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# Quorum Rules and Shareholder Voting\*

Patricia Charléty<sup>†</sup>, Marie-Cécile Fagart<sup>‡</sup> and Saïd Souam<sup>§</sup>

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## Abstract

We analyze how a minimum quorum affects shareholder voting in meetings. We show that a quorum creates an incentive for coalition formation of shareholders supporting the resolution. It works as a coordination device for possibly small shareholders allied in a winning voting coalition. It also generates an equilibrium in which the resolution is not adopted due to lack of quorum. The shareholder structure is central to the determination of the outcome of the vote. A resolution supported by the dominant shareholder is always adopted if his ownership reaches the quorum. However, allied blockholders can successfully approve a resolution opposed by the dominant owner. As a consequence, it is more effective for an active shareholder to propose and pass a resolution than to oppose a board resolution. Finally, we find that the dominant shareholder de facto controls the meeting when his share by far exceeds the second largest share.

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<sup>†</sup>ESSEC Business School and THEMA. email: [charley@essec.edu](mailto:charley@essec.edu)

<sup>‡</sup>Université Paris Descartes. email: [marie-cecile.fagart@univ-paris5.fr](mailto:marie-cecile.fagart@univ-paris5.fr)

<sup>§</sup>Université Paris Nanterre (EconomiX) and CREST. email: [souam@ensae.fr](mailto:souam@ensae.fr)

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# 1 Introduction

Recent years have seen regulatory developments aiming at empowering shareholders in annual meetings.<sup>1</sup> Although this objective is advocated by many practitioners and academics (e.g., Bebchuk, 2005), the value of shareholder democracy is regularly questioned. Shareholders are heterogeneous. When turnout is low, the outcome of the vote possibly contradicts the majority view. High participation supposedly increases legitimacy of the vote, particularly when owners disagree. To increase turnout, a minimum quorum is generally required for all or some decisions.<sup>2</sup>

In this paper, we show that requiring a quorum rule affects the voting behavior of owners and impacts the outcome of annual meetings. Anticipating that their vote is necessary to reach the quorum and pass a resolution, shareholders who would otherwise withhold decide to vote: a minimum quorum works as a coordination device for shareholders allied in a winning coalition. A minimum quorum also generates an equilibrium in which shareholders strategically do not vote when they expect the quorum will not be reached: the resolution is not adopted due to the lack of a quorum.

Moreover, our model implies that a resolution supported by the largest (or dominant) shareholder is always adopted if his ownership reaches the quorum. In all other cases, allied shareholders can successfully approve a resolution opposed by the dominant owner. Our results suggest that it is easier for an active blockholder to propose and pass a resolution than to oppose a board resolution. Shareholder proposals are therefore an important component of corporate governance. More generally, we contribute to a better understanding of the power exercised by significant shareholders in meetings. Defining control as the power to pass and to block any resolu-

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<sup>1</sup>Voting has been facilitated through electronic voting, the adoption of the "record date" or the use of proxy advisors' services. Along the same line, the scope of resolutions that have to be put to a vote has widened, the Say On Pay rule is an example.

<sup>2</sup>In Europe, the legal quorum varies across countries: for example, 25% of shares in France, no legal quorum in Germany while two shareholders are a quorum in the UK. In many countries (e.g., Germany, Italy, the UK, the USA), the quorum is defined in the corporate charter and may be modified during the meetings.

tion, we find that the dominant shareholder controls the meeting when his stake reaches the quorum and significantly exceeds the ownership of the second largest shareholder. Such a controlling interest may represent much less than 50% of the voting stock: we provide an operational interpretation of the notion of de facto control.

Two lines of literature are directly related to this research: the analysis of quorum rules, and voting in shareholder meetings.

