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# About the Wrong Proof of the Lindley Paradox

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**Abstract:** The purpose of this paper is to show that the so called Lindley Paradox is not at all a paradox. It came from a wrong application of Bayesian Theory to the Hypothesis Testing. Two hypotheses  $H_0$  and  $H_1$  are considered,  $H_0: \{\theta=\theta_0\}$  versus  $H_1: \{\theta\neq\theta_0\}$ ;  $\theta$  denotes the parameter whose value we want to test. Two theories are available: Classical Statistics and Bayesian Statistics. Lindley (in 1957) pretended to prove that the two theories provide two opposite and contradictory solutions to the same problem: that, obviously, is not scientifically acceptable. Using the “Distribution Theory (in Mathematics)” which extends the Theory of the Real Functions to the Real Functionals, where the “Dirac Delta  $\delta$ ” (named also the “Impulse Function”) is scientifically defined and it is the derivative of the Heaviside function, we find the true form of the posterior probability  $P(H_0|\text{data})$  and prove the mistake of Lindley. Researchers must be alert in order to do a good job... Everybody have to consider the methods of Logic and of the Scientific Theory (Mathematics, Probability, Statistics, Physics...). Several Professors do not practice them.

**Keywords:** Bayesian approach, Quality Methods and Statistics, Scientific Approach, Intellectual Honesty

## 1. Introduction: “The Problem Outline”

On November 2021, the author was asked to read about the Lindley Paradox in the paper “Demystify Lindley’s Paradox by Interpreting P-value as Posterior Probability” [1].

He was really astonished: wrong application of the Bayes’ Theorem.

After reading various papers, not cited in the references, on the subject (Lindley Paradox), he decided to read the original paper “A Statistical Paradox” [Biometrika], hoping to find a sound and correct use of the Bayes’ Theorem [2].

He was more astonished than before: another wrong application of the Bayes’ Theorem.

Hence, the author decided to write this paper.

Before proving the subtle errors, we need to recap the Bayes’ Theorem.

Let  $S$  be the set made of the union of other sets  $A_i$ ,  $i=1, \dots, n$ .  $S$  is named the “space (of events)”: any set  $A_i$  is a subset of  $S$  and we write  $A_i \subset S$ .  $A_i$  are called “events” and hence  $S$  is named the “certain of events”. Remember the operations on any number of sets: union, intersection, negation, mutually exclusivity and the related theorems.

To each event, say  $B$ , we attach a number, the “probability  $P(B)$  of the event  $B$ ”. The probability must satisfy the “probability Axioms”. See any good book... We also define the “conditional probability  $P(B|A_i)$  of the event  $B$ ”.

**Bayes’ Theorem for Events:** If  $\{A_1, A_2, \dots, A_n\}$  is a partition of  $S$  [ $A_i \cap A_j = \emptyset \forall i, j$ , with  $\cup A_i = S$ ] and  $B$  an arbitrary event, then

$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1) + \dots + P(B|A_n)P(A_n)}$$

This theorem can be extended to Random Variables (RV).

Let  $X$  be a RV and  $B=\{X \leq x\}$  an event; we consider the

Cumulative Probability Function  $F(x)=P\{X \leq x\}$  and  $f(x)$  the related pdf (probability density function); hence we can define the Conditional Cumulative Probability Function  $F(x|A)=P\{X \leq x|A\}$  and the related Conditional pdf (Conditional probability density function)  $f(x|A)$ .

From the identity

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

we derive

$$P(A|X \leq x) = \frac{F(x|A)P(A)}{F(x)}$$

and (with a limit process)

$$P(A|X = x) = \frac{f(x|A)P(A)}{f(x)}$$

Therefore, we get the probability of the event  $A$

$$P(A) = \int_{-\infty}^{\infty} P(A|X = x) f(x) dx$$

and hence the

**Bayes’ Theorem for continuous Random Variables:** If  $X$  is a continuous RV the conditional pdf  $f(x|A)$ , with  $A$  an arbitrary event, is

$$f(x|A) = \frac{P(A|X = x)f(x)}{\int_{-\infty}^{\infty} P(A|X = x) f(x) dx}$$

We have also

$$F(x|A) = \frac{P(A|X \leq x)F(x)}{P(A)}$$

$$\text{and } P(A) = P(A|X \leq x)F(x) + P(A|X > x)[1 - F(x)]$$

## 2. Test of Hypothesis and Distribution Theory

Now we go to the *Test of Hypotheses*, as given in the paper “Demystify Lindley’s Paradox by Interpreting P-value as Posterior Probability” [1].

We consider two hypotheses  $H_0$  and  $H_1$ , where  $\theta$  denotes the true proportion of male births, in a city, in a year; we want to test  $H_0: \theta=0.5$  versus  $H_1: \theta \neq 0.5$ ; collecting the data we have the observed proportion of male births  $y=28298/56099 \approx 0.5044297$ ; using the normal approximation (sample size  $n$  large)  $Y \sim N[\theta, y(1-y)/n]$  we calculate (in the frequentist approach) the  $p$ -value as

$$\Pr(Y \geq y|H_0) = \int_{28298/56099}^{\infty} \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(x-0.5)^2}{2\hat{\sigma}^2}\right\} dx \approx 0.01793329$$

Evidently, the exact and approximate  $p$ -values are very close. As the hypothesis test is two-sided, the final  $p$ -value is  $2 \times 0.01793329 \approx 0.03586658$ . At the typical significance level of 5%, we clearly reject  $H_0$ .

*Excerpt 1.* From the paper “Demystify Lindley’s Paradox ... [1].

Since the Excerpt is not easily readable we rewrite it here:

$$\Pr(Y \geq y|H_0) = \int_{y=28298/56099}^{\infty} \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(x-0.5)^2}{2\hat{\sigma}^2}\right\} dx \approx 0.01793329$$

Evidently, the exact and approximate  $p$ -values are very close. As the hypothesis test is two-sided, the final  $p$ -value is 0.03586658. At the typical significance level 5%, we clearly reject  $H_0$ .

*Excerpt 1b.* From the paper “Demystify Lindley’s Paradox ... [1].

The  $H_0$  hypothesis is **rejected** at  $\alpha=5\%$ .

Using the Bayesian approach, the authors of [1] have to first specify a prior distribution on  $H_0$  and  $H_1$ . Without assuming any preference, they assign an equal prior probability to  $H_0$  and  $H_1$ , i.e.,  $P(H_0)=P(H_1)=0.5$ . Under  $H_0$ ,  $\theta$  has a point mass at 0.5. Under  $H_1$ ,  $\theta$  is not equal to 0.5 and, to be fair, they assign a uniform prior distribution to  $\theta$  on  $[0, 1]$ . They find the “posterior probability”

$$\begin{aligned} P(H_0|y) &= \frac{P(y|H_0)P(H_0)}{P(y|H_0)P(H_0) + P(y|H_1)P(H_1)} \\ &= \frac{\exp\left\{-\frac{(y-0.5)^2}{2\hat{\sigma}^2}\right\}}{\exp\left\{-\frac{(y-0.5)^2}{2\hat{\sigma}^2}\right\} + \int_0^1 \exp\left\{-\frac{(y-\theta)^2}{2\hat{\sigma}^2}\right\} d\theta} \\ &\approx 0.9543474, \end{aligned}$$

*Excerpt 2.* From the paper “Demystify Lindley’s Paradox ... [1].

