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1 **Rain gauge networks' limitations and the implications** 2 **to hydrological modelling highlighted with a X-band** 3 **radar**

4
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6
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11 12 **Abstract**

13 Precipitation risk and water management is a key challenge for densely
14 populated urban areas. Applications derived from high spatio-temporal
15 resolution observation of precipitations are to make our cities more weather-
16 ready. Finer resolution data available from dual polarised X-band radar
17 measurements enhance engineering tools as used for urban planning policies
18 as well as protection (mitigation/adaptation) strategies to tackle climate-change
19 related weather events. For decades engineering tools have been developed to
20 work conveniently either with very local rain gauge networks, or with mainly C-
21 band radars that have gradually been set up for space-time remote sensing of
22 precipitation. Most of the time, the C-band radars continue to be calibrated by
23 the existing rain gauge networks. Inhomogeneous distributions of these
24 networks lead to only a partial information on the rainfall fields. Here we show
25 that the statistics of measured rainfall is strongly biased by the fractality of the

26 measuring networks and that this fractality needs to be properly taken into
27 account to retrieve the original properties of the rainfall fields, in spite of the
28 radar data calibration. In this work, we use the semi-distributed hydrological
29 modelling over the Bièvre catchment to generate a virtual rain gauges' network.
30 And, firstly, performing a fractal analysis of this network distribution, we
31 demonstrate that the semi-distributed hydrological models statistically reduce
32 the distributed (weather radar) rainfall fields into rainfall measured by a much
33 scarcer network of virtual rain gauges. Then, with the help of the Intersection
34 Theorem and multifractal theory, we statistically compare the virtual rain
35 gauges' data with the rainfall data measured by the dual-polarimetric X-band
36 radar operated at Ecole des Ponts with a spatial resolution of 250 m, providing
37 pre-factors that indicate the need of a proper re-normalisation of rain gauge
38 rainfall data when comparing (or calibrating) with radar data and the possible
39 counterproductivity of this conditioning.

40

41 **Keywords:** rain gauge network; spatio-temporal variability; X-band radar;
42 fractals; multifractals; semi-distributed hydrological model

43

44 **1. Introduction**

45 The increase in global urbanisation and population density emphasises
46 the importance and the need to improve the adaptation of urban areas mainly to
47 climate change (Pumo et al., 2017; Arnone et al., 2018). Managing extreme
48 weather events, particularly intense precipitation ones and heat waves, in these
49 areas is a major challenge for the future. The population's demand for a better
50 quality of life motivates an improvement in the ability to measure, understand,

51 model and predict hydrometeorological processes in urban environments,
52 aiming at better flood control and associated risk management. Conventional
53 local measurements in urban areas generally do not meet the World
54 Meteorological Organization (WMO) criteria for the measurement of
55 precipitation (WMO, 2014). Better spatio-temporal scales with accuracy and
56 reliability are required (Fabry et al., 1994; Berne et al., 2004). In this way, the
57 use of ground-based remote sensing has been very important in elucidating
58 complex urban environment structures, thus expanding hydrometeorological
59 challenges (NRC, 2012).

60 Weather radars have the capability to estimate rainfall fields with high
61 spatio-temporal resolutions and have been used to cope the sparseness of rain
62 gauge networks. Nevertheless, the initial measurement of reflectivities adopted
63 by those devices, not providing direct precipitation rates such as rain gauges,
64 usually generates significant uncertainties. Then, with the growing use of radars
65 to estimate rain rates, many studies have been developed to adjust and/or
66 merge weather radars (mostly considering non-polarimetric ones) with rain
67 gauge networks (Einfalt et al., 2005; Allegretti et al., 2012; Lo Conti et al.,
68 2015). On the other hand, especially for high rainfall intensities, in order to
69 reduce these uncertainties and improve the rainfall estimates, dual polarisation
70 technology using specific differential phase (KDP) has been employed, which
71 does not necessarily requires rain gauge adjustments according to some
72 authors (Bringi and Chandrasekar 2001; Illingworth and Blackman 2002;
73 Figueras i Ventura et al., 2012; Chandressekar et al., 2015). This technology
74 explores the flatness of large raindrops to analyse the phase difference of the

75 reflected vertical and horizontal signals and then directly obtain the precipitation
76 rates.

77 Coming together with rainfall data improvement, an efficient storm water
78 management also deals with the accuracy and reliability of hydrological models,
79 mainly in urban areas which present higher levels of imperviousness leading to
80 shorter response times (Berne et al., 2004; Segond et al., 2007; Furusho et al.,
81 2014; Ochoa-Rodriguez et al., 2015; Arnone et al., 2018). This requests the use
82 of high resolution models, which means reducing the size of the sub-
83 catchments in the semi-distributed models or adopting the fully-distributed
84 models (Fewtrell et al., 2011; Ichiba, 2016; Pina et al., 2016; Pumo et al., 2017;
85 Alves de Souza et al., 2018; Paz et al., 2019).

86 This work presents a discussion of the rain gauge networks' limitations
87 and the implications to hydrological modelling with the help of fractal and
88 multifractal analysis. The study was carried out over the Bièvre catchment,
89 which is a 110 km² semi-urbanised area located in the southwest of Paris
90 region, using the rainfall data of the dual-polarimetric X-band radar recently
91 installed on the roof of the École des Ponts ParisTech (ENPC).

92 The paper is organised in five sections. Section 2 describes the case
93 study and the rainfall data. The methodology of fractal and multifractal analyses
94 are presented in Section 3. Section 4 presents the results and discussion. And
95 finally, the conclusions are presented in Section 5.

96

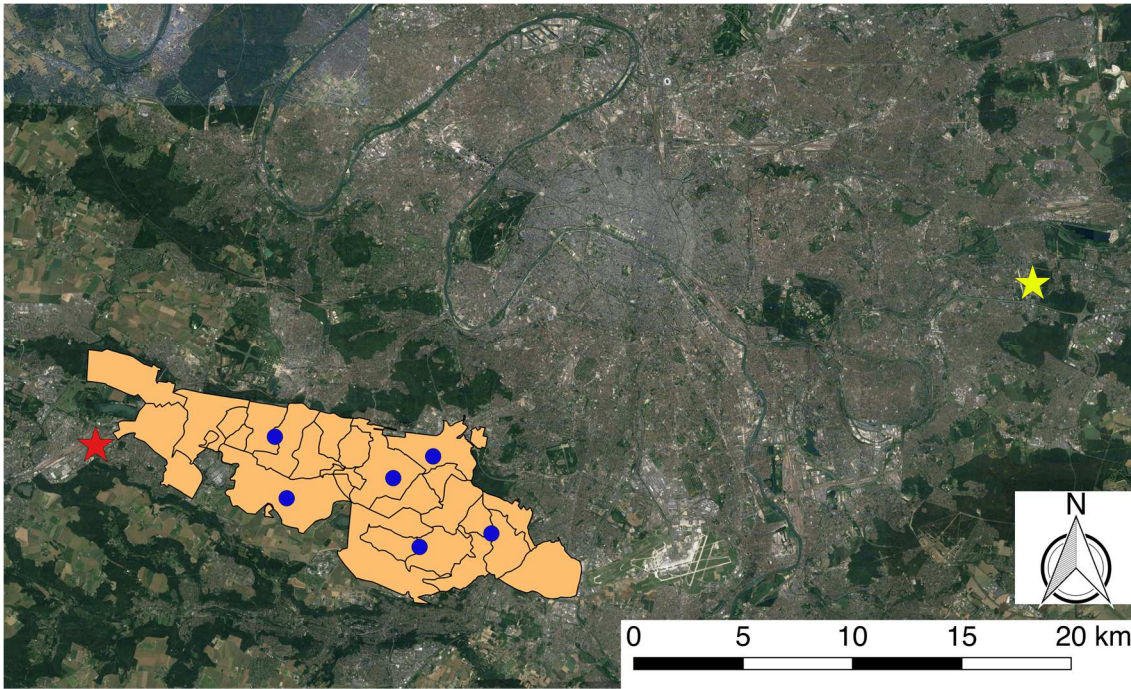
97 **2. Case Study and Rainfall Data**

98 The Bièvre Valley, which is a 110 km² semi-urbanised area in the
99 southwest of Paris region, was selected as case study. The Bièvre River is 33

