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Inverse source estimation problem in EEG

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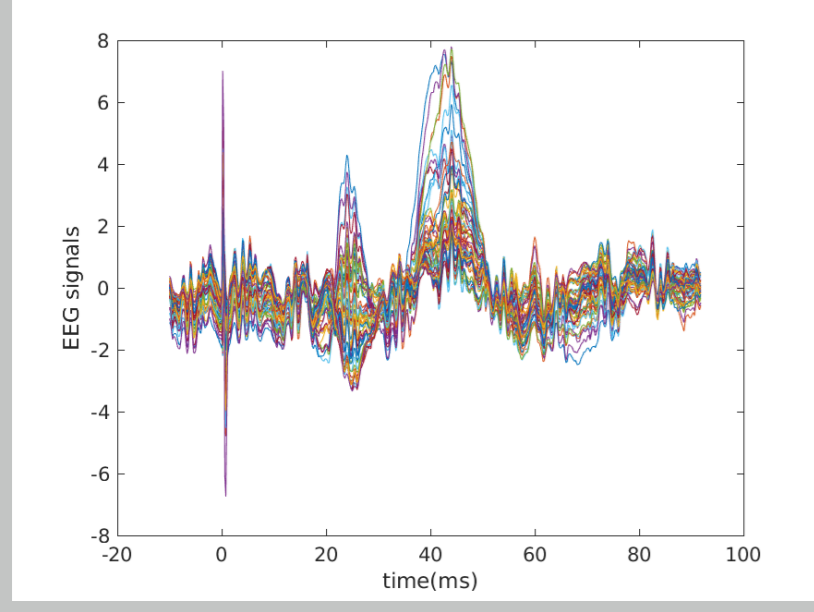
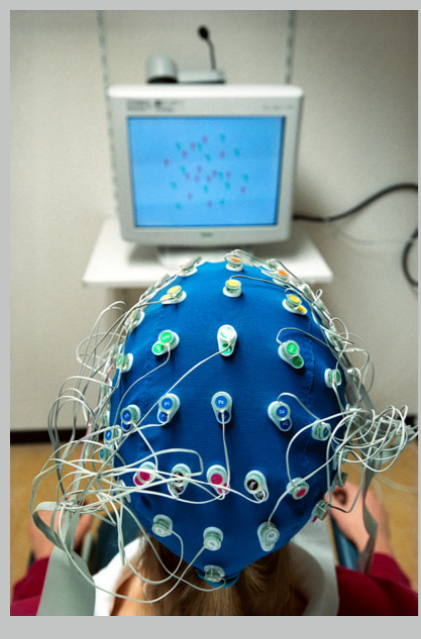
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Abstract

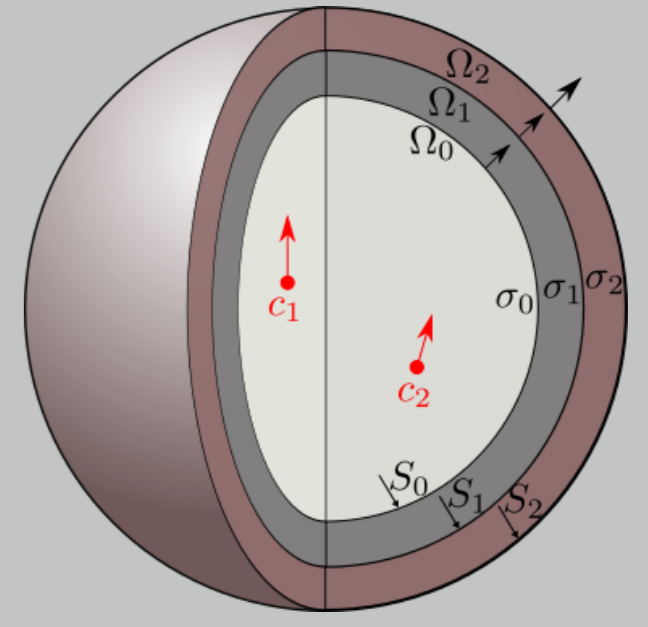
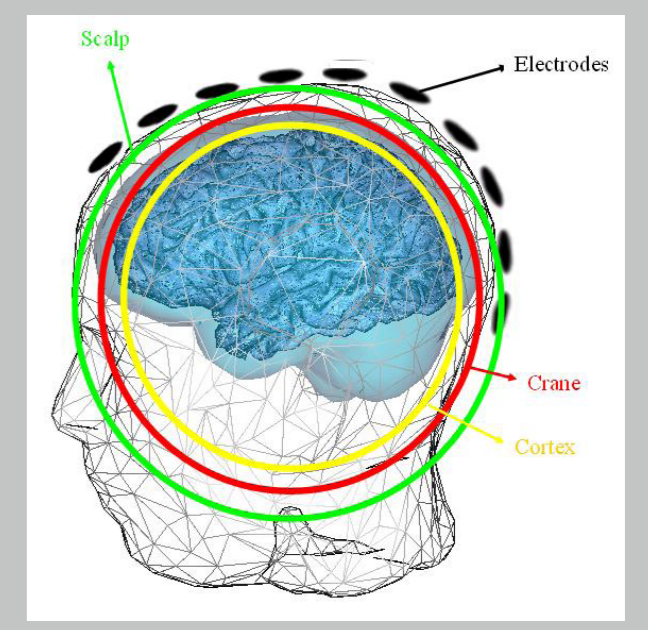


Being given pointwise measurements of the electric potential taken by electrodes on part of the scalp, the EEG (electroencephalography) inverse problem consists in estimating current sources within the brain that account for this activity. A model for the behaviour of the potential rests on Maxwell equations in the quasi-static case, under the form of a Poisson-Laplace partial differential equation (PDE). We solve the inverse problem (IP) in spherical geometry, for piecewise constant electric conductivity values, and pointwise dipolar source terms.