Since the Excerpt is not easily readable we rewrite it here:

$$\begin{aligned} P(H_0|y) &= \frac{P(y|H_0)P(H_0)}{P(y|H_0)P(H_0) + P(y|H_1)P(H_1)} \\ &= \frac{\exp\left\{-\frac{(y-0.5)^2}{2\hat{\sigma}^2}\right\}}{\exp\left\{-\frac{(y-0.5)^2}{2\hat{\sigma}^2}\right\} + \int_0^1 \exp\left\{-\frac{(y-\theta)^2}{2\hat{\sigma}^2}\right\} d\theta} \approx 0.954 \end{aligned}$$

*Excerpt 2b.* From the paper “Demystify Lindley’s Paradox ... [1].

The result is “**accept**”  $H_0$ , because 0.95 strongly supports it.

NOTICE that  $\exp\left\{-\frac{(y-0.5)^2}{2\hat{\sigma}^2}\right\}$  is a number but it is NOT a probability: it is proportional to the normal density  $\exp\left\{-\frac{(x-\theta)^2}{2\hat{\sigma}^2}\right\}/\sqrt{2\pi}\hat{\sigma}$  (computed at the value  $x=y$ ) which obviously is NOT a probability (as a matter of fact it is integrated from 0 to 1)!

To find the true form of the posterior probability  $P(H_0|y)$  one needs to use the “Distribution Theory (in Mathematics)” [3 - 7] which extends the Theory of the Real Functions to the Real Functionals, where the “Dirac Delta  $\delta$ ” (named also the “Impulse Function”) is scientifically defined and it is the derivative of the Heaviside function. The Heaviside function is the integral of the “generalised function Dirac Delta  $\delta$ ”. Derivation, integration and series are defined without any exception in the “Distribution Theory (in Mathematics)”; there is the need to differentiate functions that are classically non-differentiable. Now we are in a position to show that, in a sense, distribution theory is exactly a theory of derivatives (of arbitrary order) of continuous functions. Locally any distribution is represented as a derivative of a continuous function. The special case of the Dirac-Delta is the derivative of certain continuous functions.

We indicate with the crochet symbol the integral (on the real line  $\mathbb{R}$ ), defining a linear functional (similar to the vectors scalar product) of  $\varphi$  ( $\varphi$  is named the “test function”, with an infinite number of derivatives)

$$\langle f, \varphi \rangle = \int_{\mathbb{R}} f(x) \varphi(x) dx$$

For any function  $f$  (generating function) “locally integrable” we define the crochet, the distribution  $\langle f, \varphi \rangle$  and vice versa: the generating function  $f$  is found from the crochet.

The Dirac distribution  $\delta$ , defined by

$$\langle \delta, \varphi \rangle = \varphi(0)$$

is such that its “integral” is 1 at point 0. ( $\varphi$ =“test function”)

More generally, at point  $x_0$ , we have

$$\langle \delta_{x_0}, \varphi \rangle = \varphi(x_0)$$

The derivative  $T'$  of a distribution  $T$  is defined by the crochet

$$\langle T', \varphi \rangle = -\langle T, \varphi' \rangle.$$

The integral of a Distribution  $S$  is a Distribution  $T$  such that its derivative is  $S$ , i.e. (think to the crochet)

$$\frac{dT}{dx} = T' = S$$

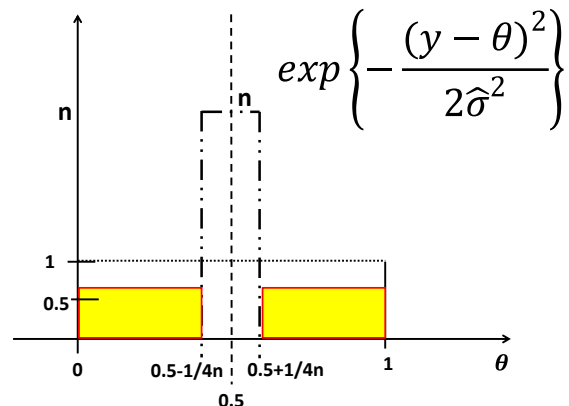


Figure 1. The pdf of the RV  $\theta$ . (for the paper [1])

Now we go back to the Bayesian approach about the density

$$f(x|\theta) = \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(x-\theta)^2}{2\hat{\sigma}^2}\right\}$$

To satisfy the prior distribution assumptions about on  $H_0$  and  $H_1$ , we consider the figure 1. The probability that  $\theta$  is in the interval  $0 \dots 1$  is 1 for any choice of the integer  $n$ : respectively, we have 0.25 for the left yellow rectangle, 0.5 for the middle rectangle, and 0.25 for the right yellow rectangle.

In the “Distribution Theory (in Mathematics)” setting, the limit for  $n \rightarrow \infty$  we have a yellow area equal to 0.5 and a Dirac Delta with “area” 0.5 at  $\theta=0.5$ ; the crochet of the Gaussian pdf is

$$\begin{aligned} \langle f(y|\theta), \theta \rangle &= \int_0^1 f(y|\theta) d\theta \\ &= \int_0^{0.5} \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(y-\theta)^2}{2\hat{\sigma}^2}\right\} d\theta \\ &\quad + \langle \delta_{0.5}, f(y|\theta) \rangle \\ &\quad + \int_{0.5}^1 \frac{1}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(y-\theta)^2}{2\hat{\sigma}^2}\right\} d\theta \\ &= 0.25\{N(0.5) - N(0)\} \\ &\quad + \frac{0.5}{\sqrt{2\pi}\hat{\sigma}} \exp\left\{-\frac{(y-\theta)^2}{2\hat{\sigma}^2}\right\} \\ &\quad + 0.25\{N(1) - N(0.5)\} \end{aligned}$$

Moreover, we have

$$f(\theta = 0.5|y) = \frac{\langle \delta_{0.5}, f(y|\theta) \rangle}{\langle f(y|\theta), \theta \rangle}$$

This is the “posterior almost-density” based on the Bayes Theorem for continuous RVs (with densities!)

$$k(z|w) = \frac{f(w|z)g(z)}{\int_{-\infty}^{\infty} f(w|z)g(z)dz}$$

It is absolutely normal that the two quantities p-value and the value of the “posterior almost-density”  $f(\theta = 0.5|y)$  do not match: NO Paradox! (see also the Excerpt 5)

Moreover, “posterior almost-density”  $f(\theta = 0.5|y)$  is not a true density: the area under the curve is  $< 1$ .

