. Clark and Houde (2013) study the case that gasoline retailers make collusion by fixing prices. Zheng et al. (2018) examine the interaction between the manufacturer's channel strategy and the downstream retailers' collusive behavior. They find that retailers’ collusion can be deterred by changing the manufacturer’s channel strategy. Hosseini-Motlagh et al. (2018) study retailers’ collusion behavior through determining credits. Seyedhosseini et al. (2019) study retailers’ collusive decisions on prices in the presence of manufacturer's investment in corporate social responsibility effort. In summary, the existing literature assume supply chain members make collusive decision in single dimension, such as the price or credit, while the research on collusion in multiple dimensions are rather limited. Recently, Johari and Hosseini-Motlagh (2019) study the case that two retailers make collusion on prices and two third-party collectors make collusion on recycling rates. Differently, we study the case that two retailers make collusion on both quantities and recycling rates.

2.2 Interactive behaviors of closed-loop supply chain members

The performances of the closed-loop supply chain under various channel structures are examined, including retailer-led, manufacturer-led, and collector-led models. In these models, the decision-making process of different stakeholders follows Stackelberg game. It is found that the retailer-led model is the most effective (Choi et al., 2013), and each member has the incentive to act as the leader (Zheng et al., 2017). The Nash model, where several stakeholders move simultaneously, is also studied (Gao et al., 2016; Maiti and Giri, 2015). Some specialized environments are considered, including the government reward-penalty mechanism (Wang et al., 2015) and the fuzzy recycling cost (Ke et al., 2018). Johari and Hosseini-Motlagh (2019) consider different game behaviors in the forward and reverse links under decentralized, centralized, and coordinated decision-making structures. Reviewing the above literature, we find more attention is paid to one-tier game between supply chain members while less attention is paid to two-tier game. Recently, Hosseini-Motlagh et al. (2019) study a two-tier Stackelberg game between a remanufacturer and two collectors. The remanufacturer acts as the first-tier Stackelberg game leader. Two collectors play the second-tier Stackelberg game where they move sequentially to determine acquisition prices in the reverse channel. We also study a two-tier Stackelberg game and our work differs in that our decisions variables in the second-tier game includes return rates in the reverse channel and selling prices in the forward channel. In addition, they assume the remanufacturer builds a direct channel to sell remanufactured products while we assume retailers act as the intermediary to distribute new and remanufactured products. The profit allocation and coordination mechanism is also different between two papers. Specifically, they propose a two-way two-part tariff contract and we allocate the profit surplus based on Nash arbitration scheme.

2.3 Sustainable supply chain management

The research about sustainable supply chain management is now attracting increasing attention. As an important aspect of sustainability, carbon emission has been taken into account by relevant research (Hugo and Pistikopoulos, 2005; Diabat and Alsalem, 2015). Das and Posinasetti (2015) integrate environmental concerns in a closed-loop supply chain model to improve overall SC performance. Gao et al. (2016) examine the influence of different channel power structures on the optimal decisions and performance of a closed-loop supply chain (CLSC) from the perspective of profit and environment. Modak et al. (2016) propose a two-stage competitive CLSC and coordination model to improve environmental and economic aspects of sustainability. Zhu and Sarkis (2007) find that environmentally friendly practices have a positive effect on economic and environmental performance based on an empirical study in China. Panigrahi and Bahinipati (2019) summarize that the research in sustainable supply chain management primarily considers the environmental issues whereas the social aspect is ignored. Wang et al. (2017) investigate the relationship between profitability and environmental goals in a reverse supply chain. The main finding is that while they often conflict, they may align under certain conditions. We can conclude that very few literature considers all perspectives of sustainability (profit, environment and society) simultaneously. In addition, collusion behavior is often ignored. Mostly related to our research, Johari and Hosseini-Motlagh (2019) propose an analytical coordination model to cover all three dimensions of sustainability. They measure the social welfare by the sum of firms' profits and consumer surplus. Different from that, we also incorporate the environment benefit when calculating the social welfare. In addition, we find the conditions under which confliction exists between profitability and environment benefit/social welfare.

2.4 Research gaps and our contributions

To better position our study, we compare this research with some closely related ones in Table 1. According to the above subsections and Table 1, the research gap and our contributions are identified as follows.

First, unlike the research of competition/cooperation in a closed-loop supply chain that considers the competition in single dimension, we study the case that retailers play dual roles in the two channels of a closed-loop supply chain. In this context, we investigates their demand competition in the forward channel and reverse competition in the reverse channel. In addition, their collusion decisions in two channels are also investigated.

Second, unlike the research of interactive behaviors of closed-loop supply chain members that usually studies one-tier Stackelberg game between different supply chain members, we also study a more complicated two-tier Stackelberg game between the manufacturer and two competing retailers.

Third, unlike the research of sustainable supply chain management that usually considers sustainability from the environment, we make a comprehensive analysis of retailers' collusion behavior from three perspectives including firms' profits, environmental benefit and social welfare. More important, we investigates the potential confliction between different perspectives. This work helps make a better understanding of the effect of retailers' collusion behavior on sustainability in the context of closed-loop supply chain.

Table 1. Comparison of our work with related literature

Research Paper	Forward channel competition	Reverse channel competition	Collusion	Two-tier Stackelberg game
Savaskan and Van Wassenhove (2006)	√	×	×	×
Mitra and Webster (2008)	√	×	×	×

Oersdemir et al. (2014)	√	×	×	×
Wu and Zhou (2017)	√	×	×	×
Yang and Zhou (2006)	√	×	√	√
Clark and Houde (2013)	√	×	√	×
Zheng et al. (2018)	√	×	√	×
Huang et al. (2013) ; Hong et al. (2013); Liu et al. (2017)	×	√	×	×
Hosseini-Motlagh et al. (2019)	×	×	×	√
Johari and Hosseini-Motlagh (2019)	√	√	√	×
Hosseini-Motlagh et al. (2018)	√	×	√	√
Seyedhosseini et al. (2019)	√	×	√	×
Our work	√	√	√	√

3. Model

We consider a closed-loop supply chain consisting of a manufacturer and two retailers. In the forward channel, the manufacturer distributes one kind of product via two retailers to consumers. In the reverse channel, two retailers collect used products (cores) and then return them to the manufacturer. These cores are then utilized for remanufacturing products.

We assume no difference exists between remanufactured and new products. This assumption is widely used in relevant literature on CLSC, including Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), Akcali and Cetinkaya (2011), Atamer et al. (2013), Bulmus et al. (2014), Hong et al. (2015), Wang et al. (2016), and Liu et al. (2017). It means that remanufactured products, with the same quality and warranty, are perfect substitutes to new ones so that consumers cannot distinguish them. We can find supporting examples such as single-use cameras (Akcali and Cetinkaya, 2011) and reusable containers for beverage and food manufacturing, packaging, and transportation (Atamer et al., 2013).

In the research, we consider that the market price is a linear function of product quantities, that is, $p_i = \alpha_i - q_i - dq_j (i, j \in \{1,2\}, i \neq j)$. This linear inverse demand function is widely used in the fields of economics, marketing, operation management, and supply chain management to model demand competition. We refer to Arya et al. (2008), Farahat and Perakis (2011), Wang et al. (2013), Bagchi and Mukherjee (2014), and Wu and Zhou (2017) for details. In this research, we consider that the competition intensity or the substitution degree is identical, that is, $d_i = d_j = d$, and this assumption can be easily extended to the case where $d_i \neq d_j$.

The return rate depends on retailers' effective investment. In the real world, firms often make investment in promotional/advertising activities to encourage consumers to return used products

(Savaskan et al., 2004; Savaskan and Van Wassenhove, 2006). We use a strictly convex increasing function to model the investment cost of cores collection. First of all, one retailer's return rate concavely increases with his investment, which has been adopted by Savaskan et al. (2004), Savaskan and Van Wassenhove (2006), Hong et al. (2015), and Hong et al. (2017). In addition, when competition for limited cores in the reverse channel is taken into account, it is obvious that one retailer's return rate increases with her own investment while decreases with the competitor's investment. Specifically, we use $\tau_1 = \sqrt{(I_1 - kI_2)/C}$ and $\tau_2 = \sqrt{(I_2 - kI_1)/C}$ to model their return rates respectively, where k denotes reverse competition intensity. This function has been used in Huang et al. (2013) and Liu et al. (2017). In fact, one can regard $I_i - kI_j (i, j \in \{1,2\}, i \neq j)$ as an effective investment of Retailer i, and this function indicates that the return rate depends on effective investment. In addition, we assume sufficiently large C to guarantee $\tau_1 + \tau_2 < 1$. Based on these two equations, we can equivalently obtain $I_1 = (C\tau_1^2 + kC\tau_2^2)/(1-k^2)$ and $I_2 = (C\tau_2^2 + kC\tau_1^2)/(1-k^2)$.

To explore whether the collusive behavior always creates more profit to retailers in the presence of the strategic manufacturer, we examine three models consisting of one manufacturer and two retailers. The three specific models are as follows: (1) Stackelberg-Collusion game (Model SCL), wherein the manufacturer acts as the Stackelberg leader and determines the common wholesale price (w) and transfer price (b). Afterward, the two retailers cooperate with each other and jointly make decisions on quantities and return rates to maximize their total profit. In this model, two retailers cooperate and build an alliance. (2) In the Stackelberg-Cournot game (Model SCN), the manufacturer is still the leader. Afterward, each self-interested retailer makes her own quantity and return rate at the same time. (3) In the Stackelberg-Stackelberg game (Model SS), the manufacturer acts as the leader and makes pricing decisions again. Afterward, one retailer moves first, and the other retailer moves after observing the first mover's action. Without loss of generality, we assume Retailer 2 is the first mover. The three models are illustrated in Figure 1.

