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Formal Models for Safety and Performance Analysis of a Data Center's System

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Abstract

Data centers have evolved dramatically to meet the new technologies demands such as cloud computing, e-commerce and social networks. Ensuring a continuous service by avoiding downtime is becoming a competitive factor for companies. Several factors affect the availability of a data center system, among them the power energy production and temperature variations within the data center room due to the cooling system which is responsible of extracting the heat from the IT room. In this paper we investigate the different interactions between these sub-systems and their impact on the Data Center reliability. We use Production Trees, a new modeling methodology for dealing with availability issues of production systems. Our results illustrate the applicability of the proposed approach and include an analysis of the system availability, reliability and performance.

Key words: Data center, Performance, Reliability, Availability, Production Trees.

1. Introduction

A Data Center (DC) is a building which purpose is to house electronic equipments (telecommunication devices). The interconnection between these equipments make up a network system which is responsible for providing various internet and cloud services such as e-commerce, storage backup, video streaming and high-performance computing. Ensuring a continuous service with high performance (in terms of throughput and delay) and avoiding downtime is becoming a competitive factor for companies. The reduction of downtime is very important and thus must be well planned.

There are many reasons for a DC downtime. The power interruption and hardware failures are the major causes. Power interruption is caused by the electrical power system inadequacy to provide sufficient energy to the telecommunication devices due to a failure of one or multiple components of the electrical system. The

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hardware failures come in many forms and can be attributed to any number of causes such as component quality issues, human intervention or the temperature variation within the DC room. Usually, operating at temperatures higher than typical working conditions can have a negative impact on the reliability of electronics components. Indeed when the temperature rises above the recommended 20 to 22 *Celsius degrees* range, the hardware inside IT components may fail more frequently [1]. Therefore the cooling system of a DC impacts the availability of the services provided by the network system.

In order to analyze reliability and availability of a DC's system, several techniques are usually used. They are classified in two categories: Boolean formalisms and States/Transitions formalisms [2]. Boolean techniques look at the system components, critical events, and system characteristics. These techniques include Fault Tree Analysis (FTA), and Reliability Block Diagrams (RBD). They have convenient graphical representations which is important for industrial models. However, they take into account neither the functional dependencies, nor temporal dependencies between events occurrences. Consequently, it is not possible to take into account the order in which events occur and events can occur any time, no matter the current state of the system. This problem is partially solved using Dynamic Fault Trees (DFT). However, the semantic of this approach is not always clear. Moreover, currently there are no effective resolution techniques. The usual techniques for resolving FTA being ineffective on DFT, in general, they are automatically converted to Markov Chain (MC) before being solved using standard resolution techniques. Another approach is to map the DFT into a high level language (I/O Interactive Markov chain, PEPA model, ...). In all these cases, we find ourselves having to address the underlying MC. Also this technique does not allow the production availability estimation.

States/Transitions techniques are able to represent dependencies between failures. Many techniques have been proposed in the literature such as Markov chains and Generalized Stochastic Petri Nets (GSPN) [3]. These techniques are very used to represent dynamic models. They have a convenient graphical representation but this representation becomes unreadable for large scale models and it is difficult to represent propagation of flows. Another major limitation is the state space explosion when dealing with large systems.

First of all, in order to find the appropriate analysis method, it is important to study the DC system characteristics. As a system of systems, one of the DC system characteristics is dependency. A DC is a complex system with dependence relationships. Each component's failure in a DC sub-system (electrical, thermal and network) can affect other components of the sub-system or the whole sub-system. Moreover, there are dependencies between the different sub-systems of the DC with a possible effect on the whole system. Additionally, a DC is a reconfigurable system. It changes state dynamically due to redundant components. Finally, to satisfy load demands in the system, it is necessary to generate sufficient power energy and cooled

air, and transport it to the load points (IT components), taking into account the maximum capacity of each component in the system (production capacity problem). Furthermore, it is necessary to offer a sufficient network capacity (bandwidth) taking into account the maximum capacity of each component in the network.

Since a DC system must be constantly provided with energy and kept in a acceptable temperature, its reliability analysis depends on the electrical sub-system which is responsible for providing power energy, and the thermal sub-system which is responsible for extracting heat. In addition, the system reliability depends not only on the internal state of its components, but also on the packets production capacity of the network sub-system components.

A new modeling methodology called Production Trees (PT) was proposed in [4]. This formalism allows modeling the relationships between the components of a system with a particular attention to the flows circulating between these components. PTs look like Fault Trees (FT) with nodes that represent components and gates that represent behaviors. A capacity flow moving from a source to a target component is also represented to provide a sound semantics to classical FT. Therefore, according to the DC system characteristics, PT models with their basic components and gates are convenient to handle component failures, their propagation and the different dependencies between these components. Moreover, the flows circulating in a PT allow not only dealing with the production capacity problem, but also making statistics to analyze the system performance. Thus, the PT technique is very suitable for reliability and performance analysis.

We have applied the PT technique to analyze the reliability of DC's thermal sub-system in [5], and the network one in [6]. In both studies, we focused only on how the electrical system production impacts the system under study. Moreover, this impact is static, that is, the system under study fails, when its demand, in terms of power energy, exceeds the electrical system production capacity announced after assessing it. In this paper, unlike in our previous works, we analyze the electrical sub-system, and we investigate a combined study of reliability and performance of the DC system. Moreover, instead of considering interactions between sub-systems as static, we consider them as dynamic, that is, these sub-systems have an impact on each others dynamically. So, when a sub-system depends on the electrical one, and its demand exceeds the electrical sub-system capacity, the former may not fail, since the latter may modify its production capacity by producing more energy to satisfy the demand. When the demand is less than the production capacity, this production can be reduced in order to optimize the energy consumption. Finally, we assess the different performance indices to analyze the impact of component failures on the network sub-system performances.

This paper is structured as follows. In Section 2, we discuss related works. Section 3 presents the system we are interested in. Section 4 presents an overview of Production Trees. Section 5 is dedicated to modeling

the DC system using Production Trees methodology. In Section 6 we analyze the obtained model. Section 7 summarizes the obtained results. Finally, Section 8 concludes this article.

2. Related work

In the last few years, some works have been developed to perform DC availability analysis.

In the context of electrical power systems, [7] proposes the Dynamic Reliability Block Diagram (DRBD) model, an extension of RBD with new blocks, which supports the reliability analysis of systems with dependence relationships. These additional blocks for modeling dependencies made the DRBD model complex. The DRBD model is automatically converted to a colored Petri net (CPN) model in order to perform behavior properties analysis, which may certify the correctness of the model [8]. But the most serious problem is that it does not take into account the maximum power capacity to evaluate adequacy. One of the methods to solve the maximum capacity problems is the application of graph theory in reliability (network flow reliability). Several works have been proposed in this context, but it concerns in general the transport problems. The most related work that concerns the system safety is in [9]. The basic disadvantage of their approach is that it generates a large number of flow combinations, that is, all possible flows that satisfy the demands for all specified components simultaneously. The method thus becomes large even for the small sized network. Wei [10] combines the advances of both RBD and GSPN for quantifying availability and reliability of Virtual Data Center (VDC). Data center cooling architectures are not the focus of this work and the proposed models are specific for modeling VDC. A methodology which combines the advantages of both Stochastic Petri nets (SPN) and RBD to assess dependability in power systems is adopted in [11]. This technique is event based, and describes interactions between components, but it is impossible to take into account the order in which events occur any time. Considering RBDs as a strong mathematical modeling techniques, Failure Modes, effects, and Critically analysis (FMECA) is employed with RBD to evaluate the reliability of DC's electrical power system and provide high system availability [12]. But it is difficult to use this technique since the failure rates are particularly difficult to estimate when human performance is involved. An approach to calculate the reliability of different topologies using SPN is presented in [13]. Continuous-time Markov chain (CTMC) models are adopted to compute the availability of DC's electrical topology in [14]. Finally a tool approach to estimate reliability and availability of DC's power system called Mercury is presented in [15]. This tool supports RBD, SPN, CTMC and Energy Flow Model (EFM). The EFM verifies the energy flow model on the electrical power system, taking into account the power capacity that each component can provide. However, in general, all techniques based on Petri Nets may partly be categorized as simulation,

since simulation methods often are necessary to solve these models.

In the context of thermal systems, Chang [16] proposes a method for estimating the exergy consumption during the material extraction, manufacturing, operational, transport and disposal phases. Many assumptions are adopted to take into account the entire device life-cycle, which may turn the results far from reality. In [17], an analysis is carried out to ensure availability by providing adequate cooling resources to match the heat load. This work did not cover the impact of cooling component failures on the availability of the IT room. Callou [18] proposes a set of models to the quantification of dependability of DC cooling system. Although the cooling system architectures are analyzed, DC room temperature was not discussed.

