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# A cycle-based formulation for the simulation of multi time-scale systems – Application to the modeling of the storage system of a fully electric ferry

S. Hmam<sup>a,b</sup>, J.-C. Olivier<sup>a</sup>, S. Bourguet<sup>a</sup>, L. Loron<sup>a</sup>, N. Bernard<sup>a</sup>, E. Schaeffer<sup>a</sup>

<sup>a</sup>IREENA, University of Nantes, 37 boulevard de l'université, Saint-Nazaire, France

<sup>b</sup>SERMA Ingénierie, 5 Parc d'Ariane, Boulevard des chênes, Guyancourt, France

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## Abstract

This paper addresses the simulation of complex systems which consider phenomena with different time scales. Such problems are encountered on studies of electrical systems which try to take into account the simulation of the power converter and its control laws, over representative operating cycles of several hours. Moreover, when storage elements are integrated into a power chain, their aging may need the designer to consider larger time scales, which can then exceed a few years. It is the reason why most studies separate the time scales between the slow dynamics for the energetic, thermal and aging phenomena, and the fast ones to study the power converter and its control laws. This paper presents an original cycle-based and multi-rate method for the simulation of power systems with a wide range of time scales and with high mutual dependency between the fast and slow state variables. This method is applied to the supercapacitor energy storage system of a full-electric ferry. The proposed simulation results take into account at the same time the switching of the power converter and the aging of the supercapacitor, with a reduction of the computational effort greater than  $10^5$ . In other words, while a full calculation of the problem takes 10 centuries on a personal computer, the proposed method permits to have the same result in only 15 days.

*Keywords:* Multirate method, Computer simulation, Supercapacitors, Full-electric ferry, Aging

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## 1. Introduction

Simulation is an important step in the optimal design of power systems. To be effective and useful, it must take into account multiphysics and multi-time scales phenomena. More specifically, the integration of the power converter in the optimization process of complex power conversion and storage systems leads to real problems of computation time (Crosbie et al., 2007; Pierquin et al., 2014; Herr et al., 2017; Dang et al.,

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*Email address:* jean-christophe.olivier@univ-nantes.fr (J.-C. Olivier)

2017). Indeed, the pulse width modulation used by the power stage requires a sampling time at a much lower value than the switching period to give accurate results. Then, even with an efficient variable step solver, this short time step reduces the reachable simulation horizon within a reasonable time. The method proposed in (Middlebrook and Cuk, 1976), which is the most used today, permits to avoid this problem by working with average values. The idea is to replace the power stage by a set of switched models. In the framework of the piecewise linear systems, it is possible to deduce a single equivalent circuit which is then only set by the value of the duty cycle and switching frequency (Olivier et al., 2010; Hernandez-Torres et al., 2010; Crow and Chen, 1996). Once the power stage is averaged, there is much less constraints on the minimum step size and the simulation horizon can be increased. Nevertheless, the same problem arises if one wishes to simultaneously simulate phenomena with very different time scales. To illustrate this difficulty, we can

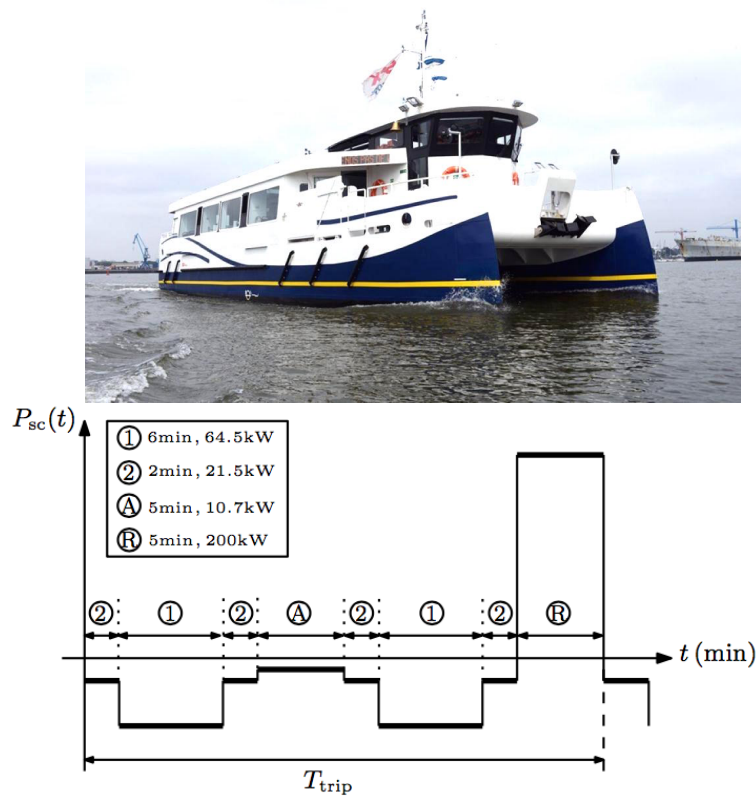


Figure 1: The ferry *Ar Vag Tredan* and its power cycle.

rely on the *Ar Vag Tredan* ship application (see Figure 1), which was introduced in recent works (Bennabi et al., 2016; Trieste et al., 2015; Bernard et al., 2017). This zero emission ferry uses supercapacitors to store the electric energy. It makes 36 trips daily and therefore, it recharges 36 times per day its supercapacitors

at a dock (in less than 5 minutes). For this application, the design optimisation mainly depends on the supercapacitor aging. Because this aging depends on the voltage and temperature cycles, a multiphysics modeling is required. To compute the aging of the supercapacitor, a time horizon of 20 years is needed. But in the same time, this aging needs an accurate voltage profile which imposes a time step lower than few seconds. Last but not least, the thermal constant is close to the daily cycle and needs a simulation of many hours to achieve a steady state. All in all, evaluating the aging of supercapacitors on this application requires many steps, such as making an average model of the power converter, simulating at least one round-trip to have the voltage profile and repeat the operation up to reach the steady state temperature.

In (Hmam et al., 2016a, 2017) the authors present an original multirate method for the simulation of such multi-physics systems, containing a wide range of time scales. This method used a cycle-based problem formulation, which consists in finding the repeating operation sequences of the system, at the different time scales. This method is applied to the supercapacitor ferry by considering round-trip cycles to build daily cycles, which is then used to form annual cycles. In order to reduce the simulation time, an average power converter model is used.