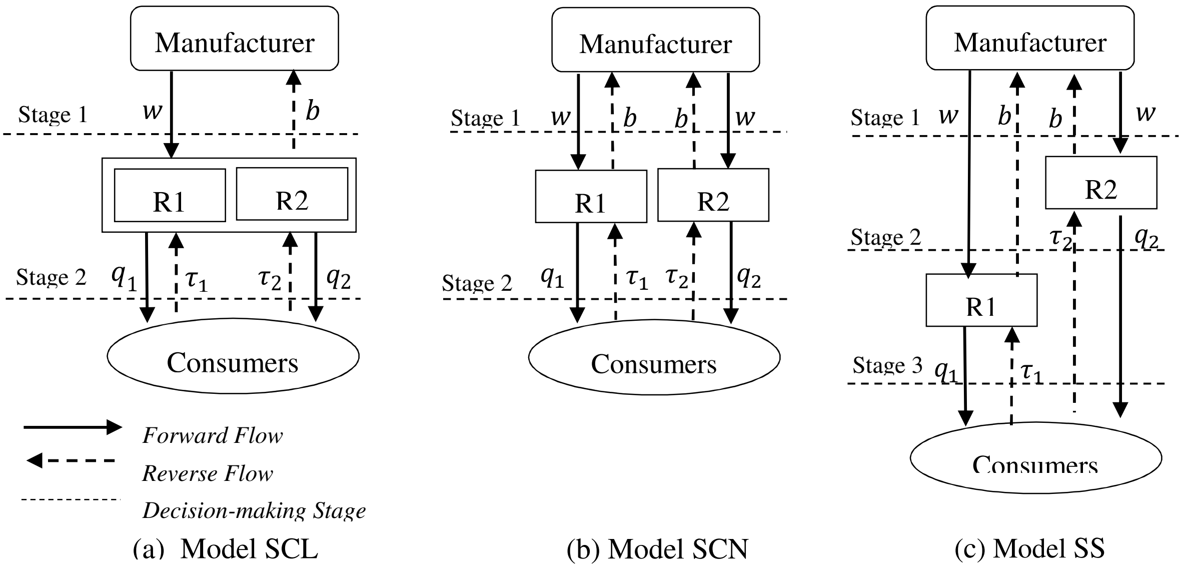


Figure 1. Three models of CLSC with competing retailers

For ease of explanation, we list all notations in the following Table 2.

Table 2. Notations

Decision variables	
w	Wholesale price
q_i	Ordering quantity of retailer i, and $i \in \{1,2\}$
b	Transfer price of unit used product from retailers to the manufacturer
τ_i	Return rate of retailer i, and $i \in \{1,2\}$
Model parameters	
α_i	Retailer i's market scale, and $i \in \{1,2\}$
p_i	Retailer i's retail price, and $i \in \{1,2\}$
d	Demand competition intensity between retailers, and $d \in (0,1)$
k	Reverse competition intensity between retailers, and $k \in (0,1)$
c_n	Unit cost of new product
c_r	Unit cost of remanufactured product and $c_r < c_n$
Δ	Cost saving due to remanufacturing, and $\Delta = c_n - c_r > 0$
I_i	Investment of retailer i, and $i \in \{1,2\}$
C	Scaling parameter between collecting investment and return rate
e	The environmental benefit for unit recycled product
π_x^y	The profit of firm x in Model y, where $x \in \{m, r_1, r_2, r\}$ and $y \in \{SCL, SCN, SS\}$
$()^*$	The notation of optimality/equilibrium

4. Equilibrium Analysis

In this section, we sequentially analyze the equilibrium results in Model SCL, Model SCN, and Model SS. In addition, we compare the manufacturer's and the retailers' equilibrium decisions among models. We use superscripts SCL, SCN, SS to differentiate the models and their corresponding variables to avoid ambiguity.

4.1 Stackelberg-Collusion Game (Model SCL)

In the Stackelberg-Collusion game, which is abbreviated as Model SCL, the sequence of events is of two stages.

(1) In anticipation of the retailers' decisions, the manufacturer decides the wholesale price w and the unit tranfer price b to maximize his profit function. Referring to Savaskan et al. (2004), we write the profit function as follows:

$$\pi_m(w,b)=[w-c_n+(\Delta-b)(\tau_1+\tau_2)](q_1+q_2), \tag{1}$$

where $w(q_1+q_2)$ is the revenue of selling products to retailers , $[c_n-\Delta(\tau_1+\tau_2)](q_1+q_2)$ is total production cost and $b(\tau_1+\tau_2)(q_1+q_2)$ is the transfer payment to collectors.

(2) Given that w and b are determined by the manufacturer, Retailer 1 and Retailer 2 jointly make decisions on q_1, q_2, τ_1, τ_2 to maximize their total profit $\pi_r=\pi_{r1}+\pi_{r2}$ as follows:

$$\begin{aligned}&\pi_r(q_1,q_2,\tau_1,\tau_2\,|\,w,b)\\ &=(\alpha_1-q_1-dq_2-w)q_1+(\alpha_2-q_2-dq_1-w)q_2+[b(\tau_1+\tau_2)](q_1+q_2)-(I_1+I_2)\end{aligned},$$

(2)

where the first term represents Retailer 1's profit in the forward channel, the second term represents Retailer 2's profit in the forward channel, the third term represents the payment of manufacturer for returned items and the last term represents the total investment costs. Note that without any designed profit allocation scheme, the profit of Retailer 1 is $\pi_{r1}=(\alpha_1-q_1-dq_2-w)q_1+b\tau_1(q_1+q_2)-I_1$ and the profit of Retailer 2 is $\pi_{r2}=(\alpha_2-q_2-dq_1-w)q_2+b\tau_2(q_1+q_2)-I_2$.

There becomes a subgame perfect equilibrium, and we apply backward induction method to solve this problem. Specifically, we first solve retailers' best-response functions of q_1, q_2, τ_1, τ_2, given w and b. The results are shown in Lemma 1.

Lemma 1. Under given w and b, the retailers' best-response functions are as follows:

$$q_1^{\mathrm{SCL}*}=\frac{\alpha_1-\alpha_2}{4(1-d)}+\frac{C(\alpha_2+\alpha_1-2w)}{4\big(C(1+d)-b^2(1-k)\big)},\quad q_2^{\mathrm{SCL}*}=\frac{\alpha_2-\alpha_1}{4(1-d)}+\frac{C(\alpha_2+\alpha_1-2w)}{4\big(C(1+d)-b^2(1-k)\big)},$$

$$\tau_1^{\mathrm{SCL}*} = \tau_2^{\mathrm{SCL}*} = \frac{b(1-k)\left(\alpha_1+\alpha_2-2w\right)}{4\left(C(1+d)-b^2(1-k)\right)}.$$

By Lemma 1, we find that the return rates of Retailer 1 and Retailer 2 are equal. Moreover, the return rate decreases linearly in the wholesale price, which reflects the interaction between the forward and the reverse channels. As the wholesale price decreases, retailers order and sell more, and therefore, more proportionally used products are available for recycling, which motivates the retailers to invest more on recycling. Interestingly, the return rate is nonlinearly increasing in b. The reason is that in addition to motivating direct recycling, a larger transfer price b can also enhance the return rate by stimulating more demand. With the dual effect of b, the return rate increases more drastically than b. Another observation is that the gap between the two retailers' quantities only depends on the gap of their market scales and the forward competition intensity. The retailer who has a larger market scale orders more quantity, which is intuitive.

By the results in Lemma 1 and the first-order conditions, we obtain the manufacturer's equilibrium wholesale price and transfer price, which are shown in Theorem 1.

Theorem 1. *In Model SCL, the manufacturer's equilibrium wholesale price and transfer price are as follows:*

$$w^{\mathrm{SCL}^*} = \frac{\alpha_1+\alpha_2+2c_n}{4}, \quad b^{\mathrm{SCL}^*} = \Delta .$$

The retailers' equilibrium decisions are: $q_1^{\mathrm{SCL}^*} = \frac{(\alpha_1-\alpha_2)}{4(1-d)} + \frac{C(\alpha_1+\alpha_2-2c_n)}{8\left(C(1+d)-(1-k)\Delta^2\right)}$,

$q_2^{\mathrm{SCL}^*} = \frac{(\alpha_2-\alpha_1)}{4(1-d)} + \frac{C(\alpha_2+\alpha_1-2c_n)}{8\left(C(1+d)-(1-k)\Delta^2\right)}$, *and* $\tau_1^{\mathrm{SCL}^*} = \tau_2^{\mathrm{SCL}^*} = \frac{\Delta(1-k)(\alpha_1+\alpha_2-2c_n)}{8\left(C\left(1+d\right)-(1-k)\Delta^2\right)}$.

Based on Theorem 1, the equilibrium profit of the manufacturer is $\pi_m^{\mathrm{SCL}^*} = \frac{C\left(-2c_n+\alpha_1+\alpha_2\right)^2}{16\left(C(1+d)+(-1+k)\Delta^2\right)}$. We now make a simple comparison with the case of no remanufacturing. According to Zhou and Yang (2006) that study duopolistic retailers' collusion behavior without remanufacturing, we can obtain the profit of the manufacturer and the total demand is $\frac{\left(\alpha_1+\alpha_2-2c_n\right)^2}{16(1+d)}$ and $\frac{\alpha_1+\alpha_2-2c_n}{4(1+d)}$, respectively. We find that both of them are smaller than that in the investigated case with remanufacturing. The comparison implies that although there is no direct economic benefit from recycling and remanufacturing in the reverse channel ($b=\Delta$), the manufacturer mainly benefits from larger demands in the forward channel.

For each retailer, the ordering quantity increases with her own market scale and decreases with the rival's. However, the return rate increases with her own market scale or the rival's. We also find that the market scale has great impact on the profit distribution. When the retailers are symmetric ($\alpha_1=\alpha_2=\alpha$), $\pi_{r1}^{\mathrm{SCL}*}=\pi_{r2}^{\mathrm{SCL}*}=\dfrac{C(\alpha-c_n)^2}{16[C(1+d)+(-1+k)\Delta^2]}$, $\pi_m^{\mathrm{SCL}*}=\dfrac{C(\alpha-c_n)^2}{4\left(C(1+d)+(-1+k)\Delta^2\right)}$, and $\dfrac{\pi_m^{\mathrm{SCL}*}}{\pi_{r1}^{\mathrm{SCL}*}+\pi_{r2}^{\mathrm{SCL}*}}=2$. That is, the manufacturer's profit is the double of the two retailers' total profit.

4.2 Stackelberg-Cournot Game (Model SCN)

In the Stackelberg-Cournot game, which is abbreviated as Model SCN, two retailers make decisions simultaneously after the manufacturer makes pricing decisions. At equilibrium, neither retailer can gain more profit if she unilaterally changes her own strategy.

The sequence of events is of two stages: (1) The manufacturer decides the wholesale price w and transfer price b . (2) Retailer 1 makes decisions on q_1 and τ_1 to maximize π_{r1}. At the same time, Retailer 2 makes decisions on q_2 and τ_2 to maximize π_{r2}.

