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# Linking the differential permeability and loss coefficients in Bertotti's approach

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Abstract:

**In this work, we investigate in Fe-Si laminations the core-loss coefficients variation with the flux density within the loss statistical approach framework. We find that the dynamical core-loss coefficients can be linked with the differential permeability defined from the low frequency normal curve. We interpret these variations as the fingerprint of the domain-wall activity (i.e. all the processes involving domain-wall movements, nucleation and annihilation) during the magnetization process. We show that the classical loss variation can be qualitatively described by the macroscopic formulation of the skin-depth, but not quantitatively.**

Keywords: core-losses, soft magnetic material, Fe-Si laminations

## I. Introduction

In soft magnetic material, the accurate calculation of iron-losses is critical to improve the performances of electromechanical devices. However, losses are commonly studied in standard conditions which strongly differ from the conditions to which the material is subjected in the final device. The impact of the fabrication process [1][2], the operating conditions [3] or the material ageing [4] are all aspects that strongly influence the final material performances. The absence of a general model that describes the losses starting from material parameters and applicable whatever the working condition makes impossible to predict the final material performances without carrying additional experiments. The scope of this paper is focused on the statistical approach of losses commonly referred as the loss separation approach developed by Bertotti and coworkers [9], with the aim to link the macroscopic and the mesoscopic level.

Iron-losses originate from the eddy-current pattern generated by the domain-wall (DW) motion of a magnetic DW structure [5]. They reflect all the dynamical processes which occur during the sample magnetization. The main issue when modelling the losses comes from the wide range of space and time scales involved in the magnetization process. Applications of soft magnetic materials for energy conversion require macroscopic models and measurements carried out at the macroscopic scale, on large sample with thicknesses up to the mm and at excitation frequencies not exceeding few kHz in the case of FeSi lamination. However, magnetic phenomena occur at a much lower level, ranging from the exchange energy (GHz and nm) to the domain-wall structure scale (MHz and hundreds of  $\mu\text{m}$ ), corresponding respectively to the microscopic and mesoscopic scales [6][7]. The used formalisms also differ: at the macroscopic scale, magnetic properties are well described by the Maxwell's equations that are commonly used for electrical machine design, whereas at the microscopic level, properties are modeled using relevant energies and Landau-Lifshitz-Gilbert-(Bloch) equation [8]. Hence, the ideal iron-losses model built on physical basis must link several scales and act as a bridge between different formalisms.

To make the link between the microscopic and macroscopic levels, the statistical approach of losses takes its roots in the Barkhausen experiment where the Barkhausen spectrum is characteristic of independent random processes. The Statistical Theory of Losses shows that a system of  $180^\circ$  domain walls placed at random positions across a lamination width (thereby having randomly distributed velocities) generates a dynamic contribution to the energy loss that is made of a background term (the classical loss), and an excess term. The background term results from the random superposition of the individually generated eddy currents [5]. The excess term is the proportional, through a term decreasing with increasing the DW density, to the average contribution of a single DW, as if it were alone. The stochasticity of the magnetization process at mesoscopic-macroscopic level should be intended in this sense. There is no need for assuming the walls moving independently one of another, nor that they reverse the same

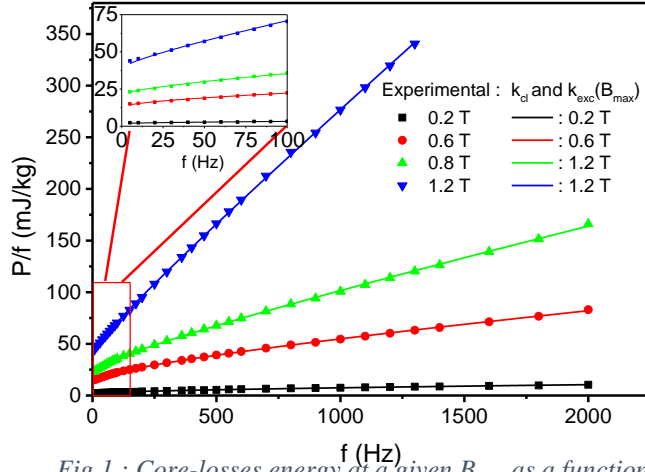


Fig 1 : Core-losses energy at a given  $B_{max}$  as a function of the frequency. The inset shows a zoom of the low frequency region. Points represent the experimental data obtained on 12 M800-50 Epstein samples cut along the rolling direction. Continuous lines are the result using the Bertotti formulation with flux-density dependent core-loss coefficients.