In this paper, we propose to extend this previous work by taking into account in the same time the switching of the power converter and the aging of the supercapacitor. Such a tool thus makes way for the development of a single model, able of handling both the problem of control (corrector adjustment) and energy management (taking into account the lifetime of the various devices). In order to do that, the whole system is modeled by a set of ordinary differential equations, without making assumptions about the coupling between physical domains. Next, by using the cycle-based formulation, the problem is decomposed into PWM cycles, used to build one round-trip, also used to build a daily and next an annual cycle. Finally, this formulation permits to calculate 20 years of operation, from few years, which are computed from few days, computed from few round-trips, themselves computed from few PWM periods.

The paper is organized as follows. In Section 2, a brief description of the system is presented, as well as the cycle-based decomposition, which describes the operation profile of this ferry during its whole lifetime. This section also presents an approach of discretization of classical solvers, to make them compatible with a cycle representation.

In Section 3, the ODE modelling of the supercapacitor ferry is presented. This model takes into account the electrical dynamics of the supercapacitor and the smooth inductor, the current controller (with a classical PI regulator), the thermal dynamics and the aging of the supercapacitor. Next, Section 4 presents some

simulation results where all the previous equations are computed together. Results are given for each layer of the formulation, from the PWM cycles to the 20 years of operation. Section 5 is the conclusion.

## 2. Description of the application and cycle-based formulation

### 2.1. Operation of the ferry

This section presents the full-electric ship *Ar Vag Tredan*, operating since September 2013 and crossing the harbour of Lorient, France. It uses supercapacitors to store all the electric energy. The power cycle for one round-trip is illustrated by Figure 1. The ferry makes 36 trips daily and therefore, it recharges 36 times per day its supercapacitors at a dock. The charging station is designed for a constant power charging strategy. The duration of this recharge is 5 min. The operation profile of this boat is particularly interesting because it can be decomposed into a recursive formulation. Indeed, as shown by Figure 2, the lifetime of the ferry can be divided into 20 successive years ( $Y$ ). Each year is composed of 335 successive operating days ( $D_o$ ), followed by 30 days of rest ( $D_m$ ) due to a planned annual maintenance (in addition of a real-time monitoring of the state of health of supercapacitors). In the same way, an operating day is composed of 36 successive round-trip cycles ( $C_o$ ), followed by a rest period of 6 hours ( $C_r$  cycles). Finally, each round-trip can be subdivided into a succession of PWM cycles ( $M$ ).

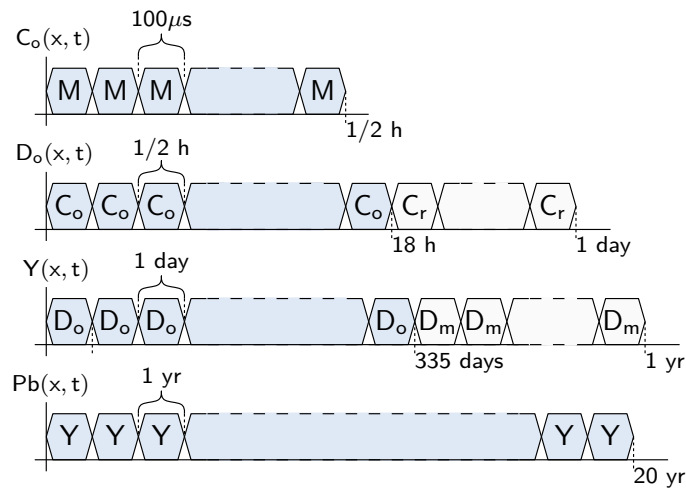


Figure 2: Cycle-based formulation of the usage of the ferry *Ar Vag Tredan*, for 20 years of exploitation, from the PWM cycle up to the lifetime.

What is interesting in this cycle-based formulation is that each layer can be computed from a partial calculation of a lower one. For instance, it is not necessary to simulate all of the 20 years of operation

to estimate the aging of the supercapacitors. By using a first order extrapolation method for example, the calculation of only one or few years permits to estimate an average degradation of the supercapacitor, which can be extrapolated over several years. The formulation makes it easy to extend these extrapolations for each layer. Thus, each year can be computed from the extrapolation of few operating days, each day from few round-trips and finally each round-trip from few PWM cycles. This example on a ferry application is easily extendable to many other applications. For instance, in (Pierquin et al., 2016), a power converter with its PWM modulation and passive smoothing filter supply a transformer which considers electromagnetic and thermal phenomenon by using finite element method. In this work, the authors manage to decouple the converter from the transformer through a waveform relaxation method. This method can be replaced by a cycle-based formulation, by considering a first scale with the PWM cycle ( $M$ ), a second scale with a sine wave period ( $S$ ) which can then be extrapolated over few hours ( $H$ ). In (Robert et al., 2017), the authors consider an exhaust gas recirculation valve, which takes into account the electrical, mechanical, thermal and magnetic domains. The simulation is achieved with an average model for the power converter and with a relaxation method for the thermal behavior. Once again, a cycle-based formulation is easy to apply on such problem, with a cycle defined at the scale of one PWM period ( $M$ ), a second at the scale of one mechanical cycle of 10 seconds ( $C$ ) and a third at the scale of few hours in order to take into account the thermal transient. It is therefore possible to apply this formalism to many problems, as soon as its description can be made on the basis of successive cycles. This method is more detailed in the next section.

## 2.2. Cycle-based formulation

In (Hmam et al., 2016a,b), an original cycle-based formulation has been proposed. It permits to organise the problem with stacked layers and to help the simulation of wide time-scale problems. In this section, a brief description of this method is given. Firstly, it assumes that the problem to be addressed can be decomposed into a finite number of elementary cycles, as explained in the previous section. It is also considered that the complete dynamic problem can be decomposed into a series of  $k$  sub-problems, each one defined by a non-autonomous ordinary differential equation (ODE) such as:

$$\dot{x} = f_i(x, t), i \in [1; k] \quad (1)$$

Each elementary problem  $f_i(x, t)$  is simulated on a time lap  $T_i$ , corresponding to an elementary cycle simulation. Applied to our problem, this time lap can be one year, one day, a half-hour or finally a PWM cycle. Now, by considering the problem (1), the simulation of one cycle on the shortest layer permits to

pass from an initial state  $x_n$  to the next state  $x_{n+1}$ , such as :