The algorithm relies on consecutive steps and is encoded in the software FindSources3D:

- (i) Singular value decomposition (SVD), in order to separate the time independent activities.
- (ii) Data transmission from scalp to cortex.
- (iii) Best quadratic rational approximation on families of 2D slices, compute singularities in circular sections.
- (iv) Clustering of these singularities in order to localize the sources.

[CLMP] Clerc, Leblond, Marmorat, Papadopoulos. Source localization in EEG using rational approximation on plane sections, *Inverse Problems* 28, 2012.



1. Model, assumptions, IP

- Potentials u , subject to pointwise dipolar J supported within the brain

$$\operatorname{div}(\sigma \operatorname{grad} u) = \operatorname{div} J = \sum_{k=1}^K p_k \cdot \operatorname{grad} \delta_{C_k}$$

- Spherical head geometry, 3 homogeneous isotropic layers $\Omega_2, \Omega_1, \Omega_0$ (scalp, skull, brain) constant conductivity values $\sigma_2 = \sigma_0 > \sigma_1 > 0$ & $\partial_n u = 0$ on S_2

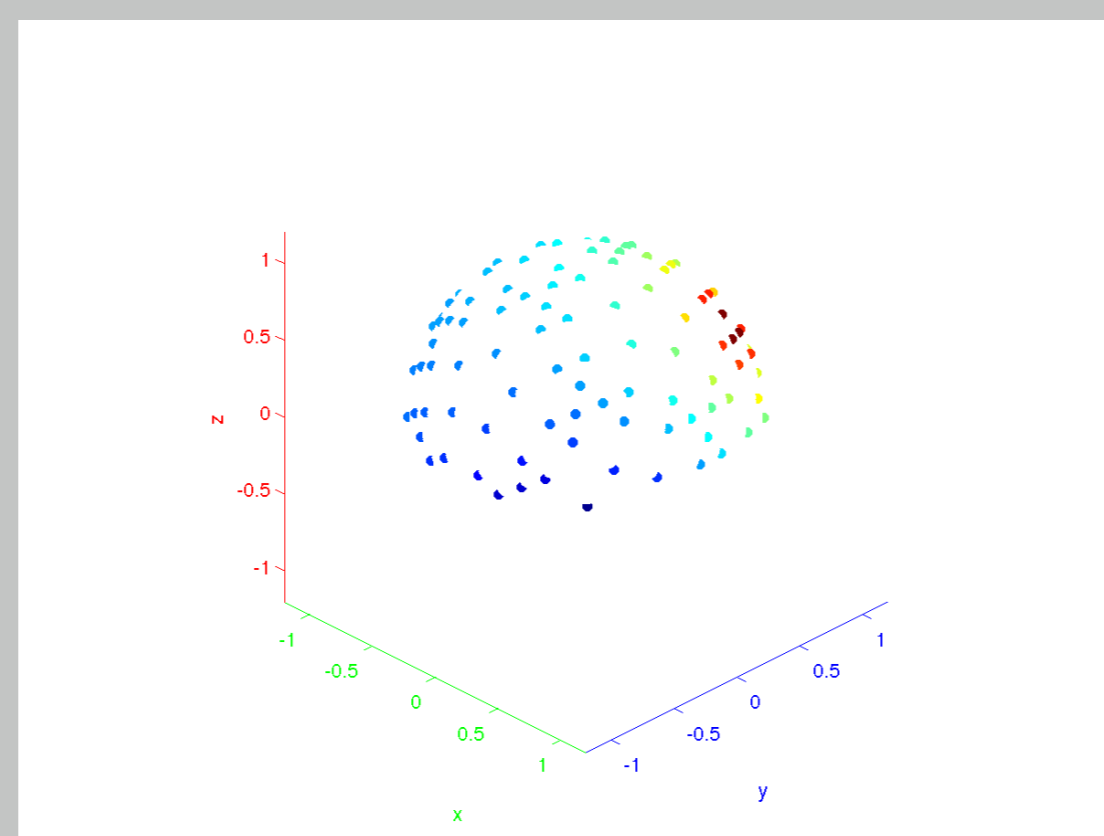
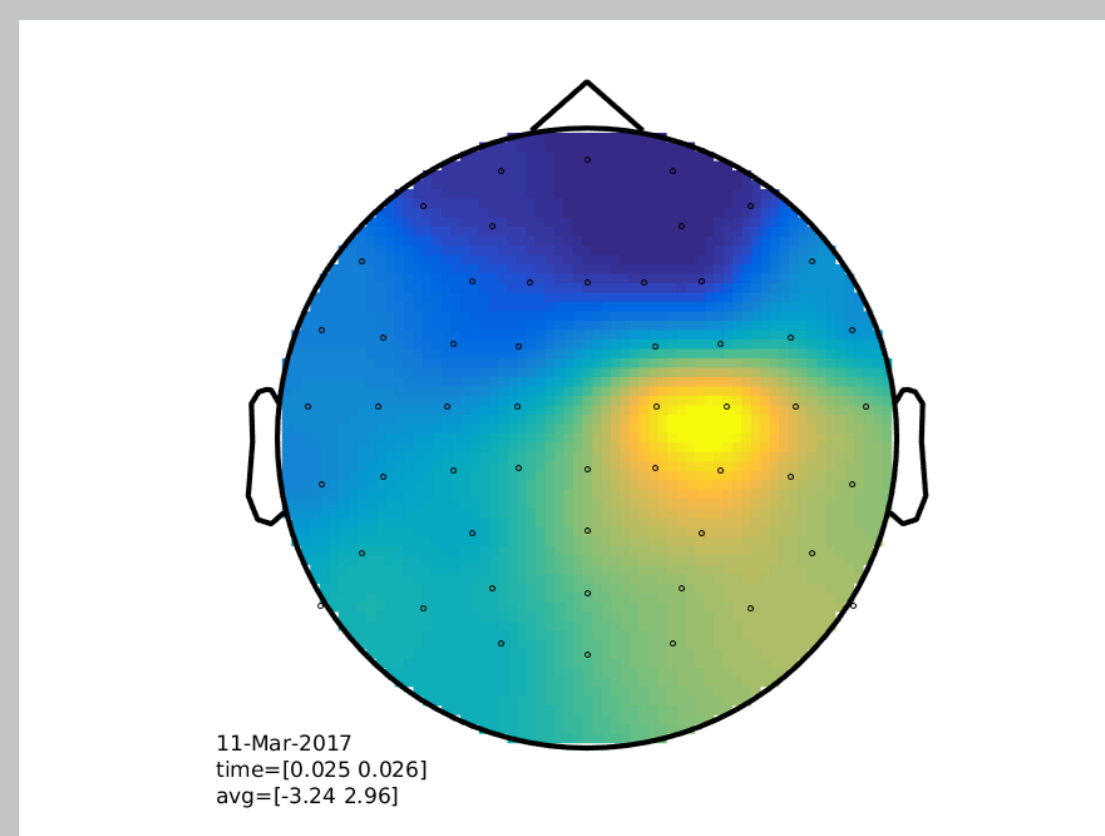
- EEG data set: u depends on time t through $|p_k(t)|$ pointwise values of potential $u = u(t_j, e_j)$ at electrodes locations e_j on S_2

