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# On some Opinion Dynamics in Multi-Agent Systems

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### MULTI-AGENT SYSTEM

MAS = set of  $n$  agents interacting between each others, each supporting some opinion.

### AGREEMENT TASK

The whole (or parts of the) system must agree on some opinion.

### LABEL PROPAGATION ALGORITHMS

LPAs = class of algorithms for community detection in MAS:

1. an initial label/opinion to each agent;
2. activation rule for each agents;
3. majority-based update rule for labels.

At the end, each community agrees on some different label.

### THE 2-CHOICES DYNAMICS

Opinion Dynamics for majority consensus studied also in non-complete topologies (e.g., expanders) [4,6].

Here, we consider it as a LPA.

Initialization: random binary state, red or blue.

Activation: all nodes active in each round.

Update: each node samples two neighbors and, if they have the same state, updates to that state.

Underlying graph:  $(2n, d, b)$ -clustered regular graph [2] with  $d > 2b$ , which exhibits nice clustered structure.

### 2-CHOICES DYNAMICS

Neighborhood of  $v$

Node  $v$

UPDATE

Neighborhood of  $v$

Node  $v$

### OPINION DYNAMICS WITH NOISE

Class of simple algorithms for the Majority Consensus Problem with noise perturbing communications. The dynamics must bring the system into a config. satisfying:

1. **Almost-Agreement**: the majority of agents agree on some opinion;
2. **Majority**: this opinion is the initial majority one;
3. **Almost-Stability**: such a configuration lasts for time  $\text{poly}(n)$ .

Applications in sensor networks [1], chemical reaction networks [3], biological systems [8], ecc.

### RESULT: with constant prob. the initialization is asymmetric and, w.h.p., it holds that [5]

Symmetry-breaking after time  $\mathcal{O}(\log n)$

Almost agreement in time  $\mathcal{O}(\log n)$ . The bias in each community is  $n - \mathcal{O}(\log n)$

Meta-stable phase of length  $\Omega(n^\lambda)$ , for some  $\lambda > 0$

### NOISE MODEL AND RESULTS

We introduce a simple model of uniform noise, bio-inspired [8].

$u$  pulls  $v$ 's state

with prob.  $1 - 3p$  it sees red

with prob.  $p$  it sees undecided

with prob.  $p$  it sees red

with prob.  $p$  it sees blue

We prove [7] the following holds w.h.p. for  $p < 1/6$ :

Symmetry-Breaking    Almost-Consensus    Meta-Stable Phase

time  $\mathcal{O}(\log n)$  to bias  $\Omega(\sqrt{n \log n})$     time  $\mathcal{O}(\log n)$  to bias  $\Theta(n)$     for  $\text{poly}(n)$  rounds bias  $\Theta(n)$

**Note**: if  $p > 1/6$  every information about the majority color is lost in time  $\mathcal{O}(\log n)$ , w.h.p.

### THE UNDECIDED-STATE DYNAMICS

Introduced by [1] as a fast, robust and simple dynamics for the majority consensus problem.

Rule:

1. each agent  $u$  samples one neighbor and look at its state  $x$ ;
2. if  $x$  is different from  $\text{state}(u)$ , then  $u$  becomes undecided;
3. if  $u$  is undecided, then it copies what it sees.

Neighborhood of  $v$

Node  $v$

UPDATE

Neighborhood of  $v$

Node  $v$

[3] has completely characterized the parallel binary case in the complete graph.

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