$$x_{n+1} = x_n + \int_{t_n}^{t_n+T_i} f_i(x, t) dt \quad (2)$$

The proposed method considers that the evolution of the state vector  $x(t)$  can be substituted by its average rate of change at the scale of an elementary cycle of length  $T_i$ . This averaging method is well known in power converter modelling (Middlebrook and Cuk, 1976; Tse and Bernardo, 2002; Maksimović et al., 2001; Banerjee and Verghese, 2001) where the electrical state variables are replaced by their mean value, at the scale of the switching frequency. Here, this averaging is not calculated from analytical expressions but is computed numerically by an ODE solver. This approach is then generalized to any kind of cycle. The average state vector derivative at the scale of  $T_i$  is given by the next equation:

$$F_i(x_n, t_n) = \frac{1}{T_i} \int_{t_n}^{t_n+T_i} f_i(x, t) dt \quad (3)$$

By this way, the system is solved for an operating cycle  $T_i$  and then only state vector variations  $F_n(x_n, t_n)$  over a time interval  $[t_n, t_n + T_i]$  are evaluated. It is equivalent to the average rate of change, or simply the slope between the endpoints of the interval. Using the formulation (3) in equation (2), the problem can be written in the form of the following recursive scheme:

$$x_{n+1} = x_n + T_i F_i(x_n, t_n) \quad (4)$$

where  $x_n$  describes the state vector at the beginning of each cycle. Because the initial problem is time continuous, and in the case of a succession of several identical cycles  $F_i$ , it is assumed that the average rate of change estimated at the scale of a cycle  $T_i$  for a given state vector  $x_n$  could be close from one cycle to the following. Then, the state vector for the next cycles can be approximated with a variable step discretization method and using extrapolation concept. In order to illustrate this point, let us consider the problem of  $m$  successive cycles  $F_i$ , starting from the initial condition  $x_n$ . The aim is to evaluate the state vector variations over a time interval  $[t_n; t_{n+m}]$ . If all the cycles are evaluated, the final state vector  $x_{n+m}$  is given by :

$$x_{n+m} = x_n + T_i \sum_{j=0}^{m-1} F_i(x_{n+j}, t_{n+j}) \quad (5)$$

In order to speed-up the simulation, the idea is to approximate the right term of the equation (5) by an approximation function  $\Phi_i$  which can be likened to a conventional numerical integration scheme (Gear and Petzold, 1984; Stoer and Bulirsch, 2002), given by the next equation :

$$x_{n+m} = x_n + h \Phi_i(x_n, t_n; h), \text{ with } h = m T_i \quad (6)$$

with  $m \in \mathbb{N}^*$ , the number of consecutive cycles  $F_i$ . The function  $\Phi_i$  depends on the discretization method and of the chosen step size  $h$ . It is noteworthy that the extrapolation step size  $h$  must be a multiple of the operating cycle  $T_i$ . Figure 3 gives an example of cycle homogenization, in the case of  $m$ -subsequent cycles of length  $T_0$ . The averaging function  $F_0$  is calculated with respect to equation (3) and is then called  $m$  times for the computation of the complete problem (from  $x_0$  to  $x_m$ ). For a faster computation, extrapolation function  $\Phi_i$  can be used to estimate the total change of the state vector from only few evaluations of  $F_0$ .

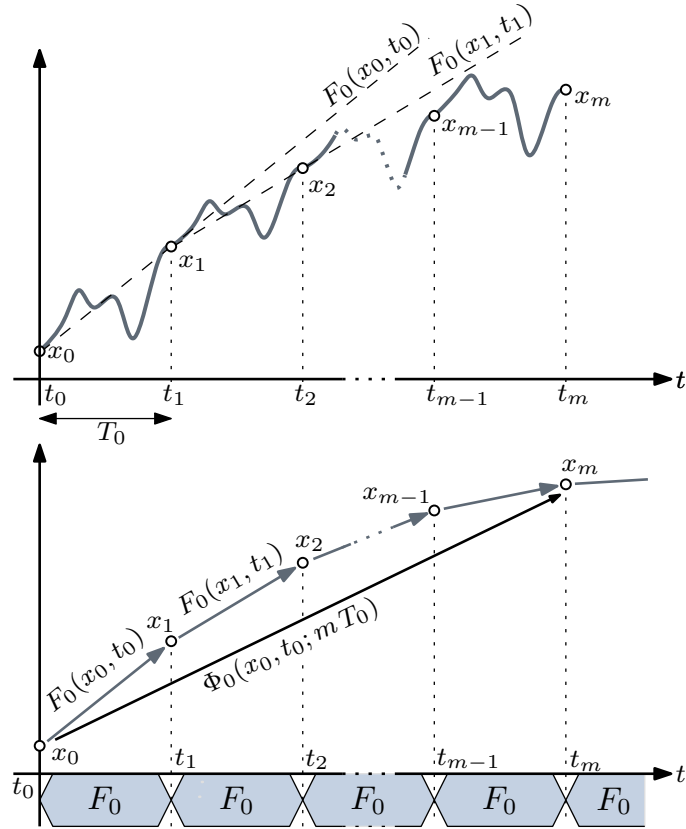


Figure 3: Example of homogenization process of  $m$  successive cycles  $F_0$  of length  $T_0$ . The extrapolation function  $\Phi$  is used to overcome a costly calculation time of all the  $m$ -subsequent cycles.

### 2.3. Multi-layer problem formulation

Now, we were able to lay down the basic principle of the method: A continuous system subject to operation cycles can be replaced by a discret homogenized system inwhere the extrapolation of cycles becomes possible. This method can then be extended with a multi-layer formulation, where cycles are described recursively from sub-cycles. The example illustrated by Figure 2 is such a multi-layer formulation, where



each cycle can be expressed from a serie of shorter sub-cycles. To illustrate this extended formulation, one can start with the computation of the average variation of the state vector over one PWM cycle, noted  $M$ , as given by equation (3):

$$M(x_n, t_n) = \frac{1}{T^M} \int_{t_n}^{t_n+T^M} f(x, t) dt \quad (7)$$

where  $f$  is the ode function of the complete system. From now on, an index is added to the state vector in order to differentiate the level of considered layers. For the description of a round-trip cycle, the index notation is  $x_n^c$  for the  $n$ th cycle. In the same way, its time length is noted  $T^c$ . With such a notation, it becomes possible to describe a round trip  $C_o$  of half an hour, from a sequence of PWM cycle  $M$  of 100  $\mu$ s:

$$C_o(x_n^c, t_n^c) = \frac{1}{T^c} \sum_{j=0}^{18,000,000} T^M M(x_j^M, t_j^M) \quad (8)$$