- IP: estimate quantity K and locations C_k in Ω_0 of sources & moments p_k

2. Step (i) SVD \rightsquigarrow static activities

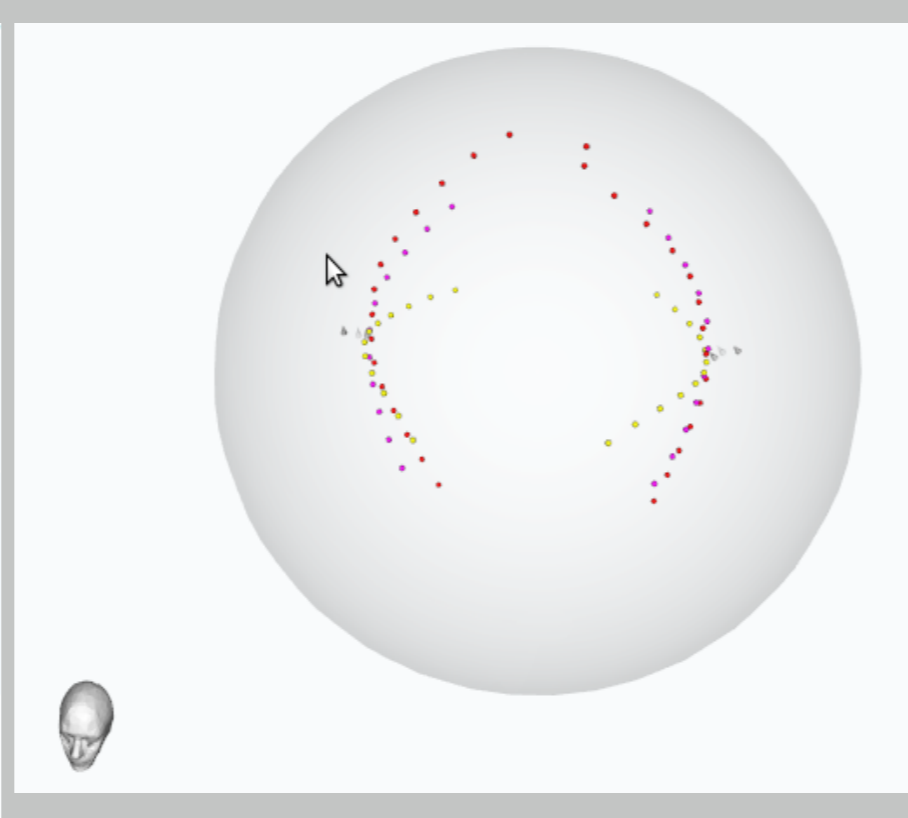
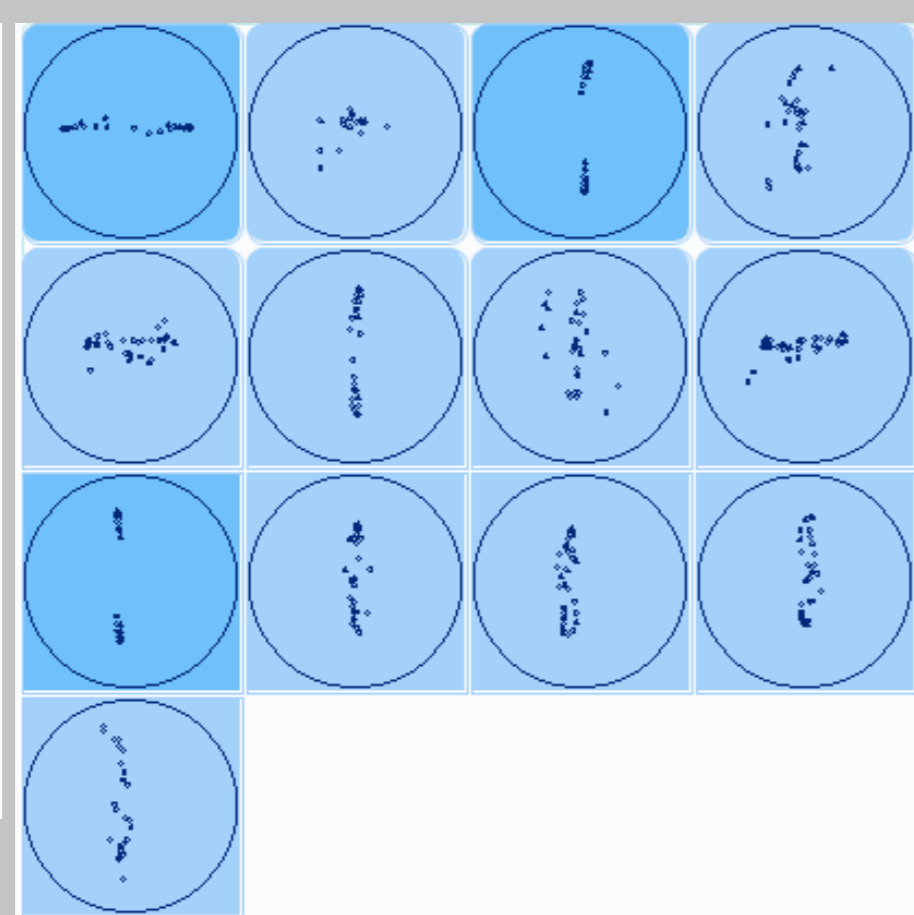
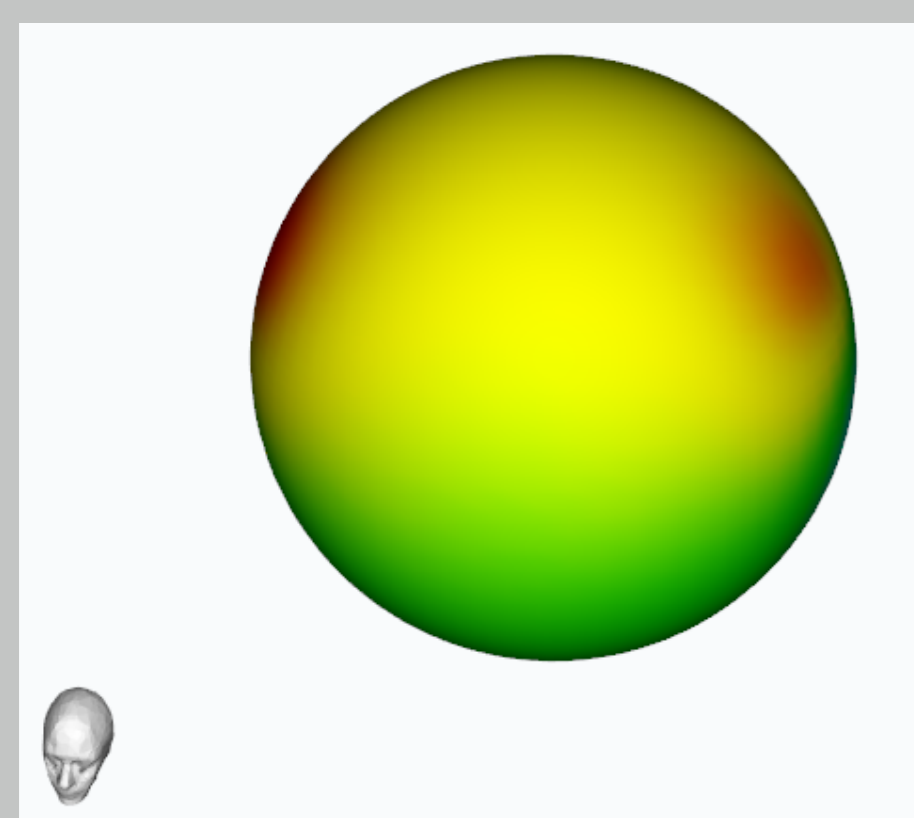
Singular value decomposition of EEG matrix time window, EEG = $U \Sigma V$, rk R

- main asynchronous activities columns u_r of U , $r = 1, \dots, R$
- best linear combination, in view of source estimation change basis in span



3. Steps (ii), (iii), (iv) $S_2 \rightsquigarrow S_0 \rightsquigarrow$ sources in Ω_0

For each activity $u = u_r = u_r(e_j)$ (P circles in 13 \rightsquigarrow 3 directions)



(ii) $S_2 \rightsquigarrow S_0$
cortical mapping

(iii) $S_0 \rightsquigarrow \Omega_0$
 $K = 2$ poles in each 2d slice

(iv) $\Omega_0 \rightsquigarrow$ sources C_k
3d clustering

3. step (ii) cortical mapping $S_2 \rightsquigarrow S_0$

Compute harmonic extension of the data on S_2 in Ω_2 , then in Ω_1

$\Delta u = 0$ in Ω_1 & Ω_2 + transmission conditions on S_i : $u, \sigma \partial_n u$ continuous
integral representation, single & double layer potentials

Best constrained quadratic approximation on spheres S_i $i = 1, 2$
by harmonic functions in Ω_i , with norm constraint on S_{i-1}

- expansions on spherical harmonics data at electrodes e_j
- transmission $S_2 \rightsquigarrow S_1$ then $S_1 \rightsquigarrow S_0$