100 km-long tributary of the Seine River and its upstream catchment is managed by
101 two local authorities: the CASQY (“Communauté d’Agglomération de Saint-
102 Quentin-en-Yvelines” – Saint-Quentin-en-Yvelines Agglomération Community)
103 in the upstream portion and the SIAVB (“Syndicat Intercommunal
104 d’Assainissement de la Vallée de la Bièvre” – Inter-municipal local authority for
105 Sanitation of the Bièvre Valley) in the downstream portion.

106 The Bièvre catchment was modelled using InfoWorks CS (Collection
107 Systems) (Paz, 2018; Paz et al., 2018), a widely used semi-distributed model
108 (Soft, 2010), with the operation responsibility given to the company Veolia since
109 1991 by the local authority SIAVB to perform a real time control of the
110 hydrological area. This hydrological model is integrated in the Optim Sim
111 platform, developed by Veolia, that mimics the actual regulation of the storage
112 basins (Paz, 2018; Paz et al., 2018) in two simulation modes: the “replay mode”
113 and the “forecasting mode”. The first one extracts rainfall data from the 6 SIAVB
114 rain gauges’ network database (using the Thiessen polygons technique) to
115 replay past events. And the “forecasting mode” uses different rainfall data, e.g.
116 from weather radar measurements, to simulate the catchment hydrological
117 behaviour. The catchment area is divided into 27 sub-catchments, as displayed
118 on Figure 1.

119



120

121 *Figure 1: Illustration of rainfall measurement devices available over the Bièvre catchment: the X-band radar (yellow*
 122 *star), the C-band radar (red star) and the six rain gauges (blue dots).*

123 This modelling had been firstly calibrated using the rainfall data from the
 124 Météo-France C-band radar of Trappes, located in a direct proximity (~ 0 - 20
 125 km) of the catchment (see Fig. 1), with a resolution of 1 km x 1 km x 5 min.
 126 These C-band radar data are firstly obtained as reflectivity (dBZ) and then, after
 127 some post-treatments and filtering process, they are transformed into rainfall
 128 intensity (R) by applying the Marshall-Palmer Z-R equation (Marshall and
 129 Palmer, 1948), with fixed parameters $a = 200$ and $b = 1.6$ (Eq. (1)), which
 130 means that the polarimetric capability of this radar of Trappes has not been
 131 used. In addition, this catchment also contains a network of six tipping bucket
 132 rain gauges (see Fig. 1), which had been used to calibrate the C-band radar
 133 data.

$$Z = aR^b \quad (1)$$

134

135 However, in this study, we used rainfall data from the ENPC dual-
136 polarimetric X-band radar of Champs-sur-Marne (see Fig. 1), with distances
137 ranging between 25 to 45 km and a 250 m x 250 m x 3.41 min resolution. The
138 ENPC X-band Dual Polarization Surface Rainfall Intensity (DPSRI) product still
139 considers the Z-R relationship for low intensities, but for high intensities it uses
140 the specific differential phase (KDP) (Selex, 2015):

$$R = 19.63|KDP|^{0.823}, \text{ for } Z > 35 \text{ dBZ and } KDP > 0.3^\circ/\text{km} \quad (2)$$

141

142 For the purpose of this work, six different rainfall events were selected:
143 12-13/09/2015 (44 hours); 16/09/2015 (11.3 hours); 05-06/10/2015 (31 hours);
144 21-23/05/2016 (59.5 hours); 28-31/05/2016 (87.7 hours); and 17/06/2016 (24
145 hours).

146

147 **3. Methods: (multi)fractals**

148 When studying some objects of very irregular or fragmented form (with
149 unusual properties in classical geometry), Mandelbrot (1974) named them as
150 “fractals” (“fractus” means irregular in Latin). Nevertheless, fractal objects must
151 not just be irregular, but also scale invariant, which means that their form
152 remains unchanged at different scales of observation. This brings the properties
153 of self-similarity (a zoomed part of the object looks similar to the object itself,
154 with an isotropic variation) and self-affinity (when the variation is anisotropic).

155 In classical geometry, dimensions are integers characterising regular
156 spaces as straight lines (1D), planar figures (2D) and volumes (3D). If we take
157 an object, embedded in an Euclidian dimension D , of linear size L , it can be
158 covered by N small objects of linear size l , with a scale ratio $\lambda (= L/l)$.

159 In the case of fractal objects and sets, the dimension is no longer an
160 integer and is called fractal dimension (D_f) (Mandelbrot, 1967, 1977;
161 Mandelbrot and Pignoni, 1983; Feder, 1988). Even if its theoretical computation
162 is rather complex, an easier way to evaluate it is through the box-counting
163 method (Hentschel and Procaccia, 1983; Lovejoy et al., 1987). It takes into
164 account that when $\lambda \rightarrow \infty$ there is a power-law relation between the fractal
165 dimension and the number of “non-empty” pixels of the set (N_λ) at the scale λ :

$$N_\lambda \approx \lambda^{D_f} \quad (3)$$

166
167 where \approx means the asymptotic equivalence.

168 Therefore, the box-counting method can be easily applied considering a
169 simple methodology to change the resolution of a given dataset. Let us define A
170 as the geometrical set (embedded in a space of dimension D ; in this paper we
171 will consider the 2-D space) to be analysed and $N_{\lambda,A}$ the number of non-
172 overlapping pixels at the resolution λ necessary to cover the set A . Thus, the
173 method consists in firstly counting the number of non-empty pixels at the
174 smallest pixel size (l), then we multiply the pixel size by two at each step of
175 process (practically, it means that in the 2-D case we merge 4 by 4 pixels at
176 each scale variation step) and count again the number of non-empty pixels at
177 this new size, and we continue this procedure until we achieve the maximum
178 pixel size (L). Once we have counted the numbers of non-empty pixels (N_λ) at
179 different scales (λ), we display them in a log-log plot (λ vs N_λ). Then, if A is a
180 fractal set, the points of this plot will be along a straight line, and, from Eq. (1),
181 we can estimate the fractal dimension D_f as its slope.

