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Track it to crack it:

Dissecting processing stages with finger tracking

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Abstract

A central goal in cognitive science is to parse the series of processing stages underlying a cognitive task. A powerful yet simple behavioral method that can resolve this problem is finger trajectory tracking: by continuously tracking the finger position and speed as a participant chooses a response, and by analyzing which stimulus features affect the trajectory at each time point during the trial, we can estimate the absolute timing and order of each processing stage, and detect transient effects, changes of mind, serial versus parallel processing, and real-time fluctuations in subjective confidence. We suggest that trajectory tracking, which provides considerably more information than mere response times, may provide a comprehensive understanding of the fast temporal dynamics of cognitive operations.

What is trajectory tracking?

A fundamental problem in Psychology involves characterizing the series of cognitive processing stages underlying mental operations such as mental arithmetic, decision-making, etc. In traditional mental chronometry, which dates back to Donders in the late 19th century [1], response times (RT) are interpreted as an index of the underlying cognitive processes. Mental chronometry can examine whether a set of mental operations run serially or in parallel (the additive factors method [2–4]) and can detect processing bottlenecks (the psychological refractory period effect [5,6]). Nevertheless, RT is only a summary measure of all operations that occurred during an experimental trial. Thus, mental chronometry cannot reveal the absolute timing and order of the processing stages, and even its ability to inform about serial versus parallel processing is limited [7].

To overcome these limitations, several methods aim to provide direct information about the time course of processing stages (Box 1). Among these, a powerful behavioral method is trajectory tracking – continuously tracking the fluctuations in the finger or mouse location as a participant makes a decision by pointing. This method has been used to study motor control processes [8–10], and since Spivey's work in 2005 [11] it is increasingly being used also to investigate high-level cognitive processes including decision making [12–20], subjective confidence estimation [13,21], cognitive control [22], executive functions [23], number processing [24–29], arithmetic [30,31], various aspects of language processing [11,32–36],

sequence processing [37], social attitudes [38] and cognition [39,40], dual processing [41], and the processing of subliminal information [36,42].

The methodological aspects of trajectory tracking have already produced several excellent review papers [39,43,52,44–51]. To take this discussion another step forward, here we review important aspects of trajectory tracking that previously received little attention: we show how continuous analysis of the trajectory in each time point can reveal the cognitive operations involved in a given task, their order, their duration, and their timing [10,24,28,29,53–56]; how single-trial analyses can detect within-trial changes of mind; and how movement speed can reveal online fluctuations in subjective confidence [13]. We focus solely on high-level cognitive processes; the motor aspects of finger/mouse trajectories, and how they interact with the decision process, are briefly addressed in Box 2 and were reviewed at length in [43].

The basics of trajectory tracking

We start with a simple example, the monitoring of a serial decision-making task. Evidence accumulation is a well-accepted mechanism for decision making, but the evidence that supports this conclusion is either indirect, based on analysis of response time distributions, or costly, based on neural recordings. Can finger tracking provide a more direct source of evidence for a real-time process of evidence accumulation during decision making? We tested this idea in a decision-making task by presenting stimuli serially in discrete steps [13]: on each trial, participants saw arrows appearing one after another, each pointing left or right, and had to move their finger to the left or right response button according to the majority of arrows (Fig. 1a). If participants process each arrow as it appears and use the accumulated information to update their finger movement, this should result in the trajectory deviating leftwards or rightwards whenever the arrow changes its direction, even before the end of the trial. Single-trial trajectories (Fig. 1b) suggested that this was indeed the case: in trials such as $\rightarrow\leftarrow\rightarrow$ (green), which include arrow direction changes, trajectories seem to fluctuate more than in trials without arrow direction changes (e.g., $\rightarrow\rightarrow\rightarrow$, orange). This pattern becomes even clearer when inspecting the full dataset (average trajectories, Fig. 1c): the tree-like branching pattern in this figure clearly indicates that each arrow was processed soon after it appeared and quickly started affecting the

trajectories. This finding is confirmed by computing the first time point when each particular arrow started having a significant effect on the trajectory (circles in Fig. 1c).

Variants of the experimental design

As illustrated by this example, in a typical trajectory tracking task, the participant moves the finger or mouse from a fixed start point to the response location. The response locations can be discrete buttons [13,14,28,29] (Fig. 1a), a continuous line [24,31] (Fig. 1e), or other spatial arrangements [10,21,35]. To obtain continuous trajectory information, the finger should move continuously without ever stopping. This can be achieved by enforcing a minimum-speed limit [10,24]. To obtain such information right from the start of the trial, the finger should start moving before the stimulus appears [10,16,25,57]. Participants respond using a mouse, or by moving their finger in space or on a touchscreen: all response modes can tap mental operations. However, moving the finger is the most natural mode, and a mouse requires additional sensorimotor transformations which increase variance [58], so we recommend using the finger. There are currently at least three software packages for trajectory tracking experimentation: MouseTracker [55] (<http://mousetracker.org>), MouseTrap [50] (<http://pascalkieslich.github.io/mousetrap>), and our own recently-released TrajTracker (<http://trajtracker.com>).

Analyzing the results

Most trajectory-tracking studies used summary measures that provide one value per trial. For example, in tasks with two response buttons, a higher degree of competition between the two responses can be indexed as larger deviation from the ideal straight line towards the correct response button [27,28,30,44,56] (see a recent review in TICS [39] about the use of trajectory tracking to investigate such response competition). However, such summary measures do not make optimal use of the full richness of trajectory information – for instance, they would miss the tree-like pattern in Fig. 1c. To reach more detailed temporal conclusions, trajectories must be analyzed as a continuous series of time points [16,24,25,31,57,59,60]. This can reveal when a particular factor (e.g., an arrow) starts affecting the trajectories (Fig. 1c), and how this effect builds up in time.

Time-by-time regression provides a simple means of analyzing the data. For each time point (e.g., the dashed line in Fig. 1b), the x coordinates are entered as the dependent variable in

a multiple linear regression with the key factors that are thought to affect the processing stages [16,24,60,61]. For instance, in our arrows decision-making study, to assess the effects of the 3 arrows, their directions (coded as +1/-1) are entered as 3 predictors (see the regression equation in Fig. 1b). The data of each participant is regressed separately, and the regression coefficients are averaged across participants for each predictor and time point, and plotted as a function of time, such that each predictor yields one curve (Fig. 1d). The regression curves of the 3 arrows are almost parallel, indicating that the 3 arrows entered into an accumulation of evidence process that started 250-400 ms after the arrow's appearance on screen. The different asymptotes of the 3 regression lines indicate that at the end of the trial, the finger was affected by earlier arrows slightly more than by late arrows.

Such a time-resolved regression method for analyzing time-series data has been used since the 1960's [62,63], but was applied to manual trajectory data much later [16]. A common practice is to normalize the time points into percentage of each trial's duration [16,55] or reach distance [52] before applying time-resolved analyses. However, this may bias the results if the normalization factor correlates with the analyzed variables [64]. Sometimes, such normalization is better avoided – e.g., when the analysis aims to discover what happens at an absolute time during the trial, or when the analysis focuses on the early part of the trajectory [24,25].

An alternative to regressions is to look, at each time point, for a significant difference between the x coordinates of different experimental conditions [10,11]. In our experiment, comparing trials starting with \rightarrow versus trials starting with \leftarrow reveals when the first arrow affects the finger movement, comparing trials starting with $\rightarrow\rightarrow$ versus $\rightarrow\leftarrow$ (or $\leftarrow\rightarrow$ versus $\leftarrow\leftarrow$) reveals the second arrow's effect, and so on (circles in Fig. 1c). More generally, such a time-resolved statistic (t-test or ANOVA) detects time windows during which different experimental conditions invoke different cognitive representations [27,28,30,52,65].

A critical assumption of time-resolved analyses is that changes in cognitive representations are quickly reflected in the pointing movement. The motor program is not launched once cognitive processing is finished, in a strictly serial manner, but keeps being updated in real-time, in parallel to the ongoing cognitive processes, such that partial information gets transmitted to the unfolding motor program in a cascaded manner. This assumption was validated by several studies [10,13,14,22,31,56,66–69]. To investigate mental operations we do not have to commit to a particular motor-control framework (Box 2 and [43]), yet the above assumption is compatible

with recent models that assume that movement selection and execution operate continuously and in parallel to each other [70].

Decomposing a cognitive task

Serial organization of covert processing stages

In the arrows experiment, seriality is imposed by the stimuli. An even more interesting method, rarely used so far, is to use trajectory tracking to reveal the serial organization of *covert* processing stages. For example, in a study that examined the processing stages in mental arithmetic [31], participants saw single-digit addition or subtraction problems (one per trial) and pointed to the estimated result location on a 0-10 number line. Average trajectories of the subtractions 9-1, 9-2, 9-3, ... 9-8 (Fig. 1e) suggested serial processing of the two operands: the finger first pointed towards the larger operand and then deviated towards the result. This serial effect was confirmed using the time-resolved regression method. Here, the dependent variable was not the x coordinates but the *implied endpoint* – the end-of-trial x coordinate that the finger would reach if it keeps its current direction. This measure improves the temporal precision of the analysis because with x coordinates, any factor that modifies the finger direction can be revealed only after the finger has traveled some distance in the new direction [24]. The predictors were the larger operand, the smaller operand (in negative value for subtractions), and the operator (coded as +1 or -1). The finger was first influenced by the larger operand (regardless of whether it appeared on the left or right of an addition problem), and only ~150 ms later by the smaller operand, suggesting that the two operands were processed serially (Fig. 1f).

Transient effects

Time-resolved analysis can also detect processing stages whose effect does not persist until the end of the trial. For example, Fig. 1f shows that on top of the two operands, the finger position was additionally affected by the operator during an intermediate time window: the finger slightly deviated rightwards for addition problems and leftwards for subtractions. This operator-driven bias, known as *operational momentum* [71,72], completely disappeared by the end of the trial. If the same task had been run without trajectory tracking, or analyzed using a trial-level summary measure of the trajectory, this effect could have been missed.

Single-subject sensitivity

Trajectory tracking is powerful enough to detect effects within individual participants. For example, in the calculation experiment (Fig. 1e), each of the 30 subjects showed serial processing of the two operands (larger operand followed by smaller operand). This single-subject sensitivity, rarely exploited so far, may have not only theoretical but also clinical importance: trajectory tracking may be turned into a diagnostic tool to assess certain cognitive disorders. For instance, we ran a single aphasic patient in a task that required pointing to the location of two-digit numbers on a number line. This patient showed serial effects of the decade and unit digits which differed strikingly from a control group, suggesting a deficit in that patient's ability to process the two digits in parallel [26].

