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Christian Urom, Julien Chevallier, Bangzhu Zhu

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## DATA IN BRIEF ARTICLE

<b>*Title:</b>	<i>Dataset for a dynamic conditional regime-switching GARCH CAPM for energy and financial markets.</i>
<b>*Authors:</b>	<i>Christian Urom<sup>1</sup>, Julien Chevallier<sup>2</sup> Bangzhu Zhu<sup>3</sup></i>
<b>*Affiliations:</b>	<i>Alex Ekwueme Federal University Ikwo (Nigeria) and University Paris 8 (LED)<sup>1</sup> IPAG Business School (IPAG Lab) and University Paris 8 (LED)<sup>2</sup> <sup>3</sup>School of Management, Jinan University, Guangzhou 510632, China and Business School, Nanjing University of Technology &amp; Information Science, Nanjing 210044, China</i>
<b>*Contact email:</b>	<a href="mailto:jpchevallier@gmail.com">jpchevallier@gmail.com</a>
<b>*Co-authors:</b>	<i>Christian Urom Julien Chevallier Bangzhu Zhu</i>
<b>*CATEGORY:</b>	<i>Economics, Econometrics and Finance</i>

### Data Article

**Title:** *Dataset for dynamic conditional regime-switching GARCH CAPM for energy and financial markets.*

**Authors:** *Christian Urom<sup>1</sup> and Julien Chevallier<sup>2</sup>*

**Affiliations:** *<sup>1</sup>Alex Ekwueme Federal University Ikwo (Nigeria) and University Paris 8 (LED) and <sup>2</sup>IPAG Business School (IPAG Lab) and University Paris 8 (LED).*

**Contact email:** *sainturom@gmail.com*

### Abstract

*In this article, we describe a large dataset comprising of 81 energy, financial and other commodity markets collected over the period from August 1999 to January 2018, with which we offer new insights into the dynamics of market risks using a conditional regime-switching GARCH CAPM with time-varying betas explaining both bull and bear markets. The static version of CAPM predicts that the relationship between expected returns across assets and their betas is linear. Despite the wide applicability of this model, it has been severally criticised especially following its failure to capture the effects of time-varying investment opportunities that may affect the calculation of an asset's risk. In parallel to this, several modifications have been offered in which an asset's risk has been set to vary conditionally*

over the business cycle. However, empirical results remain relatively divergent regarding the identification procedure of determinant factors behind the evolution of conditional asset betas. We contribute to the literature on conditional CAPM by addressing the above issue. We explore the ability of the conditional regime-switching GARCH-CAPM to capture time varying betas and conditional variance processes using a GARCH and Markov-switching framework.

### Specifications Table

Subject area	<i>Economics</i>
More specific subject area	<i>Energy Economics.</i>
Type of data	<i>Times series (Table, excel file, graphs)</i>
How data was acquired	<i>Data are acquired from Datastream international, IMF's International Financial Statistics (IFS) and the U.S. Federal Reserve databases.</i>
Data format	<i>Raw and processed</i>
Experimental factors	<i>Dataset include monthly MSCI and stock returns for 56 countries and 22 commodity indices, one-month Treasury bill rates for all the countries and Eurodollar interest rate, which are considered as a risk-free rate. Industrial production index for all the countries.</i>
Experimental features	<i>Very brief experimental description</i>
Data source location	<i>City, country, and/or latitude and longitude (and GPS coordinates) for collected samples/data, if applicable</i>
Data accessibility	<i>Data are available within this article</i>
Related research article	<i>Urom and Chevallier, 2019. A dynamic conditional regime-switching GARCH CAPM for energy and financial markets "in press."</i>

### Value of the Data

- There is a general agreement in the literature that asset prices vary over the business cycle (Maheu and McCurdy, 2000). This dataset may be useful in predicting time-varying investment opportunities that may affect the estimation of assets' risk.
- Given the increasing concern about risk management, this dataset and our permits the forecasting of important Risk Metrics such as the Value-at-Risk (VaR) and Expected Shortfall (ES). According to Engle and Manganelli (2004), VaR offers a quantitative technique through which a single number could quickly and easily convey significant information about the risk of a portfolio.
- This dataset will also guide further research in this area as it would permit the replication and advancement of empirical studies on dynamic conditional beta.

## Data

In this paper, we use an extensive dataset containing 81 monthly Stock Market Indices for 56 countries drawn from North/Latin America, Western Europe, Emerging Europe, the Middle East/Africa, Developed Asia, Emerging Asia, and Africa. The dataset also contains 22 Energy and Other Commodity indices drawn from the main classes of commodities including Metals, Energy, and Agriculture. The data was collected over the period from August 1999 to January 2018. All the data are extracted from Thomson Datastream International. Moreover, we include three aggregated stock market indices for the World, Europe, and Emerging markets. We employ the 30-day Treasury bills rate as the risk-free rate for each country while we use the 30-day Euro-Dollar interest rate for the selected commodities. Regarding the state of the economy in each country, we consider industrial production as an instrumental variable to reflect changes in the level of economic activity in each regime. For commodities and the World aggregate market, we use industrial production in the United States as a reflection of the level of global economic activity. Lastly, we use industrial production in China and Europe industrial production to measure the state of economic activities in emerging markets and Europe respectively.

Table 1 and Figure 1 present the descriptive statistics and return series plots for few countries and commodity return series. Our discussion is based on the entire 81 sampled returns series which can be found in the main article. It can be deduced that all the return series both for countries and commodities have positive mean except Italy, Latvia, and Portugal that have negative mean returns. Also, all the series are negatively skewed except Turkey, Columbia, United Arab Emirates, Chile, Malte, Gold, Ruthenium, Wool, Wheat, Cocoa, Coffee and Cotton which have positive values for the skewness while all the values for the kurtosis are above 3 as shown by positive excess kurtosis for all the series. Lastly, the \$p\$-values for the Jarque Bera and the Lagrange Multiplier (LM) test for all the series are zero. These results imply asymmetric and fat tail characteristics and that all the return series do not follow the normal distribution under 5% significance level.

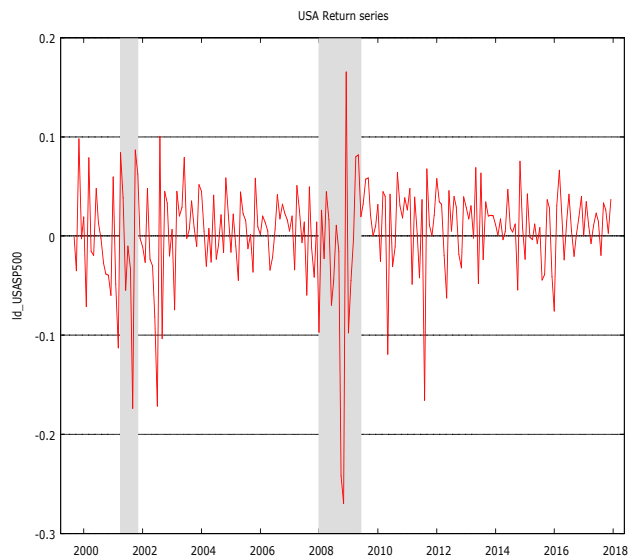
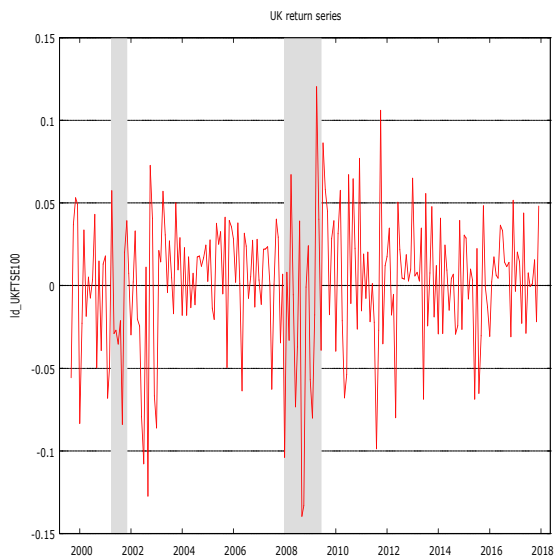
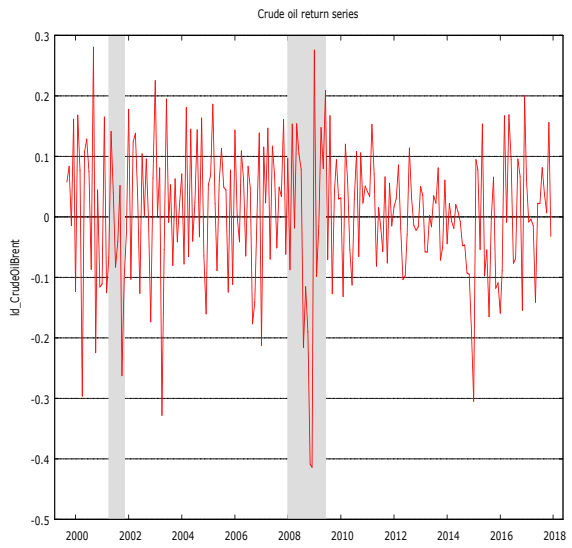
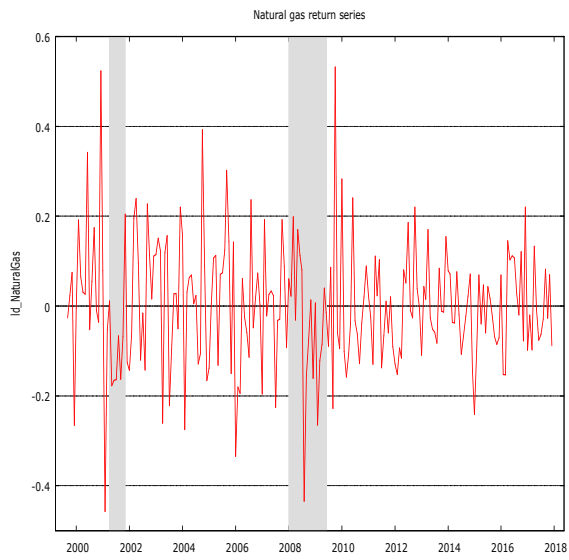
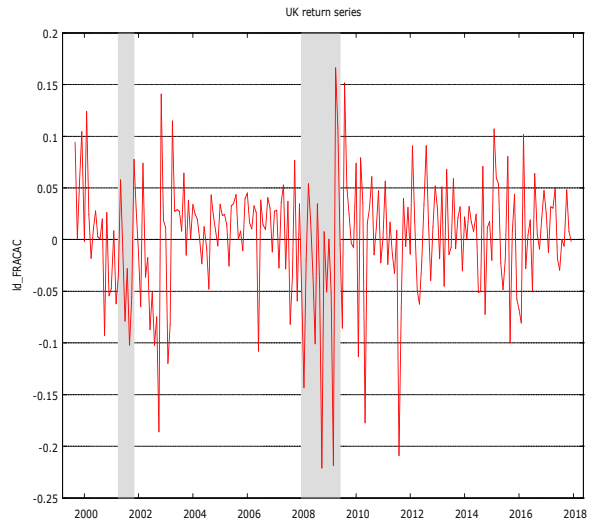
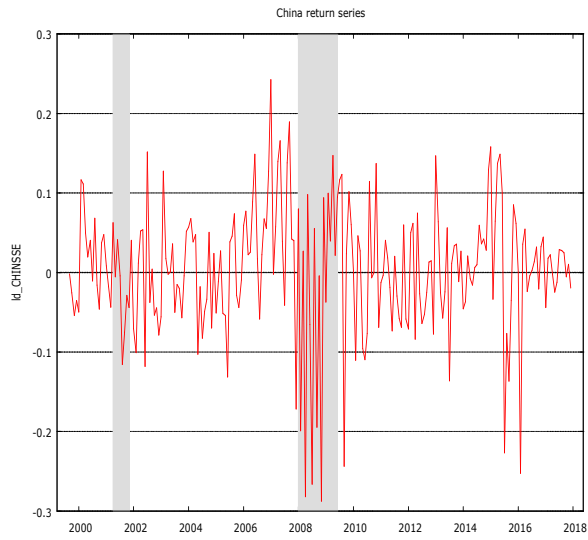
**Table 1: Descriptive statistics**

Countries	Mean	Std. Dev.	Skewness	Ex. Kurtosis	JB	LM ARCH (5)
USA	0.0032	0.0518	-1.5932	6.0584	429.52	54.985
					[0.000]	[0.000]
CANADA	0.0038	0.0404	-1.0552	3.1994	134.65	21.292
					[0.000]	[0.000]
GERMANY	0.0041	0.0623	-0.8671	3.0538	113.06	13.819
					[0.000]	[0.016]
AUSTRALIA	0.0033	0.0413	-0.4041	1.1743	18.621	47.666
					[0.000]	[0.000]
DENMARK	0.0071	0.0544	-0.5504	1.8793	43.485	36.067
					[0.000]	[0.000]
FINLAND	0.0011	0.0797	-0.2628	2.0447	40.857	45.586
					[0.000]	[0.000]
SPAIN	0.0005	0.0633	-0.4596	0.5081	10.115	22.611

					[0.000]	[0.000]
FRANCE	0.001	0.0595	-0.7812	2.1792	65.912	21.227
					[0.000]	[0.000]
UK	0.0008	0.0413	-0.6388	1.0478	25.027	15.128
					[0.000]	[0.009]
ITALY	-0.0022	0.0609	-0.3265	0.8052	9.851	5.855
					[0.007]	[0.032]
SWEDEN	0.0031	0.0587	-0.2977	0.681	7.5028	26.114
					[0.023]	[0.000]
SWITZERLAND	0.0013	0.0392	-0.7277	0.8563	28.139	21.115
					[0.000]	[0.000]
NEW ZEALAND	0.0044	0.0354	-0.5696	2.104	52.476	36.106
					[0.000]	[0.000]
NORWAY	0.0081	0.0592	-0.7901	1.5799	45.768	39.405
					[0.000]	[0.000]
NETHERLAND	0.0003	0.0574	-10723	2.2708	89.431	44.214
					[0.000]	[0.000]
JAPAN	0.0011	0.059	-0.4712	1.0824	18.882	17.889
					[0.000]	[0.003]
IRELAND	0.0041	0.0629	-0.8876	2.5519	88.586	62.801
					[0.000]	[0.000]
THAILAND	0.0062	0.0668	-1.1079	4.6719	24.508	59.109
					[0.000]	[0.000]
MYLASIA	0.0038	0.0432	-0.4117	1.6887	32.357	10.199
					[0.000]	[0.069]
INDONESIA	0.0109	0.0643	-1.0897	3.5015	35.059	16.67
					[0.000]	[0.002]
PHILIPPINES	0.0056	0.0591	-0.4426	1.6858	33.233	19.624
<b>COMMODITIES</b>						
CRUDE OIL	0.0052	0.1144	-0.6037	1.0278	23.051	33.972
					[0.000]	[0.000]
GOLD	0.0072	0.049	0.0799	1.6769	26.012	13.122
					[0.000]	[0.022]
SILVER	0.0051	0.0842	-0.3747	2.0796	44.791	19.753
					[0.000]	[0.001]
NATURAL GAS	0.0003	0.1388	-0.32151	1.7985	33.439	46.064
					[0.000]	[0.004]
COPPER	0.0066	0.0778	-0.8704	4.3231	199.1	25.678
					[0.000]	[0.000]
PLATINUM	0.0042	0.0697	-0.5932	3.1535	104.06	27.119
					[0.000]	[0.000]
PALLEDIUM	0.0049	0.1067	-0.6819	2.7276	85.251	7.044
					[0.000]	[0.054]
NICKEL	0.0027	0.1053	-0.6051	2.4071	66.541	6.311
					[0.000]	[0.037]

TIN	0.0059	0.0731	-0.3861	1.494	25.928	186.272
					[0.000]	[0.000]
ZINC	0.0046	0.0775	-0.9029	4.1319	186.39	8.1738
					[0.000]	[0.005]
RHODIUM	0.0024	0.1441	-1.6576	14.075	149.34	179.45
					[0.000]	[0.000]
RUTHERNIUM	0.0071	0.1417	0.8763	5.0058	257.85	187.68
					[0.000]	[0.000]
CORN	0.0026	0.0864	-0.3647	1.9197	38.66	181.18
					[0.000]	[0.000]
RUBBER	0.0045	0.0884	-1.2396	6.3796	429.42	178.78
					[0.000]	[0.000]
SOYABEAN	0.0032	0.0712	-0.184	0.8392	7.697	10.566
					[0.021]	[0.060]
WOOL	0.005	0.0495	0.0212	2.6458	64.186	194.85
					[0.000]	[0.000]
ALLUMINIUM	0.0017	0.0538	-1.0768	5.457	315.48	49.08
					[0.000]	[0.000]
LEAD	0.0072	0.0955	-0.3254	2.2701	51.122	37.475
					[0.000]	[0.000]
WHEAT	0.003	0.0959	0.0282	1.376	17.384	13.852
					[0.000]	[0.016]
COCOA	0.0027	0.0739	0.2582	1.1666	14.921	7.183
					[0.000]	[0.0207]
COFFEE	0.0013	0.0887	0.0474	1.9878	36.302	6.322
					[0.000]	[0.041]
COTTON	0.0016	0.0806	0.1724	0.9626	9.584	18.783
					[0.008]	[0.002]

**Figure 1: Return series plots**



**Acknowledgments:** None

**Transparency document**

**Supporting information**

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# A dynamic conditional regime-switching GARCH CAPM for energy and financial markets

Revised September 2019

## Abstract

This paper develops a methodology for estimating a time-varying conditional version of the CAPM with regime changes in conditional variance dynamics. Our research goal is related to documenting the power of the beta when it is estimated dynamically. The empirical performance is tested across a sample of 81 financial, energy, and other commodity markets for the period August 1999 - January 2018. The conditional regime-switching GARCH CAPM, with time-varying betas explaining both bull and bear markets, outperforms the unconditional (static) CAPM. Among stocks, there are significant time variations in betas across our models and regimes. This empirical feature is even more pronounced in the USA, the UK, Germany, France, China, and Malaysia. Among energy and other commodities, we find similar variations in the market price of risk. The direction of the relation with market returns for Crude Oil, Gold, Copper, Tin, Rubber, Aluminum, and Platinum is the same across our nested models. This result also holds for aggregate markets indices. Secondly, we provide a ranking by mean filtered volatility series where Natural Gas stands out at a high level. Average pricing errors are inferior in the case of the conditional model, and for Crude Oil. Lastly, we demonstrate that the regime-switching model delivers better estimates of one-day-ahead Value-at-Risk than its non-switching counterpart. Our results shed light on the supremacy of the market factor alone associated with time variation in risk premia across the energy and financial markets.

**Keywords:** Dynamic Beta; Conditional CAPM; Regime-Switching; MS-GARCH; Risk Management; Crude Oil; Natural Gas

**JEL Classification:** C32; F36; G12; Q48; Q54

# 1 Introduction

For over a half-century, the Capital Asset Pricing Model (CAPM) first proposed by Sharpe (1964) and Lintner (1965) and extended by Mossin (1966), Fama (1968a; 1968b) and Long (1972) has offered a theoretical background for the estimation of asset prices with volatile returns. As early as the 1960s, from the work of Markowitz developed some years ago, Sharpe, Lintner, and Mossin proposed the equilibrium model of financial assets (CAPM) that has served as a foundation for modern financial theory. According to this model, the expected profitability of a security is explained by a factor (the market risk premium) with a sensitivity specific to each company (the beta). The risk of a project is measured by the beta of the cash flow concerning the return on the market portfolio of all assets in the economy. This model predicts that the relationship between expected returns across assets and their betas concerning the market portfolio is linear (Morana, 2009; Tsai, Chen and Yang 2014). The crucial second prediction of the CAPM is that all investors are the Arrow-Pratt relative risk-averse utility of terminal wealth maximizers, whose choice of stocks is mainly guided by mean-variance efficiency (Frazzini and Pedersen, 2014), and that investors' risk aversion are constant over time.

Early stream of studies offered significant empirical evidence in favor of the theoretical paradigm of the CAPM, especially regarding its crucial prediction that the market portfolio must be mean-variance efficient. This pillar of academic finance laid a strong background for research for several years (see, e.g., Black, Jensen, and Scholes 1972; Blume and Friend 1973). However, Lettau and Ludvigson (2001) note that recent empirical implementations have revealed some downsides of the CAPM. We adopt a pragmatic view regarding the usefulness of the unconditional CAPM. Despite its observed drawbacks, Jagannathan and Wang (1996) note that the CAPM is still the preferred model for MBA and other managerial finance courses. Even more, Vendrame, Guermat, and Tucker (2018) note that the CAPM remains a simple, intuitive, and an economically sound theory and that the search for its replacement has led the researcher to either discard its central doctrines, or adopt some statistical approaches that prove too complicated to be replicated by researchers and practitioners. Since then, the CAPM has had many applications, has been subjected to many empirical tests on all the financial markets but remains to this day an unavoidable model despite continual attacks, both theoretical and empirical.

Two critical possibilities have been offered to explain the observed deficiencies of the CAPM. First, Lettau and Ludvigson (2001) argue that a significant explanation for the failure of CAPM is its assumption of the static specification, which has failed in accounting for the effects of time-varying investment opportunities that may affect the calculation of an asset's risk. The unconditional CAPM was derived from a hypothetical model in which investors are assumed to live for only one period. In the real world, investors live for many periods (Jagannathan and Wang, 1996), and their expectations as economic agents for future returns are conditioned on many factors (Klemkosky and Martin, 1975; Fabozzi and Francis 1978; Bos and Newbold, 1984, Collins and Ledolter, 1987; Bollerslev, Engle, Wooldridge, 1988; Bodurtha and Mark, 1991), implying that risk and risk premium are time-varying. To provide some economic intuition, the time variation in risk premia may be attributable to time variation in risk aversion, or in risk itself (Constantinides and Duffie, 1996). The second possibility is that systematic risk itself has more than one component, and that beta is not the only measure of risk. Additional factors such as the ratio of earnings to price, level of market capitalization, leverage effects and the increasing synchronizations of global finance have been found to significantly influence systematic risk (Banz, 1981; Bhandari, 1988; Arouri et al. 2011).

Perhaps, the most obstructing of empirical applications of the unconditional CAPM was its failure to capture the cross-sectional variation of average return on a portfolio containing assets with varying sizes and book-to-market equity ratios. In response to these anomalies, additional sensitivity components have been added to the CAPM such as in the famous three-factor model of Fama and French (1993), the consumption CAPM by Breeden (1979) and the four-factor model of Carhart (1997). Despite the success of these models, especially the three-factor model, they have, however not been enough to account for the central anomalies. For instance, the three-factor model has been criticized due to the controversies surrounding the interpretation of its proxies for unobserved common risk in portfolios. Harvey, Liu, and Zhu (2016) have added critics to the

inflation of such factors: the market factor alone is the main (if not the only) factor explaining the returns of securities. The consumption-based CAPM has failed in its formulation of the representative agent with time-separable power utility using U.S. data and has not done better in capturing cross-section of average returns on portfolios with assets of different sizes (Lettau and Ludvigson, 2001).

According to Vendrame et al. (2018), the most reoccurring explanation for the failure of CAPM has been that CAPM may hold conditionally rather than unconditionally. There is a general agreement in the literature that asset prices vary over the business cycle (Maheu and McCurdy, 2000). Hence, the risk premium should also change over the business cycle (Cochrane, 2001). Besides, the covariance between asset returns and the market portfolio is of central interest (Adrian and Franzoni, 2009; Avramov and Chordia, 2005; Lewellen and Nagel, 2006). Conditional beta pricing models – which offer a convenient approach to modelling the time-varying conditional variances and covariances in financial time series – have been applied several times to introduce time variations in several dimensions of the CAPM (see e.g. Campbell and Shiller, 1988; Bollerslev et al. 1988; Campbell 1991; Bodurtha and Nelson, 1991; Ferson and Harvey, 1991; Lamont 1998; Lettau and Ludvigson 2001; Cochrane 2001; Andersen et al. 2005; Ang and Chen, 2007; Morana 2009; Korkmaz et al. 2010; Billio et al., 2012; Cenesizoglu and Reevesm 2018; Tansuchat et al. 2018, Vendrame et al. 2018). For instance, Bodurtha and Mark (1991) estimated a conditional CAPM with time-varying expected risk premium, variance, and covariances using a GMM approach. They found sufficient evidence against the constant beta CAPM. Also, Ang and Chen (2007) examined a conditional CAPM with a conditional beta and time-varying risk premium using an autoregressive AR(1) latent process. They found that conditional betas were time-varying and positively correlated with the market risk premium.

Recently, new approaches have been developed which offer flexibility in capturing the dynamic aspects of conditional asset betas. In a dynamic world, investors care about hedging against a variety of risks that do not arise in a static economy. Dynamic conditional beta models allow for time-variation in betas based on predicted conditional covariances using estimates from both the univariate and multivariate DCC model of Engle (2002) applied to asset pricing and systematic risk assessment (see, e.g., Bali, Engle & Tang, 2016; Engle, 2016). Also, Ferreira et al. (2011) derived dynamic conditional betas using a non-parametric approach and Monte Carlo simulations on the conditional version of Fama and French three-factor model. Further, Darolles, Francq, and Laurent (2018) extended the use of conditional covariance matrix by applying the Cholesky decomposition under the Cholesky-GARCH framework and applied this approach to the estimation of extensive portfolio and risk management metrics. They show that the Cholesky-GARCH model performs better than the dynamic conditional beta of Engle (2016) which has been criticized due to the imposition of constancy on conditional betas, and the impossibility of identifying the deriving factors behind their evolution.

To our knowledge, the method we develop in this paper can be seen as an extension of the dynamic conditional beta of Engle (2016) and Darolles et al. (2018). We explore the ability of the conditional regime-switching CAPM with time-varying betas to capture regime changes in the conditional variance dynamics. With this approach, the time variation is captured using two dynamic processes: GARCH and Markov-switching. Our work is closely related to the novel estimation approach proposed by Ardia, Bluteau, Boudt, Catania, and Trottier (2018) to implement the Markov switching GARCH specification of Haas, Mittnik, and Paoletta (2004). A critical theoretical advantage of the Markov switching model is that it offers the opportunity to assess different GARCH behaviors in each regime. It reveals the difference in the conditional variance dynamics of low and high volatility regimes, and therefore the possibility of large swings in returns. We consider two types of markets: bull and bear, across which market risk premia are expected to vary (Pettengill, Sundaram and Mathur, 1995). We investigate the performance of this model across an international sample of 81 financial, energy and other commodity markets from August 1999 to January 2018.

In contrast to traditional derivations of the static CAPM, the first contribution of this paper is to examine temporal variation in beta and risk premium by estimating regime changes according to Markov-switching processes. As a framework of analysis, the conditional CAPM provides

indeed a convenient way to incorporate the time-varying conditional variances and covariances specific to energy and financial time series. Our modeling strategy offers the advantage to compare the time-varying risks and price of risk across three models, namely: the unconditional CAPM, the regime-switching CAPM, and the conditional regime-switching GARCH-CAPM. The regime-switching CAPM model permits us to estimate the variations in betas across regimes, as well as the market regime probabilities. The regime-switching GARCH-CAPM allows us to derive additionally, the conditional variance dynamics while using residuals from the static CAPM as demeaned time series.

Secondly, the regime-switching GARCH model permits us to derive the volatility forecasts which adapts to variations in the unconditional volatility levels for all our series using the mean fitted posterior volatility. Here, it is argued that if the evolution of volatility is heterogeneous across two regimes, the regimes may exhibit different unconditional volatility levels. Further, Ardia et al. (2018) note that one of the critical empirical applications of the MS-GARCH model is within the domain of wealth allocation among risky investment opportunities. Here, investors may wish to assess the quantile of their future distribution at given risk levels as well as the expected values below this level. It has been noted that regime-switching models have proven to offer better out-of-sample back-testing results than single-regime models.

The third contribution of this paper to the empirical asset pricing literature is to apply the MS-GARCH in the forecasting of essential Risk Metrics such as the Value-at-Risk (VaR) and Expected Shortfall (ES). According to Engle and Manganelli (2004), VaR constitutes a quantitative technique through which a single number could quickly and easily convey significant information about the risk of a portfolio. It has recently become a necessary tool for risk managers, enabling them to appraise and allocate risk more efficiently. In simple terms, the VaR represents a quantile of the log-returns distribution at a prior determined horizon and confidence level whereas ES reflects the loss expected when the loss is above the VaR level.

Estimation results from conditional beta pricing models in 81 financial, energy, and other commodity markets are remarkable. First, among stocks, there are significant variations in size and the nature of relations between systematic risks and the markets from one model to another. This finding is even more pronounced across bull/bear regimes for prominent countries such as the USA, the UK, Germany, France, China, and Malaysia. Secondly, we find variations mostly in the size of the beta parameters. The direction of the relationship between energy and other commodities (such as Crude Oil, Gold, Copper, Tin, Rubber, Aluminum, and Platinum) and the market is the same across two of our models. Variations in the relation between these commodities and the market are only witnessed in the MS-GARCH model. These results also hold for aggregate markets, where most differences are found in the conditional regime-switching GARCH model. Thirdly, we document that the mean filtered volatilities rank, in the realm of energy markets, at a high level for Natural Gas. Average pricing errors are inferior for the conditional model, especially for Crude Oil. Lastly, our risk management tests show that the regime-switching model delivers the best estimates of one-day-ahead VaR. By investigating the strength of the market factor across financial, energy and other commodity markets, we identify economically important risk-return relationships that market professionals can exploit.