In this model, the manufacturer's profit function is shown in Equation (1), and the retailers' profit functions are as follows:

$$\pi_{r1}(q_1,\tau_1\,|\,w,b)=(\alpha_1-q_1-dq_2-w)q_1+b\tau_1(q_1+q_2)-I_1, \tag{3}$$

$$\pi_{r2}(q_2,\tau_2\,|\,w,b)=(\alpha_2-q_2-dq_1-w)q_2+b\tau_2(q_1+q_2)-I_2. \tag{4}$$

The equilibrium is subgame perfect equilibrium, and backward induction method is used to solve this problem. Specifically, we first obtain retailer i's best-response functions of q_i, τ_i, given w and b . After that, we obtain the equilibrium w and b .

Lemma 2. Given w and b , retailers' best-response functions are as follows: $q_1^{\mathrm{SCN}^*}=\dfrac{\alpha_1-\alpha_2}{2(2-d)}+\dfrac{C(\alpha_1+\alpha_2-2w)}{2\left(C(2+d)-b^2(1-k^2)\right)}$, $q_2^{\mathrm{SCN}^*}=\dfrac{\alpha_2-\alpha_1}{2(2-d)}+\dfrac{C(\alpha_1+\alpha_2-2w)}{2\left(C(2+d)-b^2(1-k^2)\right)}$,

$\tau_1^{\mathrm{SCN}^*}=\tau_2^{\mathrm{SCN}^*}=\dfrac{b(1-k^2)(\alpha_1+\alpha_2-2w)}{2\left(C(2+d)-b^2(1-k^2)\right)}$.

Substituting these best-response functions into the manufacturer's profit in Equation (1) and by the first-order conditions, we can obtain w and b in equilibrium.

Theorem 2. *In Model SCN, the manufacturer's equilibrium wholesale price and transfer price are as follows:*

$$w^{\mathrm{SCN}^*}=\frac{\alpha_1+\alpha_2+2c_n}{4},\quad b^{\mathrm{SCN}^*}=\Delta.$$

The retailers' equilibrium decisions are: $q_1^{\text{SCN*}} = \dfrac{\alpha_1 - \alpha_2}{2(2-d)} + \dfrac{C(\alpha_1 + \alpha_2 - 2c_n)}{4\left(C(2+d) - \Delta^2(1-k^2)\right)}$,

$q_2^{\text{SCN*}} = \dfrac{\alpha_2 - \alpha_1}{2(2-d)} + \dfrac{C(\alpha_1 + \alpha_2 - 2c_n)}{4\left(C(2+d) - \Delta^2(1-k^2)\right)}$, *and* $\tau_1^{\text{SCN}^*} = \tau_2^{\text{SCN}^*} = \dfrac{\Delta(1-k^2)\left(\alpha_1 + \alpha_2 - 2c_n\right)}{4\left(C(2+d) - \Delta^2(1-k^2)\right)}$.

From Theorem 2, the equilibrium return rates are the same for the two retailers. The gap between the two retailers' quantities only depends on the gap of their market scales ($\alpha_2 - \alpha_1$) and the forward competition intensity (d). By Theorem 2, we can also obtain the profit of the manufacturer and the total quantity is $\pi_m^{\text{SCN*}} = \dfrac{C\left(-2c_n + \alpha_1 + \alpha_2\right)^2}{8C(2+d) + 8\left(-1+k^2\right)\Delta^2}$ and $q^{\text{SCN*}} = \dfrac{C\left(-2c_n + \alpha_1 + \alpha_2\right)}{2C\left(2+d\right) + 2\left(-1+k^2\right)\Delta^2}$. Based on the method of Yang and Zhou (2006) that study duopolistic retailers' Cournot competition without remanufacturing, we can also obtain the profit of manufacturer and total quantity is $\dfrac{\left(\alpha_1 + \alpha_2 - 2c_n\right)^2}{8(2+d)}$ and $\dfrac{\left(\alpha_1 + \alpha_2 - 2c_n\right)}{2\left(2+d\right)}$, respectively. The comparison implies that in Model SCN, as a result of increased total quantity ($q^{\text{SCN*}}$), the profit of manufacturer increases, compared with the case without remanufacturing, which is similar to the results in Theorem 1.

4.3 Stackelberg-Stackelberg Game (Model SS)

In the Stackelberg-Stackelberg game, which is abbreviated as Model SS, the sequence of events is of three stages. (1) The manufacturer decides the wholesale price w and transfer price b ; (2) Retailer 2 makes decisions on q_2 and τ_2 to maximize π_{r2}; (3) and Retailer 1 makes decisions on q_1 and τ_1 to maximize π_{r1}.

In this model, the manufacturer's profit is shown in Equation (1), and retailers' profit functions are as follows:

$$\pi_{r2}(q_2, \tau_2 \mid w, b) = (\alpha_2 - q_2 - dq_1 - w)q_2 + b\tau_2(q_1 + q_2) - I_2, \tag{5}$$

$$\pi_{r1}(q_1, \tau_1 \mid w, b, q_2, \tau_2) = (\alpha_1 - q_1 - dq_2 - w)q_1 + b\tau_1(q_1 + q_2) - I_1. \tag{6}$$

This problem is harder to solve. We also use backward induction method to analyze the supply chain members' equilibrium decisions. First, we obtain Retailer 1's best-response function given Retailer 2's and the manufacturer's decisions, as shown in the following lemma:

Lemma 3. For given w , b , q_2 , τ_2 , Retailer 1's best-response functions are $q_1^{\text{SS*}} = \dfrac{[-2Cd - b^2(-1+k^2)]q_2 + 2C(-w + \alpha_1)}{4C - b^2(1-k^2)}$ and $\tau_1^{\text{SS*}} = \dfrac{b(-1+k^2)[w + (-2+d)q_2 - \alpha_1]}{4C - b^2(1-k^2)}$.

Next, we obtain Retailer 2's best-response functions given the manufacturer's decisions, as shown in Lemma 4:

Lemma 4. Given w and b, Retailer 2's best-response functions are as follows: $\tau_2^{\mathrm{SS}*}=\frac{Z_2 w+Z_3}{2Z_1}$, $q_2^{\mathrm{SS}*}=\frac{Z_4 w+Z_5}{2Z_1}$,

where $Z_1=8C^2(d^2-2)+b^2C(-12+8d+d^2+(2-d)^2k)(k^2-1)+b^4(d-1)(k^2-1)^2$,

$Z_2=b(1-k^2)\left\{2C[8-d(4+d)]-b^2(4-3d)(1-k^2)\right\}$,

$Z_3=b(1-k^2)\{2[C(-4+d(2+d))+b^2(1-d)(1-k^2)]\alpha_1+(2-d)[b^2(1-k^2)-4C]\alpha_2\}$,

$Z_4=8C^2(2-d)-2b^2C[2+(2-d)k](1-k^2)+b^4(k^2-1)^2$, and

$Z_5=2C\{4Cd+b^2[2+\left(d-2\right)k](k^2-1)\}\alpha_1-[4C+b^2(k^2-1)]^2\alpha_2$.

By Lemma 4, we can obtain $\tau_1^{\mathrm{SS}*}=\frac{Z_2 w+Z_3}{2Z_1}$ and $q_1^{\mathrm{SS}*}=\frac{(Z_4 w+Z_5)Z_6}{2Z_1}+Z_7(\alpha_1-w)$, where $Z_6=\frac{-2Cd-b^2(k^2-1)}{4C+b^2(k^2-1)}$ and $Z_7=\frac{2C}{4C+b^2(k^2-1)}$.

Finally, knowing the retailers' best-response functions, we obtain the manufacturer's equilibrium decisions. Because of the complexity of q_1, q_2, τ_1, and τ_2 in Lemmas 3 and 4, it is difficult to obtain the manufacturer's equilibrium decisions in closed form. We present some-properties regarding them in Theorem 3.

Theorem 3. *In Model SS, the manufacturer's equilibrium wholesale and transfer prices have the following properties:*

$$b^{\mathrm{SS}*}<\Delta,\quad w^{\mathrm{SS}*}(b)=\frac{XZ_1c_n-\left(2XT_1+Z_1\right)\left(Y_1\alpha_1+Y_2\alpha_2\right)}{2X\left(XT_1+Z_1\right)}.$$

In addition, when $b=0$ *or* $b=\Delta$, $w^{\mathrm{SS}*}(b)\geq\frac{\alpha_1+\alpha_2+2c_n}{4}$.

Z_1 *is defined in Lemma 4, and* X, T_1, Y_1, *and* Y_2 *are defined as follows:*

$X=2C(8-d(4+d))-b^2(4-3d)\left(1-k^2\right)>0$, $T_1=b\left(1-k^2\right)(\Delta-b)>0$,

$Y_1=2\left(C(-4+d(2+d))+b^2(-1+d)(-1+k^2)\right)<0$, $Y_2=-(2-d)\left(4C+b^2\left(-1+k^2\right)\right)<0$.

Owing to the high complexity of the manufacturer's profit function, the closed-form expression of the optimal transfer price $b^{\mathrm{SS}*}$ is not available, which, however, can be proven smaller than Δ.

We will conduct numerical experiments to find the equilibrium decision b^{SS*} within the range of $(0,\Delta)$ in Section 5. The most important finding is that in Model SS, the manufacturer sets a lower transfer price and gets direct profit from remanufacturing cores.

The results in Theorem 3 are different from the existing literature. Yang and Zhou (2006) show that in a two-echelon supply chain without reverse channel or remanufacturing, under different retailers' behavior modes (Cournot, Collusion, or Stackelberg), the manufacturer's equilibrium wholesale price is the same when retailers have equal market scales ($\alpha_1 = \alpha_2$). However, in this research, the wholesale price in Model SS differs from that in Model SCL or Model SCN even if $\alpha_1 = \alpha_2$. This finding shows that introducing remanufacturing activity into the supply chain and considering the reverse competition change the manufacturer's pricing decisions.

5. Comparative Analysis

5.1 Analytic Comparison

We first compare the manufacturer's equilibrium decisions among the three models. The main results are presented in the following theorem:

Theorem 4. *We compare the equilibrium decisions of the manufacturer among the three models and find that:*

(1) $b^{SCL*} = b^{SCN*} > b^{SS*}$*;*

(2) $w^{SCL*} = w^{SCN*}$.

