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# A hybrid memetic-ant colony optimization algorithm for the home health care problem with time window, synchronization and working time balancing

Jérémy Decerle<sup>a,\*</sup>, Olivier Grunder<sup>a</sup>, Amir Hajjam El Hassani<sup>a</sup>, Oussama Barakat<sup>b</sup>

<sup>a</sup>Nanomedicine Lab, Univ. Bourgogne Franche-Comté, UTBM, F-90010 Belfort, France

<sup>b</sup>Nanomedicine Lab, Univ. Bourgogne Franche-Comté, F-25000 Besançon, France

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## Abstract

This paper addresses the routing and scheduling of caregivers in a home health care problem. In order to obtain a valid planning, some skill, time window, and synchronization constraints must be met. Since the increase in demand, organizations providing home health care are eager to optimize the planning of caregivers which is often performed manually. Thus, many works have emerged on this research problem, taking into account new constraints gradually. One interesting aspect is the workload balancing between caregivers. Indeed, the workload must be roughly the same to obtain fairness. Already applied successfully to similar problems, the ant colony optimization algorithm has never been applied to the home health care problem. As a result, an original hybrid algorithm combining memetic and ant colony optimization algorithm is suggested for solving the home health care problem with working time balancing. Computational results on benchmark instances from the literature highlight the efficiency of the proposed hybrid algorithm in comparison with other metaheuristics and a commercial optimization solver.

*Keywords:* home health care, memetic algorithm, ant colony optimization, workload balance

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## 1. Introduction

Home health care (HHC) services provide nursing and general hygiene cares at patient home for people in a situation of dependency. Different types of care are performed depending on the need of the patients. The caregivers of the home health care structures are often composed of nurses and auxiliary nurses, in order to provide the full range of cares. Nowadays, the planning of the caregivers is performed manually by an experienced nurse who has the role of coordinator. This complex planning often requires an extensive effort and time in order to obtain a valid planning which respects all the constraints (availability of the patients, right qualification of the caregiver, ...). Since this task is performed daily, an important improvement is possible in favor of a better planning of the caregivers.

Because of the overall aging of the population and also because of the need of patients to recover at home, which is possible due to technology advances as explained by Christensen et al. (2009), the demand for home health care is rising as reported by Bertrand (2010) in France.

In the early literature related to the home health care problem, Begur et al. (1997) and Cheng and Rich (1998)

were among the first who addressed this problem from an operations research point of view. Begur et al. (1997) have developed a decision support system to address the routing and scheduling of nurses, while Cheng and Rich (1998) have considered the HHC problem as a vehicle routing problem with time window (VRPTW) including multiple depots. Their modelization of the problem aims at minimizing the overtime of full-time nurses and the work of part-time nurses scheduled.

Subsequently, more and more works on this subject have appeared in the literature revealing new constraints and objectives to be taken into account.

In order to obtain a complete overview of the existing work on the home health care problem, the recent literature reviews of Fikar and Hirsch (2017) and Cissé et al. (2017) present a comprehensive state of the art.

Among the constraints frequently taken into account, time windows regarding patients' availability or the working hours of caregivers have been studied by Affi et al. (2016). They defined a service time window that specifies the earliest and latest possible starting time of the service for a given patient. As a result, if the caregiver arrives too early at the patient's home, the caregiver must wait until the beginning of the time window of availability to cure the patient. In opposite, late arrivals at patient's home are not possible.

In their work, some patients may require more than one caregiver to perform the care. Bredström and Rönnqvist (2008) have also studied this variant of the home health care problem by synchronizing the routes of two caregivers

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\*Corresponding author

Email addresses: [jeremy.decerle@utbm.fr](mailto:jeremy.decerle@utbm.fr) (Jérémy Decerle),  
[olivier.grunder@utbm.fr](mailto:olivier.grunder@utbm.fr) (Olivier Grunder),  
[amir.hajjam-el-hassani@utbm.fr](mailto:amir.hajjam-el-hassani@utbm.fr) (Amir Hajjam El Hassani),  
[oussama.barakat@univ-fcomte.fr](mailto:oussama.barakat@univ-fcomte.fr) (Oussama Barakat)

54 so they both arrive at the same time at the patient's home.<sup>111</sup>  
55 To do so, two visits from two different caregivers need<sup>112</sup>  
56 to be synchronized in order to have the same start time.<sup>113</sup>  
57 Synchronization of visits is frequently needed when the<sup>114</sup>  
58 care requires more than one person like lifting the patient<sup>115</sup>  
59 or when different skills are required.<sup>116</sup>

60 Furthermore, Haddadene et al. (2016) have introduced<sup>117</sup>  
61 the notion of caregivers' qualification. Indeed, all care can<sup>118</sup>  
62 not be performed by the same type of caregivers. There<sup>119</sup>  
63 fore, it is necessary to define the qualification of caregivers<sup>120</sup>  
64 to provide care. For instance, an injection must be per<sup>121</sup>  
65 formed by a nurse, and auxiliary nurses are not qualified<sup>122</sup>  
66 to perform this type of care.<sup>123</sup>

67 Similar to the caregivers' qualification, Gayraud (2015)<sup>124</sup>  
68 have introduced the notion of experience concerning the<sup>125</sup>  
69 nurses. They have focused on the nurse's know-how and<sup>126</sup>  
70 not on his choices. A nurse is characterized by his skills,<sup>127</sup>  
71 his experience, and the languages he masters. Finally, this<sup>128</sup>  
72 category also includes the problems of over-qualification.<sup>129</sup>  
73 In a home-based hospitalization structure, nurses may be<sup>130</sup>  
74 required to perform nursing care, for example, which is<sup>131</sup>  
75 more of an auxiliary nurse job.<sup>132</sup>

76 In addition, while caregivers may have a preference to<sup>133</sup>  
77 cure some patients, a patient may also prefer to be treated<sup>134</sup>  
78 by a specific caregiver. A patient may choose to be treated<sup>135</sup>  
79 by a woman rather than a man or vice versa. A patient<sup>136</sup>  
80 may express a preference to a nurse who speaks the same<sup>137</sup>  
81 language as him or always the same nurse, always at the<sup>138</sup>  
82 same time who visits him.<sup>139</sup>

83 The well-being of the patient is thus a concern taken<sup>140</sup>  
84 into account in many works in wanting to reduce the in<sup>141</sup>  
85 convenience of the patient. As a result, Braekers et al.<sup>142</sup>  
86 (2016) have proposed a bi-objective modelization of the<sup>143</sup>  
87 problem seeking to minimize the operating costs for the<sup>144</sup>  
88 service provider and the patient inconvenience. The au<sup>145</sup>  
89 thors have analyzed the trade-off between these two con<sup>146</sup>  
90 flicting objectives.<sup>147</sup>

91 As well, many works consider a planning horizon of one<sup>148</sup>  
92 or several weeks, unlike most papers in the literature fo<sup>149</sup>  
93 cus on the daily planning. Thus, Nickel et al. (2012) have<sup>150</sup>  
94 sought to obtain a weekly optimal plan by using differ<sup>151</sup>  
95 ent metaheuristics combined with methods from constraint<sup>152</sup>  
96 programming. Similarly, Duque et al. (2015) have consid<sup>153</sup>  
97 ered a planning horizon of several weeks to maximize the<sup>154</sup>  
98 service level and to minimize the distance traveled by the<sup>155</sup>  
99 caregivers.<sup>156</sup>

100 One interesting aspect of the HHC problem is the work<sup>157</sup>  
101 load balance between caregivers. Indeed, the workload<sup>158</sup>  
102 must be roughly same to obtain fairness. However, we<sup>159</sup>  
103 noticed that depending on the application, the workload<sup>160</sup>  
104 can be defined differently. Indeed, the workload has been<sup>161</sup>  
105 considered only as the sum of cares duration by Bredström<sup>162</sup>  
106 and Rönnqvist (2008) or as the sum of the routing costs by<sup>163</sup>  
107 Liu et al. (2013a). Many workload measurement systems<sup>164</sup>  
108 have been developed to satisfy patient needs in specified<sup>165</sup>  
109 areas as presented by Siew and Ghani (2006).<sup>166</sup>

110 In this study, the workload is considered equal to the<sup>167</sup>

working time of a caregiver, in order to obtain a similar  
working time among all caregivers. As a result, one of  
the objectives to optimize is to reduce the maximal work-  
ing time difference among caregivers. To the best of your  
knowledge, there is no previous work aiming to optimize  
the following three objectives: decrease the traveling time  
of the caregivers, provide the best quality of service to the  
patients and minimize the maximal working time differ-  
ence between the caregivers.

Regarding the solution methods used in the literature,  
the majority of works have developed metaheuristic solu-  
tion procedures to solve the home health care problem.  
As a result, many approximate methods such as particle  
swarm optimization by Akjiratikar et al. (2007), variable  
neighborhood search by Issaoui et al. (2015), genetic al-  
gorithm by Liu et al. (2013b), tabu search by Rest and  
Hirsch (2016), etc. have been implemented.