The same equation is used to compute a *rest* cycle  $C_r$  of the same length but with a null power profile. One can then compute an operating day  $D_o$  from the previous cycles  $C_o$  and  $C_r$ :

$$D_o(x_n^d, t_n^d) = \frac{1}{T^d} \left[ \sum_{j=0}^{17} T^c C_o(x_j^c, t_j^c) + \sum_{j=18}^{23} T^c C_r(x_j^c, t_j^c) \right] \quad (9)$$

where the index  $d$  is used for all variables relating to the day. A day of maintenance  $D_m$  is computed from a sequence of idle cycles  $C_r$ , such as:

$$D_m(x_n^d, t_n^d) = \frac{1}{T^d} \sum_{j=0}^{23} T^c C_r(x_j^c, t_j^c) \quad (10)$$

Finally, it becomes easy to build a year  $Y$  from day cycles  $D_o$  and  $D_m$ :

$$Y(x_n^y, t_n^y) = \frac{1}{T^y} \left[ \sum_{j=0}^{334} T^d D_o(x_j^d, t_j^d) + \sum_{j=335}^{364} T^d D_m(x_j^d, t_j^d) \right] \quad (11)$$

and the 20 years of exploitation from a sequence of years  $Y$ :

$$Pb(x_n^{pb}, t_n^{pb}) = \frac{1}{T^{pb}} \sum_{j=0}^{20} T^y Y(x_j^y, t_j^y) \quad (12)$$

But the set of equations (7) to (12) is just a rewrite of the problem. It does not reduce the computational effort. In order to do this, it is necessary to use approximation functions, as given by equation (6), where it is not necessary to compute all the sub-cycles of a sequence, but only few of them. The next section present such extrapolation methods.

#### 2.4. Extrapolation methods

The main idea of the extrapolation principle, presented in (Hmam et al., 2016a), is to reduce the computation time by evaluating a minimum number of cycles in a sequence. In order to solve ordinary differential equations, variable step integration schemes, such as Runge-Kutta methods, are commonly used (Holsapple et al., 2007; Herty et al., 2013; Anastassi and Kosti, 2015). In the multi-layer formulation proposed in this paper, such an integration scheme can be used to help evaluating the evolution of the state vector over many successive cycles. However, it must be adapted because the problem to solve is not an ordinary differential equation but a discrete iterative equation, as given by equation (6). The most straightforward adaptation from the continuous to discrete scheme is obtained by the second order method Heun-Euler (Dormand and Prince, 1980; Fehlberg, 1969). Indeed, this method only needs 2 evaluations, at the beginning and at the end of the interval. Figure 4 gives a discretized version of this explicit second order method (Hmam et al., 2016a). The concept of *integration* is now replaced by the concept of *extrapolation*. Then, for an extrapola-

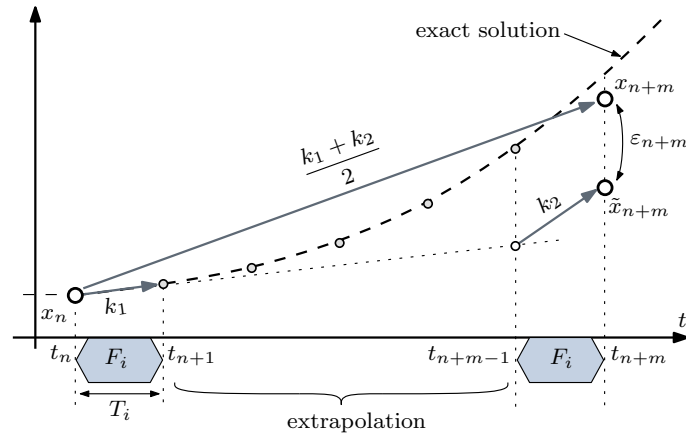


Figure 4: Illustration of the Heun-Euler embedded method, adapted to this multi-layer formulation.

tion of  $m$  successive cycles  $F_i$  and starting from a time  $t_n$ , it is possible to compute an approximation of the state vector variation with the evaluation of only two sub-cycles, which are the first and the last one. The extrapolation is then considered between  $n + 1$  and  $n + m - 1$  and is obtained considering the following equations (Hmam et al., 2016a):

$$\begin{aligned}
k_1 &= F_i(x_n, t_n) \\
k_2 &= F_i(x_n + (m-1)T_i k_1, t_n + (m-1)T_i) \\
x_{n+m} &= x_n + m T_i \frac{1}{2} (k_1 + k_2) \\
\tilde{x}_{n+m} &= x_n + T_i ((m-1)k_1 + k_2) \\
\varepsilon_{n+m} &= \tilde{x}_{n+m} - \tilde{x}_{n+m} = T_i \frac{m-2}{2} (k_2 - k_1)
\end{aligned} \tag{13}$$

Such a second order and explicit method is very efficient for nonstiff problems and for a small number of extrapolation cycles. In the case of stiff problems with large extrapolations, it is preferable to use implicit methods. The main difficulties with this kind of method are the need to evaluate the Jacobian matrix and the use of many iterations to converge toward a solution. That makes these implicit solvers highly stable, but also computationally intensive. A compromise is the use of semi-implicit methods, such as the matlab `ode23s` method, which is based on a Rosenbrock method and whose algorithm is detailed in (Ashino et al., 2000). It is called semi-implicit because it only uses one iteration to converge and it is assumed that the Jacobian matrix is constant on the extrapolation interval. Considering these assumptions and by a reformulation of the ODE solver in a discrete form, a condensed form of the implemented algorithm is given by the next set of equations (Ashino et al., 2000):

$$\begin{aligned}
f_0 &= F_i(x_n, t_n) \\
k_1 &= W^{-1} f_0 \\
f_1 &= F_i(x_n + 0.5 m T_i k_1, t_n + 0.5 m T_i) \\
k_2 &= W^{-1}(f_1 - k_1) + k_1 \\
x_{n+m} &= x_n + m T_i k_2 \\
f_2 &= F_i(x_{n+m}, t_{n+m}) \\
k_3 &= W^{-1}(f_2 - c_{32}(k_2 - f_1) - 2(k_1 - f_0)) \\
\varepsilon_{n+m} &= \frac{m T_i}{6} (k_1 - 2 k_2 + k_3)
\end{aligned} \tag{14}$$

where

$$W = \mathbb{I} - m T_i d J, \quad d = \frac{1}{2 + \sqrt{2}}, \quad c_{32} = 6 + \sqrt{2} \tag{15}$$

and

$$J \simeq \frac{\partial F(x_n, t_n)}{\partial x} \tag{16}$$

It can be noted that this method is a *first step at last* and allows to use the evaluation  $f_2$  both at the end of a step and at the beginning of the next one. Moreover, an expensive operation in computation time

is the numerical evaluation of the Jacobian matrix. It is the reason why only one evaluation is done per extrapolation step. Now that the multi-layer and cycle-based formulation is presented, it can be applied to the *Ar Vag Tredan* Ferry.