182 Furthermore, the codimension of a fractal set is usually (geometrically)
183 defined as (Mandelbrot, 1967, 1977; Feder, 1988; Falconer, 1990):

$$c = D - D_f \tag{4}$$

184

185 Then, it is also possible to probabilistically define the fractal codimension,
186 using Eqs. (3) and (4), where the probability that a cube embedded in the set
187 \mathbb{R}^D at a scale λ is contained in the fractal set is given by:

$$\lambda^{D-D_f} \approx \lambda^c \tag{5}$$

188 This last definition is more general because it enables $c > D$, which
189 would imply $D_f < 0$ from Eq. (4).

190 The concept of fractal dimension is related to the sparseness of a
191 dataset and it is well applied to binary fields (e.g., rainfall occurrence, rain
192 gauge network distribution). Many geophysical fields with different intensity
193 levels, and also irregular geometries and scale invariance, can be seen as
194 fractal fields at different thresholds (singularities). Then, for each singularity it is
195 possible to calculate its fractal dimension, which means that the field is
196 characterised by several fractal dimensions according to the imposed threshold
197 (Grassberger, 1983; Hentschel and Procaccia, 1983; Schertzer and Lovejoy,
198 1984); i.e., for each given threshold, a different fractal dimension is estimated
199 for the field that exceeds the threshold. These fields were named by Parisi and
200 Frisch (1985) as “multifractals”.

201 Multifractals rely on the assumption that a geophysical field is generated
202 through a multiplicative cascade process (Schertzer and Lovejoy, 1987, 2011)

203 produced by random multiplicative modulation of large-scale structures into
 204 small-scale ones. Furthermore, rainfall fields have been widely studied as
 205 presenting multifractal behaviour (Schertzer and Lovejoy, 1987; Ladoy et al.,
 206 1993; Tessier et al., 1993; Olsson et al., 1996; de Lima et al., 1999; Deidda,
 207 2000; Veneziano et al., 2006; García-Marin et al., 2008; Serinaldi, 2010;
 208 Ochoa-Rodriguez et al., 2015; Paz, 2018).

209 As just mentioned, the multifractal fields are characterised by a hierarchy
 210 of fractal dimensions (and codimensions). Then, by referring to the notion of
 211 fractal codimension, one can calculate, for a given multifractal field ε_λ , the
 212 probability of obtaining a singularity of order greater than or equal to γ at the
 213 scale λ :

$$\Pr(\varepsilon_\lambda \geq \lambda^\gamma) = \frac{\lambda^{D_f(\gamma)}}{\lambda^D} \approx \lambda^{-c(\gamma)} \quad 6)$$

214

215 where the codimension function $c(\gamma) = D - D_f(\gamma)$ is convex and increases with
 216 λ .

217 Additionally, multifractal fields can also be described by their statistical
 218 moments. Schertzer and Lovejoy (1987, 1991) introduced the scaling moment
 219 function $K(q)$, which is also convex and characterises the various q^{th} order
 220 statistical moments of the multifractal field ε_λ :

$$\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)} \quad 7)$$

221

222 where q is the statistical moments order and $\langle \varepsilon_\lambda^q \rangle$ is the q^{th} moment mean of the
 223 intensities at the scale λ .

224 Furthermore, Parisi and Frisch (1985) demonstrated that the two
 225 functions $c(\gamma)$ and $K(q)$ have a one-to-one relationship, which is highlighted by
 226 the Legendre transform:

$$K(q) = \max_{\gamma}\{q\gamma - c(\gamma)\} \Leftrightarrow c(\gamma) = \max_q\{q\gamma - K(q)\}$$

8)

227

228 These related expressions bring the correspondence between the orders
 229 of moments (q) and the singularities (γ), such that:

$$q_{\gamma} = c'(\gamma) ; c(\gamma) = q_{\gamma}\gamma - K(q_{\gamma}) \quad (9)$$

$$\gamma_q = K'(q) ; K(q) = q\gamma_q - c(\gamma_q) \quad (10)$$

230

231 where at the moment of order q corresponds the singularity γ_q , and conversely,
 232 at the singularity γ corresponds the order of moment q_{γ} . In addition, it is
 233 possible to see from Eq. (6) that more rare rainfall events (with very large
 234 singularities γ) correspond to bigger values of $c(\gamma)$. Consequently, due to the
 235 convexity of both $c(\gamma)$ and $K(q)$ and to the Legendre transform, for $q > 1$, these
 236 extreme events also correspond to bigger values of q and $K(q)$.

237 As both statistical functions $c(\gamma)$ and $K(q)$ have the only constraint of
 238 convexity, there is an infinity of parameters required to characterise a
 239 multifractal process. However, Schertzer and Lovejoy (1987, 1997) explored the
 240 concept of universality (usually used in physics, where among an infinite
 241 number of parameters only a few would be relevant) and developed the
 242 Universal Multifractals (UM) (see Schertzer and Lovejoy, 2011, for a more
 243 recent review). In this context, $c(\gamma)$ and $K(q)$ can be fully described by only
 244 three "UM parameters" (α , C_1 and H) as:

$$K(q) = q^H + \begin{cases} \frac{C_1}{\alpha - 1} (q^\alpha - q); & \alpha \neq 1 \\ C_1 q \log(q) & ; \alpha = 1 \end{cases} \quad (1)$$

$$c(\gamma + H) = \begin{cases} C_1 \left(\frac{\gamma}{C_1 \alpha'} + \frac{1}{\alpha} \right)^{\alpha'} & ; \alpha \neq 1 \\ C_1 \exp\left(\frac{\gamma}{C_1} - 1\right) & ; \alpha = 1 \end{cases} \quad (2)$$

245

246 where $\frac{1}{\alpha} + \frac{1}{\alpha'} = 1$, for all $\alpha \neq 1$, and:

247 – α is the Levy's multifractality index ($0 \leq \alpha \leq 2$). It measures the degree of
 248 multifractality of the process. In particular, if $\alpha = 0$ we observe a monofractal
 249 process (also entitled β -model, Mandelbrot (1974)) and $\alpha = 2$ corresponds to
 250 the maximum of multifractality for a model (improperly called lognormal);

251 – C_1 is the codimension of the mean singularity of the field. It measures the
 252 mean inhomogeneity, where $C_1 = 0$ for a homogeneous field. The more it
 253 increases, the more the singularity of the field average is dispersed. We thus
 254 observe a field rarely exceeding its mean, but which can do so in an
 255 extremely strong way;

256 – H is the Hurst's exponent, which measures the degree of non-conservation of
 257 the field. Values of H close to zero indicate higher conservativeness of the
 258 process.

259

260 While performing a multifractal analysis of a (rainfall) field, we should
 261 determine its statistical functions $c(\gamma)$ and $K(q)$. The Trace Moment (TM)
 262 method (Schertzer and Lovejoy, 1987) allows to directly determine the scaling
 263 moments function $K(q)$ for any $q > 0$.

264 As $K(q) > 0$ for $q > 1$, $\langle \varepsilon_\lambda^q \rangle \approx \lambda^{K(q)} \rightarrow \infty$ when $\lambda \rightarrow \infty$. Then, to avoid this
 265 divergence, the concept of flux (Π_λ) is used, given by:

$$\Pi_\lambda = \int \varepsilon_\lambda d^D x \quad (13)$$

266

267 And the trace of the q^{th} -power of the flux is:

$$Tr[\varepsilon_\lambda^q] = \left\langle \int (\varepsilon_\lambda)^q d^{qD} x \right\rangle \approx \lambda^{K(q)-D(q-1)} \quad (14)$$

268

269 In practice, the method consists in taking the q^{th} -power of each value of
 270 the field ε_λ at the scale λ , and then the ensemble average of q^{th} -power
 271 moments is calculated at that scale. Then, from Eq. (14), the same process is
 272 repeated for different scales λ , the resulting averages are displayed as a
 273 function of λ in a log-log plot, and a linear regression is performed to obtain the
 274 value of $K(q)$ (which will be estimated as the given slope). Finally, by repeating
 275 this process also with other values of q , we obtain the $K(q)$ function (and, via
 276 Legendre transform, the $c(\gamma)$ function).