As an example, Afifi et al. (2016) developed a simulated  
annealing based algorithm (SA-ILS) for a variant of the  
vehicle routing problem with time windows and synchron-  
ized visits (VRPTWSync).

Mathheuristics have also been used to solve the HHC  
problem. Allaoua et al. (2013) developed a matheuristic  
based on the decomposition of the integer linear program-  
ming formulation into two problems. The first one is a set  
partitioning like problem and it represents the rostering  
part. The second problem consists in the routing part.  
This latter is equivalent to a multi-depot traveling sales-  
man problem with time windows (MTSPTW)

In addition, memetic algorithm (MA) have also been  
applied to the home health care problem. Memetic algo-  
rithms is a marriage between a population-based global  
search and the heuristic local search made by each of the  
individuals according to Moscato et al. (1989). **Widely ap-  
plied to numerous optimization problems, memetic algo-  
rithms have also shown theoretical excellent performances  
in solving some machine learning and data mining tasks,  
such as pruning of Pareto optimal solutions ensemble by  
Qian et al. (2015) and influence maximization by Qian  
et al. (2018).**

As defined by Neri et al. (2012), memetic computing  
is a broad subject which studies complex and dynamic  
computing structures composed of interacting modules  
(memes) whose evolution dynamics is inspired by the dif-  
fusion of ideas. Memes are simple strategies whose har-  
monic coordination allows the solution of various prob-  
lems. **Towards this end, Ong et al. (2010) have related  
that the earliest and fastest growing areas of memetic com-  
puting research is memetic algorithm (MA). It is indeed  
one of the most direct applications of memetic comput-  
ing in problem-solving. A detailed review on the recent  
memetic algorithms, hybrids, and adaptive hybrids is pro-  
vided by Chen et al. (2011) in their survey on memetic  
computation.**

Consequently, the hybrid memetic-ant colony optimiza-  
tion algorithm suggested in this paper belongs to the cate-  
gory of memetic computing since the memes interact dur-

ing the optimization process. Following the classification suggested by Ong et al. (2006), it belongs to the category of adaptive hyper-heuristic where the coordination of the memes is performed by means of heuristic rules, actually following a randomized scheme. The literature review proposed by Neri and Cotta (2012) offers an extensive study of memetic algorithms and memetic computing optimization.

Besides approximate methods, exact solution procedures have also been implemented like the branch-and-price. As mentioned by Riazi et al. (2017), the most successful exact algorithms for VRPTW are based on branch-and-price framework, which combine column generation and branching.

In this regard, Rasmussen et al. (2012) have modeled the problem as a set-partitioning problem with side constraints and develop an exact branch-and-price algorithm, as this method has previously given solid results for classical vehicle routing problems. Temporal dependencies are modelled as generalised precedence constraints and enforced through the branching.

Similarly, Evesborn et al. (2006) utilized a set-partitioning model and solved the master problem using a repeated matching algorithm. This is a heuristic approach which has been successfully applied to real-world home care problems.

Another branch-and-price algorithm for the home health care problem has been suggested by Yuan et al. (2015). They studied the problem with stochastic service times. To solve it, the authors have transformed into a master problem and a pricing sub-problem. They developed a column generation algorithm to solve the relaxation of the master problem and obtain its lower bound, and devised a label algorithm and several effective accelerating techniques to solve the pricing sub-problem. To obtain feasible solutions, their column generation procedure is embedded within a branch and bound framework.

Among all the solution approaches already implemented, the ant colony optimization algorithm (ACO) has never been applied to the home health care problem to the best of our knowledge. However, ACO has been already successfully applied to similar problems such as the multiple traveling salesmen problem by Liu et al. (2009).

Thus, the aim of this paper is to evaluate the efficiency of the ACO on the home health care problem. It is an opportunity to evaluate its efficiency and compare its results with others solving methods.

As well, a hybrid memetic-ant colony optimization algorithm (MACO) is suggested in this paper. MACO is a hybridization of two algorithms: memetic algorithm and ant colony optimization algorithm. In previous work, Decerle et al. (2018) have suggested an efficient memetic algorithm for the home health care problem with soft time window and synchronization constraints. Since previous works on the hybridization of these two algorithms have shown their efficiency on other applications, the authors have decided to evaluate this hybrid algorithm on the home health care

problem.

To the best of our knowledge, this is the first work on memetic computing applied to the home health care problem that combines ant colony optimization and memetic algorithms. As well, both structures share the same population of solutions and update the pheromone matrix.

To summarize, the main contributions of this paper are as follows. **First, this work studies a variant of the home health care problem with soft time window and synchronization constraints, and working time balancing. Secondly, a hybrid memetic-ant colony optimization algorithm is suggested to tackle the proposed problem. Lastly, the performances of the memetic, ant colony optimization and the hybrid of the previous two algorithms are evaluated on benchmark instances from the literature. Results are compared with best solutions obtained by a commercial solver.**

This paper is outlined as follows. In the next section, characteristics of the studied system are detailed. In section 3, the mixed-integer programming formulation of the home health care problem is presented with a description of the various parameters, variables, and constraints taken into account. In Section 4, a description of the three solution approaches including the hybrid memetic-ant colony optimization algorithm is provided. The description of the test instances, experiments settings and an analysis of the computational results is presented in Section 5. Finally, some concluding remarks, as well as some perspectives on future works, will be drawn in the last section.

## 2. System definition

The home health care problem is defined as follows. Given a set of caregivers and a set of patient visits, the goal is to find a valid planning for a one-day period for each caregiver. The resulting planning must indicate which visit should be carried out by which caregiver and when the visit should begin. Each caregiver has a defined working time window, which means they only work during this time. As well, caregivers have an assigned home health care office from which they start and end their workday. Each caregiver uses the same mode of transportation (i.e. a car provided by the home health care company). Working overtime is not allowed due to increased work costs for the company. Home health care staff are either nurses or auxiliary nurses.

The time at which a caregiver must start a visit is limited by a time window. Therefore, the patient and the home health care company agree on a time window in which care must begin. However, this time window can be partially satisfied by starting the visit shortly before or after the patient's preference. Each patient time window is specific depending on the availability of the patient or the type of care to be provided. For this purpose, the level of tolerance to respect the time window is different depending on the care to be provided. Cares are performed by a single category of caregiver (for example, an injection is per-

formed only by a nurse). Therefore some visit/caregiver combinations are not valid. Some visits must be synchronized when health care needs to be done by two caregivers simultaneously. In order to penalize the potential gap in arrival time between the two staff members, we consider a fixed time window of duration zero and the quality of service decreases as the arrival times of the caregivers are different.

The quality of service is optimal if all cares begin during the patient's availability time window and the arrival times of caregivers are equal when they have to perform a synchronized visit.

Furthermore, the home health care company aims to improve the planning of caregivers' routes by balancing the working time among caregivers, in order to obtain a fair planning.

As a result, the home health care company seeks to optimize the three following objectives: decrease the traveling time of the caregivers, provide the best quality of service to the patients and minimize the maximal working time difference between the caregivers.

### 3. Model description

In this section, the mixed-integer programming formulation of the home health care problem is presented.

#### 3.1. Basic notation

The home health care problem is modeled on a graph  $G = (N, A)$  where  $N$  is the set of nodes and  $A$  the set of arcs. The set of visits to perform is denoted  $O$  and the set of home health care offices is denoted  $P$ . Thus,  $N = O \cup P$ . Each visit is represented by a separated node in the graph, whether two or more visits are associated with the same physical location or not. For example, if a given client requires two visits during the day, a node will be created for each visit and they will both have the same geographical location. Using this information, the set of arcs is defined as  $A = \{(i, j) | i, j \in N, i \neq j\}$ . Each arc  $(i, j) \in A$  has a distance  $d_{ij}$ .

The planning horizon is assumed to be a single day.

The set of caregivers is denoted  $S$ . For each caregiver  $i \in S$ , a working hard time window  $[\alpha_i, \beta_i]$  is known. The set  $R = \{r_1 = \mathcal{N}, r_2 = \mathcal{A}\}$  represents the possible job role of caregivers. Nurses have the job role  $\mathcal{N}$  and the auxiliary nurses have the job role  $\mathcal{A}$ . The assignment of a job role to a caregiver  $i$  is defined by the binary parameter  $\omega_i$ . The parameter  $\omega_i$  is equal to  $\mathcal{N}$  if the caregiver  $i$  is a nurse and equal to  $\mathcal{A}$  otherwise.

The home health care offices may be located in different places and therefore have different locations. The binary parameter  $\gamma_{ik} = 1$  if the caregiver  $i$  is assigned to the home health care office  $k$ , and zero otherwise.

Moreover, for each visit  $i \in O$  at the patient's home, a duration  $c_i$  represents the time needed to perform a care. A soft time window  $[a_i, b_i]$  represents the availability

of the patient to receive a care. The binary parameter  $\delta_{ij}$  is used when two visits  $i$  and  $j$  are synchronized.  $\delta_{ij} = 1$  if the visits  $i$  and  $j$  are synchronized, and 0 otherwise.