Model-free analyses

The regression approach examines trajectories according to a hypothesized model. However, trajectories provide enough data to allow also for model-free analyses. For example, principal components analysis (PCA) [47] reveals, without a-priori assumptions, the orthogonal factors that affect the finger/mouse movement. Plotting each factor loading at each time point can reveal how its effect builds up in time – on average (Fig. 7 in [47]) and even on single trials. The cognitive meaning of each factor can be interpreted by analyzing its temporal pattern and by finding which experimental conditions yield trajectories with high loads on this factor.

Measuring the delay between cognitive processes

When two processing stages unfold serially, we can use the time-resolved regression method to measure the delay between them. In our arithmetic example (Fig. 1f), the delay between the processing of the two operands is the horizontal distance between their regression curves. Mathematically, this delay can be computed in several ways: by finding the optimal horizontal shift of one regression curve that would minimize the overall area between the two regression curves, by fitting each curve to a predefined function with the temporal onset as a free parameter [37], or by comparing the time when each curve reaches a threshold value (e.g. half of its maximum). For Fig. 1f, the latter method estimated that the second operand of additions was processed 107 ms after the first, while for subtractions the delay was significantly larger, 207 ms.

Measuring subjective confidence in real time

Confidence is defined as our degree of belief that a certain thought or action is correct. Faster decisions are generally associated with higher subjective confidence [73]. Remarkably, trajectory tracking provides information not only about the ongoing decision (reflected in the finger/mouse *direction* relative to the various response options), but also about the instantaneous buildup of subjective confidence (reflected in the finger or mouse *speed* towards the chosen option) [13]. For instance, in the arrows task (Fig. 1c), starting from the second arrow, we observed that each new arrow affected the instantaneous speed (Fig. 2a), and crucially, final speed correlated with the subjective confidence reports at the end of each trial. Furthermore, even in the course of a trial, the specific factors that affected speed were the same factors that affected the self-reported confidence: a larger amount of instantaneous evidence, which increased confidence, also increased the instantaneous speed; and reversals in the arrow direction (relative to the previous arrow), which reduced confidence, also decreased the instantaneous speed (see the corresponding time-resolved regressions in Fig. 2b).

The use of trajectory tracking to measure confidence offers three major advantages. First, it can measure subjective confidence in real time. As far as we know, no other behavioral method is capable of doing so. Second, the method can measure how subjective confidence changes even before the participant commits to a particular response location, i.e., it measures the *pre-decision* confidence. The idea is that even before the decision, participants slow down when they feel momentarily unconfident, in order to accumulate more evidence, and tracking the manual movement can detect this slowdown. Third, unlike other measures of confidence [21,74,75], which typically involve an explicit post-decision report, the finger/mouse speed is an implicit measure. It can be measured in virtually any pointing task without any training or explicit instructions. This makes the method potentially useful in several scenarios, e.g., when experimenting with animals or with young children. Trajectory tracking is indeed applicable to children (see a review in [46]), in experiments with two response buttons [76–78] and even in experiments with multiple target locations [79,80].

Trajectory tracking can simultaneously index decision and confidence, and it can be exploited in additional ways to simultaneously record multiple measures. For example, one study [21] used four response buttons, organized as a square, with the middle of the square as the

trial's starting point. The participants were asked to report their decision by moving left or right, and simultaneously report their confidence in that decision by moving up or down. Using similar designs, participants can respond simultaneously to any two questions: horizontal and vertical movement would provide continuous indices for the two responses, and the movement speed could still be used as a third index, reflecting confidence.

Detecting changes of mind, changes of confidence, and other change points in single trials

So far, we have described time-resolved analyses that pool over many trials. These can reveal what happens *on average*, but they cannot reliably show that two effects co-exist in the same trial [13,49]. Trajectory tracking, however, is sensitive enough to provide information about cognitive changes that occur *within single trials*. Changes of mind, i.e., moments in which the planned response decision was changed, can be captured as changes in the movement direction [56,69], and are sometimes visible even in single trials (Fig. 3a). Statistically, several techniques can detect such changes – e.g., finding points with high horizontal acceleration, or points wherein the trajectory switches between clockwise and counter-clockwise movements [13]. We can focus on specific changes of mind (e.g. the first in a trial [25]), or count their total number per trial. For example, in the arrows task, more arrow reversals per trial yielded more changes of mind (indexed as clockwise-counterclockwise switches, Fig. 3b, red curve). In another study [81], participants pointed left or right to indicate whether the stimulus was a black or white face. Changes of mind showed that participants with low familiarity with mixed-color individuals showed more changes of mind (more/stronger left-right deviations per trial) in mixed-color faces than in single-color faces. The degree of change of mind can even be estimated continuously as the trajectory curvature [13].

We can similarly detect within-trial changes in confidence – e.g., by finding points with high positive or negative acceleration in the *vertical* axis, in which speed is affected by confidence but not by the left-right decision [13] (Fig. 3b, blue curve).

At the motor level, to account for within-trial changes of mind, motor control theories can assume that a trial consists of several movement plans [14,82].

The term “change of mind” supposes that a trial involves a series of interim decisions, each of which may differ from the previous one. Correspondingly, the methods presented above attempt to dissect each trial into a series of mutually exclusive sections, each reflecting a single movement plan. An alternative assumption is that a trial involves a series of temporally *overlapping* processes [83,84]. To detect them, the trajectory can be dissected into a series of overlapping sub-movements. The technique is simple: it assumes that the velocity profile of all sub-movements has the same bell-like shape, which can be mathematically modeled as a function with three free parameters (start time, duration, and amplitude). For each trial, the number of sub-movements and their parameters are fit to the trajectory’s velocity profile [14]. This method can yield very good fits with the actual trajectories (Fig. 3c). Within this model, each sub-movement may reflect a processing stage or an interim decision. The sub-movement’s start time and duration reflect the timing of the corresponding processing stage, and the relative amplitudes of different sub-movements inform about the relative magnitudes of the underlying cognitive representations.

Changes of mind may also be informative when examined at the whole-trial level (summary measures). For instance, Moher and Song [85] classified trials into "partial errors" (when the finger deviated towards the incorrect response prior to selecting the correct one) versus "direct movements", and showed that this classification predicted the dynamics of the subsequent trial. Other model-free methods can cluster trajectories based on their shape without a-priori assumptions [49].

Avoiding the pitfalls

Trajectories offer a powerful source of information, but to analyze them properly, one should avoid several potential pitfalls. We hereby describe what we see as the main difficulties.

Time versus space confound

Trajectory tracking paradigms require the participant to deviate the finger or mouse towards a target location, and they record how this movement progresses in time. Under this setting, stronger deviations and earlier deviations may sometimes produce identical trajectories. Similarly, it is sometimes hard to tell whether factor A affects the cognitive processing *more* than

factor B or *before* factor B, because the two alternatives are indistinguishable in several analysis methods.

In such cases, one should use analysis methods that can control for the time-space confound. For example, in [24] participants saw numbers and pointed to the corresponding positions on a number line (similar to Fig. 1e). Time-resolved regressions showed an effect of the target number, but also a transient effect of its logarithm (Fig. 4b). This log effect initially led us to incorrectly conclude that participants transiently activated a logarithmic representation of quantity [24]. In fact, the log effect was an artifact of averaging trials with different *temporal* characteristics [25]: the participants processed small numbers faster than large numbers (presumably due to different durations of number identification and comprehension processes), so the finger deviated sideways earlier on trials with smaller numbers (Fig. 4a), creating an artificial log effect in the regressions. To control for this artifact, we realigned each trajectory relative to the trial's initial processing duration, indexed as the first time when the trajectory showed a significant sideways deviation. With this new definition of time points, the regressions no longer showed an effect of $\log(\text{target})$ (Fig. 4c) – i.e., the apparent logarithmic effect could be completely explained by inter-trial differences in the onset of lateral finger movements. Similarly, we can align trajectories (and any time-resolved analysis) on the trial endpoint [13] or on any other measurable within-trial event.

Averaging across trials

Many of the analysis methods described above, including time-resolved regression, pool over large sets of trials. Such analyses provide information about what happens on average, but they may hide inter-trial variability or distinct single-trial events. These aggregate-level approaches may, for example, suggest simultaneous co-activation of cognitive representations that actually originate in different subsets of trials [49]. To overcome these limitations and draw reliable conclusions about within-trial processes, average-based analyses should be complemented with single-trial analyses (e.g., trial clustering [49]) and other methods described above, e.g., aligning trials.

Speed versus deviation confound

In decision tasks with trajectory tracking, the finger/mouse deviation reflects the buildup of the decision, whereas its instantaneous speed reflects subjective confidence. However, speed and

deviation may be confounded. For example, higher speed may cause larger deviation (distance) from the middle of the screen. Alternatively, sharp sideways deviations may cause the finger to slow down for purely motor reasons. Analyses that aim to distinguish between decision and confidence should control for the potential relations between speed and deviation – e.g., by adding the momentary curvature as a covariate (see additional methods in [13]).

Motor and geometric confounds

The motor response in trajectory tracking experiments is more complex than in several classical paradigms, e.g., responding by clicking a button [86,87]. This complexity introduces potential motor biases. For example, pointing towards the left side of the screen or towards the right side involves the activation of different muscles, and this creates asymmetry between left and right responses. This problem can be addressed in several ways – e.g., by swapping the response sides on half of the trials, or by recruiting both right-handed and left-handed participants [25]. Another type of artifact arises when the response location is continuous (e.g., the point-to-number-line task, Fig. 1e). In such tasks, responses close to the middle of the screen are quite different from responses close to the end of the screen: they require different motor plans and they produce trajectories with different geometrical properties (e.g., mid-screen trajectories would have lower curvatures). At least in some cases, e.g. perceptual decision making, the cost of an action may even bias the decision itself [88]. To address such artifacts, the statistical analyses in continuous-response paradigms should control for the response location – e.g., by adding the distance from the middle of the screen as covariate [25].

Concluding Remarks

Mental chronometry has been the dominant behavioral method to investigate the dynamics of cognitive operations since Donders in the late 19th century. However, RTs are only a summary measure of the entire processing chain, blind to the succession of the processing stages. Here we presented an emerging framework using trajectory tracking that is powerful enough to resolve this temporal dissection problem, revealing the order and absolute time of each processing stage, arbitrating between parallel vs. serial architectures and indexing subjective online decision confidence. Additionally, trajectory tracking has a practical advantage among other time-resolved behavioral methods (e.g. eye tracking), since it is very accessible, cheap and

scalable. The paradigm may potentially have even more advantages and uses (see Outstanding Questions), which would need to be first confirmed more thoroughly using a multi-methodological approach; these may provide new exciting directions to better understand the relationship between behavior and brain activity.

