Water management in ancient Greece: The Inopos reservoir on Delos

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Delos island, Inopos river, water management, drain time, ancient Greece

Abstract
Water resource management has played an important role in the development of ancient Greek cities. The semiarid Mediterranean environment with significant rain falling in the winter months only is the typical climate of the Cyclades Islands. In the case of Delos, the development of an elaborate water supply system allowed an efficient management of scarce water on the island. One of the most interesting hydraulic work is certainly the Inopos dam and its reservoir, named “the Inopos reservoir”. The building of the dam across the Inopos river began in the classical period from the 5th through 4th centuries BC. In the Imperial period (27 B.C.-393 A.D.), holes were drilled through the dam in order to allow water flow. In this paper, we used a simple approach based on the Bernoulli’s equation with losses to explain the performance of this hydraulic structure.
1. Introduction

The development of urban centres as well as that of agriculture during antiquity was directly related to water resource sustainability. The climatic constraints of the Mediterranean environment, the hot and semi-arid climate, led to limited and often inadequate natural water resources. These climatic conditions forced the ancient Greeks to develop advanced hydraulic technology to capture, store, and convey water (Angelakis & Koutsoyiannis 2003, Mays et al. 2007, Koutsoyiannis et al. 2008, Mays 2008, Zarkadoulas et al. 2012, Yannopoulos et al. 2016). The following examples show their remarkable mastery of hydraulic technologies.

The management of water resources began in Greece during the Early Minoan period (ca. 3500-2150 B.C.). The first wells, cisterns and fountains appeared during this period (Angelakis et al. 2013, 2016). Various technologies such as rainwater harvesting, sedimentation tanks and aqueducts are started to be used from the Early Minoan period II (ca. 2900-2300 B.C.).

The water supply system of the palace of Knossos shows the remarkable know-how of the Cretans during the Minoan period. The palace had at least three separate water-management systems: one for supply, one for drainage of runoff, and one for drainage of waste water (Angelakis et al. 2005, 2014). The water was conveyed from springs over several kilometers using terracotta pipes. The Minoan settlements used rainfall collection and cisterns over a 1,000 years before the classical and Helenistic-Greek cities.

During the Mycenaean Era (1900-1100 B.C.), large dams were built in Peloponnese. For example, a dam was erected across the Lakissa River to protect the town of Tyrins from flooding. The Tiryns dam was 10 m high, 3.5 to 4.0 m thick and 70 m long constructed with an earth core and two masonry walls. The water was diverted towards another river further south by a 1.5 km long canal (Balcer 1974, Tassios 2008).

The most famous hydraulic work of ancient Greece was the tunnel of Eupalinos on the Greek island of Samos, Asia Minor (Goodfield & Toulmin 1965, van der Waerden 1968, Burns 1971, Mitchell 1975, Apostol 2004). This tunnel was excavated through a mountain from both ends. The 1036 m long tunnel is the central part of a 2800 m aqueduct. It is estimated that between 8 and 15 years were necessary to complete the work begun in 523 B.C. (Archaic period, ca. 800-480 B.C.) and it was in use for about 1,100 years. In 1992, UNESCO declared the area of Eupalinos Aqueduct a World Cultural Heritage site.

During the Classical and Hellenistic periods (500-323 B.C. & 323-146 B.C.), the significant increase of the population in the urban centers built according to the Hippodameian system, imposes new rules for water management (Mays 2008, Koutsoyiannis et al. 2008, Angelakis et al. 2012). Advances in mathematics and physics have allowed hydraulic engineers to improve water distribution.

For example, a better understanding of the role of pressure is found in the water supply system of the citadel of Pergamon (Garbrecht 1995, Viollet 2007, Koutsoyiannis et al. 2008, Saba 2009, de Feo et al. 2013). The ancient city of Pergamon is in western Anatolia (Bergama, Turkey) and 26 km from the Mediterranean Sea. The water supply of the city was initially based on fountains and rock cisterns at the foot of the acropolis in which rainfall was stored. Probably around 200 B.C., a 50 km long triple aqueduct was built, included a vaulted tunnel of 180 m of length and several settling basins that led freshwater directly onto the acropolis. One of the aqueducts had an inverted siphon to go through a depression. This siphon of length exceeding 3 km, was certainly made of lead and withstood a maximum pressure head of about 190 m. It has been calculated that the amount of fresh
water available in Pergamon was as high as 26 000 m$^3$ per day, equal to an average of 160 L per day and person (with 160 000 inhabitants) (Berking et al. 2016).

