Simple intra-cavity beam shaping for generating a shape-invariant flat-top laser beam

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Simple intra-cavity beam shaping for generating a shape-invariant flat-top laser beam

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Abstract :

For laser users, the usual Gaussian beam (GB) may not always be the best intensity profile. Often, a preferred option is a Flat-Top (FT) beam which is generally obtained by using diffractive optics in a reshaping operation. In this case, the resulting FT profile occurs only in the focal plane of the focusing lens, and its vicinity. We consider here the generation of a shape-invariant FT laser beam resulting from the incoherent weighted (50%-50% in power) mixing of LG_{00} (GB) and LG_{01} (doughnut) beams. For that we consider the insertion inside the laser cavity of a diaphragm and an absorbing ring, and optimising their sizes makes possible the simultaneous oscillation on LG_{00} and LG_{01} modes allowing the generation of a shape-invariant laser beam..

Keywords: beam shaping, flat-top, shape invariant beam, transverse modes, multimode laser oscillation

1. Introduction

The usual Gaussian beam (GB) may not always be the best intensity profile for many laser applications. For instance, an intensity profile, in the focal plane of a focusing lens, having the shape of a flat-top can be very useful [1]. As shown in Fig. 1, the *Flat-Top* (FT) intensity profile refers to a profile which is flat over most of the beam cross section contrary to the *Top-Hat* profile for which the intensity is ideally constant over all the beam cross section. Two families of technique can be used for getting a FT intensity profile. The first one consists to transform the usual Gaussian beam into a FT intensity profile by using a diffractive technique [2-7] or an interferometric technique based on the coaxial coherent superposition of two Gaussian beams having the same width and two opposite curvatures [8], and a focusing lens. The second technique called as intra-cavity beam shaping consists to insert some phase mask inside the laser cavity so that its fundamental mode takes the shape of a FT beam [9-10]. In both cases, the FT beam is not invariant in shape with propagation distance. This means that the intensity profile evolves as the beam is propagating, and the desired FT intensity profile only occurs in the vicinity of the focal plane of the focusing lens. In contrast, it could be interesting to generate a FT beam keeping its shape along propagation. Indeed, one can guess the usefulness of such invariant FT beams in many practical applications such as, for instance, material processing, nonlinear frequency conversion, material sputtering,… The generation of shape-invariant FT laser beams has been experimentally demonstrated by using two different techniques: (i) diffractive intra-cavity beam shaping with amplitude objects [11], and (ii) by optical feedback [12] which is basically in fact an interferometric technique.

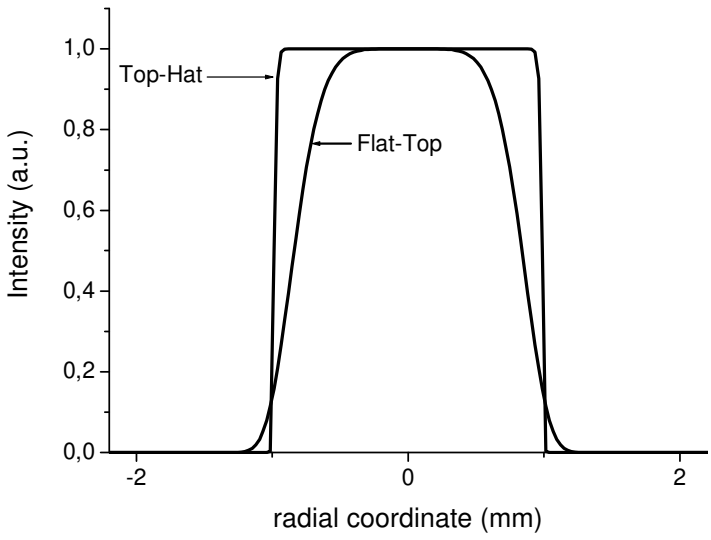

Figure 1: Flat-Top and Top-Hat intensity profiles.