Theorem 4 reveals the difference of the manufacturer's equilibrium decisions under different retailers' behavior modes. It is interesting that the manufacturer sets the same wholesale prices in Models SCL and SCN. The manufacturer transfers all cost saving of remanufacturing to retailers ($b^{SCL*} = b^{SCN*} = \Delta$). The manufacturer only benefits directly from the forward channel. The reverse channel plays a role in boosting the market demand. Model SS is different since the manufacturer also benefits directly from the reverse channel by setting a lower transfer price ($b^{SS*} < \Delta$). Therefore, the more returns, the larger benefit he obtains by remanufacturing. We further compare the wholesale prices among the three models. $w^{SCL*} = w^{SCN*}$ is straightforward by Theorems 1 and 2. With $b^{SCL*} = b^{SCN*}$, we conclude that the manufacturer adopts the same strategy when two retailers make collusion or Nash game. For Model SS, although Theorem 3 shows that the wholesale price at the thresholds of 0 and Δ is higher than w^{SCL*} and w^{SCN*}, we can numerically reveal that the equilibrium wholesale price in Model SS may be lower than w^{SCL*} and w^{SCN*}. The details are shown in the following subsection.

We further compare the retailers' total profit among different models. Let $\pi_r^{\text{SCL}*}$ and $\pi_r^{\text{SCN}*}$ be the total profits of two retailers at equilibrium in Models SCL and SCN, respectively, and we have the following theorem:

Theorem 5. *We compare retailers' total profit in Models SCL and SCN and find that* $\pi_r^{\text{SCL}*} > \pi_r^{\text{SCN}*}$.

Since Theorem 4 shows the manufacturer sets the same prices for retailers, the retailers actually make decisions in the same environments. Referring to Equations (2)–(4), we find that Theorem 5 is straightforward. The intuition behind Theorem 5 is that if the retailers make collusive decisions, they can gain more profit margin in the forward channel by reducing the total ordering quantity as well as save recycling investment cost by setting a lower return rate.

5.2 Numerical Experiments

To obtain more managerial insights, we conduct extensive numerical experiments to examine the effect of retailers' collusion in comparison with Nash game and Stackelberg game. Referring to Savaskan and Van Wassenhove (2006), we set the base parameters as $\alpha_1 = 100$, $\alpha_2 = 100$, $C = 1000$, $c_n = 20$, $d = 0.3$, and $\Delta = 15$. The numerical experiments follow the base setting unless otherwise clarified.

5.2.1 The Comparison of Equilibrium Wholesale and Transfer Prices

We compare the equilibrium wholesale and transfer prices among the three models. Intuitively, the market scale (α_1 or α_2) plays an important role in determining the wholesale price. The cost saving (Δ) plays an important role in determining the transfer price. Therefore, we design two scenarios where $\Delta = 15$ or $\Delta = 5$ and vary α_1 (with fixed α_2). The former scenario represents the base case where the cost saving is relatively high, and the latter scenario represents the case where the cost saving is relatively low. The results are shown in Table 3.

We find that the wholesale price in Model SS is higher than that in Models SCL and SCN when both Δ and α_1 are rather small (see $\Delta = 5$, $\alpha_1 = 50$ or $\alpha_1 = 60$ in Table 2). Under such conditions, Model SS is the worst in terms of the total profit of two retailers because the manufacturer sets a higher wholesale price in the forward channel and a lower transfer price in the reverse channel so that the profit margin of each retailer decreases.

The market scale and cost saving show different effects on pricing decisions. As the market scale increases, the retailers can set higher retail prices, and therefore, the manufacturer charges them a higher wholesale price. To recycle more cores to satisfy the enlarged market demand, the manufacturer also sets a higher transfer price. In summary, the market scale has a positive effect on both the transfer and wholesale prices. In contrast, the cost saving has a positive effect on the transfer price and a negative effect on the wholesale price. As the cost saving increases, the manufacturer has stronger motivation to set a higher transfer price to increase the return rate (see Lemma 1 for details). In addition, since the average production cost decreases, the manufacturer can still get considerable profit margin even if he distributes products with a lower wholesale price to retailers. Therefore, the wholesale price is nonincreasing with the cost saving.

Table 3. The comparison of the wholesale and transfer prices among three models ($k=0.5$)

Parameters		Decision variables					
Δ	α_1	w^{SC*}	w^{SN*}	w^{SS*}	b^{SC*}	b^{SN*}	b^{SS*}
15	50	47.50	47.50	46.81	15.00	15.00	11.08
	60	50.00	50.00	49.22	15.00	15.00	11.17
	70	52.50	52.50	51.63	15.00	15.00	11.25
	80	55.00	55.00	54.04	15.00	15.00	11.31
	90	57.50	57.50	56.45	15.00	15.00	11.37
	100	60.00	60.00	58.86	15.00	15.00	11.42
5	50	47.50	47.50	47.58	5.00	5.00	3.67
	60	50.00	50.00	50.04	5.00	5.00	3.70
	70	52.50	52.50	52.50	5.00	5.00	3.72
	80	55.00	55.00	54.96	5.00	5.00	3.74
	90	57.50	57.50	57.42	5.00	5.00	3.76
	100	60.00	60.00	59.88	5.00	5.00	3.78

5.2.2 The Value of Retailers' Collusion

In this part, we investigate whether two retailers' collusion is always beneficial in terms of improving their overall profit. To enhance the robustness of numerical experiments, referring to Bulmus et al. (2014), we set three different values for each parameter (with fixed $C=1000$) to represent low, medium, and high levels, respectively, as Table 3 shows. For notational convenience, we let $\theta=\alpha_1/\alpha_2$ represent the relative market scale of two retailers. We also let $\delta=\Delta/c_n$ represent the relative cost saving compared with unit cost of new product. $\delta\in(0,1)$ and a larger δ represents higher cost saving, which implies remanufacturing is more economic than manufacturing . Note that the base setting is now just one case of the full fractional design.

Table 4. Parameter setting in the full fractional design

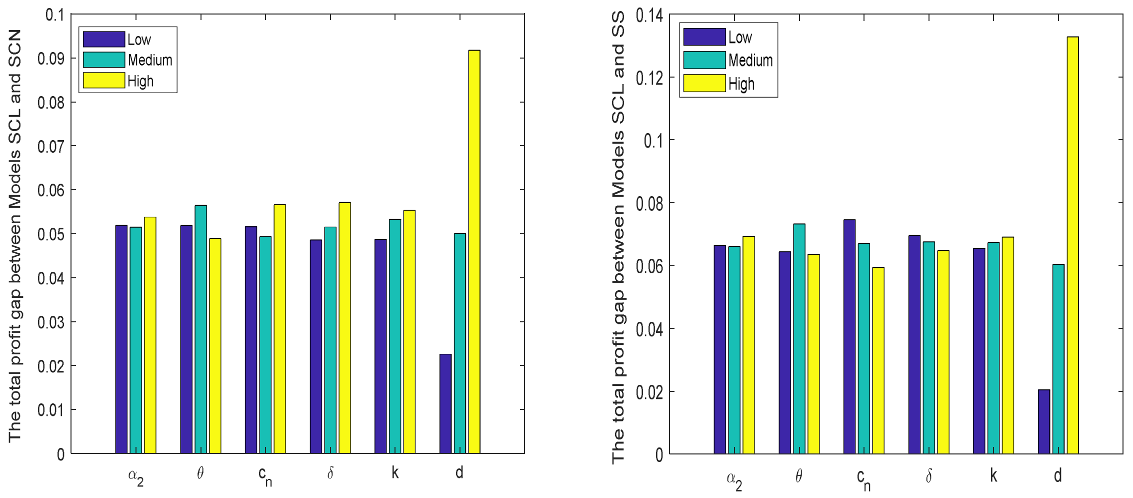
	α_2	θ	c_n	δ	k	d
Low	75	0.85	5	0.25	0.3	0.3
Medium	100	0.90	10	0.50	0.5	0.5

High	150	1.0	20	0.75	0.7	0.7

Based on the data in Table 4, there are $3^6 = 729$ cases in total and 44 cases are invalid because nonnegativity constraint is violated. For each valid case, we calculate and compare individual and total profits among the three models.

We first compare the total profit of retailers. Our numerical experiments show that for 685 valid cases, Model SCL always results in the highest total profit for retailers, so collusion is valuable to the retailers as a whole. We define the relative profit improvement to Model SCN as $\mathrm{PI}^{\mathrm{SN}} = \frac{\pi_r^{\mathrm{SC}} - \pi_r^{\mathrm{SN}}}{\pi_r^{\mathrm{SN}}} \times 100\%$. The relative profit improvement to Model SS can be defined in the same way. We record the average and maximum of the profit improvements among the 685 cases and report them as follows: For Model SCN, the average profit improvement is 5.0%, and the maximum profit improvement is 14.0%. For Model SS, the average and maximum profit improvements are 7.0% and 16.0%, respectively.

The individual effect of each parameter on profit improvement by collusion is investigated, as shown in Figure 2. The figure shows that the profit improvement is remarkable when the forward and reverse competition intensities are high, which indicates that collusion can dampen competitions to some degree. Interestingly, on average, the cost saving shows the positive effect in Model SCN, while it shows the negative effect in Model SS. Another interesting result is that the profit improvement is the largest when the market scale gap is medium, which indicates that when two retailers are too close or differentiated, they have less enthusiasm to make collusion.



