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Informal Job Search through Social Networks and Vacancy Creation

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Abstract

In this paper I study the interaction between firms’ decisions to open vacancies and workers’ investment in social networks to find jobs. The transmission of information in the network generates positive search externalities so that in equilibrium vacancy creation and socialization are strategic complements. Despite this, there is a unique interior stable equilibrium.

Keywords: Job Search; Endogenous Job Contact Network; Vacancy Creation.

JEL classification: A14; D85; E24; J64; J63.
1. Introduction

It most countries, between 30% and 50% of jobs are filled through the use of social networks (Holzer, 1987). This evidence has led to several theoretical studies, which explore the importance of social networks for labor market outcomes (Calvó-Armengol, 2004; Calvó-Armengol and Zenou, 2005; Fontaine, 2007; Galenianos, 2014; Arbex, Caetano and O’Dea, 2016).

While most papers take the use of social network and its size as given, Galeotti and Merlino (2014) (henceforth, GM14) provide a tractable model of endogenous job contact networks and show how it can replicate how informal search varies with labor market conditions. In related frameworks, Merlino (2014) study the interaction between formal and informal search, while Horvath and Zhang (2018) the implications of inequality.

While these papers take the behavior of the firms as given, the aim of this paper is to study how workers’ incentives to invest in socialization interplay with firms’ incentives to open new jobs, and how this interplay shapes labor market outcomes. In particular, firms’ incentives to create new jobs depend on the probability that vacant jobs are eventually filled. This depends on matching rate in the network, and therefore on the size of workers’ job contact networks.

The externalities generated by the transmission of information in the network imply that, in equilibrium, vacancy creation and socialization are strategic complements. As in other models with positive externalities (e.g., increasing returns to matching in Diamond, 1982, and Howitt and McAfee, 1987), this leads to multiple equilibria with the feature that the equilibrium with more vacancies also entails a higher socialization effort. Furthermore, this equilibrium is Pareto dominant and it is the only stable equilibrium.

This paper fits in the literature of strategic interactions with indirect negative externalities on endogenous networks (Galeotti and Goyal, 2010; Kinateder and Merlino, 2017). In this paper, the network formation protocol follows Cabrales, Calvó-Armengol and Zenou (2011) and GM14 to obtain a tractable model to study the interaction between vacancy creation and networking. In a related model with indirect positive externalities, Canen, Jackson and Trebbi (2017) show that only the Pareto dominated is stable. Here, the Pareto dominant equilibrium is stable due to the different network externalities and to the interaction between firms and workers.

2. Model

There is a large set of risk neutral workers $N = \{1, ..., n\}$ and free entry of firms. The productivity of each worker is 2, equally shared between the firm and the worker. Initially all workers are employed, but they lose their job
with probability \( b \in (0, 1) \). Workers directly receive an offer from a firm with probability \( a \in (0, 1) \)\(^2\) and indirectly through their social contacts (at a rate derived below). Unemployment benefits are normalized to 0.

**Workers.** Ex-ante, a worker anticipates that with probability \( b(1-a) \) she will be unemployed and without a new offer. To reduce the likelihood of unemployment, she can engage in informal search, i.e., ask her employed friends whether they hold a needless offer.\(^3\) This process is described by an undirected network. Information about jobs flows only from employed workers with a needless offer to job seekers with whom they have a link. A link between workers \( i \) and \( j \) is denoted by \( g_{ij} = 1 \), while \( g_{ij} = 0 \) indicates that \( i \) and \( j \) are not linked. Each worker \( i \) chooses a costly socialization \( s_i \in \mathbb{R}_+ \); \( s_{-i} \) indicates the socialization profiles of all workers other than \( i \). Given a profile \( s = (s_i, s_{-i}) \), a link between workers \( i \) and \( j \) forms with probability

\[
\Pr(g_{ij} = 1 | s) = \min \left\{ \frac{s_i s_j}{\sum_{j \in N} s_j}, 1 \right\} \text{ if } \sum_{j \in N} s_j > 0,
\]

and 0 otherwise (Cabrales et al., 2011; GM14). A profile \( s \) generates a multinomial random graph (or binomial in a symmetric equilibrium).