magnetization volume [10]. There is then the intrinsically stochastic DW motion at a microscopic level, which is the source of the hysteresis loss contribution.

Under a sinusoidal induction waveform, the well-known iron-loss formulation is given by:

$$\frac{P}{f} = k_h B_{max}^\alpha + k_{cl} f B_{max}^2 + k_{exc} f^{0.5} B_{max}^{1.5} \quad (1)$$

The first term refers to the hysteresis or static loss, characterized by the coefficients  $k_h$  and  $\alpha$ . To simplify the problem, we considered a frequency independant coefficient  $C_{stat}(B_{max}) = k_h B_{max}^\alpha$  for the static loss part. The two last terms represents the dynamical losses and are the main interest of this paper. The second term associated to the coefficient  $k_{cl}$ , represents the *classical losses* with a theoretical value  $k_{cl0} = \frac{\pi^2}{6} \sigma d^2$ , where  $\sigma$  is the DC electrical conductivity,  $d$  the lamination thickness, and  $\rho$  the material density. Even if it has the same formulation as the loss contribution from a homogeneous background field, it represents the loss of infinitely close moving domain-walls. The third loss contribution, associated to the coefficient  $k_{exc}$  is related to the *excess losses* and represents the correlation between domain-walls, proportional to the simultaneously active magnetic object number [5]. In expression (1),  $B_{max}$  is the peak value of a sinusoidal magnetic induction at a frequency  $f$ .

In the statistical loss approach, each dynamical term is a constant characteristic of the material, i.e.  $k_{cl}$  and  $k_{exc}$  are considered constant. However, as the domain-wall structure evolves from a strong multidomain branched domain-wall structure at zero field to a single magnetic domain at high field, the DW activity, correlations and velocities are expected to strongly vary during the magnetizing process [7]. Thus, loss coefficients must reflect these variations as one would expect with a direct modelling from different magnetic domain structure [11].

Following this reasoning, we made the assumption of loss coefficients  $k_{cl}(B_{max})$  and  $k_{exc}(B_{max})$  varying with  $B_{max}$ . Even if there is no reason that these coefficients do not vary with the frequency, we assume that their variations are already described by the frequency dependency present in the formulation. In the following, static losses are identified at 0Hz frequency and will not be further discussed as it is outside of the scope of this paper. If this approach is the same as followed empirically by Ionel et al [12] and others [13], we will show that on the same basis, a proper physical interpretation can be done, extending the framework developed in the original loss approach.

In the following, the material and experimental methods are described. Then, the main results are presented before being discussed in the last section.

## II. Material and methods

To test the induction dependent coefficient approach, we carried out extensive experiments on non oriented laminations with thicknesses ranging from 200 $\mu$ m to 650 $\mu$ m, cut at 0 $^\circ$  and 90 $^\circ$  with respect to the rolling direction.