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References

- 1 Donders, F.C. (1969) On the speed of mental processes. *Acta Psychol. (Amst)*. 30, 412–431
- 2 Dehaene, S. (1996) The organization of brain activations in number comparison: Event-related potentials and the additive-factors method. *J. Cogn. Neurosci.* 8, 47–68
- 3 Sternberg, S. (1969) The discovery of processing stages: Extensions of Donders’ method. *Acta Psychol. (Amst)*. 30, 276–315
- 4 Sternberg, S. (2013) The meaning of additive reaction-time effects: some misconceptions. *Front. Psychol.* 4,
- 5 Pashler, H. (1984) Processing stages in overlapping tasks: Evidence for a central bottleneck. *J. Exp. Psychol. Hum. Percept. Perform.* 10, 358–377
- 6 Sigman, M. and Dehaene, S. (2005) Parsing a cognitive task: a characterization of the mind’s bottleneck. *PLoS Biol.* 3, e37
- 7 Townsend, J.T. (1990) Serial vs. parallel processing: Sometimes they look like Tweedledum and Tweedledee but they can (and should) be distinguished. *Psychol. Sci.* 1, 46–54
- 8 Ghez, C. *et al.* (1997) Discrete and continuous planning of hand movements and isometric force trajectories. *Exp. Brain Res.* 115, 217–233
- 9 Goodale, M.A. *et al.* (1986) Large adjustments in visually guided reaching do not depend on vision of the hand or perception of target displacement. *Nature* 320, 748–750
- 10 Chapman, C.S. *et al.* (2010) Reaching for the unknown: Multiple target encoding and real-

- time decision-making in a rapid reach task. *Cognition* 116, 168–176
- 11 Spivey, M.J. *et al.* (2005) Continuous attraction toward phonological competitors. *Proc. Natl. Acad. Sci. U. S. A.* 102, 10393 LP – 10398
 - 12 Buc Calderon, C. *et al.* (2017) Continuous track paths reveal additive evidence integration in multistep decision making. *Proc. Natl. Acad. Sci.* 114, 10618–10623
 - 13 Dotan, D. *et al.* (2018) On-line confidence monitoring during decision making. *Cognition* 171, 112–121
 - 14 Friedman, J. *et al.* (2013) Linking cognitive and reaching trajectories via intermittent movement control. *J. Math. Psychol.* 57, 140–151
 - 15 Lepora, N.F. and Pezzulo, G. (2015) Embodied choice: How action influences perceptual decision making. *PLOS Comput. Biol.* 11, e1004110
 - 16 Scherbaum, S. *et al.* (2010) How decisions evolve: The temporal dynamics of action selection. *Cognition* 115, 407–416
 - 17 Zgonnikov, A. *et al.* (2017) Decision landscapes: Visualizing mouse-tracking data. *R. Soc. Open Sci.* 4, 170482
 - 18 McKinstry, C. *et al.* (2008) Action dynamics reveal parallel competition in decision making. *Psychol. Sci.* 19, 22–24
 - 19 Spivey, M.J. and Dale, R. (2006) Continuous dynamics in real-time cognition. *Curr. Dir. Psychol. Sci.* 15, 207–211
 - 20 Kording, K.P. and Wolpert, D.M. (2004) Bayesian integration in sensorimotor learning. *Nature* 427, 244–247
 - 21 van den Berg, R. *et al.* (2016) A common mechanism underlies changes of mind about decisions and confidence. *Elife* 5, e12192
 - 22 Erb, C.D. *et al.* (2016) Reach tracking reveals dissociable processes underlying cognitive control. *Cognition* 152, 114–126
 - 23 Weaver, S.M. and Arrington, C.M. (2013) Tracking the multitasking mind. *Z. Psychol.* 221, 51–60
 - 24 Dotan, D. and Dehaene, S. (2013) How do we convert a number into a finger trajectory? *Cognition* 129, 512–529
 - 25 Dotan, D. and Dehaene, S. (2016) On the origins of logarithmic number-to-position mapping. *Psychol. Rev.* 123, 637–666

- 26 Dotan, D. *et al.* (2014) Breaking down number syntax: Spared comprehension of multi-
digit numbers in a patient with impaired digit-to-word conversion. *Cortex* 59, 62–73
- 27 Faulkenberry, T.J. *et al.* (2016) Response trajectories capture the continuous dynamics of
the size congruity effect. *Acta Psychol. (Amst)*. 163, 114–123
- 28 Santens, S. *et al.* (2011) Distance in motion: response trajectories reveal the dynamics of
number comparison. *PLoS One* 6, Article ID e25429
- 29 Song, J.H. and Nakayama, K. (2008) Numeric comparison in a visually-guided manual
reaching task. *Cognition* 106, 994–1003
- 30 Marghetis, T. *et al.* (2014) Doing arithmetic by hand: Hand movements during exact
arithmetic reveal systematic, dynamic spatial processing. *Q. J. Exp. Psychol.* 67, 1579–
1596
- 31 Pinheiro-Chagas, P. *et al.* (2017) Finger tracking reveals the covert stages of mental
arithmetic. *Open Mind* 1, 30–41
- 32 Dale, R. and Duran, N.D. (2011) The cognitive dynamics of negated sentence verification.
Cogn. Sci. 35, 983–996
- 33 Tomlinson, J.M. *et al.* (2013) Possibly all of that and then some: Scalar implicatures are
understood in two steps. *J. Mem. Lang.* 69, 18–35
- 34 Anderson, S. *et al.* Grammatical aspect and temporal distance in motion descriptions. ,
Frontiers in Psychology, 4. (2013) , 337
- 35 Farmer, T.A. *et al.* (2007) Tracking the continuity of language comprehension: Computer
mouse trajectories suggest parallel syntactic processing. *Cogn. Sci.* 31, 889–909
- 36 Xiao, K. and Yamauchi, T. (2017) The role of attention in subliminal semantic processing:
A mouse tracking study. *PLoS One* 12, e0178740
- 37 Al Roumi, F. *et al.* (2019) Acquisition and processing of an artificial mini-language
combining semantic and syntactic elements. *Cognition* 185, 49–61
- 38 Wojnowicz, M.T. *et al.* (2009) The self-organization of explicit attitudes. *Psychol. Sci.* 20,
1428–1435
- 39 Stillman, P.E. *et al.* (2018) How mouse-tracking can advance social cognitive theory.
Trends Cogn. Sci. 22, 531–543
- 40 Freeman, J.B. and Ambady, N. (2009) Motions of the hand expose the partial and parallel
activation of stereotypes. *Psychol. Sci.* 20, 1183–1188

- 41 Freeman, J.B. and Dale, R. (2013) Assessing bimodality to detect the presence of a dual
cognitive process. *Behav. Res. Methods* 45, 83–97
- 42 Cressman, E.K. *et al.* (2007) On-line control of pointing is modified by unseen visual
shapes. *Conscious. Cogn.* DOI: 10.1016/j.concog.2006.06.003
- 43 Gallivan, J.P. *et al.* (2018) Decision-making in sensorimotor control. *Nat. Rev. Neurosci.*
19, 519–534
- 44 Song, J.H. and Nakayama, K. (2009) Hidden cognitive states revealed in choice reaching
tasks. *Trends Cogn. Sci.* 13, 360–366
- 45 Fischer, M.H. and Hartmann, M. Pushing forward in embodied cognition: may we mouse
the mathematical mind? , *Frontiers in Psychology*, 5. (2014) , 1315
- 46 Erb, C.D. (2018) The developing mind in action: measuring manual dynamics in
childhood. *J. Cogn. Dev.* 19, 233–247
- 47 Hehman, E. *et al.* (2015) Advanced mouse-tracking analytic techniques for enhancing
psychological science. *Gr. Process. Intergr. Relations* 18, 384–401
- 48 Freeman, J.B. (2018) Doing psychological science by hand. *Curr. Dir. Psychol. Sci.* 27,
315–323
- 49 Wulff, D.U. *et al.* (2019) Mouse-tracking: Detecting types in movement trajectories. In *A
Handbook of Process Tracing Methods: 2nd Edition* (Schulte-Mecklenbeck, M. *et al.*,
eds), Psychology Press
- 50 Kieslich, P.J. *et al.* (2018) Mouse-tracking: A practical guide to implementation and
analysis. *PsyArXiv Prepr.* DOI: 10.31234/osf.io/zuvqa
- 51 Spivey, M.J. *et al.* (2009) The movement of eye and hand as a window into language and
cognition. In *Oxford Handbook of Human Action* (Morsella, E. *et al.*, eds), pp. 225–249,
Oxford University Press
- 52 Gallivan, J.P. and Chapman, C.S. Three-dimensional reach trajectories as a probe of real-
time decision-making between multiple competing targets. , *Frontiers in Neuroscience*, 8.
(2014) , 215
- 53 Buc Calderon, C. *et al.* (2015) Losing the boundary: Cognition biases action well after
action selection. *J. Exp. Psychol. Gen.* 144, 737–743
- 54 Finkbeiner, M. *et al.* (2008) Engaging the motor system with masked orthographic primes:
A kinematic analysis. *Vis. cogn.* 16, 11–22

- 55 Freeman, J.B. and Ambady, N. (2010) MouseTracker: Software for studying real-time
mental processing using a computer mouse-tracking method. *Behav. Res. Methods* 42,
226–241
- 56 Resulaj, A. *et al.* (2009) Changes of mind in decision-making. *Nature* 461, 263–266
- 57 Scherbaum, S. and Kieslich, P.J. (2017) Stuck at the starting line: How the starting
procedure influences mouse-tracking data. *Behav. Res. Methods* DOI: 10.3758/s13428-
017-0977-4
- 58 Moher, J. and Song, J.-H. (2019) A comparison of simple movement behaviors across
three different devices. *Attention, Perception, Psychophys.* DOI: 10.3758/s13414-019-
01856-8
- 59 Dshemuchadse, M. *et al.* How decisions emerge: Action dynamics in intertemporal
decision making. , *Journal of Experimental Psychology: General*, 142. (2013) , American
Psychological Association, 93–100
- 60 Scherbaum, S. *et al.* (2015) Action dynamics in multitasking: the impact of additional task
factors on the execution of the prioritized motor movement. *Front. Psychol.* 6, 934
- 61 Sullivan, N. *et al.* (2014) Dietary self-control Is related to the speed with which attributes
of healthfulness and tastiness are processed. *Psychol. Sci.* 26, 122–134
- 62 Almon, S. (1965) The distributed lag between capital appropriations and expenditures.
Econometrica 33, 178–196
- 63 Eisner, R. (1960) A distributed lag investment function. *Econometrica* 28, 1–29
- 64 Whitwell, R.L. and Goodale, M.A. (2013) Grasping without vision: Time normalizing
grip aperture profiles yields spurious grip scaling to target size. *Neuropsychologia* 51,
1878–1887
- 65 Dale, R. *et al.* (2007) Graded motor responses in the time course of categorizing atypical
exemplars. *Mem. Cognit.* 35, 15–28
- 66 Berthier, N.E. (1996) Learning to reach: A mathematical model. *Dev. Psychol.* 32, 811–
823
- 67 Dotan, D. (2017) Parallel and serial processes in number-to-quantity conversion. In *Multi-
digit number processing: cognitive mechanisms and their impairment (PhD dissertation)*
pp. 95–118, Tel Aviv University
- 68 Fitts, P.M. (1954) The information capacity of the human motor system in controlling the