In the context of DC networks, Queueing Network (QN) theory has been widely used to model communication network for performance analysis. Yang et al [19] considered a finite capacity M/M/R queue with second optional channel. Using the matrix - geometric method, they obtained the steady-state probability distributions and various system performance measures. Sharma and Virtamo [20] considered a queue with finite buffer where the buffer size limits the amount of work that can be stored in the queue. They proposed algorithms to compute the stationary density of the workload process, the waiting times and the packet loss probability. In [20], the authors investigate the blocking probability of QN with a finite buffer and a Markovian arrival process.

Finally, some works have been carried out in the context of both electrical and thermal systems. Patterson [21] evaluates the impact of the temperature on energy efficiency and suggests the correct temperature for DC operation. However, the author was not concerned with the availability within the DC environment. In [22], authors present an approach to calculate the reliability of different topologies and to compare them using SPN. However, the authors did not focus on the dependency between thermal and electrical systems.

In our knowledge, no analysis approach (integrating reliability, performance and interactions between the DC sub-systems) exists in the literature. Production Trees are a simple technique that describes the interactions between components of the system taking into account the maximum capacity flow. It is useful to represent and analyze dynamic models.

3. Data Center system

The proper functioning of a DC is based on the continuity of the services provided by the equipments of the network sub-system. In order to ensure a constant service, these equipments must be provided with a sufficient and continuous power energy, and kept in a constant and acceptable temperature. The electrical sub-system provides energy to both the network and the cooling sub-systems. Thus the network sub-system

depends on both the electrical sub-system and the cooling sub-system, which itself depends on the electrical sub-system to operate properly (see Figure 1).

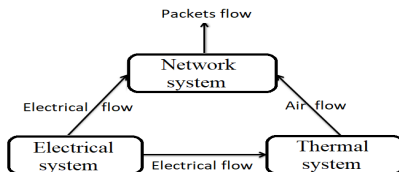


Figure 1: Dependencies between sub-systems of a DC system

In the following section, only the most important entities, that is the servers, are considered as the whole network sub-system. This one will be detailed in the dedicated section.

3.1. The Electrical sub-system

In this paper, we consider the topology illustrated in Figure 2 which combines an electrical system and a thermal system. These are typical thermal and electrical systems of a real DC. The network sub-system is represented by twelve (12) servers to deliver service to users. This topology consists of four layers: *production layer*, *transformation layer*, *storage layer* and *distribution layer*

In a normal operating mode, the servers are powered by two paths *A* and *B*. Each path is supplied by two different power sources PS_1 and PS_2 . However, if one of these power supplies fails, the power is supplied by a backup power generator (PG). Thus, initially, the generator is on standby and is only brought online after PS_1 or PS_2 becomes unavailable. Power sources provide a medium voltage, typically less than 50 kV and they represent the *production layer*. This voltage is used for distribution to two transformers Tr_A and Tr_B , one on each flow path [23]. Transformers are used to decrease the voltage of electricity (*transformation layer*). Then, from the *transformation layer*, the power enters the building (*storage layer*) with low-voltage lines going to FDP_A and FDP_B , the front low-voltage master distribution panels, to supply two Uninterruptible Power Supply (UPS) systems per path noted UPS_{iA} and UPS_{iB} , $i = 1, 2$.

Typically, an UPS combines three functions. First, it contains a transfer switch or converter ($Conv$) which chooses the active power input (either power source or power generator). After a power source failure, the transfer switch senses when the generator has started and is ready to provide power. Typically, a generator takes 10 to 15 *secs* to start and complete the full rated load [23]. Second, the UPS contains some form of energy storage (battery) Bat to bridge the time between the utility failure and the availability of power generator. Third, the UPS conditions the incoming power feed, removing voltage spikes in the alternating current. This conditioning is accomplished by the rectifier (Rec), a component included in the UPS system.

The output flow from each UPS system is finally routed to the *distribution layer* which contains a back low-voltage master distribution panel installed in the data center floor. We note BDP_{iX} the i^{th} panel on flow path $X = A, B$ and $i = 1, 2$. Then, both $BDPs$ on a path $X = A$ (respectively $X = B$) are connected to four (respectively two) Transfer Units (TU). These units are responsible of transferring the load through a Load Transfer Module (LTM) to a Distribution Table (DT). Finally, each two transfer units are connected to a Power Distribution Unit (PDU). This unit is like the breaker panel in residential house but can also incorporate transformers for final voltage adjustments. It takes a larger input feed and break it up into many smaller circuits that distribute power to the servers. A typical PDU handles 75 to 225 kW of load [23]. PDUs are the last components in the *distribution layer* to route the power to the servers or the load points. Each PDU provides the electrical flow to 4 servers, Ser_j , $j = 1, \dots, 12$, grouped in 4 racks (4 servers per rack).

3.2. The Thermal sub-system

The second part of the system is the cooling sub-system. It is designed to extract the maximum amount of heat produced by the servers. If the cooling system is unable to extract the generated heat, the temperature will increase and impact on the servers availability.

Basically, the cooling sub-system consists of Computer Room Air Conditioning (CRAC) units, chillers and Cooling Towers (CT) [24]. Note that the cooling sub-system may consume around 40% of the total power consumption of the DC [1].

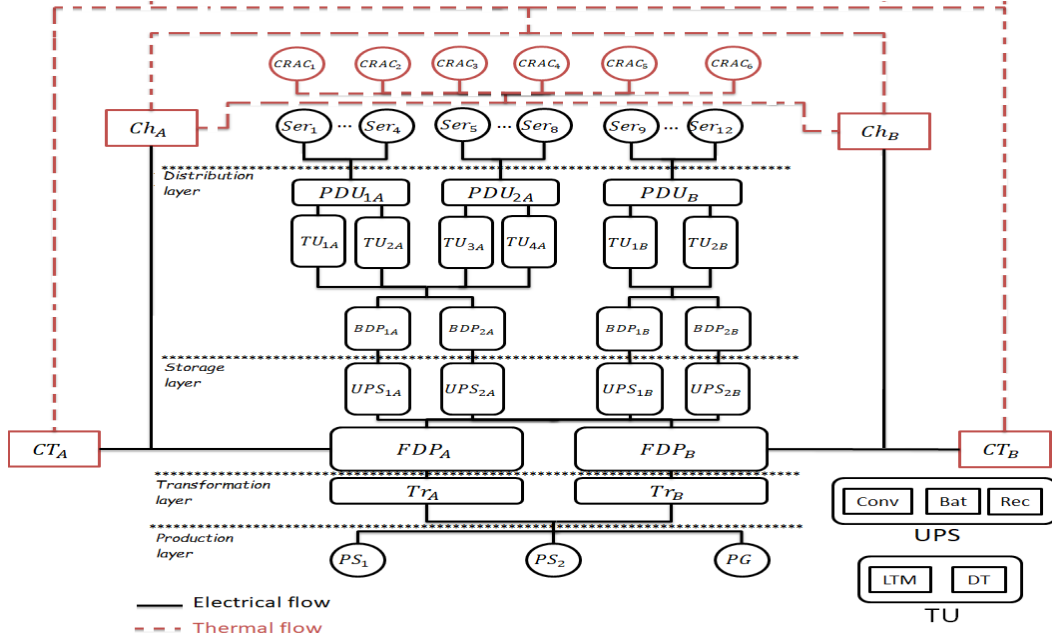


Figure 2: A typical thermal and power sub-system of a data center

The system considered in this paper consists of ten components: six CRAC units, two chillers and two CTs. These components are distributed in three layers: *production layer*, *cooling layer* and *extraction layer*. The two redundant cooling towers CT_A and CT_B in the *production layer* drive water from a source S . Each one contains pumps and needs to be powered to get the water from source S . The power energy is provided by FDP_A and FDP_B , respectively (see Figure 2). The pumped water is routed to the *cooling layer* containing two chillers Ch_A and Ch_B . The main role of a chiller is to cool the water as long as it is powered by the electrical sub-system. Chillers Ch_A and Ch_B are powered by PDU_A and PDU_B , respectively. Once the water is chilled, it is delivered to the *extraction layer* which contains CRAC units. Each CRAC unit extracts the air from the chilled water on condition that it is powered by at least one BDP. Finally, the CRAC units provide the cooled air to the servers. In this scenario each CRAC unit provides air to a rack containing four servers and the cooling system is considered to be operational if at least one of the two CRAC units $CRAC_i$, $i = 1, \dots, 6$ associated with each rack is working, the other one being in a standby mode (see Figure 3).

Electrical equipments produce heat (known in the literature as *Joule heating*), which must be removed to prevent the equipments temperature from rising to an unacceptable level. Heat is energy and the common measure of heat output rate for equipment is *Joule/second* (1 Joule/sec equals 1 Watt). In our case the power consumed by the servers is essentially all converted to heat. Consequently, the thermal output of servers is simply equal to their power consumption (in *Watts*) [24].

Moreover, the electronic components are usually designed to function only within specified temperature ranges and the temperature variation is a common cause of components failure. Once the specified temperature range is exceeded, failures rates increase significantly. In fact, the failure rate of a semiconductor device doubles for every 10 °C increase [24]. Therefore, the temperature variation has a considerable effect on servers availability. If a CRAC unit or any other component of the cooling system fails, then the cooling system will not be able to extract the amount of heat from the DC room it is supposed to extract leading to the possible rise of the temperature.