The rest of this paper proceeds as follows. Section 2 presents the methodology from where we show a detailed build-up to the model for this paper. Section 3 offers a description of the data. Section 4 compares results from the competing models and contains volatility dynamics and risk management statistics from the regime-switching models. We present the conclusions in Section 5.

## 2 Methodology

We attempt to determine the importance of changing risk premia and returns variability over time. To that end, we employ an empirical test of the conditional regime-switching GARCH CAPM that unfolds in three steps as follows. Notice the assumption that the model has a conditional mean zero usually requires the model to be applied on a demeaned time series. When the series exhibits dynamics in the conditional mean, the demeaned time series becomes the residuals of the time series model (Ardia et al. 2018).

## 2.1 Unconditional CAPM

In our view, the Capital Asset Pricing Model may serve as a useful benchmark model of relative asset returns. Its economic appeal is that any risk-averse investor would demand higher expected returns to compensate for taking higher risks. In its static (or unconditional) version, all investors are single-period risk-averse utility of terminal wealth maximizers.

Therefore, our analysis begins with the static CAPM, whose primary output is the expected return of an asset  $i$  at time  $t$  with the assumption that investors are risk averse and that the market is complete (see, e.g. Cortazar et al. 2013; Blitz, 2014). Return on asset  $i$  and the market portfolios for indices may be expressed as  $R_{i,t} = \ln \left\langle \frac{P_{i,t}}{P_{i,t-1}} \right\rangle$  where  $R_{i,t}$  is the log return on asset  $i$  in period  $t$ , for instance a given value-weighted stock index.  $P_{i,t}$  is the price of asset  $i$  at time  $t$ . In its typical form, the CAPM implies that the beta premium is positive, and equals the expected market return minus the risk-free rate expressed as follows:

$$(R_{i,t} - R_{f,t}) = \alpha + \beta(R_{M,t} - R_{f,t}) + \epsilon_t \quad (1)$$

where  $R_{i,t}$  denotes the log return on asset  $i$  at time  $t$  and  $t = 1, 2, \dots, T$  is the time horizon. Similarly,  $R_{m,t}$  is the log return on the market portfolio at time  $t$  while  $R_{f,t}$  is the risk free rate at time  $t$ . Therefore, excess return on asset  $i$  is denoted by  $(R_{i,t} - R_{f,t})$  whereas the excess return on the market portfolio is represented by  $(R_{M,t} - R_{f,t})$ .  $\alpha$  is the intercept term.  $\beta$  is the market beta, measured by the slope coefficient  $\hat{\beta}_i$ , and defined as the covariance between the excess returns on the asset and the market, divided by the variance of the excess market returns. As a measure of the systematic risk associated with asset  $i$ , the theoretical purity of the beta is unmatched by other asset pricing models (Harvey, 1989). Jagannathan and Wang (1996) further decompose the unconditional beta between the market beta itself (e.g., the average market risk), and the premium on the beta (e.g., the beta instability).  $\epsilon_t$  is the error term at time  $t$  which is assumed to be an independently and identically distributed random variable that follows the normal distribution such that  $\epsilon \sim N(0, \sigma^2)$ .

One of the classical assumptions of the CAPM according to Sharpe (1964) and Lintner (1965) is that performing the expectations operator  $E_t(\cdot)$  of equation (1) conditionally on information set up to time  $t$ , the condition below must hold:

$$E_t(r_i) = \beta E_t(r_m) \quad (2)$$

where  $r_i$  and  $r_m$  represent return on asset  $i$  and the return on the market portfolio, respectively, as defined earlier. The above condition implies that if the CAPM holds, the intercept  $\alpha$  must not be statistically different from zero (Cortazar et al. 2013).

## 2.2 Conditional regime-switching CAPM

The static CAPM is built on the assumption that investors care only about the mean and variance of single-period asset returns. However, in practice, investors revise their decisions over time based on their expectations of future investment opportunities. Although investors prefer high expected return and low return variance, they are also concerned with the covariances of asset returns with state variables (such as the business cycle) that affect future investment opportunities.

Next, we assume that the CAPM holds in a conditional sense, i.e., betas and expected returns are allowed to vary over the business cycle. Much like many economic times series, financial data exhibits abrupt changes due to sudden movements in fundamentals which show up in asset prices (Ang and Bakaert, 2002). Assets with betas that are prone to vary with the risk premium are less stable over the business cycle, and also have higher unconditional expected returns. In its classical form, the Markov-Switching models proposed in Hamilton (1989) for the non-stationary time series analysis of the business cycle, estimate regime-switching endogenously.

In this section, we use the Markov-switching model to test whether there are regime shifts in the beta of different assets within the CAPM framework. Indeed, we seek to find out if two different states exist between returns on asset  $i$  and the returns on diversified equity portfolios. The greatest

challenge for the econometrician lies in recovering the state of the economy, which is unobservable. This practical obstacle can be overcome thanks to Hamilton's Expectation-Maximization (EM) algorithm.

To do this, we follow He et al. (2018) to assume that there are two different regimes: bull and bear. The intuition is that when economic growth is expanding (contracting), the market is deemed to be bullish (bearish). Conversely, during recessionary/expansionary periods, correlations of assets with the market may increase/decrease depending on the industry.  $s_t$  represents the state variable which reflects the current regime in the market. A specific risk premium characterizes each market phase, for instance, a substantial risk premium can be associated with an extremely bearish market. The Markov-switching CAPM equation may, therefore, be expressed as follows:

$$(R_{i,t} - R_{f,t}) = \alpha_{st} + \beta_{st}(R_{M,t} - R_{f,t}) + \epsilon_{st}, \quad (3)$$

where  $s_t$  denotes the two states of the model,  $\epsilon_{s_t}$  is the error term which is assumed to be independently and identically distributed and follows the normal distribution such that  $N(0, \sigma_{s_t}^2)$ . Unlike conditional correlations, conditional betas need not be constrained (except for the stationarity condition), which makes this approach very appealing.  $s_1$  reflects one regime with the following parameters  $\alpha_{s1}, \beta_{s1}, \sigma_{s1}^2$  while  $s_2$  denotes the second regime with the following corresponding parameters  $\alpha_{s2}, \beta_{s2}, \sigma_{s2}^2$ . We allow for regime switching in the variances of the error term following Nelson et al. (2001) which notes that regime changes in economic and financial times series might be better modeled through a probabilistic process.

Following the 2-state regime-switching model of Hamilton (1989), the state variable  $s_t$  takes only binary values of 0 and 1. Therefore, the transition probabilities of the first-order Markov chain may be modeled as follows:

$$\begin{aligned} Pr[S_t = 1/S_{t-1} = 1] &= p \\ Pr[S_t = 2/S_{t-1} = 1] &= 1 - p \\ Pr[S_t = 2/S_{t-1} = 2] &= q \\ Pr[S_t = 1/S_{t-1} = 2] &= 1 - q \\ \forall p, q &\in [0, 1] \end{aligned}$$

where  $p$  and  $q$  are the fixed transition probabilities of being in low and high volatility regimes respectively. In equation (3),  $\alpha_{st}$  is assumed to vary depending on the regimes.

The estimation of equation (3) following Maximum Likelihood approach is through the Expectation-Maximization (EM) algorithm explained in Hamilton (1994), Krolzig (1997) and Korkmaz et al. (2010).

## 2.3 Conditional regime-switching GARCH CAPM

Bollerslev, Engle, and Wooldridge (1988) have established the empirical fit of the (multivariate) GARCH to model time-varying covariances across T-bills, bonds, and stocks. Beta is computed as the ratio of the conditional covariance to the conditional variance. Engle (2002) and Bali and Engle (2010) have further documented that GARCH models (with DCC) capture satisfactorily time-varying betas, that covary significantly with the market risk premium. In contrast, this article models conditional covariances and variances by resorting to the family of Markov-Switching GARCH processes (Haas et al., 2004; Ardia et al., 2018).

### 2.3.1 Conditional variance dynamics

Regarding conditional variance modeling, the method proposed in this paper relies on the GARCH equation from Ardia et al. (2018) where given  $t = 1, \dots, T$  with  $T$  the sample size, and  $r_t$  denotes the log return at time  $t$ .  $\epsilon_t$  is an MS-GARCH process if

$$r_t | (S_t = k, I_{t-1}) \sim D(0, h_{k,t}, \xi_k), \quad (4)$$

where  $D(0, h_{k,t}, \xi_k)$  represents an identically and independently distributed (i.i.d) random variable with zero mean and a unit variance while  $h_{k,t}$  is a time-varying variance and  $\xi_k$  is a vector of additional shape parameters (e.g., asymmetry).  $I_{t-1}$  is an information variable that specifies the condition of the world in up to time  $t - 1$ , that is,  $I_{t-1} \equiv (r_{t-1}, i > 0)$ . Furthermore, the latent variable  $S_t$  evolves according to an unobserved first order ergodic homogeneous Markov Chain with fixed state space  $S = 1, \dots, k$ , and a transition matrix  $P$ . Therefore, the probability of switching from one regime to another depends on the transition matrix  $P$ , expressed as follows:

$$P = \begin{pmatrix} p_{11} & \cdots & p_{1k} \\ \vdots & \cdots & \vdots \\ p_{k1} & \cdots & p_{kk} \end{pmatrix}$$

where given the probability to be in state  $i$  at time  $t - 1$ ,  $p_{ij} = p(i=j|_{t-1} = i)$  is the probability to be in state  $j$  at time  $t$ . The following conditions apply:  $0 < p_{i,j} < 1 \forall i, j \in \{1, \dots, K\}$ , and  $\sum_{j=1}^K p_{i,j} = 1, \forall i \in \{1, \dots, K\}$ . Given the parameterization of  $D(\cdot)$  and the probability of transition from state  $j$  at time  $t$  ( $s_t = j$ ) and to be in state  $i$  at time  $t - 1$  ( $s_{t-1} = i$ ), we have  $E[y_t^2 | s_t = k, \Delta_{t-1}] = h_{k,t}$ . Therefore,  $h_{k,t}$  is the variance of  $y_t$  conditional on the realization of  $s_t = k$ . However, given the difficulty in calculating the likelihood function for a sample of  $T$  observations as it requires the integration of  $k^T$  possible regime paths where  $k$  is the number of regimes, the MS-GARCH model was proposed by Gray (1996) under the assumption that the conditional variance at any state depends on the expectation of previous conditional variances.

Following Haas et al. (2004), the conditional variance of  $r_{t-1}$  is assumed to follow a GARCH-type model. Specifically, conditional on regimes  $s_t = k$ ,  $h_{k,t}$  is specified as a function of past returns and the additional regime-dependent vector of parameters  $\theta_k$ . This is expressed as follows:

$$h_{k,t} \equiv h(r_{t-1}, h_{k,t-1}, \theta_k),$$

where  $h(\cdot)$  is a  $I_{t-1}$ -measurable function that specifies the filter for the conditional variance as well as ensures that the function is positive. It is further assumed that  $h_{k,1} \equiv \tilde{h}_k (k = 1, \dots, K)$ , where  $\tilde{h}_k$  denotes the initial value of the variance recursions which is set equal to the unconditional variance for regime  $k$ . Given this, the heteroscedastic specification following Bollerslev (1986) GARCH model with which each conditional variance depends only on its own lag is given as follows:

$$h_{k,t} = w_k + \alpha_1 k \epsilon_{t-1}^2 + \beta_k h_{k,t-1} \quad (5)$$

where  $w_k$  denotes the intercept and  $\beta_k$  is the time-varying beta realized from regime  $k$ . With  $w_k > 0, \beta_k \neq 0$  and  $\alpha_k + \beta_k < 1 (k = 1, \dots, K)$ , we obtain the conditional regime switching GARCH, in which case  $\theta_k \equiv (w_k, \alpha_k, \beta_k)'$ . This model can be written in matrix form as follows:

$$h_t = w_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_{t-1},$$

where  $w_0 = [w_{01}, w_{02}, \dots, w_{0k}]'$ ,  $\alpha_1 = [\alpha_{11}, \alpha_{12}, \dots, \alpha_{1k}]'$ ,  $\beta = \text{diag}(\beta_1, \beta_2, \dots, \beta_k)$  while  $h_t$  is a vector of  $k \times 1$  components.

### 2.3.2 Conditional distribution

The conditional distribution of the standardized innovations for the above-specified models follows the Skewed Student- $t$  distribution in each regime of the Markov Chain. Given that the distribution of financial returns exhibit fat tails and is mostly skewed, the most common distribution to model the process of financial log-returns is the standardized skewed student-t which has a zero mean and unit variance. Following Bauwens and Laurent (2005) which generalized the multivariate method proposed by Fernández and Steel (1998), the standardized skewed Student- $t$  distribution in which  $\nu \leq 2$  may be defined as follows:

$$f(Z|\xi, \nu) = \left( \frac{2}{\sqrt{\pi}} \right)^k \left( \prod_{i=1}^k \frac{\xi_i S_i}{1 + \xi_i^2} \right) \times \frac{\Gamma((\nu + k)/2)}{\Gamma(\nu/2)(\nu - 2)^{k/2}} \left( 1 + \frac{Z^*{}' Z^*}{\nu - 2} \right)^{-(k+\nu)/2}, \quad (6)$$

where

$$Z^* = (z_1^*, \dots, z_k^*)',$$

$$\begin{aligned}
Z^* &= (S_i z_i + m_i) \xi_i^{l_i}, \\
m_i &= \frac{\Gamma((\nu - 1)/2) \sqrt{\nu - 2}}{\sqrt{\pi} \Gamma(\nu/2)} \left( \xi_i - \frac{1}{\xi_i} \right), \\
S_i^2 &= \left( \xi_i^2 + \frac{1}{\xi_i^2} - 1 \right) - m_i^2,
\end{aligned}$$

and

$$l_i = \begin{cases} -1 & \text{if } z_i \geq -\frac{m_i}{s_i} \\ 1 & \text{if } z_i < -\frac{m_i}{s_i} \end{cases}$$

where  $\Gamma(\cdot)$  is the Gamma function. The constants  $m_i$  and  $s_i^2$  are vectors of means and standard deviations and are functions of  $\xi$  and  $\nu$ . The density is denoted by  $SKST(0, l_k, \xi, \nu)$ , and  $\nu$  is the vector of asymmetry parameters, whereas  $\xi^2$  can be interpreted as the skewness measure. Lastly,  $z_i$  is an independently and identically distributed process with zero mean and a unitary variance, and  $\nu > 2$  is imposed to ensure that the second-order moment applies. The kurtosis of the skewed Student- $t$  distribution increases as the value of  $\nu$  diminishes. The predictive densities further motivate the choice of the skewed Student- $t$  distribution for each regime of the MS-GARCH-CAPM (see Figure 6 in the Appendix in the case of USA for illustrative purposes).

### 2.3.3 Model estimation

Given the difficulty in computing the likelihood function of the MSGARCH models specified earlier, it cannot be estimated by the Quasi-Maximum Likelihood (QML) approach but either through the Maximum Likelihood or the Markov Chain Monte Carlo (MCMC) / Bayesian techniques. State-space models offer the advantage of modeling parsimoniously the beta by considering it as an unobservable latent variable (Durbin and Koopman, 2012). In this study, we estimate our models by the MCMC/Bayesian techniques that require the evaluation of the likelihood function.

Given  $\Psi \equiv (\theta_1, \xi_1, \dots, \theta_K, \xi_K, P)$  is the vector of model parameters, the likelihood function may be stated as follows:

$$L(\Psi | I_T) \equiv \prod_{t=1}^T f(r_t | \Psi, I_{t-1}), \quad (7)$$

where  $f(r_t | \Psi, I_{t-1})$ , represents the density of  $r_t$  conditioned by information set up to  $I_{t-1}$ , and the model parameters  $\Psi$ . The conditional density of  $r_t$  for the MS-GARCH process is stated as follows:

$$f(r_t | \Psi, I_{t-1}) \equiv \sum_{i=1}^K \sum_{j=1}^K p_{i,j} z_{i,t-1} f_D(r_t | s_t = j, \Psi, I_{t-1}), \quad (8)$$

where  $z_{i,t-1} \equiv P[s_{t-1} = i | \Psi, I_{t-1}]$  denotes the filtered probability of state  $i$  at time  $t - 1$  gotten through Hamilton's filter (see Hamilton (1989), and Hamilton (1994)).

The Maximum Likelihood estimator  $\hat{\Psi}$  is evaluated by maximizing the logarithm of the likelihood function in equation (11). However, for the MCMC/Bayesian estimation, the likelihood function is pooled with an erstwhile determined value for  $f(\Psi)$  to form the kernel of the subsequent distribution  $f(\Psi | \Delta_T)$ . Following Ardia et al. (2018), we form the prior for this study using unrelated diffuse priors as shown below:

$$\begin{aligned}
f(\Psi) &= f(\theta_1, \xi_1) \cdots f(\theta_K, \xi_K) f(P) \\
f(\theta_k, \xi_k) &\propto f(\theta_k) f(\xi_k) \|\{(\theta_k, \xi_k) \in CSC_k\} (k = 1, \dots, K) \\
f(\theta_k) &\propto f_N(\theta_k; \mu_{\theta_k}, \text{diag}(\sigma_{\theta_k}^2)) \|\{\theta_k \in PC_K\} (K = 1, \dots, K) \\
f(\xi_k) &\propto f_N(\xi_k; \mu_{\xi_k}, \text{diag}(\sigma_{\xi_k}^2)) \|\{\xi_{k,1} > 0, \xi_{k,2} > 2\} (k = 1, \dots, K) \\
f(P) &\propto \prod_{i=1}^K \left\langle \prod_{j=1}^K p_{i,j} \right\rangle \|\{0 < p_{i,i} < 1\},
\end{aligned} \quad (9)$$

where  $PC_K$  denotes the positivity condition in state  $k$ ,  $CSC_K$  represents the covariance-stationarity condition while  $\xi_{k,1}$  and  $\xi_{k,2}$  are the asymmetry and tail parameters of the skewed Student- $t$  distribution in state  $k$  respectively. Also,  $\mu$  and  $\sigma^2$  represent vectors of predetermined means and variances, while  $f_N(\bullet; \mu, \Sigma)$  represents the multivariate normal density with mean vector  $\mu$  and covariance matrix  $\Sigma$ . To evaluate the Bayesian measure of fit, we compute the Deviance Information Criterion (DIC) by Spiegelhalter, Best, Carlin, and Van der Linde (2012) for all models.<sup>1</sup> Parameters estimated record the lowest posterior means of the deviance (which suggests the best fit to the data). According to the DIC, the performance of the conditional model suggested here is also robust to the change of prior distributions, and the number of iterations of the burn-in period.

### 3 Data description and test for varying betas

In this section, we present the description of the dataset used for this study as well as the preliminary analysis. We also conduct as part of our preliminary analysis, an analysis to reveal varying betas across all the markets considered in this study.

#### 3.1 Data

In this paper, we use an extensive dataset containing 81 monthly Stock Market Indices for 56 countries drawn from North/Latin America, Western Europe, Emerging Europe, the Middle East/Africa, Developed Asia, Emerging Asia, and Africa. The dataset also contains 22 Energy and Other Commodity indices drawn from the main classes of commodities including Metals, Energy, and Agriculture. The data was collected over the period from August 1999 to January 2018. All the data are extracted from Thomson Datastream International. Moreover, we include three aggregated stock market indices for the World, Europe, and Emerging markets.

We employ the 30-day Treasury bills rate as the risk-free rate for each country while we use the 30-day Euro-Dollar interest rate for the selected commodities. Regarding the state of the economy in each country, we consider industrial production as an instrumental variable to reflect changes in the level of economic activity in each regime. **We note that gross domestic product (GDP) is the traditional measure of the level of economic activity published on a quarterly basis. However, industrial production is an advanced indicator of GDP, which measures the extent to which factories are idle or running at full steam and is published monthly. Since industrial production is published at a higher frequency, we use it instead of GDP.** We use the industrial production for each country, but for commodities and the World aggregate market, we use industrial production in the United States as a reflection of the level of global economic activity. Lastly, we use industrial production in China and Europe industrial production to measure the state of economic activities in emerging markets and Europe, respectively.

**For the unconditional CAPM model, we construct the excess return for each series by subtracting the risk-free rate from the market portfolio return. Similarly, we subtract the risk-free rate from the returns on the respective stock market to get the excess return for the market. For the market portfolio, we use the capitalization-weighted index that best represents a particular stock market such as the SP 500, FTSE 100, DAX, CAC, TSX for the U.S.A, United Kingdom, Germany, France, and Canada respectively. We use the Morgan Stanley Capital International (MSCI) index for each country as the benchmark to measure stock market performance for each country. Regarding the commodities, we use the MSCI World index as a benchmark for commodities returns. For the regime-switching models, we retrieve the demeaned series from the unconditional CAPM models together with the proxy for existing economic condition namely, industrial production.**

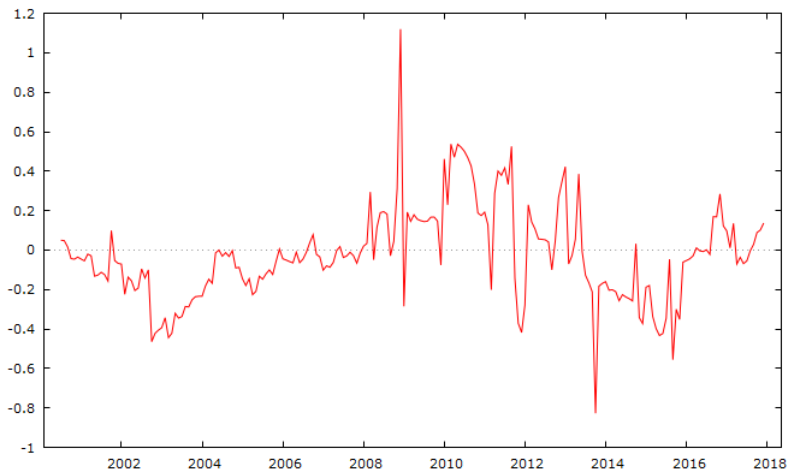
Table 7 in the Appendix presents the descriptive statistics for all the 81 return series. It can be deduced that all the return series both for countries and commodities have positive mean except Italy, Latvia, and Portugal that have negative mean returns. Also, all the series are negatively skewed except Turkey, Columbia, United Arab Emirates, Chile, Malte, Gold, Ruthenium, Wool, Wheat, Cocoa, Coffee and Cotton which have positive values for the skewness while all the values for the kurtosis are above 3 as shown by positive excess kurtosis for all the series. **Lastly, the**

<sup>1</sup>Not reproduced here to save space, but available upon request.

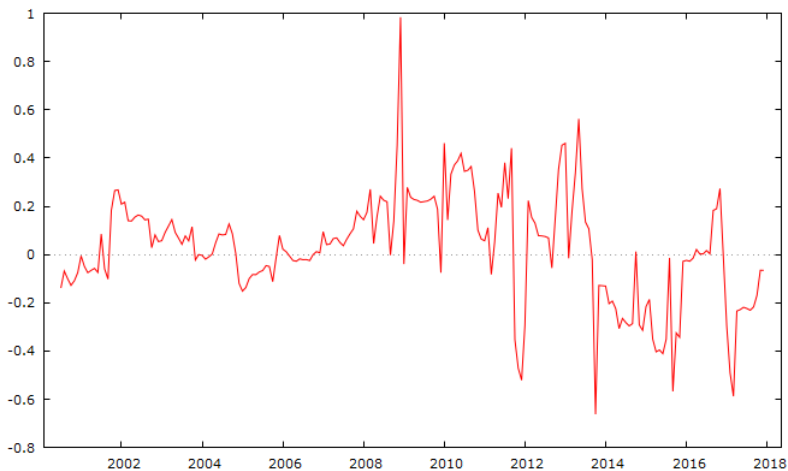
$p$ -values for the Jarque Bera and the ARCH LM test for all the series are reported. The Jarque-Bera statistic is a test for normality in the distribution of the return series, whereas the ARCH LM test of Engle (1982) tests for first-order conditional heteroscedasticity. We reject the null hypothesis of normality and no ARCH effect in all return series considered. These results imply asymmetric and fat tail characteristics and that all the return series do not follow the normal distribution under 5% significance level. These results further motivate our choice of a GARCH modeling approach for conditional variance processes. Table 8 in the Appendix presents the exact names of the series and their designations as they appear in the three models.

### 3.2 Rolling-window betas

Following previous papers in the literature on beta instability (see e.g., Tsai, Chen and Yang 2014; Engle 2014; Marrero, Puch, and Ramos-Real, 2015), we use return series for both crude oil and natural gas to compute time-varying CAPM betas using a 10-month rolling window regression. In Figures 1 below, we present the two plots of time-varying betas for crude oil and natural gas, respectively. From the plots for both markets, we can infer clear evidence of time variation in CAPM betas over the entire sample period. Specifically, results from the plots show that for crude oil, the average CAPM beta was -0.028. The CAPM beta reached its highest point of about 1.1 around 2008 – 2009 coinciding the past financial crisis. Contrarily, the lowest beta value of about -0.9 may be found around 2013 – 2014 coinciding with the period of low crude oil prices.



(a) Crude oil



(b) Natural gas

Figure 1: Rolling window betas for Crude Oil and Natural Gas.

Regarding the second plot, we can also see a similar pattern in the time variation of CAPM beta in the market for natural gas. Over the sample period, the average beta is about 0.025. Similar to the crude oil market, the highest CAPM beta for natural gas of approximately 1 may be found around the financial crisis period of 2008-2009 whereas the lowest value of about -0.7 may be found around 2013 -2014. Lastly, we also found evidence of volatility clustering beginning from the financial crisis until the end of our study period in both energy markets. The time-varying perspective to the modeling of betas adopted in this study is further motivated by the time-varying patterns as well as volatility clustering present in these markets.

## 4 Empirical results

We apply the methods outlined in the previous section to our database of 81 financial, energy, and other commodity markets. We use the  $\beta$  representation of the market factor as the basis of our empirical work. The fact that the implied unconditional model nests the static CAPM facilitates direct comparison of three asset pricing models under consideration: (i) the unconditional CAPM, (ii) the conditional regime-switching CAPM, and (iii) the conditional regime-switching GARCH CAPM. In the conditional CAPM, an asset's beta is the ratio of the conditional covariance between the asset and market returns, and in the conditional variance of the market return (Bodurtha and Mark, 1991). We compare the  $\beta$ s across these models and summarize the goodness-of-fit of each empirical specification based on sensitivity tests.

### 4.1 Results of the unconditional measures of market beta

To form a basis for comparison, we first examine the static CAPM with the traditional measure of market beta based on equation (1) by employing the linear regression technique with the assumption that the intercept in the CAPM is zero against the alternative that it is not equal to zero. Tests of the CAPM imply that (i) the static CAPM holds, and (ii) the market return is a linear function of the asset return.

To do this, we construct the excess return for each series by subtracting the risk-free rate from the index return. Similarly, we subtract the risk-free rate from the returns on the respective market portfolios to get the excess return for the portfolio index. We evaluate the validity of our model specifications using the Durbin-Watson test for residual autocorrelation and the Q-statistics for residual normality and serial correlation. Lastly, the efficiency of the respective market portfolio index in the test of CAPM is assessed using the R-squared coefficients.

Table 1 reports the intercept and beta coefficients of estimating the empirical specification of the unconditional CAPM. First, it can be observed that the assumption of no intercept holds for all the markets as shown by the estimates which are statistically insignificantly different from zero. This implies that the simple CAPM correctly predicts the risk premium in our sampled stocks. Regarding the unconditional measure of market beta, the beta coefficient is statistically significant across all countries and portfolios but with positive and negative values except in Canada, Croatia, Hungary, Egypt, Switzerland, Bulgaria, Iceland, Russia, Mexico, Sweden, and Argentina. However, the beta coefficient is positive and statistically significant across all the commodities except in Gas, Platinum, and Rhodium and all aggregate markets and portfolios, namely Europe, Emerging Markets, and World. Arshanapalli et al. (1998) note that positive beta coefficients imply that up-market movements drive a stock, while negative beta coefficients suggest that stock appears to be less sensitive to market fluctuations.

Regarding the size of the beta coefficient across these markets and portfolio proxies, the beta estimate is about 0.2 for 23 countries, but this coefficient is negative for the UK, Italy, and China. It is around 0.3 for 14 countries but negative for only the UAE. It is approximately 0.4 for four countries and 0.5 for two countries while it is highest for Chile and Germany with about 0.6. However, the beta estimate is lowest with about 0.1 for Latvia and Qatar. Concerning the commodities indices, the beta estimate is around 0.1 for 11 commodities, approximately 0.2 for two commodities and highest with around 0.3 for six commodities. The beta coefficients for all the commodities indices have a positive sign. Also, the beta estimates for our aggregated markets are

positive and around 0.2 except the emerging market index which is about 0.3. This model performed well based on the value of the Durbin-Watson test, which falls within the acceptable range of 2 for all the stock markets. Similarly, the null hypothesis of serial correlation is rejected for all the markets. However, the R squared for all the markets are tiny ranging between 0.01 and 0.35.