Water management in the Greek islands was very critical mainly because of the lack of natural water resources, such as large rivers or groundwater associated with a semi-arid climat. The sources of water for the ancient Greek cities on the islands were mainly rainwater harvesting systems connected to wells or cisterns. A dam built across a nonpermanent river also allowed the formation of a freshwater stock. Limited water resources required an efficient urban water supply. One of the most interesting water management systems is certainly the Inopos dam and its reservoir. This hydraulic work is unique in the Mediterranean world.

In this paper, we propose a simple approach based on the Bernoulli’s equation to explain the performance of this hydraulic structure. We take into account the effect of frictional head losses and minor losses on the drain time of the Inopos reservoir. This case shows how the ancient Greeks used their empirical knowledge in fluid dynamics to manage water resources.

### 2. Island of Delos and the Inopos reservoir

The island of Delos, located in the centre of the Cyclades (Greece, Aegean Sea), 150 km south-east of Athens, is characterized by a semi-arid Mediterranean climate. It is one of the most important archaeological sites in Greece. With a surface area of 3.6 km$^2$, Delos is the smallest island of the insular group Mykonos, Delos and Rhenia. Its highest point is the mount Kynthos or Cynthium at 113 m. As the Greek geographer, Strabo (64/63 B.C.-24 A.D.) says in Geography (Jones 1961): “Now the city which belongs to Delos, as also the temple of Apollo, and the Letöum$^1$, are situated in a plain; and above the city lies Cynthius$^2$, a bare and rugged mountain; and a river named Inopus flows through the island – not a large river, for the island itself is small.”

Water scarcity due to the dry climate and the insularity of the Cyclades are not favourable to fresh water resources (Desruelles 2004, Desruelles & Cosandey 2005, Desruelles 2007). Rainfalls are relatively weak and irregular. Actually, the average rainfall reached 200 mm to 400 mm per year (Brunet 2011). The quasi totally of rainfalls occurs between October and April and the months of July and August are totally dry. During the warmest months, from May to October, almost all the rain evaporates. Figure 1 shows average annual variations in temperature and precipitation during 1971-2000 (Hellenic National Meteorological Service 2018). The average rainfall was 400 mm per year during this period. The Moreover, annual precipitations may vary strongly from one year to another with a factor 1 to 5 (Moretti & Fincker 2011). The geological structure of Delos allows the formation of many aquifers. The replenishment of aquifers can be favoured by the abundance and the intensity of winter rainfall. Precipitation is the only source of water for this island and the main concern of its inhabitants is the management of the irregularity of this resource. It was assumed that the climatological conditions during the Antiquity were probably the same (Desruelles 2004, Finné et al. 2011, 2014, Benito et al. 2015).

The Delos population did not exceed 6,000 persons in the III$^{rd}$ century B.C. A bet-

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1. Temple of Leto
2. Cynthium
Figure 1

...
houses (Chamonard 1924). Their dimensions were quite proportional to the size of the roof and thus to the volume of water it collected. The storage capacities of these tanks could reach significant volumes. In the Theater quarter, the House of Dionysus (14 in Fig. 2), whose roof measured 500 m$^2$, was equipped with a 190 m$^3$ storage tank, the house of the Trident (13 in Fig. 2), with a roof of 350 m$^2$, had a cistern of 155 m$^3$ and, the Hostelry (15 in Fig. 2) had a 300 m$^3$ cistern for a roof of 1140 m$^2$ (Desruelles 2004).