3.3. The Network sub-system

We have considered the servers as the whole network sub-system. In this section, the network sub-system components are detailed. We consider the fat-tree network [25] illustrated in Figure 4. The network has four layers: the lowest layer (*layer4*) contains 80 servers Ser_j , $1 \leq j \leq 80$, distributed in four Racks $Rack_i$, $1 \leq i \leq 4$ (each rack contains 20 servers). The layer above (*layer3*) contains 4 switches ToR_i , $1 \leq i \leq 4$, each one is connected to a rack $Rack_i$ in *layer4*. The switches are connected to two aggregation switches $AggS_A$ and $AggS_B$ (*layer2*), for redundancy. The aggregated traffic is then forwarded to the top layer (*layer1*) which contains two access routers $AccR_A$ and $AccR_B$. These route the traffic to Core routers $Core_A$ and $Core_B$ which are connected to the external network (internet).

The function of each component is to route the traffic and thus has a certain communication capacity. In this paper, the component treatment capacity, known as the throughput, is considered as the component capacity in both upload and download links.

In order, for a component, to ensure its function and route the traffic to other network components, it has to be powered by the electrical system. A component is initially *idle* and becomes *active* when it receives a request. This request can be seen as a demand coming from an internet user for a service proposed by the servers of the DC. The energy consumption of each component varies according to this demand. An *active* component consumes more energy than an *idle* one, thus, the network system impacts the electrical system energy production according to the received demands. A power consumption estimate P_i of any *active* component i can be calculated if the maximum power consumption of this component (P_{max}) and its consumption when it is *idle* (P_{idle}) are known:

$$P_i = (P_{max} - P_{idle}) * U_i + P_{idle} \quad (1)$$

U_i is the component utilization rate. For example, if a switch consumes 400W when it is *active* and 200W when it is *idle*, then at 25% of utilization, it consumes approximately $P_{switch} = 250W$.

The servers and *ToR* switches are powered by the electrical component *PDU* installed in each rack, while the other switches and routers are powered by at least one *BDP* installed in the DC room.

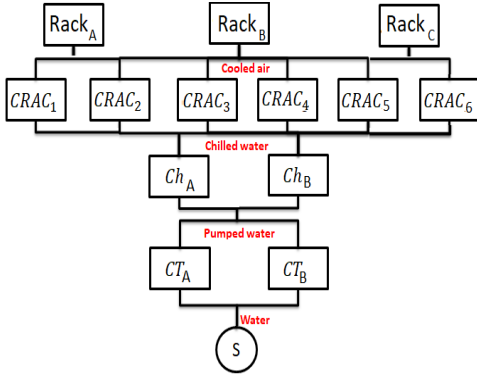


Figure 3: A typical thermal sub-system of a DC

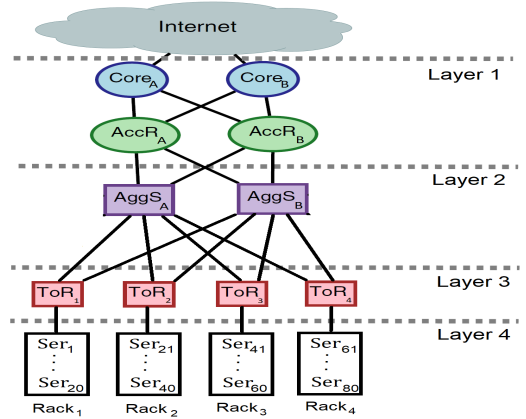


Figure 4: A conventional DC network sub-system

4. Production Trees

Production Trees (PT) are a new modeling methodology for production availability analysis [4]. They allow modeling the relationship between basic components of a system with a particular attention to the flows circulating between these components.

Production Trees provide two types of components to model a production system: basic components and gates. Basic components represent the production or treatment units of the system whereas the gates model the interactions between these units and thus the behavior of the whole system. Basic components are similar to basic events in a Fault Tree (FT). However, unlike the gates of FT, the gates of PT are not logical. They allow dealing with production flows upstream and downstream a production line, according to the type of these flows. Three types of flows circulate in a PT:

- *Capacity flow* moving forward from source to target units.
- *Demand flow* moving backward from target to source units.
- *Production flow* moving forward from source to target units.

The production depends on the demand which itself depends on the capacity. First, each component (production unit) exports its actual production capacity, noted *outCapacity*. This capacity is null if the component is failed and equal to its intrinsic capacity (*intraCapacity*) otherwise. Then, the component

receives a demand, noted $inDemand$, which, in stabilized situations, should not exceed the component capacity. Finally, the component exports a production ($outProduction$), which is the minimum of its actual capacity and the input demand. If the demand is null, the component is considered in standby mode. Figure 5 shows the flows circulating in and out a component having m parents and n children.

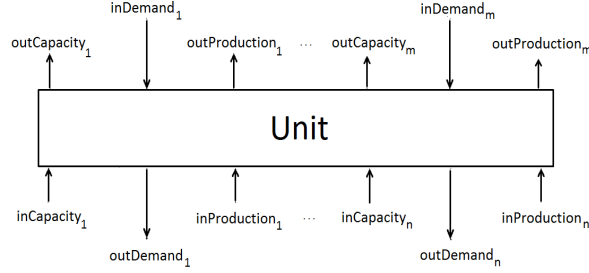


Figure 5: Flows circulating in/out a component

In PT, the gates serve to permit, inhibit or modify the passage of flows. In [4], three types of gates are defined: the *PLUS-gate*, the *MIN-gate* and the *SPLITTER-gate*.

1) The *MIN-gate*: It has one parent and two or more children. Its output capacity is the minimum of the output capacities of its children and of its intrinsic capacity (Equation 2). The input demand of the gate (coming from its parent) is propagated unchanged to its children. Finally, the output production of the gate is the minimum of the output production of its children.

$$outCapacity = \min(inCapacity_1, \dots, inCapacity_n, intraCapacity) \quad (2)$$

2) The *PLUS-gate*: It has one parent and several children. Its output capacity is the minimum of its intrinsic capacity and the sum of the output capacities of its children as specified in Equation 3. The input demand of the gate is propagated unchanged to its children. Finally, the output production of the gate is the sum of the output productions of its children. In the case where the output capacity of the gate is not equal to the output capacity of its children, the input demand of the gate is propagated to its children according to an allocation strategy. For example, the demand can be allocated according to a pro-rata of their capacities. Another strategy consists to allocate the maximum production to the first child, the maximum of the rest to the second child, etc (priority).

$$outCapacity = \min\left(\sum_{i=1}^n inCapacity_i, intraCapacity\right) \quad (3)$$

Note that for both *MIN-gate* and *PLUS-gate*, $inCapacity_i$ is equal to $outCapacity_i$ of child i , $i = 1, \dots, n$. Similarly, $inProduction_i = outProduction_i$.

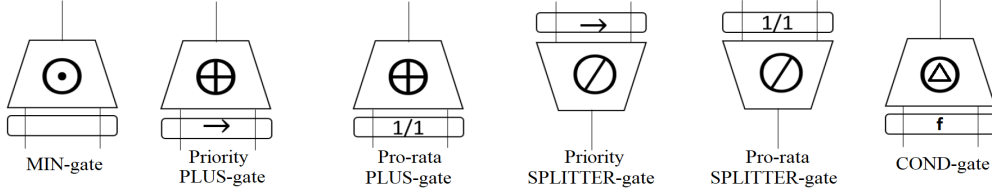


Figure 6: Graphical representation of gates of Production Trees

3) The *SPLITTER-gate*: unlike the other gates, this gate has only one child and several parents. The output capacity of the *SPLITTER-gate* is the minimum of its intrinsic capacity and the output capacity of its unique child. It is transmitted unchanged to its parents. The output demand of the gate is the sum of its parents demands. Finally, the output production of the gate is split among its parents following an allocation strategy (priority, pro-rata, ...), as for *PLUS-gate*.

Note that, *inCapacity* and *inProduction* are equal to *outCapacity* and *outProduction*, respectively, of the unique child of the gate.

Additionally to these gates defined in [4], we have introduced a new gate, namely the *COND-gate* to be able to deal with different kinds of flows. This gate defined in [5] allows modeling the functional dependencies between sub-systems .

4) The *COND-gate*: It has one parent and two children. Each child represents a specific kind of flow. Let K_1 and K_2 be these types of flows. The output flow of the gate is a flow of type K_1 and its output capacity $outCapacity_{K_1}$ depends on, the one hand, the gate intrinsic capacity, and, on the other hand, the input capacity of type K_2 flow, according of a predefined function $f(inCapacity_{K_1}, inCapacity_{K_2}) : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $inCapacity_{K_1}$ and $inCapacity_{K_2}$ are the input capacities of flow types K_1 and K_2 , respectively.

$$outCapacity_{K_1} = \min(intraCapacity, f(inCapacity_{K_1}, inCapacity_{K_2})) \quad (4)$$

It follows that the input demand of the gate is of K_1 type flow ($inDemand_{K_1}$). Since the gate has two children, this demand is forwarded unchanged to the gate children, according to their type, namely $outDemand_{K_1}$ and $outDemand_{K_2}$. These demands depend on both $inDemand_{K_1}$ and $outCapacity_{K_1}$, according to a predefined function, for example, the *min* function.