Given that the beta coefficient for the world aggregated index is about 0.2, the following stylized facts emanate from the simple CAPM result. First, the systematic risk in 23 countries is about the same as the world average. Put differently, the systematic risks and expected excess returns in these markets are equivalent to the world average risk and return. Similarly, the beta coefficient for the markets which are higher than the world average implies that the systematic risk and expected excess returns are higher in these markets than in the world average. However, for the countries whose beta coefficients are higher than the world average, the implication is that investments in these stock markets have higher systematic risk and excess return than the world average (especially in Chile and Germany). In countries with lower beta coefficient than the world average such as Latvia and Qatar, systematic risks and return are smaller than the world average.

Secondly, given that the beta coefficient for most commodities is less than the aggregate world average, this suggests that investment in most commodities carry lower systematic risks and returns. The implication is that most commodity stocks carry lower systematic risk and returns. They can be viewed as an alternative asset class, whose inclusion in an investment portfolio reduces risk, especially during periods of turbulence in stock prices. However, the remaining commodities whose beta coefficient is either the same as the world aggregate stock (or higher) suggest that they carry similar risk and return as the world average stock (or higher). This provides further empirical evidence in support of increasing financialization of commodities. Regarding the aggregate market indices, the higher beta coefficient in the emerging market than the coefficient for Europe and the world average suggests that investment in emerging stock markets carry higher expected returns but with higher systematic risks than investments in Europe and the world average stock. This means that returns on investments in emerging market stocks appear to be more volatile than in the European and world average stocks.

Table 1: Unconditional CAPM Results

STOCKS	US S&P500	CAN TSX	GER DAX	AUS ASX	DEN OMX	FIN OMXHEX	SPNIBEX	SLOVSBITOP	UKFTSE100	ITAFITSEMIB
intercept	0.0077 [0.3482]	0.0106 [0.2870]	0.0116 [0.1162]	0.0081 [0.3226]	0.0111 [0.1828]	0.0171 [0.1335]	0.0064 [0.4697]	0.0068 [0.4571]	0.0162 [0.2496]	0.0101 [0.4312]
Beta ( $\beta$ )	0.2471 [0.0002]***	0.0381 [0.5797]	0.6123 [0.000]***	0.232 [0.0006]***	0.2016 [0.0029]***	0.4876 [0.000]***	0.1978 [0.0029]***	0.3627 [0.000]***	-0.1797 [0.0074]***	-0.1745 [0.0097]***
R-squared	0.0637	0.0014	0.3539	0.0533	0.0401	0.1613	0.0402	0.1015	0.0326	0.0304
Durbin-Watson	1.9727	2.0502	2.0201	1.9666	1.9064	1.7608	1.8499	1.9124	2.0654	2.0654
Q-stat	0.0282 [0.867]	0.1603 [0.689]	0.7928 [0.373]	0.0467 [0.829]	0.4311 [0.511]	0.2076 [0.647]	1.1872 [0.276]	0.3463 [0.556]	0.268 [0.605]	0.3132 [0.576]

	THAI-SET50	MYL-FTSEKLCI	INDO-JCI	PHI-PSEI	SING-STI	CHIN-SSE	INDI-SENSEX	TAI-TWSE	KOR-KOSPI	HNGKNG-HIS
intercept	0.0086 [0.2182]	0.0054 [0.2481]	0.0101 [0.1947]	0.009 [0.3151]	0.0056 [0.5111]	-0.0103 [0.7773]	0.0086 [0.3896]	0.0043 [0.4812]	0.0101 [0.3213]	0.0069 [0.4272]
Beta ( $\beta$ )	0.2017 [0.0035]***	0.1896 [0.0081]***	0.2436 [0.0005]***	0.2158 [0.0035]***	0.3095 [0.000]***	-0.1663 [0.0160]**	0.3331 [0.000]***	0.5155 [0.000]***	0.2419 [0.0017]**	0.3572 [0.000]***
R-squared	0.0387	0.0319	0.0536	0.0387	0.096	0.0264	0.0967	0.2287	0.0445	0.1248
Durbin-Watson	2.1738	2.0695	2.1227	1.8994	2.0436	1.9824	2.0814	2.1367	1.8535	2.0097
Q-stat	1.7527 [0.186]	0.4358 [0.509]	0.8736 [0.350]	0.5167 [0.472]	0.1241 [0.725]	0.0023 [0.962]	0.3721 [0.542]	1.0419 [0.307]	1.0222 [0.312]	0.0144 [0.905]

	LAT-OMXRIGA	EST-OMXTALLIN	TUR-XU100	CRO-CROBEX	LIT-OMXVILNIUS	HUN-BUX	EGY-EGX30	POR-PS-I20	CZECH-SEPX	BRA-BOVESPA
intercept	0.0062 [0.4210]	0.0109 [0.2790]	0.0155 [0.4079]	0.0225 [0.2746]	0.0063 [0.5563]	0.0092 [0.7094]	0.0093 [0.6995]	0.0017 [0.7916]	0.0094 [0.3302]	0.0053 [0.5432]
Beta ( $\beta$ )	0.1296 [0.0446]**	0.2855 [0.000]***	0.2242 [0.0018]***	-0.012 [0.8653]	0.2874 [0.0003]***	0.0955 [0.1950]	0.1056 [0.1234]	0.2801 [0.000]***	0.3359 [0.000]***	0.3757 [0.000]***
R-squared	0.0185	0.0655	0.044	0.0001	0.0598	0.0077	0.0109	0.0689	0.094	0.0965
Durbin-Watson	1.8544	1.8302	1.8115	2.0498	1.8138	2.0543	2.0378	2.1769	1.9824	2.1489
Q-stat	0.71 [0.399]	1.3428 [0.247]	1.4529 [0.228]	0.142 [0.706]	1.8428 [0.175]	0.1662 [0.684]	0.0814 [0.775]	1.7431 [0.187]	0.0093 [0.923]	1.267 [0.260]

	SWTSMI	NZLNZX50	FRA-CAC	NOR-OSEAX	NETH-AEX	JAP-NIKKEI225	IRE-ISEQ	TUN-TUNINDEX	UKR-PFTS	BUL-SOFIX
intercept	0.03249 [0.1512]	0.0023 [0.5147]	0.0067 [0.4315]	0.0078 [0.3783]	0.0072 [0.4005]	0.0064 [0.4428]	0.0095 [0.2810]	0.0025 [0.5240]	-0.0058 [0.6126]	0.0061 [0.8041]
Beta ( $\beta$ )	-0.086 [0.2416]	0.3711 [0.000]***	0.3711 [0.0013]***	0.1973 [0.0045]***	0.1738 [0.0108]**	0.4367 [0.0121]**	0.4367 [0.000]***	0.1718 [0.0527]*	0.2589 [0.0071]***	0.0932 [0.1718]
R-squared	0.0077	0.1331	0.0463	0.0366	0.0296	0.0286	0.1614	0.0172	0.0328	0.0085
Durbin-Watson	2.0626	1.9617	1.9349	1.8167	1.8752	1.8354	1.8502	1.8821	1.9489	1.9616
Q-stat	0.1879 [0.665]	0.0377 [0.846]	0.195 [0.659]	0.6913 [0.193]	0.7412 [0.389]	1.4414 [0.230]	0.1905 [0.662]	0.7436 [0.389]	0.1316 [0.717]	0.0802 [0.777]

	POL-WIG	ICE-SEICEX	RUS-MICEX	MALT-MALTEX	ISR-TA100	COL-COLCAP	BELG-BEL20	UAE-ADXGEN	CHIL-IGPA	MEX-S&PBMVIPC
intercept	0.0057 [0.5868]	0.0096 [0.6729]	0.0123 [0.5163]	0.0081 [0.3786]	0.0052 [0.5638]	0.0128 [0.2163]	0.0057 [0.5402]	0.0085 [0.6687]	0.0017 [0.8238]	0.0127 [0.1266]
Beta ( $\beta$ )	0.2898 [0.0002]***	0.1019 [0.1346]	-0.0746 [0.3002]	0.3177 [0.000]***	0.3196 [0.000]***	0.2718 [0.000]***	0.2566 [0.0008]***	-0.2922 [0.000]***	0.5691 [0.0000]***	-0.1118 [0.1544]
R-squared	0.0623	0.0103	0.0049	0.0776	0.0926	0.0534	0.0505	0.0827	0.2848	0.0093
Durbin-Watson	1.9547	1.9107	2.0189	1.9756	2.0066	1.7914	1.8557	2.0044	1.8974	1.8764
Q-stat	0.0797 [0.778]	0.4361 [0.509]	0.0244 [0.876]	0.0207 [0.886]	0.0144 [0.905]	2.3431 [0.126]	1.0553 [0.304]	0.002 [0.964]	0.4974 [0.481]	0.8276 [0.363]

Note: Series names are given in Table 8. \*\*\*,\*\* and \* denote significance of CAPM betas at 1%, 5% and 10% levels respectively whereas Q-stat is the statistics for residual normality.

	SWEOMX30	SERB-BELEX15	ARG-MERVAL25	ROM-BET	QAT-QE	LUX-LUXX
intercept	0.0233 [0.3630]	0.0142 [0.4688]	-0.0005 [0.9838]	0.0031 [0.7515]	0.01152 [0.2708]	0.0099 [0.3264]
Beta ( $\beta$ )	0.0524 [0.4891]	0.2704 [0.000]***	-0.0805 [0.2366]	0.2003 [0.0067]***	-0.1269 [0.0663]*	0.2432 [0.0020]***
R-squared	0.0026	0.0731	0.0064	0.0334	0.0158	0.043
Durbin-Watson	1.9897	1.9432	2.0194	1.8754	2.1518	1.9197
Q-stat	0.0048 [0.945]	0.1777 [0.673]	0.0249 [0.875]	0.7531 [0.386]	1.2897 [0.256]	0.321 [0.571]

COMMODITIES	CRUDEOIL	GOLD	SILVER	GAS	COPPER	PLATINUM	PALLADIUM	NICKEL	TIN	ZINC
intercept	0.0053 [0.6061]	0.0021 [0.8374]	0.0089 [0.7096]	0.0094 [0.6820]	0.0077 [0.3987]	0.011 [0.6390]	0.0106 [0.2205]	0.0083 [0.7157]	0.0096 [0.2899]	0.0061 [0.7949]
Beta ( $\beta$ )	0.1069 [0.0880]*	0.32011 [0.000]***	0.1246 [0.0660]*	0.0922 [0.1342]	0.3706 [0.000]***	0.1077 [0.1092]	0.2398 [0.000]***	0.1257 [0.0509]*	0.3067 [0.000]***	0.1282 [0.0542]*
R-squared	0.0134	0.0732	0.0155	0.0103	0.1407	0.0118	0.0983	0.0172	0.1046	0.0169
Durbin-Watson	1.9056	2.1019	2.0125	1.9739	2.1943	1.999	1.8989	1.9564	2.1686	1.9902
Q-stat	0.4511 [0.502]	0.6095 [0.435]	0.0104 [0.919]	0.0328 [0.856]	2.2226 [0.136]	0.5387 [0.997]	0.0999 [0.463]	1.6321 [0.752]	1.6321 [0.201]	0.0046 [0.946]

	COCOA	COFFEE	COTTON	RHODIUM	RUTHENIUM	CORN	RUBBER	SOYABEAN	WOOL	ALUMINIUM
intercept	0.0059 [0.7981]	-0.0037 [0.8760]	0.0008 [0.9809]	0.01165 [0.6261]	0.0105 [0.6568]	-0.0016 [0.9432]	0.0045 [0.6329]	0.0032 [0.7332]	0.0026 [0.7740]	0.0057 [0.8042]
Beta ( $\beta$ )	0.1372 [0.0377]**	0.1636 [0.0157]**	0.1191 [0.0727]*	0.05642 [0.3968]	0.1367 [0.0392]**	0.1581 [0.0195]**	0.1728 [0.0050]***	0.3539 [0.000]***	0.3223 [0.000]***	0.1121 [0.0998]*
R-squared	0.0197	0.0266	0.0147	0.0033	0.0194	0.0245	0.0358	0.1163	0.0819	0.0124
Durbin-Watson	1.9897	2.0091	1.9569	1.8697	1.9815	2.0378	1.8445	2.2123	2.0051	1.9352
Q-stat	0.0046 [0.946]	0.0051 [0.943]	0.1027 [0.749]	0.9213 [0.337]	0.0153 [0.902]	0.082 [0.775]	1.2238 [0.269]	2.6107 [0.106]	0.0054 [0.941]	0.2305 [0.631]

	LEAD	WHEAT
intercept	0.0114 [0.6214]	-0.0052 [0.6056]
Beta ( $\beta$ )	0.1266 [0.0584]*	0.3417 [0.000]***
R-squared	0.0164	0.114
Durbin-Watson	1.9342	2.1575
Q-stat	0.234 [0.629]	1.5416 [0.214]

AGGREGATES	WORLD	EUROPE	EMERGING MARKETS
intercept	0.0056 [0.5141]	0.0053 [0.5607]	0.0039 [0.6561]
Beta ( $\beta$ )	0.2352 [0.0006]***	0.2347 [0.0006]***	0.3008 [0.000]***
R-squared	0.0531	0.0523	0.0915
Durbin-Watson	1.9604	1.9584	1.9702
Q-stat	0.0554 [0.814]	0.0616 [0.804]	0.0314 [0.859]

Note: Series names are given in Table 8. \*\*\*,\*\* and \* denote significance of CAPM betas at 1%, 5% and 10% levels respectively whereas Q-stat is the statistics for residual normality.

## 4.2 Results of regime-switching market beta

Next, we evaluate the performance of the conditional CAPM with a time-varying beta. We expect to uncover a positive and significant link between the dynamic conditional beta and asset returns.

Since up and down market movements are random variables observed with a given probability, we now proceed to estimate our next model for the MS-CAPM as stated in equation (3). Here, the intuition is to employ the Markov Switching regression technique in the estimation of intercepts and CAPM beta under a regime-switching framework. The underlining theory of CAPM maintains that a stable and linear relationship exists between asset returns and risk. Investors will expect to be compensated by a higher level of wealth through the positive correlation of the returns. However, Huang (2003) argues that evidence abounds suggesting significant variations in market beta. For instance, Jagannathan and Wang (1996) note that relative risks associated with changes in a firm's cash flow over the business cycle may induce some switching behavior in market risk. During bad economic times, when the expected market risk premium is high, leveraged firms are more likely to face financial difficulties, and thus could have higher conditional betas. Also, technology or taste shocks induce fluctuations in the betas of firms.

Given this, our MS-CAPM follows Huang (2000) by allowing the systematic risk of  $\beta$  to come from two different regimes to show whether it is unstable over the regimes. **The two-regime specification is selected as the classic "boom-bust" representation of the business cycle by Hamilton (1989). For a survey of this representation, see the paper by Maheu and McCurdy (2000). Notice that increasing the number of regimes would need to be explained on economic grounds, and would induce a greater computational burden on the regime-switching models.** This model would enable us to determine if the estimates of alpha and beta coefficients are significantly different between low and high volatility regimes, and if they are consistent with the unconditional CAPM. Finally, the transition probabilities matrix is constructed with values that indicate how difficult it is to switch from one volatility regime to the other.

From Table 2 and Figure 2, some interesting results stand out. First, following Korkmaz et al. (2010), the low and high volatility regimes are distinguished based on the size of the estimated standard errors of the regression. In the low volatility regime, the estimates of the beta coefficient of the securities of Slovenia, France, the UK, Sweden, Switzerland, Norway, Japan, Ireland, China, Singapore, and Bulgaria are statistically significant and less than one showing that the securities in these countries are less risky than the respective markets. However, estimates of the beta coefficients in this same period for Finland, Malaysia, Philippines, Taiwan, Hong Kong, Serbia, New Zealand, Portugal, and Chile are less than zero and statistically significant indicating that returns in these countries move in an opposite direction with movements in market returns. The returns on securities in Sweden is the riskiest as indicated by the highest beta coefficient of 0.59 while that of France is the least risky, as shown by a beta parameter of approximately 0.003. Lastly, the beta coefficients of the remaining markets are not statistically significant, indicating that they do not have a relation with the market during the low volatile regime.

Concerning the high volatility regime, estimates of the beta parameter for securities in Germany, Finland, France, the UK, New Zealand, Ireland, Malaysia, Philippines, and Taiwan are less than one and statistically significant. Returns on these securities are therefore less risky than the respective market returns in this regime. Whereas estimates of the beta parameter for securities from Norway and Thailand are less than zero and statistically significant, indicating that they move in the opposite direction with movements in the market returns during this period. Also in this period, returns on securities in Finland is the riskiest with a beta coefficient of 0.84 while that of France is the least risky with a beta coefficient of 0.02. The beta parameters of the remaining countries are found to be statistically insignificant suggesting that they do not have a relation with the market return or an inefficient market.

Regarding the commodity indices, in the low volatility regime, the estimates of the beta coefficient for Silver, Platinum, Rhodium, Ruthenium, Corn, Rubber, Aluminum and Cocoa are less than one and statistically significant. This suggests that returns on these commodity securities are less risky than the market return. In the same period, only the beta coefficients for Copper is less than zero but statistically insignificant, indicating that its performance moves in the oppo-

site direction with the market return. More so, Rubber is the most dangerous commodity in this regime with a beta coefficient of about 0.43, while Silver is the least risky with about 0.17 beta coefficient. The estimates of beta coefficients for the remaining commodity indices are statistically insignificant showing that they are not related to the market return in the low volatility regime.

In the high volatility regime, among the commodity indices, only the estimates of beta coefficients of Wheat and Cotton are less than one and statistically significant. This shows that in this period, only these two securities are less risky than the market returns. Similarly, only the beta coefficients for Palladium and Tin are less than zero and statistically significant, indicating that they move in the opposite direction to the movement of market returns. Whereas, the remaining commodity indices have no relation with the market return as suggested by their statistically insignificant beta coefficients. In the high volatility regime, Wheat is the riskiest commodity but with a beta coefficient of only 0.28 while Cotton is the least risky with a beta coefficient of about 0.17. The implication is that commodity indices are less risky in both low and high volatility regimes compared to country securities. This is as suggested by the estimates of beta coefficients for the least as well as the riskiest securities in both regimes.

Regarding the aggregate markets, only the World aggregate security is statistically significant and less than one with a beta coefficient of about 0.30. The remaining aggregate securities, including Europe and Emerging Markets, are statistically insignificant, suggesting that they do not have a relation with the market in the low volatility regime. More so, in the high volatility regime, all the estimates of beta coefficients for all the aggregate markets are statistically insignificant. This implies that they do not have a relation with the market during the high volatility period. Concerning the probabilities of transition from one volatility regime to the other, the probability of switching from the low volatility regime to the high volatility regime is higher than the probability of switching from the high volatility to low volatility regime for the World market security. The transition probability for the low volatility regime is 0.26, whereas it is 0.21 for the high volatility regime.

Concerning the probabilities of transition from one regime to the other for the countries, it is generally less likely to switch from low volatility regime to high volatility regime, as indicated by low values of  $P_{12}$ . The highest value of  $P_{12}$  is 0.61 for Bulgaria. However, the probability of transition from high to low volatility regime is relatively higher, as shown by higher  $P_{21}$  values. For instance, the  $P_{21}$  value for Germany is 0.99. Similarly, among commodity indices, it is also less likely to switch from low to high volatility regime as suggested by  $P_{12}$  values. The  $P_{12}$  value is highest for Copper with about 0.21, whereas it is relatively more likely to switch from high to low volatility regime. The  $P_{21}$  value is highest for Tin with 0.24. This result generally suggests that the probability of transition from any regime to the other is relatively higher in conventional financial assets class than in the commodity market.

Figure 2 reports the filtered probabilities of the bull and bear regimes for 12 selected markets, including Argentina, Bulgaria, China, Denmark, Natural Gas, Italy, Platinum, Rhodium, Silver, Turkey, the USA, and Zinc. The two regimes are quite distinct. The pattern displayed by the filtered probabilities in regime 1 suggests that, in the earlier part of the sample, high volatility levels dominate in Argentina, Bulgaria, Platinum, Silver, Turkey, and the USA. However, volatility decreases substantially towards the middle of the sample. Towards the middle of the sample, high volatility levels are incidental and transitory, whereas, towards the end of the sample, high volatility levels seem to gather more again. However, in the same regime, periods of low volatility level predominate especially among commodities such as Natural Gas, Rhodium, Zinc before periods of high volatility levels clustered towards the middle and the end of the sample. Lastly, the entire sample period was dominated by a constellation of high volatility levels throughout regime 1 for Denmark. The opposite held for Italy, where low volatility levels dominated the whole sample period, except at the beginning of the second half of the sample. Then, volatility decreased substantially until the end of the sample.

The pattern of filtered probabilities for regime 2 suggests that high volatility levels dominated the entire sample period in China, Italy, Turkey, and the USA. On the contrary, for Gas, Rhodium, and Zinc, volatility was incidental and transitory but clustered towards the end of the sample. In

Bulgaria, Platinum, and Silver, the pattern shows that low volatility levels dominated but was mixed with periods of high volatility levels in the USA until the middle of the sample. The second half of the sample is characterized by periods of high volatility levels with the persistence of low volatility levels before high volatility levels clustered towards the end of the sample. However, low volatility levels evolved throughout the first half of the sample before the appearance of high volatility levels in the second half in Bulgaria. On the contrary, in Argentina, the pattern shows that low volatility levels persisted in most of the sample. In Denmark, the pattern indicates that volatility rose slowly throughout the entire regime, except around the middle of the sample.

Generally, the filtered probabilities for both regimes show significant consistency with the empirical pattern displayed by the results of MS-CAPM. Towards the middle of our sample coincides with the period of the past financial crisis. The implication is that the years of the financial turmoil, U.S quantitative easing and the European sovereign debt crisis which influenced financial market investors' risk appetite and therefore, asset prices and returns may have increased volatility levels especially among commodities such as Natural Gas, Rhodium, and Zinc in regime 1.

Table 2: MS-CAPM Results: Conditional Betas with Time-Varying Risks and Returns

STOCKS	MSCI USA		MSCI CAN		MSCI GER		MSCI AUS		MSCI DEN		MSCI FIN		MSCI SPN	
<b>Regime 1 intercept</b>		SD												
<b>Beta (<math>\beta</math>)</b>	-0.0228	[0.0080]***	0.0078	[0.0332]	-1.0609	[0.0001]***	0.0132	[0.0137]	0.0212	[0.0182]	0.1719	[0.000]***	0.0105	[0.0136]
	-0.1606	[0.1518]	-0.1739	[0.1285]	0.3705	[0.0303]***	0.0097	[0.0972]	0.0442	[0.1123]	0.8412	[0.000]***	0.0537	[0.0853]
<b>Regime 2 intercept</b>	0.0258	[0.0185]	-0.0055	[0.0050]	-0.0517	[0.0001]***	-0.0165	[0.0065]**	-0.0158	[0.0074]***	-0.1719	[0.000]***	-0.0194	[0.0069]***
<b>Beta (<math>\beta</math>)</b>	0.0225	[0.1139]	0.1101	[0.0845]	-0.0047	[0.0058]	-0.1148	[0.1792]	0.0494	[0.1077]	-0.0391	[0.000]***	0.1398	[0.1220]
p12	0.1629		0.0249		0.01		0.1034		0.1667		0.08		0.0723	
p21	0.1582		0.0871		0.99		0.0753		0.1884		0.91		0.038	

	MSCI SLOV		MSCI FRA		MSCI UK		MSCI ITA		MSCI SWE		MSCI SWT		MSCI NZL	
<b>Regime 1 intercept</b>														
<b>Beta (<math>\beta</math>)</b>	-0.0079	[0.0078]	-0.9592	[0.4355]**	0.6456	[0.5236]	0.0511	[0.0820]	0.0088	[0.0068]	0.1278	[0.1004]	0.0038	[0.0097]
	0.3082	[0.0017]***	0.0027	[0.0010]***	0.0031	[0.0004]***	-0.1316	[0.2033]	0.5927	[0.0598]***	-0.1604	[0.1721]	0.5578	[0.2270]**
<b>Regime 2 intercept</b>	0.0076	[0.0165]	0.0326	[0.0002]***	-0.0043	[0.1137]	-0.0133	[0.0050]**	0.0775	[0.1269]	-0.022	[0.0056]	-0.0013	[0.0048]
<b>Beta (<math>\beta</math>)</b>	-0.0291	[0.0976]	0.0229	[0.0034]***	0.1667	[0.0349]***	0.0099	[0.0494]	-0.0096	[0.1771]	0.2605	[0.084]***	-0.3166	[0.1008]***
p12	0.1171		0.06		0.12		0.0656		0.0869		0.0805		0.3592	
p21	0.1606		0.94		0.88		0.3598		0.0171		0.3224		0.6839	

	MSCI NOR		MSCI NLD		MSCI JAP		MSCI IRE		MSCI THAI		MSCI MYL		MSCI INDO	
<b>Regime 1 intercept</b>														
<b>Beta (<math>\beta</math>)</b>	0.0095	[0.0187]	-0.0185	[0.0075]**	0.0039	[0.0077]	0.0037	[0.999]	-0.0017	[0.0181]	0.0083	[0.0043]	0.0088	[0.0072]
	0.459	[0.1039]***	-0.0757	[0.1393]	0.2311	[0.0886]***	0.4096	[0.0017]***	-0.6918	[0.2172]***	-0.177	[0.0554]***	-0.1044	[0.0789]
<b>Regime 2 intercept</b>	-0.012	[0.0097]	0.011	[0.0133]	-0.0062	[0.0186]	-0.0037	[0.0976]	0.0063	[0.0072]	-0.0268	[0.0155]	-0.0443	[0.0332]
<b>Beta (<math>\beta</math>)</b>	-0.2939	[0.1131]**	0.0553	[0.0868]	-0.0471	[0.1106]	0.0808	[0.0137]***	0.1103	[0.0589]	0.7869	[0.2831]***	-0.0397	[0.1739]
p12	0.4391		0.0955		0.06411		0.61		0.4545		0		0.0721	
p21	0.5341		0.1552		0.0407		0.39		0.9203		0.4119		0.1402	

	MSCI PHI		MSCI SING		MSCI CHIN		MSCI INDI		MSCI TAI		MSCI KOR		MSCI HNGKNG	
<b>Regime 1 intercept</b>														
<b>Beta (<math>\beta</math>)</b>	-0.0213	[0.0158]	0.0284	[0.0272]***	0.0263	[0.0149]	-0.0109	[0.0100]	0.0504	[0.0875]	-0.0163	[0.0082]**	0.0298	[0.0190]
	-0.3613	[0.1123]***	0.0338	[0.1375]	0.2119	[0.0710]***	-0.2112	[0.1090]	0.3817	[0.1792]**	-0.1195	[0.0981]	0.0673	[0.1197]
<b>Regime 2 intercept</b>	0.0251	[0.0216]	-0.0128	[0.0063]**	-0.059	[0.0897]	0.0153	[0.0254]	-0.0095	[0.0057]	0.0202	[0.0223]	-0.03	[0.0077]***
<b>Beta (<math>\beta</math>)</b>	0.5312	[0.1663]***	-0.1747	[0.0816]**	-0.0491	[0.1208]	0.0321	[0.1288]	-0.2688	[0.0638]***	0.1625	[0.1172]	-0.2955	[0.0926]***
p12	0.6646		0.0866		0.2534		0.2816		0.0469		0.2981		0.2877	
p21	0.6579		0.2053		0.1634		0.1741		0.808		0.2661		0.3107	

	MSCI SERB		MSCI UKR		MSCI BUL		MSCI ROM		MSCI POL		MSCI ICE		MSCI RUS	
<b>Regime 1 intercept</b>														
<b>Beta (<math>\beta</math>)</b>	0.0087	[0.049]	-0.0239	[0.0308]	0.0255	[0.0680]	-0.0081	[0.0056]	-0.0056	[0.0296]	-0.0054	[0.0095]	-0.0187	[0.0744]
	0.3258	[0.2477]	0.0329	[0.1205]	-0.0468	[0.1242]	0.1244	[0.0821]	0.1316	[0.1296]	0.2566	[0.0525]	-0.0732	[0.1534]
<b>Regime 2 intercept</b>	-0.0094	[0.0057]	0.0138	[0.0081]	-0.0008	[0.0092]	0.0437	[0.0552]	0.0038	[0.0097]	0.0139	[0.0745]	0.0036	[0.0123]
<b>Beta (<math>\beta</math>)</b>	-0.2633	[0.0226]***	-0.0234	[0.0992]	0.269	[0.0495]***	-0.0062	[0.1618]	-0.1151	[0.0909]	-0.0034	[0.1808]	0.1015	[0.0892]
p12	0.1237		0.0211		0.0401		0.3481		0.0211		0.0606		0.0355	
p21	0.2019		0.0456		0.076		0.0617		0.054		0.0333		0.1178	

Note: Series names are given are given in Table 8. \*\*\* and \*\* denote significance of regime switching CAPM betas at 1% and 5% whereas  $p_{12}$  and  $p_{21}$  refer to the probability of moving from one volatility regime to the other