As well, the caregiver qualification needed to perform the job  $i$  is defined by  $\rho_i$ . If the visit requires a nurse,  $\rho_i = \mathcal{N}$ , otherwise the visit requires an auxiliary nurse and  $\rho_i = \mathcal{A}$ .

Finally, the decision variable  $x_{ij}^k = 1$  if the caregiver  $k$  travels from  $i$  to  $j$ , and zero otherwise. As well, the variable  $t_i$  defines the starting time of the visit  $i$ .

In order to illustrate our problem, the figure 1 depicts a situation where there is one home health care office, thirteen patients and four caregivers. Among the caregivers, the paths colored in blue represent the routes of the nurses and the path in red is performed by an auxiliary nurse. In this case, the patient 6 requires two caregivers. As a result, both cares are synchronized so that the nurse and the auxiliary nurse arrive simultaneously. Another example, the patient 8 is served by a nurse. As a result,  $\rho_8 = \mathcal{N}$ . In opposite,  $\rho_5 = \mathcal{A}$ .

#### 3.2. Working time definition and balancing

In this part, the working time of the caregivers and its balancing is detailed.

The working time  $\omega_k$  of a caregiver  $k$  is defined as the interval of time between the time he or she leaves the home health care office and the time he or she returns to it. The possible waiting times on his/her route are also considered as being part of working time. The variables  $l_k$  and  $u_k$  respectively represents the time the caregiver  $k$  leaves from the home health care office and the time he/she returns to it.

To summarize, the working time  $\omega_k$  of a caregiver  $k$  is computed as follows:

$$\omega_k = u_k - l_k \quad \forall k \in S \quad (1)$$

$$l_k = \min(t_i - d_{ij} - (x_{ji}^k - 1) \times M) \quad \forall i \in O, j \in P, k \in S, \gamma_k^j = 1 \quad (2)$$

$$u_k = \max(t_i + c_i + d_{ij} + (x_{ij}^k - 1) \times M) \quad \forall i \in O, j \in P, k \in S, \gamma_k^j = 1 \quad (3)$$

In addition, the working time balancing aims to minimize the maximal working time difference among caregivers. Working time is balanced separately between nurses and auxiliary nurses. Indeed, the routes have different characteristics (number of patients treated on average, duration of cares). Consequently, the maximal working time difference among nurses is defined by the variable  $W_{\mathcal{N}}$ . Similarly, the maximal working time difference among auxiliary nurses is computed by the variable  $W_{\mathcal{A}}$ .



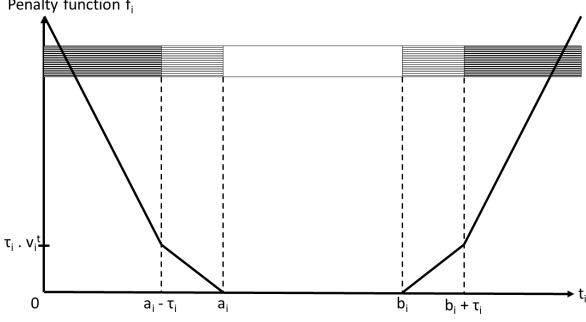


Figure 2: Value of the penalty function  $f_i$  depending on the arrival time  $t_i$

428 by the following constraints:

$$\tau_i^{e-} = \min(\tau_i, \max(0, a_i - t_i)) \quad \forall i \in O \quad (6)$$

$$\tau_i^{e+} = \max(0, a_i - t_i) - \tau_i^{e-} \quad \forall i \in O \quad (7)$$

$$\tau_i^{l-} = \min(\tau_i, \max(0, t_i - b_i)) \quad \forall i \in O \quad (8)$$

$$\tau_i^{l+} = \max(0, t_i - b_i) - \tau_i^{l-} \quad \forall i \in O \quad (9)$$

429 The figure (3) illustrates the value of the penalty func-  
430 tion  $g_{ij}$  depending on the arrival time of the caregivers  
431 and  $j$ .

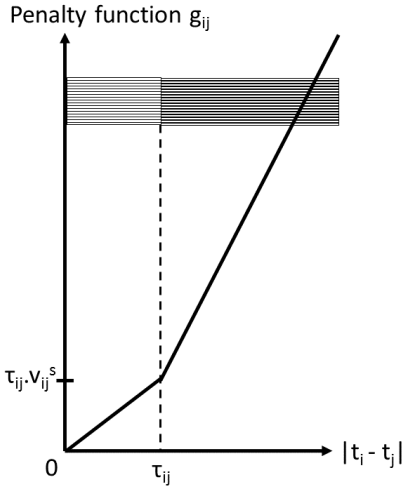


Figure 3: Value of the penalty function  $g_{ij}$  depending on the arrival times of both caregivers.

432 About the synchronized visits, the penalty function  $g_{ij}$   
433 is defined as follows:

$$g_{ij}(t_i, t_j) = \begin{cases} 0, & \text{if } t_i = t_j \\ v_{ij}^s \cdot t_{ij}^{s-}, & \text{if } |t_i - t_j| \leq \tau_{ij} \\ v_{ij}^s \cdot (t_{ij}^{s-} + p^s \cdot t_{ij}^{s+}), & \text{if } |t_i - t_j| > \tau_{ij} \end{cases}$$

437 The binary parameter  $\delta_{ij}$  is used when two cares  $i$  and  $j$   
438 need to be synchronized.  $\delta_{ij} = 1$  if the visits  $i$  and  $j$  have  
439 to be synchronized, and 0 otherwise.

440 Each minute of difference between the starting times of  
441 the synchronized visits occurs a penalty per minute of  $v_{ij}^s$   
442 until a limit of time of  $\tau_{ij}$ . Moreover, this penalty per  
443 minute is multiplied by a factor  $p^s$  in the extra zone.

444 The number of minutes of non-satisfaction of the syn-  
445 chronized visits in the first zone is denoted  $t_{ij}^{s-}$ . As well,  
446 the minutes of violations belonging to the extra zone is  
447 denoted by the variable  $t_{ij}^{s+}$ .

448 The number of minutes of violation of the synchronized  
449 visits is determined by the following constraints:

$$t_{ij}^{s-} = \min(\tau_{ij}, |t_i - t_j|) \quad \forall i \in O, j \in O, \delta_{ij} = 1, i \neq j \quad (10)$$

$$t_{ij}^{s+} = |t_i - t_j| - t_{ij}^{s-} \quad \forall i \in O, j \in O, \delta_{ij} = 1, i \neq j \quad (11)$$

450 Note that the constraints (6) to (11) are linearizable in  
451 order to obtain a MILP model.

### 3.4. Problem formulation

452 In this part, the formulation of the problem is presented.  
453 In order to facilitate the understanding of the model,  
454 the notation of the parameters for the home health care  
455 problem is summarized in the table 3.4.

456 The mixed-integer programming modelization of the  
457 home health care problem is formulated as follows:  
458

$$\begin{aligned} \min \quad & \sum_{i \in N} \sum_{j \in N} \sum_{k \in S} x_{ij}^k \times d_{ij} \\ & + \sum_{i \in O} f_i(t_i) + \sum_{i \in O} \sum_{j \in O} \delta_{ij} \times g_{ij}(t_i, t_j) \\ & + W_N + W_A \end{aligned} \quad (12)$$

459 subject to:

$$\sum_{j \in N} \sum_{k \in S} x_{ij}^k = 1 \quad \forall i \in O \quad (13)$$

$$\sum_{j \in O} x_{ij}^k = \sum_{j \in O} x_{ji}^k = \gamma_k^i \quad \forall i \in P, k \in S \quad (14)$$

$$\sum_{j \in N} x_{ij}^k = \sum_{j \in N} x_{ji}^k \quad \forall i \in N, k \in S \quad (15)$$

$$t_j \geq t_i + c_i + d_{ij} + ((\sum_{k \in S} x_{ij}^k) - 1) \times M \quad \forall i \in N, j \in O, i \neq j \quad (16)$$

$$t_i \geq l_k + ((\sum_{j \in N} x_{ij}^k) - 1) \times M \quad \forall i \in O, k \in S \quad (17)$$

Sets		Description
$N$		Set of nodes
$S$		Set of caregivers
$O$		Set of visits
$P$		Set of home health care offices
$R$		Set of caregivers' qualification

Parameters	Type	Description
$d_{ij}$	Positive real	Travel time between the nodes $i$ and $j$
$c_i$	Positive real	Duration of the visit $i$
$[\alpha_i, \beta_i]$	Positive real interval	Working time window of caregiver $i$
$[a_i, b_i]$	Positive real interval	Availability time window of visit $i$
$\delta_{ij}$	Binary	1 if visits $i$ and $j$ are synchronized, 0 otherwise
$\omega_i$	R	Qualification of the caregiver $i$
$\rho_i$	R	Qualification required to perform the visit $i$
$p^t$	Positive real	Coefficient value for time windows non-satisfaction
$p^s$	Positive real	Coefficient value for synchronized visits non-satisfaction
$\lambda$	Positive real	Percentage used for the penalties computation
$\gamma_{ik}$	Binary	1 if caregiver $i$ is assigned to the home health care office $k$

Variables	Type	Description
$x_{ij}^k$	Binary	1 if caregiver $k$ travels from $i$ to $j$ , 0 otherwise
$t_i$	Positive real	Starting time of the visit $i$

Table 1: Description of the sets, parameters and variables used in the mathematical model

$$u_k \geq t_i + c_i + d_{ij} + (x_{ij}^k - 1) \times M \quad \forall i \in O, j \in P, k \in S, i \neq j, \gamma_{kj} = 1 \quad (18)$$

$$\sum_{\substack{j \in N \\ i \neq j}} x_{ij}^k = 0 \quad \forall i \in O, k \in S, r \in R, \omega_{kr} \neq \rho_{ir} \quad (19)$$

$$x_{ij}^k \in \{0, 1\} \quad \forall i \in N, j \in N, k \in S \quad (20)$$

$$t_i \in T \quad \forall i \in O$$

In addition, constraint (19) ensures that caregivers have the requested qualification to perform the health cares planned in their route.