### 3. Application of the multi-layer formulation to the *Ar Vag Tredan* ferry

It is proposed to apply the idea of cycle-based decomposition of the operation profile of the *Ar Vag Tredan* ferry, presented earlier in this paper (Olivier et al., 2014; Trieste et al., 2015). The objective is to be able to simulate, with a single model, all the dynamics of the power storage unit and its power converter. Figure 5 illustrates a simplified diagram of the power chain, including the supercapacitors, the power converter with the smoothing inductor, and the PI controller for the current regulation. A thermal model is also considered to take into account the aging of the supercapacitor parameters. The next section is dedicated to the presentation of the multi-physics modeling.

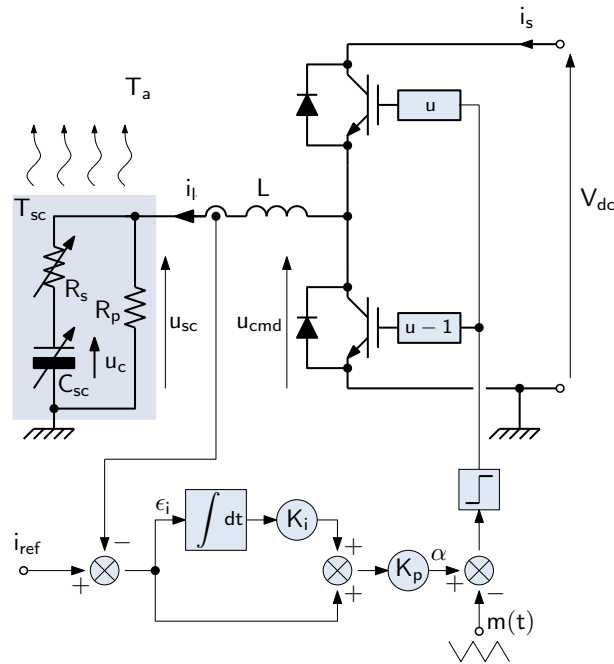


Figure 5: Diagram of the energy storage system.

#### 3.1. System modelling

As said before, this application can be divided into three main parts, which are the supercapacitors, the power converter with its smoothing inductor, and finally the current controller with its modulation function.

It should be recalled that the main objective of this work is to propose a full simulation of the system, from the PWM to the lifetime of the supercapacitors, without formulating any hypothesis or reduction of the proposed models. This is why we don't use average equivalent models, neither steady state assumption for the faster dynamics (Hmam et al., 2016a; Bouabdallah et al., 2013; Prieto et al., 2016).

For the supercapacitors, electrical, thermal and aging phenomena (Kreczanik et al., 2014; Trieste et al., 2015) are taken into account through four state variables:

$$\frac{du_c}{dt} = \frac{i_c}{C_{sc}} \quad (17)$$

$$\frac{dT_{sc}}{dt} = \frac{1}{C_{th}} \left( P_\phi - \frac{T_{sc} - T_a}{R_{th}} \right) \quad (18)$$

$$\frac{dC_{sc}}{dt} = K_{c0} C_{sc}(0) k_{sc}(u_c, T_{sc}) \quad (19)$$

$$\frac{dR_s}{dt} = K_{r0} R_s(0) k_{sc}(u_c, T_{sc}) \quad (20)$$

which need the next intermediate equations:

$$i_c = \frac{R_p i_l - u_c}{(R_s + R_p)} \quad (21)$$

$$P_\phi = R_s i_c^2 + R_p (i_l - i_c)^2 \quad (22)$$

$$k_{sc}(u, T) = 2^{\frac{u-U_0}{\Delta U}} 2^{\frac{T-T_0}{\Delta T}} \quad (23)$$

In this work, the power converter is considered perfect. This assumption seems very restrictive, but in reality it is very easy to consider a dynamic power stage and to include its losses. But to keep a certain simplicity on the writing of the model, it is preferred here to consider only the sequential aspect of the commutation law of the power stage. Then, the output voltage  $u_{cmd}$  is described by a simple switching function such as:

$$u_{cmd} = \begin{cases} V_{dc} & \text{if } u = 1 \\ 0 & \text{otherwise} \end{cases} \quad (24)$$

The control signal  $u$  is based on a natural modulation between the setting duty-cycle  $\alpha$  calculated at the output of the PI current controller and a symmetric triangular carrier  $m(t)$  (normalized to [0;1]). Written under the formalism of an ODE, the PWM signal is given by:

$$m(t) = \frac{1}{T^M} (t \bmod T^M) \quad (25)$$

$$u(t) = \frac{1}{2} [1 + \text{sgn}(\alpha - m(t))] \quad (26)$$

where  $\alpha$  is the duty cycle,  $(. \text{ mod } .)$  is the modulo operation and  $\text{sgn}(\cdot)$  the sign function. Finally, the modeling of the PI controller of the current  $i_l$  in the smoothing inductor requires two additional state variables given by:

$$\frac{di_l}{dt} = \frac{u_{cmd} - (u_c + R_s i_c)}{L} \quad (27)$$

$$\frac{d\epsilon_i}{dt} = i_{ref} - i_l \quad (28)$$

where  $i_c$  is obtained from equation (21). The control angle  $\alpha$  is then obtained by using the next equation:

$$\alpha = \epsilon_i K_p + (i_{ref} - i_l) K_i \quad (29)$$

For the current setting  $i_{ref}$ , two different strategies are considered. The first one, acting during the discharge of the supercapacitor and during the first phase of the recharge, consists in imposing the power profile given in Figure 1:

$$i_{ref} = \frac{P_{sc}(t)}{u_c} \quad (30)$$

This strategy is used while the supercapacitor voltage  $u_{sc}$  does not exceed the end of charge voltage  $U_{eoc}$ . This is why a proportional action is implemented at the end of the phase of recharge:

$$i_{ref} = \min(P_{sc}(t); (U_{eoc} - u_c)K_{eoc}) \quad (31)$$

All the system parameters are summarized in Table 1. All the different models of this powertrain are detailed in Trieste et al. (2015, 2011). In these works, the electrical, thermal and aging models are presented and compared to experimental data.