277 Thus, once we know $K(q)$, it is possible to (indirectly) determine the UM
 278 parameters α and C_1 :

$$C_1 = K'(1) \quad (15)$$

$$\alpha = K''(1)/C_1 \quad (16)$$

279

280 In the framework of UM, Lavallée et al. (1993) developed the Double
 281 Trace Moment (DTM) method to directly determine the parameters α and C_1 ,
 282 which means that there is no need to firstly obtain $K(q)$ (and $c(\gamma)$). This

283 technique is in fact a generalisation of the TM method and considers that the
 284 q^{th} -order moments of the renormalised η -power of a conservative field ε_λ
 285 remain scale invariant. Then, the idea is to apply the TM method to this
 286 renormalised field (through the $K(q, \eta)$ function):

$$\langle (\varepsilon_\lambda^{(\eta)})^q \rangle \approx \lambda^{K(q, \eta)} \Rightarrow \frac{\langle \varepsilon_\lambda^{\eta q} \rangle}{\langle \varepsilon_\lambda^\eta \rangle^q} \approx \frac{\lambda^{K(q\eta)}}{\lambda^{qK(\eta)}} = \lambda^{K(q\eta) - qK(\eta)} \quad (1)$$

7)

287 where $\varepsilon_\lambda^{(\eta)} = \frac{\varepsilon_\lambda^\eta}{\langle \varepsilon_\lambda^\eta \rangle}$.

288 Then:

$$K(q, \eta) = K(q\eta) - qK(\eta) \quad (1)$$

8)

289

290 Since this method is especially applied for UM, considering Eq. (11), Eq.
 291 (18) becomes:

$$K(q, \eta) = \eta^\alpha K(q) \quad (1)$$

9)

292 From Eq. (19), the parameter α can be directly estimated as the slope of
 293 the linear part of $K(q, \eta)$ vs. η in a log-log plot, for a given q . And in the same
 294 log-log plot, the parameter C_1 can be also estimated from the interception value
 295 of the linear part and the axis $\log(\eta) = 0$ (Hoang, 2011).

296 In addition, according to the Intersection Theorem (Schertzer and
 297 Lovejoy, 1987), considering two independent (multi)fractal fields $\varepsilon_{1,\lambda}$ and $\varepsilon_{2,\lambda}$
 298 embedded in the same space and presuming that the intersection of both ($\varepsilon_{1,\lambda} \cap$
 299 $\varepsilon_{2,\lambda}$) is not empty, for the given intersection the probability of obtaining a
 300 singularity of order greater than or equal to γ at the scale λ is:

$$\Pr(\varepsilon_{1,\lambda} \cap \varepsilon_{2,\lambda} \geq \lambda^\gamma) = \Pr(\varepsilon_{1,\lambda} \geq \lambda^\gamma) \Pr(\varepsilon_{2,\lambda} \geq \lambda^\gamma)$$

20)

301

302 Therefore, following Eq. (6), the resulting codimension is the addition of
 303 independent (multi)fractal processes:

$$c_{\varepsilon_1 \cap \varepsilon_2}(\gamma) = c_{\varepsilon_1}(\gamma) + c_{\varepsilon_2}(\gamma)$$

21)

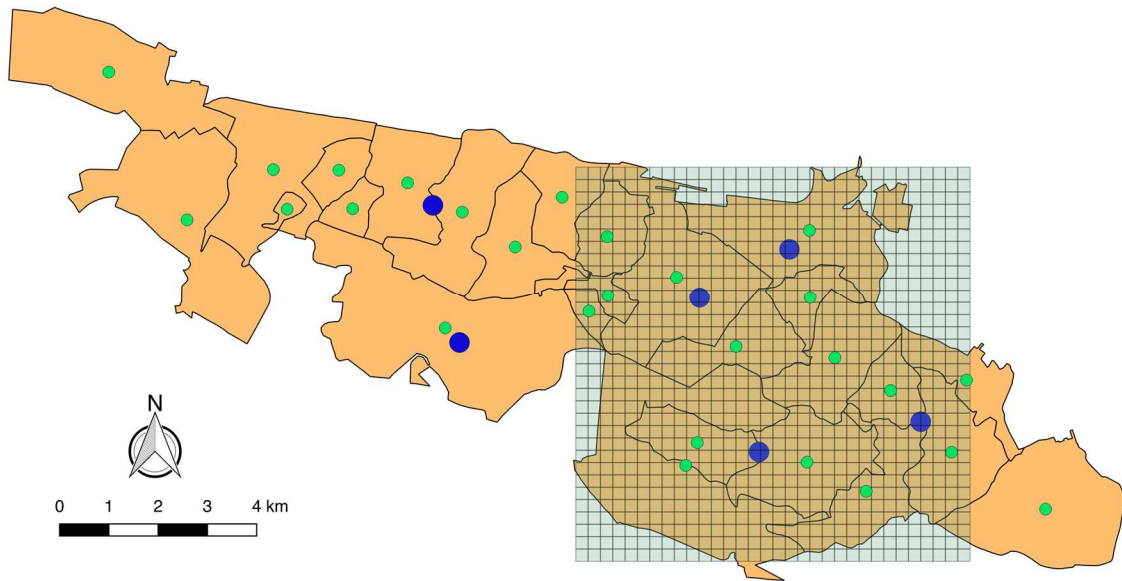
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305 In the following Section, we will use the fractal and multifractal (with the
 306 help of the DTM method and the Intersection Theorem) approaches to discuss
 307 the rain gauge networks' limitations and the implications to hydrological
 308 modelling.

309

310 **4. Results and Discussion**

311 In this work, the sub-catchments' distribution of the pilot site of Bièvre
 312 catchment, which had already been modelled with the semi-distributed model
 313 InfoWorks CS (Paz, 2018; Paz et al., 2018), was used to construct a network of
 314 virtual rain gauges located in the centre of mass of each sub-catchment. Then,
 315 an area of 8 km x 8 km was selected using the X-band radar grid (Fig. 2). This
 316 choice corresponds to the most homogeneous distribution of virtual rain gauges
 317 over a square area of Bièvre catchment, containing 15 of the 27 virtual rain
 318 gauges.