In order to understand if Lindley himself did the same error, the author read the [original paper](#) of D. V. Lindley [Stat Lab, University of Cambridge] (1957), “[A Statistical Paradox](#)”, *Biometrika*. 44(1-2): 187-192, 1957

We take Lindley’s own words:

We begin by giving the mathematical derivation of the example and later comment on it and the assumptions involved. Let  $\{x_1, x_2, \dots, x_n\}$  be a random sample from a normal distribution of mean  $\theta$  and known variance  $\sigma^2$ . Let the prior probability that  $\theta = \theta_0$ , the value on the null hypothesis, be  $c$ . Suppose that the remainder of the prior probability is distributed uniformly over some interval  $I$  containing  $\theta_0$ . We shall deal with situations where  $\bar{x}$ , the arithmetic mean of the observations, and a minimal sufficient statistic, is well within the interval  $I$ . The posterior probability that  $\theta = \theta_0$ , in the light of the sample, can be evaluated; it is

$$\bar{c} = c \exp\left[-n(\bar{x} - \theta_0)^2 / (2\sigma^2)\right] / K \quad (1), \quad \text{where}$$

$$K = c \exp\left[-\frac{n(\bar{x} - \theta_0)^2}{2\sigma^2}\right] + (1 - c) \int_I \exp[-n(\bar{x} - \theta)^2 / (2\sigma^2)] d\theta$$

by Bayes's theorem. In virtue of the assumption about  $\bar{x}$  and  $I$  the integral can be evaluated as  $\sigma\sqrt{2\pi/n}$ .

*Excerpt 3. From the paper “A Statistical Paradox” (Lindley’s paper).*

NOTICE: Excerpt 3 is wrong as it is Excerpt 2.

The function  $\bar{c}$  is neither a probability nor a density, as we can easily see by writing it as follows [where you see the “normal probability density”  $\varphi(\bar{x}|\theta_0; \sigma^2, n)$ ; from now on  $\varphi$  is not the “test function”]:

$$\begin{aligned} \varphi(\bar{x}|\theta_0; \sigma^2, n) &= \frac{1}{\sqrt{2\pi}\sigma/\sqrt{n}} \exp\left[-\frac{(\bar{x} - \theta_0)^2}{2\sigma^2/n}\right] \\ \bar{c} &= \frac{c\varphi(\bar{x}|\theta_0; \sigma^2, n)}{c\varphi(\bar{x}|\theta_0; \sigma^2, n) + (1 - c)} \end{aligned}$$

The formula  $\bar{c}$  is nonsense!

It does not conform to Theory. See the real pdf in figure 2: it was not used by Lindley and he found a wrong formula!

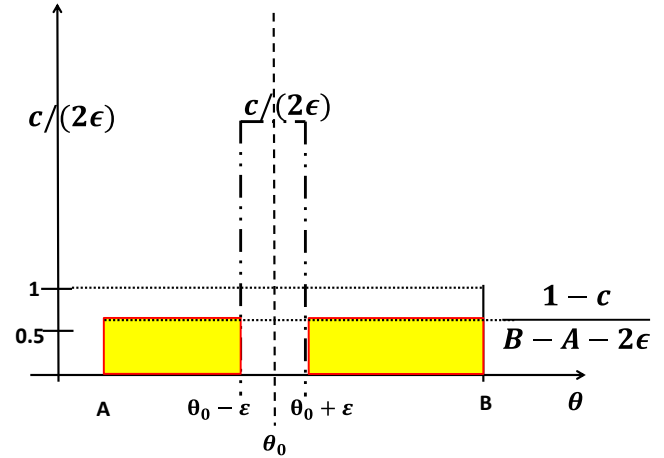


Figure 2. The pdf of the RV  $\theta$ . (for the Lindley paper [2])

To find the “posterior density” (based on the Bayes Theorem for continuous RVs)

$$k(\theta|w) = \frac{f(w|\theta)h(\theta)}{\int_{-\infty}^{\infty} f(w|\theta)h(\theta)d\theta}$$

We derive the correct posterior pdf

$$k(\theta|\bar{x}) = \frac{\varphi(\bar{x}|\theta)h(\theta)}{c\varphi(\bar{x}|\theta_0) + \frac{1-c}{B-A}}$$

Letting  $\epsilon \rightarrow 0$ , we get the delta  $\delta_{\theta_0}$ , with area  $c$ , at the abscissa  $\theta_0$  and pdf  $(1-c)/(B-A)$ , with area (yellow)  $1-c$ .

This fundamental fact was missed by Lindley: he did not use the length  $B-A$  of **his interval I**.

Using the values  $\theta_0 = 5$ ,  $n=100$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.5$ ,  $A=3$  and  $B=8$ , anybody can draw the 2 curves in figure 3 (cumulative distribution, Cdf) and 4 (probability density distribution, pdf), where “original” are the normal [Cdf (red), pdf (blue)] of the RV “mean”  $\bar{X}$ , and “Bayes” are the posterior,

Cdf (red), pdf (blue), from which we can find, the Credible Interval to be compared to the Confidence Interval.

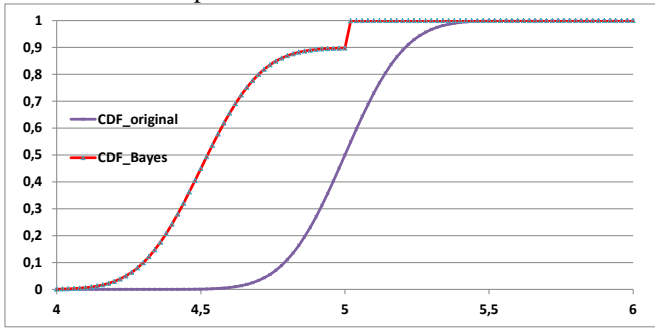


Figure 3. The 2 curves (Cdf) for the Lindley Paradox.

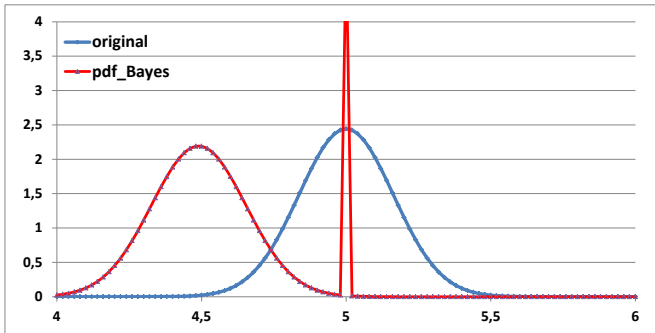


Figure 4. The 2 curves (pdf) for the Lindley Paradox.

Notice the “spike” (of Bayes) at the chosen *prior*  $\theta_0 = 5$  in the interval  $A \sim B$ ,

Lindley made the same error as Guosheng Yin and Haolun Shi (2020) did.