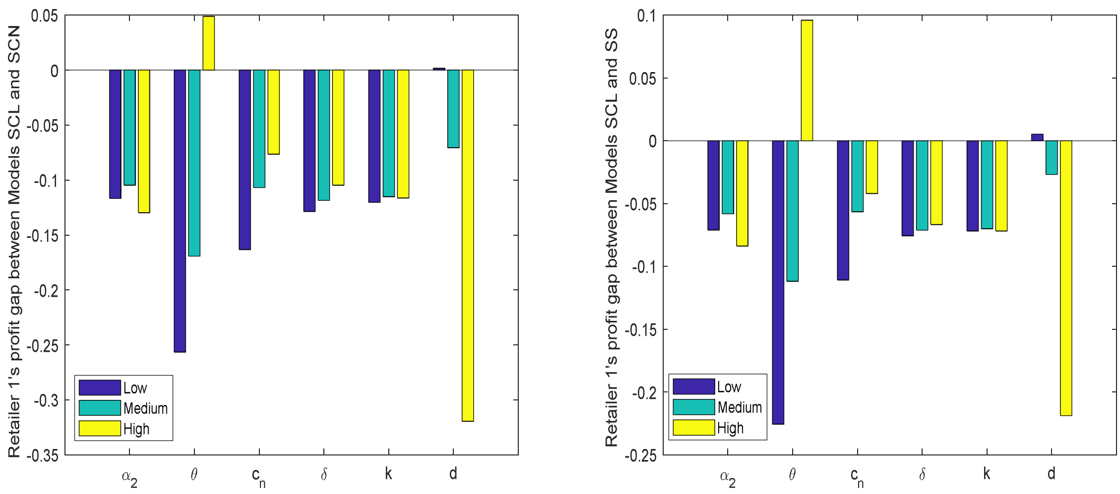
(a) Model SCL vs. SCN

(b) Model SCL vs. SS

Figure 2. Comparison of retailers' total profit between Model SCL and Model SCN/SS

A natural question arises whether both two retailers can gain more profit. To answer this question, we investigate the value of collusion to each retailer. We can define Retailer 1's profit improvement in the two models with the above method. However, numerical results show that among 685 valid cases, Retailer 1 suffers profit loss by collusion, compared with Model SCN or Model SS in 627 cases. Specifically, compared with Model SCN, Retailer 1 suffers 12% profit loss on average by collusion, and the number is 7% for Model SS. In the remaining 58 cases, Retailer 1's profit is higher than with both Models SCN and SCL. The profit improvement can reach as high as 11% for Model SCN and 20% for Model SS, respectively.

The individual effect of each parameter is shown in Figure 3. One important finding is that Retailer 1 gains positive profit improvement only when the market scale is rather close to Retailer 2 or the forward competition is rather small. In other cases, Retailer 1 suffers profit loss. In addition, we find that θ, c_n, and δ have a positive effect, while d has a negative effect on the profit improvement. The effect of α_2 is U-shape. That is, when the market scale of Retailer 2 is medium, Retailer 1 suffers the smallest profit loss. Comparing the left and right bar charter in Figure 3, we find that the effect of each parameter in two models is very similar.



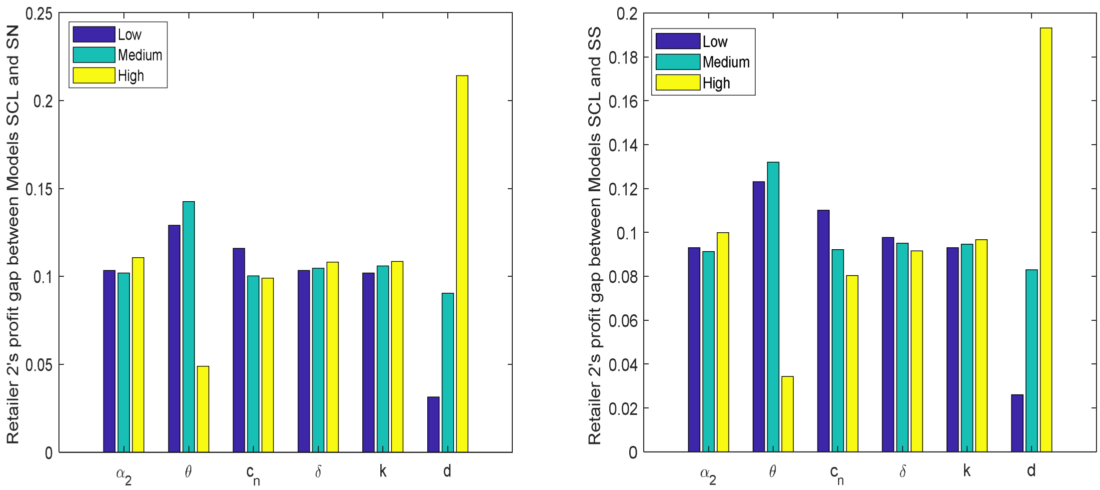
(a) Model SCL vs. SCN

(b) Model SCL vs. SS

Figure 3. Comparison of Retailer 1's profit between Model SCL and Model SCN/SS

We also investigate the value of collusion to Retailer 2. Numerical experiments show that collusion always brings Retailer 2 profit improvement. Compared with Model SCN, the minimal, average, and maximum profit improvement by collusion is 2%, 11%, and 35%, respectively. Compared with Model SS, the minimal, average, and maximum profit improvement by collusion is 1%, 9%, and 34%.

The individual effect of each parameter is shown in Figure 4. In Models SCN and SS, d and k show positive effects, while c_n shows negative effects on profit improvement. The effect of α_2 is U-shape. That is, when α_2 is medium, the improvement is the smallest. On the contrary, the effect of θ is inverse U-shape. That is, the improvement is the highest when θ is medium. Figure 4 also shows that the effect of δ is positive in Model SCN and negative in Model SS.



(a) Model SCL vs. SCN (b) Model SCL vs. SS

Figure 4. Comparison of Retailer 2's profit between Models SCL and Model SCN/SS

In summary, we find that Model SCL always yields larger total profit for two retailers compared with the case without collusion, especially when the forward competition is rather fierce. However, the smaller retailer is harmed by collusion in most cases, especially when the forward competition is rather fierce or their market scales are rather differentiated. Therefore, some elaborate coordination mechanism is necessary to achieve a win-win situation as specified below.

5.2.3 The Profit Allocation Mechanism

In the above parts, we study the retailers' collusion behavior to maximize the total profit function while the profit function of each retailer is not specified. In fact, the profit of each retailer should be higher than the case without collusion; Otherwise, the collusion collapses. We now design a mechanism that allocates the total profit by collusion to guarantee that both retailers are better-off.

For Model SCN, denote by $\tilde{\pi}_{r1}^{\text{SCN}}$ ($\tilde{\pi}_{r2}^{\text{SCN}}$) the profit of Retailer 1 (Retailer 2) after profit allocation. Obviously, $\tilde{\pi}_{r1}^{\text{SCN}}+\tilde{\pi}_{r2}^{\text{SCN}}=\pi_{r}^{\text{SCL}}$. In addition, $\tilde{\pi}_{r1}^{\text{SCN}}>\pi_{r1}^{\text{SCN}}$ and $\tilde{\pi}_{r2}^{\text{SCN}}>\pi_{r2}^{\text{SCN}}$ should hold so that both retailers would like make collusion rather than Cournot competition. Analogously, For

Model SS, denote by $\tilde{\pi}_{r1}^{SS}$ ($\tilde{\pi}_{r1}^{SS}$) the profit of Retailer 1(Retailer 2) after profit allocation. We also have $\tilde{\pi}_{r1}^{SS}+\tilde{\pi}_{r2}^{SS}=\pi_{r}^{SCL}$, $\tilde{\pi}_{r1}^{SS}>\pi_{r1}^{SS}$ and $\tilde{\pi}_{r2}^{SS}>\pi_{r2}^{SS}$.

Among those effective profit allocation mechanisms, Nash arbitration scheme is widely used (Leng and Zhu, 2009) Based on this mechanism, the objective is to maximize the following function:

$$\max_{f_1\geq f_1^0, f_2\geq f_2^0}(f_1-f_1^0)(f_2-f_2^0), \tag{7}$$

where f_i and f_i^0 ($i\in\{1,2\}$) denotes the allocated surplus and security level of Retailer i, respectively. In this research, we let $f_1^0=f_2^0=0$ without loss of generality. For Model SCN, $f_i=\tilde{\pi}_{ri}^{SCN}-\pi_{ri}^{SCN}$ holds and for Model SS $f_i=\tilde{\pi}_{ri}^{SS}-\pi_{ri}^{SS}$ holds.

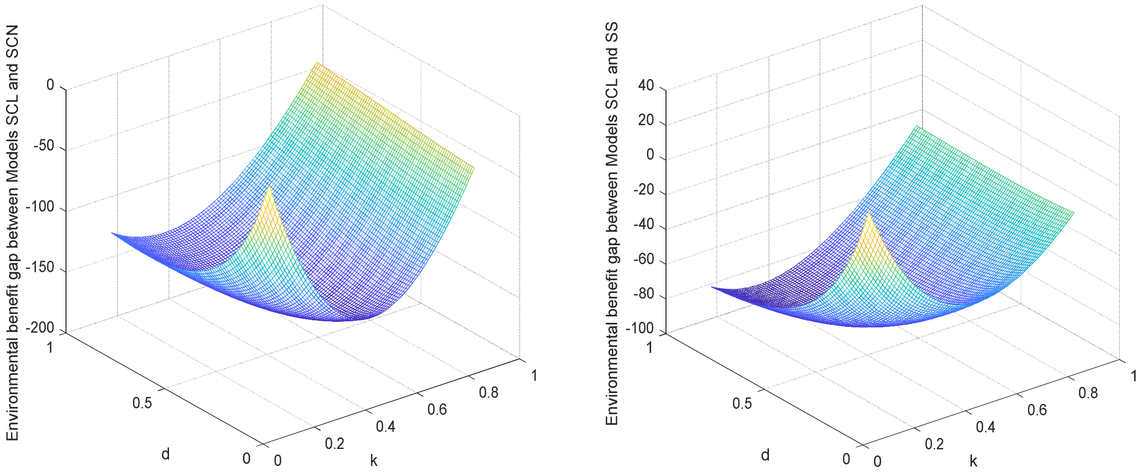
By solving the above problem, we can "fairly" allocate the profit surplus by collusion between two retailers (Gjerdrum et al., 2002). The allocated profits of two retailers for two models are presented in the following Table 5.