The source of Inopos, the main water course of Delos, is located to the south-west of
Inopos reservoir on Delos, general plan at 1/400 (Fincker & Moretti 2007).

the Cynthium (1 & 2 in Fig. 2). Contrary to what the mythology suggests, Inopos was only an intermittent stream of water. This river was subject to a number of interventions, the most important located upstream from where its original direction changes. This basin is known as the “Inopos lower reservoir”  \(^4\) or the “Inopos reservoir” (3 in Fig. 2). Its current configuration consists of a paved platform, under which is a water reservoir, bordered by a staircase and the tank itself (Fig. 3). This impoundment created by the development of the river is approximately rectangular. The free surface area of the reservoir is equal to 378 m\(^2\) (42 m x 9 m) corresponding to a maximum volume of 1700 m\(^3\). This reservoir is both a well and a collector of runoff from the surrounding slopes, which also serves as the Inopos water catchment basin, when a flow occurs (Brunet 2011). The volume of water from the Inopos

\(^4\) The “Inopos upper reservoir” is in fact, a quarry crossed by the Inopos river (4 in Fig. 2).
supplying the reservoir was estimated at 51 000 m³ for one year (2000-2001) (Brunet et al. 2003).

The history of the Inopos reservoir can be divided into three periods (Reinach 1883, Holleaux 1909, Bruneau 1990, Fincker & Moretti 2007, Moretti & Fincker 2011, Brunet 2011):

— The building of the dam began in the Classical period from the Vth through IVth centuries B.C. A channel of leakage brought the water to the terraces of culture or small reservoirs.
— During the Delos Independence (314-167 B.C.), the reservoir was deeply modified. A second wall was built to create a crawl space and thus limit water infiltration. It was at this time that the steps were built. A raising of the walls allowed to increase the maximum capacity of the tank of 300 m³.
— The dam was highly modified during the Imperial period (146 B.C.-330 A.D.), presumably around 27 B.C. The main change in its structure was the drilling of four holes in the stairs (Fig. 4).

Figure 4
Reproduction of the Inopos reservoir on Delos, Imperial period. Myriam Fincker, IRAA, CNRS, Université Lyon 2.

Figure 5
Terracotta pipe.
These holes with diameters between 5 and 7 cm are located at different levels. They are connected directly to terracotta pipes themselves linked to a vertical terracotta pipe in order to feed an aqueduct. From the base of the stonework starts a fourth pipe. Feet of amphoras were used to block the pipes. The aqueduct is divided into two branches. One feeds the thermal baths via a water tower. The route of the second branch is not clearly established (Holleaux 1909, 1912, Blackman 2000-2001, Whitley et al. 2005-2006, Fincker & Moretti 2007, Moretti & Fincker 2011).

The terracotta pipe segments were linked with mortar (Fig. 5). Generally, the terracotta in contact with the water has been glazed. Corresponding roughness is comprised 10 and 30 \( \mu m \) and up to 0.2 and 0.5 mm in the case of a calcareous deposit (Haut & Viviers 2010).

Figure 6
Sketch of the Inopos reservoir.

To model the reservoir, we have chosen to neglect the variation of surface area due to the presence of steps. The Figure 6 shows a sketch of the Inopos reservoir. As regards the terracotta pipe, as no accurate information is available about the absolute roughness of the pipe used, we will take 0.5 mm as our value.

3. Results

The drainage model of the Inopos reservoir on Delos is obtained from Bernoulli’s equation. This equation is obtained when we consider the incompressible flow of an inviscid fluid. The development of the calculations is given in appendix A.

The water level \( h(t) \) in the reservoir is given by the following relationship:

\[
h(t) = h_0 - \sqrt{\frac{2gh_0}{\alpha^2\beta - 1}} t + \frac{g}{2(\alpha^2\beta - 1)} t^2
\]

with:

- \( \beta = 1 \), without loss;
- \( \beta = 1 + f l/d \), with frictional head loss;
- \( \beta = 3/2 + f l/d \) with frictional head and entrance losses.

where \( f \) is a dimensionless coefficient called the Darcy friction factor, \( l \) and \( d \) are, respectively, the length and the hydraulic diameter of the pipe.

The time to empty the tank or drain-time \( t_d \) is given by:

\[
t_d = \sqrt{\frac{2h_0}{g(\alpha^2\beta - 1)}}
\]

8. D. Graebling
The flow rate is obtained from the time derivative of the water level \( h(t) \):

\[
Q(t) = -\frac{dV}{dt} = -A \frac{dh(t)}{dt} = A \left( \sqrt{\frac{2gh_0}{\alpha^2\beta - 1}} - \frac{gt}{\alpha^2\beta - 1} \right)
\]

where \( V \) and \( A \) are respectively, the volume and the free surface area of the reservoir.