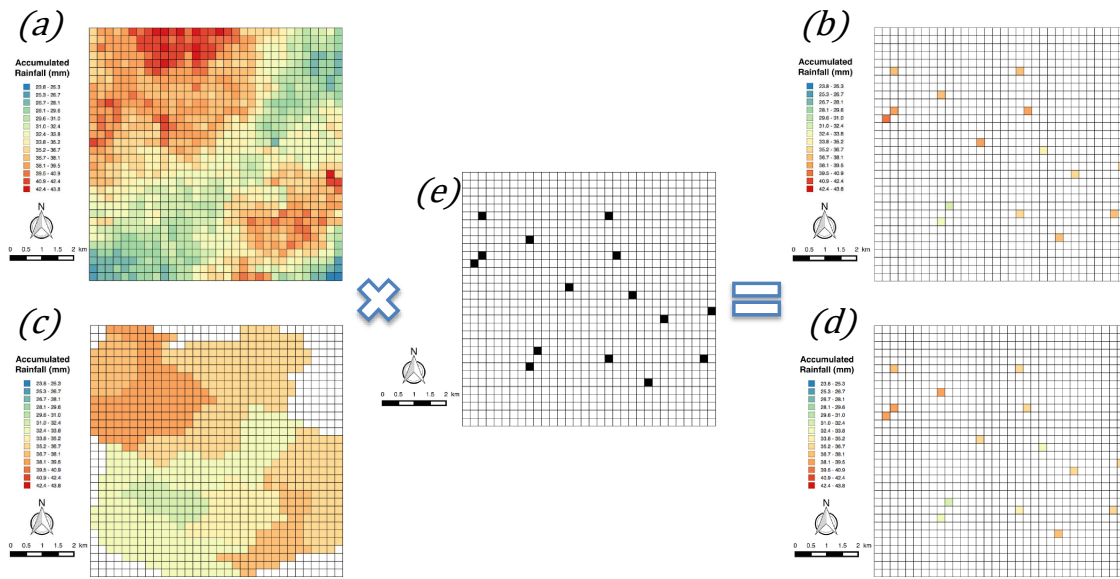


319

320 Figure 2: Virtual rain gauges' distribution (green dots) and 6 SIAVB rain gauges
 321 (blue dots) over the 27 sub-catchments, and the 8 km x 8 km selected ENPC X-band
 322 radar grid.

323

324 Four different rainfall fields have been analysed over the selected area
 325 (Fig. 3): (a) original radar data; (b) original radar data corresponding to each
 326 centroid pixel; (c) sub-catchment averaged rainfall radar data; (d) sub-
 327 catchment averaged rainfall radar data being concentrated at the corresponding
 328 centroid pixel; as well as a (e) fractal distribution of the corresponding virtual
 329 rain gauges, located at the centroid pixels.



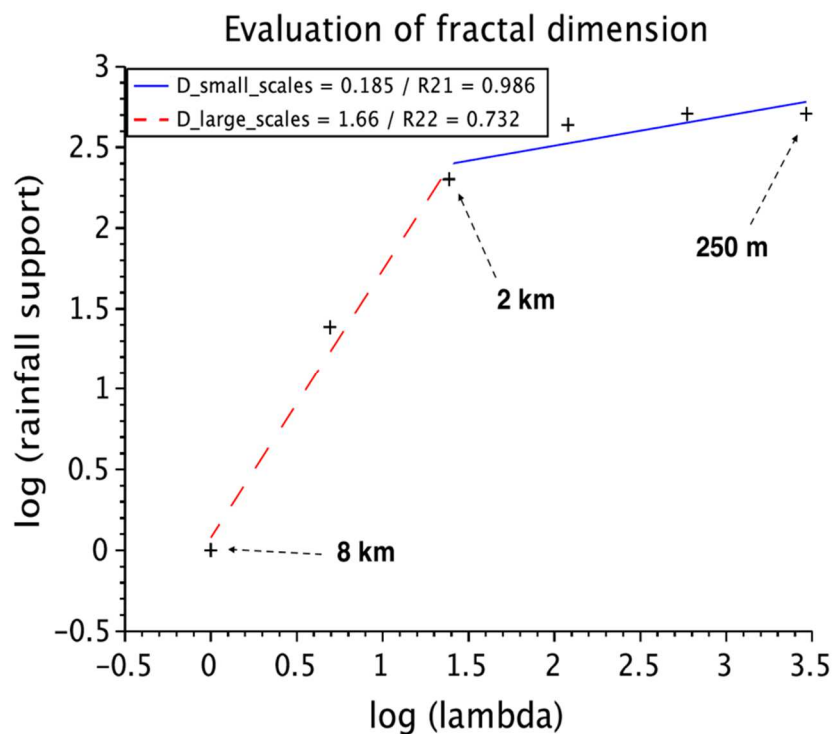
330
331

332 Figure 3: Four different rainfall radar data fields analysed over the
 333 selected area (demonstrated here with data of the 12 September 2015 event):
 334 (a) original data; (b) original data corresponding to each centroid pixel; (c) sub-
 335 catchment averaged rainfall; (d) sub-catchment averaged rainfall being
 336 concentrated at the corresponding centroid pixels. And the 15 sub-catchment
 337 centroid pixels corresponding to the virtual rain gauges' locations (e).

338

339 Firstly, the fractal analysis of the 15 virtual rain gauge pixels' distribution
 340 (which will also be called rainfall support) was performed using the box-counting
 341 method (Fig. 4) with a simple Scilab routine (Gires et al., 2017, 2018), which
 342 applies the methodology described in the previous section (see Eq. (3)) to the
 343 dataset. It is possible to identify a scaling break (with two different behaviours
 344 for the small scales and the large scales) at the spatial scale of 2 km, which is in
 345 fact the average sub-catchment's size and close to the 1-km resolution of the C-
 346 band radar rainfall data that have been used for the model calibration. Then, a
 347 relatively high fractal dimension ($D_f = 1.66$) obtained over the large-scale range

348 corresponds to the network of 15 virtual rain gauges at 2-km scale. This number
 349 of gauges remains still reasonable, although only 6 real rain gauges are
 350 available for the full Bièvre catchment (i.e. 4 over the selected area). Preserving
 351 the same fractal dimension $D_f = 1.66$ over smaller scales up to 250 m (the
 352 resolution of the X-band radar rainfall) would result in $N = 315$ virtual rain
 353 gauges (and/or sub-catchments). Thus, a smaller number of gauges ($N = 15$),
 354 characterised by much lower fractal dimension ($D_f = 0.185$), significantly
 355 reduces the captivity of spatial rainfall variability over small scales and hence its
 356 representability in the model having a constant rainfall per sub-catchment of 2-
 357 km characteristic scale.



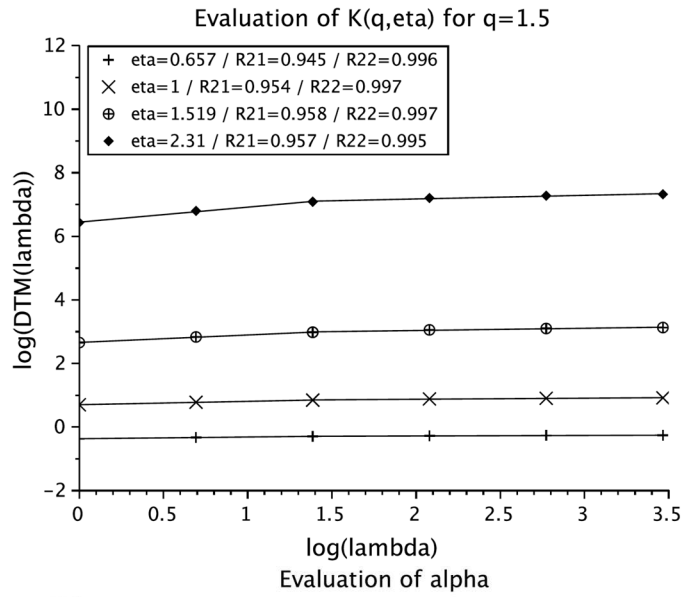
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359 *Figure 4: Fractal analysis of the centroid pixels' distribution, providing the linear regression coefficients R21 (small*
 360 *scales) and R22 (large scales).*

