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## ► To cite this version:

Hosain Zaman, Georges Zaccour. Vehicle scrappage incentives to accelerate the replacement decision of heterogeneous consumers. Omega, 2020, 91, pp.102016 -. <10.1016/j.omega.2018.12.005>. <hal-03488744>

**HAL Id: hal-03488744**

**<https://hal.science/hal-03488744v1>**

Submitted on 21 Jul 2022

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# Vehicle Scrappage Incentives to Accelerate the Replacement Decision of Heterogeneous Consumers\*

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October 26, 2018

## Abstract

Vehicle scrappage subsidy programs have been widely applied by governments to replace old cars by newer, more fuel-efficient ones. While these programs have been implemented to provide motivation for replacing vehicles earlier, they may not be as effective as expected. From a cost-benefit perspective, the consumers who would have replaced anyway, even without the program, must be considered when evaluating the net benefits of the program. This requires accounting for variations in consumers' willingness to replace. Considering consumer heterogeneity in net trade-in valuation, this study investigates a dynamic vehicle-replacement problem based on a life cycle optimization (LCO) approach. We theoretically demonstrate that although increasing the subsidy level does motivate low-value consumers to replace earlier, it also induces consumers with a high net trade-in valuation to replace later in order to become eligible for the subsidy program. We have also developed a simulation program based on real data, to demonstrate the application of our general model. According to the simulation results, ignoring consumer heterogeneity could result in an overestimation of the net benefits of the scrappage program.

**Keywords:** Vehicle scrappage program; Heterogeneous consumers; Dynamic programming; Life cycle optimization

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\*The authors would like to thank the two anonymous reviewers and the associate editor for their very helpful and constructive comments. Research supported by NSERC Canada, grant RGPIN-2016-04975.

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# 1 Introduction

Following the global recession in 2008, many countries introduced car scrappage incentive programs with the dual aim of stimulating the car market and reducing emissions, as new cars typically pollute less than older ones. Unlike some subsidy programs that provide financial incentives for retailers and recyclers (Liu et al. (2016) and Zhang et al. (2015)), scrappage subsidies are offered directly to consumers. The CARS (Car Allowance Rebate System) program in the US was introduced in 2009, and provided subsidies of up to \$ 4,500 to individuals trading in their car for a more fuel-efficient one. The program was so successful that its \$1 billion budget was used up well ahead of the expected date (Huang et al., 2014). In Canada, the Vehicle Efficiency Incentive (VEI) was introduced in 2007, and offered a \$2,000 rebate to consumers purchasing a new fuel-efficient vehicle (Walsh, 2012). Similar scrappage programs have also been adopted in European countries to incentivize consumers to take cars over a certain age off the road.

Scrappage incentive programs have been reported to be very successful at stimulating car sales, but their effectiveness in terms of environmental impact and cost is debatable (Smit, 2016). A recent concern regarding scrappage and other subsidy programs used to accelerate durable products replacement is consumer targeting. In a study on air conditioners, Kleine et al. (2011) highlighted that consumers differ in their willingness to replace, and that there is a need for further research to understand their behavior. More specifically, they argued that the design and assessment of incentive programs for air conditioner replacement must take into account those individuals who would replace even without a subsidy. Such consumers are called “free riders” in Blumstein (2010) and Skumatz et al. (2009), and they are excluded when calculating the net savings from the incentive program. Hoekstra et al. (2015) justified the need to account for free riders in their conclusion that 60% of subsidies from the Cash for Clunkers program (scrappage program in the US) went to households that would have purchased regardless while the program was ongoing. Such observations constitute an invitation to account for consumer heterogeneity, which seems a crucial feature when designing an incentive program.

We consider a dynamic life cycle optimization model (LCO) to study the vehicle scrappage problem. In the LCO approach, the environmental impact of a vehicle is determined by taking into account all the stages in the product’s life cycle, including material production, manufacturing, use, maintenance, and end-of-life disposal. However, we concentrate only on the usage phase, which accounts for the largest share of pollution in a car’s life cycle. At each period, heterogeneous consumers decide whether to replace or keep their vehicle. In order to accelerate replacement, the government offers a subsidy to consumers who replace their vehicle with a new one.

In this work, it is assumed that a new vehicle pollutes less than an older one, as a result of technological progress as well as increases in vehicle emissions levels due to age (Zachariadis et al., 2001; Pastramas et al., 2014; Trusts, 2010; Zyl et al., 2015). We develop a vintage model in which different generations of cars are differentiated in terms of their usage emissions. In such a setting, since the pollution generated from vehicle use differs for each car model, the total pollution at each period can be calculated only by knowing the age distribution of the current vehicle fleet. In other words, one needs information on the number of vehicles in use for each car model. In terms of modeling, such an information requirement creates a high burden on the dimension of the state space, making the problem of *pollution from use* much more complex than the one of *pollution from production*. When pollution is seen as an outcome of production, then the emissions in each period are usually assumed to be a function of the quantity of production in the same period (Jorgensen et al., 2010). On the other hand, pollution caused by consumption at each period is the aggregation of emissions from the use of all products (e.g., vehicles) of different ages. Consequently, in a dynamic setting, many state variables are required to keep track of the pollution generated from all products still in use in each period.

## 1.1 Literature background

Problems and issues related to environmental benefits of scrappage programs are studied widely in the literature. For instance, Dill (2004) showed that scrapped vehicles would have been driven less compared to those of the same vintage that are kept by their owners, and concluded that the reduction in emissions due to a scrappage program is lower than expected. Further, since some of the scrapped vehicles might have been retired even without the program, the subsidy in such cases is a waste of public funds. Such observations triggered cost-benefit studies of scrappage programs aimed at finding the optimal subsidy level. Assuming that a consumer will participate in a scrappage program when the subsidy exceeds the price of the old vehicle, Hahn (1995) determined the number of vehicles scrapped for different subsidy levels. Under the same assumption in Hahn (1995), Lavee and Becker (2009) also estimated a supply curve of vehicles for retirement as a function of the subsidy level and used car value. Using a cost-benefit analysis, they next computed the subsidy level that maximizes the net benefit of the scrappage program. Lavee et al. (2014) extended the study in Lavee and Becker (2009) by adding age and maintenance cost in the participation rate function, and estimated its parameters using real data.

Unlike the static models in the above-cited papers, Lorentziadis and Vournas (2011) proposed a dynamic model where the demand for new vehicles is equal to the number of scrapped old vehicles and depends on a time-varying subsidy. However, the authors did not account for the impact of vehicle age in the decision to replace. We shall also consider a dynamic model where the decision to replace is endogenous and depends on, among other things, the vehicle's age: intuitively, this factor is an important driver of such decisions.

As mentioned above, we apply an LCO approach to model the vehicle replacement problem. Kim et al. (2003) introduced a novel LCO approach and applied it to an automobile replacement problem. In addition to its application in single-vehicle analysis, the LCO approach has been used to find the optimal vehicle fleet conversion (Kim et al., 2004; Figliozi et al., 2013). Kim et al. (2004) studied a vehicle fleet optimization problem from a life cycle perspective,

where the objective was to minimize the total environmental burden of vehicles. For a comprehensive review of fleet-based life cycle approaches, see Garcia and Freire (2017). Considering such an approach, some studies, e.g., Kim et al. (2004, 2006) and Lenski et al. (2010), have raised doubts about the environmental efficiency of scrappage programs, stating that they may increase pollution emissions due to the increased production of new cars. Further, while the decisions to replace in the above-cited papers are related only to the environment, other studies, —see, e.g., Kim et al. (2006), Spitzley et al. (2005) and Kleine et al. (2011)— used an LCO approach with a focus on both the environment and the owner’s cost. Spitzley et al. (2005) showed that when ownership cost is accounted for, the replacement intervals (number of years a vehicle is kept) are larger than when one considers an emissions-damage minimization problem. Also, when the pollution damage cost is combined with ownership cost, the results are identical to those of the ownership cost optimization.

In our paper, we focus on the usage phase of the LCO approach when evaluating the environmental performance of the subsidy program, and the decisions to replace is studied from the consumer’s point of view. That is, the individual’s objective is to minimize her ownership cost, while the environmental consequences are evaluated ex-post.

In all the papers mentioned above, the decisions to replace a vehicle are made by homogeneous consumers. Consumer heterogeneity in scrappage policies has been taken into account in a few recent papers. Miao et al. (2017) and Li and Xu (2015) considered consumer heterogeneity in trade-in models. Shiraldi (2011) used a dynamic discrete-choice model in which heterogeneous consumers with different tastes decide whether to replace their automobile. He evaluated different incentive policies and checked whether or not the beneficiaries of such policies would have replaced their vehicle without the subsidy. Wei and Li (2016) retained the same discrete-choice setting to analyze vehicle scrappage programs with consumer heterogeneity. They stated that targeting marginal consumers, i.e., those who would not have replaced without a subsidy, is the key factor in designing scrappage programs. Further, they concluded that programs that do not have an environmental objective and that target marginal consumers

are more cost-effective than those with explicit environmental objectives and no targeting of marginal consumers. Unlike these two papers, where the focus is on the effect of the scrappage program on heterogeneous consumers' vehicle choice, in this paper we concentrate on the impact of the subsidy level on the timing of the replacement decisions. In this regard, the closest paper to ours is Langer and Lemoine (2017), which studies the reaction of strategic consumers to government subsidies for new products. However, they investigated the effect of subsidies on purchasing decisions rather than on replacement decisions as we do here.

In the literature on scrappage programs, a subsidy has always been considered an incentive to accelerate replacement. However, interestingly, we show that it can motivate consumers to postpone replacing, so they can become eligible for the incentive program. Similar consumer behavior has been observed in the dynamic adoption of new products and new technologies. Here, consumers wait for cheaper options and for the entry of other firms onto the market (Gowrisankaran and Rysman, 2012). Similarly, firms postpone investing in new technologies to wait for new inventions when technological progress is rapid (Feichtinger et al., 2006). Likewise, our results imply that, besides borrowing future demand, increasing the subsidy can lead some consumers to postpone their demand too, affecting net benefit calculations of scrappage subsidy programs.