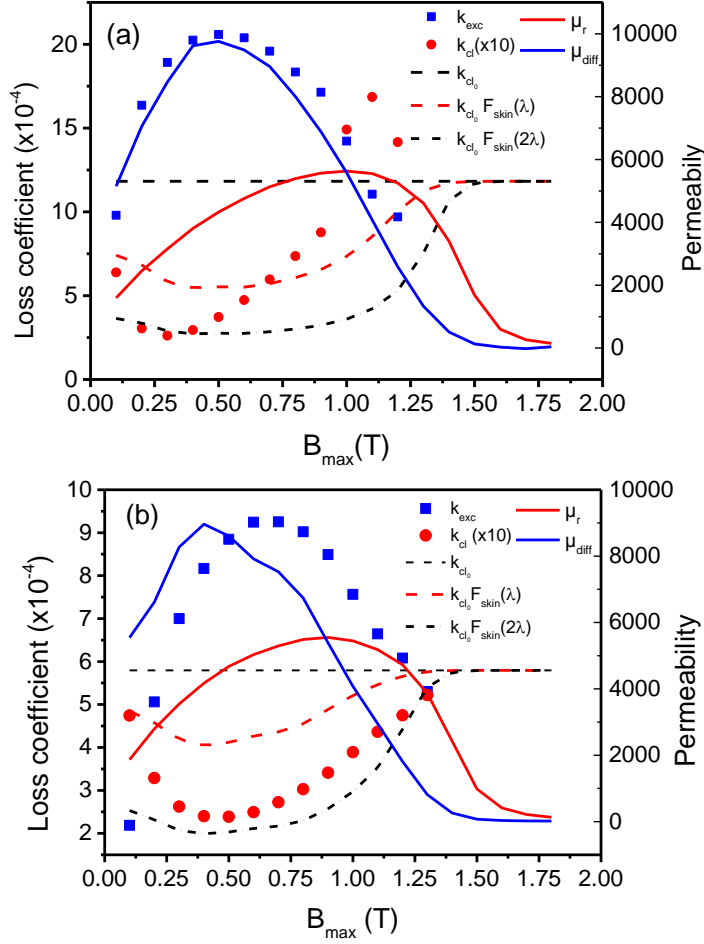


Figure 2: Excess loss,  $10*k_{cl}$  coefficient,  $\mu_r$ ,  $\mu_{diff}$  and  $F_{skin}$  variation as a function of the maximal flux density. Points represents the measured coefficients, lines the permeability. Dashed lines represent the  $10*k_{cl0}$ ,  $10*k_{cl0}*F_{skin}(\lambda)$  and  $10*k_{cl0}*F_{skin}(2\lambda)$ . Epstein laminations, (a) grade M400-50A 0deg ( $d=500\mu m$ ). (b) grade M270-35A 90deg ( $d=350\mu m$ ).

Experiments were carried out on a standard MPG200D Brockhaus measurement system, using an Epstein frame and a single sheet tester (SST), with a sinusoidal induction waveform in the frequency range 5Hz to 2kHz and  $B_{max}$  from 0.1T to 1.7T (depending on the excitation frequency) at ambient temperature. The measurement reproducibility for the loss and normal curves is obtained with a dispersion less than 1% and respects the IEC standard [14]. To calculate  $k_{cl0}$ , we considered  $\sigma=2.2$  MS/m, and  $\rho=7640$  kg/m<sup>3</sup>, which are typical values of FeSi laminations. Typical loss results are presented in Fig 1.

### III. Results

The identification of  $C_{stat}(B_{max})$ ,  $k_{cl}(B_{max})$  and  $k_{exc}(B_{max})$  from measurements was carried out by directly fitting the model, with a Levenberg-Marquardt algorithm, for all experimental data at each given value of  $B_{max}$  and using Eq.1 for all available frequencies.

Systematically, this procedure gives a very good loss description over all the frequency range as illustrated in Fig 1. In Fig 2.a and Fig 2.b, we present, for two laminations thicknesses, the loss coefficients variation. To plot on the same scale  $k_{exc}$  and  $k_{cl}$ , we multiplied  $k_{cl}$  and calculated quantities described in Sec IV which depend on  $k_{cl0}$  by a factor 10. Due to the limitation of our experimental setup regarding the sinusoidal control of the induction at high flux density and high frequency (>200 Hz), we restricted our analysis to induction levels below  $B_{max} = 1.3-1.4$  T for each sample.