Finally, the constraint (20) defines the domain of the variables.

#### 4. Solution approaches

In this section, we present three algorithms for solving the home health care problem. First, a memetic algorithm from the literature is presented. Then, an ant colony optimization algorithm is detailed. Finally, a hybridization of the two previous solutions approaches is presented as a hybrid memetic-ant colony optimization algorithm.

##### 4.1. Memetic algorithm

Memetic algorithm (MA) was first introduced by Moscato et al. (1989) as the combination of evolutionary algorithms with local search procedure. Successful implementation of memetic algorithm has been applied to the home health care problem. For instance, the authors of this work have previously developed a memetic algorithm for the home health care problem with soft time window and synchronization constraints (Decerle et al. (2018)). The proposed algorithm has shown very good performance on the test instances, whether the home health care problem is studied with hard or soft time window and synchronization constraints, several home health care offices or

Objective function (12) minimizes the sum of total traveling time of the caregivers, the patients' time window non-satisfaction and the synchronized visits non-respect and the maximal working time difference among caregivers. Constraint (13) makes sure that each visit is done by one caregiver. Constraint (14) ensures the departure and arrival of the caregivers to their assigned home health care office. The flow conservation is guaranteed by constraint (15). Constraint (16) verifies that caregivers have enough time between two consecutive visits to perform the first one and then go to the next patient's home. In addition, caregivers are allowed to work only during a hard time window as guaranteed by the constraints (17) and (18).

498 various caregiver qualifications. As a result, the suggested  
 499 memetic algorithm from the literature is briefly presented  
 500 in this section.

501 The structure of the memetic algorithm (see figure 4) is  
 502 similar to the genetic algorithm, except there is one more  
 503 step of local search during the process. As the memetic  
 504 algorithm is a population-based metaheuristic, the algo-  
 505 rithm starts with the generation of an initial population.

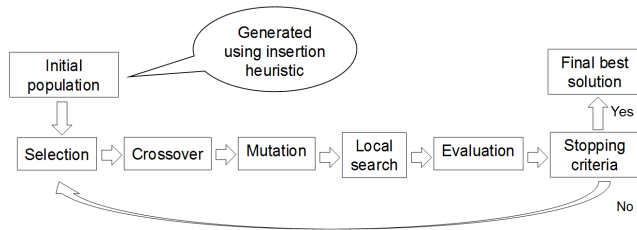


Figure 4: Structure of the memetic algorithm

506 An insertion heuristic generates the initial population.  
 507 At first, routes are composed of a single visit as a sub-tour.  
 508 Other visits are then inserted following their least cost of  
 509 insertion. Once a solution is generated, the solution is par-  
 510 tially destroyed and is randomly modified. Based on this  
 511 solution, the next one is built following the same process.  
 512 It is repeated until the stopping condition is reached. At  
 513 the end of the heuristic, the best-found solutions become  
 514 the initial population.

515 After the generation of the initial population, the  
 516 memetic algorithm performs the steps of selection,  
 517 crossover, mutation, local search, and evaluation until the  
 518 stopping criterion is reached. Each of these steps is de-  
 519 scribed in the remaining part of this section.

520 Selection process aims to select individuals from a pop-  
 521 ulation in order to breed a new generation. First, a fit-  
 522 ness proportionate selection chooses 80% of the individu-  
 523 als of the next population. Secondly, the remaining 20%  
 524 of the population is determined by rank based selection,  
 525 which tends to avoid convergence by reducing selection  
 526 pressure. The proportion of individuals selected by the  
 527 different techniques of selection has been set empirically  
 528 since it is providing a good balance between elitism and  
 529 diversity.

530 Crossover process uses several operators to create new  
 531 solutions from populations genetic information analogously  
 532 to reproduction. Crossover eases inheritance of  
 533 characteristics by an offspring from its parents. The prob-  
 534 ability of a crossover operator to occur is dynamic and  
 535 depends on the quality of its generated solution. Famous  
 536 techniques are used by the three operators used in the  
 537 crossover procedure such as one-point crossover, two-point  
 538 crossover, and one path exchange crossover.

539 Mutation process aims to preserve the genetic diversity  
 540 of the population. Mutation operators change a solution  
 541 from its initial state and greatly help the memetic algo-  
 542 rithm to avoid local optima. For each selected individual,  
 543 mutation operators are applied according to a fixed prob-

ability. The three mutation operators used are a node  
 exchange operator, node relocation operator and an ar-  
 rival time shift operator (random update of cares' starting  
 time).

After all the changes brought by the crossover and the  
 mutation operators, the individuals of the population are  
 evaluated before moving on to the local search phase.

Improvement procedure of the memetic algorithm is per-  
 formed by a local search on separate individuals. Local  
 search algorithms apply changes iteratively to the solution  
 until no improvement is found. The three operators of the  
 local search procedure use different techniques to improve  
 the solution. 2-opt and 2-opt\* are combined into the first  
 operator, the second one uses node exchange technique and  
 node relocation algorithm is used by the third operator.

Finally, the algorithm stops when the stopping criterion  
 has been reached, and the best solution found at runtime  
 is the result of the memetic algorithm.

#### 4.2. Ant colony optimization algorithm

Ant colony optimization algorithm (ACO) is a meta-  
 heuristic approach to tackle NP-hard combinatorial prob-  
 lems. Initially proposed by Dorigo (1992), ACO is in-  
 spired by the indirect communication of real ants by  
 means of trails of a chemical substance called pheromone.  
 As explained by López-Ibáñez (2004), artificial ants are  
 simple agents that use numerical information (artificial  
 pheromone information) to communicate their experience  
 while solving a particular problem to other ants.

Inspired by the foraging food behavior of real ant  
 colonies, this mechanism of ant colony has been success-  
 fully applied to resolve a wide range of optimization prob-  
 lems such as the traveling salesman problem (TSP) by  
 Dorigo and Gambardella (1997a) or the multiple traveling  
 salesmen problem (MTSP) by Liu et al. (2009), which is  
 similar to the home health care problem.

In the proposed implementation of an ant colony opti-  
 mization algorithm, an effective variant suggested by Stut-  
 zle and Hoos (2000) called MAX-MIN Ant System is ap-  
 plied. In this variant, pheromone deposit is limited by an  
 upper bound represented by the parameter  $\tau^+$  (preventing  
 a track from being over-reinforced) and a terminal lower  
 $\tau^-$  (leaving the possibility of being explored to any so-  
 lution). This algorithm achieves better results than the  
 original, and in particular avoids premature convergence.

In order to give an overview of the ant system as de-  
 scribed by Dorigo et al. (1996); Dorigo and Gambardella  
 (1997b), the following figure 5 details the different steps.