In this paper we will visit again in Section 2 the technique used in REF. [11] for producing a shape-invariant flat-top beam by introducing inside a two-mirror laser cavity an absorbing ring on the plane mirror and a diaphragm on the concave mirror. The interpretation given in [11] was that the shape-invariant FT beam results from the incoherent superposition of a LG_{00} (Gaussian beam) and LG_{10} (one central peak surrounded by one ring of light) beams. This statement is unfortunately wrong because the incoherent superposition of LG_{00} and LG_{10} beams (addition of intensity profiles) does not give any FT profile whatever the power weighting of each beam. However, one has to remember from the experimental evidence in [11] that a plano-concave laser cavity including an absorbing ring and a diaphragm can sustain a laser oscillation taking the form of a shape-invariant FT beam. We will give in Section 2 an interpretation which we think corresponds to the right one, and will make a complete modelling of the laser cavity including amplitude masks in order to force the laser oscillation to take place simultaneously on the LG_{00} and LG_{01} Laguerre-Gauss eigenmodes. Achieving a remarkable experimental observation such as the generation of a shape-invariant FT laser beam [11] is useful, but a full understanding of the ins and outs of the issue is even better. This is what has motivated the study presented in this paper.

2. Generation of a shape-invariant Flat-Top laser beam

In the following, we will consider the coaxial incoherent superposition of LG_{00} (Gaussian) and LG_{01} (doughnut) Laguerre-Gauss eigenmodes. The intensity profiles I_{00} and I_{01} associated, respectively, with the LG_{00} and LG_{01} modes are expressed as follows

$$I_{00} = \frac{2P_{00}}{\pi W^2} \times \exp\left[-\frac{2\rho^2}{W^2} \right] \tag{1}$$

$$I_{01} = \frac{2P_{01}}{\pi W^2} \times \left(\frac{2\rho^2}{W^2} \right) \times \exp\left[-\frac{2\rho^2}{W^2} \right], \tag{2}$$

where P_{00} and P_{01} are the power carried by the LG_{00} and LG_{01} beams, respectively.

A remarkable intensity profile is obtained from this beam mixing when they have the same power ($P_{00} = P_{01}$) since a Flat-Top intensity profile is obtained as shown in Fig 2. Since the LG_{00} and LG_{01} beams are eigenmodes, namely they keep their shape as they propagate, it results that their weighted (50%-50% in power) incoherent mixing leads to a shape-invariant Flat-Top beam. It is seen in Fig. 2 that the FT beam results from the filling of the doughnut LG_{01} beam by the Gaussian LG_{00} beam. It is important to note that in Fig. 2 the plot of I_{01} is

not to scale. Indeed, the maximum intensity of the ring associated with a LG_{0l} beam of power P is given by [13]:

$$I_{\max} = I_0 l^l e^{-l} \qquad (3)$$

where $I_0 = 2P/(\pi W^2)$ is the on-axis intensity of the LG_{00} Gaussian beam carrying the power P. For l=1, one has $I_{\max} = 0.36 \times I_0$.

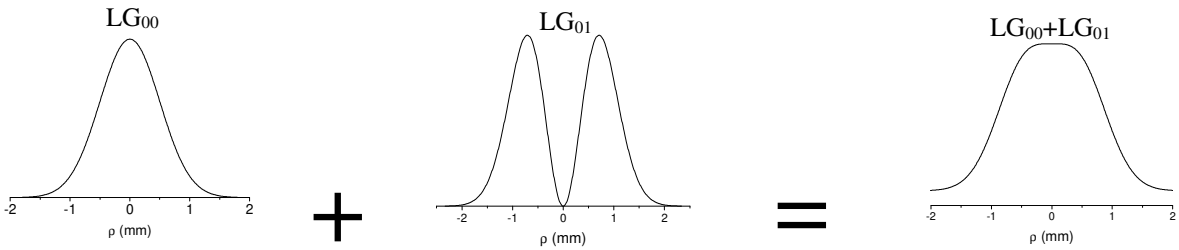

Figure 2: Generation of a flat-top intensity profile from the incoherent superposition of LG_{00} and LG_{01} eigenmodes.