- amplitude of movement. *J. Exp. Psychol.* 47, 381–391
- 69 Alonso-Diaz, S. *et al.* (2018) A threshold-free model of numerosity comparisons. *PLoS One* 13, e0195188
- 70 Cisek, P. (2007) Cortical mechanisms of action selection: the affordance competition hypothesis. *Philos. Trans. R. Soc. B Biol. Sci.* 362, 1585–1599
- 71 McCrink, K. *et al.* (2007) Moving along the number line: operational momentum in nonsymbolic arithmetic. *Percept. Psychophys.* 69, 1324–33
- 72 Knops, A. *et al.* (2009) Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Atten. Percept. Psychophys.* 71, 803–821
- 73 Kiani, R. *et al.* (2014) Choice certainty is informed by both evidence and decision time. *Neuron* 84, 1329–1342
- 74 Kiani, R. and Shadlen, M.N. (2009) Representation of confidence associated with a decision by neurons in the parietal cortex. *Science* (80-.). 324, 759–764
- 75 Meyniel, F. *et al.* (2015) The Sense of Confidence during Probabilistic Learning: A Normative Account. *PLOS Comput. Biol.* 11, e1004305
- 76 Erb, C.D. *et al.* (2018) Numerical cognition in action: Reaching behavior reveals numerical distance effects in 5- to 6-year-olds. *J. Numer. Cogn.* 4, 286–296
- 77 Erb, C.D. *et al.* (2017) Cognitive control in action: Tracking the dynamics of rule switching in 5- to 8-year-olds and adults. *Cognition* 164, 163–173
- 78 Hermens, F. (2018) When do arrows start to compete? A developmental mouse-tracking study. *Acta Psychol. (Amst)*. 182, 177–188
- 79 Feldman, A. *et al.* (2019) Following the finger: The development of the mental number line in elementary school children. *PsyArXiv* DOI: 10.31234/osf.io/qm43b
- 80 Cargill, S.A. *et al.* (2007) , Children’s online processing of complex sentences: New evidence from a new technique. , in *Proceedings of the Annual Meeting of the Cognitive Science Society*, 29(29)
- 81 Freeman, J.B. *et al.* (2016) A perceptual pathway to bias: Interracial exposure reduces abrupt shifts in real-time race perception that predict mixed-race bias. *Psychol. Sci.* 27, 502–517
- 82 Nashed, J.Y. *et al.* (2012) Influence of the behavioral goal and environmental obstacles on

- rapid feedback responses. *J. Neurophysiol.* 108, 999–1009
- 83 Meyer, D.E. *et al.* Optimality in human motor performance: Ideal control of rapid aimed movements. , *Psychological Review*, 95. (1988) , American Psychological Association, 340–370
- 84 McClelland, J.L. (1979) On the time relations of mental processes: An examination of systems of processes in cascade. *Psychol. Rev.* 86, 287–330
- 85 Moher, J. and Song, J.H. (2013) Context-dependent sequential effects of target selection for action. *J. Vis.* 13, 10
- 86 Flash, T. and Hogan, N. (1985) The coordination of arm movements: an experimentally confirmed mathematical model. *J. Neurosci.* 5, 1688–1703
- 87 Maloney, L. *et al.* (2003) Statistical decision theory and trade-offs in the control of motor response. *Spat. Vis.* 16, 255–275
- 88 Hagura, N. *et al.* (2017) Perceptual decisions are biased by the cost to act. *Elife* 6,
- 89 Fischer, B. and Weber, H. (1993) Express saccades and visual attention. *Behav. Brain Sci.* 16, 553–567
- 90 Laeng, B. *et al.* (2012) Pupillometry: A window to the preconscious? *Perspect. Psychol. Sci.* 7, 18–27
- 91 Kirchner, H. and Thorpe, S.J. (2006) Ultra-rapid object detection with saccadic eye movements: Visual processing speed revisited. *Vision Res.* 46, 1762–1776
- 92 Zylberberg, A. *et al.* Pupil Dilation: A Fingerprint of Temporal Selection During the “Attentional Blink.” , *Frontiers in Psychology*, 3. (2012) , 316
- 93 Neggers, S.F.W. and Bekkering, H. (2001) Gaze anchoring to a pointing target is present during the entire pointing movement and is driven by a non-visual signal. *J. Neurophysiol.* 86, 961–970
- 94 Gamble, C.M. and Song, J.H. (2017) Dynamic modulation of illusory and physical target size on separate and coordinated eye and hand movements. *J. Vis.* 17, 23
- 95 Johansson, R.S. *et al.* (2001) Eye–hand coordination in object manipulation. *J. Neurosci.* 21, 6917 LP – 6932
- 96 Prablanc, C. *et al.* (1979) Optimal response of eye and hand motor systems in pointing at a visual target. *Biol. Cybern.* 35, 113–124
- 97 A., G.M. (2014) How (and why) the visual control of action differs from visual

- perception. *Proc. R. Soc. B Biol. Sci.* 281, 20140337
- 98 Land, M. *et al.* (1999) The roles of vision and eye movements in the control of activities of daily living. *Perception* 28, 1311–1328
- 99 Anderson, S.E. *et al.* (2011) On the temporal dynamics of language-mediated vision and vision-mediated language. *Acta Psychol. (Amst)*. 137, 181–189
- 100 Yang, T. and Shadlen, M.N. (2007) Probabilistic reasoning by neurons. *Nature* 447, 1075–1080
- 101 Wolpert, D.M. and Landy, M.S. (2012) Motor control is decision-making. *Curr. Opin. Neurobiol.* 22, 996–1003
- 102 Miller, G.A. *et al.* (1960) *Plans and the structure of behavior.*, Henry Holt and Co.
- 103 Gallivan, J.P. *et al.* (2011) One to four, and nothing more: Nonconscious parallel individuation of objects during action planning. *Psychol. Sci.* 22, 803–811
- 104 Todorov, E. and Jordan, M.I. (2002) Optimal feedback control as a theory of motor coordination. *Nat. Neurosci.* 5, 1226–1235
- 105 Fishbach, A. *et al.* (2007) Deciding when and how to correct a movement: Discrete submovements as a decision making process. *Exp. Brain Res.* 177, 45–63

Box 1. Trajectory tracking versus other time-resolved measures of the dynamics of cognitive processes

Finger tracking is just one of the methods by which cognitive processes can be tracked in real-time – with various advantages and drawbacks:

Eye tracking

Gaze shifts during a trial can track the underlying cognitive operations [89], while pupil dilation can index the degree of cognitive effort [90]. Both eye saccades [91] and pupil dilation [92] quickly reflect cognitive changes, so they can provide a fine-grained index of cognitive processing, with a resolution of about ~100 ms.

Eye and hand movements are usually coordinated [93–95], with eye saccades preceding the movement initiation [96], but they can also dissociate [94,95]. This distinction may occur for several reasons – e.g., because separate visual mechanisms support perception and action [97], or

because moving the hand may involve a higher decision threshold than moving the eye [51]. Indeed, guiding manual movement is merely one goal of eye gaze [98]. Also when examining high-level cognitive processes, eye tracking and finger/mouse tracking may tap either similar or different processes depending on the specific experimental design [94,99], and may be used as complementary methods.

Presently, eye tracking hardware is much more expensive than trajectory tracking. In the future, cheap high-quality eye-tracking technologies (e.g. in smartphones) may increase the popularity of this method.

Brain imaging

Continuous measurement is possible using methods that measure brain activity with high temporal resolution, such as electroencephalography (EEG) and magnetoencephalography (MEG). Some experimental paradigms, which were run both with trajectory tracking [37] and with neuronal recordings [100], have obtained similar results.

EEG and MEG offer important advantages over trajectory tracking, including millisecond-accuracy, direct measurement, the ability to uncover the neural mechanisms underlying cognition, and the ability to examine what happens before, after, and even in the absence of behavioral response. However, trajectory tracking also offers advantages over brain imaging. It can show how a decision process affects behavior. The cognitive meaning of the finger deviating towards a particular response may often be easier to interpret than that of a brain activity pattern. Moreover, compared with trajectory tracking, EEG and MEG are costly, involve a lengthy acquisition procedure, require multidisciplinary teams, and are not easily scalable to large groups of participants. When both methods are appropriate for a particular research question, trajectory tracking offers a cheaper, simpler, and faster alternative to EEG/MEG.

Combining the two methods could potentially offer additional power. For example, one may detect specific events in single trajectories – e.g., changes of mind (direction) or changes in confidence (speed) – and use them to guide the analysis of brain signals.

Box 2. From decision to manual movement

Motor control can be conceptualized as a complex decision-making process [101] that involves several aspects: the **selection** of the movements that may achieve the particular goal, the

shaping and execution of a single movement, the **revision** of a given movement in case it should be adjusted to meet the goal, and the **optimization** of the sequence of required movements to maximize task performance [43].

Traditional theories of action planning assumed serial selection and execution, that is, subjects first select the target, and only then execute the movement [102]. However, more recent models challenge this serial assumption. For example, one study [10] first recorded the participants' finger trajectories as they pointed towards a target point. Then, in the critical trials, several potential target locations were shown when the finger started moving, and the specific target was indicated only later in the trial. The initial finger trajectory was the mean of the trajectories to the different possible targets; later, when the target was indicated, the finger deviated towards it. The researchers concluded that even before selecting a movement plan towards a particular target, the participants could represent several movement potential plans (subject to working memory limitations [103]) and initiate a movement according to their average. This idea that movement can start even before the final decision, is critical for experiments that examine the temporal dynamics of high-level cognitive processes, because this is what allows measuring, via the finger movement, intermediate processing stages [13,14,22,31,67,69].

An interesting property of the sequencing of movements is the fact that reaching a given goal location can be achieved with an infinite number of movement trajectories. This is known as the 'problem of redundancy'. So how does the motor system choose a particular trajectory at a given time? One of the most influential models developed to address this question is the optimal feedback control (OFC) [104]. According to the OFC model, the motor system uses an optimization algorithm with the aim of minimizing the movement's cost, which is normally considered to be related to energy consumption. Accordingly, a central principle of the OFC model is the 'minimum intervention', that is, the revision process of the movement trajectory occurs only when it is necessary in order to meet the goal of the task. A possible extension of this idea is that the finger would deviate (thereby revealing a "change of mind") only when its current direction deviates from the intended goal by a sufficient amount [25,105].