To see why, we have only to repeat the proof given previously, using figure 2: the probability  $1-c$  is the yellow area (in the limit).

$$\text{The posterior mean is } m_{\text{posterior}} = \frac{\theta_0 c \varphi(\bar{x}|\theta_0) + \bar{x} \frac{1-c}{B-A}}{c \varphi(\bar{x}|\theta_0) + \frac{1-c}{B-A}}$$

With the same ideas we can compute the posterior variance that has to be used for computing the Credibility Intervals (m is the posterior mean):

$$\text{Var}_{\text{posterior}} = \left( \frac{[\theta_0 - m]^2 c \varphi(\bar{x}|\theta_0) + [\sigma^2/n + (m - \bar{x})^2] \frac{1-c}{B-A}}{c \varphi(\bar{x}|\theta_0) + \frac{1-c}{B-A}} \right)$$

Lindley proceeds:

Now suppose that the value of  $\bar{x}$  is such that, on performing the usual significance test for the mean  $\theta_0$  of a normal distribution with known variance, the result is significant at the  $\alpha$  percentage point. That is,  $\bar{x} = \theta_0 + \lambda_\alpha \sigma/\sqrt{n}$ , where  $\lambda_\alpha$  is a number dependent on  $\alpha$  only and can be found from tables of the normal distribution function. Inserting this value for  $\bar{x}$  in (1) we have the following value for the posterior probability that  $\theta = \theta_0$

$$\bar{c} = c e^{-\lambda_\alpha^2/2} / \{c e^{-\lambda_\alpha^2/2} + (1-c) \sigma \sqrt{2\pi/n}\} \quad (2)$$

(Note that  $\bar{x} - \theta_0$  tends to zero as  $n$  increases so that  $x$  will lie well within the interval  $I$  for sufficiently large  $n$ .) From (2) we see that as  $n \rightarrow \infty$   $\bar{c} \rightarrow 1$ . It follows that whatever the value of  $c$ , a value  $n$  can be found, dependent on  $c$  and  $\alpha$

such that

- (i)  $\bar{x}$  is significantly different from  $\theta_0$  at the  $\alpha\%$  level;
  - (ii) the posterior probability that  $\theta = \theta_0$  is  $(100 - \alpha)\%$ .
- This is the paradox.

Excerpt 4. From the paper “A Statistical Paradox” (Lindley’s paper).

From the outlined Theory [of Mathematical Distributions] (and from figure 2, 3, 4, for any sample size  $n$ ) it is clear that the conclusion of Lindley in Excerpt 4 is NONSENSE!

### 3. Further points from Lindley’s paper

Let’s compute the Confidence Interval for the case given in figure 4. Classical Statistics provides ( $z_\alpha$  is the abscissa of the Normal pdf leaving the upper tail area  $\alpha$ ; same as  $\lambda_\alpha$ )

$$LCL = \bar{x} - z_{1-\alpha} \frac{\sigma}{\sqrt{n}} \leq \theta \leq \bar{x} + z_{1-\alpha} \frac{\sigma}{\sqrt{n}} = UCL$$

The previous relation can be written as

$$-z_{1-\alpha} \leq (\bar{x} - \theta_0) / (\sigma/\sqrt{n}) \leq z_{1-\alpha}$$

It is well known that  $z_{1-\alpha}$  is the quantity able to reject the null hypothesis  $H_0$  in favour of the alternative  $H_1$ , using the statistic  $(\bar{x} - \theta_0) / (\sigma/\sqrt{n})$ .

Putting  $\theta_0 = 5$ ,  $n=100$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.5$ ,  $A=3$  and  $B=8$ , we can compute the Confidence Interval 4.27~4.73, with  $\alpha=5\%$ . From it we see that we should decide that the empirical mean  $\bar{x} \neq \theta_0$ , the hypothesised value.

With the same data and the TRUE posterior density (figure 4), we can compute the Credibility Interval 4.25~4.78 (larger than the Confidence Interval), with  $\alpha=5\%$ . From it we see that we should decide that  $m_{\text{posterior}} \neq \theta_0$ , as before, BUT we can say that  $\theta$  has only a probability 0.5 [the prior chosen value  $c=0.5$ ] of being  $\theta = \theta_0$  (see figure 4).

Moreover, the posterior mean  $m_{\text{posterior}}$  is included into the Confidence Interval 4.27~4.73; therefore Classical Statistics provides the same decision as the Bayesian Statistics, when  $\theta_0 = 5$ ,  $n=100$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.5$ ,  $A=3$  and  $B=8$ .

In a different case, putting  $\theta_0 = 5$ ,  $n=75$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.9$ ,  $A=3$  and  $B=8$ , we can compute the Confidence Interval 4.23~4.77, with  $\alpha=5\%$ . From it we see that we should decide that  $\bar{x} \neq \theta_0$ .

With the same data and the TRUE posterior density (figure 5 and 6 for CDF), we can compute the Credibility Interval 4.30~5.20 (larger than the Confidence Interval), with  $\alpha=5\%$ . From it we see that we should decide the opposite, that is  $\bar{x} = \theta_0$ .

Moreover, the posterior mean  $m_{\text{posterior}}$  is AGAIN included into the Confidence Interval 4.23~4.77; therefore Classical Statistics provides a different decision with respect to the Bayesian Statistics, when  $\theta_0 = 5$ ,  $n=75$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.9$ ,  $A=3$  and  $B=8$ .

The different decisions depend on the pdfs (NOT on the silly Excerpt 5...), Classical and Bayesian methods are contradictory because the pdfs are different as they must be. But this is not the Paradox of Lindley, which depends on a wrong application of the Bayes Theorem for densities!!! Notice that Lindley himself provided the proof of his error, by writing (see excerpt 5):

the paradox arises because the significance level argument is



*based on the area under a curve and the Bayesian argument is based on the ordinate of the curve. However, the above interpretation through the likelihood involves no mention of alternative hypotheses which seem basic to any approach to the problem.*

*Excerpt 5. From the paper "A Statistical Paradox" (Lindley's paper).*

We used several values of the sample size to check the Lindley's statements

- (i)  $\bar{x}$  is significantly different from  $\theta_0$  at the  $a\%$  level;
- (ii) the posterior probability that  $\theta = \theta_0$  is  $(100 - a)\%$ .

*This is the paradox.*

*The usual interpretation of the first result is that there is good reason to believe  $\theta \neq \theta_0$ ; and of the second, that there is good reason to believe  $\theta = \theta_0$ . The two interpretations are in direct conflict, and the conflict may apparently be made even stronger ...*

*With  $a = 5$  we are 95 % confident that  $\theta \neq \theta_0$ ; but have 95 % belief that  $\theta = \theta_0$ .*

*Excerpt 6. From the paper "A Statistical Paradox" (Lindley's paper).*

You can graphically (fig. 5, 6) see what happens if one decides to "believe strongly on  $H_0: \theta = \theta_0$  versus  $H_1: \theta \neq \theta_0$ ":  $\theta_0 = 5$ ,  $n=75$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.9$ ,  $A=3$  and  $B=8$ .