It is important to note that the coherent coaxial mixing of Gaussian LG_{00} and doughnut LG_{0l} beams, correctly weighted, leads to a space-invariant flat-top beam only for l=1. Now, it remains to look at the possibilities to achieve the incoherent coaxial mixing of LG_{00} and LG_{01} beams. First, it is worth noting that the necessary incoherence between LG_{00} and LG_{01} beams, in order to add the intensities I_{00} and I_{01} given by Eqs. (1) and (2) without interference, is fulfilled when their frequencies are different. The authors of REF. [14] have experienced the generation of a FT intensity profile by using two Nd:YLF lasers with one operating in LG_{01} (doughnut) mode, and the other in LG_{00} (Gaussian) mode. By adjusting the power ratio of the two lasers, a FT intensity profile can be reached [14]. Here, in this paper, and elsewhere in the experimental generation of a shape-invariant Flat-Top laser beam [11,12] we consider a single solid-state laser forced to oscillate simultaneously in the LG_{00} (Gaussian) and LG_{01} (doughnut) Laguerre-Gauss eigenmodes. For doing that we have to make sure that their loss levels are equal. In other words, we have to insert inside the cavity some amplitude masks meeting the condition of equal losses in order that these two modes should oscillate

simultaneously as soon as the laser threshold is reached. Before proceeding, it is worthwhile to note the two following points:

(i) The transverse LG_{00} mode does not compete very much against LG_{01} mode in the amplifying medium because they extract energy from different regions of the active medium: central for the LG_{00} mode, and peripheral for the LG_{01} mode.

(ii) It is worthwhile to note that the mixing of transverse LG_{00} and LG_{01} modes of a given laser is necessarily incoherent since the different transverse modes oscillate on distinct frequencies.

The envisaged laser cavity allowing the simultaneous oscillation of LG_{00} and LG_{01} modes is illustrated in Fig. 3. The amplitude masks mentioned above are a circular aperture set against the concave mirror and a circular absorbing ring against the plane mirror.

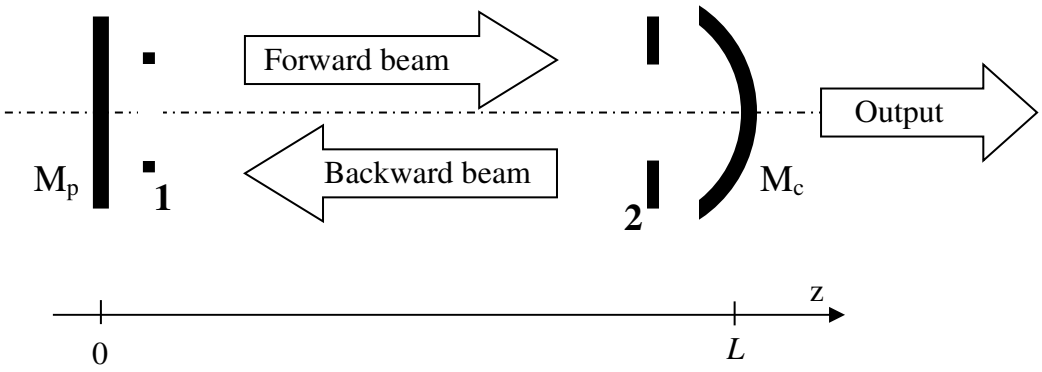

Figure 3: Scheme of the laser cavity of length L made up of a plane mirror M_p and a concave mirror M_c of radius of curvature noted R. The transverse mode selection is achieved by the combined effects of a circular aperture (2) against M_c and a circular absorbing ring (1) set on M_p.

We have to recognise that the ring and the diaphragm are lossy devices, and there might be a temptation to believe that the use of a phase mask should be preferable for selecting a high order Laguerre-Gauss mode as a fundamental mode. Before to proceed, we would like to bring a very widespread but false idea which states that the use of a phase mask in place of an amplitude mask is always more efficient because it is intrinsically transparent, and therefore less lossy. This point has been discussed in details elsewhere [15], and it has been

demonstrated that the use of an absorbing mask can be less lossy than a phase mask for the same transverse mode selection. It is important to note that the losses introduced by an absorbing ring can be reduced by its segmentation [16] while its ability to discriminate the desired transverse mode is maintained. In addition, it can be noted that a basic amplitude mask such as diaphragm or ring is probably more simple to fabricate, and less expensive than any phase mask.

Another possibility for selecting a high order LG mode without any intra-cavity mask is to reshape the laser beam pump when the pumping is longitudinal. This has been already achieved by pumping a solid-state laser with a light ring forcing the laser to oscillate on a LG_{0l} mode [17,18]. In our case this concept of transverse mode control is not applicable because an annular gain region does not sustains laser oscillation on a Gaussian beam which is absolutely essential for finally getting a shape-invariant FT laser beam. We can also note that simultaneous oscillation of LG_{00} and LG_{0l} modes has been recently reported [19] with a particular laser configuration. The latter is made up of a single laser crystal in the main cavity shared by two secondary cavities in which the desired transverse mode selection is made independently. However, authors of REF [19] have not considered the mixing of doughnut and Gaussian beams, and consequently did not observe any Flat-Top beam.