Finally, the output production of the gate is a K_1 flow type and its value is according to the function $f(inProduction_{K_1}, inProduction_{K_2}) : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ where $inProduction_{K_1}$ and $inProduction_{K_2}$ are the input productions of flow types K_1 and K_2 , respectively.

Figure 6 gives a graphical representation for *MIN-gate*, *PLUS-gate*, *SPLITTER-gate* and *COND-gate*.

5. Modeling the system using PT

The electrical sub-system is responsible for providing power to both cooling and network sub-system. In order to model the global system which consists of three sub-systems (electrical, thermal and network), we model first the electrical sub-system, because it does not depend on any other sub-systems of the DC. Then we model the thermal and networks sub-systems individually.

5.1. Modeling the electrical sub-system

In general, building the PT model goes through 3 steps: the transmission of the capacity (step 1), the transmission of the demand (step 2) and the transmission of the production (step 3). However, the electrical sub-system has a particular behavior. The electrical production components will not export their capacities. The load points (the servers in our case) export directly their demand, in terms of energy, to the other components of the sub-system. Then the power sources produce the energy taking into account their maximum capacities (intrinsic capacities). This energy is transmitted to the servers, through the other components of the sub-system. If the demand exceeds the intrinsic capacity of a component on the path, the circuit breaker of this component cuts-off the electrical flow, and the system will not be able to supply the load point. Therefore building the PT model goes through 2 steps only: the exportation of the demand by the load points (servers) (step 2) and the transmission of the energy production according to the demand received by the energy sources (step 3).

Step 2: this step is the transmission of the demand in the system. It starts with the servers. They send their power demand $outDemand_{ser_i}$, $i = 1, \dots, 12$ to the *PDU*s of the *distribution layer*. This is modeled using the *SPLITTER-gate* in order to combine the total demand coming from servers.

From each *PDU*, the demand is propagated to a pair of *TU*s (block of *LTM* and *DT*). This is modeled using a *PLUS-gate* with a pro-rata strategy according to each one can treat. Since a *TU* block contains components in series, we use a *MIN-gate* to combine them.

The demand continues its traversal in the *distribution layer* to BDP_{iX} , $i = 1, 2$ and $X = A$. Each BDP_{iA} sends its demand to the *storage layer* which contains UPS_{iA} . A *MIN-gate* is used to model it. We model similarly the other path ($X = B$) of the system.

As each UPS_{iX} , $i = 1, 2$, $X = A, B$ has to send its demand to FDP_A and FDP_B in the same layer, a *SPLITTER-gate* is used to collect the sum of demands $outDemand_{UPS_{iX}}$. Then a *PLUS-gate* is used to propagate the total demand between two redundant paths (FDP_A and FDP_B).

Since FDP_A and FDP_B are in series with transformers Tr_A and Tr_B in the *transformation layer*, respectively, they are combined using a *MIN-gate*. Finally the demand is transmitted to the *production layer*.

The demand $outDemand_{Tr_A}$ from Tr_A is sent to power source PS_1 and power generator PG with a priority to PS_1 . This is modeled using a *PLUS-gate* with priority strategy. We model similarly the second path through Tr_B ; a demand $outDemand_{Tr_B}$ is sent to power source PS_2 and PG with a priority to PS_2 . Thus, the PG is initially in standby mode.

Step 3: this step is the transmission of the power sources production in the system. According to the demand received by each power source, the required quantity is sent to the servers (bottom-up) through the gates. The complete model is presented in Figure 7.

Note that as the electrical sub-system does not depend on any other sub-system of the DC, no *COND-gate* is required in the PT model.

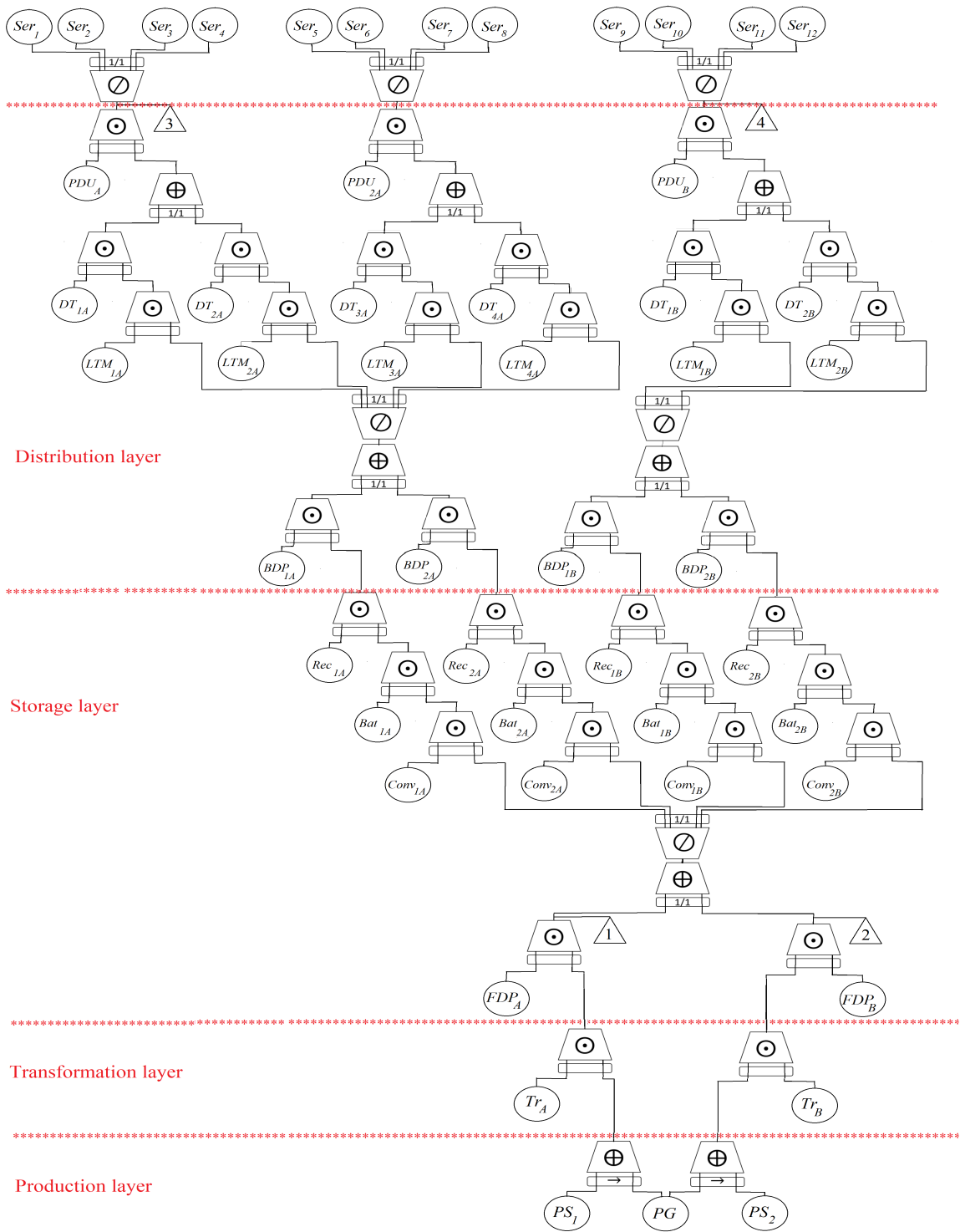


Figure 7: PT of the Electrical sub-system

5.2. Modeling the thermal sub-system

The production tree modeling the thermal sub-system has been discussed in [6]. Unlike the electrical sub-system PT model, building the thermal sub-system PT model goes through the 3 defined steps, since this sub-system has no specific behavior. Moreover, we have taken into account the dependencies between the electrical sub-system and the thermal one, because the thermal components become operational only if they are powered by the electrical sub-system. For that, we have used *COND-gates* to model these dependencies with two different kinds of flows: air flow (K_1) and electrical flow (K_2). By definition, the role of this type of gate is to allow, modify or inhibit the passage of the air flow according to a predefined set of electrical flow values coming from the PT modeling the electrical sub-system after assessing it. Thus the output air flow of the gate depends on these predefined values of electrical flow and thus the dependency is static. In this paper, the dependency is dynamic, because instead of having a predefined electrical flow values set, these values change dynamically according to the air flow. In other words, when an air flow demand is routed through a *COND-gate*, the required power demand is sent to the PT modeling the electrical sub-system. The latter sends a response (electrical flow) which corresponds to the received demand, unless this demand exceeds its production capacity. In this case, the complete model is presented in Figure 8.