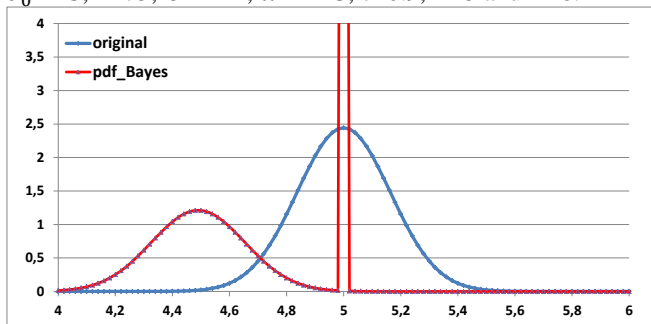


Figure 5. The 2 curves (pdf, with  $c=0.9$ ) for the Lindley Paradox.

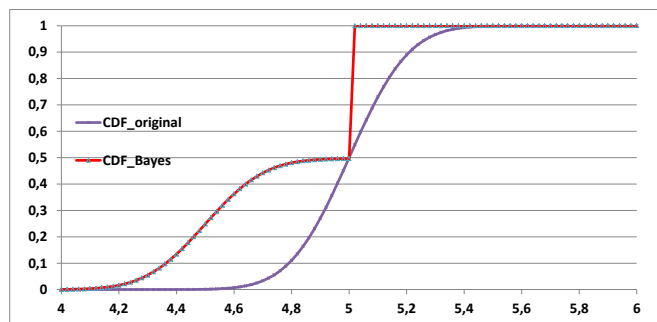


Figure 6. The 2 curves (CDF, with  $c=0.9$ ) for the Lindley Paradox.

We end this section with another excerpt:

*My point in both these examples is that the value  $\theta_0$  is fundamentally different from any value of  $\theta \neq \theta_0$ , however near to  $\theta_0$  it might be. Unquestionably there exist situations (perhaps they are the more common) in which this is not so; where we are interested in testing the approximate validity of the null hypothesis, such as that the treatment has no (or very little) effect.*

*Omissis ...  
I am much indebted to Profs. Pearson and Barnard for helpful comments on the first draft of this paper.*

*Excerpt 7. From the paper "A Statistical Paradox" (Lindley's paper).*

As a final example:

when  $\theta_0 = 5$ ,  $\sigma^2 = 2$ ,  $\bar{x} = 4.5$ ,  $c=0.95$ ,  $A=2$  and  $B=10$ ,  $\theta_0$  is outside the Credibility Interval until the sample size  $n$  is  $<119$ ;  $\theta_0$  is within the Credibility Interval for  $n > 119$ .

## 4. Scientific Attitude is needed

What we showed is the clear fact that "Mathematics is the Science which draws necessary conclusions".

The same can be found in the papers in the References: wrong papers and good papers.

The problem of Science and Quality was originated (in the author's mind) at least 48 years ago, when the author was working in big Corporations [General Electric, Siemens] and continued in [Fiat (later FCA and now Stellantis), Philco, IVECO] and at the Politecnico of Turin (Italy): many "false Scholars" were writing wrong papers defeating their readers (managers, researchers and students) of their right of getting good ideas. UNFORTUNATELY the Peer-Reviewers were worse than the authors: very few of the readers took care of this big problem! This was the opposite of Fausto Galetto who, ever fond of Scientific Management (the *Management of taking sound decisions using Scientific Methods*) tried to make people aware of their need of learning Scientific Ideas...

He had little success: ignorance increased constantly.

The reader can find in the References quite a number of wrong papers and several papers showing those errors (FG).

If the reader acted sometime as a Peer-Reviewer he must admit that very few Journals asked him to evaluate the SCIENTIFIC content of the articles. The Journals prefer the "literature review of the problem" as though that is a symptom of Quality.... The author showed many times how a superficial assessment of the quality of the methods had led to the publication in the literature of wrong information which might mislead inattentive readers and diffuse wrong ideas; the author showed the correct way to operate with respect to the concerned topic.

We do not present a literature review of the problem, because it will need at least hundreds of pages to be settled, both for Quality, Reliability, Statistics, Confidence Intervals and for Design of Experiments that are subjects very little known by managers and professors; we list here only few docs in the references, taken from the hundreds of papers read by F. Galetto in his 48 years of experience/activity in the Quality Field (big and medium Corporations and Universities).

We prefer to present the ideas about Science and the Scientific Attitude and Behavior (SAB).

Every "author's opinion" is based on this long experience in the Quality Field: *they are not only opinions, they are hard facts*. See, for example, the figure 7, 8 and especially 9: Fausto Galetto during the "students' defense of their final thesis" (to get their degree in Engineering) used to open the written thesis

at a page and to ask the future graduate what he meant with some statements found in there. 90%-98% of the students did not know how to provide answer to the questions: moreover, 50%-60% said "I copied it from the web!" That was not the biggest problem: it always astonished me the fact that the (Professor) Referee of the thesis did not know the matter/answer himself! These are *hard facts*, not opinion; the same were for Deming and Gell-Mann..., and Einstein...

We start with the important statement of the great scientist Isaac Newton "If I have seen farther than others, it is because I have stood on the shoulders of giants"; the process of Science is such that the discoveries of one people generation serve for the next one, by knowledge accumulation. This is true for any discipline (e.g. Logic, Mathematics, Physics, Probability, Statistics, Medicine, Economics, Reliability...): any building needs *sound foundations*.

When using other people words (like those of Newton, Galilei, Einstein, Deming, Gell-Mann...) the author tries to show that very great scholars have been providing correct hints to the readers in order to help them increasing their knowledge...

The Knowledge-Making process and the Knowledge itself must have Quality got through Quality Tools and Methods; this is depicted in the figure 1, *Quality Tools and Quality Methods to avoid the Disquality*.

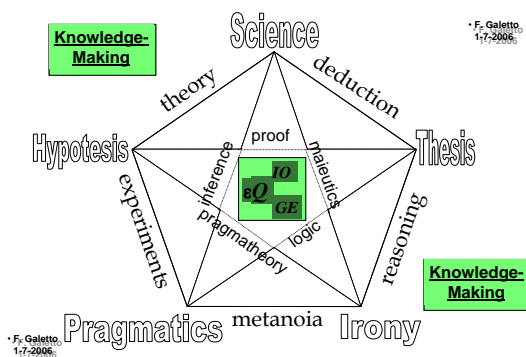


Figure 7. *Quality Tools and Quality Methods to avoid the Disquality*.

To build Science we need to deduce by Logic and Methods some results to be verified by Experiments in order to confirm our Hypotheses, with "high" Confidence Level: then we have the Theory [figure 8].

The word "science" is derived from the Latin word "scire" (to know for certain) {derived from the Greek words μαθησις, επιστημη, meaning learning and knowledge, which, at that time, were very superior to "opinion" [δοξα], while today opinion of many is considered better than the knowledge of very few! (See the "like attitude": people now are used to push the like-dislike buttons, instead of using their own intelligence!)}; knowledge is strongly related to "logic reasoning" [λογικος νους], as it was, for ages, for Euclid, who's Geometry was considered the best model of "scientificness". Common (good) sense is not science! Common sense does not look for "understanding", while science looks for "understanding"! "Understanding" is related to "intelligence" (from the Latin verb "intelligere" ([intus+legere: read into]: "intellige ut credas" i.e. understand to

believe. Unfortunately "none so deaf as those that won't hear".