The mean flow rate is given by the following relationship:

\[
\overline{Q} = \frac{V}{t_d} = \int_0^{t_d} Q(t) \, dt = A \left( \sqrt{\frac{2gh_0}{\alpha^2\beta - 1}} - \frac{gt_d}{2(\alpha^2\beta - 1)} \right)
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Pipe} & 1 & 2 & 3 & 4 \\
\hline
h_0 (m) & 1.32 & 0.39 & 0.63 & 0.55 \\
L (m) & 0.5 & 1.0 & 1.5 & 2.0 \\
\Phi (m) & 0.055 & 0.074 & 0.062 & 0.065 \\
f & 0.037 & 0.033 & 0.035 & 0.035 \\
\hline
\end{array}
\]

Table 1  Geometric characteristics and Darcy friction factors of the pipes.

Table 1 gives the geometric characteristics and Darcy friction factors of the pipes. We assume that the flow is fully developed turbulent flow, and so the Darcy friction factors was calculated from the van Kármán’s equation. This assumption is valid for \( Re > 2.10^5 \) (Moody chart (Moody 1944)).

The results obtained for the three flows are summarised in Figure 7 and Table 2. The successive opening of the pipes (solid line with symbol) is compared to the opening of the fourth pipe (dashed line).

Regardless of the assumptions used, the drain with the successive opening of the pipes allows a better flow rate control. The split procedure of the flow leads to a considerable increase of the drain-time. If we take into account the head and minor losses, the drain-time almost doubles with the split, \( \approx 38 \rightarrow 76 \) h. This effect is clearly visible on the mean flow rates, \( 467 \rightarrow 240 \) L.min\(^{-1}\).

As expected, the variation of the absolute roughness makes only a marginal difference in the drain time. A doubling of the roughness \( 0.5 \rightarrow 1 \) mm, only implies a few percent increase in drain time \( \approx 76 \rightarrow 80 \) h, in the case where the head and minor losses were taken into account.

The Reynolds number decreases with the flow rate and thereby with the time. The transitionally rough range appears for \( Re > 2.10^5 \) and so, the assumption of fully developed turbulent flow is valid for nearly 90% of the drain time. This does not alter our conclusions about the design principles of the dam.

In the entrance region where a nearly inviscid upstream flow converges and enters the tube, the pressure decreases as fluid velocity increases. In all cases, the entrance pressure is quite sufficient to prevent the cavitation.
4. Discussion

The main challenge for Delians was to manage the urban water-supply system from a weak resource. This requires a good control of the Inopos outflow.

During antiquity, very few devices were used to regulate the water distribution. Reservoirs and cisterns could serve as retaining ponds but the most commonly used control device in aqueduct channels is the vertical sluice-gate or cofferdams (Chanson 2000, Bossy et al. 2000, Chanson 2002, 2008).

The castellum divisorum at Nîmes shows at its entrance two vertical grooves which suggest that it was equipped with a movable sluice-gate, consisting of two parts or plates (Kessener 1995, Duval 1989, Liardet 2010). The function of this sluice-gate has originally been associated with a device for controlling the inflow of water into the castellum. The gate could be completely closed to clean the castellum(Pelet 1844, 1876, pp 304-305).
<table>
<thead>
<tr>
<th>Flow</th>
<th>$h_0$ (m)</th>
<th>Without Head</th>
<th>Head</th>
<th>Head &amp; minor</th>
<th>Without Head</th>
<th>Head</th>
<th>Head &amp; minor</th>
</tr>
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<tbody>
<tr>
<td>0→1</td>
<td>1.32</td>
<td>22:56</td>
<td>26:30</td>
<td>30:04</td>
<td>363</td>
<td>314</td>
<td>268</td>
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<td></td>
<td></td>
<td></td>
<td>725</td>
<td>627</td>
<td>535</td>
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<tr>
<td>1→2</td>
<td>0.39</td>
<td>6:53</td>
<td>8:17</td>
<td>9:36</td>
<td>357</td>
<td>297</td>
<td>256</td>
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<td></td>
<td></td>
<td>712</td>
<td>593</td>
<td>511</td>
</tr>
<tr>
<td>2→3</td>
<td>0.63</td>
<td>12:28</td>
<td>16:56</td>
<td>19:06</td>
<td>318</td>
<td>234</td>
<td>208</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>636</td>
<td>468</td>
<td>415</td>
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<tr>
<td>3→4</td>
<td>0.55</td>
<td>10:36</td>
<td>15:16</td>
<td>17:00</td>
<td>327</td>
<td>227</td>
<td>204</td>
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<td></td>
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<td></td>
<td></td>
<td>710</td>
<td>453</td>
<td>443</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td>52:53</td>
<td>66:59</td>
<td>75:46</td>
<td>344</td>
<td>272</td>
<td>240</td>
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<td>696</td>
<td>535</td>
<td>476</td>
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<tr>
<td>0→4</td>
<td>2.89</td>
<td>24:18</td>
<td>35:13</td>
<td>38:59</td>
<td>750</td>
<td>520</td>
<td>467</td>
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<td></td>
<td></td>
<td>1,498</td>
<td>1,040</td>
<td>937</td>
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</table>