5.3. Modeling the network sub-system

As in the electrical sub-system model, the users send directly their demands (packets) to the network sub-system components. In this case, only the last 2 steps are required and the same process is applied as for the electrical system. However, the production tree modeling the network sub-system has to take into account dependencies between this sub-system and the electrical one. Indeed, the network sub-system components become operational only if they are powered by the electrical sub-system (see Figure 1). Thus, the PT model has to take into account two different kinds of flows: packets flow (K_1) and electrical flow (K_2). For that, we use the *COND-gate* to model the dependency.

The PT model is presented in Figure 9, where, due to a high number of servers, these are not represented. The racks (containing 20 servers each) are represented. Moreover, to simplify the graphical representation, only production flows are represented.

Note that, the production tree modeling the network sub-system has been also discussed in [5]. However, as for the thermal sub-system, the dependencies between the network sub-system and the electrical one are dynamic .

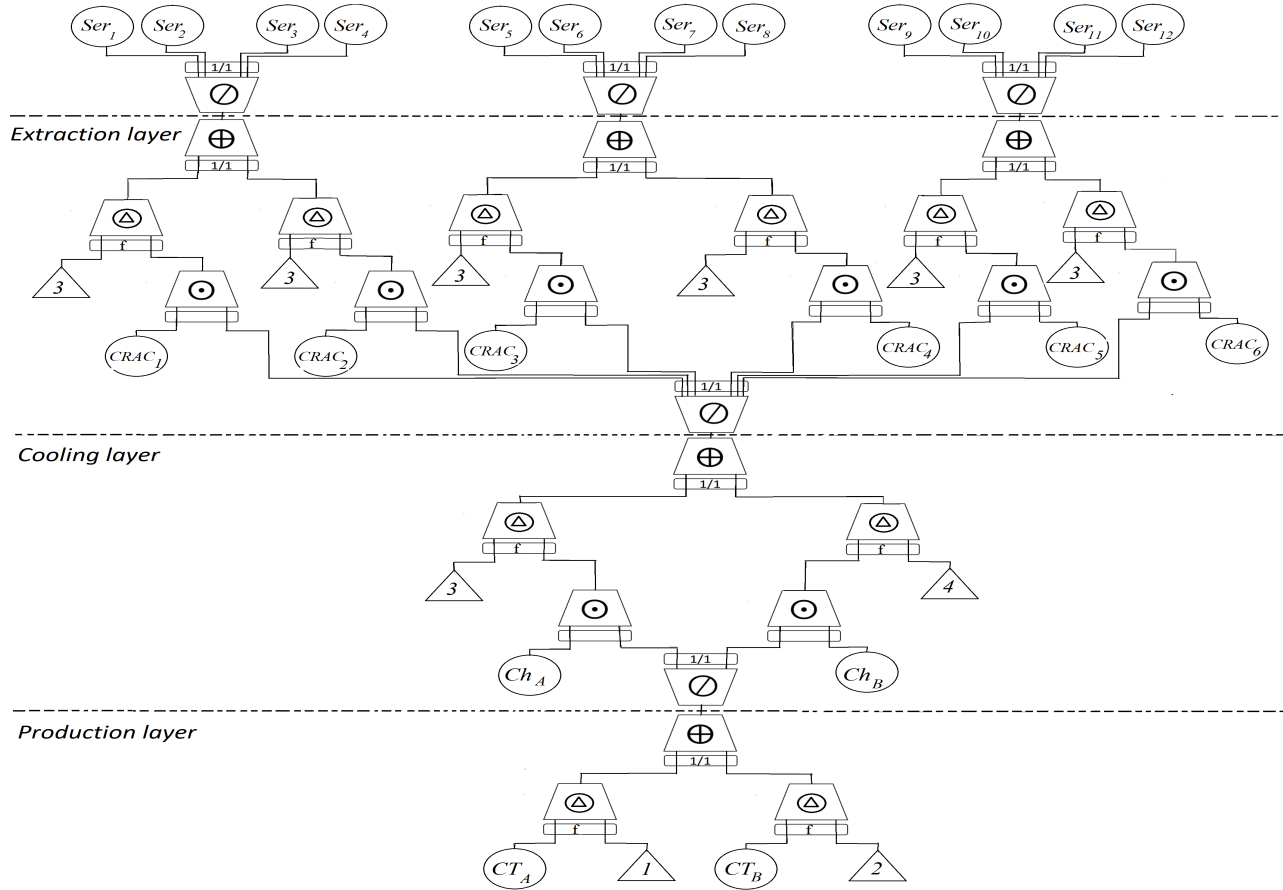


Figure 8: PT of the thermal sub-system

6. Performance and Safety Analysis of the system

Firstly, to analyze the availability and reliability of the whole system depicted in Figure 2, we analyze individually the PT model of the electrical sub-system (see Figure 7), the thermal subsystem (see Figure 8) and network sub-system (see Figure 9). For that, a new analytical approach we have recently developed based on the Probability Distributions of Capacity (PDC) is applied. This approach calculates both system reliability and availability using a set of predefined formulas. It is more concise (values range) and provides more accuracy in terms of system availability than simulation methods. The basic idea of this assessment algorithm of the PTs is inspired from [26]. In this approach, a Production Probability (PP) is associated with each component of the system. Then, once all sub-systems identified, the component's PPs of each sub-system are combined and a new PP is associated with each sub-system. The same process is applied to

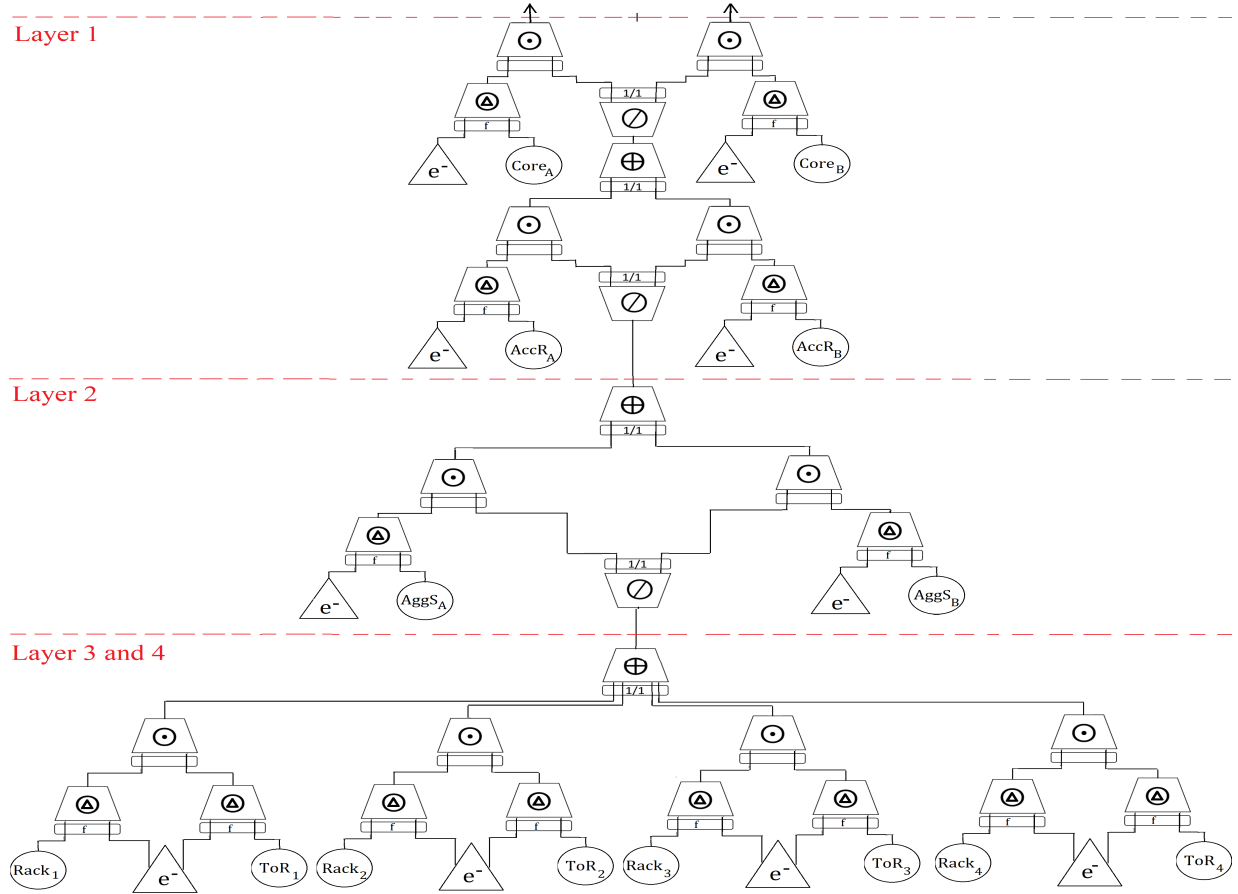


Figure 9: PT of the network sub-system (production flow only)

all sub-systems and the PP of global system is returned. However, in our method, instead of identifying all basic sub-systems and combining them like in [26], in our approach we associate with each component of a PT a production probability distribution which values depend on the failure probability of the component. Then, according to the type of each gate in the PT, we combine the different probability distributions associated with its children using a rule defining how to combine these entries. Finally, once all the rules are applied, the system availability and reliability are estimated.