A few articles analyze the consequences of the introduction of quorums on voter behavior, turnout and the outcome of the vote in referenda. Herrera and Mattozzi (2010) and Aguiar-Conraria and Magalhães (2010) find that the equilibrium expected turnout may actually be lower with a participation quorum. Most models in the political science literature consider many voters, with one vote each. Ritzberger (2005) analyzes strategic voting in annual meetings when stockholders with different shares disagree on a resolution. He concludes that an equilibrium exists if and only if the largest shareholder supports the resolution. Cvijanovic et al. (2017) analyze participation in a model where a group of "regular voters" always vote while the second group consists of a very large number of identical "discretionary voters" who choose to vote strategically. They show that similar preferences across groups lower the participation rate. As in Ritzberger (2005), we consider several strategic owners with different shares and analyze their power. By adding a minimum quorum, our results differ substantially.

The paper is organized as follows. Section 2 describes the model and details shareholders' equilibrium strategies. Section 3 analyzes the power of blockholders and derives the conditions for de facto control. Section 4 concludes.

## **2 Shareholder voting**

We model the annual meeting as a simultaneous game in which each shareholder decides to vote for or against a resolution or withhold based on his expectations about other shareholders' strategies. Hereafter, we detail the structure of the game, the properties of equilibrium strategies and the Nash

equilibria.

## Shareholders: preferences and strategies

Shareholders have different ownerships, and hence, different voting powers. They may be in favor of a resolution, or opposed to it. They incur a small voting cost and are not required to vote. Some shareholders never vote: holding a very limited ownership, their (private) benefit from obtaining their preferred outcome is smaller than their voting cost. The others decide strategically to vote, or to not participate (to withhold their vote). The letter  $F$  stands for "in favor of the resolution", and  $A$  stands for "against the resolution".  $\mathcal{S}^F = \{\alpha_1^F, \alpha_2^F, \dots\}$  represents the set of the voting shares of these favorable owners (hereafter shareholders  $F$ ), with  $1 \geq \alpha_1^F \geq \alpha_2^F \dots$ . Similarly,  $\mathcal{S}^A = \{\alpha_1^A, \alpha_2^A, \dots\}$  represents the set of voting shares of the shareholders against the resolution (shareholders  $A$ ), with  $1 \geq \alpha_1^A \geq \alpha_2^A \geq \dots$ . Note that  $\sum_{\alpha_i^F \in \mathcal{S}^F} \alpha_i^F + \sum_{\alpha_j^A \in \mathcal{S}^A} \alpha_j^A \leq 1$  since some shareholders never vote.

With costly voting, voting against one's preferred alternative is strictly dominated by withholding. Therefore, we consider only two possible actions: voting according to the preferred alternative, or withholding the vote. All shareholders are assumed to know all others' voting shares and preferences (perfect information). They simultaneously choose their best strategy given their expectations about other shareholders' strategies. Our objective is to predict a clear outcome, and analyze the conditions under which one or several shareholders control the meeting. This is why we focus on pure strategies, as in Ritzberger (2005). Indeed, in mixed strategy equilibria, the result of the meeting is uncertain.<sup>3</sup> We thus do not need to specify the payoffs explicitly. In our analysis, it is enough to use the following ranking of

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<sup>3</sup>The analysis of costly voting in political elections has focused on mixed strategy equilibria because pure strategy equilibria arise only in degenerate cases under one-head-one-vote (Palfrey and Rosenthal 1983). In our framework with asymmetric voting shares, interesting pure strategy equilibria exist. Moreover, our very general model with different voting weights, the most common case in practice, would be untractable in mixed strategies.

the different outcomes for shareholders. When the private gain from obtaining the preferred result exceeds the voting cost, the best outcome for any shareholder  $F$  is adoption without participation, the second-best is adoption with participation, which is better than rejection without participation, and the worst is rejection with participation. A symmetric reasoning applies to shareholders  $A$ . This very general formalization encompasses any gain from winning the vote and any voting cost.

Two conditions must be verified for a resolution to be adopted. First, a minimum proportion, denoted  $Q$ , of total shares must be voted (quorum rule). Second, the resolution must obtain a strict majority of favorable votes (simple majority rule). When both conditions hold, the resolution is adopted. If either the quorum or the strict majority is not met, the resolution is rejected.