Boundary Element Methods, Bounded Extremal Problems BEM, BEP

$$\min_{u_{ap}} \left(\|u - u_{ap}\|_{L^2(S_i)} + \lambda \|\operatorname{grad} u_{ap}\|_{L^2(S_{i-1})} \right)$$

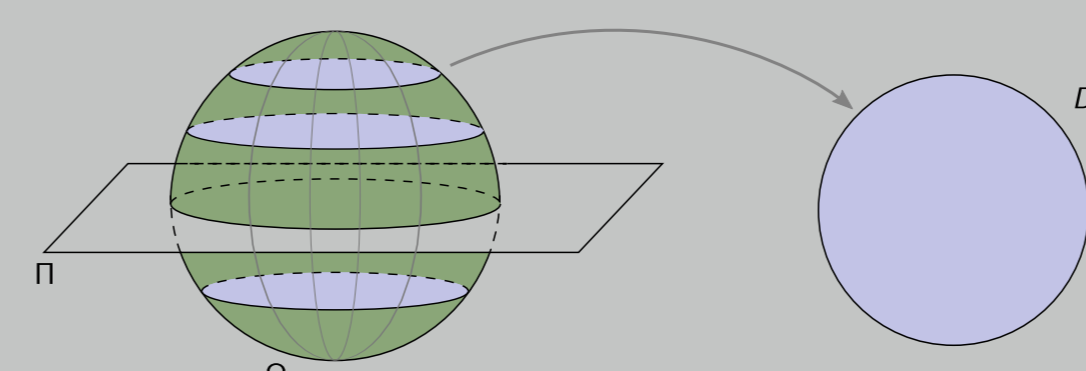
u_{ap} on spherical harmonic basis $r^l Y_l^m(\theta, \varphi)$, Lagrange parameter $\lambda > 0$

3. step (iii) 2d slices of $S_0 \rightsquigarrow$ poles \simeq singularities

Best quadratic rational approximation on circles

- $\sigma_0 \Delta u = \operatorname{div} J$, convolution by fundamental solution $\rightsquigarrow u = u_s +$ harmonic, $u_s(X) \simeq \sum_{k=1}^K \frac{p_k X - C_k}{|X - C_k|^3}$

- from $u, \partial_n u$ on planar sections of S_0

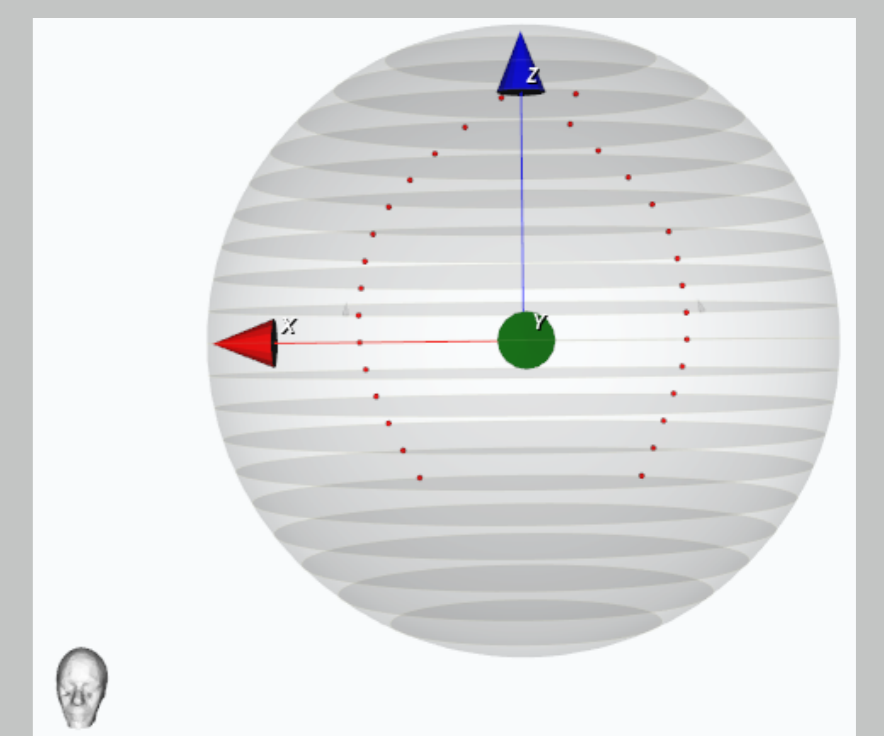
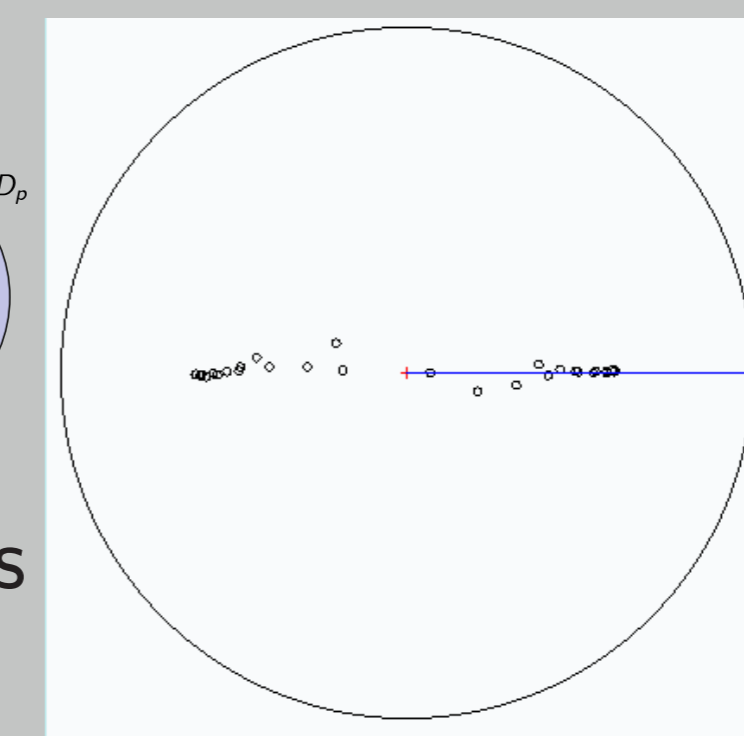