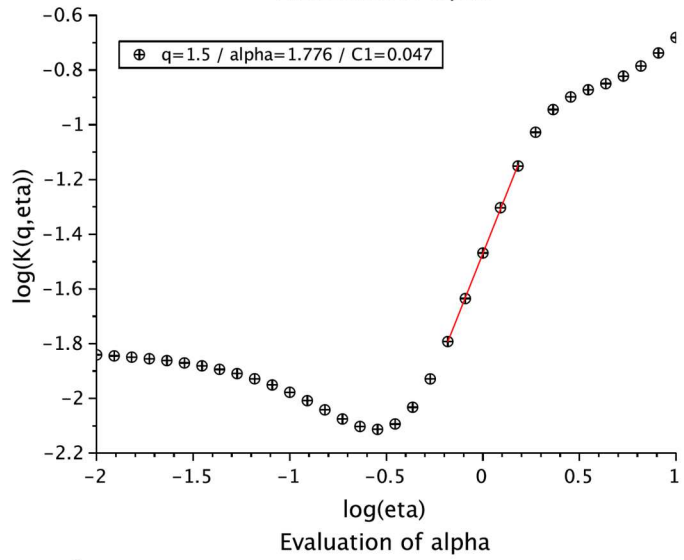
361 This result from the fractal analysis of the 15 virtual rain gauge network
 362 distribution corroborates the researches from Austin (1987), Schertzer and
 363 Lovejoy (1987), Seed and Austin (1990), Vieux and Vieux (2005), Villarini et al.

364 (2008), Peleg et al. (2013), which identify the difficulties of sparsely distributed
365 rain gauge networks to capture the high spatial and temporal variability of
366 precipitation fields (specially high convective ones), while comparing to weather
367 radars. On the other hand, when dealing with the sub-catchment sizes, it also
368 confirms the findings of Thorndahl et al. (2017), based on a review of the state
369 of the art about the use of weather radar rainfall data in urban hydrology
370 (Schilling, 1991; Berne et al., 2004; Einfalt et al., 2004; Ochoa-Rodriguez et al.,
371 2015), in which they associate the better spatial and temporal resolution of
372 radar rainfall data to decreasing catchment sizes.

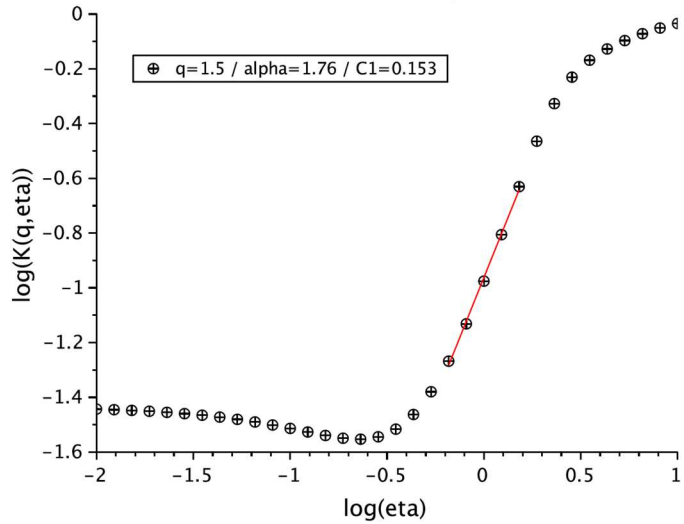
373 Then, to better evaluate the unfortunate consequences of sparse rain
374 gauge networks, the multifractal analyses were performed over the studied
375 rainfall fields in conditions (a), (b), (c) and (d), and then related to the fractal
376 analysis of the rain gauge's distribution with the help of the Legendre transform.
377 Firstly, the DTM method was applied (Fig. 5), following the methodology
378 described in Section 3 (Eq. (19)), on ensemble of data over the whole rainfall
379 event (each time step being considered as an independent realisation) with
380 scaling break. The values obtained for α and C_1 for the six studied events are
381 presented at Tables: 1 (rainfall fields (a)), 2 (rainfall fields (b)), 3 (rainfall fields
382 (c)) and 4 (rainfall fields (d)).



383



384



385

386

Figure 5: Application of DTM method (demonstrated here with data of the 12-

387

13 September 2015 event): Evaluation of empirical $K(q, \eta)$ with scaling break at 2 km,

388 providing the linear regression coefficients R21 (small scales) and R22 (large scales)
 389 (top); Evaluation of α at small scales (centre); Evaluation of α at large scales (bottom).

390

391

Events	Small Scales		Large Scales	
	α	C_1	α	C_1
12-13/09/2015	1.776	0.047	1.76	0.153
16/09/2015	1.676	0.027	0.922	0.094
05-06/10/2015	2.029	0.05	1.775	0.189
21-23/05/2016	1.324	0.041	1.242	0.161
28-31/05/2016	1.656	0.045	1.665	0.199
17/06/2016	1.305	0.127	1.058	0.574

392

Table 1: Estimated UM parameters for rainfall fields in condition (a).

393

Events	Small Scales		Large Scales	
	α	C_1	α	C_1
12-13/09/2015	-0.009	1.757	0.746	0.659
16/09/2015	0.023	1.769	0.244	0.539
05-06/10/2015	0.07	1.788	0.636	0.623
21-23/05/2016	0.015	1.778	0.295	0.571
28-31/05/2016	-0.003	1.761	0.613	0.64
17/06/2016	-0.009	1.753	0.487	1.016

394

Table 2: Estimated UM parameters for rainfall fields in condition (b).

395

Events	Small Scales		Large Scales	
	α	C_1	α	C_1
12-13/09/2015	0.642	0.087	1.397	0.131

16/09/2015	0.247	0.075	0.602	0.092
05-06/10/2015	0.684	0.09	1.421	0.152
21-23/05/2016	0.309	0.083	0.844	0.127
28-31/05/2016	0.493	0.084	1.31	0.149
17/06/2016	0.661	0.123	0.969	0.341

396

Table 3: Estimated UM parameters for rainfall fields in condition (c).

397

Events	Small Scales		Large Scales	
	α	C_1	α	C_1
12-13/09/2015	-0.014	1.751	0.66	0.6
16/09/2015	0.011	1.762	0.153	0.514
05-06/10/2015	0.039	1.769	0.515	0.591
21-23/05/2016	-0.003	1.763	0.278	0.545
28-31/05/2016	-0.019	1.75	0.587	0.601
17/06/2016	-0.006	1.749	0.545	0.871

398

Table 4: Estimated UM parameters for rainfall fields in condition (d).