We look for the pure strategy Nash equilibria of this game. Throughout the paper, equilibrium  $\mathcal{F}$  refers to a pure strategy Nash equilibrium in which the resolution is adopted. Symmetrically, equilibrium  $\mathcal{A}$  refers to a pure strategy Nash equilibrium in which the resolution is rejected.

## Nash equilibria with a Quorum Rule

Before fully characterizing the Nash equilibria of the voting game, we establish two properties verified at equilibrium, and that simplify the analysis. They follow directly from the above ranking of the outcomes.

**Property 1.** **No shareholder  $A$  votes at equilibrium  $\mathcal{F}$ . Similarly, no shareholder  $F$  votes at equilibrium  $\mathcal{A}$ .**

Indeed, suppose that a shareholder opposed to the resolution votes at equilibrium  $\mathcal{F}$ . In that case, he is better off not voting, since the outcome is the same without bearing the cost of voting. Thus the situation in which he votes is not an equilibrium. The same reasoning applies to equilibrium  $\mathcal{A}$ .

**Property 2.** **At equilibrium  $\mathcal{F}$ , a shareholder  $F$  participates in the vote if and only if he is pivotal, i.e., his vote is necessary to obtain his preferred outcome. Similarly, at equilibrium  $\mathcal{A}$ , a shareholder  $A$  casts a vote if and only if he is pivotal.**

Suppose a shareholder voting for the resolution expects its adoption. If, given the others' strategies, the result remains adoption if he does not vote, his best strategy is to withhold, since voting is costly. Thus, at equilibrium  $\mathcal{F}$ , no shareholder  $F$  participates when his preferred outcome (adoption) emerges without his vote. The same reasoning applies to equilibrium  $\mathcal{A}$ .

These properties have two consequences for equilibria, when they exist. First, an equilibrium  $\mathcal{A}$  is necessarily "non voting" with zero turnout. Indeed, suppose one or several shareholders vote against the resolution (shareholders  $F$  never vote at equilibrium  $\mathcal{A}$ ). All of them are better off withholding since the resolution is still rejected without their vote: no voter is pivotal. Second, on the contrary, an equilibrium  $\mathcal{F}$  is necessarily "voting" since the quorum must be reached. Moreover, since no shareholder  $A$  votes at equilibrium  $\mathcal{F}$ ,  $\sum_{\alpha_i^F \in \mathcal{S}^F} \alpha_i^F > Q$  is necessary. We assume it is verified for non-triviality.

Before characterizing the equilibria, let us define a threshold that plays an important role. Consider the coalitions of shareholders  $F$  that cannot be challenged by any shareholder  $A$ . Let  $V^F \subset \mathcal{S}^F$  be the corresponding sets of ownerships, thus  $\sum_{\alpha_i^F \in V^F} \alpha_i^F > \alpha_1^A$ . Among those coalitions, take one that minimizes aggregate shares after removing the smallest ownership from the coalition. Let  $\alpha_m^F$  denote the total shares of this coalition minus the smallest one.

**Proposition 1**

(1) *There exists a unique equilibrium  $\mathcal{A}$  in which no shareholder votes if and only if  $\alpha_1^F < Q$ ,*

(2) *When  $\alpha_1^A \geq Q$ , equilibria  $\mathcal{F}$  exist if and only if  $\alpha_m^F < Q$ . Voters are necessarily blockholders owning more than  $\alpha_1^A - Q$ .*

*When  $\alpha_1^A < Q$ , there always exists at least one equilibrium  $\mathcal{F}$ . Voters may own shares of any size.*

A formal proof of Proposition 1 is provided in the Appendix.