As defined by Dorigo and Gambardella (1997b), ant  
 system works as follows. Each ant generates a com-  
 plete solution by choosing the visits according to a prob-  
 abilistic state transition rule; ants prefer moving to visits  
 which are connected by short edges with a high amount  
 of pheromone. Once all ants have completed their tours  
 a global pheromone updating rule is applied; a fraction of  
 the pheromone evaporates on all edges (edges that are not

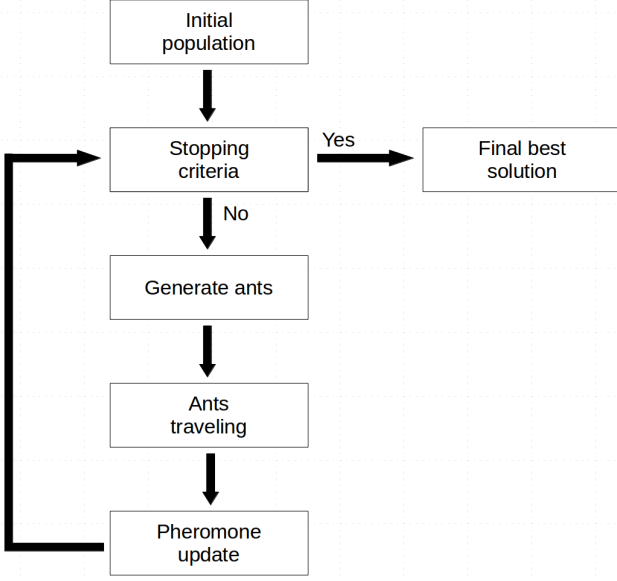


Figure 5: Structure of the ant system

where:

•

$$\Delta\tau_k(r, s) = \begin{cases} \frac{Q}{L_k}, & \text{if } (r, s) \in \text{tour done by ant } k \\ 0, & \text{otherwise} \end{cases}$$

- $\rho$  is a pheromone decay parameter with  $0 < \alpha < 1$
- $L_k$  is the value of the objective function of the solution obtained by ant  $k$
- $m$  is the number of ants
- $Q$  is a constant

At each iteration, newly generated ants will create new solutions based on the edge selection rule. Pheromone placed on the edges plays the role of a long-term memory to allow an indirect form of communication. As well, a portion of the pheromone evaporates on all edges at each iteration.

refreshed become less desirable), and then each ant deposits an amount of pheromone on edges which belong to its tour in proportion to how short its tour was. The process is then iterated until the stopping criteria are reached.

Thus, the movement of the ants is defined by two major rules: the edge selection and the pheromone update.

Edge selection is performed by a state transition rule, which gives the probability  $p_k(r, s)$  with which ant  $k$  at visit  $r$  chooses to move to the city  $s$ :

$$p_k(r, s) = \begin{cases} \frac{\tau(r, s)^\alpha \times \eta(r, s)^\beta}{\sum_{u \in J_k(r)} \tau(r, u)^\alpha \times \eta(r, u)^\beta}, & \text{if } s \in J_k(r) \\ 0, & \text{otherwise} \end{cases}$$

where:

- $\tau(r, s)$  is the pheromone level on the edge between the visits  $r$  and  $s$  with  $\tau^- \leq \tau(r, s) \leq \tau^+$
- $\eta(r, s)$  is equal to  $1/d_{rs}$  is the inverse of the traveling time from the visit  $r$  to  $s$
- $\alpha$  is a parameter that determines the importance of the pheromone ( $\alpha > 0$ )
- $\beta$  is a parameter that determines the importance of traveling time ( $\beta > 0$ )
- $J_k(r)$  is the set of visits that remain to be visited by ant  $k$  positioned on visit  $r$  to make the solution feasible

Subsequently, once all ants have built a solution, pheromones are updated on all edges according to the following rule:

$$\tau(r, s) \leftarrow (1 - \rho) \times \tau(r, s) + \sum_{k=1}^m \Delta\tau_k(r, s)$$

#### 4.3. Hybrid memetic-ant colony optimization algorithm

Hybrid memetic-ant colony optimization algorithm (MACO) is a mono-objective algorithm that we propose in this paper.

MACO is a hybridization of the two algorithms presented previously: MA and ACO. The aim of this hybridization is to couple the structure of MA and ACO to combine a new algorithm. Previous works on the hybridization of these two algorithms have shown their efficiency on other applications. Interesting results have emerged from Zuhri and Paputungan (2013) to solve the traveling salesman problem (TSP), where their hybrid genetic ant colony optimization performed better than genetic algorithm and ant colony optimization algorithm. In addition, Mavrovouniotis and Yang (2013) have developed an efficient ACO-based MA for the dynamic traveling salesman problem.

In order to give an overview of the structure of the proposed hybrid memetic-ant colony optimization algorithm, the figure 6 represents the flow chart of the algorithm.

In the hybrid algorithm, an individual represents a solution. As shown in the figure 7, an individual is composed of a table of routes, that may have different lengths. Also used by Shi et al. (2017), the strength of this encoding is that each individual does not need to be decoded and encoded in the optimization process.

The algorithm starts with a set of feasible solutions which can be randomly generated. At each iteration of the algorithm, either the MA or the ACO operators will be applied, depending on a fixed probability represented by the parameter  $proba_{ACO}$ . Both algorithms share the same population during the resolution.

If the memetic algorithm is chosen for the current iteration, then the selection, crossover and mutation operators are applied on the current population. In the other case,

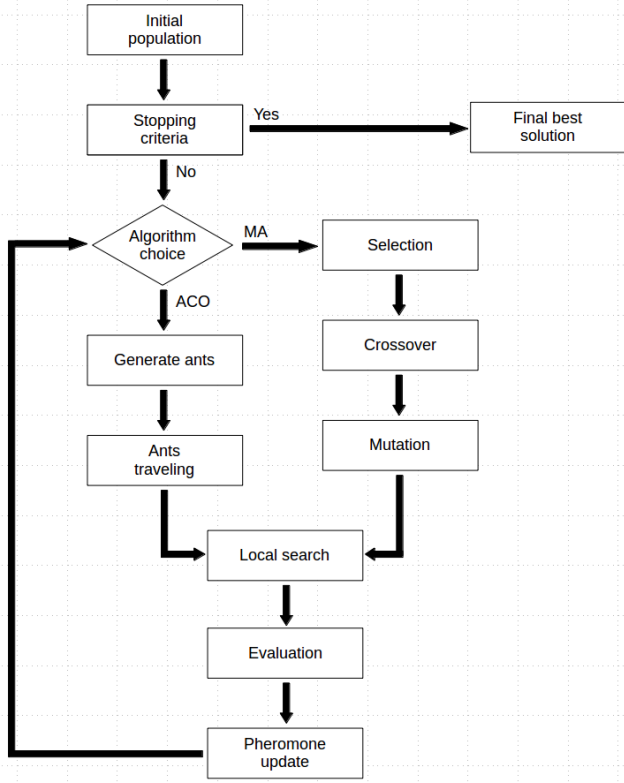


Figure 6: Structure of the hybrid memetic-ant colony optimization algorithm

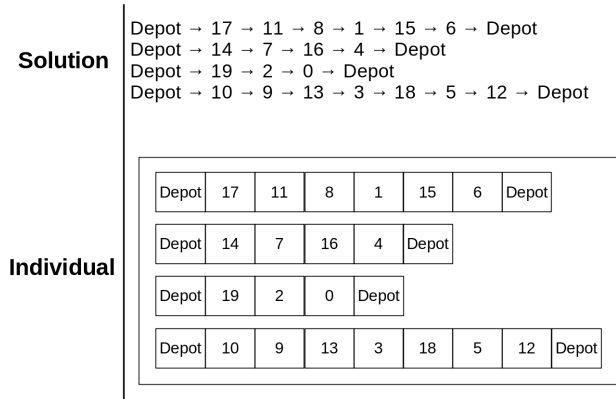


Figure 7: Representation of an individual

670 new individuals are created following the ACO process pre-  
 671 sented in the section 4.2.

672 During all its execution, the algorithm works only with  
 673 feasible solutions since there is no correction procedure.  
 674 Consequently, the crossover, mutation and local search op-  
 675 erators must produce only feasible solutions.

676 In the process of the hybrid algorithm, the crossover,  
 677 mutation and local search operators implemented for the  
 678 purpose of the experiments are detailed hereafter. As ex-  
 679 plained earlier, the following operators are similar as the  
 680 ones developed previously by Decerle et al. (2018).

681 Crossover operators aim at creating new offspring so-  
 682 lutions from population's genetic information analogously  
 683 to reproduction. Crossover operators are applied to the  
 684 selected individuals according to a pre-defined probability  
 685  $proba_C$ . The three operators used in the algorithm are  
 686 presented here after.

687 One-point crossover operator randomly selects a  
 688 crossover point and the tails of its two parents are  
 689 swapped to get new off-springs

690 Two-point crossover works similarly except that two  
 691 crossover points are chosen and the inner segment be-  
 692 tween the two crossover point is swapped to get new  
 693 off-springs

694 One path exchange crossover exchanges one randomly  
 695 chosen path between the two candidate solutions to  
 696 create off-springs

697 Mutation operators are essentials to preserving the ge-  
 698 netic diversity of the population. They can change signifi-  
 699 cantly a solution from its initial state and greatly help the  
 700 algorithm to escape from a local optimum. Mutation op-  
 701 erators are applied to the selected individual according to  
 702 a pre-defined probability  $proba_M$ . If a solution is selected  
 703 for the mutation process, then any of the three following  
 704 mutation operators will be applied to it randomly. The  
 705 mutation operators that have been implemented are the  
 706 node exchange operator, node relocation operator, and ar-  
 707 rival time shift operator. While the first two operators will  
 708 focus on the routing part (either exchanging or relocating  
 709 some randomly chosen visits), the arrival time shift op-  
 710 erator focus on the scheduling of the solution by updating  
 711 the arrival times of the visits within a certain interval of  
 712 time.