## 1.2 Research questions and contributions

To the best of our knowledge, this paper is the first to study the impact of the level of incentive payment on the supply of vehicles retired for scrappage in the presence of consumer heterogeneity. More specifically, our objective is to answer the following research questions:

1. Does the subsidy affect all consumers' replacement decisions in the same way?
2. What is the effect of a subsidy on the environment when consumer heterogeneity is taken into account?

3. What is the impact of varying the parameter values on the results?

We believe that answering these questions can help governments design a more efficient policy while also providing a better understanding of the real benefits of scrappage programs. Our main results can be summarized as follows:

1. Subsidies affect high-value consumers differently than low-value consumers. While the subsidy induces low-value consumers to replace their vehicle earlier, it motivates high-value consumers to delay replacing until their cars become eligible for the subsidy. This implies that investing in high-value consumers is a waste of funds. However, direct price discrimination between different consumer groups is not possible for the regulator. Instead, she must determine the subsidy level considering both groups, i.e., high-value and low-value consumers. On the one hand, the subsidy must be high enough to accelerate purchases by low-value consumers. On the other hand, it should be lower than the threshold above which high-value consumers delay replacing their vehicles in order to qualify for the subsidy.
2. When consumer heterogeneity is taken into account, the estimated benefits of the scrap-page program are lower than when all consumers are assumed homogeneous. Specifically, the pollution reduction and new vehicle purchases resulting from the program are over-estimated when consumer heterogeneity is ignored. This finding clarifies the real benefits of a scrappage programs. This can be useful when assigning budgets for future subsidy programs. Also, with a better understanding of advantages of subsidy programs, policy makers are able to choose among different options to reach their objective in the most efficient way.

The remainder of the paper is organized as follows: In Section 2, we introduce our general model, and we analytically investigate two particular instances in Section 3. In Section 4, we report and discuss our empirical results. In Section 5, we briefly conclude.

## 2 The Model

Denote by  $t$  the time (year) index, with  $t \in \mathcal{T} = \{0, \dots, T^s, \dots, T\}$ , where  $T^s$  corresponds to the terminal date of the subsidy program, which is assumed to be shorter than the planning horizon  $T$ . Let  $j, j = 0, \dots, \omega$ , be the vehicle's age, where  $\omega$  is the maximum life span of a vehicle. We assume  $\omega$  to be constant for all vehicles, irrespective of their date of introduction onto the market. At time period  $t$  a vehicle of age  $j$  is of the year model  $t - j$ . Consumers are heterogeneous in terms of their valuation of the environment and are indexed by  $i \in I$ , where  $I$  represents the set of types. Denote by  $n_t, t \in \mathcal{T}$ , the exogenously given number of consumers buying a car for the first time, and by  $N_t$  the number of new cars purchased at  $t$ . The difference  $N_t - n_t$  is the demand for replacement vehicles at year  $t$  and will be determined endogenously.

Denote by  $p_t$  the price of a new car at time  $t$ , and let  $\theta_i$  be the “utility” that a consumer of type  $i$  derives from replacing her used car by a new one. We shall refer to  $\theta_i$  as the net trade-in valuation, which is measured by the difference between the environmental valuation of a new car minus the valuation of a retained car. Consequently, we define by  $p_t - \theta_i$  the effective price of a vehicle sold at  $p_t$  to a consumer of type  $i$ . This definition of an effective vehicle-replacement price is in line with Huang et al. (2014), where a parameter  $\theta$  is added to the consumer's utility function. The probability  $P_i$  of a first-time buyer being of type  $i$  is constant over time. We denote the distribution of first-time buyers by  $(P_i)_{i \in I}$ . The distribution of all buyers, including those replacing their vehicle at period  $t$ , is denoted by  $(\pi_{i,t})_{i \in I, t \in \mathcal{T}}$ , and will be determined endogenously.

In deciding whether or not to replace a vehicle one must consider an investment cost (effective price of the new vehicle) and the usage cost. We assume that the usage cost of a vehicle of age  $j$  at time  $t$  can be well approximated as follows:

$$C_t(j) = f(t - j) \cdot D \cdot O_t + m(j), \quad (1)$$

where  $D$  is the number of kilometers traveled per year (km/y),  $f(t - j)$  is the per-kilometer

fuel consumption,  $O_t$  is the unit fuel cost at time  $t$ , and  $m(j)$  is the maintenance cost per year of a car of age  $j$ . In our formulation, fuel consumption varies by car vintage (given by  $t - j$ ), while maintenance cost is only affected by age  $j$ . Also, consumers are assumed to be homogeneous in terms of their yearly travel demand.

The government (or the regulator) can affect the vehicle replacement decision through a scrappage subsidy. The subsidy, denoted by  $r$ , is given to any consumer who scraps a vehicle older than a given age  $\eta$  (measured in years), independently of the type of consumer and of the actual age of the vehicle (assuming eligibility, of course, that is,  $j \geq \eta$ ).

We let  $q_t$  be the environmental quality index of a new vintage vehicle at time  $t$ , i.e., vintage  $t - 0$ , defined by

$$q_t = 1 - \left( \frac{(f(t) - f(T))}{f(T)} \right). \quad (2)$$

This index reflects the empirical observation that the per-kilometer fuel consumption increases over time and reaches its maximum value at the terminal date, i.e.,  $q_T = 1$ . This means that consumers give more value to newer, more fuel-efficient vehicles. Note that in practice,  $\frac{f(t)}{f(T)} \leq 2$  which means that the fuel efficiency of new vehicles would at most double in the short and mid terms. Consequently,  $q_t$  is non-negative.

Given the usage cost, new vehicle price, and government policy, the replacement decision problem of consumer  $i$  owning a vehicle of age  $j$  can be modeled by the following dynamic optimization program:

$$V_{i,t}(j) = \begin{cases} C_t(j) + \beta \cdot V_{i,t+1}(j+1) & \text{if } 0 \leq j < \zeta, \\ \min \{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \zeta \leq j < \eta, \\ \min \{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) - r + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \eta \leq j \leq \omega \text{ and } 0 \leq t \leq T^s, \\ \min \{C_t(j) + \beta \cdot V_{i,t+1}(j+1), (p_t - q_t \cdot \theta_i) + C_t(0) + \beta \cdot V_{i,t+1}(1)\} & \text{if } \eta \leq j \leq \omega \text{ and } t > T^s, \end{cases} \quad (3)$$

where  $V_{i,t}$  is consumer  $i$ 's cumulative cost of keeping or of replacing the vehicle from period  $t$

to  $T$ , and where  $\zeta = 1, \dots, \eta$  is the minimum car age below which no individual will consider replacing her vehicle. The terminal condition of the problem is  $V_{i,T+1}(j) = 0$  for all consumers and all vehicles. The above program has four parts. If the car is still brand new, that is, if its age is below  $\zeta$ , then replacing is not considered, and the total cost is the sum of the current cost and the discounted future cost. In the second case ( $\zeta \leq j < \eta$ ), no subsidy is yet available. Therefore, the consumer of type  $i$  compares the total cost of replacing her vehicle to the cost of keeping the same car. The replacement cost is given by the sum of the effective price of a new car ( $p_t - q_t \cdot \theta_i$ ) and its usage cost. The same type of comparison is done for  $\eta \leq j \leq \omega$ : however, here, the price includes the subsidy. In other words, the comparison is given by  $(p_t - q_t \cdot \theta_i) - r$ . Recall that the subsidy is only available in periods  $t = 0, \dots, T^s$ . The case  $\zeta < \eta$  allows us to assess the government program's impact on those consumers who would have replaced their vehicle before becoming eligible for a subsidy, but who postpone their purchase until they become eligible. As alluded to in the introduction, one issue with subsidies being offered to all individuals satisfying a given condition is that some of them would have purchased the product anyway, without the subsidy. Our framework will allow us to assess the size of this group.

Let  $d_{i,t}(j)$  be the decision made at time  $t$  by consumer  $i$  owning a vehicle of age  $j$ , defined as follows:

$$d_{i,t}(j) = \begin{cases} 0 & \text{if keep,} \\ 1 & \text{if replace.} \end{cases}$$

In order to keep track of consumers corresponding to each vehicle vintage at each period, let  $K_t(j-1)$  be the set of consumers who have a vehicle of age  $j-1$  at time  $t$  and let  $R_t(j-1)$  be the set of consumers who replace their vehicles of age  $j-1$  at time  $t$ . Consequently, we have  $K_{t+1}(j) = K_t(j-1) \setminus R_t(j-1)$ . In other words, the set of individuals who have a vehicle of age  $j$  at period  $t+1$  is made up of those who had a vehicle of age  $j-1$  at period  $t$ , minus those who replaced their vehicles at period  $t$ . Further, let  $W_t(i)$  be the set **corresponding to the age of vehicles that are replaced by consumers of type  $i$  at time  $t$** .

