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# Transient Stabilization of a Synchronous Generator in a Multimachine Power Network

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*Abstract:* The transient stabilization problem of a synchronous generator in a multimachine power system is addressed. A robust adaptive nonlinear feedback control algorithm is designed on the basis of a third order model of the synchronous machine in a multimachine power network which takes into account the effect of transfer conductances and of the remote network dynamics: it does not require the machine internal voltage measurement and relies on the knowledge of only three model parameters and of upper and lower bounds on the uncertain ones. Sufficient conditions to be satisfied by the remaining part of the network for guaranteeing  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$  disturbance attenuation and relative speed regulation are derived which are more general than those required by the single machine-infinite bus approximation.

*Key-Words:* Power system control, robust nonlinear control, adaptive control,  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$  disturbance attenuation.

## 1 Introduction

The transient stabilization problem of power networks, which are large-scale interconnected nonlinear systems, consists in the design of a suitable excitation feedback control keeping each generator close to the synchronous speed when mechanical and electrical perturbations, such as load shedding, generation tripping or short circuits, occur. The standard linear controllers which are designed on the basis of linear approximations around operating conditions may not be able to handle the severe disturbances and contingencies typically occurring in power systems so that, in the recent years, several nonlinear control algorithms have been proposed, leading to significant advances in power system control ([6], [8], [11], [12], [4], [9]).

In this paper we extend the analysis presented in [9] by taking into account the effects of transfer conductances and of remote network dynamics: the resulting control for the transient stabilization of a synchronous generator in a multimachine power network, does not assume the availability of the machine internal voltage measurement as well as the knowledge of the model parameters excepting for machine damping and inertia constants and synchronous speed.  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$

disturbance attenuation and relative speed regulation are guaranteed under certain assumptions on the network dynamics which are more general than those required by the single machine-infinite bus approximation. Simulation results with reference to a 3-machine, 9-bus system illustrate the closed loop performance.

## 2 Problem Statement and Nonlinear Control Design

A large-scale power system consisting of  $n$  generators interconnected through a transmission network is described by the nonlinear third order model in [10] [ $1 \leq i \leq n$ ]

$$\begin{aligned} \dot{\delta}_i &= \omega_i \\ \dot{\omega}_i &= -\frac{D_i}{2H_i}\omega_i + \frac{\omega_0}{2H_i}P_{mi} - \frac{\omega_0}{2H_i}P_{ei} \\ \dot{E}'_{qi} &= \frac{k_{ci}}{T'_{d0i}}u_{fi} - \frac{E'_{qi}}{T'_{d0i}} - \frac{(x_{di} - x'_{di})}{T'_{d0i}}I_{di} \\ P_{ei} &= E'_{qi}{}^2 G_{ii} + E'_{qi} \sum_{j=1, j \neq i}^n \left[ E'_{qj} G_{ij} \cos(\delta_{ij}) \right. \\ &\quad \left. + E'_{qj} B_{ij} \sin(\delta_{ij}) \right] \end{aligned} \quad (1)$$

$$\begin{aligned}
Q_{ei} &= -E'_{qi}{}^2 B_{ii} + E'_{qi} \sum_{j=1, j \neq i}^n \left[ E'_{qj} G_{ij} \sin(\delta_{ij}) \right. \\
&\quad \left. - E'_{qj} B_{ij} \cos(\delta_{ij}) \right] - x'_{di} (I_{di}^2 + I_{qi}^2) \\
I_{di} &= -E'_{qi} B_{ii} + \sum_{j=1, j \neq i}^n \left[ E'_{qj} G_{ij} \sin(\delta_{ij}) \right. \\
&\quad \left. - E'_{qj} B_{ij} \cos(\delta_{ij}) \right] \\
I_{qi} &= E'_{qi} G_{ii} + \sum_{j=1, j \neq i}^n \left[ E'_{qj} G_{ij} \cos(\delta_{ij}) \right. \\
&\quad \left. + E'_{qj} B_{ij} \sin(\delta_{ij}) \right] \\
\delta_{ij} &= \delta_i - \delta_j \\
V_{ti} &= \sqrt{v_{di}^2 + v_{qi}^2} = \sqrt{(x'_{di} I_{qi})^2 + (E'_{qi} - x'_{di} I_{di})^2}
\end{aligned}$$