Let \( \Psi_i(s, a) \) be the probability that worker \( i \) who loses her job accesses at least one offer from the network, i.e., her network matching rate. The expected utility to a worker \( i \in N \) is

\[
EU_i(s, a) = 1 - b(1-a)[1 - \Psi_i(s, a)] - cs_i.
\]

The last term represents the cost of socialization given a constant marginal cost \( c \). The reminder represents the probability that worker \( i \) will be employed earning a wage equal to 1.

**Firms.** Firms open vacancies at rate \( a \). The probability that the job is filled is \( b + (1-b)\Psi_f(s, a) \), where \( \Psi_f(s, a) \) is the probability that a vacancy is filled via the network. Since the number of filled vacancies equals the number of workers who find a job, I get

\[
\Psi_f(s, a) = \frac{b(1-a)}{a(1-b)} \Psi(s, a).
\]

Given the cost of opening a vacancy \( k \in [0, 1] \), the profits to a firm opening a vacancy are:

\[
\Pi_f(a, s) = b + (1-b)\Psi_f(a, s) - k.
\]

**Equilibrium.** An equilibrium is a profile \( s^* \) and a vacancy rate \( a^* \) with \( n \to \infty \) such that: 1) \( s^* \) is a symmetric Nash equilibrium given \( a^* \), and 2) \( \Pi_f(a^*, s^*) = 0 \).

\(^2\)Under mild regularity conditions, a more sophisticated matching function, i.e., Cobb-Douglas, does not affect the main results, but it may affect the Pareto ranking of the equilibria.

\(^3\)Socialization takes place before job separation to capture that it takes time to establish connections.
3. Results

When \( n \to \infty \), the network matching rate of \( i \) when all other agents are playing \( s_{-i} \) is (GM14)

\[
\Psi_i(s, a_{-i}) = 1 - e^{-\frac{n(1-b)}{n} s_i(1-e^{-sb})}.
\]

Then:

Proposition 1. When \( n \to \infty \) and \( k > b \), there exists \( \bar{c}(k, b) > 0 \) such that an interior equilibrium \((s^*, a^*)\) exists if and only if \( c \leq \bar{c}(k, b) \). An interior equilibrium solves:

\[
\begin{align*}
    b(1-a^*)\frac{\partial \Psi_i}{\partial s_i}(s^*, s^*, a^*) &= c, \quad (2) \\
    \frac{b}{a^*}[a^* + (1-a^*)\Psi(s^*, s^*, a^*)] &= k. \quad (3)
\end{align*}
\]

When \( c = \bar{c}(k, b) \), there is one interior equilibrium, while if \( c < \bar{c}(k, b) \) there are two interior equilibria, \((\hat{s}^H, \hat{a}^H)\) and \((\hat{s}^L, \hat{a}^L)\), where \( \hat{s}^H > \hat{s}^L \) and \( \hat{a}^H > \hat{a}^L \).

The socialization condition (2) is non-monotonic in \( a \) as in GM14, while vacancy creation condition (3) is increasing in \( a \) (Figure 1). When the cost of socialization is sufficiently low, there are two interior equilibria. In one equilibrium, firms create many new jobs under the expectation of a high network matching rate. Since under a high vacancy rate the job network supply is also high, workers heavily invest in connections and this self-fulfills firms’ expectation of a high network matching rate. In the other equilibrium, the opposite occurs. So, the endogenous formation of job contact networks determines strategic complementarities between the decision of firms to open new vacancies and workers’ investment in socialization.

As this discussion suggests, the network matching rate is higher under equilibrium \((\hat{s}^H, \hat{a}^H)\) than under equilibrium \((\hat{s}^L, \hat{a}^L)\). Since in the former equilibrium it is also more likely that a worker accesses a direct offer than in the latter, unemployment rate is lower in the equilibrium with dense networks and high vacancy rate. Hence, labor market tightness, \( a/u \), is higher in the high activity equilibrium. Furthermore, the two equilibria can be ranked in the Pareto sense and, between them, only the Pareto dominant equilibrium is stable. These results are stated in the following proposition.