The geometry of the diaphragm and absorbing ring is given in Fig. 4.

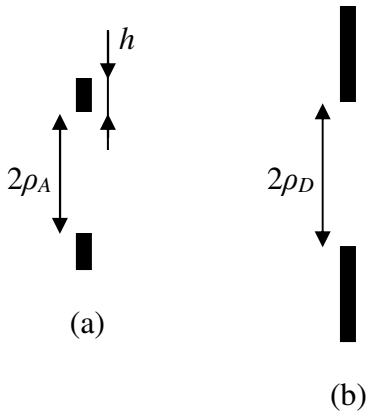

Figure 4: Geometry of (a): absorbing ring, and (b): circular aperture

For convenience, we will use the normalised radius $Y_D = \rho_D / W_c$ for the diaphragm (circular aperture), and $Y_A = \rho_A / W_p$ for the absorbing ring; where W_p (W_c) is the beam radius of the

fundamental Gaussian beam on the plane (concave) mirror of the bare cavity, i.e. without the amplitude masks. The widths W_p and W_c are given by

$$W_p^2 = \left(\frac{\lambda L}{\pi}\right)\sqrt{\frac{g}{1-g}} \quad \text{and} \quad W_c^2 = \left(\frac{\lambda L}{\pi}\right)\sqrt{\frac{1}{g(1-g)}}, \tag{4}$$

where $g = (1 - L/R)$ is the stability parameter of the resonator. For convenience we will note $\rho_B = (\rho_A + h)$ and $Y_B = \rho_B / W_p$, where h is the width of the absorbing ring.

The eigenmodes of the bare cavity (without the masks) are assumed to be the Laguerre-Gaussian LG_{pl} modes characterised by a width W_{pl} increasing with the mode orders p and l [20]

$$W_{pl} = W\sqrt{2p + m + 1} \tag{5}$$

where W represent the width of the Gaussian LG_{00} mode. The mechanism of transverse mode discrimination is based on the losses resulting from the beam truncation achieved by the ring and diaphragm. As it will be shown below, the behaviour of the ring and diaphragm in term of resulting losses is very different. It is important to recall that when selecting the fundamental Gaussian beam of a cavity using one or two diaphragms, it results a perturbation by diffraction so that it is no longer Gaussian in shape. This has been theoretically [21-23] and experimentally [24] demonstrated. Consequently, the study of the transverse mode selection using some amplitude masks inside a cavity needs the determination of the resonant field using for instance the Fox and Li method [25], or a modal decomposition method [26]. Such methods are necessary but relatively numerically cumbersome so as at first sight we can start by considering the single pass properties of the diaphragm or absorbing ring when they are enlighten by a pure LG_{pl} beam. This approach will be followed hereafter.

2.1 Single pass transmission of the ring and diaphragm

By analogy with the case of a Gaussian beam incident on some amplitude mask [27] we can expect that diffraction of a LG_{pl} beam upon a diaphragm or an absorbing ring will affect the beam characteristics which are:

(i) the longitudinal intensity distribution;

(ii) the transversal intensity distribution in the near-field and far-field regions;

(iii) the far-field angular divergence;

(iv) the beam quality factor M^2.

In this paper, we will be mainly interested in the dependence of the *single pass* transmission of a diaphragm and absorbing ring upon their geometry characteristics, and LG_{pl} mode orders. This will give a first glance about the selection issue of LG_{00} and LG_{01} modes having almost the same loss level. The calculation of the single pass transmission of the diaphragm and absorbing ring is made using a power ratio between the transmitted and incident powers. The relation between losses L and transmission T is assumed to be given by the equation $L=1-T$.

Role of the diaphragm: The variations of the diaphragm transmission, as a function of Y_D, its normalised size, is shown in Fig. 5. On can deduce that the losses due to the diaphragm increases with the mode orders p and l.