Using the above-defined sets, the expected total number of used cars being replaced at time  $t$  is then given by

$$\sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,t-j} \cdot N_{t-j}.$$

The expected total number of new vehicles purchased at  $t$  is

$$N_t = n_t + \sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,t-j} \cdot N_{t-j},$$

where  $\pi_{i,t}$  is the probability that new car buyers (all buyers including first-time buyers as well as those who replace their car) at time  $t$  will be of type  $i$ . This probability is determined as follows:

$$\pi_{i,t} = \frac{P_i \cdot n_t + \sum_{j \in W_t(i)} \pi_{i,t-j} \cdot N_{t-j}}{N_t}.$$

To determine the pollution dynamics, denote by  $e$  the emissions **in kilograms** generated by the consumption of one liter of fuel, and let  $E_t$  be the accumulated stock of pollution (a state variable) by time  $t$ . To capture the fact that the vehicle emissions rate increases with age (Zyl et al., 2015), we introduce the increasing deterioration factor  $\mu(j)$ . The dynamics of pollution accumulation are given by the following difference equation:

$$\begin{aligned} E_{t+1} = & E_t + \sum_{j=1}^{\omega} \sum_{i \in K_t(j) \setminus R_t(j)} \pi_{i,t-j} \cdot N_{t-j} \cdot e \cdot f(t-j) \cdot D \cdot \mu(j) \\ & + \sum_{j=1}^{\omega} \sum_{i \in R_t(j)} \pi_{i,t-j} \cdot N_{t-j} \cdot e \cdot f(0) \cdot D + n_t \cdot e \cdot f(0) \cdot D. \end{aligned}$$

The three terms in the above equation represent the stock of pollution from the last period, the pollution from vehicles kept in the current period, and the emissions from new cars, respectively. As we are considering a short-term planning horizon, we do not account for emissions absorption into nature. As car sales depend on the subsidy program, so does the stock of pollution.

Consequently, the government can easily assess the environmental impact of the subsidy.

Table 1 summarizes the notations used throughout the paper.

Notation	Description
Related to Consumers	
$i$	Index of consumers
$\theta_i$	Net trade-in value of consumer type $i$
$P_i$	Distribution of new buyers
$\pi_{i,t}$	Distribution of consumers who buy a vehicle of age 0 at time $t$ and who are affected by replacement decisions
$V_{i,t}(j)$	Value function related to consumer $i$ who has a vehicle of age $j$ at period $t$
$d_{i,t}(j)$	Decision variable related to consumer $i$ who has a vehicle of age $j$ at period $t$
$K_t(j)$	Set of consumers who have a vehicle of age $j$ at period $t$
$R_t(j)$	Set of consumers who replace a vehicle of age $j$ at period $t$
$s_i$	Incremental threshold related to the subsidy's adverse effect
$I$	Set of all consumers
$n_t$	Number of new buyers at time $t$
$N_t$	Total number of consumers who buy a new car at time $t$ including those who replace their car
Related to Vehicles	
$j$	Index of vehicles (age)
$C_t(j)$	Ownership cost of a car of age $j$ at time $t$
$D$	Travel demand per year
$f(t - j)$	Per-kilometer fuel use for a car of age $j$ at time $t$
$O_t$	Unit fuel price at time $t$
$m(j)$	Maintenance cost of a car of age $j$
$\omega$	Maximum life span of a vehicle
$W_t(i)$	Set of vehicles (ages) replaced at time $t$ by consumer type $i$
$p_t$	Price of the vehicle at time $t$
$\mu(j)$	Parameter related to emissions deterioration by age
$q_t$	Fuel economy coefficient of a new car produced at time $t$

Notation	Description
Related to Government	
$\zeta$	The age after which replacement is allowed
$\eta$	Eligibility age threshold
$r$	Subsidy level
Other Parameters	
$t$	Index for each period
$T$	The last period
$T^s$	The last period of the subsidy program
$\beta$	Discount rate
$\tau$	Number of periods remaining till the end of the horizon
$E_t$	Accumulated pollution at time $t$
$\delta$	Parameter representing absorption of pollution by nature

Table 1: Notation for the general model

### 3 A Simplified Model

The dynamic program defined in (3) cannot be solved analytically in its full generality. We shall solve the model numerically in the next section, using real data. To gain qualitative hints about its solution, we analytically investigate a simplified version of the model. Specifically, instead of having the detailed cost function in (1), we omit specifying  $C_t(j)$  and simply assume that it is an increasing concave function in  $j$ . We also assume that the pollution emitted per year by a vehicle of age  $\tau$  is  $e_\tau$ , which is increasing in  $\tau$ .

#### 3.1 Two-period setting

First, we consider a two-period setting, with the subsidy being available in both periods. In the first period, the vehicles that can either be kept or replaced by a new one are of age 0 to  $\omega$ . Vehicles of age  $j$  that are replaced in the first period are vehicles of age 1 in the second period, and those that are kept will be of age  $j+1$ . The terminal value of all vehicles at the end of the second period is assumed to be zero. The value functions in the first and second periods are as follows:

$$\begin{aligned}
V_{i,1}(j) &= \begin{cases} C_1(j) + \beta \cdot V_{i,2}(j+1) & \text{if } 0 \leq j < \zeta, \\ \min\{C_1(j) + \beta \cdot V_{i,2}(j+1), p - q_1 \cdot \theta_i + C_1(0) + \beta \cdot V_{i,2}(1)\} & \text{if } \zeta \leq j < \eta, \\ \min\{C_1(j) + \beta \cdot V_{i,2}(j+1), p - q_1 \cdot \theta_i + C_1(0) - r + \beta \cdot V_{i,2}(1)\} & \text{if } j \geq \eta, \end{cases} \\
V_{i,2}(j) &= \begin{cases} C_2(j) & \text{if } 0 \leq j < \zeta, \\ \min\{C_2(j), p - q_2 \cdot \theta_i + C_2(0)\} & \text{if } j < \eta, \\ \min\{C_2(j), p - q_2 \cdot \theta_i + C_2(0) - r\} & \text{if } j \geq \eta. \end{cases} \quad (5)
\end{aligned}$$

In the first period, strategic consumers make their vehicle replacement decision based not only on the cost in the first period, but also on the cost in the second period. Consequently, the effect of the subsidy on different groups of consumers may vary. On the one hand, offering a subsidy to those who replace only after age  $\eta$  in the absence of a subsidy induces them to replace their vehicle earlier. This result is quite intuitive since higher subsidies make the purchase of new vehicles cheaper, thereby motivating the consumer to replace her vehicle when it is younger. On the other hand, increasing the government payment level may have the opposite effect on the replacement decision of those who would have replaced before  $\eta$  if the subsidy did not exist. This adverse effect occurs because forward-looking consumers find it more cost-efficient to delay their purchase so they can become eligible for the subsidy in the next period, rather than replacing the vehicle in the current period and receiving no subsidy.

To analyze this adverse effect in the subsidy, consider the consumers who replace their vehicle of age  $\eta - 1$  in the first period. **Assuming**  $\zeta > 1$  such consumers satisfy the following inequality:

$$C_1(\eta - 1) + \beta \cdot \min\{C_2(j), p - q_2 \cdot \theta_i + C_2(0) - r\} \geq p - q_1 \cdot \theta_i + C_1(0) + \beta \cdot C_2(1), \quad (6)$$

where the left-hand side (LHS) is the cost of keeping a vehicle of age  $\eta - 1$  in the first period plus the discounted cost-to-go of the vehicle of age  $\eta$  in the second period. The right-hand side (RHS) is the cost of replacing the vehicle in the first period plus the discounted cost-to-go of the vehicle of age 1 in the second period. Next, we show that, for some parameter values, increasing the subsidy level incites consumers to postpone the replacement of their vehicles aged  $\eta - 1$ , when they would have replaced them if the subsidy level had been lower.

**Proposition 1** *Let  $\tilde{r}$  be a subsidy level satisfying*

$$\tilde{r} \geq p - q_2 \cdot \theta_i + C_2(0) - C_2(\eta),$$

that is, a consumer of type  $i$  replaces her vehicle of age  $\eta$  in period 2. Suppose also that the inequality in (6) holds true for such  $\tilde{r}$ .

If the subsidy level is set at  $\tilde{r} + s_i$ , where

$$s_i \geq \frac{1}{\beta}(C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r} - C_2(1)) - p + q_1 \cdot \theta_i - C_1(0)), \quad (7)$$

then the same consumer would no longer replace the vehicle aged  $\eta - 1$  in the first period but would postpone the replacement from period 1 to period 2.

**Proof.** Suppose that for a given subsidy, there are consumers who replace their vehicle of age  $\eta$  in the second period. Also, assume that there is a subgroup of these customers who replace their vehicle aged  $\eta - 1$  in the first period with the same subsidy, that is, the inequality in (6) holds true. We need to show that when the subsidy exceeds a certain level, some consumers in this subgroup prefer to keep their vehicle aged  $\eta - 1$  in the first period and to replace it at age  $\eta$  in the second period.

Suppose that the subsidy is  $\tilde{r}$ . For consumers who replace their vehicle at age  $\eta$  in period 2 at this subsidy level  $\tilde{r}$ , it holds that

$$C_2(\eta) \geq p - q_2 \cdot \theta_i + C_2(0) - \tilde{r},$$

or equivalently,

$$\tilde{r} \geq p - q_2 \cdot \theta_i + C_2(0) - C_2(\eta). \quad (8)$$

Note that if, in the second period, the replacement happened for a subsidy level  $r$ , then it would also happen for higher levels. Therefore, (8) holds for all  $r \geq \tilde{r}$ .

Besides, since vehicle  $\eta$  is replaced at subsidy level  $\tilde{r}$ , then  $V_{i,2}(\eta)$  reads

$$V_{i,2}(\eta) = p - q_2 \cdot \theta_i + C_2(0) - \tilde{r}.$$

Therefore, inequality (6) turns into the following:

$$C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r}) \geq p - q_1 \cdot \theta_i + C_1(0) + \beta \cdot V_{i,2}(1)$$

If the value of  $\tilde{r}$  increases enough—say, to  $\tilde{r} + s_i$ —the direction of the above inequality changes.