Moreover, since the temperature variations affect both availability and reliability of the system, we consider the Arrhenius model to analyze this effect.

Arrhenius model relates the lifetime of an electronic component to the operating temperature [27]. The following equation estimates the relationship between such a temperature and the Mean Time To Failure

(MTTF) of the device:

$$r = A * e^{\left(\frac{-E_a}{K * TP}\right)}. \quad (5)$$

Where:

- r is the reaction rate.
- TP is the temperature (in degrees Kelvin) at which components breakdown.
- K is the Boltzmann constant.
- A is a pre-exponential constant.
- E_a is the activation energy usually within the range $0.3eV - 0.7eV$ [28].

In this paper, we consider the activation energy $E_a = 0.642eV$ which is generally used for cooling components [29]. Moreover, in order to assess the impact of the temperature on the MTTF of the servers, we consider temperature TP_0 the initial temperature of the IT room, such that $TP > TP_0$. This leads to the following MTTF expression:

$$MTTF_{TP} = MTTF_{TP_0} * e^{\left(\frac{E_a}{k} * \left(\frac{1}{TP} - \frac{1}{TP_0}\right)\right)} \quad (6)$$

Equation 6 allows us to compute a new value of MTTF at elevated temperature [30]. The revised MTTF provided by the Arrhenius model is inserted into the PT model.

Using this approach, we have already analyzed the availability of the DC's electrical and thermal sub-systems depicted in Figure 2 in [6]. However, this work focuses only on the impact of the electrical sub-system on the thermal one. Furthermore, this impact is static, that is, when the thermal sub-system demand, in terms of power energy, exceeds the electrical sub-system production capacity, this demand is not satisfied, and the thermal sub-system fails.

Unlike in [6], in this paper, we take into account not only the thermal sub-system impact but also the electrical one in terms of power energy demand. Therefore, the system is analyzed by considering the impact of the thermal sub-system on the electrical one, and this impact is dynamic. So when the thermal sub-system demand exceeds the electrical sub-system capacity, the former may not fail, since the latter may adapt its production capacity by producing more energy to satisfy the demand. When the demand is less than the production capacity, this production can be reduced in order to optimize the energy consumption by the thermal sub-system.

In order to validate the results of our approach, we implement the production tree model of the system using the AltaRica 3.0 modeling language [31] and use its dedicated stochastic simulator. Using the high level language AltaRica 3.0, it is possible to design the model with a structure that is close to the functional and the physical architecture of the system under study. AltaRica 3.0 implements the prototype-oriented

paradigm [31]. This paradigm fits well with the level of abstraction reliability and safety analysis standards. As for mathematical foundations, AltaRica 3.0 is based on Guarded Transition Systems (GTS) [32].

Once the system reliability is estimated, we analyze the performance of the most important part of DC system responsible for providing services which is the network sub-system (see Figure 4), taking into account its dependencies with other sub-systems (electrical and thermal). Generally Queueing Network (QN) theory is used to analyze performance of such systems. However, these techniques does not take into account the components failure. In our case, the performance is estimated knowing that each component of the network sub-system can fail. For that, we enrich the PT modeling the network sub-system depicted in Figure 9 by introducing performance measures on flows circulating in the model. PT models, with their basic components and gates, are sufficient to deal with both DC reliability and performance issues. Instead of having a deterministic flow propagation like in a QN model, the propagation is dynamic in a PT model, according to the state of each network sub-system component (working or failed) or according to its treatment capacity. Moreover, a PT model with its *COND-gates* allows modeling the functional dependency between the network, the electrical and thermal sub-systems. The analysis has been applied in our previous work [5]. However, we have considered only the static impact of the electrical sub-system on the network one. In this paper, all the interactions between the sub-systems are taken into account (electrical, thermal and network), and are dynamic.

Some performance statistics are estimated. The most important are the total network throughput, the mean end-to-end delay and packet loss probabilities. At each basic component i in the PT model, packets arrive at rate λ_i and leave with rate μ_i which corresponds to the precessing rate P_{r_i} . Two cases are considered:

1) Case 1: when the processing rate P_{r_i} at component i is greater than the arrival rate λ_i , the queueing delay at any component of the network is null. The processing delay is constant for all packets $D = D_i = 1/P_{r_i}$ (assuming that all packets have the same treatment time).

2) Case 2: when the arrival rate λ_i is greater than the processing rate P_{r_i} , packets experience queueing delay. And since the buffer size is bounded, packets are lost. We note:

- the sending interval $S_i = 1/\lambda_i$
- the processing time $T_i = 1/P_{r_i}$
- the number of packets $N_i(t)$ in the buffer at instant t .
- the size of the buffer K_i .
- the packet loss probability $P(t)$ at instant t .

A component i sends NP packets every S_i time interval. The first of these packets reaches another component at instant t_d where t_d is the transmission delay. Then the instant of arrival at component i for packet j is $A_{ij} = t_d + j * S_i$ where $0 \leq j \leq NP - 1$.

The instant of processing of a packet j at component i is $P_{ij} = \max(A_{ij}, j * T_i)$. Thus for $j = 0$, $P_{ij} = A_{ij}$.

At instant t_K the number of packets $N_i(t_K)$ in the buffer i reaches the size limit K_i , and the packets arrival after this instant are lost. Therefore, the packet loss probability is $P(A_{ij} \geq t_k)$.

Queueing delay for packet j at component i is $Q_{ij} = P_{ij} - A_{ij}$ and the total delay is $D_{ij} = Q_{ij} + T_i$. The average delay D :

$$\begin{aligned} D &= (P_0 - A_0) + \sum_{j=0}^{NP-1} (j * (T_i - S_{ij}) - t_d + T_i) / NP \\ &= (P_0 - A_0) + ((NP - 1) * NP / 2) * (T_i - S_{ij}) - t_d + T_i / NP \end{aligned}$$

And since $P_0 = A_0$:

$$D = ((NP - 1) * NP / 2) * (T_i - S_{ij}) - t_d + T_i / NP$$

Finally, in order to validate the performance results of our approach, we compare them to simulation results. We have implemented a simulation tool for an open finite QN where each queue is a $M/M/1/K$ [20]. We use a confidence interval for the admission decision of 95%.

7. Numerical Results

We consider the components reliability data in [28]. The servers in each rack are connected to *ToRs* via a *1Gbps* link. The processing rates (treatment capacity) of the network components are real data (provided in Table 1). For confidentiality reasons, the reference cannot be provided. In the PT model, we consider the mechanism of Skipping the Unavailable Nodes (SUN) [33]. Flows are not allowed to enter to a failed component and jump to the next one according to the routing table (redundancy case).

Component i	P_{r_i} (Mbps)
<i>Core</i> and <i>AccR</i>	450
<i>AggS</i> and <i>ToR</i>	400
<i>Server</i>	716

Table 1: Components treatment capacities

In this paper, to compute the availability A , we consider six operational modes.

- OM_1 : in this mode, the availability is computed assuming that the system is operational if at least one server in a rack is operational, the others servers and racks are in standby mode (knowing that the server demands $10kW$ of power energy).

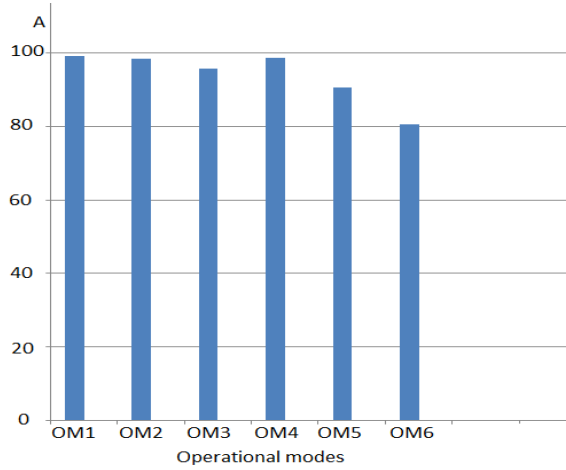


Figure 10: System availability according to the operational modes

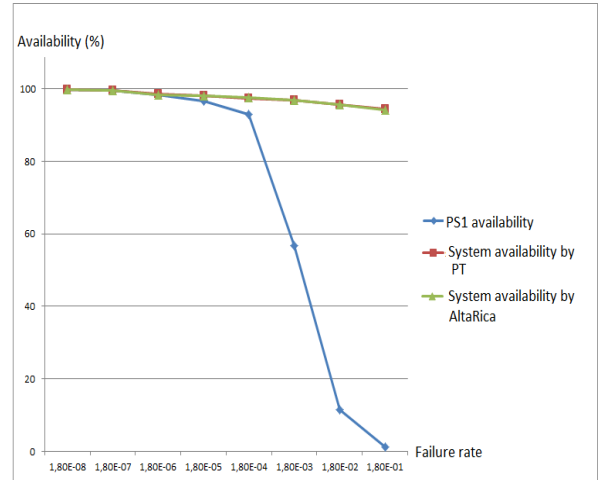


Figure 11: PS1 and system availability in OM_5

- OM_2 : the system is considered to be functioning if at least one rack is working ($10kW$ for each server in the rack).
- OM_3 : the system is considered to be functioning if all racks are functional.
- OM_4 : the system is operational if at least one server in a rack is functional with $30kW$ of energy demand.
- OM_5 : the system is considered to be functioning if at least one rack is operational ($30kW$ of demand in each server).
- OM_6 : the system is considered to be functioning if all racks are working with the same energy demand from each server.