in which (for the  $i$ -th generator):  $\delta_i$ (rad) is the power angle,  $\omega_i$ (rad/s) is the relative angular speed,  $E'_{qi}$ (p.u.) is the transient EMF in the quadrature axis,  $P_{ei}$ (p.u.) is the active electrical power,  $P_{mi}$ (p.u.) is the mechanical input power,  $Q_{ei}$ (p.u.) is the reactive electrical power,  $I_{di}$ (p.u.) is the direct axis current,  $I_{qi}$ (p.u.) is the quadrature axis current,  $V_{ti}$ (p.u.) is the terminal voltage,  $u_{fi}$ (p.u.) is the input to the SCR amplifier,  $\omega_0$ (rad/s) is the synchronous speed,  $D_i$ (p.u.) is the damping constant,  $H_i$ (s) is the inertia constant,  $T'_{d0i}$ (s) is the direct axis transient open circuit time constant,  $x'_{di}$ (p.u.) is the direct axis reactance,  $k_{ci}$  is the gain of the excitation amplifier,  $G_{ij}$ (p.u.) and  $B_{ij}$ (p.u.) are the  $i$ -th row and the  $j$ -th column element of nodal conductance and susceptance matrixes, respectively, at the internal nodes after eliminating all physical buses, which depend on the direct axis transient reactance  $x'_{di}$ (p.u.), on the transformer reactance  $x_{Ti}$ (p.u.), on the loads and on the transmission line reactance  $x_{ij}$ (p.u.) between the  $i$ -th generator and the  $j$ -th generator. Direct computation of  $P_{ei}(t)$  time derivative [ $G_{ij}$  and  $B_{ij}$ ,  $1 \leq j \leq n$ , are assumed to be constant] allows us to rewrite the dynamic model (1) as [ $I_{qi} \geq c_{Ii} > 0$  ([7])]

$$\begin{aligned}
\dot{\delta}_i &= \omega_i \\
\dot{\omega}_i &= -\frac{D_i}{2H_i} \omega_i + \frac{\omega_0}{2H_i} P_{mi} - \frac{\omega_0}{2H_i} P_{ei} \\
\dot{P}_{ei} &= \frac{k_{ci}}{T'_{d0i}} \left[ \frac{I_{qi}^2 + P_{ei} G_{ii}}{I_{qi}} \right] u_{fi} - \frac{P_{ei}}{T'_{d0i}} - \frac{G_{ii}}{T'_{d0i}} \frac{P_{ei}^2}{I_{qi}^2} \\
&\quad - \frac{(x_{di} - x'_{di})}{T'_{d0i}} I_{di} I_{qi} - \frac{G_{ii}(x_{di} - x'_{di})}{T'_{d0i}} \frac{I_{di} P_{ei}}{I_{qi}} \\
&\quad - Q_{ei} \omega_i - B_{ii} \frac{P_{ei}^2}{I_{qi}^2} \omega_i - x'_{di} (I_{di}^2 + I_{qi}^2) \omega_i \\
&\quad + \frac{P_{ei}}{I_{qi}} R_i
\end{aligned} \tag{2}$$

in which  $\delta_i$ ,  $\omega_i$ ,  $P_{ei}$  are the state variables,  $u_{fi}$  is the control input and the term

$$\begin{aligned}
R_i &= \sum_{j=1, j \neq i}^n \left\{ \dot{E}'_{qj} \left[ G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij}) \right] \right. \\
&\quad \left. + \omega_j \left[ E'_{qj} G_{ij} \sin(\delta_{ij}) - E'_{qj} B_{ij} \cos(\delta_{ij}) \right] \right\}
\end{aligned}$$

represents the effect of the network remote dynamics on the  $i$ -th generator.

In this paper, we restrict our analysis to a particular power system  $\mathcal{PS}$ , which consists of a group  $G_M$  of  $n - 1$  generators [tied together by a strong network of transmission lines] which is linked to a single generator  $g_m$  (referred to as  $r$ -th generator) by a comparatively weak set of tie lines: the transient stabilization problem of the synchronous generator  $g_m$  in the multi-machine power network  $\mathcal{PS}$  is addressed. Let us state the theoretical result of the paper (the proof is omitted).