Let us give a first example, the Pythagoras Theorem: "In a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides." Is this statement scientific? It could be scientific because it concerns the science of Geometry and it can be proven true by mathematical arguments and *logic*. It is not-scientific because we did not specify that we were dealing with the "Euclidean Geometry" (based, among others, on the "parallel axiom": from this only, one can derive that the sum of the interior angles of a triangle is always  $\pi$ ): we did not deal "scientifically" with the axioms; we assumed them implicitly.

So we see that "scientificness" is present only if the set of statements (concerning a given "system") are non-contradictory and deductible from stated principles (as the rules of Logic and the Axioms).

Let us give a second example, the 2<sup>nd</sup> law of Mechanics: "The force and the acceleration of a body are proportional vectors:  $F=ma$ , (m is the mass of the body)". Is this statement scientific? It could be scientific because it concerns the science of Mechanics and it can be proven "true" by well-designed experiments. It is not-scientific because we did not specify that we were dealing with "frames of reference moving relatively one to another with constant velocity" [inertial frames (with the so called "Galilean Relativity": the laws of Physics look the same for inertial systems)] and that the speed involved was not comparable with [i.e. not very near] the "speed of light in the vacuum [which is the same for all the observers]" (as proved by the Michelson-Morley experiment: in the Special Relativity (SR) Theory,  $F=d(mv)/dt$  is true, not  $F=ma!$ ) and not involving atomic or subatomic particles. We did not deal "scientifically" with the hypotheses; we assumed them implicitly. From the laws of Special Relativity we can derive *logically* the conservation laws of momentum and of energy, as could Newton for the "Galilean Relativity".

The Special Relativity Theory had a problem: it gave to the inertial frames of reference a "special position", assuming that the laws of Physics must have the same formulation in all the inertial frames of reference; this was unsatisfactory. So the General Relativity (GR) arose with the more general assumption that "the laws of Physics must have the same formulation in all the inertial frames of reference (equivalence principle)". [Inertial motion provided by the geodesic of the Space-Time] Both SR and GR were great innovations, but that was not enough...

For atomic or subatomic particles "quantum Mechanics" is needed (with Schrödinger equation as fundamental law) [weak and strong forces act in the interior of atoms].

To conceal the four forces (two: gravitation and electromagnetic) of Relativity Theory and (two: weak and strong forces) of Quantum Theory something new is needed... Will that be the "the Strings Hypothesis"? Only if experiments will confirm it...

So we see that "scientificness" is present only if the set of statements (concerning a given "system") do not contradict the observed data, collected through well-designed experiments ["scientific" experiments]: only in the XVII century, due to Galilei, Descartes, Newton... we learned that. Since that time

only, science could really grow, using *LOGIC* (figures 7 and 8).

When we start trying to learn something, generally, we are in the “clouds”; reality (and truth) is hidden by the clouds of our ignorance, the clouds of the data, the clouds of our misconceptions, the clouds of our prejudices; to understand the phenomena we need to find out the reality from the clouds: we make hypotheses, then we deduct *logically* some consequences, predicting the results of experiments: if predictions and experimental data do match then we “confirm” our idea and if many other are able to check our findings we get a theory. To generate a theory we need Methods. Eric Berne, the psychologist father of “Transactional Analysis”, stated that everybody interacts with other people through three states P, A, C [Parent, Adult, Child, (not connected with our age, fig. 8)]; the Adult state is the one that looks for reality, makes questions, considers the data, analyses objectively the data, draws conclusions and takes logic decisions, coherent with the data, methodically. Theory [θεωρία] comes from the Adult state! Methods [μεθοδος from μετα+οδος = the way through (which one finds out...)] used to generate a Theory come from the Adult state!

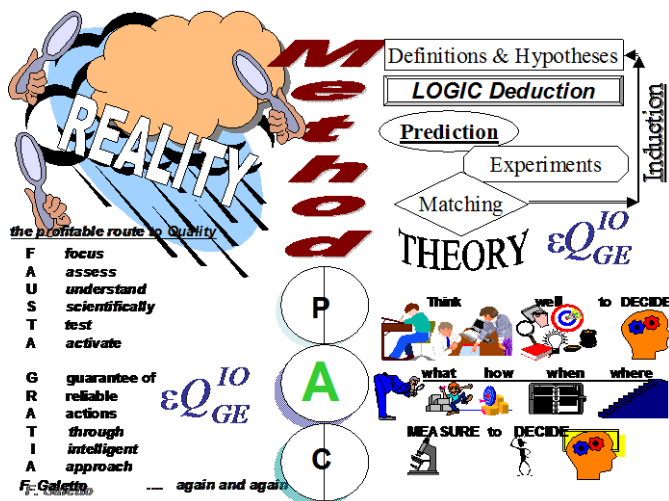


Figure 8. FAUSTA GRATIA for Quality in order to avoid the Disquality.

You can find a sample of anti\_Scientific Attitude and Behavior (aSAB) by looking at some very few cases, taken from some published documents found on the web: Montgomery, Rade, M. A. El-Damcese, et al. (and other from books [M. Xie, Y-S Dai, K-L Poh, (2004), T. Nakagawa, (2005), H. Pham, (2003), H. Wang, H. Pham, (2006), M. Lazzaroni, L. Cristaldi, L. Peretto, P. Rinaldi, M. Catelani, (2003), MONTGOMERY D.C. (2006)] and theses!). See also all the other cases in the references, about Design of Experiments AND Control Charts. The problem of ignorance is so huge that a profound change of mind (metanoia, Deming) is needed. Deming is widely regarded as the father of quality management. It was his teaching and example that guided the amazing rise of Japanese industry after the desolation of the Pacific War. In the first edition of Deming’s book *The New Economics*, a short section that described a fundamental change of mind-set on which all subsequent organizational transformation depended, was titled “*Metanoia*” [a Greek

term meaning “*Change of Mind*”], (in later editions retitled “The First Step”).

Managers, Professors, Scholars, Students, ..., need to change their mind in order to learn Scientific Attitude.

In this paper we suggest to read the case [114] «*On State Budget in the North Central of Nigeria: Analysis of Variance (ANOVA) Approach*. Journal of Investment and Management. Vol. 4, No. 6, 2015, pp. 296-300. doi: 10.11648/jjim.20150406.12» to see that “Professors and Referees (Peer-Reviewers) *do not know the basics of ANOVA*” and therefore take WRONG decisions.. Is the paper to be considered «*Science Innovation*»? Absolutely not!

My students are able to find the right solutions because they had the Sound Theory! Muritala Abdulkabir, Raji Surajudeen Tunde [114] did not have it.

Citing Muritala Abdulkabir, Raji Surajudeen Tunde, Fausto Galetto, WHO found them wrong, improves the indexes of Muritala Abdulkabir, Raji Surajudeen Tunde! Where is the Quality?