	MSCI MALT		MSCI ISR		MSCI LAT		MSCI EST		MSCI TUR		MSCI CRO		MSCI LIT	
<b>Regime 1</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	-0.0143	[0.0071]	-0.0192	[0.0093]**	0.0029	[0.0436]	0.0374	[0.0365]	0.1821	[0.1072]	0.0059	[0.0093]	0.0097	[0.0418]
	-0.092	[0.0997]	0.0914	[0.1181]	0.0734	[0.1628]	0.1196	[0.1463]	0.0252	[0.1866]	0.1112	[0.0680]	0.2079	[0.1321]
<b>Regime 2</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	0.0114	[0.0165]	0.0285	[0.0191]	0.0013	[0.0043]	-0.0172	[0.0093]	-0.035	[0.011]***	-0.009	[0.0536]	-0.0084	[0.0075]
	0.0226	[0.0960]	-0.0687	[0.1123]	-0.0286	[0.0803]	-0.0111	[0.0875]	-0.0754	[0.0733]	-0.0472	[0.1186]	0.0081	[0.0531]
p12	0.1095		0.2678		0.0169		0.068		0.0467		0.1267		0.1107	
p21	0.1284		0.2533		0.0553		0.1719		0.21		0.0819		0.3952	

	MSCI HUN		MSCI EGY		MSCI POR		MSCI CZECH		MSCI BRA		MSCI ARG		MSCI COL	
<b>Regime 1</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	0.0025	[0.0819]	0.0064	[0.0094]	0.0075	[0.0114]	-0.0036	[0.0091]	-0.0082	[0.0160]	-0.0225	[0.0939]	0.0136	[0.0267]
	-0.0352	[0.1219]	0.2024	[0.0508]	-0.3737	[0.0544]***	-0.0371	[0.0877]	0.042	[0.1578]	0.024	[0.1489]	0.1584	[0.1425]
<b>Regime 2</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	-0.0001	[0.0105]	-0.0081	[0.0624]	-0.0096	[0.0128]	0.0087	[0.0289]	0.0039	[0.0126]	0.005	[0.0117]	-0.0079	[0.0123]
	0.0211	[0.1075]	-0.0659	[0.1223]	0.3844	[0.2819]	0.033	[0.1342]	-0.1621	[0.1006]	0.0102	[0.0931]	0.0269	[0.0965]
p12	0.0258		0.07652		0.719		0.2346		0.2609		0.0284		0.2583	
p21	0.0529		0.0454		0.4484		0.1125		0.5859		0.1062		0.4076	

	MSCI BEL		MSCI UAE		MSCI CHIL		MSCI MEX		MSCI QAT		MSCI TUN		MSCI LUX	
<b>Regime 1</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	-0.0057	[0.0078]	0.0387	[0.0483]	0.0036	[0.0056]	0.0072	[0.0218]	-0.0115	[0.0499]	-0.001	[0.0085]	-0.0125	[0.007]
	0.0678	[0.0944]	0.0535	[0.1145]	-0.162	[0.0686]**	0.0734	[0.1207]	-0.4087	[0.2358]	0.0732	[0.1819]	-0.0429	[0.0930]
<b>Regime 2</b>														
<b>intercept</b>														
<b>Beta (<math>\beta</math>)</b>	0.0277	[0.0447]	-0.0193	[0.0149]	0.0049	[0.0848]	-0.0049	[0.0064]	0.0036	[0.0073]	0.001	[0.0049]	0.0147	[0.0161]
	0.0637	[0.1728]	0.1173	[0.0931]	0.4321	[0.4117]	0.0073	[0.0809]	0.1029	[0.0615]	0.0513	[0.1237]	0.0613	[0.0891]
p12	0.2068		0.0479		0.3024		0.0245		0.0549		0.4656		0.1061	
p21	0.0506		0.0757		0.0276		0.9754		0.2131		0.5198		0.1705	

AGGREGATES	MSCI WORLD		MSCI Europe		MSCI EM	
<b>Regime 1</b>						
<b>intercept</b>						
<b>Beta (<math>\beta</math>)</b>	0.0255	[0.0151]	0.0128	[0.0149]	-0.0142	[0.0074]
	-0.1677	[0.1131]	0.001	[0.3035]	0.0426	[0.1204]
<b>Regime 2</b>						
<b>intercept</b>						
<b>Beta (<math>\beta</math>)</b>	-0.0246	[0.0071]***	-0.018	[0.0068]**	0.0123	[0.0149]
	0.307	[0.0938]***	-0.0319	[0.0409]	-0.0055	[0.1001]
p12	0.2689		0.0698		0.0311	
p21	0.2142		0.0538		0.0429	

Note: Series names are given are given in Table 8. \*\*\* and \*\* denote significance of regime switching CAPM betas at 1% and 5% whereas  $p_{12}$  and  $p_{21}$  refer to the probability of moving from one volatility regime to the other

COMMODITIES	MSCI OIL		MSCI GOLD		MSCI SILVER		MSCI GAS		MSCI Copper		MSCI Platinum	
<b>Regime 1</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	0.0206	[0.0159]	0.014	[0.0126]	0.0029	[0.0640]	0.0053	[0.0639]	0.0312	[0.0204]	0.0016	[0.0097]
	0.1201	[0.0970]	-0.1263	[0.0907]	-0.041	[0.1247]	0.016	[0.1181]	-0.0467	[0.1336]	0.2433	[0.0530]***
<b>Regime 2</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	-0.0347	[0.0214]	-0.0397	[0.0628]	0.0058	[0.0102]	0.001	[0.0121]	-0.0235	[0.0144]	0.0121	[0.0610]
	-0.1318	[0.1022]	-0.0286	[0.1711]	0.1795	[0.0594]***	0.0122	[0.0931]	-0.2927	[0.0984]***	-0.036	[0.1204]
p12	0.1966		0.08143		0.06012		0.0267		0.2128		0.0516	
p21	0.8033		0.02208		0.1093		0.0529		0.2951		0.0305	

	MSCI Palladium		MSCI Nickel		MSCI Tin		MSCI Zinc		MSCI Rhodium		MSCI Ruthernium	
<b>Regime 1</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	-0.0267	[0.0092]***	0.008	[0.0123]	-0.0425	[0.0143]***	-0.0006	[0.0089]	0.0199	[0.0643]	-0.0037	[0.0094]
	-0.2215	[0.1085]**	0.0189	[0.0822]	-0.2589	[0.1035]**	0.0301	[0.0806]	0.026	[0.1236]	0.209	[0.0485]
<b>Regime 2</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	0.0134	[0.0145]	0.0003	[0.0263]	0.0348	[0.017]**	0.0117	[0.0674]	-0.0014	[0.0092]	0.0229	[0.0635]
	0.0761	[0.1024]	0.031	[0.1208]	-0.089	[0.1074]	0.0094	[0.1184]	0.2742	[0.0472]***	-0.0303	[0.1227]
p12	0.0814		0.0557		0.2145		0.0628		0.0383		0.0587	
p21	0.1217		0.0268		0.2427		0.0313		0.0683		0.0331	

	MSCI Corn		MSCI Rubber		MSCI Soyabean		MSCI Wool		MSCI Aluminium		MSCI Lead	
<b>Regime 1</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	0.0216	[0.0562]	0.0044	[0.0201]	0.0027	[0.0070]	0.0038	[0.0191]	-0.0004	[0.0080]	0.0092	[0.0096]
	-0.0561	[0.1157]	-0.0076	[0.1040]	-0.0091	[0.1002]	-0.0648	[0.1043]	0.218	[0.0481]***	0.2115	[0.0530]
<b>Regime 2</b>												
<b>intercept</b>												
<b>Beta (<math>\beta</math>)</b>	-0.0028	[0.0091]	-0.0012	[0.0062]	-0.0007	[0.0255]	-0.0014	[0.0067]	0.0151	[0.0565]	-0.0028	[0.0655]
	0.2342	[0.0485]***	0.4376	[0.0895]***	-0.1324	[0.1019]	0.261	[0.1363]	0.0103	[0.1154]	-0.0037	[0.1086]
p12	0.038		0.0359		0.0705		0.0349		0.0449		0.0903	
p21	0.0561		0.0539		0.0563		0.0502		0.0295		0.0477	

	MSCI Wheat		MSCI Cocoa		MSCI Coffee		MSCI Cotton	
<b>Regime 1</b>								
<b>intercept</b>								
<b>Beta (<math>\beta</math>)</b>	0.0086	[0.0227]	-0.0077	[0.0108]	0.0216	[0.0658]	0.0293	[0.0565]
	-0.2631	[0.1124]	0.1882	[0.0578]***	0.015	[0.1168]	-0.0003	[0.0322]
<b>Regime 2</b>								
<b>intercept</b>								
<b>Beta (<math>\beta</math>)</b>	-0.0032	[0.0081]	0.0303	[0.0623]	-0.0094	[0.0100]	-0.0105	[0.0099]
	0.2886	[0.1156]**	-0.0284	[0.1213]	-0.077	[0.0819]	0.1726	[0.0566]***
p12	0.0462		0.0431		0.0285		0.0206	
p21	0.0828		0.0244		0.0536		0.0321	

Note: Series names are given are given in Table 8. \*\*\* and \*\* denote significance of regime switching CAPM betas at 1% and 5% whereas  $p_{12}$  and  $p_{21}$  refer to the probability of moving from one volatility regime to the other

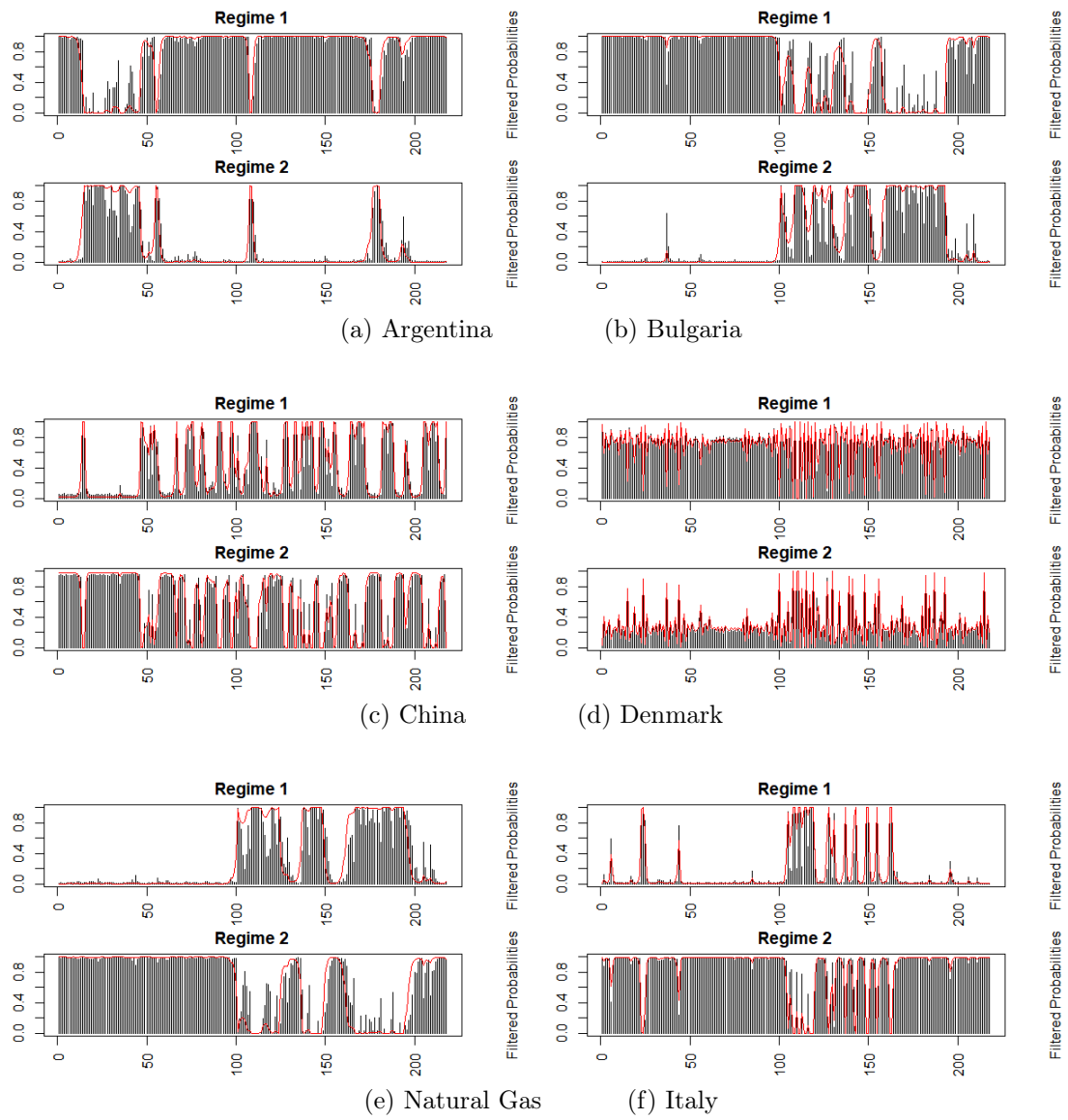


Figure 2: MS-CAPM state probabilities for twelve selected high-volatility markets

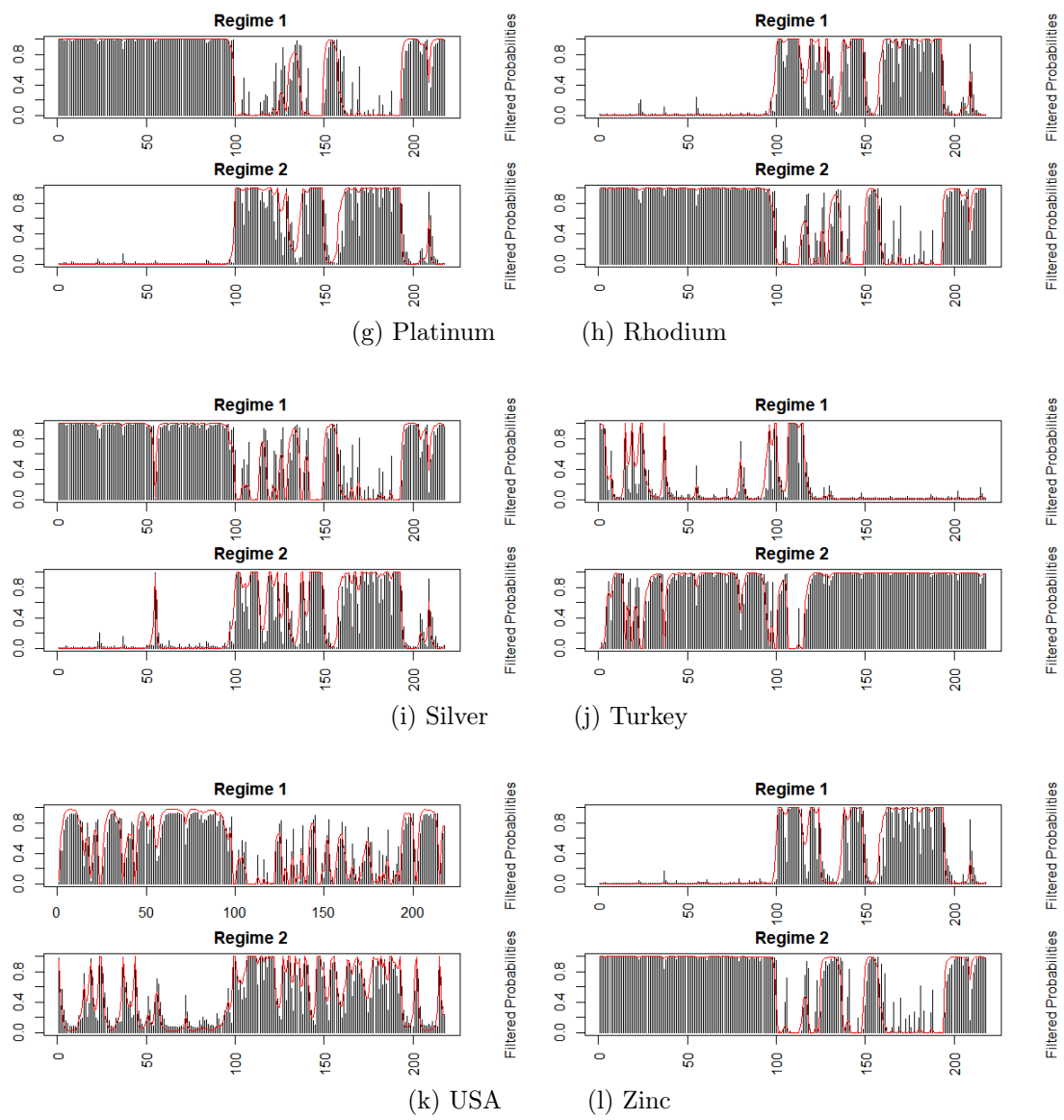


Figure 2: MS-CAPM state probabilities for twelve selected high-volatility markets (continued)

### 4.3 Results of the dynamic conditional regime-switching GARCH CAPM

Given the observed advantage of the MS-CAPM over the unconditional CAPM in accounting for instability of systematic risk especially by allowing beta coefficients to evolve through two volatility regimes, we proceed to estimate the MS-GARCH-CAPM as stated in equations (4) to (6) expecting that more realistic results could be acquired. The Markov Switching GARCH model is reputed in the analysis of systematic risks for several reasons. For instance, in addition to allowing the measure of systematic risks to be estimated from two regimes and the respective periods of duration in both volatility regimes to varying over time, under the Haas et al. (2004) specification, the conditional variance is set to change depending on the past data as well as the current regime. Also, it returns estimates of the posterior mean stable probabilities and the Bayesian predictive conditional volatility forecasts which have significant implications for risk management.

An acceptable way of comparing the two regimes from a regime-switching GARCH model is through the means of the regimes variables, obtained by averaging the regimes, which are the posterior mean stable probabilities of the states. Bauwens et al. (2010) note that a mean state close to 1 corresponds to a high probability to be in the second regime. To see this, we present in Table 3 and Figure 4 results of parameter estimates and the mean filtered volatility from the MS-GARCH-CAPM. This model performs substantially better than the previous two models in providing a more comprehensive range of insights into most of the stock markets in our sample, as shown by the higher number of statistically significant estimates of time-varying conditional betas in both the low and high volatility regimes.

Specifically, in the low volatility period, all the country stocks have estimates of beta coefficients that are statistically significant, positive, and less than one except in Finland, the United Kingdom, and Italy. This suggests that in all these markets, the stocks are less risky than the respective markets. The beta coefficients for Finland, the UK, and Italy are not found to be statistically significant. The beta coefficient for Ireland is statistically significant but negative, suggesting that the securities move in the opposite direction with the market. Generally, in this period, the values of beta coefficients varied widely across these markets. To cite few, Taiwan and Mexico exhibit the least beta of about 0.002 and 0.004, respectively. Belgium and the UAE record the highest beta with 0.97 and 0.93, respectively.

Regarding the high volatility regime, the model also offers broad insight into most of the markets in our sample as the estimates of beta coefficients are statistically significant, positive and less than 1 in all the country stocks except in Finland, the UK, and Ireland (suggesting that the stocks of these countries are less risky than the respective markets). The beta coefficients for Finland, the UK, and Ireland are statistically insignificant, suggesting that they do not follow movements in the market. The beta coefficients for Germany and France are statistically significant, but less than 0, indicating that the securities move in the opposite direction to movements in the market. Similarly, the beta coefficients vary widely throughout the markets with Tunisia and Hong-Kong having the least beta of about 0.08 each while Poland has the highest beta coefficient of about 0.94. The estimates of the predictive conditional volatility forecasts for both volatility periods reveal that regime 2 is generally more volatile than the first regime. The conditional volatility forecast for regime 2 is highest in Russia with about 187.1, whereas, in regime 1, it is highest in Turkey with about 11.8.

Concerning the low volatility regime in the commodity market, the estimates of beta coefficients are statistically significant, positive, and less than 1 in all the markets indicating that they are less risky than the respective markets. However, the beta coefficient for Ruthenium is about 1, suggesting that its securities risk is equivalent to that of the market. Similarly, Results from the high volatility regime in the commodity market corroborates that of the low volatility period as all estimates of beta coefficients are all statistically coefficient, positive and less than 1. However, the beta coefficients for Lead and Coffee are about one, implying that there are as risky as the market in this period. In both volatility regimes, the conditional volatility forecasts remained predominantly higher in regime 2 with the highest being about 163.8 for Rhodium, whereas it is 0.5 for Zinc.

Lastly, the beta coefficients in both the low and high volatility periods for the aggregate markets are statistically significant, positive, and less than 1. The world aggregate securities have the least beta coefficients of about 0.12 and 0.32 in low and high volatility periods, respectively. Emerg-

ing Markets' stocks have the highest beta of 0.42 in the low volatility period. Securities for the European market has the highest beta coefficient of 0.77 in the high volatility period. Similarly, regime 1 is characterized by low conditional volatility forecast levels, with the World aggregate stocks being the highest (at about 4.43). Regime 2 is characterized by high conditional volatility forecasts with securities in Europe being the most volatile (at about 131).

Let us contrast for now our comments on dynamic conditional betas relative to the unconditional measures of beta. We document the existence of higher dynamic conditional betas that must be compensated by higher expected returns. The unconditional measure of market beta does not capture this empirical finding. Therefore, the difference between the conditional and unconditional beta premia constitute economically significant features of market data in energy and financial markets that we bring up to the practitioners' attention.

Concerning the probabilities of transition from one beta regime to the other as represented by  $P_{12}$  and  $P_{21}$ , among the countries in our sample, it is generally more likely to move from high volatility regime to low volatility regime in most of the markets. It is only more likely to move from low volatility regime to high volatility regime in Finland, Slovenia, France, the UK, Sweden, Ireland, Thailand, Malaysia, Philippines, Hong-Kong, Bulgaria, Poland, Latvia, Turkey, Croatia, Czech, Argentina, Qatar, and Tunisia. It may be observed that Belgium that exhibited the highest distribution of systematic risks in both beta regimes, the transition probabilities of both regimes are large. This result suggests that the beta process has a little chance of staying for an extended period in any of the beta regimes. Conversely, Taiwan exhibits the least beta across both regimes. Its transition probabilities show that there is a very high chance of moving to the low beta regime and a relatively small chance of moving to the high beta regime. This result implies that there is a very high chance of staying relatively longer in the low beta regime.

Further, among the commodity stocks, the transition probabilities suggest that it is also more likely to switch from the high-volatility regime to the low-volatility regime in all the markets except in Platinum, Palladium, Ruthenium, Rubber, and Wool. These results imply that most commodity stocks have a higher chance of staying longer in the low beta regime than in the high beta better regime. For instance, both Lead and Coffee demonstrate unusually high beta coefficients. Their transition probabilities show that there is relatively a very high chance of moving to the low beta regime, while it may take a longer time in the low beta regime before switching to a high beta regime. Similarly, for the aggregate stocks representing the World and Europe, the transition probabilities suggest that both markets demonstrate very high chances of switching to the low beta regime, whereas the likelihood of moving to the high beta regime is relatively small. This implies that these markets have higher chances of staying for a longer period in the low beta regime process than the high beta regime. In contrast, the transition probabilities for emerging market suggest that the chance of switching to the high beta regime is higher than that of moving to the low beta regime.

Empirically, it is well documented that the estimation of betas and risk premia appears volatile. By way of comparison, it is evident that the CAPM beta is unstable over the three models, namely unconditional CAPM, MS-CAPM, and MS-GARCH-CAPM. Even more, the instability of beta can also be seen across different regimes in the regime-switching models. This violates the prediction of the traditional model of CAPM that the beta of risky assets is constant over time. For instance, notable structural changes may be found in frontline markets such as the USA, the UK, Germany, France, Finland, Ireland, China, Malaysia, Philippines, Taiwan, Oil, Copper, Palladium, and Tin. In the USA and Oil markets, results from the unconditional CAPM and MS-GARCH-CAPM suggest that stock returns are risky and move in the same direction with the market. The MS-GARCH model indicates that these stocks do not have a relation with the market.

Further, in the UK and China, results from the CAPM suggest that these stocks move in an opposite relation with the market. Both the MS-GARCH and the MS-GARCH-CAPM models suggest these securities move in the same direction with the markets. Also, in Germany, France, and Ireland, the CAPM and MS-CAPM imply that these securities move in the same direction with the markets. The MS-GARCH-CAPM suggest that these stocks move in an opposite relation with the market. In the rest of the markets, the CAPM and MS-GARCH-CAPM results show that

these securities move in the same direction with the market whereas the MS-CAPM suggest that these stocks move in the opposite direction with the market.

The instability of beta can also be noticed across regimes in the regime-switching models. For instance, the MS-CAPM reports that stocks in Germany and China move in the direction with the market in regime 1, but do not have a relation with the market in regime 2. Palladium and Tin securities move in the opposite direction with the market in regime 1, but also have no connection to the market in regime 2. Also, in this model, stocks in Finland and Taiwan move in the same direction with the market in regime 1, but move in the opposite direction in regime 2. Stocks in Malaysia and the Philippines move in the opposite direction with the market in regime 1, while they move in the same direction with the market in regime 2. Copper does not have relations with the market in regime 1, but moves in the opposite direction in regime 2. Results from the MS-GARCH-CAPM suggest in most of the markets, the size of the beta coefficient changes, but the relationship between the stocks and the market remains stable across both regimes. However, this is not always the case (Germany, France, and Ireland can be seen as exceptions).

Table 3: MS-GARCH-CAPM Results: Conditional Betas with Time-Varying Risks and Returns

MSCI USA_MS_GARCH			MSCI CAN_MS_GARCH			MSCI GER_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0005	[0.000]***	intercept	0.0065	[0.0007]***	intercept	-0.0272	[0.0024]***
Beta ( $\beta$ )	0.4361	[0.0141]***	Beta ( $\beta$ )	0.4845	[0.0543]***	Beta ( $\beta$ )	-1.6969	[0.1697]***
nu_1	82.39	[3.2578]***	nu_1	24.47	[7.6849]***	nu_1	0.324	[0.0985]***
xi_1	8.3793	[0.5317]***	xi_1	1.1288	[0.0639]***	xi_1	0.6759	[0.0919]***
posterior mean stable probability_1	0.4735		posterior mean stable probability_1	0.7163		posterior mean stable probability_1	0.6851	
Volatility	0.5158		Volatility	0.7335		Volatility	0.02	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.0108	[0.0016]***	intercept	0.0085	[0.0011]***	intercept	6.6109	[0.2252]***
Beta ( $\beta$ )	0.4269	[0.0538]***	Beta ( $\beta$ )	0.766	[0.0364]***	Beta ( $\beta$ )	0.0588	[0.1113]
nu_2	24.32	[2.4409]***	nu_2	45.12	[3.1213]***	nu_2	1.267	[5.8906]
xi_2	0.9363	[0.0101]***	xi_2	11.48	[1.2579]***	xi_2	0.7516	[0.2795]***
p12	0.3707		p12	0.9365		p12	0.0708	
p21	0.566		p21	0.1603		p21	0.0298	
posterior mean stable probability_2	0.5265		posterior mean stable probability_2	0.2837		posterior mean stable probability_2	0.3104	
Volatility	6.4406		Volatility	3.1636		Volatility	0.19	
cc.p-value	0.5987		cc.p-value	0.0152		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.0074		dq.p-value	0.9998	
<b>MSCI AUS_MS_GARCH</b>			<b>MSCI DEN_MS_GARCH</b>			<b>MSCI FIN_MS_GARCH</b>		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0009	[0.0001]***	intercept	0.0095	[0.0014]***	intercept	0.0034	[0.0000]***
Beta ( $\beta$ )	0.7568	[0.0257]***	Beta ( $\beta$ )	0.4464	[0.0371]***	Beta ( $\beta$ )	0.003	[0.1750]
nu_1	11.42	[2.9584]***	nu_1	9.6682	[1.1795]***	nu_1	0.2505	[0.0646]***
xi_1	1.76	[0.0486]***	xi_1	1.6655	[0.0940]***	xi_1	0.7494	[0.0683]***
posterior mean stable probability_1	0.6917		posterior mean stable probability_1	0.8746		posterior mean stable probability_1	0.6344	
Volatility	1.2957		Volatility	2.4321		Volatility	0.2102	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.0205	[0.0056]***	intercept	0.3331	[0.0622]***	intercept	-0.3186	[0.0130]***
Beta ( $\beta$ )	0.3188	[0.0494]***	Beta ( $\beta$ )	0.3581	[0.0198]***	Beta ( $\beta$ )	-0.0028	[0.0034]
nu_2	55.65	[3.2174]***	nu_2	6.4586	[1.6323]***	nu_2	0.6257	[0.4084]
xi_2	1.152	[0.1219]***	xi_2	9.7138	[1.5358]***	xi_2	0.7701	[0.0543]***
p12	0.9137		p12	0.9724		p12	0.0173	
p21	0.1937		p21	0.1929		p21	0.0282	
posterior mean stable probability_2	0.3083		posterior mean stable probability_2	0.1254		posterior mean stable probability_2	0.3611	
Volatility	3.6197		Volatility	10.111		Volatility	0.7953	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	
<b>MSCI HUN_MS_GARCH</b>			<b>MSCI EGY_MS_GARCH</b>			<b>MSCI POR_MS_GARCH</b>		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0071	[0.0001]***	intercept	0.0099	[0.0001]***	intercept	0.0037	[0.0001]***
Beta ( $\beta$ )	0.5664	[0.0028]***	Beta ( $\beta$ )	0.0258	[0.0001]***	Beta ( $\beta$ )	0.2729	[0.0027]***
nu_1	5.4554	[0.0746]***	nu_1	99.24	[0.0180]***	nu_1	73.57	[0.4166]***
xi_1	1.0787	[0.0020]***	xi_1	1.5153	[0.0057]***	xi_1	4.1175	[0.1044]***
posterior mean stable probability_1	0.8469		posterior mean stable probability_1	0.6753		posterior mean stable probability_1	0.508	
Volatility	3.2939		Volatility	1.7575		Volatility	1.4935	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.5485	[0.0051]***	intercept	0.0982	[0.0007]***	intercept	0.0068	[0.0001]***
Beta ( $\beta$ )	0.1815	[0.0039]***	Beta ( $\beta$ )	0.2196	[0.0022]***	Beta ( $\beta$ )	0.2837	[0.0032]***
nu_2	45.74	[0.6060]***	nu_2	99.36	[0.0273]***	nu_2	67.76	[0.4824]***
xi_2	9.1403	[0.1606]***	xi_2	0.9369	[0.0027]***	xi_2	2.8034	[0.0801]***
p12	0.9659		p12	0.9628		p12	0.7679	
p21	0.1889		p21	0.0773		p21	0.2397	
posterior mean stable probability_2	0.1531		posterior mean stable probability_2	0.3247		posterior mean stable probability_2	0.492	
Volatility	147.4		Volatility	11.61		Volatility	2.2976	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.7175	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.4824	

Note: Series names are given in Table 8. \*\*\*,\*\* and \* represent significance of regime switching GARCH CAPM betas at 1%, 5% and 10% respectively whereas  $p_{12}$  and  $p_{21}$  refer to the probability of moving from one volatility regime to the other. nu and xi are values of logistic functions used in determining the values of the transition probabilities. CC and DQ p-values are probability values for Conditional Coverage (CC) and Dynamic Quantile (DQ) which are tests for parameter restrictions on the transition probability matrix.