399

400 Due to the fact that the rainfall fields (condition (a)) and the virtual rain
401 gauge's network distribution (condition (e)) are fully independent, the
402 Intersection theorem (see Section 3) implies that the codimension function of
403 their product (condition (b)) will correspond to the following sum of the
404 codimension functions:

$$c_{(b)=(a)\cap(e)}(\gamma) = c_{(a)}(\gamma) + c_{(e)}(\gamma)$$

22)

405

406 And then, by Legendre transform (see Section 3), the scaling moment
407 function of the product (condition (b)) will correspond to the sum of the scaling

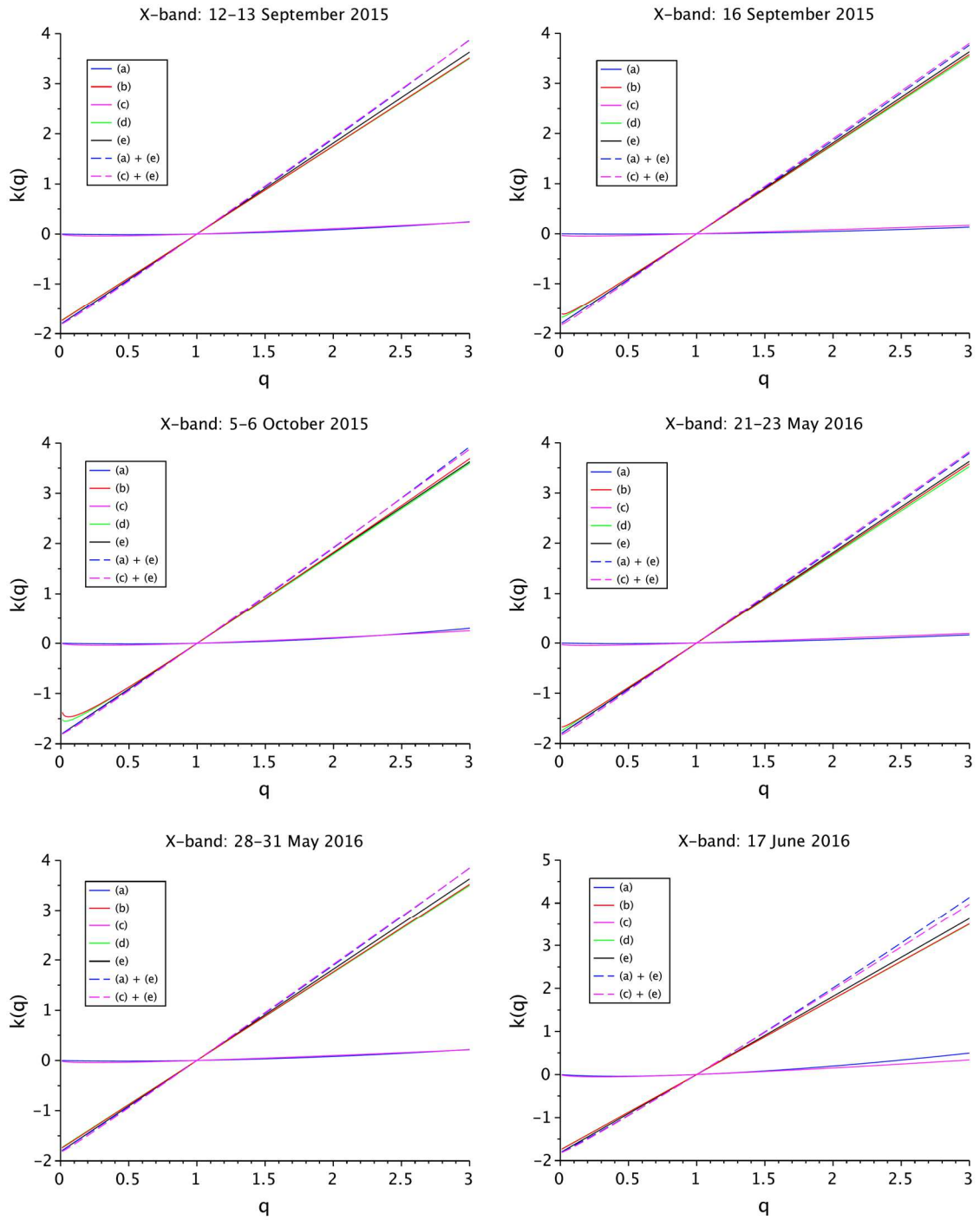
408 moment functions of the rainfall field (condition (a)) and the virtual rain gauge
409 network (condition (e)):

$$K_{(b)=(a)\cap(e)}(q) = K_{(a)}(q) + K_{(e)}(q) \quad 23)$$

410

411 The relationships similar to Eqs. (22) and (23) stand for the rainfall fields
412 (condition (c)), virtual rain gauge network (condition (e)) and their product
413 (condition (d)) as well. Figures 6 and 7 present the theoretical $K(q)$ curves for
414 all fields taking into account the estimated values (from Fig. 4 and Tables 1, 2, 3
415 and 4), for small scales (250 m – 2 km) and large scales (2 km - 8 km)
416 respectively; considering that α was simulated as 0 when the estimated $\alpha < 0$,
417 and as 2 when the estimated $\alpha > 2$.