Fausto Galetto, in his 45 years of experience, met so many incompetents (managers, professors, researchers, ...) that *he always was used to say to his students: «*IF a guy suggests books and papers written by incompetents he is TWICE incompetent, because he does not recognize wrong ideas and suggests to read wrong ideas*»*. Unfortunately several Professors do not practice the two important methods used here, the Logic and the Scientific Theory (Mathematics, Probability, Statistics, Physics...). See the references...

Researchers shall use their intelligence in order to make knowledge for the improvement of people and their life.

Researchers must not cheat people and act according to the figures 7 and 8.

Any Intellectually hOnest person that loves Quality and hates DISquality will Focus on the problems [potential and/or actual], Assess their importance (money, impact, consequences, risks, ...), Understand all the previous items SCIENTIFICALLY and SCIENTIFICALLY Test for finding the causes; when a solution is found anybody will Activate to implement the solution, in order to Guaranty that Reliable Actions (preventive and corrective) are taken Through an Intelligent Approach (approach that uses intelligence, ingenuity and science, avoiding misdeeds) [figure 8].

Eric Berne devised the Transactional Analysis “Theory” [that actually is not a theory in the scientific sense] with the 3 EGO\_States: Parent, Adult, Child.

The Parent ego\_state is a set of thoughts, feelings, and behaviours that are learned or “borrowed” from our parents or other caretakers. Two parts are comprised: the Nurturing Parent ego\_state soft, loving, and permission giving, and Prejudiced Parent, the part of our personality that contains the prejudged thoughts, feelings, and beliefs that we learned from our parents.

The Adult ego\_state is our data processing centre. It is the part of our personality that formulate hypotheses to be verified by experiments, uses Logic and Science, invents Methods to test ideas and to process data accurately, that sees, hears, thinks, and can come up with solutions to problems [potential and/or actual] based on the facts and not solely on our pre-judged



thoughts or childlike emotions: it denounces misdeeds. You can see its capacities on the right hand of the figure 8. Qualitatis FAUSTA GRATIA is related to the Adult ego\_state.

In the excerpt 1 (from the paper [56]) you read the following statements [nonsense!]:

The research work was designed to investigate “The Comparative analysis of state budget in the north central of Nigeria”.

A review of related theories on comparative analysis of state budget in North central of Nigeria backed up with a related theoretical framework was carried out chapter four, hypothesis were stated for the research study.

A well structured data collected from the field while the sources used for the collected of data were secondary source. The data collected from the field were presented in a tabular form and expressed in randomized block design, while two ways (ANOVA) table was used to test the stated hypothesis at the significant level of 0.05 percent, with F-distribution  $V_1, V_2$ . And from the test it was obvious that comparative analysis of state budget in the North central of Nigeria is significant different.

*Excerpt 8. Conclusions of [114]: analysis of budget data.*

NOTICE the words «««Significant different»»»!!! We will see that are NONSENSE!

WITHOUT making any calculation, from the ANOVA in the excerpt 4, any student can have doubts about the conclusion and make the following statements (the Peer-Reviewers could not!)

1. In the ANOVA, the “Degrees of freedom” (df) in the “Total” (last row) are wrong: they must be 53 (=5\*9-1) NOT 33
2. The F value 2.18 is with 8 and 40 df, NOT 3 and 40
3. In the ANOVA, the 3<sup>rd</sup> column “SUM of freedom” is ACTUALLY named, in all books and papers known to Fausto Galetto, «««SUM of SQUARES»»» (SS); the software Minitab, used by the “statisticians” uses SS!

To get the correct analysis of the data, anybody has to use the Theory.

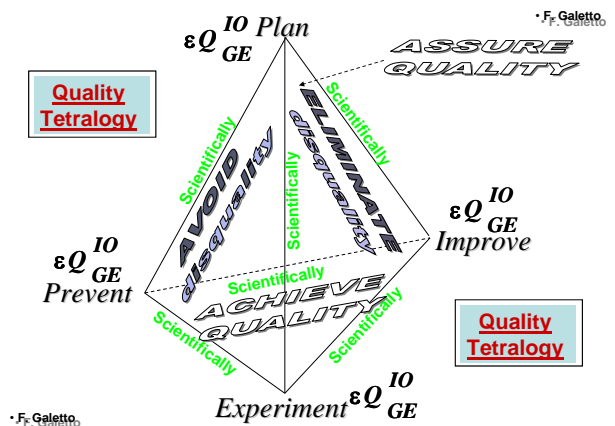
When Fausto Galetto was in the Research Gate Questions&Answers forum (2014) there were two debate points: the Least Squares Estimates [LSE] and the Maximum Likelihood Method [MLM] to find the Maximum Likelihood Estimates [MLE]; it is important to have the correct ideas on the areas of applications, because there the ignorance was prevalent.

This is especially important for YOUNG Researchers: YOU, researcher, MUST be ALERT, in order NOT to be cheated! Please meditate on the figure 9.

## 6. Conclusion

Reading the papers in the References, any intelligent person can acknowledge that without knowing the Basics of the discipline one wants to innovate it is very improbable (if not impossible) to improve the status of Science. Citations, impact factors, the various indexes are not a proof of Science. (see the Fausto Galetto experience in the Research Gate

“database”)



**Deming** The result is that **hundreds of people** are learning what is wrong. **Deming**  
 I make this statement on the basis of experience seeing every day the devastating effects of incompetent teaching and faulty applications.  
**M. Gell-Mann** In my university studies ... **M. Gell-Mann**  
 in most of the cases, it seemed that students were asked simply to regurgitate at the exams what they had swallowed during the courses. Once that such a misunderstanding has taken place in the publication it tends to become perpetual because the various authors simply copy one each other.

Figure 9. Statements from Deming, Gell-Mann, Galetto ideas.

Citations are NOT an Index of Quality!!!

Citing Muritala Abdulkabir and Raji Surajudeen Tunde, and the others (D. V. Lindley, E. Santiago, J. Smith, Dovoedo Y. H., Chakraborti S., Jones LA, Champ CW., Fang Y. Y., Khoo M. B., Lee M. H., N. Kumar, Liu J., Xie M., Sharma P., Sim, C. H., Gan, F. F., and Chang, T. C. Frisén, M., Woodall, W. H., Kittlitz, R. G., Schilling, E. G., Nelson, P. R. (1976). Zhang, H. Y, Goh, T. N., Shamsuzzaman, M., M. A. El-Damcese, A. N. Salem, and N. S. Temraz Y-S Dai, K-L Poh, T. Nakagawa, H. Wang, H. Pham, M. Lazzaroni, L.

Cristaldi, L. Peretto, P. Rinaldi, M. Catelani, K. Bouacha, M. A. Yaltese, S. Khmel, S. Belhadi, D. Jurkó, M. S. Sukumar, B. V. Sudheer Reddy, P. Venkataramaiah, J. Martinez, M. Georgallides, F. Kutsanedzie, S. Achio, E. Ameko, PARK S., YIN G. Z., JILLIE D., W. W. Eureka, N. E. Ryan, C. Casciano, (and L. Peretto referee), J. Z. Zhang et al.) Fausto Galetto, WHO found all them WRONG, improves the indexes of them, by citing them! WHERE is their Quality?