Figure 10 is a summary of the results of system availability evaluated separately in each operational mode.

In terms of availability, the system in OM_1 , OM_2 and OM_4 has the highest availability. In this case the system generates a sufficient power allowing the servers to operate properly, and there is redundancy between racks. The system in OM_3 and OM_5 has a low availability compared to the one with OM_1 , OM_2 and OM_4 . In OM_3 , this is due to no redundancy between components, because if one rack fails, no other rack will take over. However, in OM_5 , this is due to the increase of power consumption (from $10kW$ to $30kW$), and the power sub-system reaches the maximum of its production. The system in OM_6 has the lowest availability (80%), because of the increase of the power demand and no redundancy. The system produces more power by activating the PG initially in standby mode, and if one rack fails, there is no rack that will take over.

Figures 12 (a) and (b) show the probability distribution of the total electrical sub-system production and the probability distribution of air production by the CRAC units when OM_5 is considered, respectively. According to Figure 12 (a), the electrical sub-system produces $60kW$ of energy with a high probability. This is sufficient to satisfy the servers demand. However, according to Figure 12 (b), the CRAC units are able to extract $50kW$ with a high probability and $60kW$ with a null probability. Therefore, the servers power demand is satisfied but the total of heat is not extracted from the data center room. This is consistent with the results obtained in Figure 10 and explains why the availability of the system, when OM_5 is considered, decreases.

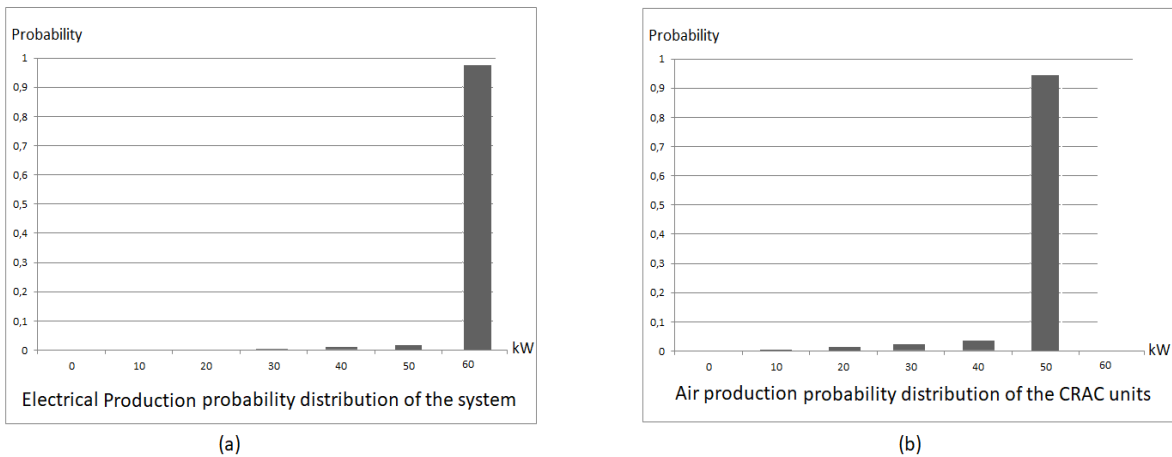


Figure 12: Probability distribution of the production capacity in OM_5

In order to identify the different dependencies between components and sub-systems, we study the variation of component's failure rates and its impact on the global availability of the system in OM_5 . Let's start by the sources of power production (PS_1 and PS_2). Figure 11 provides the variation of failure rates of the power source PS_1 (same for PS_2) and its impact on the system availability. According to these results, it is clear that the availability of the power source decreases when the failure rate increases. However, this does not affect the system availability since the power production is ensured by both PS_2 and PG (PS_1 and PG if PS_2 is considered). The simulation results of the AltaRica model match those obtained using PT.

The PDU s represent the points of distribution of the power flow to the servers. This is why it is important to analyze their failure impact on the system in OM_5 . According to the results presented in Figure 13, the variation of failure rates of a PDU unit does not affect too much the system availability due to redundancy. Once again, the simulation results of the AltaRica model match those obtained using PT.

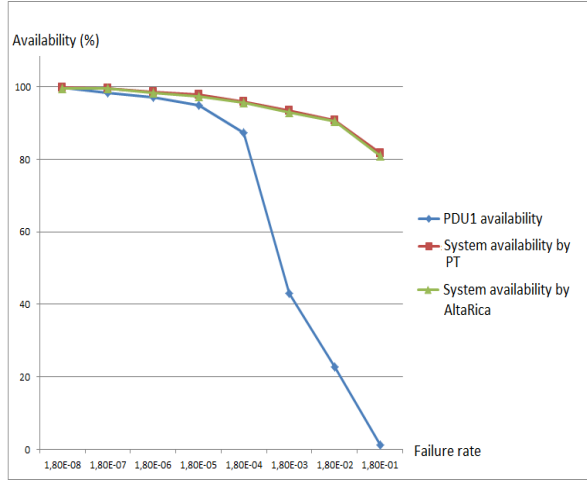


Figure 13: PDU and system availability in OM_5

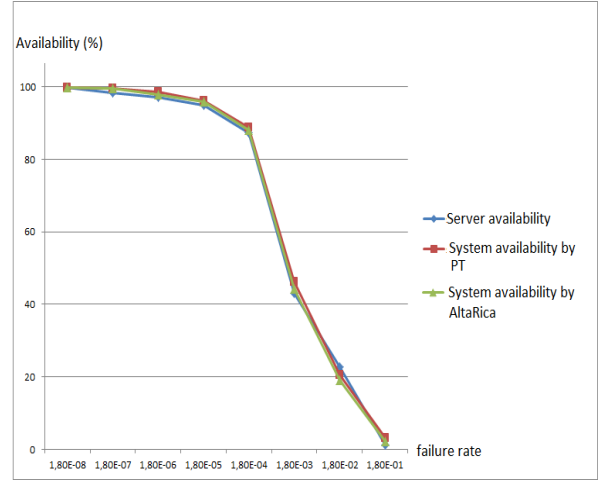


Figure 14: server and system availability in OM_5

Finally, if we analyze the impact of the servers failure rates variation on the system in OM_5 (Figure 14), it is clear that the system availability depends on the servers availability. The simulation results of the AltaRica model match those obtained using PT.

Figure 15 shows the impact of the servers demand variation on the CRAC unit's production and thus the temperature of the IT room. The servers demand varies from $30kW$ to $240kW$. When the demand is less than $120kW$, the temperature remains unchanged because the CRAC units are working well and extract the total heat from the IT room. When the demand exceeds $120kW$, the CRAC units production start decreasing progressively because the electrical sub-system is not able to produce a sufficient energy to satisfy the CRAC (thermal) demands, but can produce sufficient energy for the servers. In this case, the servers are fully powered, but the CRAC units demands are not satisfied. For example, the servers need $180kW$ of power, and the electrical sub-system produces $200kW$. The servers consume $180kW$ then generate $180kW$ of heat and the CRAC units will need $180kW$ of power to extract this heat. However, only $20kW$ (the rest of energy produced) is transmitted to the CRAC units. Therefore, only $20kW$ of heat are extracted and $160kW$ remain within the data center room which exceeds the normal Data Center temperature ($22^\circ C$). Clearly, the increase of temperature affects the servers and the system availability.