In other words, the consumer will no longer replace the vehicle aged  $\eta - 1$  in the first period. The value of  $s_i$  can be simply characterized as follows:

$$s_i \geq \frac{1}{\beta}(C_1(\eta - 1) + \beta(p - q_2 \cdot \theta_i + C_2(0) - \tilde{r} - C_2(1)) - p + q_1 \cdot \theta_i - C_1(0)).$$

■

This result illustrates the idea that a subsidy program may have the opposite effect than the one intended. It may also benefit those who would have replaced their vehicles without the subsidy.<sup>1</sup> This opposite effect of the subsidy may impact estimates of the net benefits of the scrappage program, both in terms of the environment and the car market. In order to investigate this issue, let us call the consumers mentioned in Proposition 1 “high-value” consumers. **In fact high-value consumers are those whose trade-in valuations are high enough to replace their car even in the absence of the subsidy program. In Proposition 1** these consumers would replace their vehicle aged  $\eta - 1$  in the first period without cashing in the subsidy,  $\tilde{r}$ . Assume that the number of vehicles aged  $\eta - 1$ , and aged  $\eta$  or older, owned by these consumers at the beginning of the first period is  $\nu_{\eta-1}$  and  $\nu_{\geq\eta}$ , respectively. The subscript  $\geq\eta$  represents vehicles aged  $\eta$  or older. We disregard cars younger than  $\eta - 1$ , because they are not eligible for the subsidy in either of the two periods under study.

Next, we show that increasing the subsidy does not necessarily result in more car replacements and in less pollution by high-value consumers.

**Proposition 2** *Increasing the subsidy level from  $\tilde{r}$  to  $\tilde{r} + s_i$  in Proposition 1 increases the pollution emitted by high-value consumers by  $\nu_{\eta-1}(e_{\eta-1} - e_1)$ . Also, it has no effect on the total number of cars replaced by these consumers.*

**Proof.** The proof is straightforward. When the subsidy level is  $\tilde{r}$ , consumer  $i$  replaces vehicles aged  $\eta$  and  $\eta - 1$  in the first period. The total pollution emitted by these consumers in both periods is

$$(\nu_{\geq\eta} + \nu_{\eta-1})e_0 + (\nu_{\geq\eta} + \nu_{\eta-1})e_1. \tag{9}$$

When the subsidy increases to  $\tilde{r} + s_i$ , consumers with cars aged  $\eta - 1$  delay their car replacement

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<sup>1</sup>This result is familiar in the area of price promotions on frequently purchased products. Indeed, a discount that is offered to attract non-users of the product or buyers of competitive brands also benefits regular users, who would have bought the product anyway. What is more, these regular users can also buy during the promotion for later consumption, inflicting a double loss in revenues to the firm.

to the second period. In this case, the total pollution is

$$\nu_{\geq \eta} e_0 + \nu_{\eta-1} e_{\eta-1} + \nu_{\geq \eta} e_1 + \nu_{\eta-1} e_0. \quad (10)$$

The difference between (9) and (10) is  $\nu_{\eta-1}(e_{\eta-1} - e_1)$ . Also, in both cases, the total number of replacements in both periods is  $\nu_{\eta} + \nu_{\eta-1}$ . ■

This result shows that offering a higher subsidy to high-value consumers may degrade the environment as a result of delayed replacements. More precisely, increasing the subsidy only shifts the purchase of cars by high-value consumers from the first period to the second period. Therefore, while the total number of car replacements remains constant, the environment is degraded, since old vehicles stay on the road for one more period.

This adverse effect of the subsidy program would not have been identified if all the consumers were assumed to be homogeneous. In a homogeneous setting, high-value consumers who would have replaced their car even without the program are ignored, and all consumers have the same valuation. In other words, car owners are all treated as “low-value” consumers who need the subsidy to scrap their old car. In order to examine the problem in such a setting, we study homogeneous consumers’ reaction to the subsidy, as well as its impact on the environment and on car replacements.

Assume that the net trade-in valuation, i.e.,  $\theta_i$  in our model, is equal to value  $x$ , which is the same for all consumers. Since all consumers are the same, the index  $i$  is omitted in value functions (4) and (5). Proposition 2 describes the effect of the subsidy program on the environment and on total car replacements in a homogeneous setting.

**Proposition 3** *Suppose that the net trade-in valuation of homogeneous consumers satisfies the following inequality:*

$$x \leq \frac{1}{q_1}(p + C_1(0) + \beta \cdot C_2(1) - C_1(\eta - 1) - \beta \cdot \min\{C_2(\eta), p - q_2 \cdot \theta_i + C_2(0) - r\}), \quad (11)$$

**that is, consumers do not replace their vehicle aged  $\eta - 1$  in the first period.** Also, assume that consumers do not replace their vehicle aged  $\tau \in [\eta, \omega]$  at subsidy  $\hat{r}$ . Increasing the subsidy from  $\hat{r}$  to  $\hat{r} + s$  reduces pollution and increases car replacements by at least  $\nu_{\tau}(e_{\tau} - e_1)$  and  $\nu_{\tau}$ , respectively, where

$$s = p - q_1 \cdot x + C_2(0) + \beta \cdot C_2(1) - \hat{r} - C_1(\tau) - \beta \cdot \min\{C_2(\tau + 1), p - q_2 \cdot \theta_i + C_2(0) - r\}.$$

**Proof.** Condition (11) can be transformed into

$$C_1(\eta - 1) + \beta \cdot V_2(\eta) \leq p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1),$$

that is, consumers do not replace their vehicle aged  $\eta - 1$  in the first period. Note that, since these cars are not eligible for the subsidy program, increasing the subsidy from  $\hat{r}$  to a higher level has no effect on the decision to replace them.

Suppose that consumers keep the vehicles aged  $\tau \in [\eta, \omega]$  at subsidy level  $\hat{r}$ . The following inequality holds:

$$C_1(\tau) + \beta \cdot V_2(\tau + 1) \leq p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1) - \hat{r}.$$

If the value of  $\hat{r}$  increases enough—say, to  $\hat{r} + s$ —the direction of the above inequality changes. In other words, the consumer will no longer keep the vehicle aged  $\tau$  in the first period. The value of  $s$  can be simply characterized as follows:

$$s = p - q_1 \cdot x + C_2(0) + \beta \cdot C_2(1) - \hat{r} - C_1(\tau) - \beta \cdot V_2(\tau + 1).$$

Therefore, vehicles aged  $\tau$  in the first period are replaced at level  $\hat{r} + s$ . This results in the following inequality:

$$C_1(\tau) + \beta \cdot \min\{C_2(\tau + 1), p - q_2 \cdot \theta_i + C_2(0) - r\} \geq p - q_1 \cdot x + C_1(0) + \beta \cdot C_2(1) - (\hat{r} + s). \quad (12)$$

Note that, while the LHS of (12) is increasing in  $\tau$ , the RHS is constant with respect to  $\tau$ . Consequently, vehicles older than  $\tau$  are replaced at level  $\hat{r} + s$ , if they have not already been replaced at level  $\hat{r}$ . Therefore, at least  $\nu_\tau$  more vehicles are replaced at level  $\hat{r} + s$ , as compared to  $\hat{r}$ . Also, in a similar way to the argument in Proposition 2, one can easily obtain that the pollution decreases by at least  $\nu_\tau(e_\tau - e_1)$ . ■

**Condition (11) ensures that consumers would not replace their vehicle aged  $\eta - 1$  in the first period. In other words, consumers whose cars are not eligible for the scrappage program in the first period (aged  $\eta - 1$  or younger) would not replace their car in the first period. Also, note that  $x$  is the government's belief regarding the average valuation of all consumers, which is as low as it is characterized in (11). This assumption basically implies that the government only launches a scrappage**

program because it believes that consumers would not replace their vehicle without a subsidy. Unless if (11) did not hold, the government would not launch the program as consumers would already replace cars aged  $\eta - 1$  (and apparently older) without receiving any subsidy. Therefore, (11) is the necessary condition for the existence of the scrappage program in the homogeneous setting. The result in Proposition 3 illustrates that, in a homogeneous setting, unlike in a heterogeneous setting, increasing the subsidy always benefits the environment and the car market. Hence, ignoring heterogeneity could result in an overestimation of the net benefits of the program. In the next section, using an example, we will clarify how the advantages of the program could be exaggerated if consumers are assumed homogeneous.

### 3.2 $T$ -period setting

The focus of this subsection is to characterize the threshold in Proposition 1 for more than two periods. We extend the model to  $T$  periods, with the subsidy being available until  $T^s < T$ . Suppose that in period  $t$ , consumer  $i$  has a vehicle of age  $j < \eta$ —say,  $j = \eta - \tau$ —where  $\tau$  is the number of periods the consumer must wait to become eligible for the subsidy program, with  $\tau = 1, \dots, \eta - 1$ . Assume that the consumer's valuation is so high that she replaces the vehicle in period  $t$ , while she is not eligible to cash in the subsidy, i.e., she is a high-value consumer. Also, assume that a subsidy is available in period  $t + \tau$  and that the consumer is forward-looking, meaning that she anticipates her eligibility for the subsidy,  $r = \tilde{r}$ , in period  $t + \tau$ . We are interested in characterizing a threshold for the subsidy at which the consumer postpones replacing her vehicle from period  $t$  to  $t + \tau$  in order to benefit from the subsidy.