$\rightsquigarrow K$ poles \simeq 2d singularities

- on circle Γ_p $u, \partial_n u \rightarrow f = u_s^2$
polynomials π, q complex variable

$$\min_{\pi, q} \left\| f - \frac{\pi}{q} \right\|_{L^2(\Gamma_p)}$$

iterate on degree $q \nearrow K$



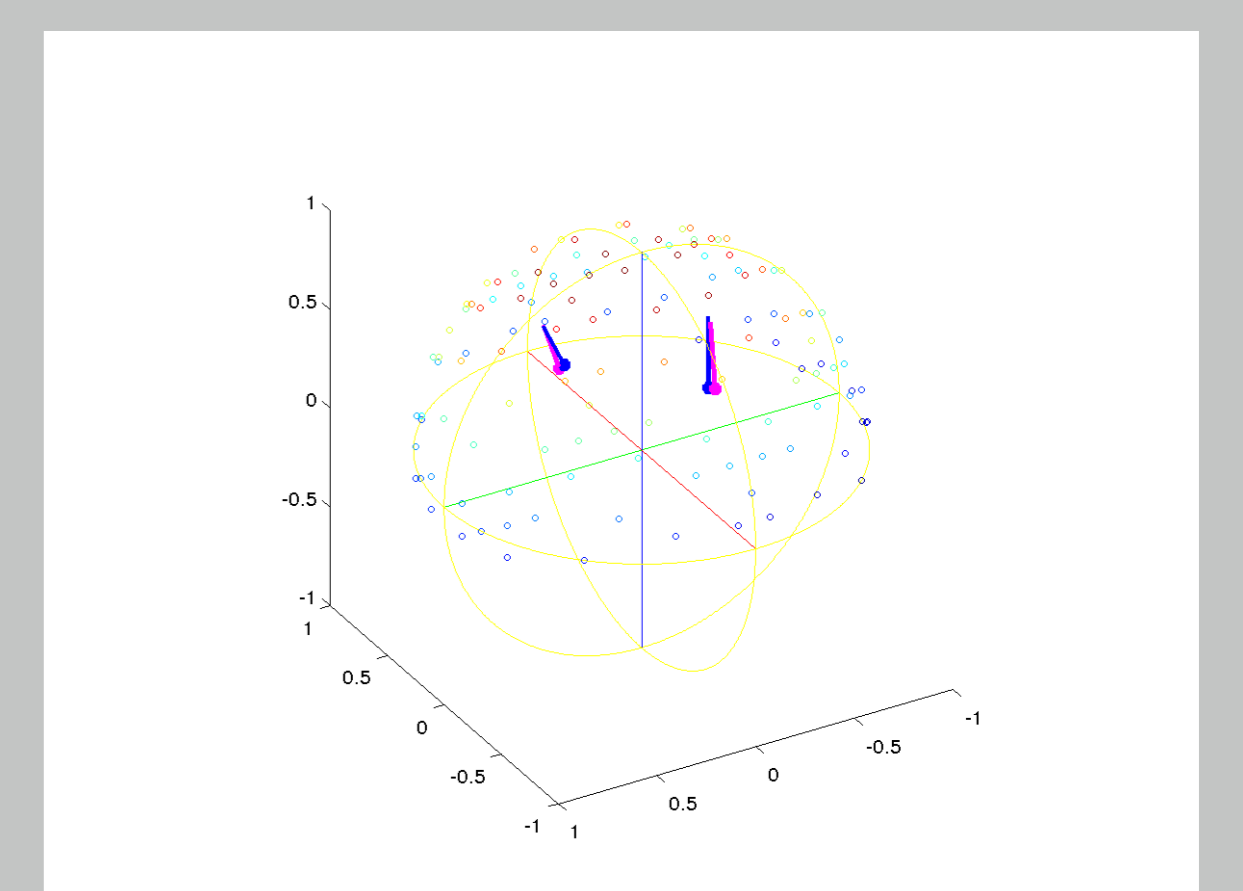
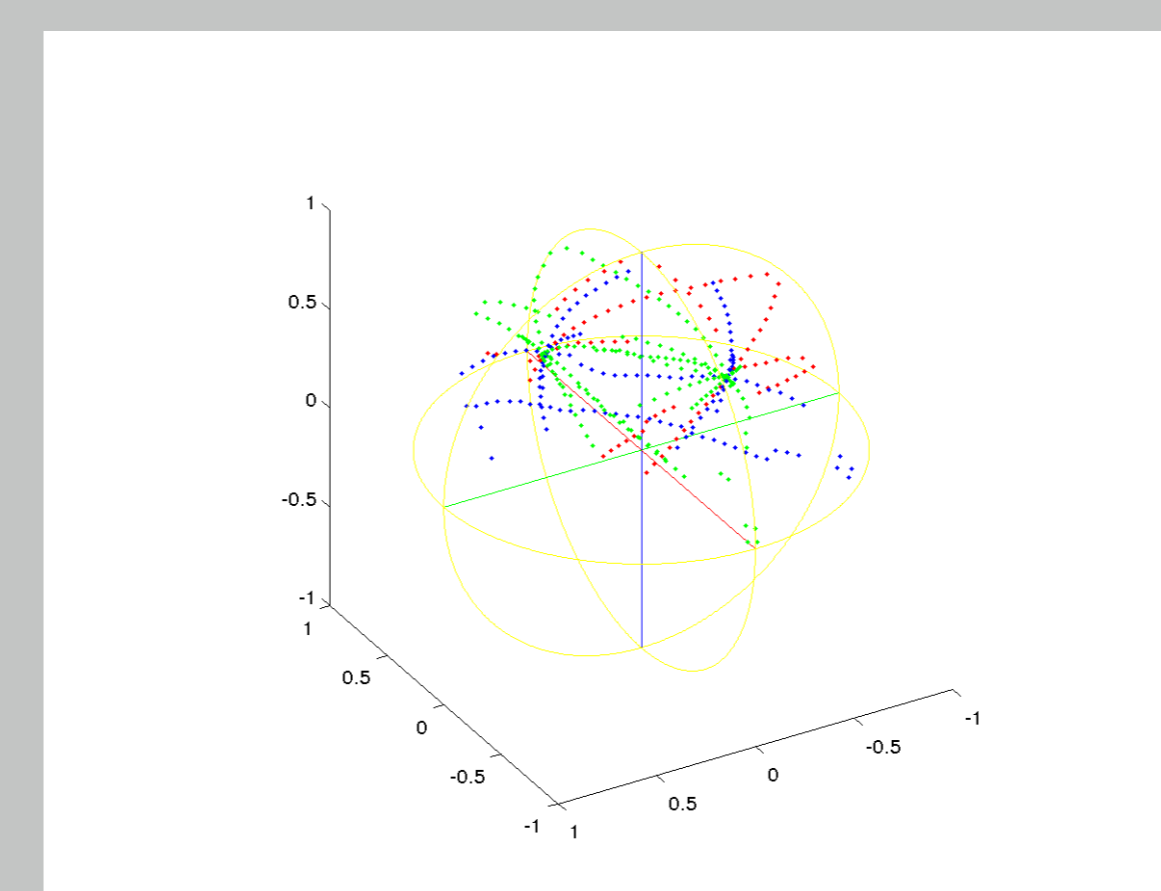
- $K = 2$ poles in P disks D_p
 $p = 1, \dots, P$, plane Π
 \rightsquigarrow roots of q in $D_p =$ poles