Figure Captions

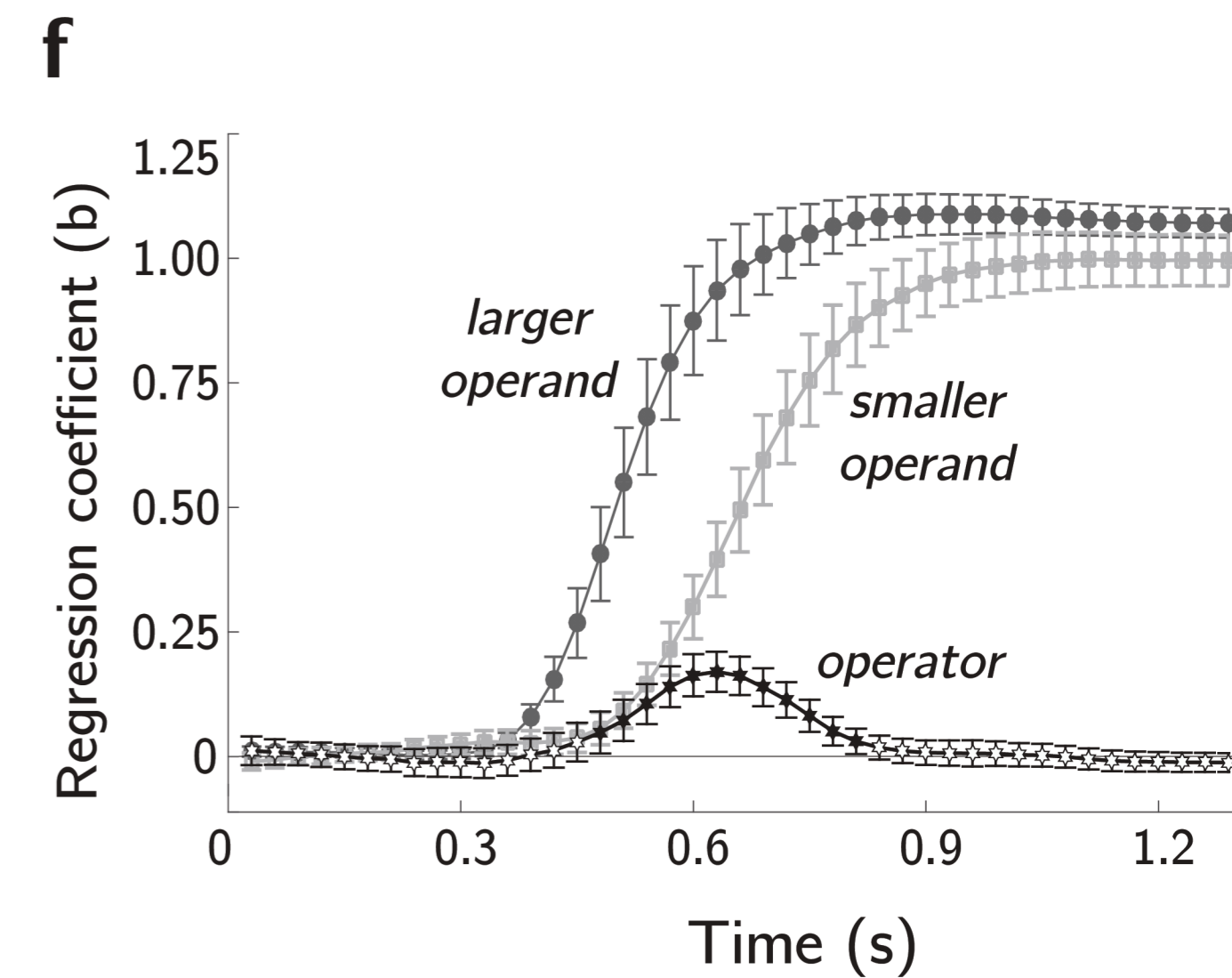
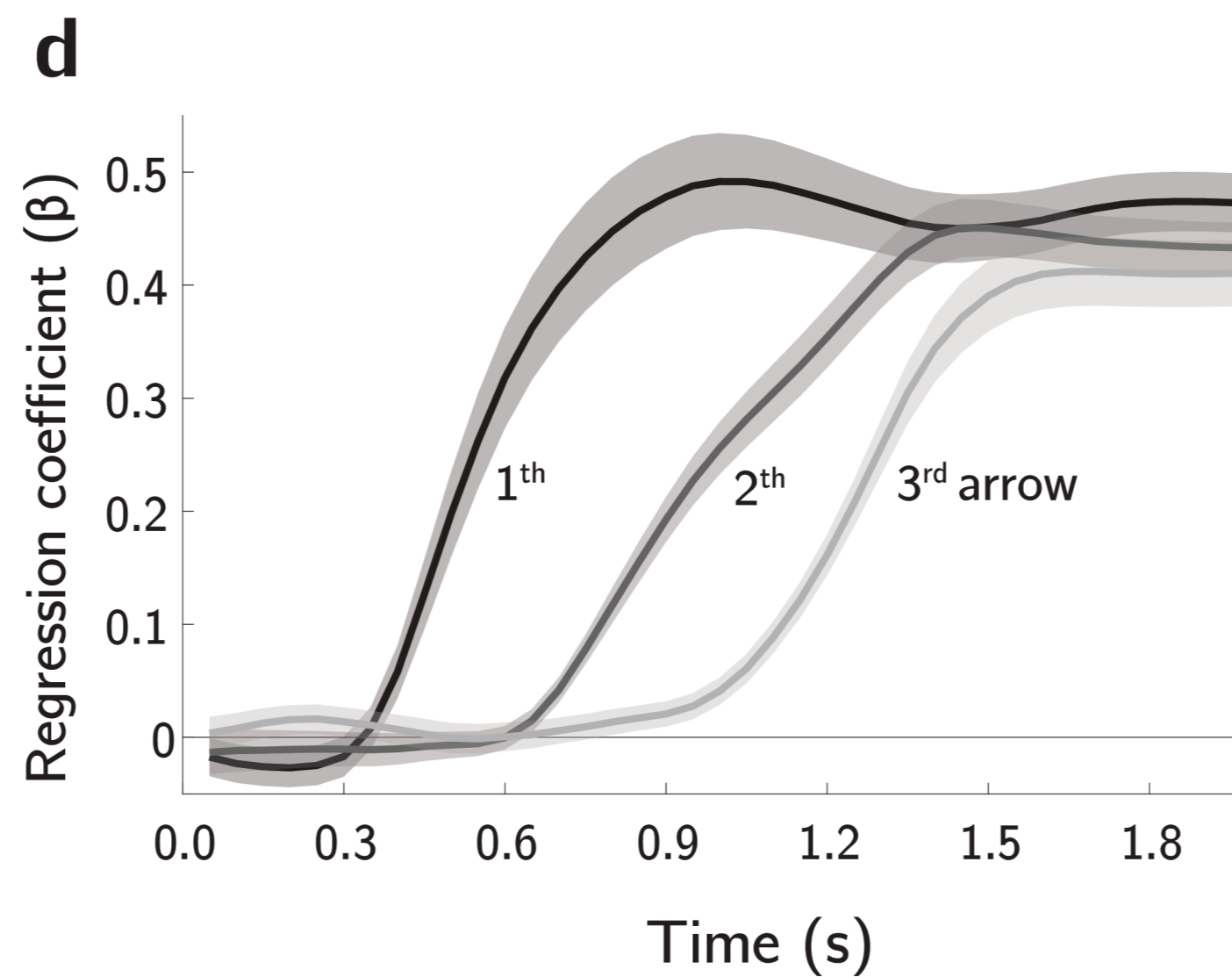
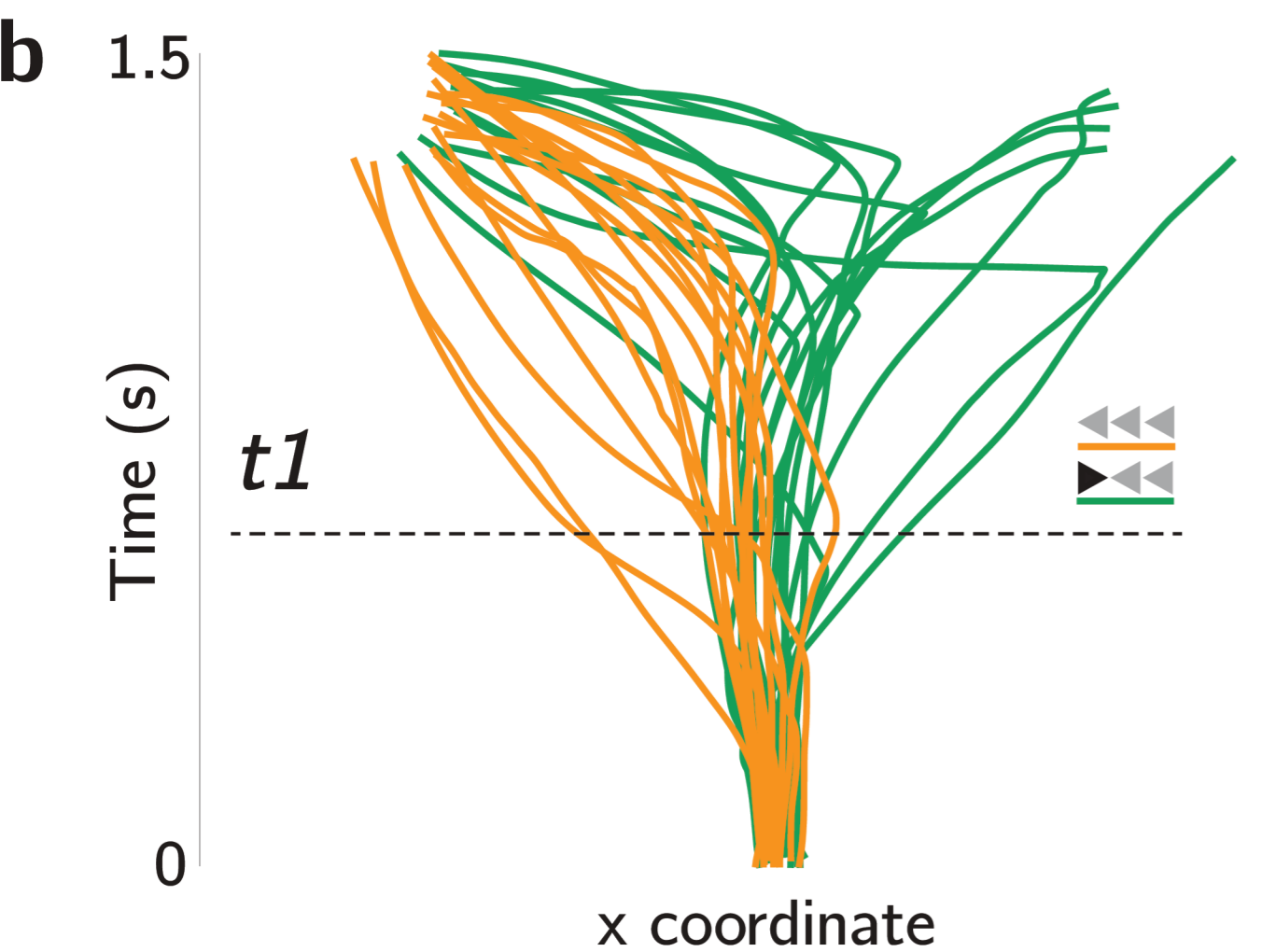
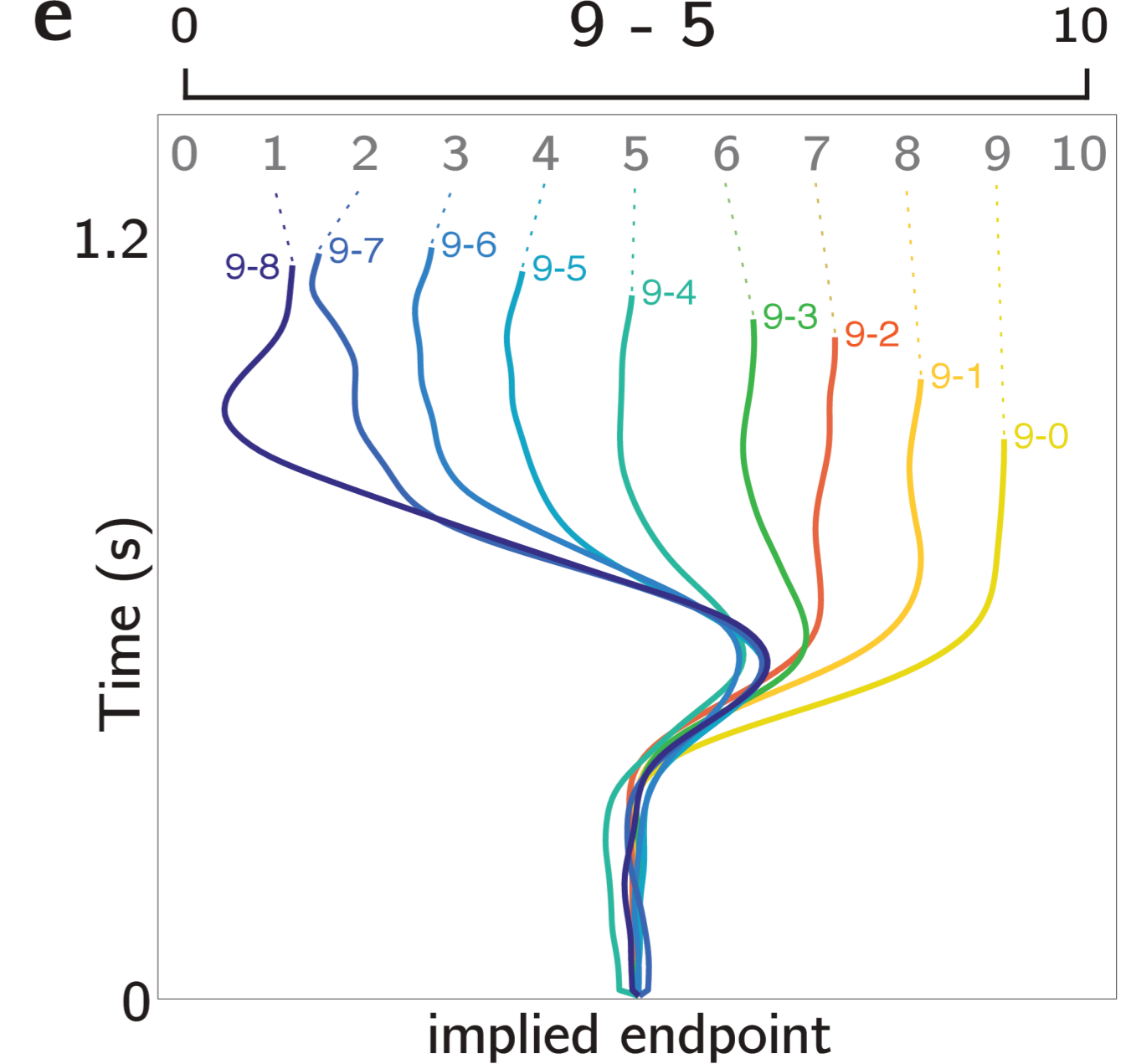
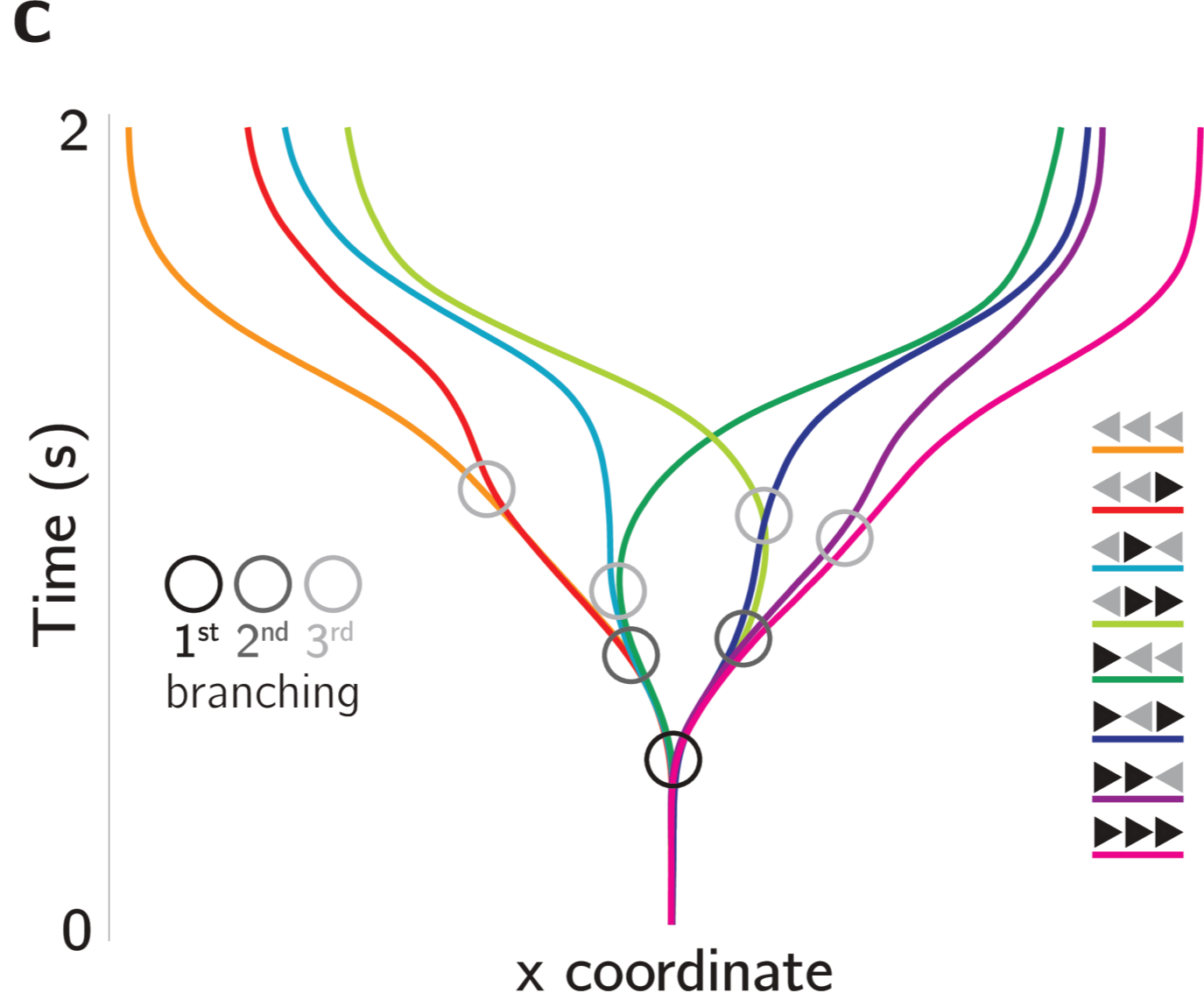
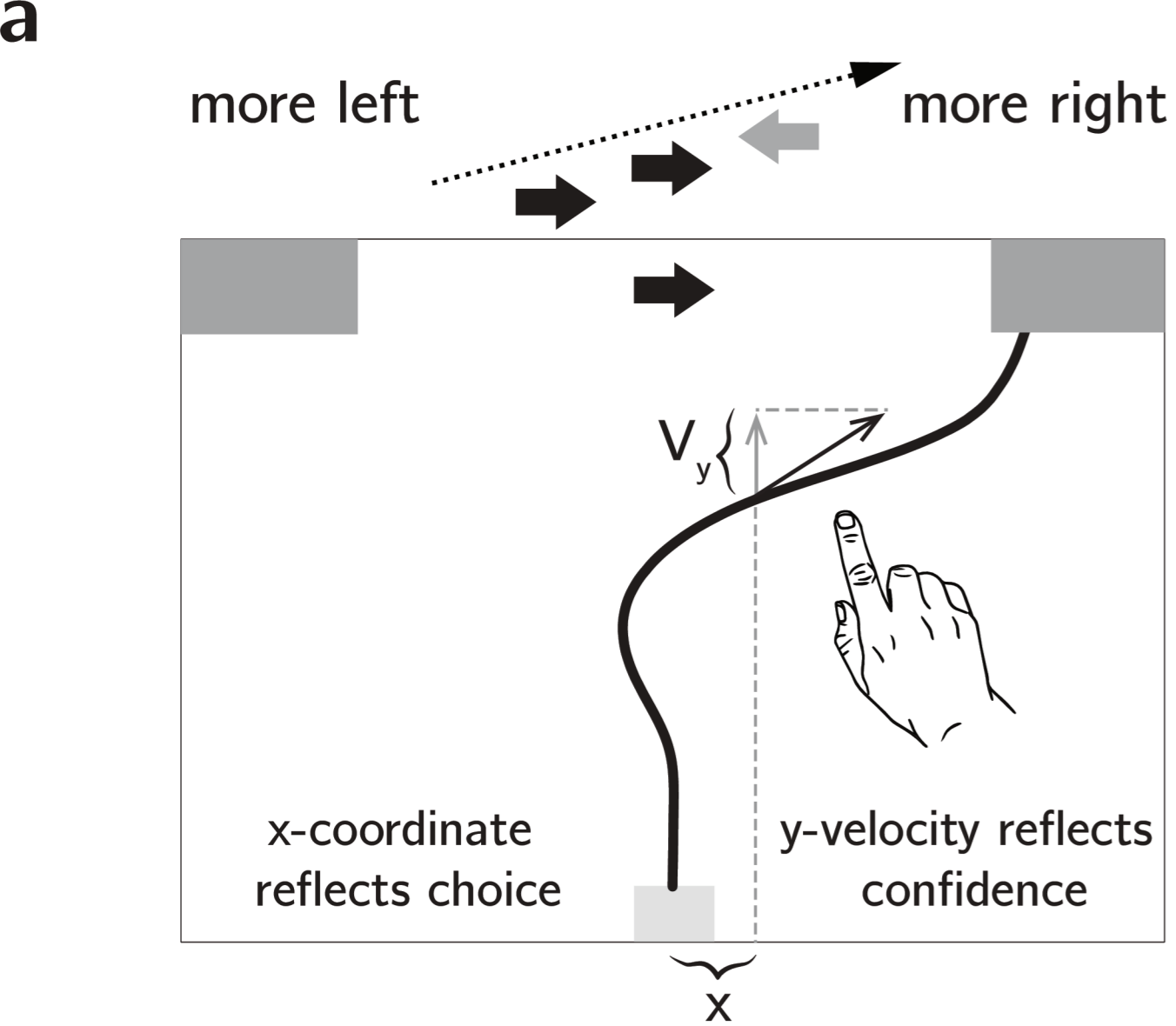
Fig. 1. Finger trajectories can reveal a series of cognitive processes. (a) Arrows decision-making task. On each trial, participants saw sequentially-presented arrows (unknown to the participant, 1, 3, or 5 arrows; SOA = 300 ms), each pointing left or right, and dragged their finger on a touchscreen to a left or right response locations according to the majority of arrows [13]. (b) X coordinates as a function of time for sample trials of one participant. Time-resolved analyses examine which factors affect the finger in each time-point (dashed line). (c) X coordinates as a function of time, averaged over trials and participants for each possible 3-arrow sequence. Circles indicate when the trajectories branched apart according to each new arrow. (d) Time course of the arrows' effects. For each subject and time-point, across trials, x coordinates were regressed against three predictors coding the arrow directions. We plotted the regression weights (β) averaged across subjects, with their standard error. Each line reflects the buildup of a particular arrow's effect. (e) Here, on each trial the participants saw a single-digit addition or subtraction and pointed to the result location on an unmarked number line [31]. Average x coordinates were plotted as a function of time for 9 of the exercises. (f) Time course of the effect of each operand, analyzed with time-resolved regressions (same method as in panel (d)): for each time point, implied endpoints (the location where the finger would land if it keeps its current direction) were regressed against the larger operand, the smaller operand (in negative value for subtractions), and the operator (+1 or -1). The first operand effect builds up before that of the second operand, indicating that they were processed serially.