MSCI SPN_MS_GARCH			MSCI SLOV_MS_GARCH			MSCI FRA_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.1253	[0.05077]***	intercept	0.041	[0.0024]***	intercept	0.061	[0.0000]***
Beta ( $\beta$ )	0.8696	[0.0083]***	Beta ( $\beta$ )	0.5578	[0.0191]***	Beta ( $\beta$ )	-0.7325	[0.9516]
nu_1	7.8557	[0.4288]***	nu_1	99.983	[0.0016]***	nu_1	0.1273	[0.0397]***
xi_1	1.0939	[0.0144]***	xi_1	10.449	[0.5087]***	xi_1	0.8508	[0.0419]***
posterior mean stable probability_1	0.7853		posterior mean stable probability_1	0.3247		posterior mean stable probability_1	0.3801	
Volatility	0.6135		Volatility	0.6797		Volatility	0.032	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.1965	[0.0062]***	intercept	0.0055	[0.0012]***	intercept	-0.8216	[0.4016]**
Beta ( $\beta$ )	0.7134	[0.0051]***	Beta ( $\beta$ )	0.693	[0.0299]***	Beta ( $\beta$ )	-0.6169	[0.1181]***
nu_2	99.339	[0.0601]***	nu_2	65.638	[2.5985]***	nu_2	0.1678	[0.0673]**
xi_2	10.197	[1.3141]***	xi_2	1.0604	[0.0064]***	xi_2	0.8431	[0.0492]***
p12	0.9342		p12	0.5165		p12	0.6178	
p21	0.2406		p21	0.2324		p21	0.3822	
posterior mean stable probability_2	0.2147		posterior mean stable probability_2	0.6753		posterior mean stable probability_2	0.6155	
Volatility	3.5814		Volatility	3.655		Volatility	0.075	
cc.p-value	0.7175		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.0074		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI UK_MS_GARCH			MSCI ITA_MS_GARCH			MSCI SWE_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.1208	[0.0000]***	intercept	0.0773	[0.0008]***	intercept	0.0112	[0.0002]***
Beta ( $\beta$ )	0.0531	[0.2670]	Beta ( $\beta$ )	0.1786	[0.0034]***	Beta ( $\beta$ )	0.1807	[0.0037]***
nu_1	0.3818	[0.0770]***	nu_1	7.6729	[0.4463]***	nu_1	10.142	[0.3218]***
xi_1	0.6181	[0.0988]***	xi_1	1.9299	[0.0671]***	xi_1	0.5606	[0.0107]***
posterior mean stable probability_1	0.897		posterior mean stable probability_1	0.6538		posterior mean stable probability_1	0.4475	
Volatility	0.3743		Volatility	3.7448		Volatility	2.4272	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	-0.6208	[0.0488]***	intercept	0.0435	[0.0005]***	intercept	0.0257	[0.0005]***
Beta ( $\beta$ )	-0.1585	[0.1085]	Beta ( $\beta$ )	0.0491	[0.0016]***	Beta ( $\beta$ )	0.2213	[0.0041]***
nu_2	1.2173	[0.7969]	nu_2	83.905	[0.5037]***	nu_2	3.2532	[0.0210]***
xi_2	0.3587	[0.1553]**	xi_2	12.508	[0.1440]***	xi_2	0.7026	[0.0108]***
p12	0.0345		p12	0.5163		p12	0.9876	
p21	0.4133		p21	0.9134		p21	0.0101	
posterior mean stable probability_2	0.0982		posterior mean stable probability_2	0.3462		posterior mean stable probability_2	0.5525	
Volatility	1.0832		Volatility	9.5149		Volatility	3.6173	
cc.p-value	0.5987		cc.p-value	0.0034		cc.p-value	0.1384	
dq.p-value	0.9998		dq.p-value	0.000003		dq.p-value	0.0074	

MSCI TUR_MS_GARCH			MSCI CRO_MS_GARCH			MSCI LIT_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1	height	SE
intercept	0.1571	[0.0023]***	intercept	0.1004	[0.0029]***	intercept	0.0043	[0.0002]***
Beta ( $\beta$ )	0.2977	[0.0040]***	Beta ( $\beta$ )	0.653	[0.0051]***	Beta ( $\beta$ )	0.0178	[0.0014]***
nu_1	2.1208	[0.0012]***	nu_1	41.87	[0.6060]***	nu_1	95.87	[0.1613]***
xi_1	6.0725	[0.1400]***	xi_1	0.8551	[0.0164]***	xi_1	3.3727	[0.0827]***
posterior mean stable probability_1	0.5304		posterior mean stable probability_1	0.1388		posterior mean stable probability_1	0.4216	
Volatility	11.87		Volatility	7.9041		Volatility	1.3141	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.2666	[0.0041]***	intercept	0.0231	[0.0015]***	intercept	0.0045	[0.0001]***
Beta ( $\beta$ )	0.2638	[0.0040]***	Beta ( $\beta$ )	0.8981	[0.0033]***	Beta ( $\beta$ )	0.7461	[0.0024]***
nu_2	2.1073	[0.0004]***	nu_2	12.15	[0.4227]***	nu_2	66.19	[0.4270]***
xi_2	5.4584	[0.1117]***	xi_2	1.0856	[0.0066]***	xi_2	0.8918	[0.0072]***
p12	0.3222		p12	0.8667		p12	0.3428	
p21	0.7656		p21	0.0215		p21	0.4791	
posterior mean stable probability_2	0.4696		posterior mean stable probability_2	0.8612		posterior mean stable probability_2	0.5784	
Volatility	16.89		Volatility	5.6969		Volatility	4.3352	
cc.p-value	0.1889		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.3074		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI SWT_MS_GARCH			MSCI NZL_MS_GARCH			MSCI NOR_MS_GARCH					
Regime 1		SE	Regime 1		SE	Regime 1		SE			
intercept		0.0493	[0.0019]***	intercept		0.1475	[0.0016]***	intercept		0.0194	[0.0005]***
Beta ( $\beta$ )		0.8407	[0.0060]***	Beta ( $\beta$ )		0.3673	[0.0034]***	Beta ( $\beta$ )		0.4044	[0.0042]***
nu_1		2.1959	[0.0042]***	nu_1		80.942	[0.3445]***	nu_1		50.995	[0.3874]***
xi_1		12.949	[0.1437]***	xi_1		0.9679	[0.0246]***	xi_1		0.9544	[0.0023]***
posterior mean stable probability_1		0.5643		posterior mean stable probability_1		0.9902		posterior mean stable probability_1		0.6509	
Volatility		6.0847		Volatility		0.9873		Volatility		1.3817	
Regime 2			Regime 2			Regime 2					
intercept		0.1898	[0.0022]***	intercept		0.2244	[0.0031]***	intercept		0.0381	[0.0002]***
Beta ( $\beta$ )		0.492	[0.0054]***	Beta ( $\beta$ )		0.118	[0.0024]***	Beta ( $\beta$ )		0.0875	[0.0015]***
nu_2		2.3414	[0.0033]***	nu_2		93.863	[0.2165]***	nu_2		64.548	[0.2122]***
xi_2		3.9149	[0.0925]***	xi_2		6.1689	[0.0465]***	xi_2		0.957	[0.0035]***
p12		0.8579		p12		0.9923		p12		0.9755	
p21		0.184		p21		0.7844		p21		0.0457	
posterior mean stable probability_2		0.4357		posterior mean stable probability_2		0.0098		posterior mean stable probability_2		0.3491	
Volatility		11.721		Volatility		22.098		Volatility		3.2359	
cc.p-value		0.7176		cc.p-value		0.7176		cc.p-value		0.5987	
dq.p-value		0.9998		dq.p-value		0.3673		dq.p-value		0.9998	

MSCI NLD_MS_GARCH			MSCI JAP_MS_GARCH			MSCI IRE_MS_GARCH					
Regime 1		SE	Regime 1		SE	Regime 1		SE			
intercept		0.13	[0.0014]***	intercept		0.0056	[0.00001]***	intercept		-0.0113	[0.0064]***
Beta ( $\beta$ )		0.8022	[0.0018]***	Beta ( $\beta$ )		0.2042	[0.0026]***	Beta ( $\beta$ )		0.0778	[0.4686]
nu_1		11.851	[0.1752]***	nu_1		7.5191	[0.0564]***	nu_1		0.2163	[0.0559]***
xi_1		1.1142	[0.0025]***	xi_1		1.1636	[0.0031]***	xi_1		0.7836	[0.0563]***
posterior mean stable probability_1		0.8235		posterior mean stable probability_1		0.6556		posterior mean stable probability_1		0.8561	
Volatility		1.0917		Volatility		1.3148		Volatility		0.046	
Regime 2			Regime 2			Regime 2					
intercept		0.0353	[0.0010]***	intercept		0.0088	[0.0002]***	intercept		-0.0319	[0.0079]***
Beta ( $\beta$ )		0.2286	[0.0032]***	Beta ( $\beta$ )		0.626	[0.0033]***	Beta ( $\beta$ )		-0.5687	[0.0462]***
nu_2		57.257	[0.4198]***	nu_2		4.8333	[0.0256]***	nu_2		34.847	[10.14]***
xi_2		1.1571	[0.0157]***	xi_2		1.0234	[0.0026]***	xi_2		0.7878	[0.0679]***
p12		0.8705		p12		0.9822		p12		0.1625	
p21		0.6043		p21		0.0338		p21		0.8374	
posterior mean stable probability_2		0.1765		posterior mean stable probability_2		0.3444		posterior mean stable probability_2		0.1393	
Volatility		5.4342		Volatility		4.9535		Volatility		0.146	
cc.p-value		0.5987		cc.p-value		0.7175		cc.p-value		0.5987	
dq.p-value		0.9998		dq.p-value		0.7074		dq.p-value		0.9998	

MSCI ISR_MS_GARCH			MSCI LAT_MS_GARCH			MSCI EST_MS_GARCH					
Regime 1		SE	Regime 1		SE	Regime 1		SE			
intercept		0.0026	[0.0001]***	intercept		0.0023	[0.0002]***	intercept		0.1429	[0.0081]***
Beta ( $\beta$ )		0.2327	[0.0032]***	Beta ( $\beta$ )		0.3932	[0.0050]***	Beta ( $\beta$ )		0.3643	[0.0047]***
nu_1		21.63	[0.4126]***	nu_1		80.3	[0.4103]***	nu_1		8.9251	[0.4945]***
xi_1		2.6745	[0.0721]***	xi_1		0.9093	[0.0042]***	xi_1		1.4412	[0.0220]***
posterior mean stable probability_1		0.3523		posterior mean stable probability_1		0.4185		posterior mean stable probability_1		0.85	
Volatility		1.0664		Volatility		0.99		Volatility		5.6219	
Regime 2			Regime 2			Regime 2					
intercept		0.0166	[0.0003]***	intercept		0.2287	[0.0051]***	intercept		0.2679	[0.0046]***
Beta ( $\beta$ )		0.1169	[0.0029]***	Beta ( $\beta$ )		0.0052	[0.0005]***	Beta ( $\beta$ )		0.1038	[0.0031]***
nu_2		34.18	[0.5750]***	nu_2		2.2538	[0.0047]***	nu_2		90.43	[0.5035]***
xi_2		1.8378	[0.0564]***	xi_2		1.1137	[0.0038]***	xi_2		9.3742	[0.1311]***
p12		0.0847		p12		0.9246		p12		0.9807	
p21		0.4979		p21		0.0543		p21		0.1092	
posterior mean stable probability_2		0.6477		posterior mean stable probability_2		0.5815		posterior mean stable probability_2		0.15	
Volatility		2.9427		Volatility		7.314		Volatility		153.4	
cc.p-value		0.7175		cc.p-value		0.7175		cc.p-value		0.5987	
dq.p-value		0.9998		dq.p-value		0.9998		dq.p-value		0.9998	

MSCI THAI_MS_GARCH			MSCI MYL_MS_GARCH			MSCI INDO_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0027	[0.0001]***	intercept	0.001	[0.0001]***	intercept	0.2714	[0.0020]***
Beta ( $\beta$ )	0.4302	[0.0054]***	Beta ( $\beta$ )	0.3374	[0.0037]***	Beta ( $\beta$ )	0.5257	[0.0025]***
nu_1	89.162	[0.3384]***	nu_1	53.687	[0.6078]***	nu_1	55.835	[0.5014]***
xi_1	0.6854	[0.0144]***	xi_1	0.4453	[0.0262]***	xi_1	0.6199	[0.0019]***
posterior mean stable probability_1	0.3723		posterior mean stable probability_1	0.2839		posterior mean stable probability_1	0.9859	
Volatility	1.4547		Volatility	0.6575		Volatility	2.0485	
Regime 2			Regime 2			Regime 2		
intercept	0.0112	[0.0005]***	intercept	0.0058	[0.0001]***	intercept	0.0524	[0.0024]***
Beta ( $\beta$ )	0.5316	[0.0064]***	Beta ( $\beta$ )	0.1756	[0.0035]***	Beta ( $\beta$ )	0.5780	[0.0055]***
nu_2	79.331	[0.4330]***	nu_2	12.598	[0.4456]***	nu_2	33.957	[0.5144]***
xi_2	0.9716	[0.0129]***	xi_2	0.8671	[0.0064]***	xi_2	1.6748	[0.0610]***
p12	0.6097		p12	0.6701		p12	0.9958	
p21	0.2315		p21	0.1308		p21	0.2905	
posterior mean stable probability_2	0.6277		posterior mean stable probability_2	0.7161		posterior mean stable probability_2	0.0141	
Volatility	2.9873		Volatility	1.4281		Volatility	0.1426	
cc.p-value	0.1483		cc.p-value	0.7176		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI PHI_MS_GARCH			MSCI SING_MS_GARCH			MSCI CHIN_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.017	[0.0003]***	intercept	0.0029	[0.0002]***	intercept	0.4134	[0.0031]***
Beta ( $\beta$ )	0.2117	[0.0039]***	Beta ( $\beta$ )	0.7178	[0.0022]***	Beta ( $\beta$ )	0.4442	[0.0023]***
nu_1	60.109	[0.6903]***	nu_1	30.188	[0.5131]***	nu_1	99.816	[0.0121]***
xi_1	1.042	[0.0149]***	xi_1	0.8138	[0.0048]***	xi_1	0.7488	[0.0026]***
posterior mean stable probability_1	0.4273		posterior mean stable probability_1	0.54		posterior mean stable probability_1	0.8014	
Volatility	3.4551		Volatility	2.0606		Volatility	1.612	
Regime 2			Regime 2			Regime 2		
intercept	0.0547	[0.0008]***	intercept	0.1437	[0.0012]***	intercept	0.5769	[0.0052]***
Beta ( $\beta$ )	0.4349	[0.0041]***	Beta ( $\beta$ )	0.2333	[0.0034]***	Beta ( $\beta$ )	0.0972	[0.0021]***
nu_2	12.006	[0.5131]***	nu_2	2.919	[0.1394]***	nu_2	81.29	[0.4540]***
xi_2	1.0945	[0.0052]***	xi_2	5.0358	[0.0956]***	xi_2	0.1915	[0.0084]***
p12	0.6849		p12	0.6418		p12	0.8188	
p21	0.2351		p21	0.4204		p21	0.7312	
posterior mean stable probability_2	0.5727		posterior mean stable probability_2	0.46		posterior mean stable probability_2	0.1986	
Volatility	4.2098		Volatility	4.3937		Volatility	48.5137	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI ICE_MS_GARCH			MSCI RUS_MS_GARCH			MSCI MALT_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0038	[0.0001]***	intercept	0.0118	[0.0007]***	intercept	0.0098	[0.0002]***
Beta ( $\beta$ )	0.7108	[0.0031]***	Beta ( $\beta$ )	0.5834	[0.0026]***	Beta ( $\beta$ )	0.421	[0.0091]***
nu_1	3.7096	[0.0326]***	nu_1	4.9777	[0.1661]***	nu_1	6.0382	[0.1633]***
xi_1	1.0941	[0.0025]***	xi_1	0.979	[0.0258]***	xi_1	1.0793	[0.0033]***
posterior mean stable probability_1	0.7629		posterior mean stable probability_1	0.9532		posterior mean stable probability_1	0.6076	
Volatility	2.1245		Volatility	5.706		Volatility	1.9578	
Regime 2			Regime 2			Regime 2		
intercept	0.1428	[0.0034]***	intercept	0.3273	[0.0036]***	intercept	0.0351	[0.0013]***
Beta ( $\beta$ )	0.2537	[0.0026]***	Beta ( $\beta$ )	0.2683	[0.0031]***	Beta ( $\beta$ )	0.4459	[0.0083]***
nu_2	7.6092	[0.0450]***	nu_2	45.82	[0.4945]***	nu_2	7.774	[0.1654]***
xi_2	1.0697	[0.0087]***	xi_2	10.53	[0.1230]***	xi_2	1.3302	[0.0222]***
p12	0.9772		p12	0.9768		p12	0.9323	
p21	0.0733		p21	0.473		p21	0.1049	
posterior mean stable probability_2	0.2371		posterior mean stable probability_2	0.0468		posterior mean stable probability_2	0.3924	
Volatility	19.02		Volatility	187.1		Volatility	4.2847	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI INDI_MS_GARCH			MSCI TAI_MS_GARCH			MSCI KOR_MS_GARCH		
Regime 1			Regime 1			Regime 1		
	SE			SE			SE	
intercept	0.0021	[0.0001]***	intercept	0.0127	[0.0001]***	intercept	0.0001	[0.0000]***
Beta ( $\beta$ )	0.1004	[0.0023]***	Beta ( $\beta$ )	0.1111	[0.0013]***	Beta ( $\beta$ )	0.767	[0.0033]***
nu_1	69.23	[0.4938]***	nu_1	68.32	[0.4141]***	nu_1	73.13	[0.4521]***
xi_1	0.9579	[0.0040]***	xi_1	1.2648	[0.0117]***	xi_1	0.1405	[0.0054]***
posterior mean stable probability_1	0.575		posterior mean stable probability_1	0.9663		posterior mean stable probability_1	0.1475	
Volatility	1.1222		Volatility	3.2053		Volatility	0.2329	
Regime 2			Regime 2			Regime 2		
intercept	0.0142	[0.0003]***	intercept	2.6288	[0.1156]***	intercept	0.2625	[0.0023]***
Beta ( $\beta$ )	0.5887	[0.0056]***	Beta ( $\beta$ )	0.0019	[0.0005]***	Beta ( $\beta$ )	0.5083	[0.0035]***
nu_2	60.511	[0.5294]***	nu_2	44.77	[0.6137]***	nu_2	5.899	[0.0941]***
xi_2	0.936	[0.0046]***	xi_2	7.3515	[0.1154]***	xi_2	0.9898	[0.0018]***
p12	0.5457		p12	0.98		p12	0.1782	
p21	0.6147		p21	0.5732		p21	0.1422	
posterior mean stable probability_2	0.425		posterior mean stable probability_2	0.0337		posterior mean stable probability_2	0.8525	
Volatility	4.2805		Volatility	32.489		Volatility	3.3963	
cc.p-value	0.5987		cc.p-value	0.00816		cc.p-value	0.7175	
dq.p-value	0.9998		dq.p-value	0.00003		dq.p-value	0.9998	

MSCI HNGKNG_MS_GARCH			MSCI SERB_MS_GARCH			MSCI UKR_MS_GARCH		
Regime 1			Regime 1			Regime 1		
	SE			SE		height	SE	
intercept	0.0023	[0.0001]***	intercept	0.0023	[0.0001]***	intercept	0.0155	[0.0003]***
Beta ( $\beta$ )	0.6739	[0.0053]***	Beta ( $\beta$ )	0.5023	[0.0020]***	Beta ( $\beta$ )	0.5357	[0.0043]***
nu_1	65.9	[0.5758]***	nu_1	57.39	[0.3387]***	nu_1	14.29	[0.4357]***
xi_1	4.0321	[0.0957]***	xi_1	0.9078	[0.0029]***	xi_1	0.8198	[0.0026]***
posterior mean stable probability_1	0.4715		posterior mean stable probability_1	0.7872		posterior mean stable probability_1	0.5365	
Volatility	1.6692		Volatility	1.35508		Volatility	2.8122	
Regime 2			Regime 2			Regime 2		
intercept	0.016	[0.0001]***	intercept	0.0142	[0.0001]***	intercept	0.0181	[0.0003]***
Beta ( $\beta$ )	0.0828	[0.0046]***	Beta ( $\beta$ )	0.9006	[0.0007]***	Beta ( $\beta$ )	0.6671	[0.0035]***
nu_2	33.1	[0.5766]***	nu_2	99.22	[0.0102]***	nu_2	50.93	[0.7081]***
xi_2	2.4808	[0.0561]***	xi_2	0.1545	[0.0023]***	xi_2	0.8415	[0.0029]***
p12	0.487		p12	0.8586		p12	0.947	
p21	0.4577		p21	0.5231		p21	0.0614	
posterior mean stable probability_2	0.5285		posterior mean stable probability_2	0.2128		posterior mean stable probability_2	0.4635	
Volatility	4.4913		Volatility	39.17		Volatility	4.2827	
cc.p-value	0.8133		cc.p-value	0.0392		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.0074		dq.p-value	0.9998	

MSCI BUL_MS_GARCH			MSCI ROM_MS_GARCH			MSCI POL_MS_GARCH		
Regime 1			Regime 1			Regime 1		
	SE			SE			SE	
intercept	0.0086	[0.0001]***	intercept	0.0096	[0.0003]***	intercept	0.0534	[0.0010]***
Beta ( $\beta$ )	0.3175	[0.0050]***	Beta ( $\beta$ )	0.3463	[0.0044]***	Beta ( $\beta$ )	0.9365	[0.0013]***
nu_1	99.98	[0.0010]***	nu_1	13.32	[0.3193]***	nu_1	99.8	[0.0270]***
xi_1	0.1049	[0.0041]***	xi_1	0.6105	[0.0065]***	xi_1	3.9519	[0.0822]***
posterior mean stable probability_1	0.273		posterior mean stable probability_1	0.5196		posterior mean stable probability_1	0.6481	
Volatility	1.7923		Volatility	2.0349		Volatility	1.2698	
Regime 2			Regime 2			Regime 2		
intercept	0.1308	[0.0017]***	intercept	0.0849	[0.0013]***	intercept	0.0242	[0.0002]***
Beta ( $\beta$ )	0.1388	[0.0017]***	Beta ( $\beta$ )	0.3072	[0.0041]***	Beta ( $\beta$ )	0.0483	[0.0015]***
nu_2	3.9007	[0.0298]***	nu_2	13.61	[0.4512]***	nu_2	44.24	[0.5055]***
xi_2	0.9822	[0.0020]***	xi_2	3.4099	[0.1038]***	xi_2	0.5177	[0.0103]***
p12	0.8913		p12	0.697		p12	0.5766	
p21	0.0408		p21	0.3276		p21	0.7798	
posterior mean stable probability_2	0.727		posterior mean stable probability_2	0.4804		posterior mean stable probability_2	0.3519	
Volatility	14.4		Volatility	4.8426		Volatility	4.7326	
cc.p-value	0.5987		cc.p-value	0.7175		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI CZECH_MS_GARCH			MSCI BRA_MS_GARCH			MSCI ARG_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0103	[0.0001]***	intercept	0.0081	[0.0002]***	intercept	0.0746	[0.0013]***
Beta ( $\beta$ )	0.1651	[0.0029]***	Beta ( $\beta$ )	0.4827	[0.0052]***	Beta ( $\beta$ )	0.1407	[0.0060]***
nu_1	35.22	[0.4474]***	nu_1	47.56	[0.8788]***	nu_1	42.59	[0.5671]***
xi_1	11.9435	[0.1350]***	xi_1	2.1339	[0.0563]***	xi_1	2.3377	[0.0579]***
posterior mean stable probability_1	0.3429		posterior mean stable probability_1	0.6086		posterior mean stable probability_1	0.3641	
Volatility	1.7706		Volatility	2.0993		Volatility	9.3749	
Regime 2			Regime 2			Regime 2		
intercept	0.0038	[0.0001]***	intercept	0.4163	[0.0508]***	intercept	0.1043	[0.0043]***
Beta ( $\beta$ )	0.4317	[0.0027]***	Beta ( $\beta$ )	0.3411	[0.0059]***	Beta ( $\beta$ )	0.7609	[0.0072]***
nu_2	46.39	[0.5918]***	nu_2	61.63	[0.9352]***	nu_2	12.59	[0.5062]***
xi_2	1.1538	[0.0524]***	xi_2	5.723	[0.1180]***	xi_2	1.7831	[0.0672]***
p12	0.0196		p12	0.7439		p12	0.8508	
p21	0.5117		p21	0.3981		p21	0.0854	
posterior mean stable probability_2	0.6571		posterior mean stable probability_2	0.3914		posterior mean stable probability_2	0.6359	
Volatility	3.4982		Volatility	13.86		Volatility	15.15	
cc.p-value	0.5987		cc.p-value	0.8133		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI COL_MS_GARCH			MSCI BEL_MS_GARCH			MSCI UAE_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0037	[0.0002]***	intercept	0.0845	[0.0011]***	intercept	0.0002	[0.0004]
Beta ( $\beta$ )	0.6897	[0.0028]***	Beta ( $\beta$ )	0.8811	[0.0017]***	Beta ( $\beta$ )	0.7246	[0.0012]***
nu_1	98.94	[0.0770]***	nu_1	45.53	[0.3822]***	nu_1	3.3434	[0.0103]***
xi_1	1.0225	[0.0019]***	xi_1	0.9218	[0.0028]***	xi_1	1.1475	[0.0211]***
posterior mean stable probability_1	0.9459		posterior mean stable probability_1	0.907		posterior mean stable probability_1	0.6309	
Volatility	2.4364		Volatility	1.1693		Volatility	1.1366	
Regime 2			Regime 2			Regime 2		
intercept	0.1144	[0.0034]***	intercept	0.0183	[0.0005]***	intercept	0.0121	[0.0002]***
Beta ( $\beta$ )	0.0414	[0.0013]***	Beta ( $\beta$ )	0.9741	[0.0005]***	Beta ( $\beta$ )	0.9311	[0.0009]***
nu_2	17.98	[0.2987]***	nu_2	62.55	[0.4158]***	nu_2	3.5332	[0.0169]***
xi_2	6.1804	[0.1127]***	xi_2	0.8138	[0.0411]***	xi_2	12.09	[0.1288]***
p12	0.9963		p12	0.9107		p12	0.6406	
p21	0.0647		p21	0.8701		p21	0.6143	
posterior mean stable probability_2	0.0541		posterior mean stable probability_2	0.093		posterior mean stable probability_2	0.3691	
Volatility	116.3		Volatility	10.09		Volatility	7.5209	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.0392	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.0074	