Proposition 2. Consider the interior equilibria \((\hat{s}^H, \hat{a}^H)\) and \((\hat{s}^L, \hat{a}^L)\) of Proposition 1. Then,

\[\text{Note that if } k < b, \text{ the only equilibrium is } (s^*, a^*) = (0, 1). \] Furthermore, when \( k > b \), there always exists an equilibrium \((s^*, a^*) = (0, 0). \) Since the probability that the job is filled is lower than the costs of a vacancy, if workers do not socialize, firms do not create new jobs, and vice versa. Furthermore, given a symmetric socialization profile \( s \), \( \Psi_f(s, a) \) is increasing in \( s \) and \( b \), and decreasing in \( a \), while the opposite holds for \( \Psi(s, a) \).

\[\text{This implies that the corner equilibrium } (s^*, a^*) = (0, 0) \text{ is also stable.}\]
1) \((\hat{s}^H, \hat{a}^H)\) Pareto dominates \((\hat{s}^L, \hat{a}^L)\);
2) the network marching rate in \((\hat{s}^H, \hat{a}^H)\) is higher than in \((\hat{s}^L, \hat{a}^L)\);
3) the unemployment rate in \((\hat{s}^H, \hat{a}^H)\) is lower than in \((\hat{s}^L, \hat{a}^L)\);
4) \((\hat{s}^H, \hat{a}^H)\) is a stable equilibrium, while \((\hat{s}^L, \hat{a}^L)\) is not.

So, despite the strategic complementarity between socialization and vacancy creation, there exists a unique interior stable equilibrium: the Pareto dominant one.

To conclude, Figure 2 shows that socialization effort in the stable equilibrium responds non-monotonically to an increase in job destruction \(b\) as in GM14. The vacancy creation condition (3) shifts down when \(b\) increases, while the effect on the socialization condition (2) is ambiguous. More precisely, when \(b\) is low enough, workers react socializing more in order to increase the probability of receiving job offers from friends. Vacancy creation however increases.

4. Conclusion

This paper studies the interaction between firms’ decisions to open vacancies and workers’ decision to use their social networks to find jobs. The model highlights that there is a unique interior stable equilibrium, characterized by a high vacancy rate and a high socialization effort.

This paper complements the analysis of Merlino (2014), in which workers decide whether to use formal or informal search when firms are passive. Together, these works show the need of a full-fledged model of the labor market in which the use of social networks is endogenous. Such a model would allow to better understand how informal job search reacts to different economic conditions, as well as how workers’ socialization interacts with firms’ decisions. Furthermore,
in a dynamic model with rational expectations, multiple stable equilibria might exist (Howitt and McAfee, 1988).

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Appendix

Proof of Proposition 1 An interior equilibrium \((s^*, a^*)\) solves conditions (2) and (3), which can be rewritten as

\[
\bar{s}(a^*, c, b, k) = \frac{b - ka^*}{c} \ln \left[ \frac{b - ka^*}{b(1 - a^*)} \right],
\]

(A-1)

\[
\tilde{s}(a^*, b, k) = \frac{1}{b} \ln \left[ 1 + \frac{b}{a^*(1 - b)} \ln \left[ \frac{b - ka^*}{b(1 - a^*)} \right] \right].
\]

(A-2)

An interior equilibrium is \((a^*, s^*)\) such that \(\tilde{s}(a^*, c, b, k) = \bar{s}(a^*, c, b, k) = s^*\). Now, \(\bar{s}(a^*, c, b, k)\) is a well-defined function when (1.) \(b - ka^* > 0\) and (2.) \(b - ka^* < b(1 - a^*)\), which holds because \(k > b\). If \(a^* \in (0, b/k]\), \(\bar{s}(a^*, c, b, k) > 0\), \(\bar{s}(0, c, b, k) = \bar{s}(b/k, c, b, k) = 0\) and \(\bar{s}(a^*, c, b, k)\) is concave in \(a^*\), since

\[
\partial^2 \bar{s}(a^*, c, b, k) / \partial a^2 = -(b - k)^2 / [(1 - a^*)^2(b - ka^*)] < 0.
\]