Fig. 2. Instantaneous finger speed reflects subjective confidence. (a) Average y-velocity on 3-arrow trials of the arrows task (Fig. 1a) [13], plotted as a function of time for each sequence type (pooling over pairs of mirror sequences). Each new arrow affects speed in two ways: first, the finger speeds up following arrows that increase the absolute amount of evidence ($\Delta|\text{Evidence}| > 0$) and slows down following arrows that decreased evidence. Second, the finger slows down when an arrow differs in the direction from the previous arrow. Crucially, the same two factors also affect the explicit post-decision ratings of subjective confidence. This indicates that the y speed is a good index of online subjective confidence. (b) The same two effects ($\Delta|\text{Evidence}|$ and arrow-direction-changes) are revealed in the time-resolved regression analysis. For each participant and each time point, the instantaneous y speed was regressed on the $\Delta|\text{Evidence}|$ provided by the 2nd and 3rd arrows, and on a third predictor coding whether these two arrows

pointed in the same direction or not. The regression coefficients were averaged and plotted like in Fig. 1d.

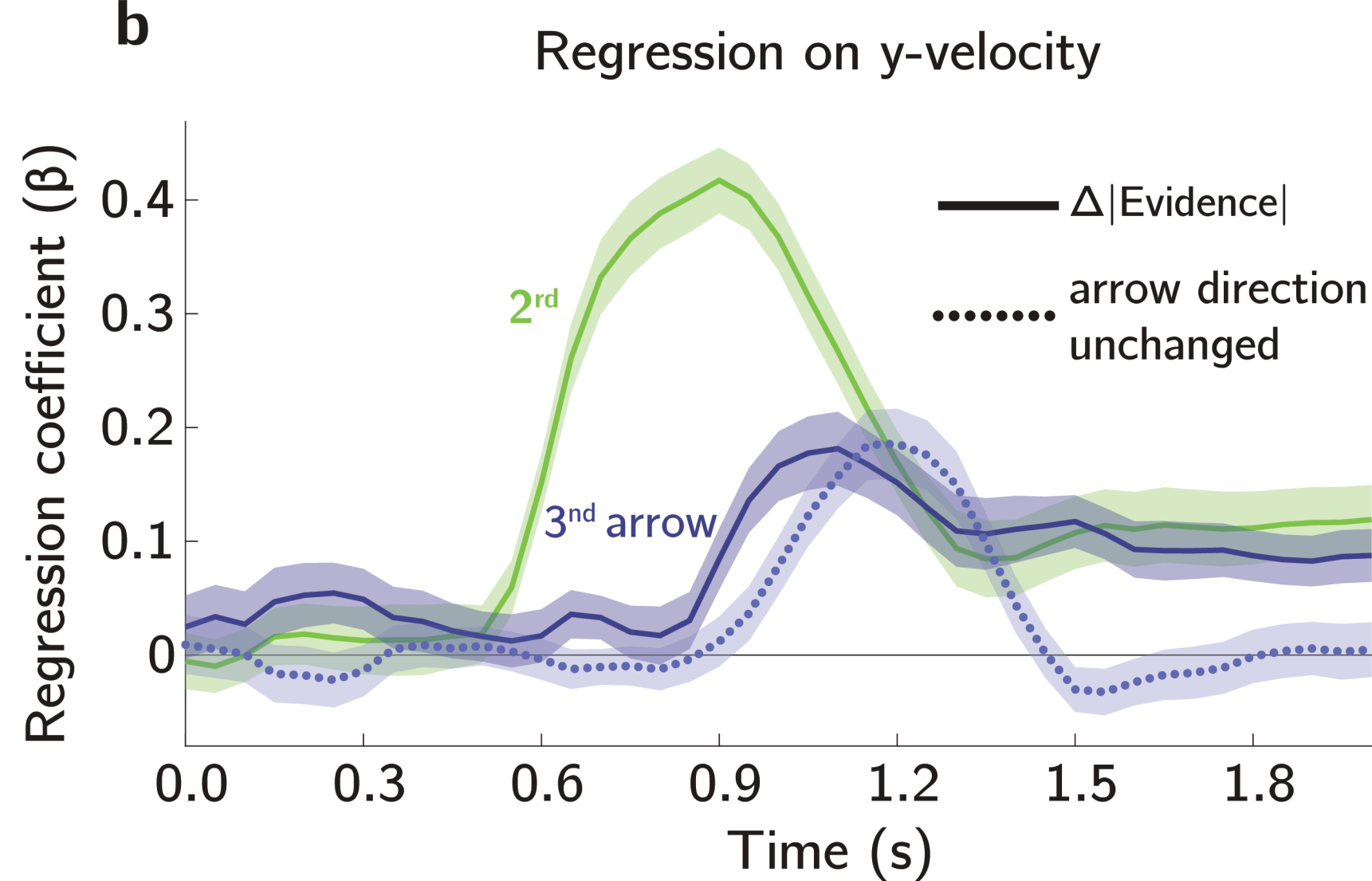
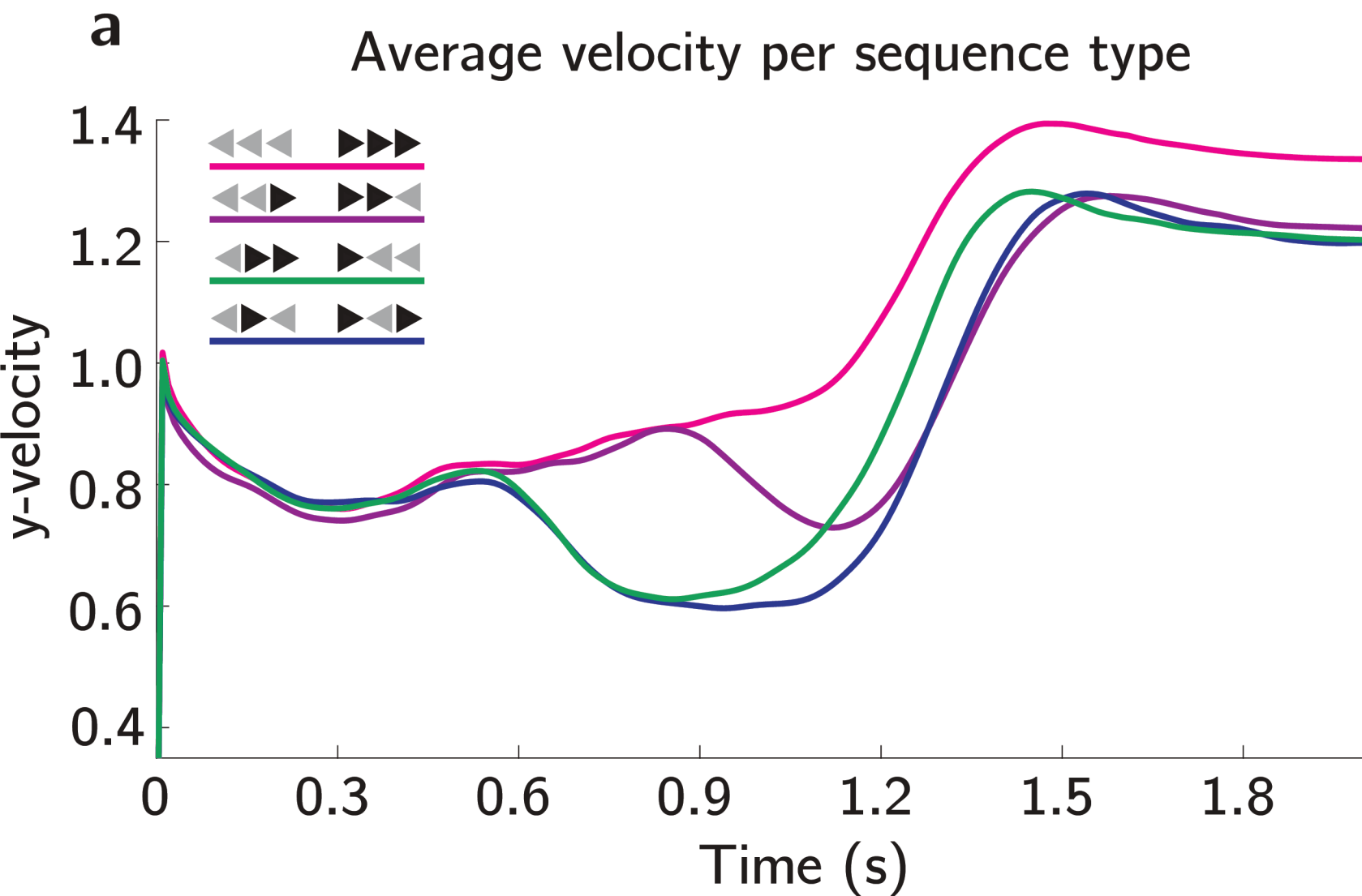
Fig. 3. Within-trial changes of mind can be detected in single trials. (a) Number comparison task. On each trial, participants move their finger left or right to indicate whether the digit presented is smaller or larger than 5. Plotting single trials indicates that in harder trials (target closer to 5), the participants tend to transiently point towards the incorrect response button and then change their mind and deviate towards the correct response (reprinted with permission from [29]). (b) Such deviations can be quantified by counting the number of bends (consecutive clockwise or counterclockwise movement) per trial (red curve). Here, in the arrows task (Fig. 1a) [13], trials with more changes in the arrow direction induce more bends and more speed fluctuations (blue curve; a speed fluctuation was defined as a time window with strong vertical acceleration). (c) An alternative approach assumes that each trial involves several discrete and temporally-overlapping processing stages. To capture these processing stages, we can break down each trajectory into a series of overlapping sub-movements. This is done by fitting the number of sub-movements and the parameters of each sub-movements (direction, speed, time window) to the observed trajectory (reprinted with permission from [14]).