We said that the problem of ignorance is so huge that a profound change of mind (metanoia, Deming) is NEEDED.

The following statements of great scientists and managers are important for any person who wants to make QUALITY Decisions on QUALITY matters. They are not opinions; they are hard facts! *Contra factum non valet argumentum*.

We think that the YOUNG Researchers MUST be ALERT if they want to LEARN: THEY MUST know the THEORY!

The author Galetto always invited people to be intellectually honest in teaching and taking decisions: THEORY is fundamental in both cases. [see the F. Galetto documents, in the references, in the RG database, and in his books]

From above we see that Fausto Galetto taking into account the following statements by great people, as always did, could provide a sensible advice for any Researcher, in any university, and any Manager, in any Company.

W. E. DEMING "It is a hazard to copy". "It is necessary to understand the theory of what one wishes to do or to make." "Without theory, experience has no meaning." "A figure without a theory tells nothing". <<<The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications. >>>

M. GELL-MANN "In my university studies ..., in most of the cases, it seemed that students were asked simply to regurgitate at the exams what they had swallowed during the courses." Some of those students later could have become researchers and then professors, writing "A\_scientific" papers and books ... For these last, another statement of the Nobel Prize M. Gell-Mann is relevant: <<<<<"Once that such a misunderstanding has taken place in the publication, it tends to become perpetual, because the various authors simply copy one each other.">>>>> similar to "Imitatores, servum pecus" [Horatius, 18 B.C.] and "Gravior et validior est decem virorum bonorum sententia quam totius multitudinis imperitiae". [Cicero]

P. B. CROSBY Paraphrasing P. B. CROSBY one could say "Professors may or may not realize what has to be done to achieve quality. Or worse, they may feel, mistakenly, that they do understand what has to be done. Those types can cause the most harm."

What do have in common Crosby, Deming and Gell-Mann statements? The fact that professors and students betray an important characteristic of human beings: rationality [the "Adult state" of E. Berne]

A. EINSTEIN "Only two things are infinite: the Universe and the Stupidity of people; and I'm not sure about the former".

GALILEO GALILEI Before EINSTEIN, GALILEO GALILEI had said [in the *Saggiatore*] something similar "Infinite is the mob of fools".

The scientific community as a whole must judge [κρίνω] the work of its members by the objectivity and the rigor with which that work has been conducted; in this way the scientific method should prevail. Any professor and any Statistical Consultant should know Probability Theory and Statistics!

*I always was used to say to my students: ««««IF a guy suggests books and papers written by incompetents he is TWICE incompetent, because he does not recognize wrong ideas and suggests to read wrong ideas»»»»*. Unfortunately several Professors do not practice the two important methods used here, the Logic and the Scientific Theory (Mathematics, Probability, Statistics, Physics,...). See the references...

Please see well the figures and see IF ...

Researchers shall use their intelligence in order to make knowledge for the improvement of people and their life.

Researchers MUST not cheat people and act according to the figures 7, 8 and 0.

Remember that any Intellectually hOnest person that loves QUALITY and hates DISquality will Focus on the problems [potential and/or actual], Assess their importance (money, impact, consequences, risks, ...), Understand all the previous items SCIENTIFICALLY and SCIENTIFICALLY Test for finding the causes; when a solution is found anybody will Activate to implement the solution, in order to Guaranty that Reliable Actions (preventive and corrective) are taken Through an Intelligent Approach (approach that uses intelligence, ingenuity and science, avoiding misdeeds). [figures 7 and 10]

Remember also that the Parent ego\_state is the set of thoughts, feelings, and behaviours that are learned or "borrowed" from our parents or other caretakers. Two parts are comprised: the Nurturing Parent ego\_state soft, loving, and permission giving, and Prejudiced Parent, the part of our personality that contains the prejudged thoughts, feelings, and beliefs that we learned from our parents.

Remember also that the Adult ego\_state is our data processing centre: it formulates hypotheses to be verified by experiments, using Logic and Methods to test ideas and to process data accurately: it denounces misdeeds. You can see its capacities on the right hand of the figure 7. Qualitatis FAUSTA GRATIA is related to the Adult ego\_state.

The Adult ego\_state is embodied in the  $\varepsilon Q_{GE}^{IO}$  symbol (the epsilon-Quality, see also figure 10).

Intellectually hOnest people use as much as possible their rationality and Logic, in order not to deceive other people.

Deming, Einstein, Gell-Mann are beacons for the Quality Journey.

If we want to achieve Quality, MANAGERS (now students) NEED to be EDUCATED on Quality  $\varepsilon Q_{GE}^{IO}$  by Quality Professors, EDUCATED on Quality.

I could, at last, paraphrase ST John "And there are also many other things, the which, if they should be written everyone, I suppose that even the world itself could not

contain the books that should be written."



Figure 10. The epsilon-Quality to avoid the Disquality.

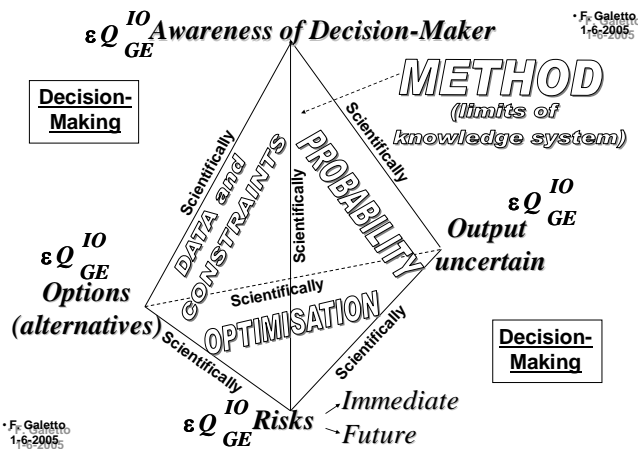


Figure 11. The Decision-Making Tetrahedron.

Will someone want to see the truth? Only God knows that ...

The personal conclusion is left to the Intellectually Honest reader to whom is offered the Quality Tetralogy: Prevent, Experiment, Improve, Plan, SCIENTIFICALLY to avoid disquality, to eliminate disquality, to achieve Quality, to assure Quality, using Intellectual Honesty: we wish them to use correctly the Decision-Making Tetrahedron (fig. 11).

Quality Tetralogy and Decision-Making are much better than ISO 9004:2008 (and 2015, as well) because Quality Tetralogy and Decision-Making Tetrahedron take into account explicitly the need for scientific behavior either of people or of organizations that really want to make Quality. Moreover they show clearly that prevention is very important for Quality and Good Management is strongly related to Good Knowledge for Business Excellence.

Brain is the most important asset: let's not forget it, IF we want that our students (Future Managers or Future Researchers) be better than their professors.

We repeat

YOUNG Researchers MUST be ALERT

If they want to LEARN:

THEY MUST know the THEORY!

««The truth sets you free»»

Professors and researchers who do not are Intellectually hOnest will not grow students and researchers fond of Quality.

If they want that the Science improve they must know at

least the BASICS!

Contra factum non valet argumentum.

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