### 3.2. Cycle-based formulation

The simulation of this system is done using the cycle-based formulation presented in Section 2. The first step of this method is to write the problem into classical ODEs functions. Indeed, it permits to use classical solvers on the lower layer, which corresponds in this example to the PWM cycle. The considered state vector for this problem is given by

$$x = [u_c \ T_{sc} \ C_{sc} \ R_s \ i_l \ \epsilon_i]^T \quad (32)$$

with, as a reminder,  $u_c$  the supercapacitor voltage,  $T_{sc}$  its temperature,  $C_{sc}$  its capacitance,  $R_s$  its equivalent serial resistance,  $i_l$  the current in the smoothing inductor and  $\epsilon_i$  the integrator state of the PI controller.

Table 1: System parameters.

Symbol	Quantity	Value
$L$	Smoothing inductance	1 mH
$R_p$	Balancing resistor	500 $\Omega$
$R_s$	Serial resistor	1.45 m $\Omega$
$C_{sc}$	Equivalent capacitance	400 F
$N_s$	Number of cells	216
$K_i$	Integrator gain	700
$K_p$	Proportional gain	1.5
$K_{eoc}$	Proportional gain	30 kW/V
$U_{eoc}$	End of charge voltage	583.2 V
$V_{dc}$	DC-bus voltage	600 V
$U_0$	Nominal voltage	583.2 V
$\Delta U$	Aging sensitivity	43.2 V
$T_0$	Nominal temperature	25 $^{\circ}\text{C}$
$\Delta T$	Aging sensitivity	10 $^{\circ}\text{C}$
$K_{c0}$	capacitance degradation rate	-1.5%/yr
$K_{r0}$	resistance degradation rate	+10%/yr
$R_{th}$	thermal resistance	5 $^{\circ}\text{C}/\text{kW}$
$C_{th}$	thermal capacitance	500 kJ/ $^{\circ}\text{C}$
$T^M$	PWM period	100 $\mu\text{s}$

Then, the set of previous equations is rewritten to obtain three ODE:

$$\frac{dx}{dt} = f_M(x, t) \quad (33)$$

$$\frac{dx}{dt} = f_{Cr}(x, t) \quad (34)$$

$$\frac{dx}{dt} = f_{Dm}(x, t) \quad (35)$$

These three fundamental ODE correspond to the system equations during respectively the working phases, the 8 hours of rest per day and the 30 days of maintenance per year (see Figure 2). The second step consists in the calculation of the average state vector variations at the scale of each cycle. The only difference between these three ODEs is the power profile  $P_{sc}$  considered on the cycle. For the ODE given by equation (33), this power profile corresponds to one round-trip, as shown in Figure 1. For the ODEs given by equations (34) and (35), the power profile  $P_{sc}$  is null over the cycle and the power converter is off. We can

therefore consider that:

$$f_{Cr}(t, x) = f_{Dm}(x, t) \quad (36)$$

Then, a PWM cycle is computed with the system given by equation (33) and by using a classical ODE solver, such as (Hmam et al., 2016a):

$$M(t, x) = \frac{1}{T^M} \int_t^{t+T^M} f_M(x, t) dt \quad (37)$$

where  $T^M$  is called the *Horizon* of the cycle. Here, it corresponds to the period of the PWM signal. The solver is here a classical explicit Runge-Kutta method based on Dormand-Prince pair (Dormand and Prince, 1980; Sanchez-Gasca et al., 1995). The same calculation is done for the daily and annual rest periods, with respectively the functions  $f_{Cr}$  and  $f_{Dm}$ :

$$C_r(t, x) = \frac{1}{T^c} \int_t^{t+T^c} f_{Cr}(x, t) dt \quad (38)$$

$$D_m(t, x) = \frac{1}{T^d} \int_t^{t+T^d} f_{Dm}(x, t) dt \quad (39)$$

$$(40)$$

with  $T^d$  for one day and  $T^c$  for one cycle of a half hour. The last step of the method is the extrapolation of these *mean* variations over few or several cycles. These extrapolations are fulfilled by modified continuous solvers given by the sets of equations (13) and (14). Different discrete solvers can be used for each layer, i.e. for the extrapolation of PWM cycles, round-trips, days and years. When a sequence is composed of a small number of cycles, an explicit and low order method should be preferred. The main reason is that whatever the number of extrapolated cycles, a second order explicit solver needs two cycles, while an implicit third order solver needs at least 9 function evaluations (including 6 evaluations for the Jacobian in the case of this 6th order ODE problem). The efficiency of the method can then be highly reduced if the extrapolations are short too. It is why the implicit method is used only for the extrapolation of the PWM cycles. Indeed, this layer involves an high number of consecutive cycles to pass from the switching period (which is 100  $\mu$ s in this study) to one round-trip (of 1/2 h). Then, only implicit formulations are stable and efficient enough to give accurate results even with these very large extrapolation steps. Conversely, for the upper layers, the number of successive cycles is relatively small. Low order explicit extrapolation methods are then used for the computation of the days, years and lifetime cycles.



#### 4. Simulation results

Figure 6 gives the simulation results obtained for the first round-trip of the ferry.

These results are obtained by using a classical `ode45` matlab solver on the PWM cycle with a maximum relative tolerance of  $1E-7$ . The extrapolation of PWM cycles is computed with a relative tolerance of  $1E-2$  with an implicit third order solver given by the set of equations (14). A focus of this result is given by Figure 7, where only the first 4 ms are illustrated (i.e. first 40 PWM cycles). We can see that the step size is automatically adapted to the states variation speed. During the transient response of the current controller, the extrapolation is of few PWM cycles only, to keep the extrapolation error of the current  $i_l$  and the PI integrator  $\epsilon_i$  under the maximum relative tolerance. As soon as this transient state is over, the step size is highly increased to reach extrapolations of more than 100 seconds (ie more than 1,000,000 PWM cycles). It is then obvious that such extrapolations are possible only by using an implicit solver and only if the lower layer is computed with a small tolerance. All-in-all, for a relative tolerance of  $1E-7$  for the PWM layer and a tolerance of  $1E-2$  for the round-trip layer, the number of PWM cycles that are computed (including rejected steps) is 20,062. The calculation effort is then reduced by a factor of 9000 for this layer.

Thanks to the recursive form of this formulation, it becomes easy to simulate one day from the extrapolation of round-trip cycles. This result is given by Figure 8 and by considering for this layer a relative tolerance of  $1E-2$ .