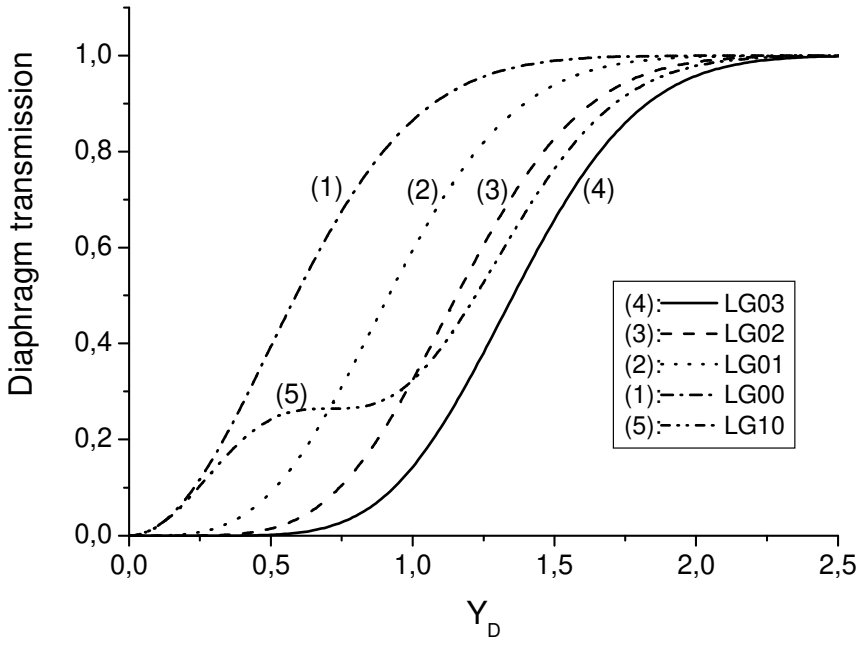

Figure 5: Variations of the single pass transmission for several LG_{pl} beams through a diaphragm of normalised radius Y_D.

Consequently, one can reasonably expect that for a diaphragmed laser cavity, by an adequate choice of the diaphragm diameter, it could be possible for LG_{00} and LG_{01} modes to reach the laser threshold unfortunately with unequal losses. We will see below that it is possible to balance the losses of LG_{00} and LG_{01} modes by a judicious choice of the ring size.

Role of the absorbing ring:

It is clear from Fig. 5 that the losses, due to the diaphragm, are greater for the LG_{01} beam than for the LG_{00} beam for a given diaphragm. The ring transmission of the LG_{00} and LG_{01} beams is shown in Fig. 6. as a function of Y_A its normalised radius. Consequently, depending upon the value of Y_A with respect to the crossing point it is possible to make the losses of the LG_{01} beam greater or smaller than the losses of the LG_{00} beam. This is the basic idea for balancing the losses of the LG_{00} and LG_{01} modes of the cavity shown in Fig. 3.

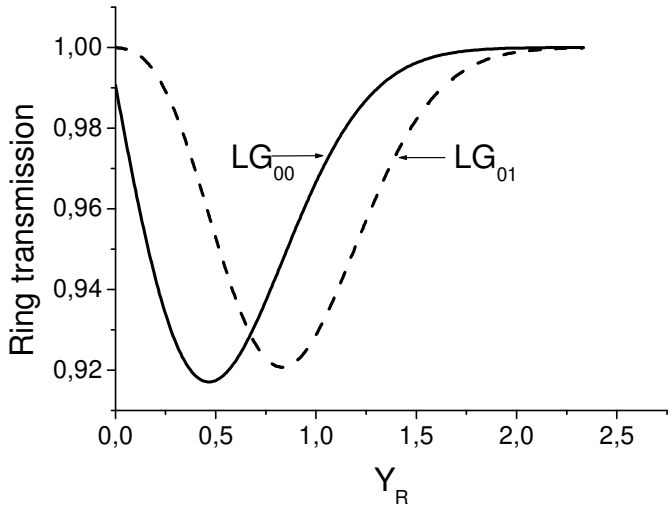

Figure 6: Variations of the single pass transmission for LG_{00} and LG_{01} beams through an absorbing ring versus the normalised radius Y_A.