Noticeably, we systematically observed the same core-loss coefficients variation for each measured sample. This observation was further supported by the work of Ionel *et al.* [12] who observed the same trends for their

variations. Qualitatively,  $k_{cl}$  starts at a value not far from its theoretical value  $k_{cl0}$  for  $B_{max} < 0.1$  T and begin to slightly decrease to reach a minimum close to  $B_{max} = 0.4$  T. Then it increases again with the magnetic flux density. The coefficient  $k_{exc}$  has the opposite variation, it begins to strongly increase to reach a peak value between  $B_{max} = 0.4$  T and  $0.6$  T before decreasing.

In Fig 2.a and Fig 2.b, we also plot, in function of  $B_{max}$  and for two lamination thicknesses, the relative permeability  $\mu_r = (1/\mu_0)B_{max}/H_{max}$  and differential permeability  $\mu_{diff} = (1/\mu_0) dB_{max}/dH_{max}$ . Both permeabilities are calculated from the normal curve obtained at the lowest measured frequency (5 Hz). From these curves, we can observe that a direct link exists between the differential permeability and the loss coefficients. In particular, the excess loss coefficient directly follows the variation of  $\mu_{diff}$ .

#### IV. Discussion

It is well known from Barkhausen experiment [15] that the signature of the domain-wall activity can be captured by macroscopic quantities such as  $\mu_{diff}$ . This is because in the microscopic approach, the central quantity describing the magnetization dynamic and the domain-wall motion is  $d\mathbf{M}/dt$  where  $\mathbf{M}$  is the magnetization of a given magnetic structure or alternatively  $d\mathbf{B}/dt$  with  $\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$ . This former quantity can be simply rewritten in the form :

$$\frac{d\mathbf{B}}{dt} = \frac{d\mathbf{B}}{d\mathbf{H}} \frac{d\mathbf{H}}{dt} = \mu_0 \left( \frac{d\mathbf{H}}{dt} + \frac{d\mathbf{M}}{dt} \right) \quad (2)$$

Strictly, to obtain the evolution of the magnetic structure with an excitation field, one has to solve the Landau-Lifshitz-Gilbert [8] equation to calculate  $d\mathbf{M}/dt$ , coupled to the Maxwell's equations to account for the conductivity [5], over the whole sample and then integrates the result with a proper time constant over a certain volume. To avoid this delicate calculation, one can rewrite the Equation (2) in the form :

$$\frac{d\mathbf{B}}{dt} = \mu_0 \left( 1 + \frac{d\mathbf{M}}{d\mathbf{H}} \right) \frac{d\mathbf{H}}{dt} = \mu_0 \mu_{diff} \frac{d\mathbf{H}}{dt} \quad (3)$$

with  $\mu_{diff} = (1/\mu_0) d\mathbf{B}/d\mathbf{H} = 1 + d\mathbf{M}/d\mathbf{H}$  which represents the variation of a magnetic structure with a variation of the applied field. This is the closest macroscopic quantity representing  $d\mathbf{M}/dt$  which can be further approximated by  $d\mathbf{B}_{max}/d\mathbf{H}_{max}$  from the low frequency normal curve.

From Fig 2.a and Fig 2.b, we can see that the peak of domain-wall activity (all the processes involving domain-wall movements, nucleation and annihilation) almost coincides with the peak of excess losses. This is in perfect agreement with the statistical losses approach because, in this framework, the excess loss coefficients represent the correlations between active domain walls and are proportional to the quantity of active domain walls. In a first approximation, for a given material, the higher the differential permeability  $\mu_{diff}$ , the higher the domain-wall activity will be and, consequently, the higher the coefficient  $k_{exc}$  will be. Regarding the variation of  $k_{cl}$ , its physical interpretation can be viewed as a counter-effect of the increase in the domain-wall activity because of the mutual interaction between DWs through the eddy field generated by moving DWs [16]. Interactions between walls are expected to lower their velocity and hence to reduce the losses that each wall is producing. Hence, the loss coefficient variation can be regarded as a fingerprint of the DW dynamic during the magnetizing process.