Here, the dominant phenomenon is the temperature variation. This time, the used method is a second order with an explicit formulation. Then, each cycle needs two evaluations. We can see that 15 cycles are computed, then the speed-up factor is of 1.6, which is a much less value than for the previous layer. It is mainly due to the small number of consecutive cycles, which do not permits large extrapolations. Moreover, the thermal constant imposes to calculate the transient states with cycles close to each other. Nevertheless, the gain are cumulative and the total computation effort for both layers is close to 15,000.

The last result presented in this paper is the simulation of one year from daily-cycles (see Figure 9). It is obtained with a second order and explicit solver, with a relative tolerance of  $1E-2$ . This time, 18 daily-cycles evaluations have been necessary to simulate 365 days. The speed-up factor is then close to 20.

When we compute the 20 years of operation of the ferry (not illustrated here) by using annual-cycles, a gain of 2.5 is obtained for a tolerance of  $1E-2$ .

All-in-all, while a full-computation of the problem needs the evaluation of more than  $10^{12}$  PWM cycles, the proposed method permits to reduce this number of evaluation to about  $10^7$ . Another point of view is to

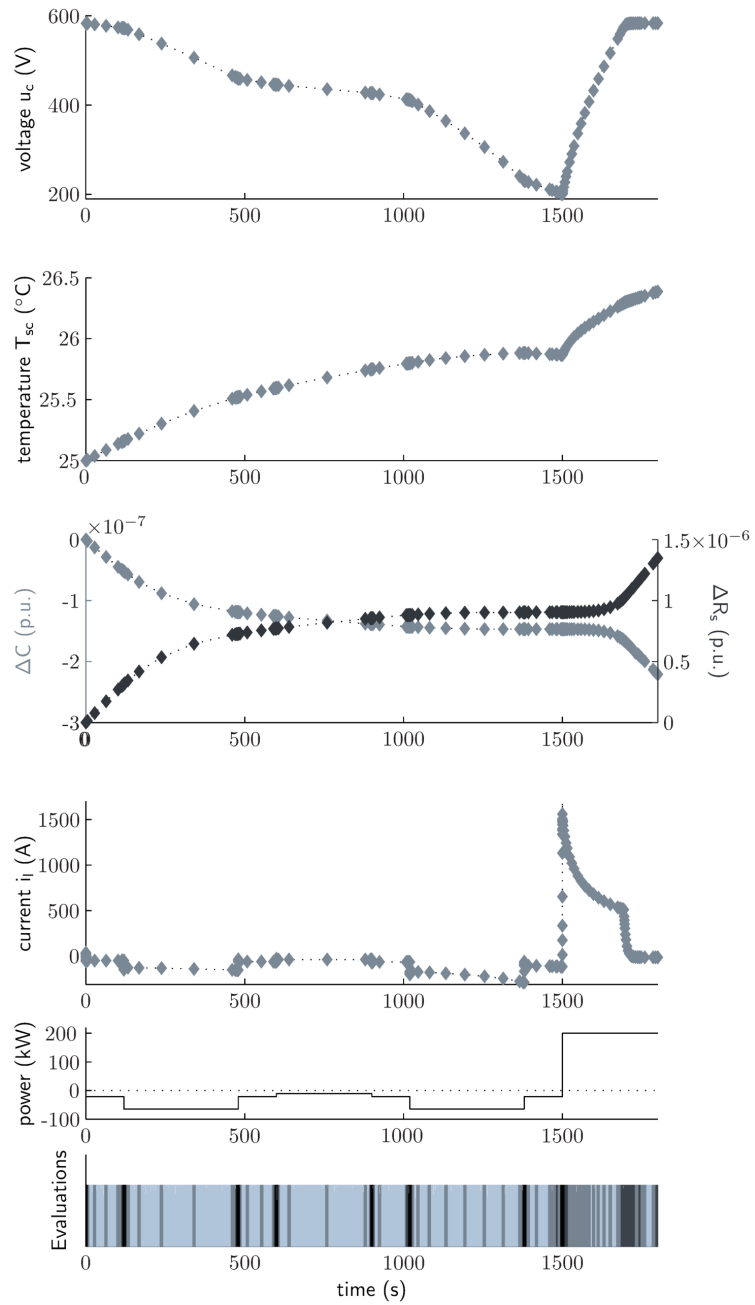


Figure 6: Simulation result for the round-trip layer, composed of PWM cycles.

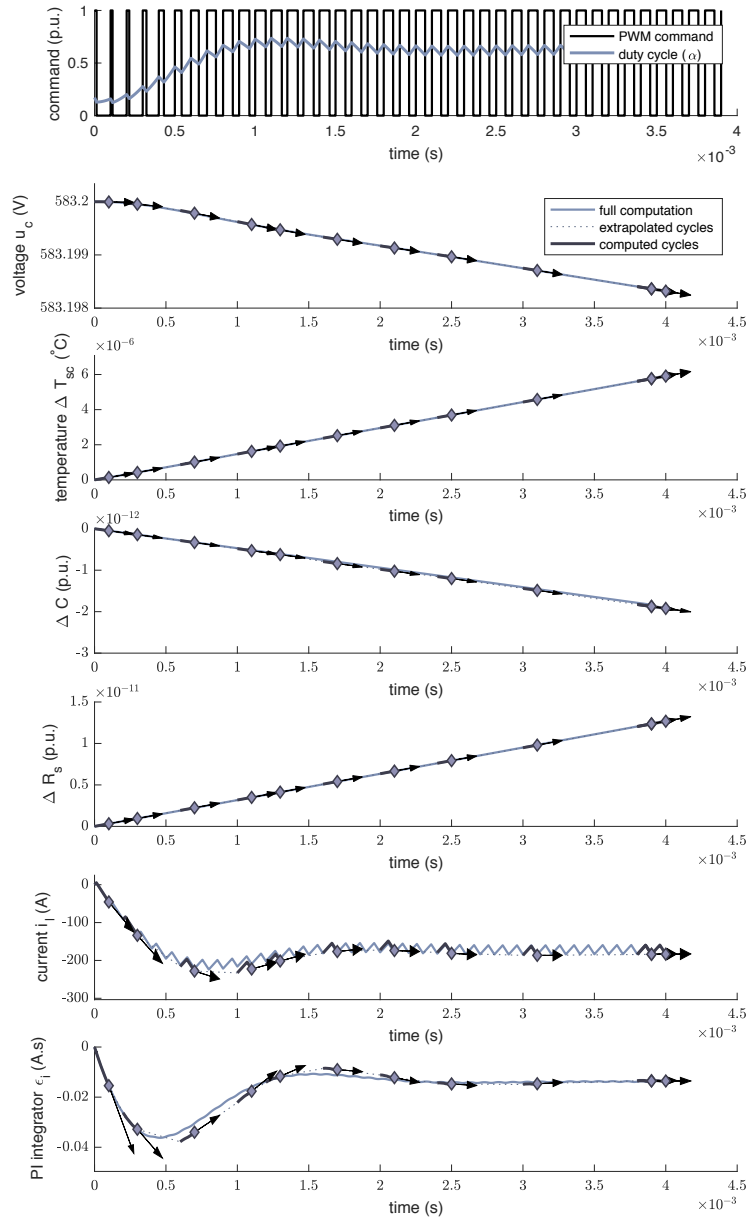


Figure 7: Focus on the first 4 ms of the simulation of one round-trip, composed of PWM cycles.