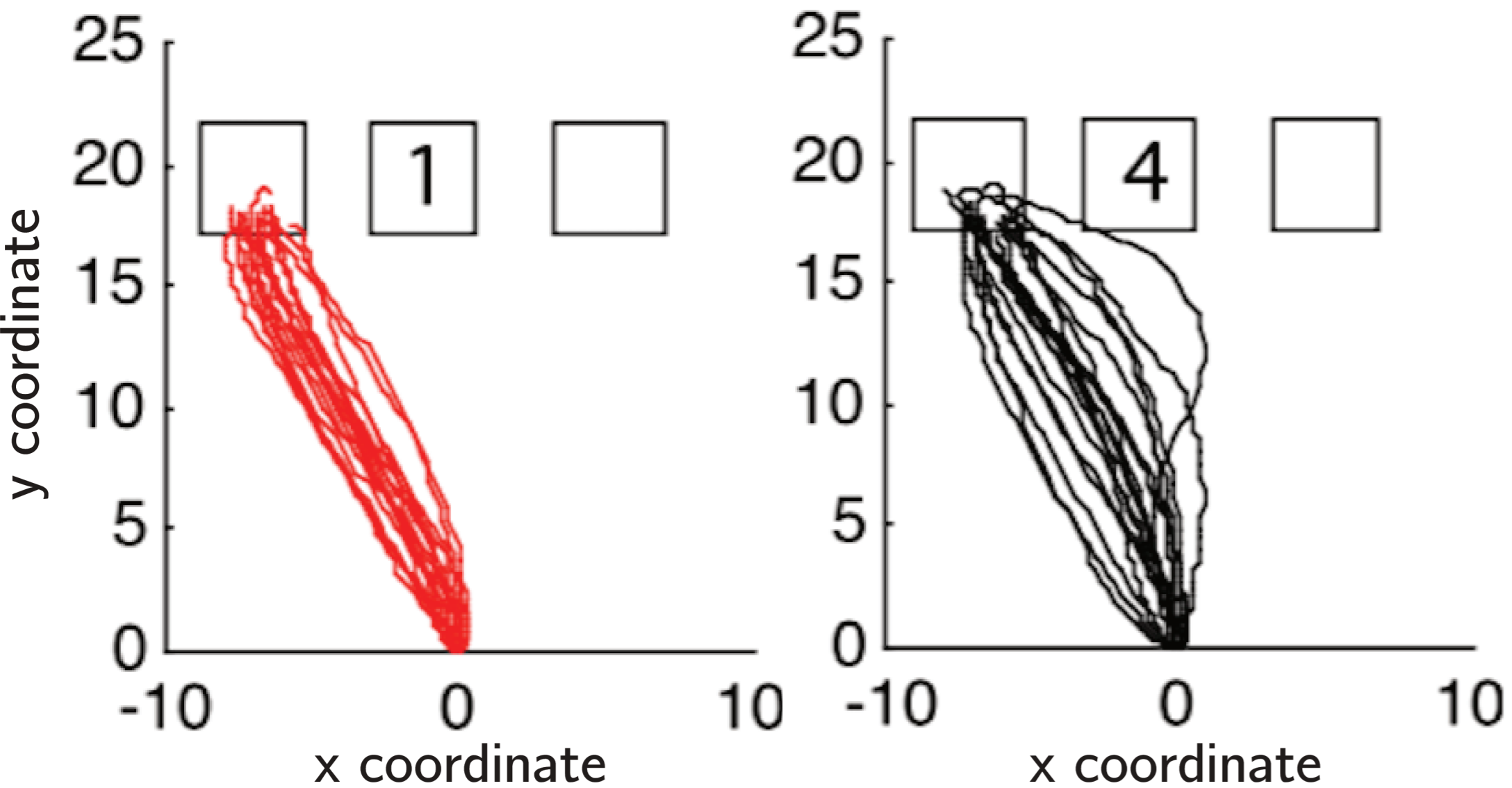
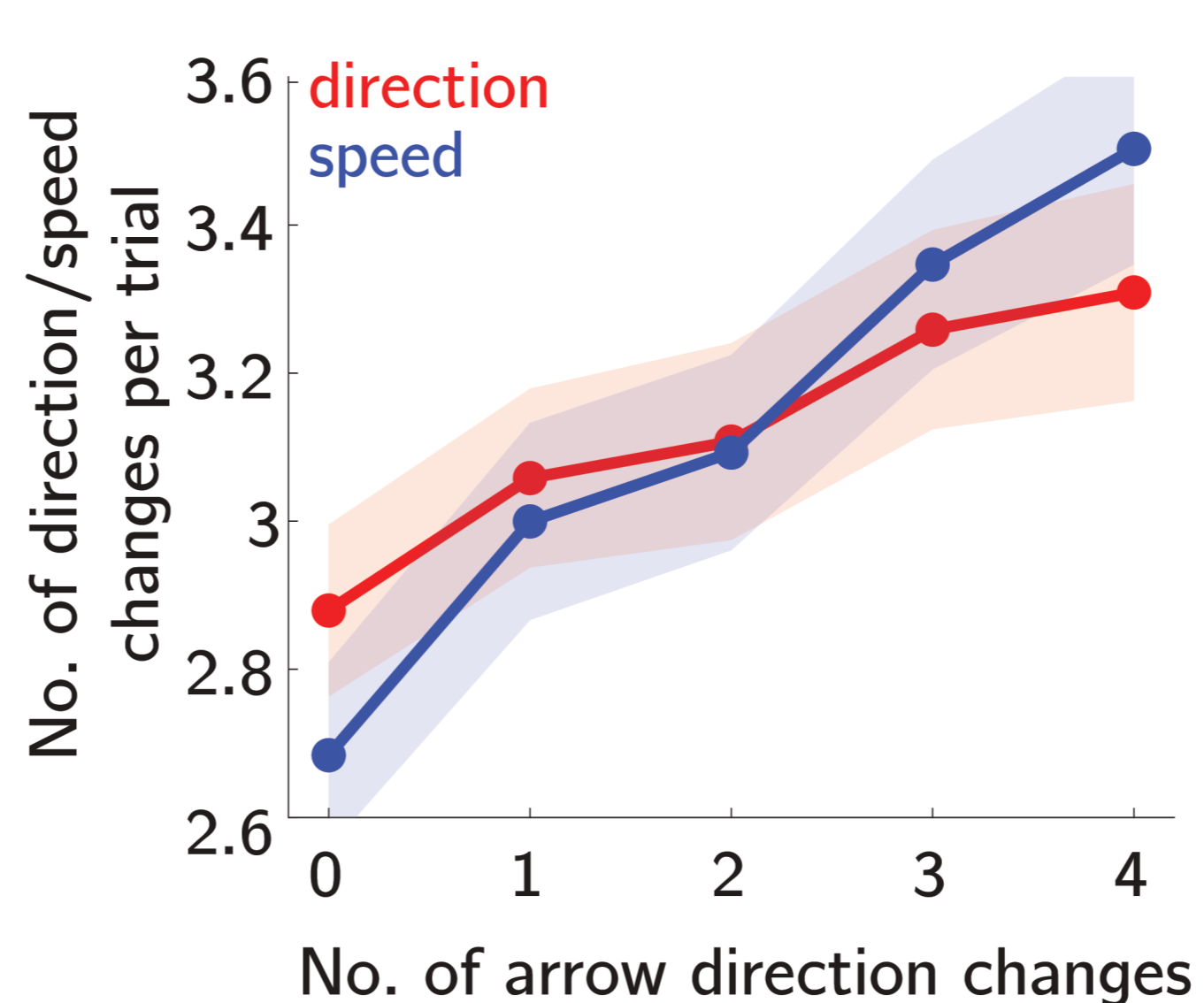
Fig. 4. Distinguishing between time delays and transient effects. (a) Average x coordinates, plotted as a function of time, in a task in which participants pointed to the estimated position of a 2-digit number on a number line [25]. During an early time-window, the trajectories of small target numbers are more spaced apart than the large-number trajectories. This can be interpreted either as a transient effect of non-linear representation of the target quantity, or as faster processing of small target numbers. (b) Time-resolved regressions (same plot type as Fig. 1d): for each time-point and subject, the implied endpoints were regressed against the target number and its logarithm. The regression coefficients were averaged across subjects and plotted as a function of time. The transient effect of $\log(\text{target})$ suggests a transient activation of log number magnitude. (c) Similar time-resolved regressions, which differ only in how trials were temporally aligned. Here, instead of target onset, trials were aligned starting from the trial's first significant sideways deviation (X movement onset). In this analysis, the effect of $\log(\text{target})$ was no longer significant. This refutes the nonlinear-quantity representation hypothesis: the transient log effect in (b) is completely reducible to between-trial differences in the target number's initial processing duration.



Time-resolved regressions:

$$x(t_1) = b_{1(t_1)} \text{arrow1} + b_{2(t_1)} \text{arrow2} + b_{3(t_1)} \text{arrow3} + c$$



a**b****c**