From Proposition 1, requiring a quorum has two consequences. On the one hand, it generates an equilibrium  $\mathcal{A}$  in which the resolution is not adopted

due to lack of quorum. Indeed, both supporters and opponents are deterred from voting when they believe that the quorum will not be reached. On the other hand, a minimum quorum works as a coordination device for supporters of the resolution who believe that their vote is necessary to win. It creates an incentive to form voting coalitions, gathering possibly small shareholders. These results contrast with the no quorum case in which at most one large enough (ownership greater than  $\alpha_1^A$ ) shareholder votes in favor in equilibrium, and no equilibrium  $\mathcal{A}$  ever exists (see Ritzberger 2005).<sup>4</sup> Example 1 illustrates the consequences of a quorum for the existence and nature of equilibria ( $\mathcal{F}$  vs.  $\mathcal{A}$ ) in the presence of blockholders.

**Example 1** *Consider the following shareholding structure:*

$$\mathcal{S}^A = \{14\%, 12\%\}$$

$$\mathcal{S}^F = \{11\%, 9\%, 8\%, 7\%, 4\%, 3\%, 2\%, 1\%, 1\%, 1\%, \dots\}$$

When  $Q = 25\%$ , there exists an equilibrium  $\mathcal{A}$  in which no shareholder votes because the largest shareholder  $F$  cannot change the outcome by voting ( $\alpha_1^F = 11\% < Q$ ). Simultaneously, any coalition that just reaches the quorum (e.g., with shares  $\{11\%, 9\%, 8\%\}$ ,  $\{9\%, 8\%, 3\%, 2\%, 1\%, 1\%, 1\%\}$ , ...) supports a voting equilibrium  $\mathcal{F}$  since the largest shareholder  $A$  cannot overturn any of those coalitions ( $\alpha_1^A = 14\% < Q$ ) and each voter is pivotal.

When  $Q = 10\%$ , coalitions with voting shares  $\{9\%, 8\%\}$ ,  $\{9\%, 7\%\}$ ,  $\{8\%, 7\%\}$  support an equilibrium  $\mathcal{F}$  ( $\alpha_1^F = 11\% < Q$ ,  $\alpha_m^F = 8\% < Q$ , voters must hold more than  $\alpha_1^A - Q = 4\%$ ). No equilibrium  $\mathcal{A}$  exists because the largest shareholder  $F$  can change the result from rejection with no turnout to adoption by voting for ( $\alpha_1^F = 11\% \geq Q$ ).

Finally, when  $Q = 0\%$  (cf. Ritzberger 2005), no equilibrium in pure strategies exists ( $\alpha_1^F = 11\% \geq Q$ ,  $\alpha_m^F = 8\% \geq Q$ ).

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<sup>4</sup>Note that the necessary and sufficient condition for the existence of an equilibrium  $\mathcal{A}$  is independent of the structure of the set of shareholders  $A$ . The resolution may be rejected even when opponents together do not reach  $Q$ . Thus a minimum quorum does not necessarily increase representativeness or turnout.

### 3 Shareholder power

The ownership structure is central to the determination of the outcome of the vote. Some firms are widely held, others have one large dominant shareholder in addition to smaller ones, and others multiple blockholders.<sup>5</sup> We first analyze the role played by large supporters and large opponents. In light of our results, we compare the efficiency of two mechanisms available to blockholders to oppose the board. We then define control and examine when the dominant shareholder controls a company.

#### The role of blockholders

Major shareholders play a decisive role in meetings. However, given the asymmetry between equilibria  $\mathcal{F}$  and  $\mathcal{A}$ , their influence varies depending on whether they support or oppose a resolution.

#### Blockholders supporting the resolution

Large supporters facilitate the adoption of a resolution in two complementary ways. First, a large enough supporter ( $\alpha_i^F \geq Q$ ) prevents rejection due to lack of quorum (Proposition 1). Second, when the dominant shareholder supports the resolution ( $\alpha_1^F > \alpha_1^A$ ), equilibria  $\mathcal{F}$  always exist when  $\sum_{\alpha_i^F \in \mathcal{S}^F} \alpha_i^F \geq Q$ . Indeed, a coalition constructed according to decreasing size until  $Q$  is reached cannot be contested since it includes  $\alpha_1^F > \alpha_1^A$ .