Because the magnetic losses are linked to the microscopic eddy-currents generated by the domain-wall motion, a direct correlation between the loss coefficient variation and the differential permeability also gives a link between microscopic and macroscopic quantities. Here, the concerned mechanism is the generation of eddy-currents which can be viewed as the sum of all microscopic eddy-currents. This gives a bridge between the macroscopic and the microscopic scale.

At the macroscopic level, it is well-known that eddy-currents are modeled from Maxwell's equations which permits one to describe and explain the skin-effect [17]. Alternatively, the skin-effect can also be understood from the domain-wall motion point of view by considering, on a single wall for simplicity<sup>1</sup>, the mutual interaction of each point of the moving wall which reduces the speed at the domain-wall center position [18].

It is often found in literature that the way to improve the Bertotti's model consists in adding empirically the

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<sup>1</sup> To be complete, the interaction between all parts of neighboring walls must be accounted.

skin-effect by multiplying the classical coefficient terms with a function  $F_{skin}$ , derived from Maxwell's equations, as given by [17]:

$$F_{skin} = \frac{3 \sinh(\lambda) - \sin(\lambda)}{\lambda \cosh(\lambda) - \cos(\lambda)} \quad (4)$$

where  $\lambda = \frac{d}{\delta}$  and  $\delta$  is the skin-depth  $\delta = \sqrt{\frac{1}{\pi f \sigma \mu_{diff}}}$ .

We tried to model the  $k_{cl}(B_{max})$  coefficient variation using  $F_{skin}$ . The calculation of  $F_{skin}$  requires the knowledge of the frequency. However, because the coefficient  $k_{cl}(B_{max})$  is frequency-independent in the present approach, we arbitrarily fix the frequency at  $f=2000$  Hz which represents our highest measured frequency. Qualitatively, we find that in the studied flux density range,  $k_{cl}(B_{max})$  may be described by the  $F_{skin}$  function except for the thicker sample (M400-50A Fig 2.a). Quantitatively, the effect of  $F_{skin}$  on  $k_{cl}$  appears to be negligible with respect to the observed variation and one must consider an enhanced skin-effect by multiplying  $\lambda$  approximately by a factor 2 to obtain a fairly good description as observed in Fig 2.a and Fig 2.b. Moreover, if only our thinnest samples are approximately well described, samples with a thickness above  $350\mu\text{m}$  present a strong deviation in the high field regime. A possible way of improvement can be given by considering the non-linear skin-effect as proposed in [19] but there will still remain a quantitative discrepancy.

Previous observations showed that the use of  $F_{skin}$  is only giving a partial empirical answer to the loss description. The presence of  $\mu_{diff}$  in the formulation which represents the DW activity, permits to qualitatively model the coefficient  $k_{cl}(B_{max})$  because it represents the domain-wall structure dynamics. It accounts for all microscopic effects at the macroscopic level with their variations, without precisely modeling them. However, the strong quantitative disagreement between the coefficient  $k_{cl}$  and the  $F_{skin}$  function, which needs to be amplified, demonstrates that the classical coefficient can not be simply understood as a background field which can be treated only macroscopically. This was already pointed out theoretically by Bishop [16] and mainly results from the fundamental hypothesis behind the statistical approach of losses: the perfect independence of DW, which is most probably a limit case valid in Barkhausen experiments but not in a general approach of losses which could use the natural link between the different involved scales provided by  $\mu_{diff}$ .

## V. Conclusion

In this work, we studied the core-loss variation dependency with the magnetic flux density using the statistical approach of losses in FeSi laminations. Using standards measurements methods, we measured the losses on a large number of laminations and observed systematically the same kind of variation for the coefficients  $k_{cl}(B_{max})$  and  $k_{exc}(B_{max})$ . We show that a direct link exists between these variations and the differential permeability. This permits to understand the loss variations as due to the evolution of the domain-wall activity during the magnetizing process. Using the interpretation that  $k_{cl}(B_{max})$  represents a background field, we tried to model our variation using the skin effect formulation. We demonstrate that this variation can not be modeled by this formulation without a strong enhancement of the effect.

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