MSCI CHIL_MS_GARCH			MSCI MEX_MS_GARCH			MSCI QAT_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0027	[0.0008]***	intercept	0.0031	[0.0001]***	intercept	0.0119	[0.0003]***
Beta ( $\beta$ )	0.3701	[0.0020]***	Beta ( $\beta$ )	0.5501	[0.0042]***	Beta ( $\beta$ )	0.2368	[0.0039]***
nu_1	46.18	[0.4538]***	nu_1	6.6427	[0.1169]***	nu_1	3.436	[0.0630]***
xi_1	1.0775	[0.0025]***	xi_1	0.893	[0.0019]***	xi_1	7.4993	[0.1268]***
posterior mean stable probability_1	0.9615		posterior mean stable probability_1	0.6944		posterior mean stable probability_1	0.3286	
Volatility	1.4642		Volatility	1.329		Volatility	2.086	
Regime 2			Regime 2			Regime 2		
intercept	0.5105	[0.0040]***	intercept	0.0293	[0.0002]***	intercept	0.0567	[0.0020]***
Beta ( $\beta$ )	0.22	[0.0033]***	Beta ( $\beta$ )	0.0036	[0.0002]***	Beta ( $\beta$ )	0.0892	[0.0037]***
nu_2	2.117	[0.0005]***	nu_2	69.91	[0.2821]***	nu_2	11.4	[0.1739]***
xi_2	6.8683	[0.1028]***	xi_2	0.9799	[0.0050]***	xi_2	3.2811	[0.1195]***
p12	0.9858		p12	0.9674		p12	0.4555	
p21	0.355		p21	0.0741		p21	0.2664	
posterior mean stable probability_2	0.0385		posterior mean stable probability_2	0.3056		posterior mean stable probability_2	0.6714	
Volatility	27.98		Volatility	3.1159		Volatility	5.1521	
cc.p-value	0.1483		cc.p-value	0.7175		cc.p-value	0.8133	
dq.p-value	0.0003		dq.p-value	0.3262		dq.p-value	0.9998	

MSCI LUX_MS_GARCH			MSCI TUN_MS_GARCH		
Regime 1		SE	Regime 1		SE
intercept	0.0217	[0.0002]***	intercept	0.1358	[0.0014]***
Beta ( $\beta$ )	0.3598	[0.0039]***	Beta ( $\beta$ )	0.085	[0.0023]***
nu_1	3.7802	[0.0251]***	nu_1	99.42	[0.0120]***
xi_1	0.9839	[0.0032]***	xi_1	1.0099	[0.0036]***
posterior mean stable probability_1	0.5141		posterior mean stable probability_1	0.6778	
Volatility	3.4573		Volatility	0.5758	
Regime 2		SE	Regime 2		SE
intercept	0.0213	[0.0002]***	intercept	0.665	[0.0039]***
Beta ( $\beta$ )	0.527	[0.0033]***	Beta ( $\beta$ )	0.1296	[0.0017]***
nu_2	2.9543	[0.0312]***	nu_2	1.00	[0.0000]***
xi_2	1.0749	[0.0035]***	xi_2	1.9622	[0.0482]***
p12	0.9772		p12	0.5527	
p21	0.0241		p21	0.9408	
posterior mean stable probability_2	0.4859		posterior mean stable probability_2	0.3222	
Volatility	3.2952		Volatility	3.576	
cc.p-value	0.7175		cc.p-value	0.7175	
dq.p-value	0.1752		dq.p-value	0.9998	

MSCI OIL_MS_GARCH			MSCI GOLD_MS_GARCH			MSCI SILVER_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0063	[0.0001]***	intercept	0.006	[0.0001]***	intercept	0.0206	[0.0005]***
Beta ( $\beta$ )	0.6329	[0.0030]***	Beta ( $\beta$ )	0.3877	[0.0042]***	Beta ( $\beta$ )	0.3979	[0.0035]***
nu_1	51.029	[0.4939]***	nu_1	37.08	[0.3483]***	nu_1	8.4641	[0.2525]***
xi_1	0.2589	[0.0070]***	xi_1	1.135	[0.0031]***	xi_1	1.5206	[0.0536]***
posterior mean stable probability_1	0.4616		posterior mean stable probability_1	0.7935		posterior mean stable probability_1	0.813	
Volatility	2.1285		Volatility	1.8572		Volatility	4.838	
Regime 2		SE	Regime 2		SE	Regime 2		SE
intercept	0.0048	[0.0001]***	intercept	0.0014	[0.0001]***	intercept	0.4271	[0.0044]***
Beta ( $\beta$ )	0.5625	[0.0026]***	Beta ( $\beta$ )	0.9845	[0.0001]***	Beta ( $\beta$ )	0.2538	[0.0038]***
nu_2	23.93	[0.3775]***	nu_2	23.64	[0.3234]***	nu_2	54.15	[0.5598]***
xi_2	1.123	[0.0053]***	xi_2	14.61	[0.1257]***	xi_2	6.7181	[0.1193]***
p12	0.3737		p12	0.9653		p12	0.9151	
p21	0.5369		p21	0.1331		p21	0.3693	
posterior mean stable probability_2	0.5384		posterior mean stable probability_2	0.2065		posterior mean stable probability_2	0.187	
Volatility	4.13		Volatility	12.63		Volatility	52.28	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI GAS_MS_GARCH			MSCI COPPER_MS_GARCH			MSCI Platinum_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0192	[0.0006]***	intercept	0.0045	[0.0001]***	intercept	0.1873	[0.0026]***
Beta ( $\beta$ )	0.4427	[0.0029]***	Beta ( $\beta$ )	0.4617	[0.0035]***	Beta ( $\beta$ )	0.7791	[0.0028]***
nu_1	11.18	[0.4455]***	nu_1	65.96	[0.4915]***	nu_1	95.34	[0.2335]***
xi_1	2.204	[0.0823]***	xi_1	1.689	[0.0111]***	xi_1	0.5643	[0.0071]***
posterior mean stable probability_1	0.8972		posterior mean stable probability_1	0.4809		posterior mean stable probability_1	0.4195	
Volatility	5.4432		Volatility	1.7101		Volatility	1.2108	
Regime 2		SE	Regime 2		SE	Regime 2		SE
intercept	0.2298	[0.0044]***	intercept	0.0049	[0.0003]***	intercept	0.1168	[0.0098]***
Beta ( $\beta$ )	0.1798	[0.0040]***	Beta ( $\beta$ )	0.6655	[0.0024]***	Beta ( $\beta$ )	0.0307	[0.0010]***
nu_2	80.53	[0.5688]***	nu_2	22.01	[0.4340]***	nu_2	3.116	[0.0423]***
xi_2	10.06	[0.1394]***	xi_2	0.9937	[0.0036]***	xi_2	1.1319	[0.0031]***
p12	0.9239		p12	0.9909		p12	0.5747	
p21	0.6639		p21	0.0084		p21	0.3074	
posterior mean stable probability_2	0.1028		posterior mean stable probability_2	0.5191		posterior mean stable probability_2	0.5805	
Volatility	57.6		Volatility	4.1555		Volatility	8.3061	
cc.p-value	0.5987		cc.p-value	0.0392		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	2.76E-06		dq.p-value	0.9998	

MSCI Palladium_MS_GARCH			MSCI Nickel_MS_GARCH			MSCI Tin_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0024	[0.0001]***	intercept	0.0108	[0.0001]***	intercept	0.0133	[0.0003]***
Beta ( $\beta$ )	0.1859	[0.0062]***	Beta ( $\beta$ )	0.268	[0.0029]***	Beta ( $\beta$ )	0.2832	[0.0044]***
nu_1	66.19	[0.4744]***	nu_1	16.11	[0.1207]***	nu_1	15.78	[0.4855]***
xi_1	12.65	[0.1826]***	xi_1	0.9052	[0.0028]***	xi_1	1.625	[0.0556]***
posterior mean stable probability_1	0.3122		posterior mean stable probability_1	0.591		posterior mean stable probability_1	0.6445	
Volatility	1.7234		Volatility	2.1687		Volatility	2.7413	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.0032	[0.0001]***	intercept	0.067	[0.0009]***	intercept	0.0875	[0.0030]***
Beta ( $\beta$ )	0.499	[0.0065]***	Beta ( $\beta$ )	0.3505	[0.0017]***	Beta ( $\beta$ )	0.1975	[0.0038]***
nu_2	89.27	[0.3836]***	nu_2	14.17	[0.2335]***	nu_2	24.996	[0.6272]***
xi_2	1.5083	[0.0528]***	xi_2	1.0373	[0.0028]***	xi_2	3.9992	[0.1232]***
p12	0.549		p12	0.9846		p12	0.7296	
p21	0.2047		p21	0.0222		p21	0.4902	
posterior mean stable probability_2	0.6878		posterior mean stable probability_2	0.409		posterior mean stable probability_2	0.3555	
Volatility	2.9641		Volatility	10.83		Volatility	6.8819	
cc.p-value	0.5987		cc.p-value	0.7175		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.0074		dq.p-value	0.9998	

MSCI Zinc_MS_GARCH			MSCI Rhodium_MS_GARCH			MSCI Ruthenium_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.284	[0.0023]***	intercept	0.0094	[0.0001]***	intercept	0.0005	[0.0003]*
Beta ( $\beta$ )	0.7035	[0.0024]***	Beta ( $\beta$ )	0.4687	[0.0038]***	Beta ( $\beta$ )	0.9988	[0.0005]***
nu_1	41.32	[0.4921]***	nu_1	10.29	[0.3267]***	nu_1	12.51	[0.2408]***
xi_1	1.0102	[0.0028]***	xi_1	1.105	[0.0015]***	xi_1	0.9902	[0.0022]***
posterior mean stable probability_1	0.6039		posterior mean stable probability_1	0.9935		posterior mean stable probability_1	0.3827	
Volatility	0.5341		Volatility	5.1636		Volatility	1.4593	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.1347	[0.0014]***	intercept	0.4255	[0.0056]***	intercept	0.0809	[0.0005]***
Beta ( $\beta$ )	0.0982	[0.0019]***	Beta ( $\beta$ )	0.0296	[0.0012]***	Beta ( $\beta$ )	0.1887	[0.0011]***
nu_2	2.3088	[0.0082]***	nu_2	14.06	[0.2737]***	nu_2	19.59	[0.2585]***
xi_2	3.7363	[0.1147]***	xi_2	1.806	[0.0616]***	xi_2	0.9673	[0.0027]***
p12	0.7701		p12	0.9948		p12	0.9817	
p21	0.3505		p21	0.8012		p21	0.0113	
posterior mean stable probability_2	0.3961		posterior mean stable probability_2	0.0065		posterior mean stable probability_2	0.6173	
Volatility	5.8197		Volatility	163.8		Volatility	12.12	
cc.p-value	0.7175		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.0074		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI Corn_MS_GARCH			MSCI Rubber_MS_GARCH			MSCI Soyabean_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
intercept	0.0595	[0.0019]***	intercept	0.0034	[0.0009]***	intercept	0.001	[0.0008]***
Beta ( $\beta$ )	0.8379	[0.0026]***	Beta ( $\beta$ )	0.0075	[0.0004]***	Beta ( $\beta$ )	0.6632	[0.0031]***
nu_1	99.91	[0.0014]***	nu_1	31.06	[0.5437]***	nu_1	78.65	[0.3432]***
xi_1	1.1383	[0.0029]***	xi_1	1.2316	[0.0033]***	xi_1	1.3275	[0.0046]***
posterior mean stable probability_1	0.7818		posterior mean stable probability_1	0.4688		posterior mean stable probability_1	0.8704	
Volatility	1.8948		Volatility	1.2635		Volatility	1.2471	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
intercept	0.1605	[0.0016]***	intercept	0.0131	[0.0131]***	intercept	0.0484	[0.0003]***
Beta ( $\beta$ )	0.2738	[0.0021]***	Beta ( $\beta$ )	0.5178	[0.0027]***	Beta ( $\beta$ )	0.2944	[0.0035]***
nu_2	62.36	[0.4566]***	nu_2	6.1099	[0.0596]***	nu_2	100	[0.0000]***
xi_2	0.7306	[0.0046]***	xi_2	1.1837	[0.0041]***	xi_2	11.56	[0.1251]***
p12	0.9441		p12	0.9921		p12	0.8811	
p21	0.2004		p21	0.007		p21	0.799	
posterior mean stable probability_2	0.2182		posterior mean stable probability_2	0.5312		posterior mean stable probability_2	0.1296	
Volatility	20.33		Volatility	3.2247		Volatility	9.7134	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI Wool_MS_GARCH			MSCI Aluminium_MS_GARCH			MSCI Lead_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
<b>intercept</b>	0.0014	[0.0007]***	<b>intercept</b>	0.0024	[0.0001]***	<b>intercept</b>	0.0017	[0.0008]***
<b>Beta (<math>\beta</math>)</b>	0.7547	[0.0032]***	<b>Beta (<math>\beta</math>)</b>	0.9218	[0.0008]***	<b>Beta (<math>\beta</math>)</b>	0.9981	[0.0008]***
nu_1	9.7623	[0.1505]***	nu_1	5.6443	[0.0594]***	nu_1	4.5562	[0.0139]***
xi_1	7.6973	[0.0924]***	xi_1	0.9892	[0.0027]***	xi_1	1.1497	[0.0025]***
posterior mean stable probability_1	0.3844		posterior mean stable probability_1	0.6055		posterior mean stable probability_1	0.6912	
Volatility	1.3135		Volatility	1.3771		Volatility	1.6446	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
<b>intercept</b>	0.0034	[0.0001]***	<b>intercept</b>	0.1012	[0.0010]***	<b>intercept</b>	0.1118	[0.0011]***
<b>Beta (<math>\beta</math>)</b>	0.7465	[0.0039]***	<b>Beta (<math>\beta</math>)</b>	0.2242	[0.0022]***	<b>Beta (<math>\beta</math>)</b>	0.2794	[0.0029]***
nu_2	10.78	[0.1541]***	nu_2	27.95	[0.4730]***	nu_2	46.35	[0.4735]***
xi_2	2.7203	[0.0846]***	xi_2	0.9156	[0.0026]***	xi_2	0.898	[0.0027]***
p12	0.6536		p12	0.9867		p12	0.9543	
p21	0.2163		p21	0.0204		p21	0.1023	
posterior mean stable probability_2	0.6156		posterior mean stable probability_2	0.3945		posterior mean stable probability_2	0.3088	
Volatility	2.3017		Volatility	10.41		Volatility	12.1	
cc.p-value	0.5987		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.9998		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI Wheat_MS_GARCH			MSCI Cocoa_MS_GARCH			MSCI Coffee_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
<b>intercept</b>	0.1673	[0.0012]***	<b>intercept</b>	0.0034	[0.0003]***	<b>intercept</b>	0.0015	[0.0008]***
<b>Beta (<math>\beta</math>)</b>	0.7936	[0.0016]***	<b>Beta (<math>\beta</math>)</b>	0.7322	[0.0042]***	<b>Beta (<math>\beta</math>)</b>	0.9979	[0.0010]***
nu_1	64.79	[0.5892]***	nu_1	4.8824	[0.0498]***	nu_1	7.3977	[0.0341]***
xi_1	4.2024	[0.0793]***	xi_1	0.9808	[0.0024]***	xi_1	1.4034	[0.0031]***
posterior mean stable probability_1	0.6325		posterior mean stable probability_1	0.599		posterior mean stable probability_1	0.726	
Volatility	1.1785		Volatility	1.9547		Volatility	2.1782	
<b>Regime 2</b>			<b>Regime 2</b>			<b>Regime 2</b>		
<b>intercept</b>	0.0255	[0.0003]***	<b>intercept</b>	0.0977	[0.0007]***	<b>intercept</b>	0.0566	[0.0005]***
<b>Beta (<math>\beta</math>)</b>	0.0356	[0.0016]***	<b>Beta (<math>\beta</math>)</b>	0.0687	[0.0013]***	<b>Beta (<math>\beta</math>)</b>	0.3275	[0.0012]***
nu_2	88.13	[0.2769]***	nu_2	87.11	[0.2403]***	nu_2	99.83	[0.0018]***
xi_2	0.7131	[0.0057]***	xi_2	0.9323	[0.0030]***	xi_2	0.8316	[0.0025]***
p12	0.7975		p12	0.9668		p12	0.9819	
p21	0.3485		p21	0.0496		p21	0.048	
posterior mean stable probability_2	0.3675		posterior mean stable probability_2	0.401		posterior mean stable probability_2	0.274	
Volatility	3.3071		Volatility	11.47		Volatility	14.57	
cc.p-value	0.1889		cc.p-value	0.5987		cc.p-value	0.5987	
dq.p-value	0.0074		dq.p-value	0.9998		dq.p-value	0.9998	

MSCI Cotton_MS_GARCH		
Regime 1		SE
<b>intercept</b>	0.0058	[0.0002]***
<b>Beta (<math>\beta</math>)</b>	0.7806	[0.0019]***
nu_1	5.5234	[0.1112]***
xi_1	1.4926	[0.0050]***
posterior mean stable probability_1	0.7457	
Volatility	2.8921	
<b>Regime 2</b>		
<b>intercept</b>	0.0808	[0.0009]***
<b>Beta (<math>\beta</math>)</b>	0.1672	[0.0017]***
nu_2	44.99	[0.5238]***
xi_2	0.8435	[0.0029]***
p12	0.9812	
p21	0.0552	
posterior mean stable probability_2	0.2543	
Volatility	12.69	
cc.p-value	0.5987	
dq.p-value	0.9998	

MSCI WORLD_MS_GARCH			MSCI Europe_MS_GARCH			MSCI EM_MS_GARCH		
Regime 1		SE	Regime 1		SE	Regime 1		SE
<b>intercept</b>	0.0489	[0.0020]***	<b>intercept</b>	0.1842	[0.0009]***	<b>intercept</b>	0.3156	[0.0044]***
<b>Beta (<math>\beta</math>)</b>	0.3249	[0.0044]***	<b>Beta (<math>\beta</math>)</b>	0.7789	[0.0023]***	<b>Beta (<math>\beta</math>)</b>	0.4285	[0.0066]***
nu_1	4.1263	[0.0587]***	nu_1	84.82	[0.2312]***	nu_1	20.38	[0.4032]***
xi_1	1.1845	[0.0075]***	xi_1	1.0696	[0.0042]***	xi_1	1.8196	[0.0562]***
posterior mean stable probability_1	0.8022		posterior mean stable probability_1	0.9922		posterior mean stable probability_1	0.4897	
Volatility	4.4324		Volatility	1.9692		Volatility	1.0731	
Regime 2			Regime 2			Regime 2		
<b>intercept</b>	0.7318	[0.0028]***	<b>intercept</b>	0.3363	[0.0043]***	<b>intercept</b>	0.189	[0.0027]***
<b>Beta (<math>\beta</math>)</b>	0.121	[0.0023]***	<b>Beta (<math>\beta</math>)</b>	0.1875	[0.0023]***	<b>Beta (<math>\beta</math>)</b>	0.6559	[0.0049]***
nu_2	2.1591	[0.0077]***	nu_2	90.74	[0.2601]***	nu_2	20.93	[0.3663]***
xi_2	2.5491	[0.0606]***	xi_2	3.0898	[0.0660]***	xi_2	2.0555	[0.0567]***
p12	0.9598		p12	0.9967		p12	0.6298	
p21	0.1632		p21	0.4185		p21	0.3553	
posterior mean stable probability_2	0.1978		posterior mean stable probability_2	0.0078		posterior mean stable probability_2	0.5103	
Volatility	12.99		Volatility	131.04		Volatility	3.0296	
cc.p-value	0.5987		cc.p-value	0.7175		cc.p-value	0.7175	
dq.p-value	0.9998		dq.p-value	0.0074		dq.p-value	0.0074	

Note: Series names are given in Table 8. \*\*\*,\*\* and \* represent significance of regime switching GARCH CAPM betas at 1%, 5% and 10% respectively whereas  $p_{12}$  and  $p_{21}$  refer to the probability of moving from one volatility regime to the other. nu and xi are values of logistic functions used in determining the values of the transition probabilities. CC and DQ p-values are probability values for Conditional Coverage (CC) and Dynamic Quantile (DQ) which are tests for parameter restrictions on the transition probability matrix.

#### 4.4 Average pricing errors

The statistical evidence of average risk price for (un)conditional betas across 81 markets lends support to the view that the conditional regime-switching GARCH CAPM is our preferred specification. In this section, we compare the fit between the unconditional and conditional versions of the CAPM using the average pricing errors from the competing models.

As an illustration for energy price series, Figure 3 presents a visual impression of the empirical performance of the models for Crude Oil and Natural Gas. Blue diamonds and orange squares represent the average pricing errors from, respectively, the unconditional and the conditional CAPM models. Each scatter point in the graph represents an asset price, with the realized average return as the horizontal axis, and the fitted expected return as the vertical axis. If the fitted expected returns and the realized average returns are the same, then all the points should lie on the 45-degree line through the origin. The straight line is the regression across the conditional CAPM estimates.

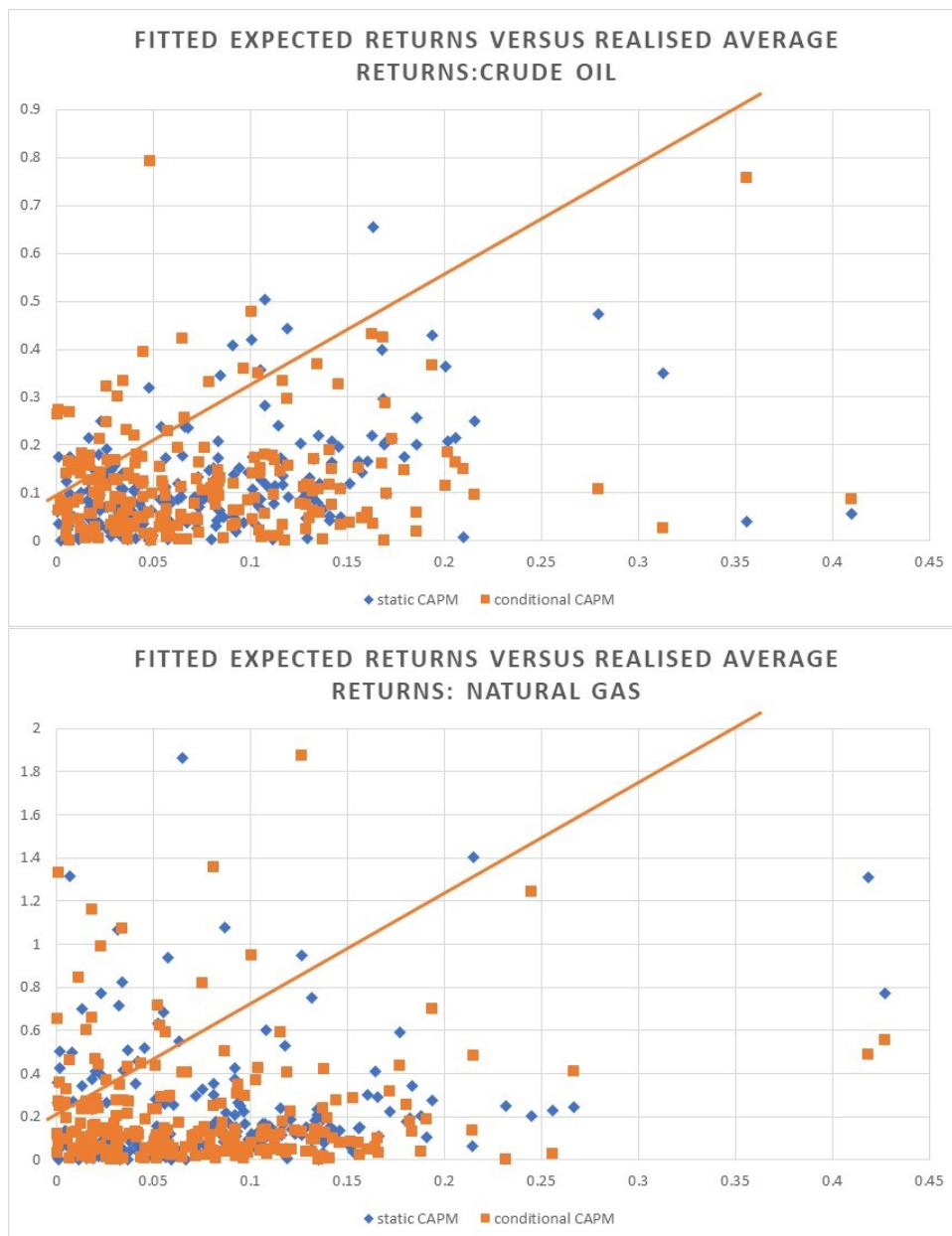


Figure 3: Pricing errors: static CAPM versus conditional CAPM for Crude Oil and Natural Gas.

Neither the static nor the conditional CAPM models offer a perfect fit for crude oil and natural gas price returns, especially in predicting outliers. Although both plots exhibit clusters around the

bottom of the fitted line, it may be observed that the static CAPM underestimates most of the realized returns in terms of level (especially for Crude Oil).

On the one hand, in the case of the static CAPM, the relationship between fitted and observed returns is almost entirely flat, due to the presence of small and insignificant risk premium. On the other hand, the performance substantially improves when considering the conditional CAPM specification we suggest in this paper. Indeed, the conditional fit is closer to that of realized average returns. Apart from outliers, the slope of the regression is steeper for the conditional CAPM relative to the static CAPM. We may, therefore, infer that the conditional CAPM with high and significant risk premia, as well as time-varying betas, correctly predicts returns in several periods.

We corroborate this finding in Table 4 which displays the mean pricing errors for both the static and conditional CAPM, as well as specification tests. The first part of the table presents the average squared pricing errors from both models. In the case of Crude Oil and Natural Gas, it is evident that the mean pricing errors from the static CAPM model are higher than those from the conditional CAPM. This lends credence to the claim that the conditional fit improves on the static fit.

Generally, mean pricing errors from both models are more significant for Natural Gas than Crude Oil. For instance, the mean pricing errors for Natural Gas from the static model is about 8.6%, whereas, from the conditional model, it is equal to 6.4%. For Crude Oil, the mean pricing errors from the static model are about 5.9%, whereas, from the conditional model, it is much less at approximately 1.6%. Therefore, average pricing errors indicate that the conditional model ranks better than the static model in predicting returns, especially for Crude Oil.

Table 4: Mean pricing errors and specification tests for Crude oil and Natural gas

Series	Mean Pricing Error	% Pricing Error		
<b>Crude Oil</b>				
Static CAPM	0.059	5.90%		
Conditional CAPM	0.016	1.60%		
<b>Natural Gas</b>				
Static CAPM	0.086	8.60%		
Conditional CAPM	0.064	6.40%		
			$\chi^2$	p-value
<b>Specification tests for Crude Oil</b>				
Are pricing errors from Static CAPM null? - $H_o : \lambda_i = 0$			26.035***	[0.0000]
Are pricing errors from Static CAPM constant? - $H_o : \lambda_i = 1$			528.01***	[0.0000]
Are pricing errors from Conditional CAPM null? - $H_o : \lambda_i = 0$			4.245**	[0.0405]
Are pricing errors from Conditional CAPM constant? - $H_o : \lambda_i = 1$			322.05***	[0.0000]
Are pricing errors from both models jointly zero? - $H_o : \lambda_s = \lambda_c = 0$			4.375**	[0.0137]
<b>Specification tests for Natural Gas</b>				
Are pricing errors from Static CAPM null? - $H_o : \lambda_i = 0$			12.686***	[0.0000]
Are pricing errors from Static CAPM constant? - $H_o : \lambda_i = 1$			44.11***	[0.0000]
Are pricing errors from Conditional CAPM null? - $H_o : \lambda_i = 0$			12.626***	[0.0004]
Are pricing errors from Conditional CAPM constant? - $H_o : \lambda_i = 1$			43.01***	[0.0000]
Are pricing errors from both models jointly zero? - $H_o : \lambda_s = \lambda_c = 0$			6.325***	[0.0021]

Note: The first panel reports the mean pricing errors while the second panel reports estimates from Wald test. \*\*\*, \*\* correspond to significance at 1% and 5% respectively .

In the second part of Table 4 (below the mean pricing errors), we present estimates of the Wald tests using the asymptotic  $\chi^2$  under the null hypothesis that all pricing errors are jointly

zero. Results show that we may reject the null hypotheses for both Crude Oil and Natural Gas. Consequently, we conclude that the pricing errors from both models are relevant, that they change over time, and there are jointly not equal to zero on average.

## 4.5 Volatility dynamics

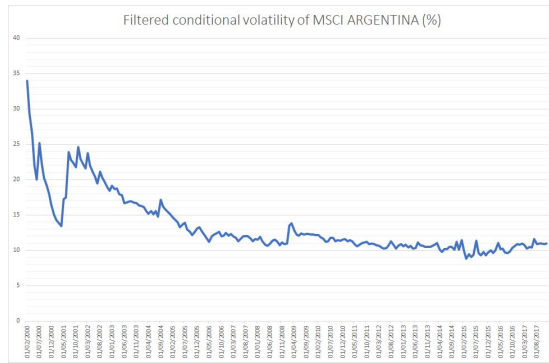
On volatility dynamics, Table 5 presents the mean filtered volatility from the MS-GARCH-CAPM for all the series. As can be seen, among the countries, the mean filtered volatility is highest in Turkey followed by Denmark and Argentina with a volatility of about 16.5, 14.9 and 13.4, respectively. In contrast, Tunisia, Portugal, and New Zealand exhibit the least mean filtered volatility of about 0.98, 1.67 and 1.85, respectively. Among the commodities, Rhodium and Natural Gas possess the highest mean filtered volatility of 7.03 and 6.08 respectively, whereas Wool and Palladium have the least mean filtered volatility of 2.07 and 2.14 respectively. Lastly, among the aggregate indices, the World aggregate stock has the highest mean filtered volatility, whereas Emerging markets have the least with about 4.8 and 1.89, respectively. On average, conventional asset classes exhibit higher volatility than the commodity securities as can be seen by the relatively larger values of the mean filtered volatility for countries compared to those of commodities.