- lines of singularities
maximum modulus in D_p
containing sources

3. step (iv) 2d singularities \rightsquigarrow 3d sources

Clustering 2d singularities for suitable directions

\rightsquigarrow source locations C_k among 13 $k = 1, 2$



\rightsquigarrow up to $K = 3$ or 4 pointwise sources

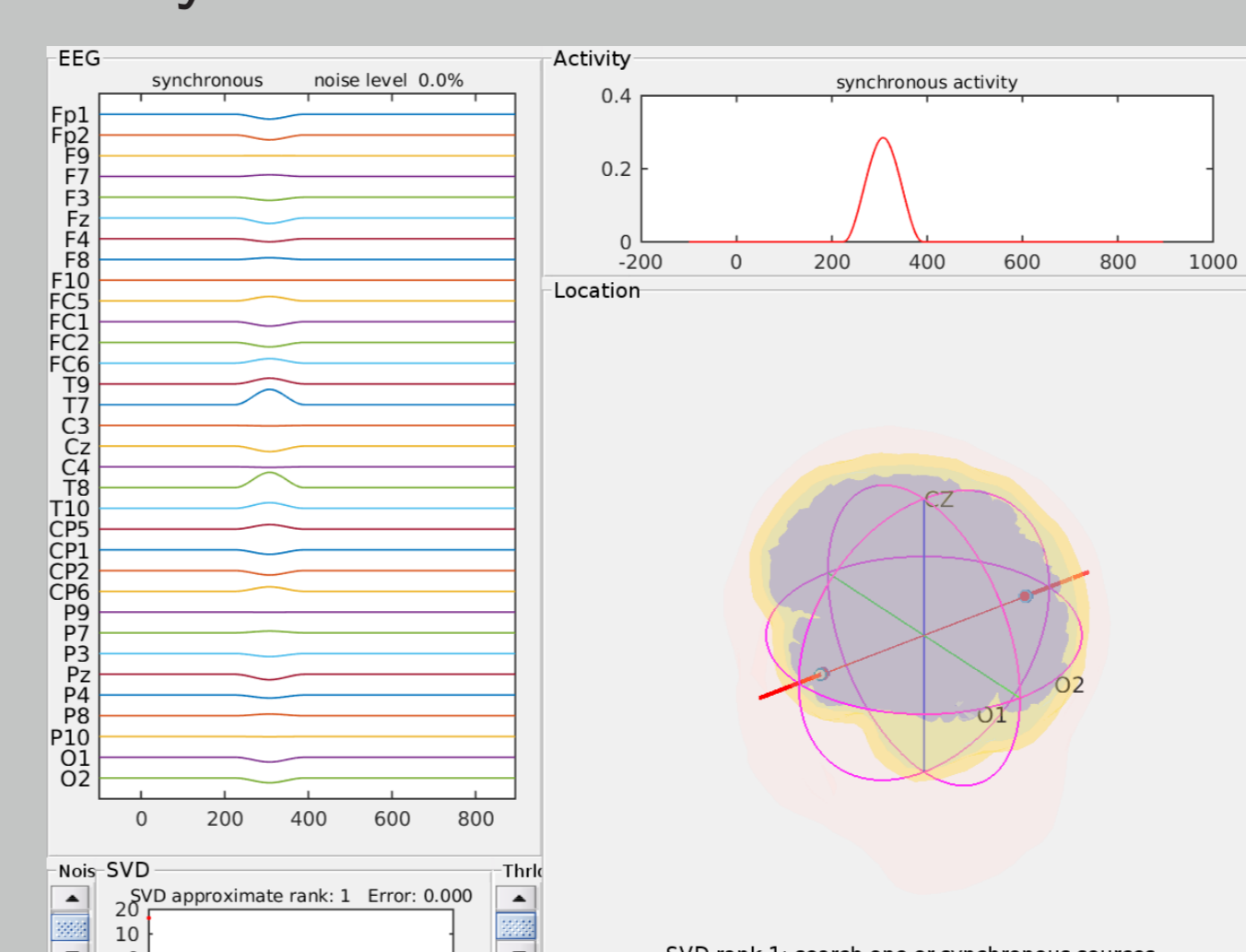
+ various activities & dipole fit

+ moments recovery (linear system)

4. Results, solutions to IP

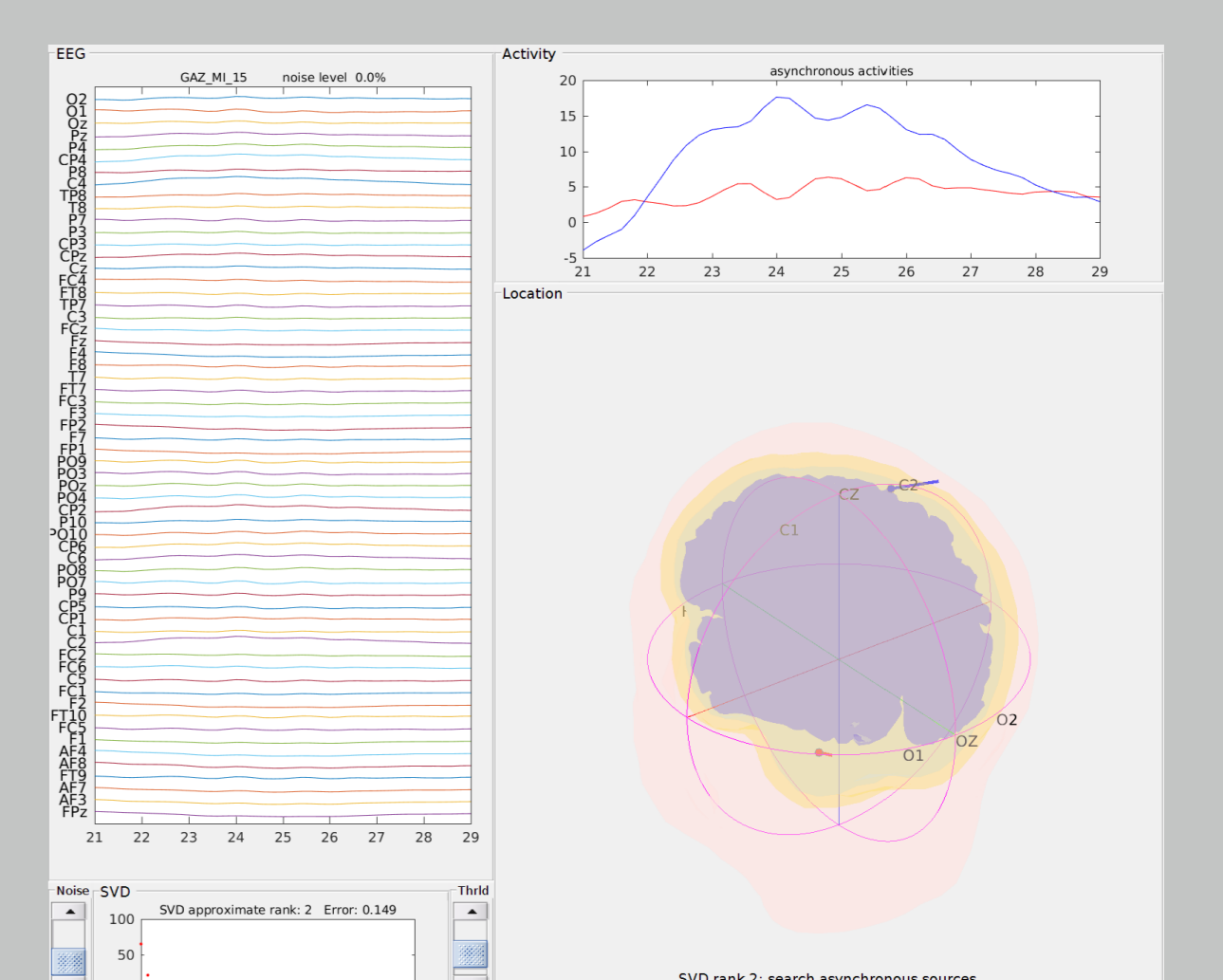
FindSources3D

- synthetic data



2 synchronous sources

- actual data



somatosensory evoked potentials (SEP)
left wrist, hospital la Timone, Marseille

<http://www-sop.inria.fr/apics/FindSources3D/>

(matlab)

5. Ongoing & further work

- Other datasets, MEG, EIT magnetoencephalography, Electrical Impedance Tomography (Calderón)
- Realistic geometries, more layers
- Other inverse problems in brain & medical imaging (e.g. estimation of skull conductivity, j.w.w. BESA GmbH)
- Other application fields planetary sciences, paleomagnetism, ...