418

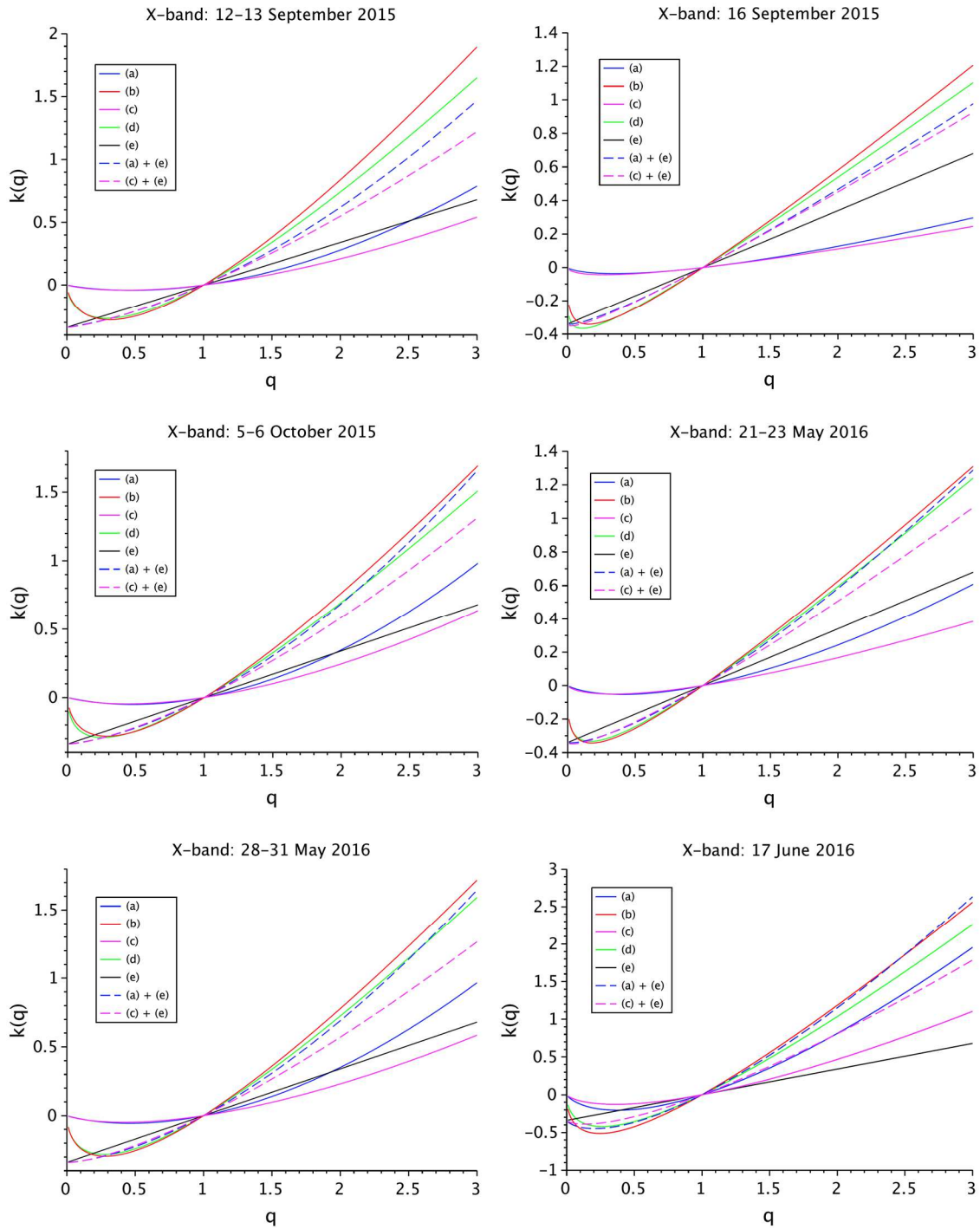


419

420

Figure 6: $K(q)$ relations at small scales for the six studied events.

421



422

423

Figure 7: $K(q)$ relations at large scales for the six studied events.

424

425 From Figure 6 one may note that the monofractality of fields (b) and (d)

426 at small scales (denoted by the estimated α values close to 0 in Tables 2 and 4,

427 respectively) affects the suitability of the Intersection Theorem (see Section 3

428 and Eq. (23)). On the other hand, for the large scales in Figure 7 the theoretical
 429 expression given by Eq. (23) works better for some of the events than for
 430 others: the $K_{(a)}(q) + K_{(e)}(q)$ curves fit better the $K_{(b)}(q)$ ones for 05-
 431 06/10/2015, 21-23/05/2016, 28-31/05/2016 and 17/06/2016 events than those
 432 for 12-13/09/2015 and 16/09/2015 events. This could be easily understood by
 433 evaluating the linear term of the Eq. (23). Indeed, the scaling moment function
 434 of the network will correspond to the β -model, using Eq. (4):

$$K_{(e)}(q) = c(q - 1) \tag{24}$$

435
 436 where over the large scales: $c = 2 - 1.66 = 0.34$, and this is independently
 437 either of rainfall events, or rainfall fields.

438 Then:

$$K_{(b)}(q) = \frac{C_1}{\alpha - 1}(q^\alpha - q) + c(q - 1) = \frac{C_1}{\alpha - 1}q^\alpha - \left(\frac{C_1}{\alpha - 1} - c\right)q - c \tag{25}$$

439
 440 The resulting scaling moment function is no longer UM function, but still
 441 could be well approximated by it, at least between statistical moments of the
 442 orders 1-3, when the pre-factor $\left(\frac{C_1}{\alpha - 1} - c\right)$ of the linear term remains positive.
 443 With $c = 0.34$ and UM parameters from Tables 1 and 3 over large scales one
 444 may obtain the following estimates (Tab. 5) of these pre-factors for conditions
 445 (a) and (c):

446

Events	Rainfall field (a)	Rainfall field (c)

12-13/09/2015	-0.139	-0.01
16/09/2015	-1.545	-0.571
05-06/10/2015	-0.096	0.021
21-23/05/2016	0.325	-1.154
28-31/05/2016	-0.041	0.141
17/06/2016	9.557	-11.34

447 *Table 5: Estimated pre-factors over large scales for rainfall fields in conditions (a) and (c).*

448 Finally, the results from the multifractal analyses presented in this section
 449 indicate a twofold discussion: (i) the events with stronger negative values of the
 450 pre-factor give less empirical agreement with the theoretical expression of Eq.
 451 (23); (ii) and, on the contrary, a larger positive pre-factor leads to a much
 452 stronger convergence between the theoretical and empirical curves.

453

454 **5. Conclusions**

455 In this study we performed (multi)fractal analysis on rain gauge network
 456 and X-band radar rainfall data in the context of (urban) hydrological modelling.
 457 The main objectives of this work are to analyse the impacts of the rainfall data
 458 spatio-temporal variability on hydrological modelling, presenting the limitations
 459 of sparse rain gauge networks compared to high-resolution radar data, and to
 460 discuss the suitability of radar-rain gauge conditioning considering the fractality
 461 of rain gauge networks.

462 This paper takes the the Bièvre catchment, which is a semi-urbanised
 463 area of 110 km² located in the southwest of Paris region, as a case study. In
 464 this work, we used the semi-distributed modelling distribution of this area
 465 performed with the InfoWorks CS and considered a virtual rain gauge in the
 466 centre of mass of each sub-catchment. Then, we compared the rainfall data

467 distributions of the virtual rain gauge network with those obtained from the
468 ENPC dual-polarised X-band radar. Firstly, we performed a fractal analysis of
469 the virtual rain gauge network distribution and identified a scaling break at 2 km.
470 Then we took advantage of the Intersection Theorem associated to the
471 multifractal theory to analyse the relationship between both rain gauge and
472 radar data.

473 The obtained results suggest that the semi-distributed hydrological
474 models statistically reduce the rainfall fields into rainfall measured by a much
475 scarcer network of virtual rain gauges and that inhomogeneous distributions of
476 rain gauging networks lead to only partial information on the rainfall fields. A
477 scaling break is retrieved in the fractal analysis of the virtual rain gauge
478 distribution at the scale of 2 km, which is close to the average sub-catchment's
479 size. The fractality of the rainfall support (here represented by the fixed number
480 of 15 virtual rain gauges over the selected 8 km x 8 km area) is biased by this
481 distribution, where the small-scale behaviour is clearly different of the large-
482 scale one. This implies that, to respect the rainfall support fractality and its
483 capability to consider the rainfall variability over the small scales, the same
484 fractality of the large scales should be found at the small scales. In fact, the
485 statistics of measured rainfall are strongly biased by the fractality of the
486 measuring networks. This fractality needs to be properly taken into account to
487 retrieve the original properties of the rainfall fields, in spite of the radar data
488 calibration.

489 Additionally, a proper rainfall data re-normalisation is needed when
490 comparing gauged rainfall with the radar data, and consequently when
491 quantifying the impacts of space-time variability within hydrological modelling.

492 One may also note that a conditioning by the rain gauges could be rather
493 counterproductive for rainfall events with $\frac{C_1}{\alpha-1}$ being weaker than the
494 codimension of the fractal rain gauge networks.

495 Furthermore, since the Bièvre catchment was calibrated to C-band radar
496 data, in case of the number and distribution of the virtual rain gauges would be
497 reliable in comparison to the C-band radar resolution (1 km²) – actually the
498 SIAVB network has only 6 rain gauges –, the number of rain gauges to perform
499 the calibration of better-resolution radar data (e.g. non polarimetric X-band
500 radar) should be big enough to respect the same fractality of the big scales.
501 This also means that the size of the sub-catchments should be comparable to
502 the resolution of the rainfall data used, which would drastically increase the
503 number of sub-catchments and become unmanageable.

504

505

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522 Writing - review & editing; Ioulia Tchiguirinskaia: Conceptualization, Formal
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524 administration, Resources, Supervision, Writing - review & editing; Daniel
525 Schertzer: Conceptualization, Formal analysis, Funding acquisition,
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528

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