Table 5. Profit Allocation Based on Nash Arbitration Scheme

Parameters		Collusion Profit	Profits before Allocation				Profits after Allocation			
			Model SCN		Model SS		Model SCN		Model SS	
θ	d	π_r^{SCL}	π_{r1}^{SCN}	π_{r2}^{SCN}	π_{r1}^{SS}	π_{r2}^{SS}	$\tilde{\pi}_{r1}^{SCN}$	$\tilde{\pi}_{r2}^{SCN}$	$\tilde{\pi}_{r1}^{SS}$	$\tilde{\pi}_{r2}^{SS}$
0.85	0.3	593.47	146.94	421.77	149.71	433.56	159.32	434.15	154.81	438.66
	0.5	529.79	104.01	388.76	100.87	399.95	122.52	407.27	115.35	414.43
	0.7	507.63	69.77	372.37	55.69	385.79	102.51	405.11	88.76	418.86
0.90	0.3	609.96	197.35	386.89	202.39	397.99	210.21	399.75	207.18	402.78
	0.5	531.76	149.84	346.22	148.72	356.89	167.69	364.07	161.80	369.96
	0.7	484.58	111.41	320.10	97.28	333.00	137.95	346.63	124.43	360.15
1.0	0.3	673.68	322.53	322.53	332.69	331.97	336.84	336.84	337.20	336.48
	0.5	576.58	269.56	269.56	273.95	278.73	288.29	288.29	285.90	290.68
	0.7	503.94	228.65	228.65	218.58	239.75	251.97	251.97	241.38	262.55

5.2.4 The Comparison of Environmental Benefit

We compare the environmental impact among the three models. Following Esenduran et al. (2017), the environmental benefit can be defined as a function of total recycled products, which implies the more recycled products, the less environmental burden. Specifically, e denotes the environmental benefit for unit recycled product, and the total environmental benefit can be written as $E=e(\tau_1 q_1+\tau_2 q_2)$. Referring to Esenduran et al. (2017) that sets $e\in\{0.001, 0.5\}$ to represent different levels for the economic measure of environmental benefit with the premise that the manufacturing cost is approximately \$0.25, we think it is acceptable to set $e \in \{0.08, 40\}$, considering the value of c_n in this research. Different values of e do not change the relative rank of the three models regarding environmental benefit, so in this numerical experiment, we take $e = 40$ as an example. We also let k and d change from 0.1 to 0.9 with a step of 0.01, and the results are presented in Figure 5.



(a) Models SCL vs. SCN (b) Models SCL vs. SS

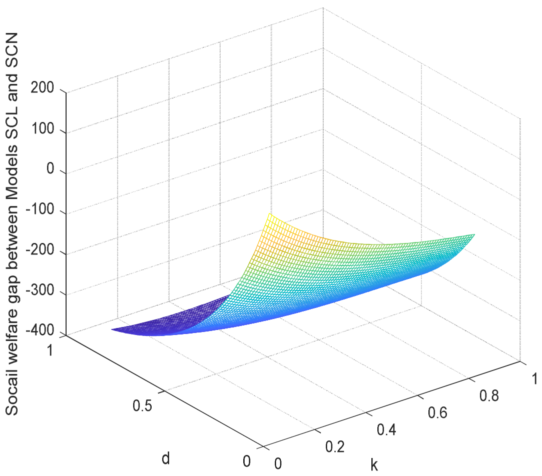
Figure 5. Comparison of the environmental benefit between Models SCL and SCN/SS

We find that in the left panel, the environmental benefit is always negative, which means that Model SCL always yields smaller environmental benefit than Model SCN. Specifically, the environmental benefit gap between Models SCL and SCN ranges from -150.32 to -3.07. The loss by collusion is the smallest when $k = 0.1$, $d = 0.1$ and is the largest when $k = 0.41$, $d = 0.35$. On the contrary, the environmental benefit of Model SCL may be smaller or larger than that in Model SS, depending on the competition conditions. Specifically, the environmental benefit gap between Models SCL and SS ranges from -80.25 to 22.51. In addition, when $k = 0.1$, $d = 0.1$, collusion brings the largest improvement; when $k = 0.31, d = 0.76$, collusion brings the largest loss. These numbers indicate that the largest loss occurs under different conditions of forward competition for two models, i.e., relatively low d ($d = 0.35$) for Model SCN while relatively high d ($d = 0.76$) for Model SS. From the perspective of environmental protection, collusion behavior is not encouraged under such conditions.

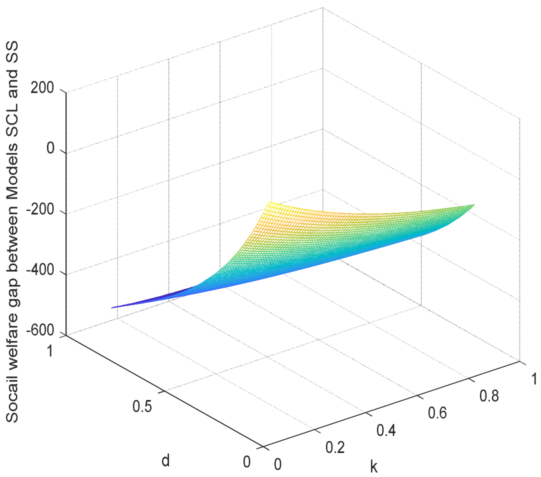
5.2.5 The Comparison of Social Welfare

We also compare the social welfare, which consists of firms' profits, environmental benefit, and consumer surplus, among the three models. It can be written as $SW = \pi_m + \pi_{r1} + \pi_{r2} + E + CS$, where E has been defined in Section 5.2.4. Following Singh and Vives (1984) and Yu et al. (2018), the consumer surplus can be defined as $CS = \alpha_1 q_1 + \alpha_2 q_2 - \frac{q_1^2}{2} - \frac{q_2^2}{2} - dq_1q_2 - (p_1q_1 + p_2q_2)$.

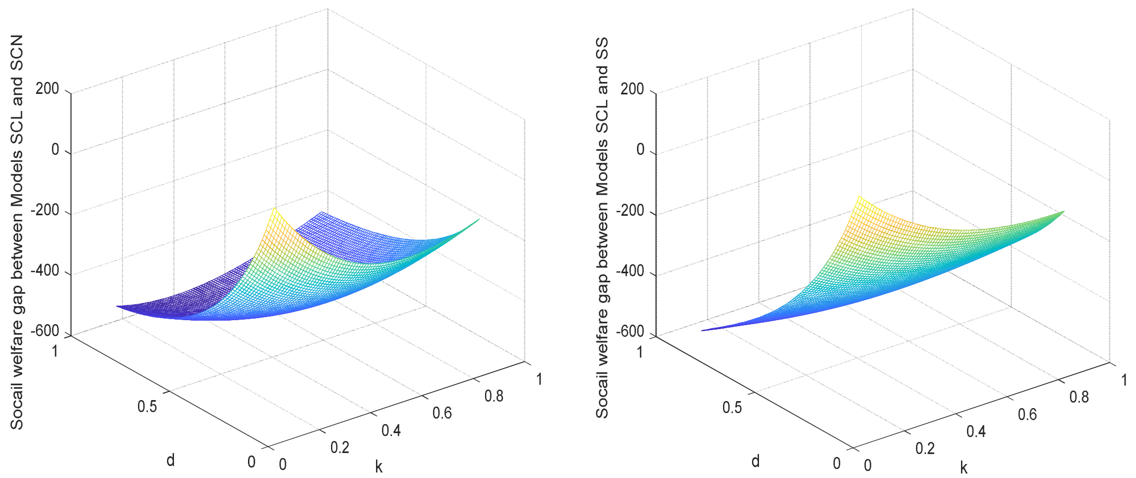
We seek to investigate whether collusion increases social welfare. To this end, we compare the social welfare between Model SCL and the other two models. In Figure 6, the left panel represents the comparison between Models SCL and SCN, and a positive gap implies that the social welfare in Model SCL is larger. The right panel represents the comparison result between Models SCL and SS, and the meaning of gap is the same. We set $e \in \{0.08, 40\}$ and also let k and d change from 0.1 to 0.9 with a step of 0.01.



(a) Model SCL vs. SCN ($e = 0.08$)



(b) Model SCL vs. SS ($e = 0.08$)



(c) Model SCL vs. SCN ($e = 40$) (d) Model SCL vs. Model SS ($e = 40$)

Figure 6. Comparison of the social welfare between Models SCL and Models SCN/SS

We find the gap between Models SCL and SCN (or SS) may be positive or negative. Specifically, when the unit environmental benefit is low ($e = 0.08$), the gap with Model SCN ranges from -393.02 to 128.47. The largest gap occurs when $k = 0.1$, $d = 0.1$, and the smallest gap occurs when $k = 0.39, d = 0.9$; the gap with SS ranges from -521.87 to 142.89. The largest gap occurs when $k = 0.1$, $d = 0.1$, and the smallest gap occurs when $k = 0.52$, $d = 0.90$. When the unit environmental benefit is high ($e = 40$), the gap with Model SCN ranges from -521.92 to 125.41. The largest gap occurs when $k = 0.1$, $d = 0.1$, and the smallest gap occurs when $k = 0.33$ with $d = 0.90$; the gap with Model SS ranges from -597.55 to 165.35. The largest gap occurs when $k = 0.1$, $d = 0.1$, and the smallest gap occurs when $k = 0.36$, $d = 0.90$. In summary, the largest gap always occurs when $k = 0.1, d = 0.1$, which means that the social welfare improvement by collusion is the most remarkable when the competitions in forward and reverse channels are very mild. The numbers means the social welfare loss by collusion is severest when the forward channel competition is very fierce ($d = 0.9$ for both models) and the reverse channel competition is relatively low ($k=0.33$ for Model SCN and $k = 0.36$ for Model SS).

5.3 Managerial Insights

In a closed-loop supply chain where two competing retailers collect used items, their competition exists not only in the forward channel but also in the reverse channel. Faced with such a case, they have stronger motivation to make collusion to jointly determine corresponding decisions variables in both channels. Our research investigates such a case and suggests the following managerial insights.

(1) Faced with retailers' different behavior, the manufacturer should adopt differentiated pricing strategy to maximize his profit. If two retailers move at the same time (collusion or Cournot competition), the manufacturer should set the transfer price equal to the cost saving of remanufacturing. In this case, he only benefits directly from the forward channel and the reverse channel plays a role in boosting the market demand. If two retailers move sequentially (Stackelberg competition), the manufacturer should set lower transfer price and he benefits from both channels.

(2) From the perspective of profit maximization, collusion is indeed the optimal choice for two retailers. Collusion always brings retailers profit improvement, which is very remarkable if the forward/reverse competition is fierce or the gap between retailers' market scale is medium.

(3) The smaller retailer may suffer profit loss by collusion. To resolve this problem, we can design an elaborate profit allocation contract based on Nash arbitration scheme to guarantee that both retailers are better-off so that collusion is achieved.

(4) From the perspective of environment protection or social welfare, collusion is not the optimal choice for two retailers under some conditions. In such cases, collusion is not encouraged to two retailers who concern environment protection or social welfare.