#### Dominant shareholder opposed to the resolution

The case of a dominant shareholder opposed to the resolution is not symmetric to the case of a dominant supporter. Indeed, from Proposition 1, rejection depends solely on the ownership of the largest shareholder  $F$  relative to the quorum. Even if the largest opponent commands more votes than the largest

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<sup>5</sup>Several contributions provide a theoretical basis for these empirical regularities. See Zwiebel (1995), Bennedsen and Wolfenzon (2000) and Dhillon and Rossetto (2015).

supporter ( $\alpha_1^A \geq \alpha_1^F$ ), coalitions of shareholders may still pass the resolution in equilibrium, as illustrated in example 1.

### **The right to propose and the right to oppose**

The board of directors is responsible for organizing the annual meeting and proposes most resolutions. They generally pass by a very large majority (Lafarre, 2017). Blockholders also have the right to initiate resolutions.<sup>6</sup> When they disagree with management, they have the right to oppose resolutions submitted by the board, and the right to put resolutions to the vote. Given the fundamental asymmetry between equilibria  $\mathcal{F}$  and  $\mathcal{A}$ , the right to propose is more effective.<sup>7</sup> Indeed, even a large shareholder has no power to force rejection at equilibrium: from Proposition 1, only a high quorum requirement ( $Q > \alpha_1^F$ ) constitutes a credible threat to adoption. However, even a mid-sized blockholder may pass resolutions thanks to the vote of other shareholders united in a coalition as in example 1 for  $Q = 25\%$ .

### **Control of the meeting**

The ability to pass/block a resolution does not imply the ability to pass/block other resolutions as the same shareholders may agree on some issues while disagreeing on others. The power of a shareholder can be ascertained by his capacity to influence the result of the vote on all resolutions. So far, we examined the power of a shareholder in relation with his ownership and preference regarding a specific resolution. We now analyze power only as a function of shares held. We define the ownership structure by the set of voting shares  $\mathcal{S} = \{\alpha_1, \alpha_2, \dots\}$  with  $1 \geq \alpha_1 \geq \alpha_2 \geq \dots$ . By law, under the majority

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<sup>6</sup>In practice, a shareholder needs to own a minimum stake, usually around 5%, to have the right to put a resolution on the proxy statement. Proposals include changes in bylaws, nomination of directors, and even replacement of the board. About 25% of shareholder proposals are adopted in France (Charl  ty et al., 2009) and the UK (Renneboog and Szilagyi, 2011).

<sup>7</sup>Matusaka and Ozbas (2017) reach a similar conclusion: by submitting a proposal, the blockholder induces the board to move the agenda in his own interest.

rule, 50% plus one voting share is a controlling interest. In reality, minority but dominant shareholders often control companies de facto, particularly when the remaining equity is in the hands of small shareholders.<sup>8</sup> What is effective control? When does the dominant shareholder control the meeting? There is no consensus on the answers to these questions.

### **Controlling dominant shareholder**

We consider that the dominant shareholder controls the meeting when the unique equilibrium outcome of the vote is to adopt any resolution he supports and to reject any resolution he opposes. A dominant shareholder owning the majority of voting rights among potential voters<sup>9</sup> can, alone, consistently pass or block any resolution provided that he reaches the quorum. However, smaller dominant shareholders may possibly control the meeting. The following corollary of Proposition 1 provides a sufficient condition for control.

**Corollary 1** *The dominant shareholder controls the meeting if  $\alpha_1 - Q \geq \alpha_2$  and  $Q > \alpha_2$ .*

Suppose  $\alpha_1 \geq Q > \alpha_2$ . The dominant shareholder passes any resolution he supports by voting alone. If he opposes a resolution, the latter is always rejected. Indeed, the second largest shareholder, possibly favorable, does not reach the quorum. Moreover, no shareholder is large enough to belong to an equilibrium coalition  $F$  since  $\alpha_1 - Q \geq \alpha_2$ .