2.2 Cavity with a ring and a diaphragm

The determination of the resonant field of the cavity shown in Fig. 3 including an absorbing ring and a circular aperture involves a decomposition into two progressive components: a *forward beam* propagating in the z>0 direction, and a *backward beam* in the opposite direction. Hereafter, the subscripts f and b refers to forward and backward quantities, respectively. We should note in passing that these two progressive beams should be characterised by two different transverse field distributions due to the diffraction on the ring and aperture. This has been experimentally demonstrated in the case of a laser cavity including a single diaphragm [24]. The numerical calculation of the resonant field in the resonator including the amplitude masks (ring and diaphragm) is based on its expansion on

the basis of the eigenfunctions of the bare cavity consisting solely of the two mirrors. This method has been initially perfected for the cavity assumed to be axially symmetric and including one diaphragm [28]. Later, this method has been applied to a cavity including: (i) two circular apertures [27], (ii) a binary phase object and a diaphragm, and (iii) an absorbing ring and a diaphragm [26]. However, in these works [22, 26, 28] the resonant field was decomposed on the set of the radial Laguerre-Gauss eigenmodes ($p \neq 0, l = 0$) owing the assumed cylindrical symmetry. Here, in this paper we will made up the field decomposition upon the Laguerre-Gauss eigenmodes ($p \neq 0, l \neq 0$) in order to consider the possibility of oscillation over any high-order LG_{pl} transverse modes.

The orthormalised basis is formed by the Laguerre-Gauss functions $G_{fpm}(\rho, z)$, for the forward wave, and $G_{bpm}(\rho, z) = G^{*}_{fpm}(\rho, z)$:

$$G_{fpm}(\rho,z) = \sqrt{\frac{2}{\pi}} \times \sqrt{\frac{p!}{(p+m)!}} \times \frac{1}{W(z)} \times \exp\left\{ i\left[\frac{k\rho^2}{2R_c(z)} - (2p+m+1)\theta(z) \right] \right\}$$
$$\times \exp(-X/2) \times X^{m/2} \times L_p^m(X) \qquad (6)$$

where $k = 2\pi/\lambda$ and L_p^m is an associated Laguerre polynomial [30], m and p are the azimuthal and radial mode numbers, respectively. The Gaussian mode of the bare cavity is characterised by its beam diameter $2W(z)$, and its radius of curvature R_c at point z. These quantities as well as the Gouy phase shift θ are z dependent and are expressed as

$$W^2(z) = W_p^2\left[1 + (z/z_R)^2\right] \qquad (7)$$

$$R_c(z) = z\left[1 + (z_R/z)^2\right] \qquad (8)$$

$$\theta(z) = \arctan(z/z_R) \qquad (9)$$

where $z_R = \pi W_p^2/\lambda$ is the Rayleigh range.

The forward and backward fields are assumed to be linearly polarised and are expressed as linear combination of the basis functions:

$$E_f(\rho,z,\varphi) = \exp\{i[kz - \omega t]\}\sum_{p,m} f_{pm} G_{fpm}(\rho,z)\exp(im\varphi) \qquad (10)$$

$$E_f(\rho,z,\varphi) = \exp\{i[k(2L - z) - \omega t]\}\sum_{p,m} b_{pm} G_{bpm}(\rho,z)\exp(im\varphi)$$

We are considering the stationary field for t=0 and then $\exp(-i\omega t)=1$. The knowledge of the resonant fields components E_f and E_b requires the determination of the coefficients f_{pm} and b_{pm} which are related to each other by the boundary conditions imposed by the mirrors, the absorbing ring and the diaphragm. The boundary conditions write as follows:

At the plane mirror of reflectance (field ratio) r_1

$$E_f(\rho,0,\varphi)=\begin{cases}-r_1E_b(\rho,0,\varphi) & for\ \rho<\rho_A \text{ and } \rho>\rho_B\\ 0 & for\ \rho_A<\rho<\rho_B\end{cases} \tag{11}$$

and at the concave mirror of reflectance (field ratio) r_2

$$E_b(\rho,L,\varphi)=\begin{cases}-r_2E_f(\rho,L,\varphi) & for\ \rho<\rho_D\\ 0 & for\ \rho>\rho_D\end{cases} \tag{12}$$