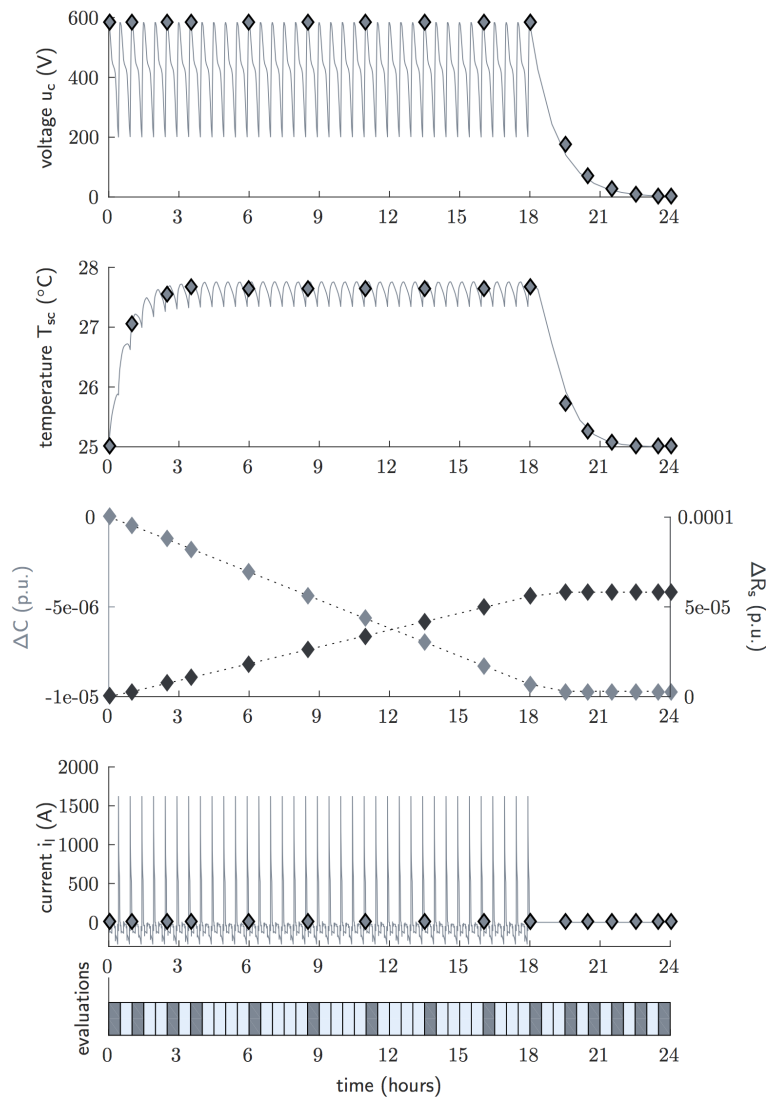


Figure 8: Simulation for one day calculated from round-trip and rest cycles.

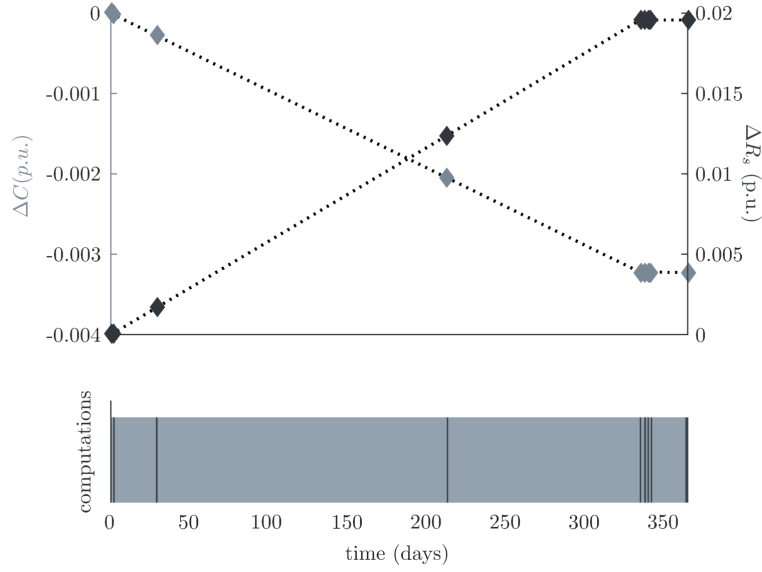


Figure 9: Simulation for one year calculated from operating days and rest days.

consider the effort on the simulation time. Even if a full computation of this problem is impossible with a classical personal computer, a quick estimation from the full simulation of one hour of operation leads to a time of 10 centuries to solve the 20 years of lifetime. The proposed method permits to reduce this time to only 15 days.

## 5. Conclusion

In this paper, a cycle-based formulation of initial value dynamical problems is presented. It has been proposed to address the problem of the simulation of large timescale and multi-physics applications, especially when it is possible to split the problem in a sequence of few elementary initial value problems. By also offering a multi-layer formulation, it becomes possible to build recursive cycles at different time scales, and thus to take into account all periodicities in the operation of a system. This method is applied to the simulation of a full-electric ferry, which only uses supercapacitors as energy storage device. With this original approach, it becomes possible to simulate in the same time and with the same set of equations, the electrical, thermal and aging behaviour of the supercapacitor and its current controller, from the scale of the PWM cycle ( $100 \mu\text{s}$ ), until its total operating lifetime (20 years). A speed up factor of about  $10^5$  on the simulation time is obtained. As a conclusion, this work proposes an original framework that makes the simulation of dynamic and multi timescale systems more affordable, mainly when the operating profile can

be reduced to a sequence of recursive cycles. A possible evolution of this work is to improve its integration with classical multi-physics modeling software such as Simscape or Modelica. This would drastically speed up the simulation time of multi-physics systems without spending time to reformulate the model at the different time scales.

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