The ownership structure for control is asymmetric: one controlling blockholder and smaller powerless shareholders who are typically dispersed (taken together, the two conditions imply  $\alpha_1 > 2\alpha_2$ ).

### **Relative control with complex ownership structures**

The control exercised by the dominant shareholder is relative when  $\alpha_1 \geq Q > \alpha_2 > \alpha_1 - Q$ . While he passes any resolution he supports, resolutions

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<sup>8</sup>La Porta et al. (1999) consider control may be effective with as little as 20% equity when no other large (at least 10% equity) shareholder exists.

<sup>9</sup>This need not represent a majority of voting rights as defined by law since some shareholders never vote.

he opposes may be adopted in equilibrium. This is possible if there exists a coalition  $\mathcal{C}$  of blockholders with shares  $\alpha_i > \alpha_1 - Q$  for all  $i$  in  $\mathcal{C}$  and  $\sum_{i \in \mathcal{C}} \alpha_i > \alpha_1$ .

Similar to the case of control, the ownership structure associated with relative control is asymmetric but complex with three categories of voters: the dominant shareholder, several blockholders, and smaller powerless shareholders. Blockholders thus interact, they can form voting coalitions and pass a resolution to counter the dominant shareholder. In this case, voting blockholders need not differ substantially ( $\alpha_1 > \alpha_i > \alpha_1 - Q$ ). Edmans and Holderness (2017) point out that 5% or 10% are considered as high enough stakes to impact the company’s governance, but that there is no theoretical basis to define the relevant thresholds. Our analysis partly fills this gap in the context of shareholder meetings.

## 4 Conclusion

The exercise of shareholder power through the vote is central in corporate governance. However, when shareholders have conflicting interests and heterogeneous voting powers, active minorities may impose their will upon a passive majority. Quorum rules supposedly limit this bias. We show that quorums do not necessarily increase representativeness. They discourage voting when the ownership of the dominant shareholder is above the quorum and by far exceeds the second largest stake. However, a quorum facilitates the emergence of coalitions of significant shareholders that can act as an effective counterweight to the dominant shareholder. We therefore explain how blockholders interact in the context of meetings, which is largely overlooked by the theoretical literature despite its practical importance. Our analysis is inconclusive regarding the merit of empowering shareholders. Nevertheless, it contributes to a better understanding of the consequences of voting rules under different ownership structures.

## 5 Appendix: Proof of Proposition 1

### Equilibrium $\mathcal{A}$

From Property 1, no shareholder  $F$  votes in equilibrium  $\mathcal{A}$ . Since the resolution is always rejected when no shareholder  $F$  votes, no voting shareholder  $A$  is pivotal. Thus from Property 2, if it exists, an equilibrium  $\mathcal{A}$  is necessarily "non voting" with zero turnout. Indeed, suppose one or several shareholders vote against the resolution. All of them have an incentive to withhold since the outcome would not be affected. Consider the strategy profile with no vote. Withholding is optimal for each shareholder  $A$  as the resolution is rejected at no cost. If  $\alpha_1^F < Q$ , withholding is also optimal for each shareholder  $F$  whose vote alone cannot overturn the outcome. Conversely, if  $\alpha_1^F \geq Q$ , at least the largest shareholder  $F$  should switch to voting since he can pass the resolution alone.

### Equilibrium $\mathcal{F}$

From Property 1, no shareholder  $A$  votes in equilibrium  $\mathcal{F}$ . Let  $V^F \subset \mathcal{S}^F$  represent the subset of the ownerships of the shareholders  $F$  who vote at equilibrium. Three conditions must be verified:

(i) aggregated votes in favor must reach the quorum,  $\sum_{\alpha_i^F \in V^F} \alpha_i^F \geq Q$  (**quorum rule**); thus  $\sum_{\alpha_i^F \in \mathcal{S}^F} \alpha_i^F \geq Q$  is necessary for equilibria  $\mathcal{F}$ .