When the consumer replaces her vehicle at period  $t$ , her current and discounted cost-to-go from period  $t$  to  $t + \tau$  is as follows:

$$C_t(0) + p - q_t \cdot \theta_i + \sum_{a=1}^{\tau} \beta^a C_{t+a}(a) \quad (13)$$

On the other hand, if she decides to wait until period  $t + \tau$ , to be able to cash in the subsidy when she replaces at age  $\eta$ , then she will incur the following cost:

$$\beta^{\tau} (p - q_{t+\tau} \cdot \theta_i - \tilde{r} + C_{t+\tau}(0)) + \sum_{a=0}^{\tau-1} \beta^a C_{t+a}(\eta - \tau + a), \quad (14)$$

where the first part is the discounted cost of purchasing a new vehicle in period  $t + \tau$  and the second part is the discounted cost of keeping the vehicle for  $\tau - 1$  periods. Note that at period  $t + a$ ,  $a = 1, \dots, \tau - 1$ , the age of the vehicle is  $\eta - \tau + a$ . Since we assumed that the consumer will replace the vehicle at period  $t$  when the subsidy is  $\check{r}$ , the cost in (13) is lower than in (14). However, this changes if the government increases the subsidy from  $\check{r}$  to  $\check{r} + s(i, \tau)$  where  $s(i, \tau)$  is given by

$$s(i, \tau) \geq \frac{1}{\beta^\tau} \left( \beta^\tau (p - q_{t+\tau} \cdot \theta_i - \check{r} + C_{t+\tau}(0)) + \sum_{a=0}^{\tau-1} \beta^a C_{t+a}(\eta - \tau + a) \right. \quad (15)$$

$$\left. - C_t(0) - p + q_t \cdot \theta_i - \sum_{a=1}^{\tau} \beta^a \cdot C_{t+a}(a) \right). \quad (16)$$

Then, the consumer keeps the vehicle for  $\tau$  periods and replaces at period  $t + \tau$ .

To recapitulate, the above discussion shows that in a  $T$ -period setting, as in the two-period case, increasing the subsidy triggers a delay in vehicle replacement which in turn, leads to higher emissions and lower car sales. **In a  $T$ -period setting, the threshold (7) in Proposition 1 (two-period model) is replaced by (16) for  $\tau = 1, \dots, \eta - 1$ . Similarly, having the information about the number of vehicles with age  $1, \dots, \eta - 1$ , we can rewrite Proposition 2 for the general case. However, here, in order to be able to rank  $s(i, \tau)$  for different values of  $\tau$ , we need the full information about the shape of the function  $C_t(j)$ . In addition, since the value function is not monotone in age, the RHS of (11) is also not monotone in  $\tau = 1, \dots, \eta - 1$ . This adds to the complexity of characterizing an upper bound for  $x$  in the  $T$ -period setting. Nevertheless, Proposition 3 holds for different values of  $\tau$  separately. In a nutshell, all the qualitative results obtained with 2 periods seem to carry over to  $T$  periods.**

## 4 Empirical Results

In this section, we illustrate our general model using available data, and run a sensitivity analysis to assess the impact of the main model's parameter values on the results.

## 4.1 Parameters setting

To run our model, we need to estimate some parameter values and to fit some functions. To begin with, we shall solve the problem for a base-case scenario characterized by the following parameter values:

$$\begin{aligned} T^s &= 3, \quad T = 10, \quad \omega = 14, \quad \beta = 0.9, \quad \eta = 10, \\ p &= 33.5, \quad O_t = 1, \quad e = 2.6, \quad n_t = 0, \quad D = 25000, \end{aligned}$$

where we let the subsidy rate and trade-in valuation have different values in order to capture the impact of the subsidy program on different market segments, that is,

$$r \in \{0, 100, \dots, 4,000\}, \quad \theta \in \{22, 23, \dots, 38\}.$$

In order to account for consumer heterogeneity, we follow Huang et al. (2014), where the net trade-in valuation is assumed to be normally distributed, with a mean  $E(\theta)=30,000$  and standard deviation  $\sigma=4,000$ . We assume that  $\theta$  is deterministic and constrain its values to  $E(\theta) \pm 2\sigma$  and, also as in Huang et al. (2014), we fix  $p = 33,500$ . In accordance with scrappage programs in some European countries such as France, Spain, and Ireland, we assume that the vehicle's age of eligibility for the subsidy is 10 years. Note that, while the duration of the subsidy program is 3 years, the (consumer's and government's) planning horizon is longer, i.e., 10 years. Further, in order to identify the number of vehicles of different ages at the initial date, we use the density distribution of vehicles aged 1 to 14 provided in Engers et al. (2012).

To run the model, we need to give values to  $f(t-j)$ ,  $m(j)$  and  $\mu(j)$ . To estimate  $f(t-j)$ , we follow Spitzley et al. (2005), where the miles per gallon (MPG) for passenger cars is assumed to increase 1% yearly with respect to 22.8 MPG in 1995. Then, we use the MPG data to estimate  $f(t-j)$  as follows:

$$f(t-j) = \frac{3.8}{1.6} \left( \frac{1}{MPG(t-j)} \right),$$

which results in fuel consumption in liters per kilometer. To estimate the maintenance cost  $m(j)$ , we take data from YourMechanic (2016), which gives this cost per block of 25,000 miles

driven (see Table 2).

Mileage	Total Maintenance Cost per 25k Miles
0-25,000	\$1,400
25,000-50,000	\$2,200
50,000-75,000	\$3,000
75,000-100,000	\$3,900
100,000-125,000	\$4,100
125,000-150,000	\$4,400
150,000-175,000	\$4,800
175,000-200,000	\$5,000

Table 2: Maintenance cost for each 25,000 miles driven

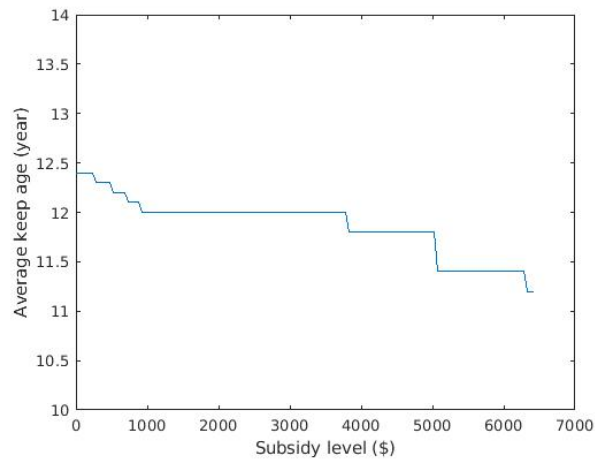
The emissions deterioration factor is assumed to increase by 0.09 from age 1 to 10 and then to remain constant for the rest of the vehicle’s life (Zyl et al., 2015):

$$\mu(j) = \begin{cases} 1 & \text{for } j = 0, \\ \mu(j - 1) + 0.09 & \text{for } 1 \leq j \leq 10, \\ \mu(10) & \text{for } j \geq 11. \end{cases}$$

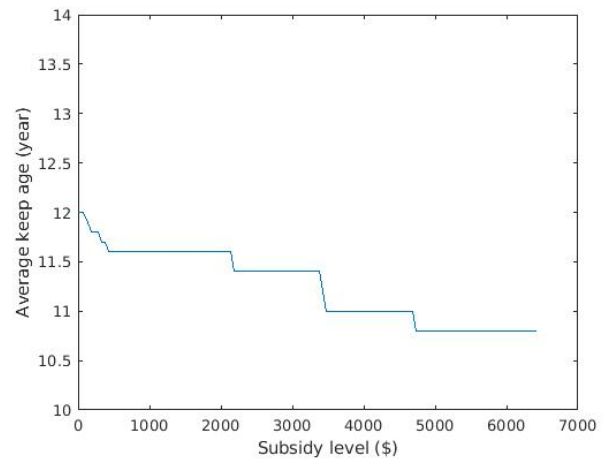
## 4.2 Heterogeneous consumers’ decisions

Considering the functions and parameters described above, we evaluate the effect of the subsidy on the decision to replace, by solving the dynamic program. Figures 1 and 2 show how the subsidy affects the replacement decisions of the different groups of consumers. The average keep age demonstrates how long vehicles are kept by consumers on average during the horizon.

One main takeaway is that increasing the subsidy does not necessarily result in earlier replacements for all groups of consumers. This surprising result stems from the subsidy program having an indirect effect on some consumers, who will prefer to postpone replacing their car in order to benefit from the subsidy, instead of paying the full price themselves. By now we are able to answer our first research question, namely, what effect does the subsidy have on

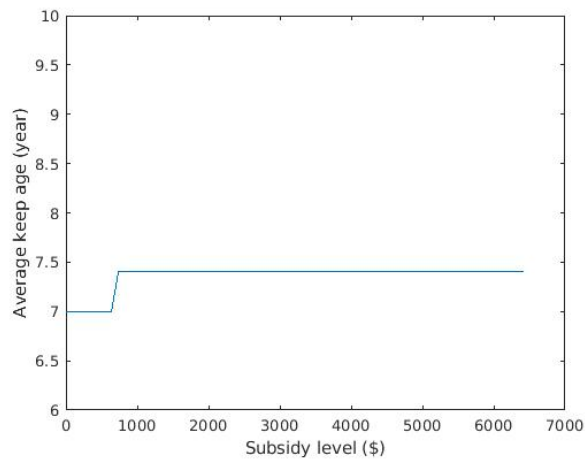


(a)

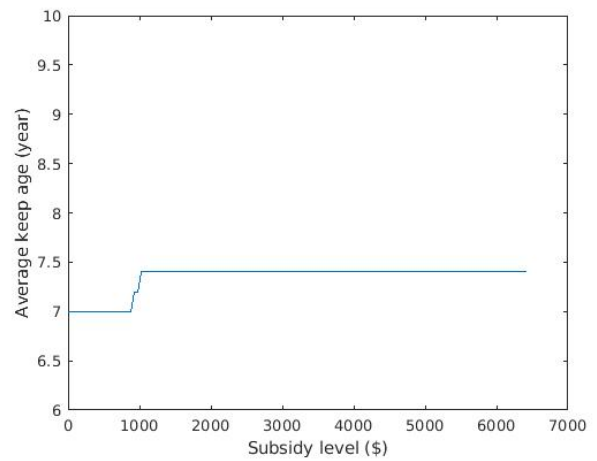


(b)

Figure 1: Average keep age decreases with subsidy for consumers with a valuation of (a) 25,000 and (b) 27,000.