6. Concluding Remarks

In this paper, we consider a closed-loop supply chain consisting of a manufacturer and two competing retailers. These retailers compete for demand in the forward channel and for the cores in the reverse channel. Three models are proposed in terms of the manufacturer's and retailers' behaviors, such as Stackelberg-collusion game (Model SCL), Stackelberg-Cournot game (Model SCN), as well as Stackelberg-Stackelberg game (Model SS). We obtain the manufacturer's and retailers' equilibrium decisions in different model and find that the market scale and cost saving show different effects on the manufacturer's pricing decisions. We make further comparison among different models analytically and numerically. We set a full fractional design to test the value of retailers' collusion and find that the average profit improvement is noticeable, compared with the cases that they are under Cournot competition or Stackelberg competition. As for each retailer, collusion cannot be achieved automatically unless some profit allocation contract is designed so that both of them can obtain profit surplus. We also find that under some conditions, there exists confliction between profit maximization and environment protection or social welfare maximization. In summary, we study the case that two competing retailers' collusion behavior in the presence of the bidirectional competitions in the forward and reverse channels and make a comprehensive analysis on the impact from different perspectives.

Our research provides new theoretical findings and managerial insights to the practitioners in the remanufacturing industry.

For future research, this paper can be extended in the following directions. In this study, we only consider the interaction between two retailers and the manufacturer in a decentralized system. We can further study the centralized supply chain to maximize the total profit of all supply chain members. It is interesting to design some supply chain contracts to coordinate the contracts so that the performance of decentralized/centralized is identical. In this research, we focus on the single case that retailers make collusive decisions on forward and reverse channels, which can be named total collusion. In contrast, we may investigate the other cases that they cooperate in one channel and compete in the other channel, which can be called partial collusion. We can further compare the decision variables and system performance among different collusion models. It is also interesting to consider the case that the manufacturer or third-party collector is involved in recycling and results in more competition. In this research, we assume that new and remanufactured products are sold with the same price. However, in practice, the willingness of consumers to pay for remanufactured and new products may be different, which implies that the prices of new and remanufactured products are different. We also assume that all information is public and there is no private information. In the practice, there may exist information asymmetry between the manufacturer and retailers. For example, it is interesting to investigate the case that the manufacturer does not know retailers' investment cost or competition parameters.

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Appendix

The Proof of Lemma 1.

In Model SCL, it is easy to prove the profit function in Eq. (2) is concave in corresponding decision variables. The Hessian matrix is as follows.

$$H=\begin{pmatrix} \frac{\partial^2\pi_r}{\partial q_1^2} & \frac{\partial^2\pi_r}{\partial q_1\partial q_2} & \frac{\partial^2\pi_r}{\partial q_1\partial \tau_1} & \frac{\partial^2\pi_r}{\partial q_1\partial \tau_2} \\ \frac{\partial^2\pi_r}{\partial q_2\partial q_1} & \frac{\partial^2\pi_r}{\partial q_2^2} & \frac{\partial^2\pi_r}{\partial q_2\partial \tau_1} & \frac{\partial^2\pi_r}{\partial q_2\partial \tau_2} \\ \frac{\partial^2\pi_r}{\partial \tau_1\partial q_1} & \frac{\partial^2\pi_r}{\partial \tau_1\partial q_2} & \frac{\partial^2\pi_r}{\partial \tau_1^2} & \frac{\partial^2\pi_r}{\partial \tau_1\partial \tau_2} \\ \frac{\partial^2\pi_r}{\partial \tau_2\partial q_1} & \frac{\partial^2\pi_r}{\partial \tau_2\partial q_2} & \frac{\partial^2\pi_r}{\partial \tau_2\partial \tau_1} & \frac{\partial^2\pi_r}{\partial \tau_2^2} \end{pmatrix} = \begin{pmatrix} -2 & -2d & b & b \\ -2d & -2 & b & b \\ b & b & -\frac{2C}{1-k} & 0 \\ b & b & 0 & -\frac{2C}{1-k} \end{pmatrix}$$

If $C>\frac{b^2(1-k)}{1+d}$, We obtain $H_1=-2<0$, $H_2=4-d^2>0$, $H_3=\frac{4(1-d)[b^2(1-k)-2C(1+d)]}{1-k}<0$, $H_4=\frac{16C(1-d)[C(1+d)-b^2(1-k)]}{(1-k)^2}>0$. To summarize, π_r is concave with the decision variables.

Further, by the first-order conditions, we can obtain optimal q_1, q_2, τ_1 and τ_2 under given w and b. The results are shown as follows.

$$\begin{cases} q_1=\frac{\alpha_1-\alpha_2}{4(1-d)}+\frac{C(\alpha_2+\alpha_1-2w)}{4[C(1+d)-b^2(1-k)]} \\ q_2=\frac{\alpha_2-\alpha_1}{4(1-d)}+\frac{C(\alpha_2+\alpha_1-2w)}{4[C(1+d)-b^2(1-k)]} \\ \tau_1=\frac{b(1-k)\left(\alpha_1+\alpha_2-2w\right)}{4[C(1+d)-b^2(1-k)]} \\ \tau_2=\frac{b(1-k)\left(\alpha_1+\alpha_2-2w\right)}{4[C(1+d)-b^2(1-k)]} \end{cases}$$

The proof of Lemma 1 is completed.

The Proof of Theorem 1.

Substituting these results in Lemma 1 into the profit function in Eq. (1), we obtain

$$\pi_m = \frac{C\left(\alpha_1+\alpha_2-2w\right)\left(2w[C(1+d)-b(1-k)\Delta]-2[C(1+d)-b^2(1-k)]c_n-b(1-k)(b-\Delta)\left(\alpha_1+\alpha_2\right)\right)}{4\left(C(1+d)+b^2(-1+k)\right)^2}.$$

For given b, the profit function is concave with w under the assumption of $C > \frac{b(1-k)\Delta}{1+d}$. By the first-order condition, we obtain $w^* = \frac{\alpha_1+\alpha_2+2c_n}{4} - \frac{b(1-k)(\Delta-b)(\alpha_1+\alpha_2-2c_n)}{4C\left(1+d\right)-4b\left(1-k\right)\Delta}$.

Substituting w^* into π_m, we obtain $\pi_m = \frac{C(-2c_n+\alpha_1+\alpha_2)^2}{16[C\left(1+d\right)-b\left(1-k\right)\Delta]}$. It is straightforward that $b^{SC*}=\Delta$ since $b \le \Delta$.

Therefore, $w^{\mathrm{SCL}*} = \frac{\alpha_1+\alpha_2+2c_n}{4}$, $q_1^{\mathrm{SCL}^*} = \frac{(\alpha_1-\alpha_2)}{4(1-d)} + \frac{C(\alpha_1+\alpha_2-2c_n)}{8[C(1+d)-(1-k)\Delta^2]}$,

$q_2^{\mathrm{SCL}^*} = \frac{(\alpha_2-\alpha_1)}{4(1-d)} + \frac{C(\alpha_2+\alpha_1-2c_n)}{8[C(1+d)-(1-k)\Delta^2]}$, *and* $\tau_1^{\mathrm{SCL}^*} = \tau_2^{\mathrm{SCL}^*} = \frac{\Delta(1-k)(\alpha_1+\alpha_2-2c_n)}{8[C\left(1+d\right)-(1-k)\Delta^2]}$.

Note that to guarantee $0 \le \tau_1^{SC^*} + \tau_2^{SC^*} \le 1$, $C \ge \frac{\Delta(1-k)(\alpha_1+\alpha_2-2c_n)}{4(1+d)} + \frac{(1-k)\Delta^2}{1+d}$ is necessary.

The proof of Theorem 1 is completed.

The Proof of Lemma 2.

In Model SCN, for retailer 1's profit, the Hessian matrix given as follows is negative definite.

$$H = \begin{pmatrix} \frac{\partial^2\pi_{r1}}{\partial q_1^2} & \frac{\partial^2\pi_{r1}}{\partial q_1 \partial \tau_1} \\ \frac{\partial^2\pi_{r1}}{\partial \tau_1 \partial q_1} & \frac{\partial^2\pi_{r_1}}{\partial \tau_1^2} \end{pmatrix} = \begin{pmatrix} -2 & b \\ b & -\frac{2C}{1-k^2} \end{pmatrix}$$

The profit function in Equation (3) is concave in q_1 and τ_1. Therefore, for retailer 1, the first-order conditions are given as follows:

$$\begin{cases} \frac{\partial \pi_{r1}}{\partial q_1} = \alpha_1 - 2q_1 - dq_2 - w + b\tau_1 = 0 \\ \frac{\partial \pi_{r1}}{\partial \tau_1} = b(q_1+q_2) - \frac{2C\tau_1}{1-k^2} = 0 \end{cases}.$$

Again, if $C > \frac{b^2(1-k^2)}{4}$, the profit function in Equation (4) is concave in q_2 and τ_2. Therefore, for retailer 2, we can also obtain the first-order conditions.

$$\begin{cases} \frac{\partial \pi_{r2}}{\partial q_2} = \alpha_2 - 2q_2 - dq_1 - w + b\tau_2 = 0; \\ \frac{\partial \pi_{r2}}{\partial \tau_2} = b(q_1+q_2) - \frac{2C\tau_2}{1-k^2} = 0. \end{cases}$$

Solving these four equations, we obtain the best-response functions given w and b.

$$\begin{cases} q_1 = \frac{\alpha_1-\alpha_2}{2(2-d)} + \frac{C(\alpha_1+\alpha_2-2w)}{2[C(2+d)-b^2(1-k^2)]}, \\ \tau_1 = \frac{b(1-k^2)(\alpha_1+\alpha_2-2w)}{2[C(2+d)-b^2(1-k^2)]}, \\ q_2 = \frac{\alpha_2-\alpha_1}{2(2-d)} + \frac{C(\alpha_1+\alpha_2-2w)}{2[C(2+d)-b^2(1-k^2)]}, \\ \tau_2 = \frac{b(1-k^2)(\alpha_1+\alpha_2-2w)}{2[C(2+d)-b^2(1-k^2)]}. \end{cases}$$

The proof of Lemma 2 is completed.

The Proof of Theorem 2.