By a calculation easily adapted from Refs. [22,31,32], one can express the change of the forward coefficients after a round trip inside the resonator by a relation involving a matrix $\boldsymbol{M}$ called as *round-trip operator* [22,31,32] which typical element is written as:

$$M_{rpsm}=r_1r_2\exp[2ikL]\sum_{n,q}A_{pqmn}.B_{rpsm}\exp[-i(2p+2q+m+n+2)\theta(L)] \tag{13}$$

where

$$A_{pqmn}=\int_0^{\pi}\int_0^{2Y_D^2}C_{pqmn}.\cos(m-n)\varphi.dX.d\varphi \tag{14}$$

$$B_{rpsm}=\int_0^{\pi}\cos[(m-s)\varphi]\left[\int_0^{2Y_A^2}D_{rpom}dX+\int_{2Y_B^2}^{\infty}D_{rpom}dX\right]d\varphi \tag{15}$$

$$D_{rpsm}=\frac{1}{2\pi}\left[\frac{p!r!}{(p+m)!(r+s)!}\right]^{1/2}\times\exp(-X)\times X^{(m+s)/2}\times L_p^m(X)\times L_r^s(X) \tag{16}$$

The integrals in the above equations are numerically calculated by using a FORTRAN routine based on a numerical integrator from the International Mathematics and Statistics Library (IMSL). The round-trip operator is a complex matrix of sizes $r\times p\times s\times n$ and its complex eigenvectors represent the eigenmodes of the cavity including a diaphragm on the concave mirror and an absorbing ring on the plane mirror. Associated Laguerre-Gauss polynomials are used up to the orders $m_{\max}=s_{\max}=10$ and $p_{\max}=r_{\max}=80$. Each of eigenvectors $\boldsymbol{u}$ of the round-trip operator is characterised by a complex eigenvalue Γ such that $\boldsymbol{Mu}=\Gamma\boldsymbol{u}$. Determination of eigenvectors of $\boldsymbol{M}$ is done numerically by using a FORTRAN routine based

on IMSL subroutines. The eigenvector of $\boldsymbol{M}$ having the largest eigenvalue $|\Gamma_{00}|$, calculated for an azimuthal order fixed to zero, corresponds to the fundamental mode TEM_{00} whose power losses per round-trip is given by

$$L_{00} = 1 - |\Gamma_{00}|^2 \tag{17}$$

The eigenvector of $\boldsymbol{M}$ having the largest eigenvalue $|\Gamma_{01}|$ calculated for an azimuthal order fixed to unity corresponds to the TEM_{01} whose power losses per round-trip is given by

$$L_{01} = 1 - |\Gamma_{01}|^2 \tag{18}$$

The aim of the calculation is to find the geometrical parameter Y_A associated to the ring for which L_{00} and L_{01} are equal or very close. An important point which has been raised in Section 2.1 is that the role of the ring is to rebalance the losses L_{00} and L_{01} since the diaphragm has the tendency to make L_{01} larger than L_{00}. The other geometrical parameters of the cavity (Y_D, L, R and h) are held fixed when varying the ring radius ρ_A, i.e. Y_A.

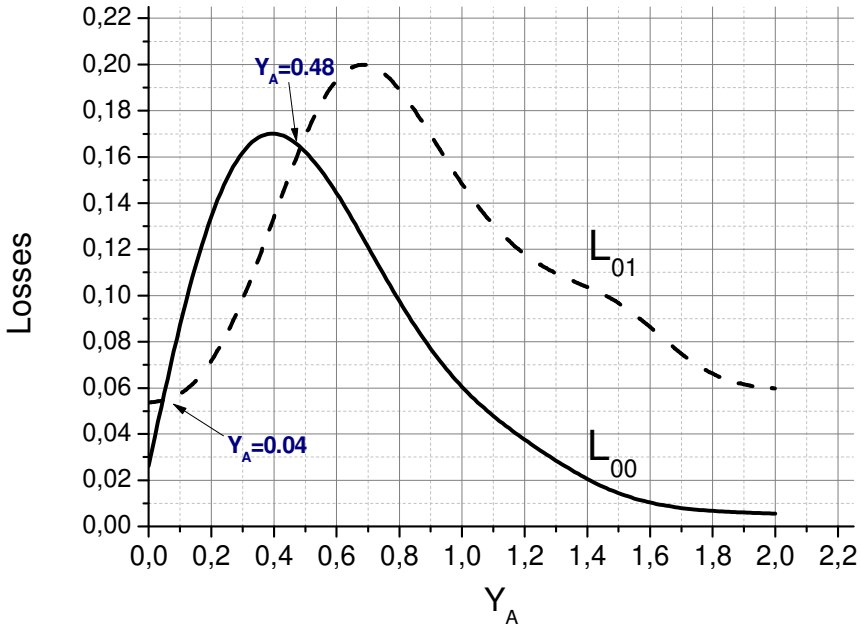

Figure 7: Variations of losses L_{00} and L_{01} versus parameter Y_A for L=252mm, g=0.5, $Y_D = 1.6$, h=20µm. $W_p = 292\mu m$, $W_c = 649\mu m$.