Next, \(\tilde{s}(a^*, b, k)\) is a well-defined function when (1.) above holds, (1a.) \(1 + b \ln[(b - ka)/(b(1 - a))]/(a(1 - b)) > 0\) and (2a.) \(\ln[1 + b \ln[(b - ka)/(b(1 - a))]/(a(1 - b))] > 0\).
Since proof of Proposition 1. While for all there exists a \(\bar{a}\) strictly decreasing in interior equilibria. Since for fixed \(G\) there is only one interior equilibrium, while if \(G(\bar{a}, k, b, k) = 0\) there exists a unique \(\bar{a}\). Note here that: \(\tilde{s}(0, b, k) = -\ln((1 - a^*)/(b(1 - a^*)))\). Hence, (1a.) implies (1.). So, an interior equilibrium exists if and only if \(a < \bar{a}(b, k)\). The properties of \(\bar{a}\) and that \(\tilde{s}(a^*, b, k)\) is increasing and convex in \(a \in [0, \bar{a}(b, k)]\). The equilibrium exists if and only if \(G(\bar{a}, c, b, k) \geq 0\). Furthermore, if \(G(\bar{a}, c, b, k) = 0\), there is only one interior equilibrium, while if \(G(\bar{a}, c, b, k) > 0\) there are two interior equilibria. Since for fixed \(k > b\) and \(a^* \in [0, \bar{a}(b, k)]\), \(G(a^*, c, b, k)\) is strictly decreasing in \(c\), \(\lim_{c \to 0} G(a^*, c, b, k) = \infty\) and \(\lim_{c \to \infty} G(a^*, c, b, k) = 0\), there exists a \(\bar{c}(k, b)\) such that if \(c = \bar{c}(k, b)\) there exists one interior equilibrium, while for all \(c < \bar{c}(k, b)\) there are two interior equilibria. This concludes the proof of Proposition 1.

**Proof of Proposition 2** Under equilibrium \((s^*, a^*)\), \(\Psi_f(s^*, a^*) = [k - b]/(1 - b)\). Since \(\Psi_f(s^*, a^*) = \Psi(s^*, a^*)b(1 - a^*)/[a^*(1 - b)]\), \(\Psi(s^*, a^*) = a^*(k - b)/[b(1 - a^*)]\), which is increasing in \(a^*\). So, \(\Psi(s^*, a^*) = a^*(k - b)/[b(1 - a^*)] = \Psi(\bar{s}^L, \bar{a}^L)\). Second, under equilibrium \((s^*, a^*)\), \(u(s^*, a^*) = b(1 - a^*)\phi(s^*, a^*) = b - a^*k\), which is decreasing in \(a^*\). So, \(u(s^*, a^*) < u(\bar{s}^L, \bar{a}^L)\). Finally, using (A-1), the expected utility under equilibrium \((s^*, a^*)\) is \(EU(s^*, a^*) = 1 - b(1 - a^*)\phi(s^*, a^*) - cs^* = 1 - b + a^*k - cs^* = 1 - b + a^*k + (b - ka^*)\ln((b - ka^*)/(b(1 - a^*)))\), which is increasing in \(a^*\). So, \((s^*, a^*)\) Pareto dominates \((\bar{s}^L, \bar{a}^L)\). If we assume that (3) is always satisfied and that workers revise their strategies together, changes in \(s^*\) outside equilibrium are described by \(ds/dt = G(a, c, b, k)\). As proved above, \(G(a, c, b, k)\) is concave in \(a^*\), \(G(0, c, b, k) < 0\) and \(G(\bar{a}(b, k), c, b, k) < 0\), so \((\bar{s}^L, \bar{a}^L)\) is unstable while \((s^*, a^*)\) is stable. This concludes the proof of Proposition 2.

**References**