Figure 4 contains graphs of mean filtered volatility for twelve selected countries and commodities including Argentina, Bulgaria, China, Denmark, Natural Gas, Crude Oil, Platinum, Rhodium, Silver, the USA, World, Zinc.

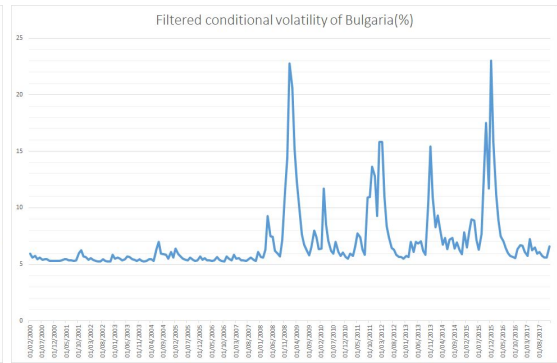
Table 5: Mean filtered MS-GARCH-CAPM volatility for all series

<b>US S&amp;P500</b>	1.962619	<b>CRO-CROBEX</b>	5.015678
<b>CAN TSX</b>	2.554394	<b>LIT-OMXVILNIUS</b>	2.648331
<b>GER DAX</b>	4.802231	<b>HUN-BUX</b>	5.337474
<b>AUS ASX</b>	2.651945	<b>EGY-EGX30</b>	4.802231
<b>DEN OMX</b>	14.96079	<b>POR-PS-I20</b>	1.677671
<b>FIN OMXHEX</b>	4.802231	<b>CZECH-SEPX</b>	2.10555
<b>SPNIBEX</b>	1.967885	<b>BRA-BOVESPA</b>	2.794947
<b>SLOVSBITOP</b>	2.135627	<b>ARG-MERVAL25</b>	13.37952
<b>UKFTSE100</b>	4.828254	<b>COL-COLCAP</b>	4.809118
<b>ITAFITSEMIB</b>	7.406845	<b>BELG-BEL20</b>	2.17729
<b>SWEOMX30</b>	3.742947	<b>UAE-ADXGEN</b>	5.40287
<b>SWTSMI</b>	2.613581	<b>CHIL-IGPA</b>	5.298427
<b>NZLNZX50</b>	1.849826	<b>MEX-S&amp;PBMVIPC</b>	1.901675
<b>FRA-CAC</b>	1.981416	<b>QAT-QE</b>	3.054392
<b>NOR-OSEAX</b>	1.932275	<b>LUX-LUXX</b>	3.481971
<b>NETH-AEX</b>	1.942495	<b>CRUDEOIL</b>	2.450924
<b>JAP-NIKKEI225</b>	1.928034	<b>GOLD</b>	2.962834
<b>IRE-ISEQ</b>	1.928034	<b>SILVER</b>	5.660347
<b>TUN-TUNINDEX</b>	0.981831	<b>GAS</b>	6.088891
<b>THAI-SET50</b>	1.851169	<b>COPPER</b>	2.332829
<b>MYL-FTSEKLCI</b>	1.851169	<b>PLATINUM</b>	5.019397
<b>INDO-JCI</b>	1.91567	<b>PALLADIUM</b>	2.135101
<b>PHI-PSEI</b>	1.91567	<b>NICKEL</b>	4.555906
<b>SING-STI</b>	2.236415	<b>TIN</b>	3.075775
<b>CHIN-SSE</b>	8.930896	<b>ZINC</b>	5.54257
<b>INDI-SENSEX</b>	2.572912	<b>RHODIUM</b>	7.026534
<b>TAI-TWSE</b>	2.003599	<b>RUTHENIUM</b>	4.528716
<b>KOR-KOSPI</b>	2.368377	<b>CORN</b>	4.902439
<b>HNGKNG-HIS</b>	2.381495	<b>RUBBER</b>	2.200272
<b>SERB-BELEX15</b>	6.445615	<b>SOYABEAN</b>	2.279543
<b>UKR-PFTS</b>	2.381495	<b>WOOL</b>	2.074668
<b>BUL-SOFIX</b>	7.443544	<b>ALUMINIUM</b>	4.334008
<b>ROM-BET</b>	5.62009	<b>LEAD</b>	4.407331
<b>POL-WIG</b>	2.616998	<b>WHEAT</b>	2.34754
<b>ICE-SEICEX</b>	4.447006	<b>COCOA</b>	4.532433
<b>RUS-MICEX</b>	3.948479	<b>COFFEE</b>	4.551753
<b>MALT-MALTEX</b>	2.52231	<b>COTTON</b>	4.800368
<b>ISR-TA100</b>	2.10099	<b>WORLD</b>	4.800368
<b>LAT-OMXRIGA</b>	1.896108	<b>EUROPE</b>	3.494552
<b>EST-OMXTALLIN</b>	5.67773	<b>EMERGING MARKETS</b>	1.890573
<b>TUR-XU100</b>	16.5353		

Note: Series names are given in Table 8.



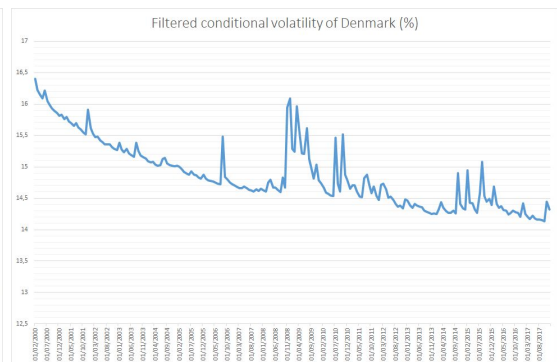
(a) Argentina



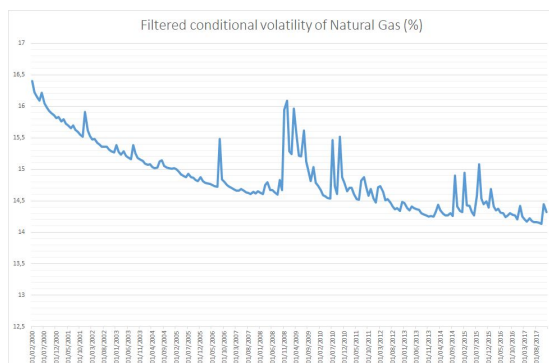
(b) Bulgaria



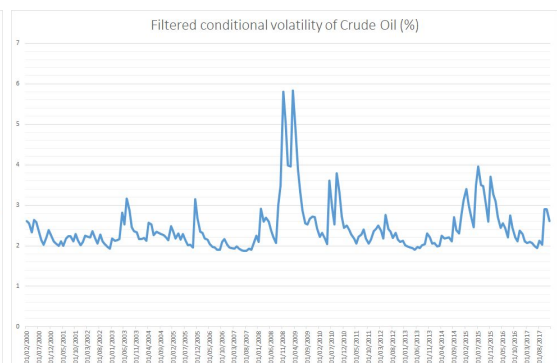
(c) China



(d) Denmark

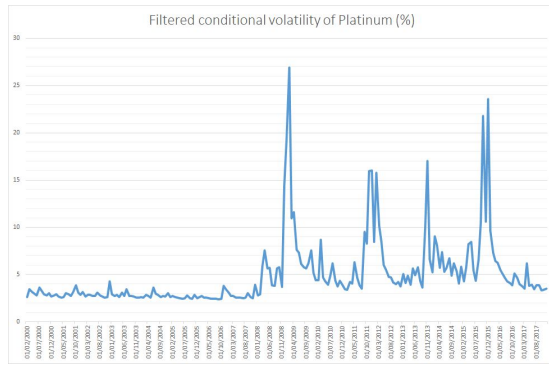


(e) Natural Gas

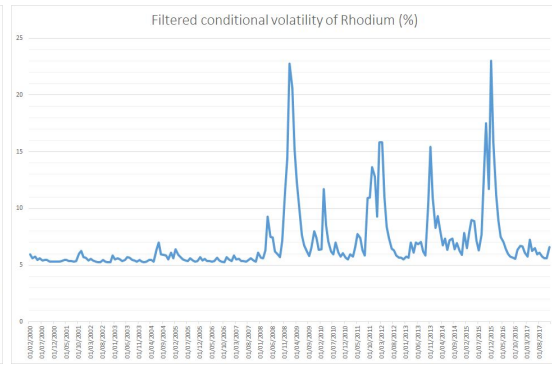


(f) Crude Oil

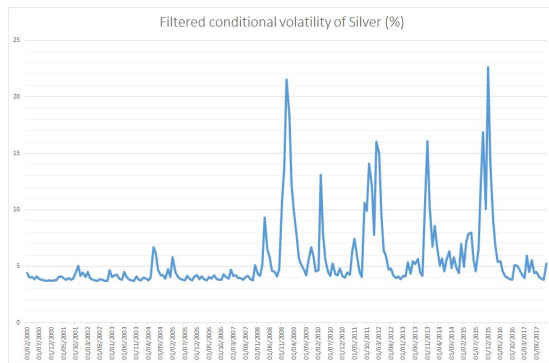
Figure 4: MS-GARCH-CAPM selected plots of filtered volatilities



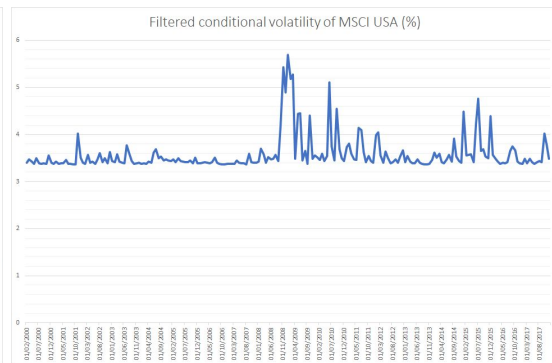
(g) Platinum



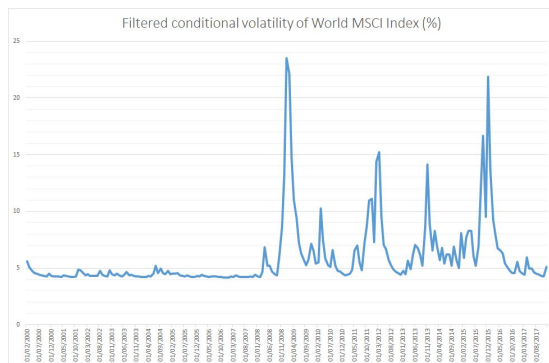
(h) Rhodium



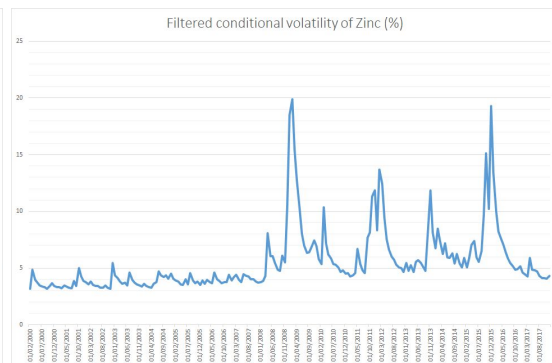
(i) Silver



(j) USA



(k) World



(l) Zinc

Figure 4: MS-GARCH-CAPM selected plots of filtered volatilities (continued)

Looking at the graphs, some patterns can be discerned in most of the markets. For instance, periods of low volatility persisted throughout the beginning parts of the sample until the middle when periods of transitory high volatility clustered till the later part of the sample in Argentina, Bulgaria, China, Crude oil, Platinum, Rhodium, Silver, the USA, World, and Zinc. In contrast, the pattern changes for both Denmark and Natural Gas. Their beginning part of the sample exhibits high volatility levels which decline continuously. Until the end of the sample, high volatility levels appear short-lived. The last few months in the sample exhibit low volatility levels except in Natural Gas, Crude oil, and Denmark in which volatility levels seems to build up. These findings, especially in the case of the energy market, are consistent with the results of previous studies (see, e.g., Ma et al., 2018; Ma et al., 2019). As observed earlier, the middle of our sample coincides with the period of the 2008 financial crisis, which triggered significant turbulence in assets prices and returns. The immediate period after the sub-prime crisis witnessed a series of unconventional macroeconomic policies such as the US Quantitative Easing which altered investors' risk appetite.

## 4.6 Quantitative risk-management

This section aims to illustrate the usefulness of the conditional regime-switching GARCH CAPM in a practical risk management exercise (Jorion, 2006). Beta pricing models are used extensively for risk-management purposes, with the central interest to compute the beta risk or market prices of risk. These estimates are indeed useful as the risk of any financial, energy or other commodity markets will change as the  $\beta$ s change. In equilibrium, investors are compensated in terms of expected returns for bearing market risk. Risk-averse utility maximizers will attempt to hedge against the risk of unfavorable shifts in their investment opportunity set. *Ceteris paribus*, they will demand more of an asset if the *ex post* opportunity set is less favorable than anticipated.

The importance of modeling financial risk in stock markets for useful risk measurement has never been more significant given the recent global financial disasters. Ardia et al. (2018) argue that regime-switching GARCH models have proven to be increasingly useful in the field of quantitative finance, where investors are interested in the allocation of wealth among a wide array of risky investment opportunities. Regarding returns of financial assets, investors are interested in understanding the quantile of their future distribution of returns at a specific risk level. Two quantities of interest when measuring this distribution include the Value at Risk (VaR) and the expected shortfall (ES). VaR remains the standard method of measuring financial risks as it yields forecasts for the likely losses which may arise following changes in price over a pre-defined time horizon and a given confidence level (Sajjad et al. 2008). In this section, we present and compare the performance of risk metrics such as the Expected Shortfall and Out-of-Sample forecasts from the GARCH and MS-GARCH models. Besides, we show which model brings about considerable improvements in correctly forecasting one-day-ahead VaR using an innovative back-testing procedure for 12 selected stock markets including Bulgaria, Columbia, China, Japan, Latvia, Korea, Portugal, Qatar, Turkey, Wool, Cocoa, and World. Although the expected shortfall is not a conventional tool for validating the VaR forecasts or evaluating models' performance, it remains an acceptable tool for risk managers as it is a suitable candidate for quantifying how much is likely to be lost in case of a failed model.

We use the innovative Generalized Autoregressive Score (GAS) models proposed by Ardia et al. (2019) for VaR evaluation, prediction and back-testing under a rolling window on a 95% confidence interval with the assumption that the distribution of returns is left-skewed and fat-tailed, and its variance is time-varying. The GAS models have found broad application in financial econometrics given their ability to link many volatility modeling frameworks, especially the GARCH models. The Conditional Coverage (CC) first proposed by Christoffersen (1998) evaluates the correct coverage of the conditional left-tail distribution of log-returns. The Dynamic Quantile (DQ) of Engle and Manganelli (2004) tests some restrictions in a linear model that links the violations to a set of explanatory variables. Ardia et al. (2016) note that the DQ has more power and provides a holistic testing procedure for identifying when VaR back-testing model is misspecified.

The  $p$ -values for the CC and DQ tests of parameter restrictions on the transition probabilities matrix for the regime-switching process in our MS-GARCH-CAPM is presented in Table 3. The null hypothesis for the CC test is that the hits variable is uncorrelated with its own lagged values

and with the lag of any other variable including past log returns. Past VaR and its expected value must be equal to zero. The DQ test is that of the correct model specification at our chosen confidence level  $\alpha = 5\%$  for the VaR model. As can be seen the table, the p-value for the CC test is more significant than the conventional significance level for most of the markets. These results suggest that the assumptions of the CC test hold for most of the markets in our sample. However, this is not the case for Canada, Italy, Taiwan, Serbia, Argentina, UAE and Copper where the p-value is less than 5% suggesting that this assumption is violated, and we can reject the null hypothesis for these markets. Similarly, the large  $p$ -values of DQ for most of the markets is an indication that the null hypothesis of the correct model specification for the 5% significant level. In contrast, the p-values for DQ test in Canada, Spain, Italy, Sweden, Taiwan, Serbia, UEA, Chile, Nickel, Zinc, Wheat, Europe, and Emerging markets are smaller than 5% and in this case, against the assumption of the correct model specification for the 5% VaR level.

Figure 5 is composed of two panels for each series. The upper panel contains the Out-of-Sample returns. The lower panel contrasts the VaR computed at 5% level for the GARCH and MS-GARCH models, respectively. Table 6 provides the remaining expected shortfall (ES) estimates. Ma et al. (2019) note that the Out-of-Sample predictability of a model proves very useful for both researchers and market participants who are concerned with the model's ability to predict future rather than past outcomes. Looking at the upper panel, it can be seen that among our twelve selected markets, the out-of-sample returns forecasts is highest in Bulgaria with about 0.43 while it is least in Turkey with about -0.76. The highest mean out-of-sample returns forecast is 0.09 for Korea and Japan, followed by 0.08 for China, whereas the least is -0.69 and -0.04 for Turkey and Latvia respectively. Out-of-sample returns forecasts are less than zero for Turkey, as well as for most of the periods in Portugal and Latvia. Out-of-sample returns forecasts are almost positive in all the periods for Korea, Columbia, Japan, and Wool. It is unclear how much weight to place on the ability of out-of-sample forecasts on predicting stock returns. Campbell and Thompson (2007) note, however, that out-of-sample forecasts do have some ability to predict stock returns and are economically important, especially to mean-variance investors, because they can generate significant improvements in portfolio performance.

Regarding the lower panel, the VaR at 5% level for the GARCH model is represented by the blue color while the orange color represents that of the MS-GARCH. From the VaR plots in this panel, it can be seen that back-testing test discriminated between the VaR for the GARCH and MS-GARCH especially in Bulgaria where the plots never met at the point throughout the sample period. Here, the critical finding is that in all the markets considered, the mean VaR at 5% for the MS-GARCH model is either higher or equal to VaR at 5% from the GARCH model except in Turkey. Specifically, the mean VaR from the GARCH model is greatest in Columbia while it is least in China. Similarly, the mean from the MS-GARCH model is most significant in Columbia and Wool but least in China.

There were some sections of the sample where the plot of VaR at 5% forecasts from the GARCH model was identical with those from MS-GARCH model. For instance, the back-testing test failed to discriminate between the plots from both models until after the first half of the sample in the following markets: China, Columbia, Cocoa, Japan, Latvia, Portugal, Qatar, and World. In most of the markets (especially in China and Latvia), the plot for the MS-GARCH was over that of the GARCH model. Conversely, the plot for the GARCH model was over that of the MS-GARCH model throughout the sample period in Bulgaria and at some point in Columbia, Turkey, and World.

Concerning the back-testing estimates for Expected Shortfalls (ES) as presented in Table 6, the last line of the table reports the average expected shortfall estimates. ES, as a financial risk measurement tool, estimates the average of  $100p\%$  worst losses where  $p$  is a chosen confidence level (Acerbi and Tasche, 2002a). ES is widely applicable in stocks returns evaluation despite the underlying sources of risks. It offers a unique global approach to portfolio selection when assets are exposed to different sources of uncertainty. The ES can be viewed as an alternative to the VaR approach to stocks returns evaluation, given that it can provide more reliable estimates even when the VaR estimators fail (Acerbi and Tasche, 2002b). Further, Taylor (2008) notes that there is no significant difference between the two approaches but that ES is an appropriate approach for

GARCH models estimated through the skewed- $t$  distribution. In our results, it can be seen that the average ES is highest in China, followed by Bulgaria with about -1.61, whereas the least is Latvia is -0.17. These results suggest that stocks from the Chinese market seem riskier, whereas securities from the Latvian market appear as the least risky among the selected markets.

In summary, the performance of the MS-GARCH model compared to the GARCH in the above Risk Metrics lends credence to the claim that regime-switching models bring about more significant improvement in forecasting stock returns, especially in back-testing the one-day-ahead VaR at 5% level for the selected markets as shown earlier. For instance, our results suggest that the single regime GARCH specification mostly underestimates the returns (risk) as demonstrated by the plots of VaR at 5% from both models. These results are complemented by the findings of Kuester et al. (2006), Sajjad et al. (2008) and Taylor (2008) who favor the use of switching models especially in back-testing VaR and Expected Shortfall. These authors argued that systematic risks might vary depending on the volatility regime.

Table 6: Back-tests: Expected Shortfall (ES) estimates

<b>BULGARIA</b>	<b>CHINA</b>	<b>COCOA</b>	<b>COLUMBIA</b>	<b>JAPAN</b>	<b>KOREA</b>	<b>LATVIA</b>	<b>PORTUGAL</b>	<b>QATAR</b>	<b>TURKEY</b>	<b>WOOL</b>	<b>WORLD</b>
-0.3696935	-1.59896481	-0.34339205	-0.28504994	-0.22241096	-0.2939785	-0.1342876	-0.23577201	-0.40365985	-0.23577201	-0.20240418	-0.20894887
-0.43337183	-1.67769711	-0.35420113	-0.2933303	-0.22812045	-0.30578616	-0.15315286	-0.23428059	-0.45654336	-0.23428059	-0.21547615	-0.22150739
-0.53007848	-1.60642624	-0.3701448	-0.30702414	-0.23707951	-0.3067333	-0.16160187	-0.2390468	-0.47023146	-0.2390468	-0.22253285	-0.2306819
-0.61036632	-1.44400855	-0.35833353	-0.30345896	-0.25069577	-0.31563426	-0.16564428	-0.23918478	-0.48149671	-0.23918478	-0.23316302	-0.2380991
-0.6900595	-1.5166263	-0.37778933	-0.30160493	-0.25903908	-0.31825599	-0.18481907	-0.24140153	-0.5168024	-0.24140153	-0.24519744	-0.26974967
-0.65571115	-1.58169698	-0.38602474	-0.29859613	-0.26478727	-0.32096735	-0.17300207	-0.23705698	-0.508046	-0.23705698	-0.23973005	-0.26329892
-0.73043507	-1.76632596	-0.40561767	-0.3097635	-0.270502	-0.3329765	-0.18535075	-0.24114021	-0.53040442	-0.24114021	-0.25219612	-0.2678908
-0.7859926	-1.75388221	-0.44246984	-0.31257195	-0.2800636	-0.33564166	-0.19322463	-0.23907795	-0.53696968	-0.23907795	-0.25639462	-0.26651705
-0.85848732	-1.5137559	-0.45070069	-0.318138	-0.28029283	-0.33197371	-0.20509865	-0.23770699	-0.58598317	-0.23770699	-0.26242186	-0.30719896
-0.85124058	-1.67578901	-0.47499313	-0.31710507	-0.28898245	-0.32554309	-0.20926281	-0.23596116	-0.54152891	-0.23596116	-0.25136108	-0.29112545
-0.65154364	-1.61351731	-0.39636669	-0.30466429	-0.25819739	-0.31874905	-0.17654446	-0.2380629	-0.5031666	-0.2380629	-0.23808774	-0.25650181

Note: The last line of the Table reports the average Expected Shortfall estimates.



(a) Bulgaria

(b) China



(c) Cocoa

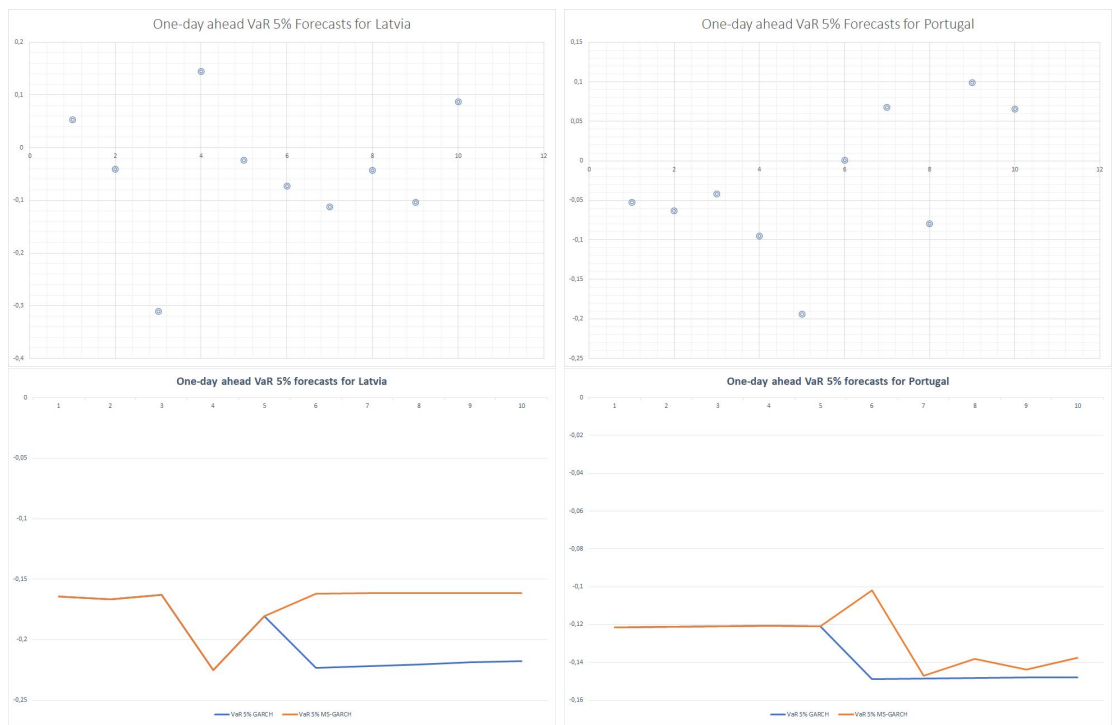
(d) Columbia

Figure 5: Back-tests: Out-of-Sample returns (top), GARCH *vs.* MS-GARCH VaR 5% Forecasts (bottom) for selected markets



(e) Japan

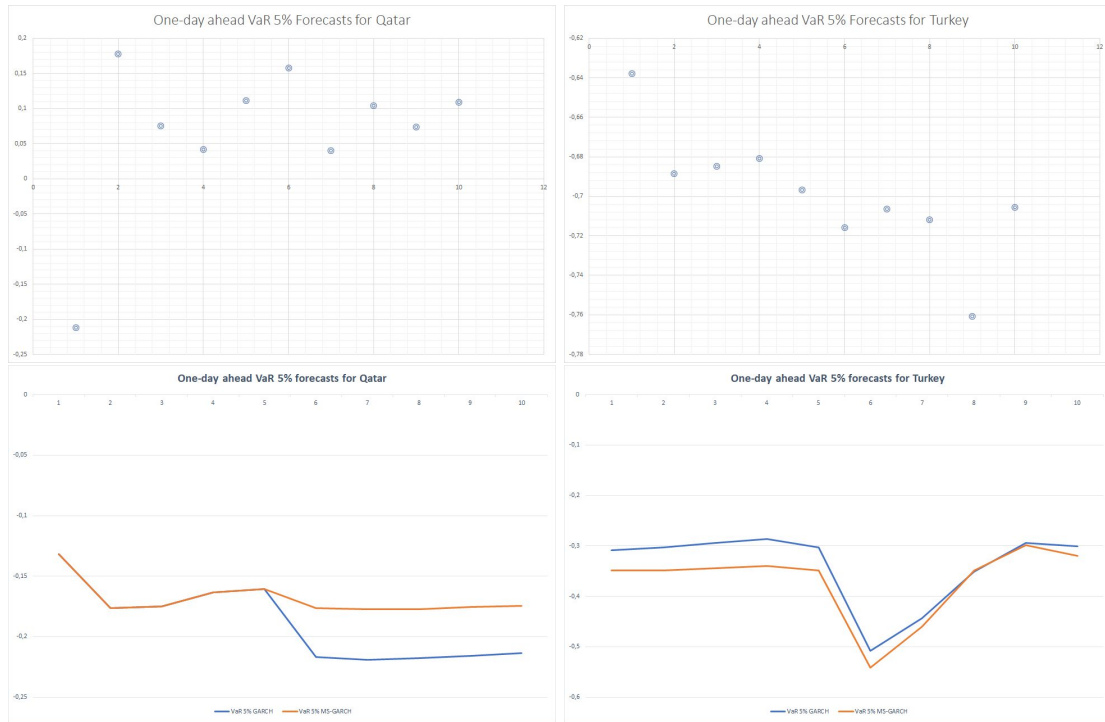
(f) Korea



(g) Latvia

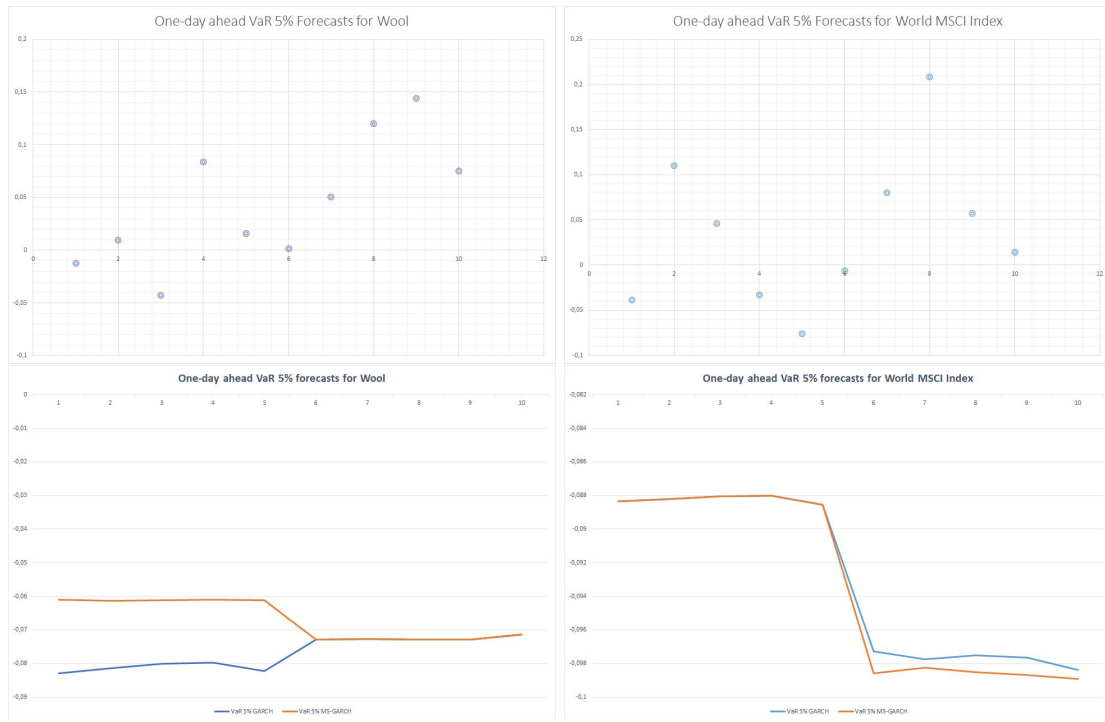
(h) Portugal

Figure 5: Back-tests: Out-of-Sample returns (top), GARCH vs. MS-GARCH VaR 5% Forecasts (bottom) for selected markets (continued)



(i) Qatar

(j) Turkey



(k) Wool

(l) World

Figure 5: Back-tests: Out-of-Sample returns (top), GARCH *vs.* MS-GARCH VaR 5% Forecasts (bottom) for selected markets (continued)

## 5 Conclusion

The conditional regime-switching GARCH CAPM accommodates the essential characteristics of time-varying conditional variances and covariances in the energy and financial time series. This theoretically-based asset pricing model captures predictable time-variation in both the conditional mean and the conditional volatility of the market excess return. A critical aspect of this framework lies in decomposing the series into two distinct economic regimes: bull and bear. Since the market regime is unobservable, the Markov-Switching process allows estimating regime probabilities and inferring time-varying betas and risk premia. This technique has been applied to study substantial time variations (both in size and potentially different sign) in the conditional betas and market returns. The time-varying risk premium can be inferred as well from the market regimes.