<table>
<thead>
<tr>
<th>t_d (h : min)</th>
<th>$\bar{Q}$ (L/min)</th>
<th>$Q_{max}$ (L/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0→1</td>
<td>363</td>
<td>268</td>
</tr>
<tr>
<td>1→2</td>
<td>357</td>
<td>256</td>
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<tr>
<td>2→3</td>
<td>318</td>
<td>208</td>
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<td>3→4</td>
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<td>Σ</td>
<td>344</td>
<td>240</td>
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<tr>
<td>0→4</td>
<td>750</td>
<td>467</td>
</tr>
</tbody>
</table>

Table 2  Drain times $t_d$, Mean flow rate $\bar{Q}$ & Max flow rate $Q_{max}$.

One part was fixed and sealed to the floor of the channel, the other part was movable, sliding up and down in the vertical grooves partly behind the fixed part, and operated from above.

Figure 8
Tebourba (Tunisia). Distribution chamber (Germain de Montauzan 1908, p. 316).
In Tebourba, “Thuburbo minus” (Tunisia), Montauzan discovered in 1906, a small distribution basin (2.8 m long by 1.5 m wide and 1.5 m deep) with one inlet and three outlets equipped with sluice gates (Fig. 8) (Germain de Montauzan 1908, pp 316-317). One of these outlets (C) consists of a plate with two rectangular holes that can be closed to regulate outflow which fed cisterns.

The ancient Greeks understood that the velocity, i.e. the flow rate, of a liquid flowing under the force of gravity out of an opening in a tank is proportional to the square root of the water height. The clepsydra or water clock is the oldest use of this principle. Simple water clocks date back to perhaps 1500 B.C. in Egypt and Mesopotamia (Pogo 1936, Neugebauer 1947, Landels 1979, Mills 1982, Cotterell et al. 1986, Mays et al. 2007).

In the absence of an adjustable valve, the water-level control is the most appropriate way to regulate the flow rate.

![Figure 9](image.jpg)

**Figure 9**

Delos. “Inopos upper reservoir”. Marble plate pierced with four holes (Moretti & Fincker 2011)

At Delos, a marble plate pierced with four holes was discovered in a masonry conduit connected to the “Inopos upper reservoir” (15 in Fig. 2 and Fig. 9) (Fincker & Moretti 2007, Moretti & Fincker 2011). It would have served to regulate the flow and filter the water. Its construction seems to date from 88 B.C. (Desruelles 2004). It is amazing to find two similar hydraulic structures in the same area.

**5. Conclusion**

The climatic constraints of the Mediterranean environment, the hot and semiarid climate, led to limited and often inadequate natural water resources. These climatic conditions forced the ancient Greeks to develop advanced hydraulic technology to manage the scarce water resources.

At Delos, water scarcity due to the dry climate and its insularity are not favourable to fresh water resources. The “Inopos reservoir” was designed to fight the effects of pluviometric irregularity. During the Imperial period, the Delians drilled four holes in the stair of the dam.