(a)



(b)

Figure 2: Average keep age decreases with subsidy for consumers with a valuation of (a) 35,000 and (b) 37,000.

heterogeneous consumers’ decisions? The answer is different for consumers whose trade-in valuation is high than for those with a low valuation. When consumers have a low valuation—say 25,000 or 27,000 (as in Figure 1)—increasing the subsidy induces earlier replacements, which results in a decreasing trend in the average keep age. This intuitive result holds true for all “low-value” consumers whose trade-in valuation ranges between 22,000 and 31,000. In contrast, “high-value” consumers with a trade-in valuation higher than \$31,000—say 35,000 or 37,000 (as in Figure 2)—respond to an increase in the subsidy by replacing later. Tables 3 and 4 clarify this behavior in high-value consumers more precisely. Consumers with a valuation of 35,000 replace vehicles aged 8 or 9 at subsidy levels under 1,230 and 680, respectively. However, above the thresholds, they keep these vehicles until age 10 so they can benefit from the subsidy. Similarly, consumers with a valuation of 37,000 postpone replacing, as shown in Table 4.

Vehicle age	Subsidy range to replace	Subsidy range to keep
8	$r < 1230$	$r \geq 1230$
9	$r < 680$	$r \geq 680$

Table 3: Replacement decision of consumers with a valuation of 35,000, for a vehicle aged 8 and 9

Vehicle age	Subsidy range to replace	Subsidy range to keep
8	$r < 1830$	$r \geq 1830$
9	$r < 930$	$r \geq 930$

Table 4: Replacement decision of consumers with a valuation of 37,000 for a vehicle aged 8 and 9

### 4.3 Stock of pollution and total car replacements

So far, we have studied separately the reaction of different groups of consumers to a variation in the subsidy level. The next step is to mix consumer groups to find the subsidy’s environmental effect and market impact in the presence of heterogeneity. These effects are shown in Figures 3 and 4. According to Figure 3a, the effect of increasing the subsidy on the total amount of pollution by all consumer groups is decreasing, despite some ups and downs. When consumers are divided into low-value and high-value groups, this result does not necessarily

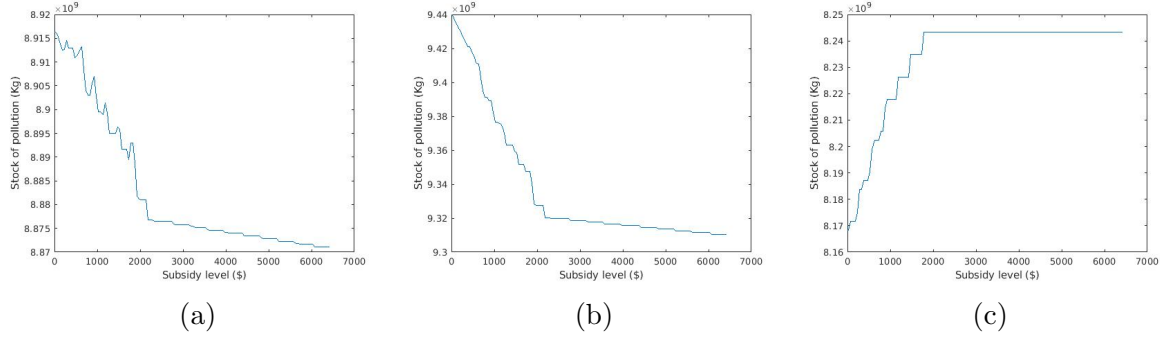


Figure 3: Effect of subsidy on stock of pollution, considering consumers with (a) both high and low valuations, (b) only a low valuation and (c) only a high valuation.

hold true. More precisely, when consumers have a low valuation, ranging between 22,000 and 31,000, the trend is downward (Figure 3b). However, when we only take into account high-value consumers, whose valuation is between 32,000 and 38,000, the subsidy has a negative effect on the environment (Figure 3c).

Similarly, the effect of the subsidy on the total number of replacements during the planning horizon is not the same for high-value and low-value consumers. While increasing the subsidy increases replacements in low-value consumers (Figure 4a), it has no effect on new vehicle purchases by those with a high valuation (Figure 4b). Indeed, the subsidy only delays new vehicle purchases by high-value consumers and does not have any impact on the total number of cars replaced by them. Consequently, older vehicles remain on the roads longer, which is environmentally harmful.

Now, we turn to the paper's second question, namely, what is the effect of the subsidy on the environment when consumer heterogeneity is taken into account? The rationale behind this question is both methodological and practical. If consumers are indeed heterogeneous, then retaining a homogeneous representation would lead to, at best, a very rough approximation of their behavior. At worst, it would lead to a completely wrong result. Practically speaking, predicting numbers of replacements when assuming a homogeneous market will be erroneous and misleading to the public decision-maker attempting to design a subsidy program. To obtain a hint of how the results differ under a homogeneous and a heterogeneous market, we run our model under the two scenarios. To do so, suppose that heterogeneous consumers are uniformly distributed over the set  $\Theta \in \{22,000; 23,000; \dots; 38,000\}$  of trade-in values. Also assume that,

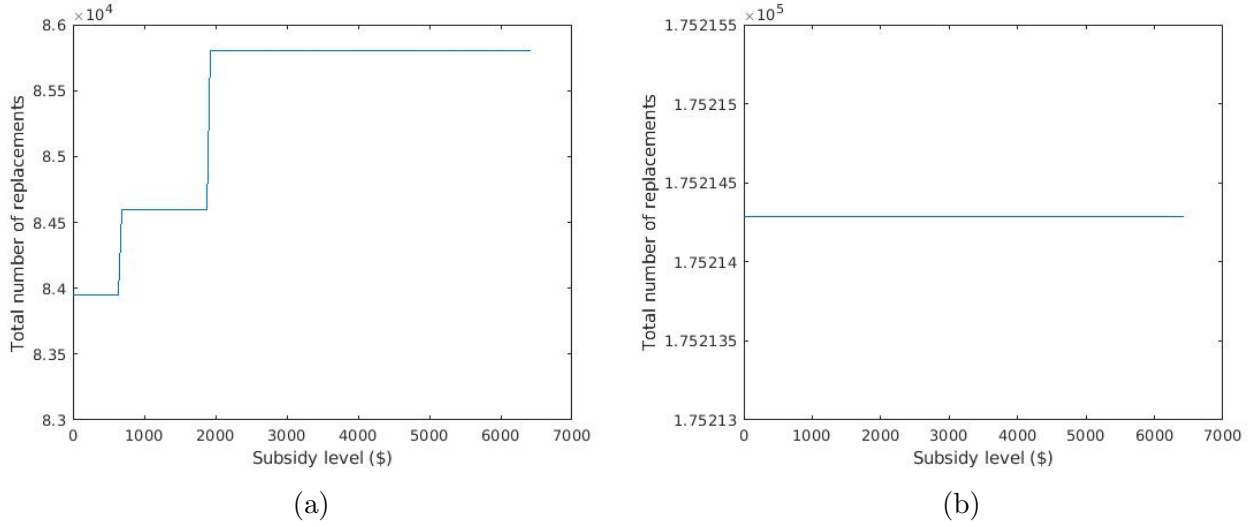


Figure 4: Effect of subsidy on total replacements during the horizon, considering consumers with (a) only a low valuation and (b) only a high valuation.

in a homogeneous setting, the only consumer group has a net trade-in value of  $\tilde{\theta} = 30,000$ , which is the middle point in the set  $\Theta$ . In Figure 5, we plot the values of the stock of pollution (panel (a)) and the number of vehicle replacements (panel (b)) for different subsidy level values. From Figure 5a, we see that giving a \$2,000 subsidy leads to a reduction in pollution of  $2 \times 10^8$  kilograms (from  $8.7 \times 10^9$  to  $8.50 \times 10^9$ ) when we assume consumer homogeneity. It also leads to a much lower decrease, of  $3 \times 10^7$  kilograms (from  $8.82 \times 10^9$  to  $8.79 \times 10^9$ ) under the heterogeneity assumption. Such a large difference is observed for all other subsidy values. Similarly, we obtain very different estimates for the number of vehicle replacements under the two market compositions (see Figure 5b). In particular, for a subsidy of \$2,000, the number of additional new vehicle purchases, as compared to a no-subsidy case, is estimated at 18,500 and 1,088 when consumers are assumed to be homogeneous and heterogeneous, respectively. By any measure, this difference is sizeable. The immediate implication of these comparative results is that the reduction in pollution and the number of new vehicle purchases resulting from a scrappage program would be highly exaggerated if the regulator used a mean trade-in valuation to predict the impact of the program.

We also compared the cost of the program under both assumptions: the results are exhibited in Figure 6. As previously discussed, accounting for heterogeneity makes it possible to assess the magnitude of high-value consumers who, even without the program, would replace their vehicle before it reached the program eligibility age. This enables us to measure the money wasted on these consumers who do not need the subsidy as an incentive to replace their vehicle.