(ii) moreover, they must exceed the share of the largest shareholder  $A$ ,  $\sum_{\alpha_i^F \in V^F} \alpha_i^F > \alpha_1^A$  (**non-contestability condition**). Otherwise at least the largest shareholder  $A$  should switch to voting since he would, alone, represent a majority (simple majority rule); thus  $\sum_{\alpha_i^F \in \mathcal{S}^F} \alpha_i^F > \alpha_1^A$  is also necessary for equilibria  $\mathcal{F}$ .

(iii) every voting shareholder  $F$  and the smallest one in particular must be pivotal, i.e., necessary to meet the quorum (**pivotal voting condition**). The condition is either (a)  $\sum_{\alpha_i^F \in V^F} \alpha_i^F - \alpha_j^F < Q$  for any  $\alpha_i^F \in V^F$  for  $Q > 0$  (i.e.  $\sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F}(\alpha_j^F) < Q$ ) or (b)  $V^F = \{\alpha_i^F\}$  for  $Q = 0$ . The minimum quorum is not reached if any shareholder (therefore the smallest) in

the coalition does not participate in the vote (a). With no minimum quorum ( $Q = 0$ ), voting coalitions must be composed of only one shareholder  $F$  so that the turnout is zero when he does not vote (b) as in Ritzberger (2005).

Let us consider the coalitions of shareholders  $F$  that cannot be challenged, i.e., that satisfy condition (ii) and consider a coalition that minimizes

$$\sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F). \text{ This is one of the coalitions that gather the least votes}$$

when removing its smallest member.  $\alpha_m^F \equiv \text{Min} \left\{ \sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F) \right\}$ .

Depending on the value of  $Q$ , three cases emerge.

**Case 1 -  $Q = 0$**  As stated above, with no quorum, if an equilibrium coalition exists, it contains only one shareholder  $F$  (iii - b). Condition (i) is always met (no quorum requirement). Therefore, there exists an equilibrium  $\mathcal{F}$  if and only if  $\alpha_i^F > \alpha_1^A$  (iii) for some  $i$ . The necessary and sufficient condition simplifies to  $\alpha_1^F > \alpha_1^A$ . The situation in which the largest shareholder  $F$  votes alone is an equilibrium; there exist as many equilibria as the number of shareholders  $F$  such that  $\alpha_i^F > \alpha_1^A$  (cf. Ritzberger, 2005).

**Case 2 -  $\alpha_1^A \geq Q > 0$**  The necessary and sufficient conditions simplify to  $\alpha_1^A - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F) < \sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F) < Q$ .

Suppose there exists a coalition (with ownership  $V^F$ ) that supports an equilibrium  $\mathcal{F}$ . By definition, we have  $\alpha_m^F \leq \sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F)$ . Therefore,  $\alpha_m^F < Q$ .

Conversely, suppose that  $\alpha_m^F < Q$ . Thus every member of the coalition is pivotal. In addition, this coalition is, by definition, non-contestable. Thus, it is an equilibrium coalition: at least one equilibrium coalition exists.

Coalitions that cannot be overturned ( $\sum_{\alpha_i^F \in V^F} \alpha_i^F > \alpha_1^A$ ) automatically reach the quorum. Moreover, all shareholders are pivotal in equilibrium ( $\sum_{\alpha_i^F \in V^F} \alpha_i^F - \text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F) < Q$ ). As a consequence,  $\text{Min}_{\alpha_j^F \in V^F} (\alpha_j^F) > \alpha_1^A - Q$ . Therefore, shareholders in a winning coalition are necessarily blockholders.

**Case 3** -  $\alpha_1^A < Q$  (i) implies (ii), and a voting equilibrium always exists. Indeed, simply add up shareholders  $F$  in a coalition according to decreasing size until  $Q$  is reached (i). As the smallest voter is pivotal by construction, so are other voters in the coalition since they own a larger share (iii). Voting coalitions gather shareholders of any size as long as the quorum is met ( $\sum_{\alpha_i^F \in V^F} \alpha_i^F \geq Q$ ). They possibly encompass many small shareholders and the turnout is close to the minimum quorum in that case.

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