To capture the true dynamics of risk premia, this paper formulates a conditional regime-switching GARCH CAPM with time-varying risk and expected returns, from where we assess the stability of the  $\beta$ s. To assess the asset pricing performance of different versions of the CAPM, we estimate three models, including the pure form of CAPM, the conditional regime-switching CAPM, and the MS-GARCH-CAPM. Within this framework, we estimate the time-varying vector of market prices of risk for a large dataset comprising a total of 81 financial, energy, and other commodity markets during the period August 1999 - January 2018. Our approach has two potential advantages: (i) we adopt a Bayesian volatility estimation through the MCMC method, and (ii) we fit a skewed Student- $t$  conditional distribution. Also, because volatility may be heterogeneous and vary across different regimes, we studied the evolution of volatility using the mean filtered volatility from the regime-switching GARCH-CAPM model. Risk-averse rational investors living in a dynamic economy will typically anticipate and hedge against the possibility that investment opportunities in the future may change adversely. That is why, as robustness checks, we also compute Risk Metrics from the regime-switching GARCH, and compare the risk forecasting performance with that of the single regime GARCH using the back-testing technique.

Our chief concern in this paper lies in providing a successful implementation of the conditional CAPM version of pricing the systematic risk (e.g., the beta) that is positively rewarded by the market. Throughout our empirical testing procedure of three nested models, we achieve our final specification under the form of the conditional regime-switching CAPM augmented by GARCH volatility dynamics. The data strongly reject the CAPM specification with constant betas for two main reasons. First, it fails to take into account the effects of time-varying investment opportunities in the calculation of an asset's risk. Second, the bull and bear regimes capture statistically different market conditions (e.g., market contraction and expansion periods with distinct switching dynamics). We also provide a further empirical basis to the assessment of the validity of the CAPM, especially its assumption about the stability of the betas. The proposed model can indeed be added to the set of econometric tools employed by practitioners for the modeling of beta variability.

The results gathered from the conditional regime-switching GARCH CAPM offer convincing evidence against the prediction of the traditional model. The CAPM beta varies across the three models and bull-bear regimes. This feature is even more pronounced in frontline stocks and commodities. Specifically, among the stocks such as the USA, the UK, Germany, France, China, and Malaysia, we find significant variations not only in the size of beta from one model to another and across regimes but also changes in the direction of the relationship between risks and market returns. For instance, beta parameter estimates from the unconditional CAPM model suggest that stocks in the US, Germany, France, and Malaysia move together with the market. Beta estimates from the MS-CAPM model only confirm these results for stocks in France in both regimes, in regime 1 for Germany and regime 2 for Malaysia. They, however, suggest that in regime 1, stocks in Malaysia move in the opposite direction with the market. Stocks in Germany, the USA have no relation with the market in regime two, and both regimes respectively.

In sum, across the various models, it is striking that conditional betas exhibit significant time-variation and correlations with market risk premia. Further, beta estimates from the regime-switching GARCH-CAPM agree with the result of the unconditional CAPM in both regimes for the USA and Malaysia. They yield contradictory results for Germany and France in both regimes.

In the UK and China, beta estimates from the CAPM model suggests these stocks move in the opposite direction. This result is validated by the MS-GARCH model in both regimes for the UK and only in regime 1 in China. The estimates of beta from the regime-switching GARCH-CAPM only agrees with that of MS-CAPM for China but suggests no relation exists between returns on UK stocks and the market.

Regarding energy and other commodities indices, we also find that these variations exist but not in equal magnitude with stocks. We are considering prominent commodities such as Crude Oil, Gold, Copper, Tin, Rubber, Aluminum, Natural Gas, and Platinum. Beta parameter estimates from the regime-switching GARCH-CAPM model suggest that all the commodity indices move in the same direction with the market during both volatility regimes. This is similar to the results from the simple CAPM model except for Natural Gas and Platinum, where this model suggest that these commodities have no relation with the market. Beta estimates from the MS-GARCH model suggest that Crude oil, Gold and Natural Gas do not have ties with the market in both regimes. Aluminum and Platinum move in the same direction with the market in regime 1, but have no relation in regime 2. Also, Tin and Copper move in the opposite direction with the market respectively in regime 1 and 2 whereas Rubber moves in the same direction with the market. These commodities are, however, not related to the market in other regimes. Concerning the aggregate markets, both the simple CAPM and the regime-switching GARCH-CAPM models suggest that these markets move together with the market in all regimes. In the MS-GARCH model, the World aggregate stock moves along with the market only in regime 2, while Europe and Emerging markets aggregate stocks do not have a relation with the market in both regimes.

Results regarding the volatility dynamics (using the mean filtered volatility from the regime-switching GARCH-CAPM) suggest that among stocks, Turkey is the most volatile with about 16.53. Among commodities, Rhodium is the most volatile with about 7.02 (followed by Natural Gas at 6.08). The World is the most volatile with about 4.8 among aggregates. This implies that stocks are the most volatile asset class with the most volatile stock being more than twice and thrice as volatile as the most volatile commodity and aggregate, respectively. Lastly, results from the quantitative risk management tests suggest that the regime-switching model delivers better estimates of one-day-ahead VaR at 5% forecasts than the single regime GARCH model. We have reached these conclusions by using a methodology that is significantly more robust than the unconditional CAPM.

At this stage, policy implications and some extensions to this paper can be considered. First, given the success of Markov switching models in capturing the switching behavior of risks and returns volatility across regimes as well as its superior forecast of RiskMetrics, this paper recommends that risk managers can improve on their risk management strategy by extending their single-regime-type models with a regime-switching mechanism to better manage portfolio risks. Although this paper employs a large dataset, it only considers the risks dynamics and monitoring process for individually traded stocks. This paper could, therefore, be extended by considering exceptions and regulatory-based tests such as the Basel traffic light regulation to compute capital requirements for banks and other financial institutions. Lastly, given that it has become a widespread practice to separately assess the VaR for the left and right tails of the returns distributions, our paper could also be extended by using our nested model to evaluate and compare the VaR for the long and short positions to equip risk managers and traders depending on their position.

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# Appendix

Table 7: Return Series Descriptive Statistics

Countries	Mean	Std. Dev	Skewness	Ex. Kurtosis	JB	ARCH LM (5)
ESTONIA	0.0064	0.0679	-0.7785	3.447	131.14 [0.000]	42.509 [0.000]
TURKEY	0.0132	0.1138	0.0614	1.2233	95.765 [0.000]	22.931 [0.000]
CROATIA	0.0046	0.0759	-0.5607	5.8042	32.346 [0.000]	42.334 [0.000]
LITHUANIA	0.0086	0.0681	-0.8171	7.0969	49.754 [0.000]	21.493 [0.001]
HUNGARY	0.0075	0.0661	-0.4716	1.5901	31.336 [0.000]	9.616 [0.008]
EGYPT	0.0122	0.0947	-0.1093	0.825	6.677 [0.035]	10.572 [0.061]
PORTUGAL	-0.0027	0.0556	-0.5994	1.4686	32.945 [0.000]	61.897 [0.022]
CZECH REPUBLIC	0.0034	0.0634	-0.9317	3.735	59.713 [0.000]	43.224 [0.000]
BRAZIL	0.0089	0.073	-0.3023	0.7116	7.993 [0.018]	6.209 [0.028]
ARGENTINA	0.0184	0.1054	-0.0611	1.0401	10.053 [0.006]	11.383 [0.044]
COLUMBIA	0.0108	0.0633	0.1285	1.3558	17.457 [0.001]	10.411 [0.064]
BELGIUM	0.0012	0.0519	-0.346	3.3225	67.622 [0.000]	57.899 [0.000]
UAE	0.0077	0.0625	0.3206	3.8689	140.98 [0.000]	40.78 [0.000]
CHILE	0.0076	0.0397	0.0967	1.0918	11.269 [0.003]	20.848 [0.072]
MEXICO	0.0021	0.0881	-0.476	2.0912	61.237 [0.000]	35.408 [0.000]
QATAR	0.0101	0.0581	-0.396	1.0943	16.729 [0.000]	40.807 [0.000]
LUXEMBOURG	0.0018	0.063	-0.8922	2.7717	99.612 [0.000]	15.929 [0.007]
ISRAEL	0.0056	0.0569	-1.2835	3.3747	164.79 [0.000]	20.783 [0.000]
MALTEX	0.0039	0.0497	1.0272	4.5366	227.34 [0.000]	22.746 [0.000]
TUNISIA	0.0077	0.0355	-0.0506	2.2332	45.809 [0.000]	16.522 [0.005]

Note: Std. Dev. stands for standard deviation, Ex. Kurtosis for excess kurtosis, JB for Jarque-Bera statistics, and LM ARCH for Lagrange Multiplier ARCH test.

Countries	Mean	Std. Dev	Skewness	Ex. Kurtosis	JB	ARCH LM (5)
USA	0.0032	0.0518	-1.5932	6.0584	429.52 [0.000]	54.985 [0.000]
CANADA	0.0038	0.0404	-1.0552	3.1994	134.65 [0.000]	21.292 [0.000]
GERMANY	0.0041	0.0623	-0.8671	3.0538	113.06 [0.000]	13.819 [0.016]
AUSTRALIA	0.0033	0.0413	-0.4041	1.1743	18.621 [0.000]	47.666 [0.000]
DENMARK	0.0071	0.0544	-0.5504	1.8793	43.485 [0.000]	36.067 [0.000]
FINLAND	0.0011	0.0797	-0.2628	2.0447	40.857 [0.000]	45.586 [0.000]
SPAIN	0.0005	0.0633	-0.4596	0.5081	10.115 [0.000]	22.611 [0.000]
FRANCE	0.001	0.0595	-0.7812	2.1792	65.912 [0.000]	21.227 [0.000]
UK	0.0008	0.0413	-0.6388	1.0478	25.027 [0.000]	15.128 [0.009]
ITALY	-0.0022	0.0609	-0.3265	0.8052	9.851 [0.007]	5.855 [0.032]
SWEDEN	0.0031	0.0587	-0.2977	0.681	7.5028 [0.023]	26.114 [0.000]
SWITZERLAND	0.0013	0.0392	-0.7277	0.8563	28.139 [0.000]	21.115 [0.000]
NEW ZEALAND	0.0044	0.0354	-0.5696	2.104	52.476 [0.000]	36.106 [0.000]
NORWAY	0.0081	0.0592	-0.7901	1.5799	45.768 [0.000]	39.405 [0.000]
NETHERLAND	0.0003	0.0574	-1.0723	2.2708	89.431 [0.000]	44.214 [0.000]
JAPAN	0.0011	0.059	-0.4712	1.0824	18.882 [0.000]	17.889 [0.003]
IRELAND	0.0041	0.0629	-0.8876	2.5519	88.586 [0.000]	62.801 [0.000]
THAILAND	0.0062	0.0668	-1.1079	4.6719	24.508 [0.000]	59.109 [0.000]
MYLASIA	0.0038	0.0432	-0.4117	1.6887	32.357 [0.000]	10.199 [0.069]
INDONESIA	0.0109	0.0643	-1.0897	3.5015	35.059 [0.000]	16.67 [0.002]
PHILIPPINES	0.0056	0.0591	-0.4426	1.6858	33.233 [0.000]	19.624 [0.008]
SINGAPORE	0.002	0.0544	-0.9535	2.0021	80.16 [0.000]	61.908 [0.000]
CHINA	0.0032	0.0824	-0.667	1.8309	47.043 [0.000]	32.67 [0.000]
INDIA	0.0098	0.0738	-0.6006	2.0889	53.227 [0.000]	12.843 0.0248
TAIWAN	0.0012	0.0647	-0.1855	1.6735	26.932 [0.000]	31.461 [0.000]
KOREA	0.0046	0.0642	-0.0967	0.752	5.527 [0.063]	33.064 [0.000]
HONG KONG	0.0036	0.0618	-0.555	1.2433	25.463 [0.000]	16.839 [0.004]
SERBIA	0.0002	0.0815	-2.0224	15.449	111.21 [0.000]	30.031 [0.000]
UKRAINE	0.0024	0.0508	-0.5851	2.6786	76.321 [0.000]	70.932 [0.000]
BULGARIA	0.0084	0.0905	-1.9048	15.15	122.51 [0.000]	25.71 [0.000]
ROMANIA	0.012	0.0804	-0.4141	2.2836	54.09 [0.000]	22.652 [0.000]
SLOVANIA	0.0006	0.056	-0.3511	0.961	12.986 [0.001]	11.508 [0.042]
POLAND	0.0059	0.0638	-0.546	1.2584	25.449 [0.000]	12.324 [0.030]
ICELAND	0.0001	0.1044	-8.3349	2.644	84.658 [0.000]	7.05 [0.005]
RUSSIA	0.0134	0.1017	-0.8937	4.6675	228.99 [0.000]	16.973 [0.004]
LATVIA	-0.0001	0.0789	-0.3563	3.9627	148.59 [0.000]	75.24 [0.000]

COMMODITIES	Mean	Std. Dev	Skewness	Ex. Kurtosis	JB	ARCH LM (5)
CRUDE OIL	0.0052	0.1144	-0.6037	1.0278	23.051	33.972
					[0.000]	[0.000]
GOLD	0.0072	0.049	0.0799	1.6769	26.012	13.122
					[0.000]	[0.022]
SILVER	0.0051	0.0842	-0.3747	2.0796	44.791	19.753
					[0.000]	[0.001]
NATURAL GAS	0.0003	0.1388	-0.32151	1.7985	33.439	46.064
					[0.000]	[0.004]
COPPER	0.0066	0.0778	-0.8704	4.3231	199.1	25.678
					[0.000]	[0.000]
PLATINUM	0.0042	0.0697	-0.5932	3.1535	104.06	27.119
					[0.000]	[0.000]
PALLEDIUM	0.0049	0.1067	-0.6819	2.7276	85.251	7.044
					[0.000]	[0.054]
NICKEL	0.0027	0.1053	-0.6051	2.4071	66.541	6.311
					[0.000]	[0.037]
TIN	0.0059	0.0731	-0.3861	1.494	25.928	186.272
					[0.000]	[0.000]
ZINC	0.0046	0.0775	-0.9029	4.1319	186.39	8.1738
					[0.000]	[0.005]
RHODIUM	0.0024	0.1441	-1.6576	14.075	149.34	179.45
					[0.000]	[0.000]
RUTHERNIUM	0.0071	0.1417	0.8763	5.0058	257.85	187.68
					[0.000]	[0.000]
CORN	0.0026	0.0864	-0.3647	1.9197	38.66	181.18
					[0.000]	[0.000]
RUBBER	0.0045	0.0884	-1.2396	6.3796	429.42	178.78
					[0.000]	[0.000]
SOYABEAN	0.0032	0.0712	-0.184	0.8392	7.697	10.566
					[0.021]	[0.060]
WOOL	0.005	0.0495	0.0212	2.6458	64.186	194.85
					[0.000]	[0.000]
ALLUMINIUM	0.0017	0.0538	-1.0768	5.457	315.48	49.08
					[0.000]	[0.000]
LEAD	0.0072	0.0955	-0.3254	2.2701	51.122	37.475
					[0.000]	[0.000]
WHEAT	0.003	0.0959	0.0282	1.376	17.384	13.852
					[0.000]	[0.016]
COCOA	0.0027	0.0739	0.2582	1.1666	14.921	7.183
					[0.000]	[0.0207]
COFFEE	0.0013	0.0887	0.0474	1.9878	36.302	6.322
					[0.000]	[0.041]
COTTON	0.0016	0.0806	0.1724	0.9626	9.584	18.783
					[0.008]	[0.002]

Table 8: Series names

	Countries	CAPM	MS-CAPM	MS-GARCH-CAPM
1	USA	US S&P500	MSCI USA	MSCI USA_MS_GARCH
2	CANADA	CAN TSX	MSCI CAN	MSCI CAN_MS_GARCH
3	GERMANY	GER DAX	MSCI GER	MSCI GER_MS_GARCH
4	AUSTRALIA	AUS ASX	MSCI AUS	MSCI AUS_MS_GARCH
5	DENMARK	DEN OMX	MSCI DEN	MSCI DEN_MS_GARCH
6	FINLAND	FIN OMXHEX	MSCI FIN	MSCI FIN_MS_GARCH
7	SPAIN	SPNIBEX	MSCI SPN	MSCI SPN_MS_GARCH
8	FRANCE	FRA-CAC	MSCI FRA	MSCI FRA_MS_GARCH
9	UK	UKFTSE100	MSCI UK	MSCI UK_MS_GARCH
10	ITALY	ITAFNSEMIB	MSCI ITA	MSCI ITA_MS_GARCH
11	SWEDEN	SWEOMX30	MSCI SWE	MSCI SWE_MS_GARCH
12	SWITZERLAND	SWTSMI	MSCI SWT	MSCI SWT_MS_GARCH
13	NEW ZEALAND	NZLNZX50	MSCI NZL	MSCI NZL_MS_GARCH
14	NORWAY	NOR-OSEAX	MSCI NOR	MSCI NOR_MS_GARCH
15	NETHERLAND	NETH-AEX	MSCI NLD	MSCI NLD_MS_GARCH
16	JAPAN	JAP-NIKKEI225	MSCI JAP	MSCI JAP_MS_GARCH
17	IRELAND	IRE-ISEQ	MSCI IRE	MSCI IRE_MS_GARCH
18	THAILAND	THAI-SET50	MSCI THAI	MSCI THAI_MS_GARCH
19	MYLASIA	MYL-FTSEKLCI	MSCI MYL	MSCI MYL_MS_GARCH
20	INDONESIA	INDO-JCI	MSCI INDO	MSCI INDO_MS_GARCH
21	PHILIPPINES	PHI-PSEI	MSCI PHI	MSCI PHI_MS_GARCH
22	SINGAPORE	SING-STI	MSCI SING	MSCI SING_MS_GARCH
23	CHINA	CHIN-SSE	MSCI CHIN	MSCI CHIN_MS_GARCH
24	INDIA	INDI-SENSEX	MSCI INDI	MSCI INDI_MS_GARCH
25	TAIWAN	TAI-TWSE	MSCI TAI	MSCI TAI_MS_GARCH
26	KOREA	KOR-KOSPI	MSCI KOR	MSCI KOR_MS_GARCH
27	HONG KONG	HNGKNG-HIS	MSCI HNGKNG	MSCIHNGKNG_MS_GARCH
28	SERBIA	SERB-BELEX15	MSCI SERB	MSCI SERB_MS_GARCH
29	UKRAINE	UKR-PFTS	MSCI UKR	MSCI UKR_MS_GARCH
30	BULGARIA	BUL-SOFIX	MSCI BUL	MSCI BUL_MS_GARCH
31	ROMANIA	ROM-BET	MSCI ROM	MSCI ROM_MS_GARCH
32	SLOVANIA	SLOVSBITOP	MSCISLOV	MSCI SLOV_MS_GARCH
33	POLAND	POL-WIG	MSCI POL	MSCI POL_MS_GARCH
34	ICELAND	ICE-SEICEX	MSCI ICE	MSCI ICE_MS_GARCH
35	RUSSIA	RUS-MICEX	MSCI RUS	MSCI RUS_MS_GARCH
36	LATVIA	LAT-OMXRIGA	MSCI LAT	MSCI LAT_MS_GARCH
37	ESTONIA	EST-OMXTALLIN	MSCI EST	MSCI EST_MS_GARCH
38	TURKEY	TUR-XU100	MSCI TUR	MSCI TUR_MS_GARCH
39	CROATIA	CRO-CROBEX	MSCI CRO	MSCI CRO_MS_GARCH
40	LITHUANIA	LIT-OMXVILNIUS	MSCI LIT	MSCI LIT_MS_GARCH
41	HUNGARY	HUN-BUX	MSCI HUN	MSCI HUN_MS_GARCH
42	EGYPT	EGY-EGX30	MSCI EGY	MSCI EGY_MS_GARCH
43	PORTUGAL	POR-PS-I20	MSCI POR	MSCI POR_MS_GARCH
44	CZECH REPUBLIC	CZECH-SEPX	MSCI CZECH	MSCI CZECH_MS_GARCH
45	BRAZIL	BRA-BOVESPA	MSCI BRA	MSCI BRA_MS_GARCH
46	ARGENTINA	ARG-MERVAL25	MSCI ARG	MSCI ARG_MS_GARCH
47	COLUMBIA	COL-COLCAP	MSCI COL	MSCI COL_MS_GARCH
48	BELGIUM	BELG-BEL20	MSCI BEL	MSCI BEL_MS_GARCH
49	UAE	UAE-ADXGEN	MSCI UAE	MSCI UAE_MS_GARCH
50	CHILE	CHIL-IGPA	MSCI CHIL	MSCI CHIL_MS_GARCH
51	MEXICO	MEX-S&PBMVIPC	MSCI MEX	MSCI MEX_MS_GARCH
52	QATAR	QAT-QE	MSCI QAT	MSCI QAT_MS_GARCH
53	LUXEMBOURG	LUX-LUXX	MSCI LUX	MSCI LUX_MS_GARCH
54	ISRAEL	ISR-TA100	MSCI ISR	MSCI ISR_MS_GARCH
55	MALTEX	MALT-MALTEX	MSCI MALT	MSCI MALT_MS_GARCH
56	TUNISIA	TUN-TUNINDEX	MSCI TUN	MSCI TUN_MS_GARCH

<b>Commodities</b>				
57	CRUDEOIL	CRUDEOIL	MSCI OIL_	MSCI OIL_MS_GARCH
58	GOLD	GOLD	MSCI GOLD_	MSCI GOLD_MS_GARCH
59	SILVER	SILVER	MSCI SILVER_	MSCI SILVER_MS_GARCH
60	GAS	GAS	MSCI GAS_	MSCI GAS_MS_GARCH
61	COPPER	COPPER	MSCI Copper_	MSCI COPPER_MS_GARCH
62	PLATINUM	PLATINUM	MSCI Platinum_	MSCI Platinum_MS_GARCH
63	PALLADIUM	PALLADIUM	MSCI Palledium_	MSCI Palledium_MS_GARCH
64	NICKEL	NICKEL	MSCI Nickel_	MSCI Nickel_MS_GARCH
65	TIN	TIN	MSCI Tin_	MSCI Tin_MS_GARCH
66	ZINC	ZINC	MSCI Zinc_	MSCI Zinc_MS_GARCH
67	RHODIUM	RHODIUM	MSCI Rhodium_	MSCI Rhodium_MS_GARCH
68	RUTHENIUM	RUTHENIUM	MSCI Ruthernium_	MSCI Ruthernium_MS_GARCH
69	CORN	CORN	MSCI Corn_	MSCI Corn_MS_GARCH
70	RUBBER	RUBBER	MSCI Rubber_	MSCI Rubber_MS_GARCH
71	SOYABEAN	SOYABEAN	MSCI Soyabean_	MSCI Soyabean_MS_GARCH
72	WOOL	WOOL	MSCI Wool_	MSCI Wool_MS_GARCH
73	ALUMINIUM	ALUMINIUM	MSCI Aluminium_	MSCI Aluminium_MS_GARCH
74	LEAD	LEAD	MSCI Lead_	MSCI Lead_MS_GARCH
75	WHEAT	WHEAT	MSCI Wheat_	MSCI Wheat_MS_GARCH
76	COCOA	COCOA	MSCI Cocoa_	MSCI Cocoa_MS_GARCH
77	COFFEE	COFFEE	MSCI Coffee_	MSCI Coffee_MS_GARCH
78	COTTON	COTTON	MSCICotton_	MSCI Cotton_MS_GARCH
<b>Aggregates</b>				
79	WORLD	WORLD	MSCI WORLD_	MSCI WORLD_MS_GARCH
80	EUROPE	EUROPE	MSCI Europe_	MSCI Europe_MS_GARCH
81	EMERGING MARKETS	EMERGING MARKETS	MSCI EM_	MSCI EM_MS_GARCH

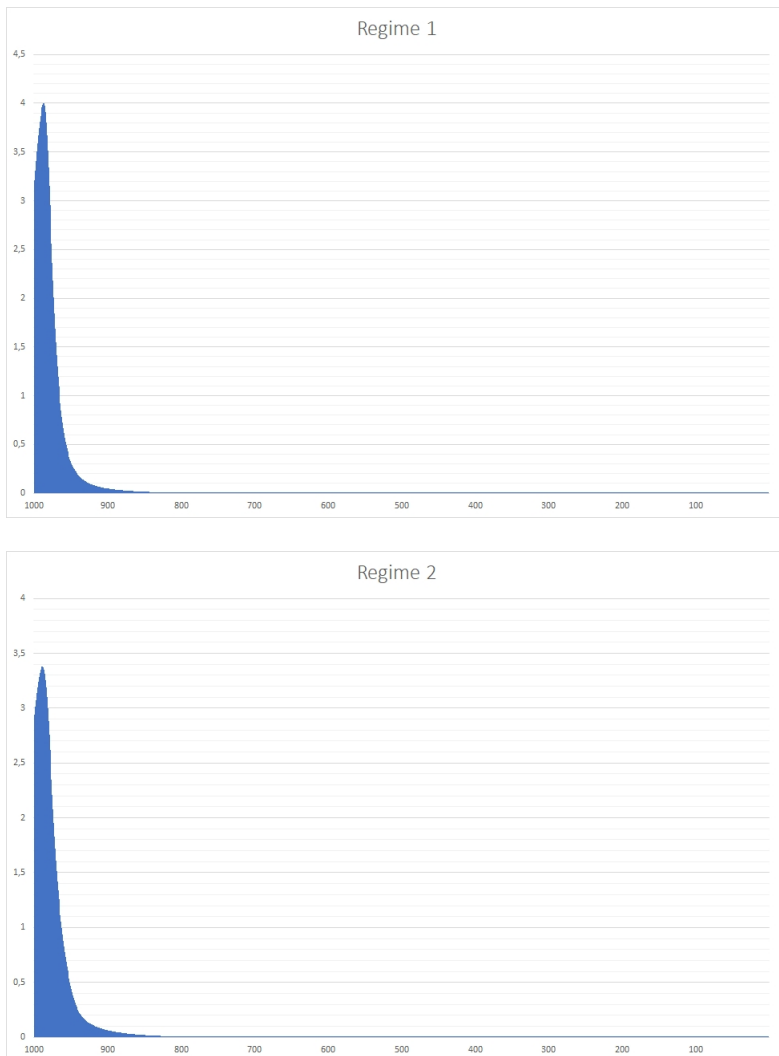


Figure 6: Histogram of the predictive distribution in each regime of the MS-GARCH-CAPM for MSCI USA