713 Finally, the local search operators apply local changes  
 714 iteratively to the solution in order to improve the objec-  
 715 tive function. Local search operators are applied to the selected  
 716 individual according to a pre-defined probability  $proba_{LS}$ .  
 717 The local search operators implemented for the experi-  
 718 ments are a combination of 2-opt and 2-opt\* operators, a  
 719 node exchange operator and a node relocation operator.  
 720 The same operator combines the 2-opt procedure with the  
 721 2-opt\* procedure (suggested by Potvin et al. (1996)) since  
 722 they work similarly except that 2-opt\* performs changes  
 723 on different routes. It is particularly effective on the home

health care problem since it is hardly possible for the 2-opt operator to find an improvement because of the preserved order of visits from the time windows. In addition, the node exchange operator and node relocation operator respectively exchange two visits with each other or relocate a visit to a different position in the same or another route.

During the local search process, each operator is applied once. For each operator, the first feasible improvement is performed and the procedure keeps going until no improvements are possible.

Once each solution is evaluated, the pheromones are updated on all edges, according to the rule explained in the section 4.2. This hybridization of the two algorithms, which share the same population, must allow the algorithm to obtain better solutions by taking advantage of the characteristics and specificities of the two algorithms taken individually. Thus, even when the memetic algorithm is chosen, the pheromone matrix will also be updated, which would not have been the case in the classic version of the memetic algorithm. This continuously updated pheromone matrix will then be used by the ACO section of the hybrid algorithm when it is selected. Similarly, the local search operator specific to the memetic algorithm will also be applied even if the ACO is chosen during the current iteration, thus potentially improving some solutions of the population.

Finally, the algorithm stops when the stopping criteria have been reached, and the best solution found during the resolution is the result of the algorithm.

## 5. Numerical experiments

In this section, the test instances and the settings applied to the hybrid memetic-ant colony optimization algorithm during the experiments are presented. Subsequently, the results obtained with different settings of the MACO are analyzed.

### 5.1. Test instances and experiments settings

In order to evaluate the efficiency of the suggested algorithm, a set of instances from the literature on the HHC problem is used. The test instances have been suggested by Bredström and Rönnqvist (2008).

The test instances are grouped into three categories depending on the number of visits. The first category contains five small-size instances of 20 visits, the second one contains three mid-size instances of 50 visits and the last category contains two instances of 80 visits.

For all instances, one day is considered nine hours long. Therefore, all the data related to the time in the instances have to be normalized on a daily nine-hour scale.

Moreover, in each instance, 10% of the visits are synchronized visits.

Finally, in order to cover the different time window possibilities, five groups of time window restrictions have been

created respectively fixed time window (F), small time window (S), medium time window (M), large time window (L) and no time window restrictions (A).

In the tables presenting the results of the experiments, the instance "3S" represents the instance 3 with small patients time window. Similarly, the instance "8A" represents the instance 8 with no time window restrictions.

As a summary, the characteristics of the test instances are presented in the table 2.

Concerning the experiments settings, each parameter has been set empirically since it showed good performance during the experiments. However, it should be pointed out to the reader that the algorithms are sensitive to the parameter settings. Therefore, results of the experiments may change with different parameters settings.

The computational time allowed for each instance is defined to 10 minutes. It represents the maximal allowed running time by Bredström and Rönnqvist (2008) who initially proposed the instances. In addition, the same computational time has been allowed regardless of the instance size since in practice, the coordinating nurse will have no more or less available time depending on the number of patients and caregivers to determine the schedule.

About the patient time window and synchronization soft constraints, the value of the limit separating the two zones of penalty denoted  $\tau_i$  has been set to  $0.1 \times c_i$ . Similarly, the time limit separating the two zones of penalty in the case of synchronized visits denoted  $\tau_{ij}$  for the visits  $i$  and  $j$  is set to  $0.1 \times c_i$  assuming  $c_i = c_j$  in the experiments. The aim is to strongly penalize the large violations of the soft constraints.

The computation of the penalty value for soft patients time window non-satisfaction  $v_i^t$  depends on the average distance between all patient locations  $AvD$  and on the time window length of the visit. If the time window of the patients is small, they have some important impediments that prevent them from being available for a longer time. Consequently, the more the patient time window is small, the greater the penalty value is.

In addition, the more the average distance between patient locations is important and the greater the value of the penalty will be. Since a high value of  $AvD$  tends to induce a high value of the total travel time needed by the staff members (that is to say the objective function), the value of the penalty has to be proportional to the total travel time of the staff members in order to have a roughly similar impact on the penalization of the objective function.

Consequently, the penalty value for each minute of non-satisfaction of the patients soft time window  $v_i^t$  for the visit  $i$  is defined as follows:

$$v_i^t = \begin{cases} \frac{AvD}{2 \times (b_i - a_i)} & \text{if } 2 \times (b_i - a_i) \geq 1 \\ AvD & \text{otherwise} \end{cases}$$

The penalty factor for soft time windows  $p^t$  is set to the same as the penalty factor for synchronized visits  $p^s$ .

Instance	O	S	Sync	AvD (h)	F(h)	S(h)	M(h)	L(h)	A(h)
1	20	4	2	0.3	0	1.5	2.1	2.9	9
2	20	4	2	0.27	0	1.7	2.2	3.0	9
3	20	4	2	0.25	0	1.5	2.4	3.0	9
4	20	4	2	0.39	0	1.8	2.9	3.9	9
5	20	4	2	0.27	0	1.3	2.1	3.2	9
6	50	10	5	0.33	0	1.4	2.3	3.1	9
7	50	10	5	0.31	0	1.6	2.5	3.4	9
8	50	10	5	0.32	0	1.5	2.4	3.2	9
9	80	16	8	0.29	0	1.5	2.3	2.9	9
10	80	16	8	0.23	0	1.6	2.6	3.6	9

Table 2: Bredström and Rönnqvist test instances. The columns are: the number of visits |O|, the number of routes |S|, the number of synchronized visits |Sync|, the average distance between the visits  $AvD$  and the last five columns are the average time window size for each category

Consequently in the experiments,  $p^t = p^s = 2$ .

Similarly, the same penalty value (i.e. penalty applied for each minute of non-satisfaction of the soft constraint) is used for the soft time windows and the synchronization constraints non-satisfaction. As a result, for a job  $i$ ,  $v_i^t = v_{ij}^s$ .

Regarding the solution approach, the crossover, mutation and local search operators presented in section 4.1 are used for the hybrid memetic-ant colony optimization algorithm. Similarly, the edge selection and pheromone update rules in section 4.2 are used by the MACO.

Concerning the settings of the crossover and mutation processes, the parameters  $proba_C$  and  $proba_M$  have been respectively defined to 1 and 0.1.

As well, the population size  $F_{max}$  of the MACO is set to 20 individuals since it shows a good balance during the experiments between the solution accuracy and the computational time needed to perform one iteration of the algorithm. The use of larger populations would increase the needed computational resources.

In regard to the pheromone update, the parameter  $Q$  influencing the amount of pheromone deposited on an edge is set to 1. As well, the coefficient of pheromone evaporation defined by the parameter  $\rho$  is set to 0.1. Consequently, only 10% of the previously deposited pheromones remains at each iteration.

In addition, experiments have been performed with different levels of probability to apply either MA or ACO each iteration of the MACO algorithm. As a result, the parameter  $proba_{ACO}$  is set to either 0, 0.5 or 1 depending on the experiment. If  $proba_{ACO} = 0$ , it means that the ACO part of the MACO is never applied. In opposite, if  $proba_{ACO} = 1$ , then the MA part of the MACO is never executed.

As well, various values for the parameters  $\alpha$  and  $\beta$  are used to control the relative influence of the intensity of the pheromone and the visibility of an edge. A high value of  $\alpha$  increases the importance of the pheromone in the edge selection rule. Similarly, a high value of  $\beta$  increase the importance of the visibility of an edge (i.e. the visibility

of an edge is inversely proportional to the travel time of the edge).

In the experiments, the parameters  $\alpha$  and  $\beta$  are defined with different values to identify a possible combination with better results.

For each experiment using the version "v1", the value of  $\alpha$  and  $\beta$  are equally set to one meaning that the influence of the intensity of the pheromone and the visibility of an edge is the same to determine the next edge. In the version "v2", ten times more influence is given to the visibility of an edge compared to the intensity of the pheromone while the version "v3" does the opposite case.

A summary of the settings applied to the MACO for each experiment is presented in the table 3.

Experiment version	$proba_{ACO}$	$\alpha$	$\beta$
<b>MA</b>	0	-	-
<b>ACO v1</b>	1	1	1
<b>ACO v2</b>	1	0.1	1
<b>ACO v3</b>	1	1	0.1
<b>MACO v1</b>	0.5	1	1
<b>MACO v2</b>	0.5	0.1	1
<b>MACO v3</b>	0.5	1	0.1

Table 3: Experimental settings applied to the hybrid memetic-ant colony optimization algorithm

Please note in table 3 that when  $proba_{ACO} = 0$ , the pheromones are not used during the execution of the algorithm. Therefore, the value of the parameters  $\alpha$  and  $\beta$  are not indicated because in this case.