Figure 16 shows the system reliability according to the arrival rate λ_i (demand). The probability that the system will perform its function, which corresponds to produce a sufficient throughput to treat demands from users, is high due to the redundancy. Then the probability starts decreasing slightly until 0.96 approximately,

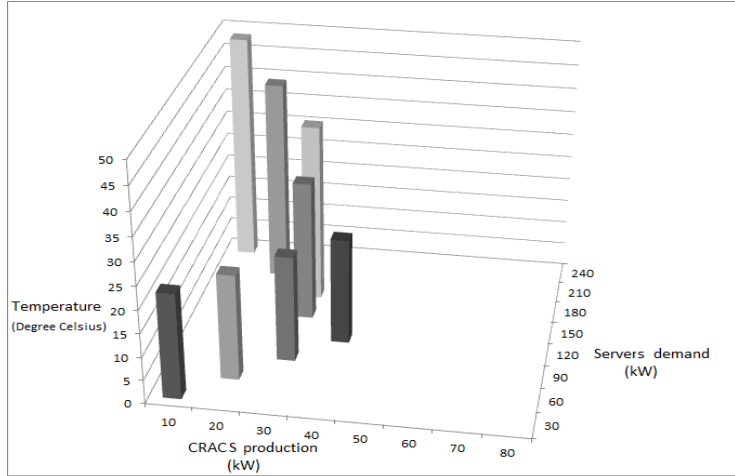


Figure 15: Electrical sub-system impact on thermal sub-system

which corresponds to $400Mbps$ of demand. This is due to the electrical sub-system impact. As shown above, a component is initially *idle* and becomes *active* when it receives a request. Therefore, when the demand increases, the power consumption increases too, and this leads to the decrease of the system reliability. Indeed, a switch can handle a number of packets according to its maximum capacity ($400Mbps$). When this capacity is reached, the second switch, initially in standby mode, is activated and starts receiving packets. Thus, the failure probability increases because no other switch will take over in case of failure (no redundancy). This explains why the reliability starts decreasing strongly from $400Mbps$ of demand. The results obtained using PT match very well those obtained using simulation.

Figure 17 shows the total throughput of the network. The throughput increases according to the demand load, and reaches its maximum ($450Mbps$) which corresponds to the maximum capacity of both routers $Core_A$ and $Core_B$. The results obtained using PT match very well those obtained using simulation.

Figure 18 and Figure 19 show the packet loss probability and the mean delay, according to demand load, respectively. Clearly, the mean delay as well as the packet loss probability increase when the demand increases. From a load of 0.86, which corresponds to approximately $400Mbps$, the curves have the same behavior change as they start increasing strongly. This is the point where the network reliability decreases strongly in Figure 16. This shows that the switches are responsible for the packets losses. And since there is a retransmission of lost packets, the mean delay increases too. Once again the results obtained using our approach match very well those obtained using simulation.

Clearly, the obtained results show that the switches impact the reliability as well as the mean delay of

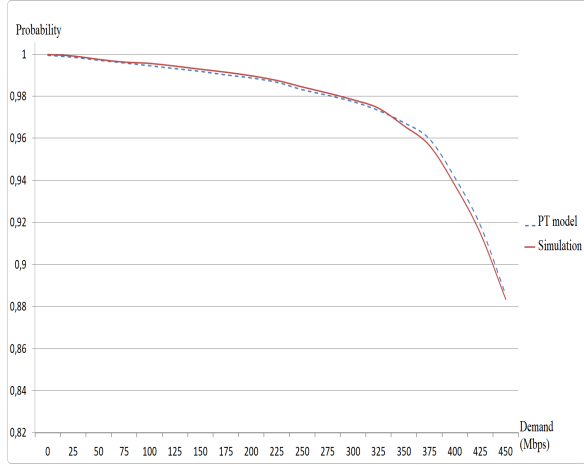


Figure 16: System reliability

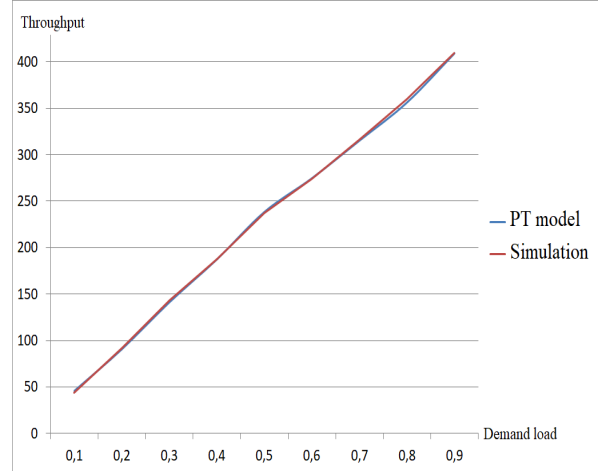


Figure 17: The total system throughput

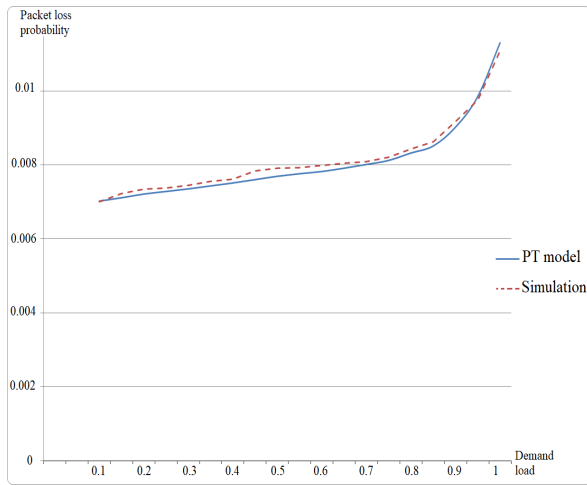


Figure 18: System packet loss probability

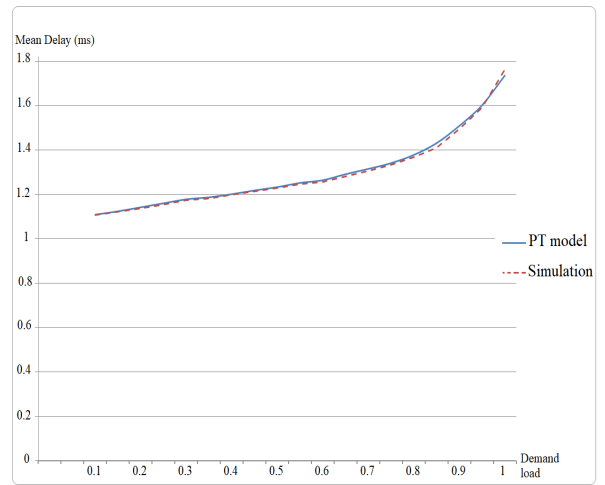


Figure 19: The Mean delay in the system

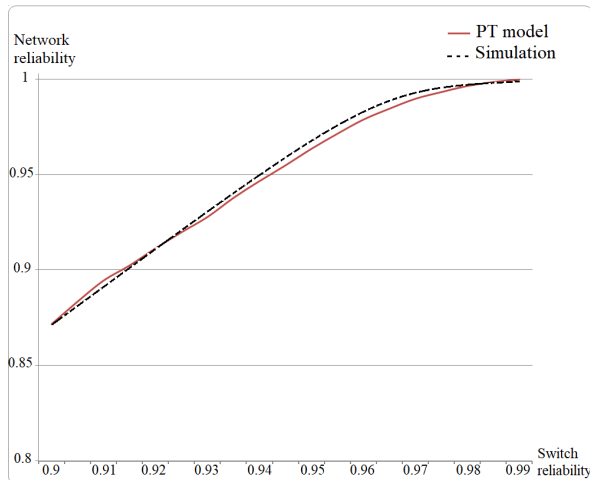


Figure 20: Switch reliability impact on the system reliability

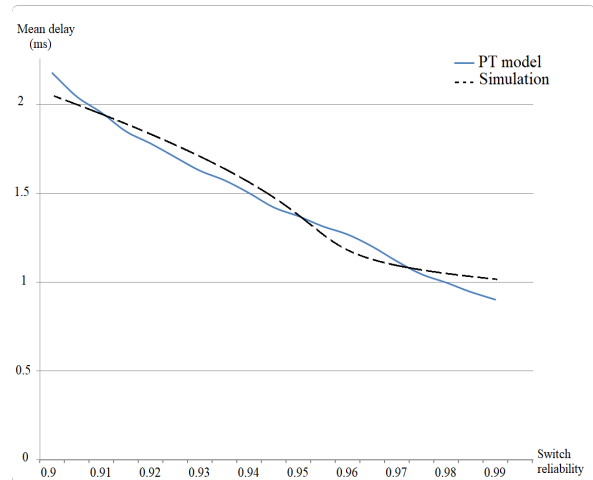


Figure 21: Switch reliability impact on delay

the system. This conclusion is confirmed by the results in Figure 20 and Figure 21 which provide the impact of the switches reliability on the global system reliability and the mean delay, respectively. The impact is important and the network sub-system depends essentially on the switches reliability.

8. Conclusion

In this paper, we investigated the reliability and availability of a DC system using Production Trees. We showed how easily this modeling technique allows taking into account, not only the different flows circulating in a DC, but also the dependencies between its sub-systems (electrical, thermal and network). Unlike in our previous works, we took into account the dynamic aspect of the dependencies, that is, the DC's sub-systems have an impact on each others dynamically. We have also demonstrated how this technique helps analyzing both reliability and performance of the DC's system. The comparison with the simulation results shows a promising effectiveness of this integrated methodology. As future work, we intend to study the global system functional architecture. This study will be important, not only to validate our obtained results, but also to enrich our methodology by identifying new interactions in the DC's system, if they are any.

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