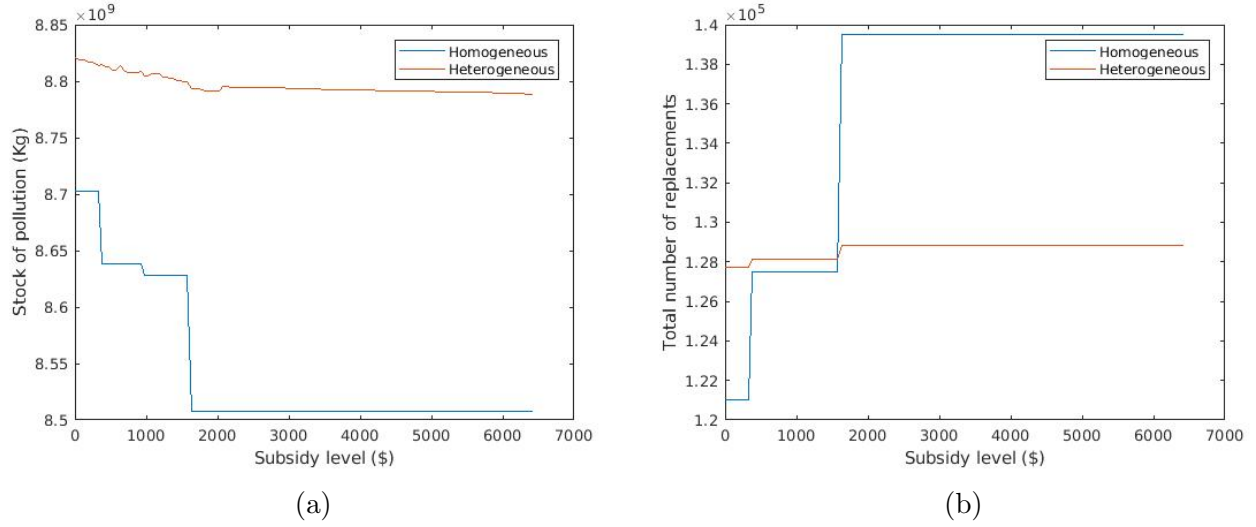


Figure 5: Effect of subsidy on a) the stock of pollution and b) total replacements, for homogeneous and heterogeneous consumers.

Figure 6b illustrates that, in our case study, up to 40% of the total subsidy spending is invested on consumers with a high valuation. This result is in line with those in Hoekstra et al. (2015) and Li and Linn (2013), which empirically show that 60% and 45%, respectively, of the subsidies in Cash for Clunkers went to households that would have replaced their vehicle even without the program. As stated in Dill (2004), Kim et al. (2004), and Lenski et al. (2010), although scrappage programs improve both the environment and the car market overall, they might not be as efficient as could be imagined. These papers identified pollution from car production as the main cause of scrappage program inefficiency. Here, by considering consumer heterogeneity, we are able to better estimate how such programs positively impact pollution emissions from vehicle usage. This provides scholars and policy makers with a greater understanding of scrappage program efficiency.

#### 4.4 Sensitivity analysis

Finally, we conduct a sensitivity analysis to assess the robustness of our results with respect to changes in some of the parameter values. The retained parameters and their values are listed below:

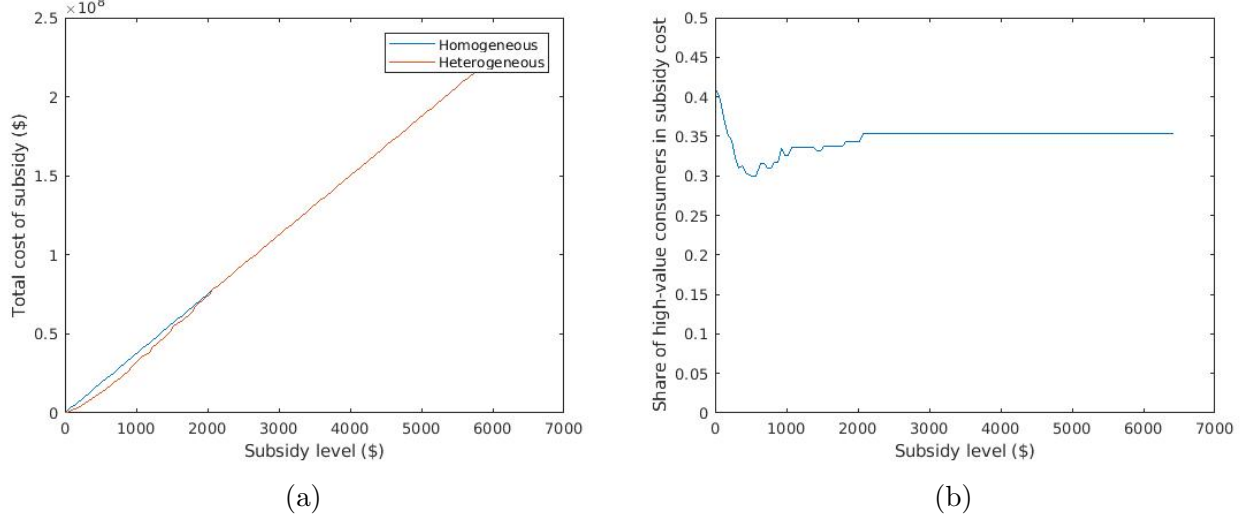


Figure 6: a) Total subsidy cost for homogeneous and heterogeneous consumers, and b) Share of total subsidy spent on high-value consumers

$$T \in \{9, 10, 11\}, T^s \in \{2, 3, 4\}, \beta \in \{0.85, 0.9, 0.95\}, \eta \in \{9, 10, 11\}, \\ \zeta \in \{7, 8, 9\}, D \in \{24000, 25000, 26000\}, MPG(t-j) \in \{0.008, 0.01, 0.012\}.$$

The results can be summarized as follows:

1. Increasing the value of  $T$  leads to an increase in the stock of pollution (see Figure 7). This figure calls for two observations. First, the incremental pollution is almost the same for all subsidy levels. From a subsidy-program design perspective, this result tells the regulator that the main effects of these two parameters are not correlated, and hence, can be considered independently. Second, adding one year to the planning horizon, i.e., shifting  $T$  from 9 to 10 or from 10 to 11 (an increase of 11%) leads to an increase in the pollution stock of roughly  $0.9 \times 10^9$  kg or 8.65%. Although this is far from negligible, the increase in pollution is of a lower magnitude than the increase in period length. The intuition behind this result is as follows: old vehicles are scrapped at the beginning of the planning horizon (subsidy period), and for the rest of the horizon, the vehicle fleet is new. As pollution from a new, fuel-efficient vehicle fleet is lower than from older vintages, the impact on pollution of adding one period is not as high. The parallel curves in Figure 7 mean that varying  $T$  only has a quantitative impact on the stock of pollution, without affecting the qualitative behavior.

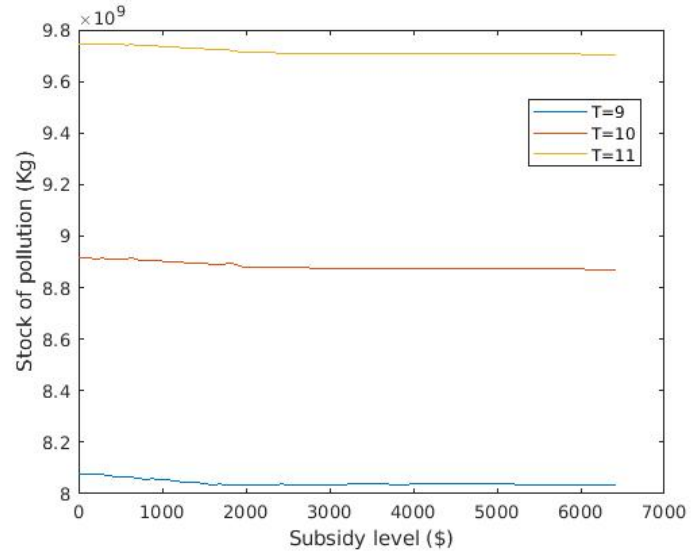


Figure 7: Effect of subsidy on stock of pollution for different values of  $T$

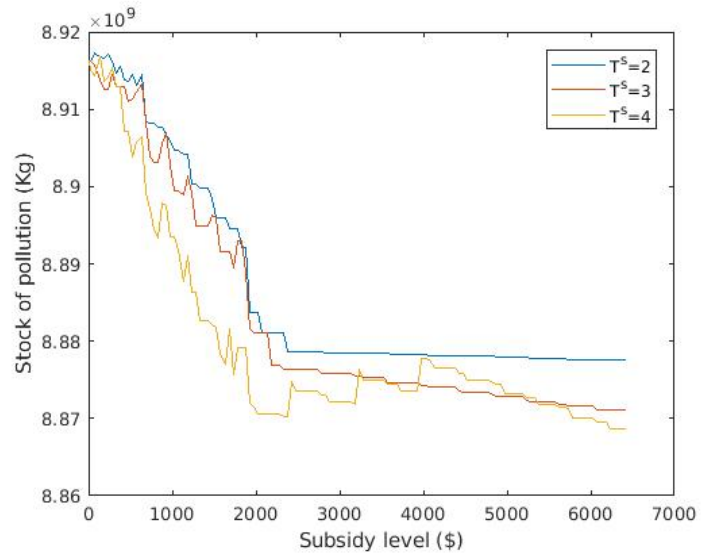


Figure 8: Effect of subsidy on stock of pollution for different values of  $T^s$

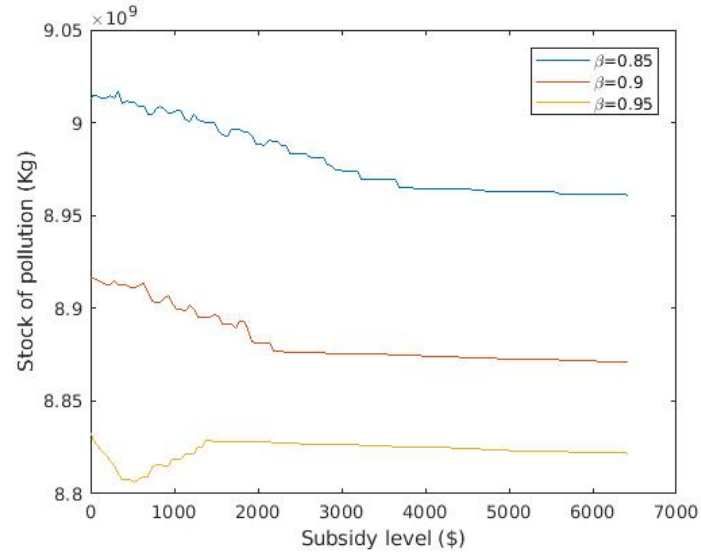


Figure 9: Effect of subsidy on stock of pollution for different values of  $\beta$