A simple approach based on the Bernoulli’s equation with losses is enough to explain
the performance of the “Inopos reservoir”. Regardless of assumptions used, the drain with the successive opening of the pipes allows a better flow rate control. This procedure allows efficient management of the weak and irregular water resource due to intermittent rainfall.

6. Acknowledgement

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A. Appendix: Flow pipe

The drainage model of the Inopos reservoir on Delos is obtained from Bernoulli’s equation when we consider the incompressible flow of an inviscid fluid. A common form of Bernoulli’s equation, valid at any two arbitrary points along a streamline where gravity is constant, is:

\[ p_1 + \rho \frac{v_1^2}{2} + \rho g h_1 = p_2 + \rho \frac{v_2^2}{2} + \rho g h_2 + \rho g (h_f + h_s) \]

where \( p \) is the pressure, \( \rho \) is the water density, \( v \) is the mean velocity, \( g \) is the acceleration due to gravity and \( h \) is the elevation. \( h_f \) and \( h_s \) represent the head loss and the minor loss, respectively.

As the free-surface pressure \( p_1 \) and the pipe outlet pressure \( p_2 \) are both atmospheric pressure, Bernoulli’s equation becomes:

\[ \frac{v_1^2}{2g} + h = \frac{v_2^2}{2g} + h_f + h_s, \quad h = (h_1 - h_2) \]

The head loss is usually calculated by the Darcy-Weisbach equation (Moody 1944):

\[ h_f = f \frac{l}{d} \frac{v_2^2}{2g} \]

where \( f \) is a dimensionless coefficient called the Darcy friction factor, \( l \) and \( d \) are, respectively, the length and the hydraulic diameter of the pipe. For a circular pipe, the hydraulic diameter is equal to the diameter of the inner pipe.

The friction factor depends on the Reynolds number, \( \mathcal{R} \), and on the relative roughness of the pipe, \( k = \epsilon/d \). For fully turbulent flow, \( \mathcal{R} > 4000 \), the friction factor is usually determined by the Colebrook-White equation (Colebrook & White 1937, Colebrook 1939, Sonnad & Goudar 2007, Brkić 2011):

\[ \frac{1}{\sqrt{f}} = -2 \log \left( \frac{k}{3.71} + \frac{2.52}{\mathcal{R} \sqrt{f}} \right) \]

For a fully developed turbulent flow, the friction factor depends only on the relative roughness of the pipe and the Colebrook-White equation is equivalent to the van Kármán equation:

\[ \frac{1}{\sqrt{f}} = 2 \log \left( \frac{3.71}{k} \right) \]
Skin friction is normally considered as the major factor causing the energy loss and form drag as the minor factor. But form drag caused by pipe entrance, section variation and pipe curves can overshadow skin friction in the dissipation of energy, when the pipe length is relatively short (White 1999, Yoo & Singh 2010).

In our case, we assume that the minor losses are only due to the entrance of the pipe. This loss is given by the following relationship:

\[ h_s = \frac{1}{2} \frac{v_2^2}{g} \]

In the case of an incompressible flow, we have: \( v_2 = \frac{S_1}{S_2} v_1 = \alpha v_1 \). \( S_1 \) and \( S_2 \) are respectively the area of the reservoir’s free surface and the cross section of the pipe. In our case, \( \alpha \gg 1 \).

As fluid drains from the pipe, the level of the free-surface decreases with the time: \( v_1 = -\frac{dh}{dt} \) and we finally obtain the following relationship:

\[ \frac{1}{2g} \left( 1 - \alpha^2 \left( \frac{3}{2} + f \frac{l}{d} \right) \right) \left( \frac{dh}{dt} \right)^2 + h = 0 \]

The solution of this equation is:

\[ h(t) = h_0 - \sqrt{\frac{2gh_0}{\alpha^2 \beta - 1}} t + \frac{g}{2(\alpha^2 \beta - 1)} t^2 \]

\[ t_d = \sqrt{\frac{2h_0 g}{\alpha^2 \beta - 1}} \]

with:
- \( \beta = 1 \), without loss;
- \( \beta = 1 + f l/d \), with frictional head loss;
- \( \beta = 3/2 + f l/d \) with frictional head and entrance losses.