As a result, the settings applied to the first experiment correspond to the behavior of a memetic algorithm as presented in section 4.1. Experiments "ACO v1", "ACO v2" and "ACO v3" reproduce an ant colony optimization algorithm including an extra step of the local search. Finally, experiments "MACO v1", "MACO v2" and "MACO v3" correspond to the MACO where memetic and ant colony optimization algorithm have equal probabilities to be applied at each iteration.

The results presented in an upcoming section are ob-

tained among 10 runs on each instance.

In addition, the commercial optimization solver Gurobi Optimization (2016) (version 6.0.5) is used with a time limit of 2 hours to perform a comparison of the performance in front of the MACO on the home health care problem.

Finally, experiments are performed on a computer using an Intel<sup>®</sup> Core<sup>™</sup> i7-7820HQ CPU with 16 GB of RAM memory.

## 5.2. Computational results

In this part, the computational results of the experiments are presented.

The results of the hybrid approach are compared to different methods to confirm the effectiveness of the proposed method. Three resolution approaches are used to compare their results with those of the hybrid algorithm. Indeed, the hybrid algorithm is compared to a memetic algorithm inspired by the work of Decerle et al. (2018), three versions of the ant colony optimization algorithm and the commercial solver Gurobi.

In addition, no experiment was carried out by previous works of the literature on the same problem so the comparison was not possible with existing results.

In order to obtain a fair comparison between the various configurations of the hybrid memetic-ant colony optimization algorithm, the average computational results over the ten runs are detailed first. Then, the best results obtained by each configuration of the MACO over the ten runs are compared with the results obtained by the commercial solver Gurobi.

First, the average computational results over the ten runs are presented in the table 4.

In each row, the lowest value is highlighted in gray to distinguish which MACO parameters provide the best results for that instance.

The "Objective function" column represents the average sum of traveling time, penalties occurred by the non-satisfaction of the soft constraints and maximal working time difference among the ten runs. The results are presented for each MACO settings variant.

Furthermore, the column "Gap (%)" indicates the gap between the best-found solution by any variant of the MACO among "MACO v1", "MACO v2" and "MACO v3" compared to the best-found solution obtained by other solving methods such as MA and ACO. The higher the gap is, the better the hybrid MACO performs. As well, the value is highlighted in gray when the MACO obtains the best solution on the considered instance.

Consequently, the gap is computed as follows:

$$Gap = \frac{\min(ACO, \min_{i \in [1..3]}(ACOV_i)) - \min_{i \in [1..3]}(MACOV_i)}{\min_{i \in [1..3]}(MACOV_i)}$$

Regarding the results presented in the table 4, the hybrid memetic-ant colony optimization algorithm performs better than the memetic algorithm or the ant colony optimization algorithm. Indeed, the MACO represented by the

experiments "MACO v1", "MACO v2", and "MACO v3" obtain the lowest average objective function for most instances. From the benchmark of Bredström and Rönnqvist (2008), the best average results are obtained by the versions "MACO v1", "MACO v2", and "MACO v3" in 45 out of 50 instances. In opposite, the ACO represented by the versions "ACO v1", "ACO v2", and "ACO v3" obtain the best result in 4 out of 50 instances and only one instance for the MA.

Similarly, it should be noted that the outstanding results of MACO compared to MA and ACO are valid for all settings of the MACO. In fact, no matter which settings of the MACO is used (v1, v2 or v3), the average value obtained among all instances is lower than the average value using the MA or the ACO. For instance, using the settings "MACO v1", the MACO obtains an average value of the objective function among all instances of 533.38 while this value reaches 546.61 for the MA and 557.25 in the best case for the ACO (using the setting "ACO v1").

Overall, the computational results highlight the efficiency of the hybrid memetic-ant colony optimization algorithm. The proposed algorithm provides better results than the memetic algorithm and the ant colony optimization algorithm, which for the first had very good results in a previous work by Decerle et al. (2018). As well, it can be emphasized that the ACO provides the worst results with the highest average value for the objective function. Finally, it should be noted that among the three configurations of the MACO (v1, v2, and v3), there is no particular configuration that stands out, and all provide good results.

Secondly, the best results obtained by the MACO algorithm for each experiment over the tens runs are presented in the table 5. Best results are compared with the best solution obtained by the commercial optimization solver Gurobi Optimization (2016).

In the column "Objective function (m)", the best solutions obtained by the MACO algorithm out of the ten runs are presented and the results obtained by the optimization solver are given by the column "MIP". A star mark (\*) is used in the column "MIP" to indicate that the solution has been proven to be optimal by the solver. When the indicated value contains an inequality (greater than), it means that Gurobi could not find a feasible solution in the available time. Only a lower bound has been obtained and is indicated as an inequality.

The column "Gap (%)" indicates the gap between the best-found solution by any variant of the MACO among "MACO v1", "MACO v2" and "MACO v3" compared to the optimization solver. The lower the deviation ratio is and the better the result is. Consequently, the deviation ratio is computed as follows:

$$Gap = \min_{i \in [1..3]} \left( \frac{MACOV_i - MIP}{MIP} \right)$$

Regarding the computational results, the hybrid memetic-ant colony optimization algorithm provides good results. Indeed, the deviation ratio remains low for small

Instance name	Objective function (m)							Gap (%)	
	MA	ACO v1	ACO v2	ACO v3	MACO v1	MACO v2	MACO v3		
1	F	358.78	357.16	357.22	357.16	356.19	355.78	356.91	0.39
2	F	369.85	355.26	355.26	355.26	357.40	354.81	356.27	0.13
3	F	338.28	326.92	326.92	326.92	328.01	326.80	326.91	0.04
4	F	499.48	501.91	501.91	501.91	500.99	499.47	500.82	0
5	F	395.49	391.36	391.88	391.62	393.96	388.14	391.94	0.83
6	F	959.91	1032.91	1065.91	1033.09	919.06	933.93	908.12	5.70
7	F	886.78	934.19	942.14	932.89	856.26	862.51	856.87	3.56
8	F	1390.54	1427.50	1437.15	1432.53	1385.77	1400.31	1390.23	0.34
9	F	1561.19	1633.70	1643.87	1644.54	1556.45	1575.44	1569.06	0.30
10	F	2431.94	2972.73	3009.48	2989.36	2702.60	2705.36	2735.88	-10.01
1	S	239.48	229.18	228.93	228.85	230.78	228.13	231.79	0.32
2	S	274.97	255.22	258.01	256.37	264.52	254.47	262.40	0.29
3	S	250.04	240.26	240.59	240.22	240.38	239.38	240.54	0.35
4	S	383.64	377.09	376.48	376.98	374.86	374.13	373.74	0.73
5	S	258.47	241.68	241.38	239.79	242.82	237.82	239.87	0.83
6	S	598.78	584.59	588.29	588.98	559.75	560.95	559.72	4.44
7	S	596.72	596.64	601.40	593.49	571.91	572.97	578.75	3.77
8	S	658.12	653.96	658.45	652.01	631.68	640.24	637.59	3.22
9	S	852.87	869.49	864.14	866.28	846.02	840.92	845.07	1.42
10	S	1341.67	1370.73	1366.66	1356.52	1268.85	1319.07	1312.62	5.74
1	M	250.29	239.40	239.64	238.92	239.02	238.71	239.51	0.09
2	M	239.65	225.88	227.19	225.64	233.36	226.14	231.41	-0.22
3	M	233.77	223.47	222.88	220.68	220.85	218.36	221.55	1.06
4	M	368.49	354.77	353.38	354.59	356.36	353.98	354.18	-0.17
5	M	238.76	238.28	238.67	238.93	236.22	229.48	235.59	3.84
6	M	551.65	541.70	547.79	546.45	524.31	525.97	521.46	3.88
7	M	539.99	533.79	532.00	532.13	512.08	509.23	510.37	4.47
8	M	606.77	599.03	602.23	603.03	580.01	586.01	581.82	3.28
9	M	774.84	793.25	794.69	795.18	778.22	772.39	766.69	1.06
10	M	912.69	941.71	945.22	970.18	794.76	841.70	784.55	16.33
1	L	233.77	218.20	218.20	218.20	221.30	217.96	221.98	0.11
2	L	230.31	213.90	214.05	213.70	213.59	212.79	213.03	0.42
3	L	227.28	216.45	215.65	215.46	210.91	211.17	209.81	2.69
4	L	340.25	326.07	327.10	326.12	330.15	326.67	330.19	-0.18
5	L	221.29	214.38	215.30	214.34	214.87	213.50	214.03	0.39
6	L	540.49	521.45	523.43	521.25	505.89	507.63	507.79	3.04
7	L	518.45	503.66	512.05	506.11	486.93	492.06	488.96	3.44
8	L	564.94	566.06	561.81	564.54	545.52	548.65	543.87	3.30
9	L	754.38	765.60	762.66	763.54	739.67	745.18	735.90	2.51
10	L	652.29	624.87	614.13	614.51	584.60	588.30	588.38	5.05
1	A	213.62	199.92	202.87	202.33	202.98	200.46	202.49	-0.27
2	A	189.32	186.04	185.97	185.75	185.89	185.75	185.67	0.04
3	A	189.47	180.08	179.46	179.85	180.03	179.15	179.62	0.17
4	A	283.24	273.31	273.12	272.69	268.64	269.32	267.57	1.91
5	A	196.69	183.01	183.01	183.16	182.56	182.56	183.01	0.25
6	A	480.67	474.75	478.10	476.31	454.25	465.19	459.32	4.51
7	A	446.60	446.59	453.13	452.87	426.42	427.85	425.78	4.89
8	A	494.25	484.75	486.63	486.06	470.93	476.08	467.91	3.60
9	A	657.33	675.30	677.12	666.86	651.03	654.13	656.47	0.97
10	A	531.97	544.29	552.03	547.76	529.16	529.04	529.57	0.55
<b>Average</b>		<b>546.61</b>	<b>557.25</b>	<b>559.91</b>	<b>558.04</b>	<b>533.38</b>	<b>536.12</b>	<b>534.67</b>	<b>2.48</b>