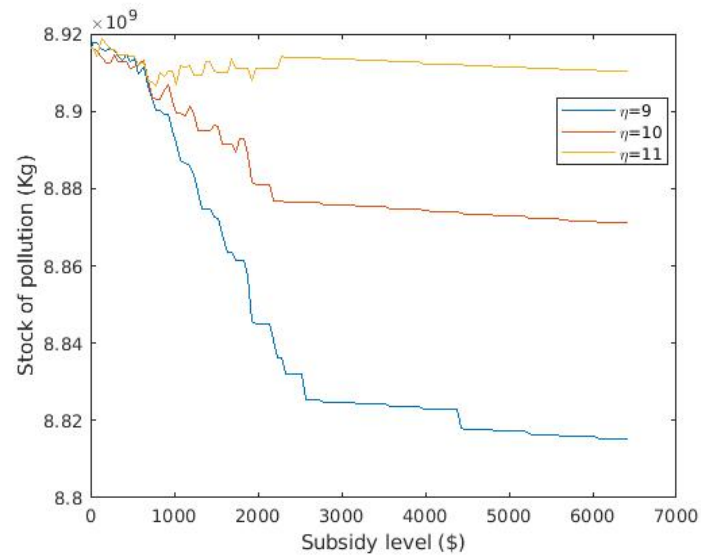


Figure 10: Effect of subsidy on stock of pollution for different values of  $\eta$

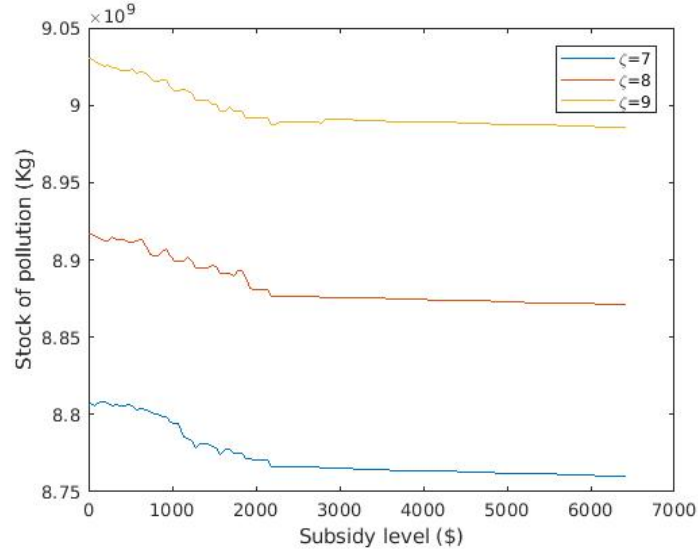


Figure 11: Effect of subsidy on stock of pollution for different values of  $\zeta$

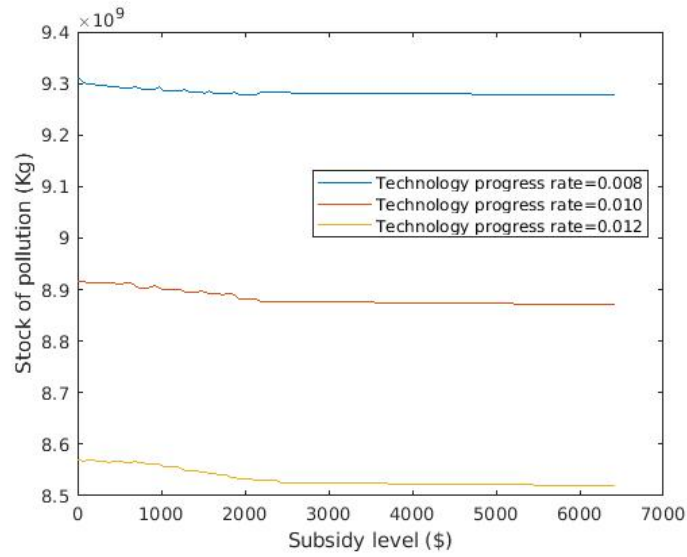


Figure 12: Effect of subsidy on stock of pollution for different values of  $f(t - j)$

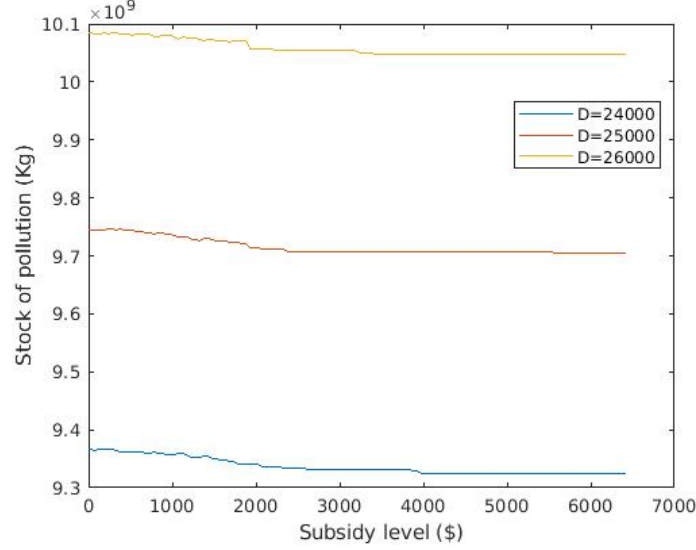


Figure 13: Effect of subsidy on stock of pollution for different values of  $D$

2. Varying the duration of the subsidy program seems to have very little effect on the stock of pollution (see Figure 8). This is because old vehicles, which are the main source of pollution, are eligible for the subsidy program in the first or second period of the subsidy program. Therefore, prolonging the subsidy period would have little effect on the replacement of cars, because the old ones are already off the road. This result should be of great interest to policymakers wishing to design scrappage programs. Indeed, it states that the duration of the subsidy program itself has little importance when it comes to environmental performance.
3. Despite some non-monotonic behavior for low subsidy values, varying the discount factor  $\beta$  essentially has an impact of the same magnitude on the pollution stock (see Figure 9). When consumers attach a higher importance to future costs, they are more inclined to purchase earlier to benefit from a new fuel-efficient car longer. In turn, the stock of pollution is lower.
4. From Figure 10, we see that varying the eligibility age  $\eta$  (of course naturally within our given range) has no effect on the stock of pollution when the subsidy is low. For higher levels of subsidy, the impact of  $\eta$  on this stock is very low in relative terms. Indeed, adding one year (an increase of 11%) leads to an increase in the pollution stock of at most 1%.
5. The relative impact of increasing the minimum age of replacement on the pollution stock

is rather small. Indeed, decreasing  $\zeta$  from 9 years to 7, which is a large decrease, leads to a reduction in the stock of pollution of less than 3% (see Figure 11). This result may seem counterintuitive, but it is not. Indeed, the result means that lowering further the minimum age of replacement does not help in reducing pollution. The reason is that consumer is anyway not interested in changing her vehicle if it is still “too” young.

6. Increasing the technology progress rate by 50%, that is, from 0.008 to 0.012, leads to an increase in the stock of pollution of roughly 10% (see Figure 12). On the one hand, a higher technological progress results in a newer vehicle fleet, which in turn, reduces pollution. On the other hand, it takes time for a whole fleet to be changed. This result is along the same line as the one just above.
7. Finally, for all subsidy levels, a larger travel demand induces an increase in pollution of the same magnitude (see Figure 13).

In summary, the main takeaway of our sensitivity analysis is that varying the parameter values does not much affect the qualitative results: we only see parallel shifts in the curves. Quantitatively speaking, the impact on pollution is at most of the same magnitude as the change in the parameter values. The conclusion is that our results are clearly robust, and should therefore be helpful for designers of scrappage programs.

## 5 Conclusion

In this paper, we applied a dynamic programming model to investigate the effect of vehicle scrappage programs on strategic consumers’ decision to replace their vehicle. Assuming that consumers are heterogeneous in terms of their net trade-in valuation, we examined the impact on different consumer groups of changing the government payment level. We theoretically show that under certain circumstances, increasing the government subsidy may have an adverse effect on some consumers. In particular, when the government payment exceeds a certain threshold, forward-looking consumers with a high net trade-in valuation will prefer to delay their replacement until their vehicle become eligible for the program.

Our computational experiments also demonstrate that, while a scrappage program induces low-value consumers to discard their old vehicle earlier, it may also motivate high-value consumers, who would have replaced their vehicle even without the subsidy, to replace it later.

Therefore, in addition to wasting the subsidy, encouraging high-value consumers to delay replacing their car is also harmful to the environment. These results emphasize the importance of accounting for heterogeneity in consumer valuations when a subsidy program is designed. We find that the net benefits of scrappage programs could be overestimated when consumers are taken to be homogeneous. More precisely, ignoring heterogeneity and relying only on the mean group of consumers results in higher estimates of pollution reduction and of new vehicle purchases when calculating program benefits. These results are in line with Li and Linn (2013), where 45% of the Cash for Clunkers program’s expenditure is estimated to be spent on those who would have replaced even without the program.

Two challenging extensions to our work are worth considering. First, our approach aimed at providing the regulator with results describing the impact of different subsidy levels on the outcomes (pollution and replacements). It would be interesting to give the regulator a more strategic role by assuming, for instance, that it plays as the leader in a non-cooperative game, where consumers are followers reacting to the leader’s subsidy announcement. The main challenge when computing Stackelberg equilibria is in solving the resulting high-dimensional dynamic program. Second, it is worth attempting to relax the assumption that the subsidy is constant over time and to introduce a time-varying one. Where needed, this would give the regulator a tool to smooth replacements over time.

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