Substituting the result in Lemma 2 into the manufacturer's profit function, we obtain the following result:

$$\pi_m = \frac{C\left(-2w+\alpha_1+\alpha_2\right)\left(\left(w-c_n\right)\left(C(2+d)+b^2\left(-1+k^2\right)\right)+b\left(1-k^2\right)(\Delta-b)\left(-2w+\alpha_1+\alpha_2\right)\right)}{\left(C(2+d)+b^2\left(-1+k^2\right)\right)^2}.$$

By the first-order condition, we obtain the stationary point as $(w,b) = (\frac{\alpha_1+\alpha_2+2c_n}{4}, \Delta)$. Define $L_1 = C\left(2+d\right)-(1-k^2)\Delta^2$ and $L_2 = C\left(2+d\right)+3(1-k^2)\Delta^2$. Therefore, the Hessian matrix at the stationary point $(w,b) = (\frac{\alpha_1+\alpha_2+2c_n}{4}, \Delta)$ is as follows:

$$H = \begin{pmatrix} \frac{\partial^2 \pi_m}{\partial w^2} & \frac{\partial^2 \pi_m}{\partial w \partial b} \\ \frac{\partial^2 \pi_m}{\partial b \partial w} & \frac{\partial^2 \pi_m}{\partial b^2} \end{pmatrix} = \begin{pmatrix} -\frac{4C}{L_1} & \frac{2C\left(-1+k^2\right)\Delta\left(2c_n-\alpha_1-\alpha_2\right)}{L_1^2} \\ \frac{2C\left(-1+k^2\right)\Delta\left(2c_n-\alpha_1-\alpha_2\right)}{L_1^2} & \frac{C\left(-1+k^2\right)L_2\left(-2c_n+\alpha_1+\alpha_2\right)^2}{4L_1^3} \end{pmatrix}$$

If $C > \dfrac{(1-k^2)\Delta^2}{2+d}$, the Hessian matrix is negative definite and the profit function is quasi-concave with w and b. The stationary point is also the optimal solution, i.e., $w^* = \dfrac{\alpha_1 + \alpha_2 + 2c_n}{4}, b^* = \Delta$. Note that to guarantee $0 \le \tau_1^{SCN*} + \tau_2^{SCN*} \le 1$, we have $C \ge \dfrac{\Delta(1-k^2)(\alpha_1+\alpha_2-2c_n)}{2(2+d)} + \dfrac{\Delta^2(1-k^2)}{2+d}$.

The proof of Theorem 2 is completed.

The Proof of Lemma 3.

Given w, b, q_2, τ_2, we first prove that Retailer 1's profit function is concave with q_1 and τ_1. The Hessian matrix is as follows.

$$H = \begin{pmatrix} \dfrac{\partial^2 \pi_{r1}}{\partial q_1^2} & \dfrac{\partial^2 \pi_{r1}}{\partial q_1 \partial \tau_1} \\ \dfrac{\partial^2 \pi_{r1}}{\partial \tau_1 \partial q_1} & \dfrac{\partial^2 \pi_{r1}}{\partial \tau_1^2} \end{pmatrix} = \begin{pmatrix} -2 & b \\ b & -\dfrac{2C}{1-k^2} \end{pmatrix}$$

For $C > \dfrac{b^2(1-k^2)}{4}$, the profit function is proven to be concave. Therefore, Retailer 1's optimal decisions are obtained by the first-order conditions.

$$\begin{cases} q_1 = \dfrac{[-2Cd - b^2(-1+k^2)]q_2 + 2C(-w+\alpha_1)}{4C + b^2\left(-1+k^2\right)}, \\ \tau_1 = \dfrac{b(-1+k^2)\left(w + (-2+d)q_2 - \alpha_1\right)}{4C + b^2\left(-1+k^2\right)}. \end{cases}$$

The proof of Lemma 3 is completed.

The Proof of Lemma 4.

Substituting these two equations in Lemma 3 into Retailer 2's profit function, it is sufficient to prove that it is concave with q_2 and τ_2 if $C > \dfrac{b^2\left(12 - 8d - d^2 - (2-d)^2 k\right)\left(1-k^2\right)}{8\left(2-d^2\right)}$.

The first-order conditions are given as follows.

$$\begin{cases} \dfrac{\partial \pi_{r2}}{\partial q_2} = -\dfrac{2b^2C(-2+d)k(1-k^2)^2\left(w-(2-d)q_2-\alpha_1\right)}{(1-k^2)\left(4C-b^2(1-k^2)\right)^2} - \dfrac{2bC(-2+d)\tau_2}{4C-b^2(1-k^2)} = 0; \\ \dfrac{\partial \pi_{r2}}{\partial \tau_2} = -\dfrac{2bC\left(w-(2-d)q_2-\alpha_1\right)}{4C-b^2(1-k^2)} - \dfrac{2C\tau_2}{1-k^2} = 0. \end{cases}$$

For the sake of simplicity, some symbols are predefined as follows.

$Z_1 = 8C^2(d^2-2)+b^2C(-12+8d+d^2+(2-d)^2k)(k^2-1)+b^4(d-1)(k^2-1)^2$,

$Z_2 = b(1-k^2)\left\{2C[8-d(4+d)]-b^2(4-3d)(1-k^2)\right\}$,

$Z_3 = b(1-k^2)\{2[C(-4+d(2+d))+b^2(1-d)(1-k^2)]\alpha_1+(2-d)[b^2(1-k^2)-4C]\alpha_2\}$,

$Z_4 = 8C^2(2-d)-2b^2C[2+(2-d)k](1-k^2)+b^4(k^2-1)^2$,

$Z_5 = 2C\{4Cd+b^2[2+\left(d-2\right)k](k^2-1)\}\alpha_1-[4C+b^2(k^2-1)]^2\alpha_2$,

$Z_6 = \dfrac{-2Cd-b^2(k^2-1)}{4C+b^2(k^2-1)}$, and $Z_7 = \dfrac{2C}{4C+b^2(k^2-1)}$. Based on these definitions, the optimal decisions of q_2 and τ_2 are rewritten as $\tau_2 = \dfrac{Z_2w+Z_3}{2Z_1}$, $q_2 = \dfrac{Z_4w+Z_5}{2Z_1}$. Further, we obtain $q_1 = \dfrac{(Z_4w+Z_5)Z_6}{2Z_1}+Z_7(\alpha_1-w)$, $\tau_1 = \dfrac{Z_2w+Z_3}{2Z_1}$.

The proof of Lemma 4 is completed.

Proof of Theorem 3.

Substituting these expressions of Lemmas 3 and 4 into the manufacturer's profit function, we obtain $\pi_m = C\left(Xw+Y_1\alpha_1+Y_2\alpha_2\right)\left(w-c_n+T_1\left(Xw+Y_1\alpha_1+Y_2\alpha_2\right)/Z_1\right)/Z_1$, where

$X = 2C(8-d(4+d))-b^2(4-3d)\left(1-k^2\right)>0$, $Y_1 = 2\left(C(-4+d(2+d))+b^2(-1+d)(-1+k^2)\right)<0$,

$T_1 = b\left(1-k^2\right)(\Delta-b)>0$, $Y_2 = -(2-d)\left(4C+b^2\left(-1+k^2\right)\right)<0$.

It is easy to prove that for given b, the profit function is concave in w. Based on the first-order condition, we can obtain $w^*(b) = \dfrac{XZ_1c_n-\left(2XT_1+Z_1\right)\left(Y_1\alpha_1+Y_2\alpha_2\right)}{2X\left(XT_1+Z_1\right)}$.

Now we prove $b^*<\Delta$. Substituting $w^*(b)=\dfrac{XZ_1c_n-\left(2XT_1+Z_1\right)\left(Y_1\alpha_1+Y_2\alpha_2\right)}{2X\left(XT_1+Z_1\right)}$ into the manufacturer's profit function, we obtain $\pi_m=-\dfrac{C\left(Xc_n+Y_1\alpha_1+Y_2\alpha_2\right)^2}{4X\left(XT_1+Z_1\right)}$. Therefore,

$$\frac{\mathrm{d}\pi_m}{\mathrm{d}b}\Big|_{b=\Delta}=\frac{C(1-k^2)D_1D_2}{4},$$

where

$$D_1=4C^2\left(64+4d(2+d)(-9+4d)+(-2+d)^2(-8+d(4+d))k\right)\Delta-(8+9(-2+d)d)\left(-1+k^2\right)^2\Delta^5$$
$$-4C\left(2(-1+d)(-16+d(10+d))+(-2+d)^2(-4+3d)k\right)\left(-1+k^2\right)\Delta^3$$

and

$$D_2=1+\left(2C(-8+d(4+d))+(-4+3d)\left(-1+k^2\right)\Delta^2\right)c_n+2\left(C(4-d(2+d))-(1-d)\left(1-k^2\right)\Delta^2\right)\alpha_1$$
$$+(2-d)\left(4C+\left(-1+k^2\right)\Delta^2\right)\alpha_2.$$

Since $\alpha_2\geq\alpha_1>c_n$, it is easy to prove $D_2>0$.

As for D_1, when $8-9(2-d)d<0$, by contraction of k, we obtain

$$D_1<f_1(C)\equiv\begin{cases}4C^2\left(64-4d(2+d)(9-4d)-(2-d)^2(8-d(4+d))\right)\Delta-(8-9(2-d)d)\Delta^5\\-4C\left(2(1-d)(16-d(10+d))\right)\Delta^3\end{cases}.$$

When $8+9(-2+d)d>0$, by contraction of k, we obtain:

$$D_1<f_2(C)\equiv\begin{cases}4C^2\left(64-4d(2+d)(9-4d)-(2-d)^2(8-d(4+d))\right)\Delta\\-4C\left(2(1-d)(16-d(10+d))\right)\Delta^3\end{cases}.$$

With $C>\Delta^2$, it is easy to prove $f_1(C)<0$ and $f_2(C)<0$. Therefore, $D_1<0$ and

$$\frac{\mathrm{d}\pi_m}{\mathrm{d}b}\Big|_{b=\Delta}=\frac{C(1-k^2)D_1D_2}{4}<0.$$

This result indicates that for the manufacturer, it is optimal to set $b<\Delta$ to maximize his profit.

Then, we prove when $b=0$ or $b=\Delta$, $w^*(b)>\dfrac{\alpha_1+\alpha_2+2c_n}{4}$ by simple algebra as follows.

$$w^*(0)-\frac{\alpha_1+\alpha_2+2c_n}{4}=\frac{d^2\left(\alpha_2-\alpha_1\right)}{4\left(8-d\left(4+d\right)\right)}>0;$$

$$w^*(\Delta)-\frac{\alpha_1+\alpha_2+2c_n}{4}=\frac{d\left(2Cd-(1-k^2)\Delta^2\right)\left(\alpha_2-\alpha_1\right)}{8C\left(8-d\left(4+d\right)\right)-4\left(4-3d\right)(1-k^2)\Delta^2}>0.$$

By substituting the optimal b and W into the functions of return rates, we can also obtain the conditions regrading C to guarantee that the total return rates is no more than 1.

Therefore, the proof of Theorem 3 is completed.