Table 4: Average computational results of the hybrid memetic-ant colony optimization algorithm - Bredström and Rönnqvist (2008) benchmark

size instances (instances 1-5) where it remains below 8% for most instances. In addition, when the size of the problem increase (instances 6-8 and 9-10), the commercial solver becomes unable to find even one feasible solution for most instances while the MACO finds a feasible solution for each instance.

It should be highlighted that within its two hours time limit, the commercial solver finds the optimal solution for only four instances (the instances "1M", "2M", "1L" and "2L"). Compared to the previous work of Decerl et al. (2018) who studied a similar problem except that the working time balancing was not part of the objective function, they showed that the same commercial solver was able to find the optimal solution in 13 out of the 50 instances of the benchmark suggested by Bredström and Rönnqvist (2008), and only with a time limit of one hour. Consequently, the addition of working time balancing to the objective function has substantially complicated the problem.

Finally, the hybrid memetic-ant colony optimization algorithm presented in this paper shows great efficiency compared to memetic and ant colony optimization algorithms by obtaining for most instances the lowest average value for the objective function. As well, the suggested MACO maintains a low gap compared to the commercial optimization solver on benchmark instances. It should also be pointed out that these good results are valid, regardless of the setting applied to the MACO during the experiments.

## 6. Conclusion

In this publication, we have studied a variant of the home health care problem with soft time window and synchronization constraints, as well as working time balancing. The problem is modeled in order to decrease the traveling time of the caregivers, to provide the best quality of service to the patients and to minimize the maximal working time difference between the caregivers. Indeed, a balanced working time between caregivers is essential to obtain fairness and ensure the applicability of the resulting planning.

In addition, we propose a novel hybrid memetic-ant colony optimization algorithm (MACO) in order to solve this problem. The algorithm is compared with a memetic algorithm from the literature as well as an ant colony optimization algorithm, which is for the first time applied on the home health care problem to the best of our knowledge. In order to evaluate the efficiency of the proposed algorithm, best results from the experiments are also compared with the commercial optimization solver Gurobi. Experiments are conducted on benchmark instances from the literature.

The results clearly show the efficiency of our hybrid memetic-ant colony optimization algorithm. Indeed, the MACO obtains the lowest average objective function for most instances. As well, no matter which settings are used the average value obtained among all instances is lower

than the average value using the memetic or ant colony optimization algorithms. However, it should be pointed out that the ACO provides the worst results with the highest average value for the objective function.

In addition, the MACO performs well overall when comparing the best result obtained with the commercial solver. Indeed, the deviation ratio remains low for small size instances where it remains below 7-8% for most instances. In addition, when the size of the problem increases, the commercial solver becomes unable to find even one feasible solution for most instances while the MACO finds a feasible solution for each instance.

In future works, other real-life characteristics could be addressed in the model such as patient preference for a given caregiver or the continuity of care by considering a longer planning horizon. In addition, we plan to extend our work by adjusting dynamically the probability to apply either ACO or MA during the execution of the hybrid algorithm. Similarly, the optimal value of ACO related parameters may be determined by a sensitivity analysis.

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Instance name		Objective function (m)				Gap (%)
		MIP	MACO v1	MACO v2	MACO v3	
<b>1</b>	<b>F</b>	325.43	354.64	349.02	355.15	7.25
<b>2</b>	<b>F</b>	340.93	352.49	353.98	352.74	3.39
<b>3</b>	<b>F</b>	320.38	326.40	326.24	326.21	1.82
<b>4</b>	<b>F</b>	482.30	499.33	497.83	498.91	3.22
<b>5</b>	<b>F</b>	365.61	385.16	386.59	386.59	5.35
<b>6</b>	<b>F</b>	1554.29	882.32	889.57	886.32	-43.23
<b>7</b>	<b>F</b>	> 532.55	833.81	838.57	832.38	-
<b>8</b>	<b>F</b>	> 583.56	1367.03	1365.40	1362.43	-
<b>9</b>	<b>F</b>	> 683.14	1522.42	1522.91	1538.59	-
<b>10</b>	<b>F</b>	> 519.92	2367.74	2362.78	2606.11	-
<b>1</b>	<b>S</b>	220.66	226.57	226.41	227.03	2.61
<b>2</b>	<b>S</b>	238.84	250.94	250.34	250.34	4.81
<b>3</b>	<b>S</b>	221.81	238.98	233.46	238.85	5.25
<b>4</b>	<b>S</b>	359.67	368.53	368.53	372.33	2.46
<b>5</b>	<b>S</b>	220.02	234.92	236.52	235.64	6.77
<b>6</b>	<b>S</b>	595.99	545.05	547.78	552.05	-8.55
<b>7</b>	<b>S</b>	> 379	552.45	558.97	553.05	-
<b>8</b>	<b>S</b>	> 437.16	619.73	628.10	628.06	-
<b>9</b>	<b>S</b>	> 517.63	820.87	818.60	829.10	-
<b>10</b>	<b>S</b>	> 392.38	1152.96	1247.09	1204.42	-
<b>1</b>	<b>M</b>	205.13*	235.09	237.76	236.20	14.60
<b>2</b>	<b>M</b>	213.70*	227.22	222.71	225.88	3.97
<b>3</b>	<b>M</b>	207.78	216.91	217.29	216.52	4.20
<b>4</b>	<b>M</b>	332.49	352.64	350.82	351.25	5.51
<b>5</b>	<b>M</b>	207.84	227.91	224.21	227.91	7.88
<b>6</b>	<b>M</b>	562.66	515.50	512.54	507.35	-9.83
<b>7</b>	<b>M</b>	756.70	507.03	491.69	505.37	-35.02
<b>8</b>	<b>M</b>	> 386.18	559.72	563.54	568.31	-
<b>9</b>	<b>M</b>	> 487.92	760.45	754.62	743.38	-
<b>10</b>	<b>M</b>	> 362.18	597.27	589.51	582.08	-
<b>1</b>	<b>L</b>	199.26*	217.59	215.87	217.42	8.33
<b>2</b>	<b>L</b>	203.81*	208.24	210.48	210.44	2.17
<b>3</b>	<b>L</b>	198.99	206.15	203.00	203.00	2.01
<b>4</b>	<b>L</b>	307.48	322.05	322.05	322.05	4.74
<b>5</b>	<b>L</b>	196.37	212.28	211.10	211.53	7.50
<b>6</b>	<b>L</b>	494.26	496.74	498.82	498.39	0.50
<b>7</b>	<b>L</b>	> 317.02	474.40	476.15	474.84	-
<b>8</b>	<b>L</b>	> 360.25	533.75	538.43	530.06	-
<b>9</b>	<b>L</b>	> 465.97	730.20	737.57	714.79	-
<b>10</b>	<b>L</b>	> 346.11	569.18	573.32	563.15	-
<b>1</b>	<b>A</b>	183.77	195.80	197.35	195.80	6.55
<b>2</b>	<b>A</b>	180.41	185.67	185.67	185.67	2.92
<b>3</b>	<b>A</b>	176.15	178.46	178.46	178.46	1.31
<b>4</b>	<b>A</b>	252.63	262.20	264.15	262.20	3.79
<b>5</b>	<b>A</b>	171.00	181.51	181.51	183.01	6.14
<b>6</b>	<b>A</b>	> 338.60	440.36	453.21	450.00	-
<b>7</b>	<b>A</b>	411.43	406.45	416.41	415.58	-1.21
<b>8</b>	<b>A</b>	> 333.98	464.19	467.13	460.23	-
<b>9</b>	<b>A</b>	> 425.98	635.52	640.78	645.29	-
<b>10</b>	<b>A</b>	> 327.46	523.11	522.36	519.00	-

Table 5: Best results obtained by the hybrid memetic-ant colony optimization algorithm compared to the optimization solver - Bredström and Rönnqvist (2008) benchmark

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