It is seen in Fig. 7 that the ring behaviour depicted in Fig. 6 in a single-pass situation is also present inside a resonant cavity. We find that there are two values of Y_A (0.04 and 0.48) for which the losses L_{00} and L_{01} are equal. For $Y_A = 0.48$ the losses are four times the losses for $Y_A = 0.04$. These numbers mean that the power of the beam inside the laser cavity reduces, due to diffraction, by about 5% (16.5%) for $Y_A = 0.04$ ($Y_A = 0.48$) after a round-trip in the resonator. One can conclude that while we use amplitude mask (ring and diaphragm) the diffraction losses are not necessarily large. However, our study involves a bare cavity, i.e. without gain medium, which could put into question the above findings obtained on the assumption that the nonlinear properties (saturation for instance) of the gain medium are ignored. In fact, the resonant field of the "cold" cavity (without gain medium) including amplitude masks is not significantly different from the resonant experimental field probed in a "hot" cavity, i.e. including a gain medium [24]. However, one particular property of a "hot" cavity cannot be ignored. This property concerns the induced thermal effect by the pump absorption in the gain medium. At least, the resulting thermal lensing effect will change the operating point of the cavity so that the beam widths given by Eq. (4) will drift causing a change in Y_A and Y_D. In return, this could break the weighting of LG_{00} and LG_{10} beams power leading to the loss of the targered beam, i.e. the shape-invariant Flat-Top beam. In fact, this "catastrophic scenario" is going a bit too far since experimentally [11] the shape-invariant Flat-Top laser beam was observed to be very stable at different level of power pump. However, in case of important thermal effects in a laser cavity, it is possible to stabilise the cavity against thermal perturbations [33-37].

Let us now come back to the experimental observations [11] which demonstrate that insertion of an absorbing ring together with a circular aperture arranged similarly to Fig. 3 inside a laser cavity can force the oscillation to take the form of a shape-invariant FT beam. The interpretation given in [11] was that the shape-invariant FT beam should result from the incoherent superposition of a LG_{00} (Gaussian beam) and LG_{10} (one central peak surrounded by one ring of light) beams. This statement is wrong because the incoherent superposition of LG_{00} and LG_{10} beams (addition of intensity profiles) does not give any FT profile whatever the weighting of each beam power. In fact, the study presented in this paper shows that the correct interpretation of the shape-invariant FT beam observed experimentally in [11] results rather from an incoherent superposition of LG_{00} (Gaussian mode) and LG_{01} (doughnut mode) beams. It is worth noting that the two oscillating modes are here necessarily incoherent because their oscillation frequencies are different since depending on the mode orders [38].

3. Conclusions

We have considered the generation of a shape-invariant flat-top (FT) laser beam. Such focused laser beams are very interesting for most of laser applications since the FT intensity profile remains unchanged on both sides of the focal plane. It is worth noting that this contrasts with flat-top beams obtained by diffractive beam shaping since in this case the intensity profile evolves on both sides of the focal plane. This property of shape invariance in a FT beam can be very useful in many practical applications such as, for instance, material processing, nonlinear frequency conversion, material sputtering,…

Finally, we have modelled the laser cavity able to sustain the simultaneous oscillation of the LG_{00} and LG_{01} modes. For that, we have considered a plano-concave cavity including two amplitude masks: an absorbing ring set on the plane mirror, and a circular aperture on the concave mirror. By optimising the size of these two masks it is possible to set equal the losses associated with LG_{00} and LG_{01} modes, and consequently to allow the simultaneous oscillation of the two modes and consequently to get a shape-invariant flat-top (FT) inside and outside the laser cavity. These theoretical results give a correct interpretation of some experimental observations previously done [11] but with a misinterpretation.

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