**Theorem:** Consider the  $r$ -th generator dynamics. Denote by  $\delta_{rs}$  the pre-fault constant value for the power angle  $\delta_r$  and assume that for all  $t \geq 0$

- i) for each  $j \neq r$ ,  $\delta_j(t)$  and  $E'_{qj}(t)$  are continuous functions of time  $t$  and boundedness of  $\delta_r(t)$ ,  $\omega_r(t)$ ,  $P_{er}(t)$  implies boundedness of  $E'_{qj}(t)$ ;
- ii)<sub>a</sub>) there exist  $\mu_r, \nu_r, \rho_r$  non-negative reals,  $\psi_{\mu r}(\cdot)$ ,  $\psi_{\nu r}(\cdot)$ ,  $\psi_{\rho r}(\cdot)$   $K_\infty$  functions and  $g_r(t)$  bounded non-negative real-valued function of time  $t$  such that
$$\begin{aligned}
|R_r(t)| &\leq \sup_{0 \leq \tau \leq t} \{g_r(\tau)\} + \mu_r \psi_{\mu r} \left( \max_{0 \leq \tau \leq t} \{|\delta_r(\tau) - \delta_{rs}|\} \right) \\
&\quad + \nu_r \psi_{\nu r} \left( \max_{0 \leq \tau \leq t} \{|\omega_r(\tau)|\} \right) \\
&\quad + \rho_r \psi_{\rho r} \left( \max_{0 \leq \tau \leq t} \{|P_{er}(\tau) - P_{mr}(\tau)|\} \right);
\end{aligned}$$
- ii)<sub>b</sub>) there exist  $\alpha_{lr}$  known non-negative reals and  $\varphi_{lr}(\cdot)$  known  $K_\infty$  functions such that for  $l = \mu, \nu, \rho$ 

$$\psi_{lr}(\cdot) \leq \alpha_{lr} + \varphi_{lr}(\cdot) \doteq \bar{\varphi}_{lr}(\cdot)$$

iii)  $P_{er}(t) \geq c_{Pr} > 0$ ;

iv) the unknown (possibly time-varying) parameters  $T'_{d0r}(t)$ ,  $x'_{dr}(t)$ ,  $x'_{dr}(t)$ ,  $k_{cr}(t)$  and the unknown constant parameter  $G_{rr}$  are within the corresponding known positive physical bounds  $(T'_{d0rm}, T'_{d0rM})$ ,  $(x_{drm}, x_{drM})$ ,  $(x'_{drm}, x'_{drM})$ ,  $(k_{crm}, k_{crM})$ ,  $(G_{rrm}, G_{rrM})$ ;

v) the unknown constant parameter  $B_{rr}$  is within the corresponding known physical bounds  $(B_{rrm}, B_{rrM})$ ;

vi)  $|\dot{P}_{mr}(t)| \leq \dot{P}_{Mr}$  with known  $\dot{P}_{Mr}$ .

Then the nonlinear adaptive feedback control algorithm  $[k_{\delta r}, k_{\omega r}, k_{pr}, k_{per}, k_{\omega pr}, k_r, k_{Rr}, \beta_{jr}, \beta_{xr}, \beta_{Br}, \varepsilon_{jr}, \varepsilon_{xr}, \varepsilon_{Br} (1 \leq j \leq 6)]$  are positive control parameters]

$$\begin{aligned}
u_{fr} &= \frac{I_{qr}}{(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er})} v_r \\
&\quad - \frac{k_r I_{qr} (2I_{qr}^4 + 2P_{er}^2)}{4(\hat{\theta}_{5r}I_{qr}^2 + \hat{\theta}_{6r}P_{er})^2 (\theta_{5rm}I_{qr}^2 + \theta_{6rm}P_{er})} \cdot v_r^2 (P_{er} - P_{er}^*) \\
v_r &= -\frac{5}{4}k_{pr}(P_{er} - P_{er}^*) + \frac{\omega_0}{2H_r}(\omega_r - \omega_r^*) \\
&\quad - \frac{k_{Rr}}{4} \frac{P_{er}^2}{I_{qr}^2} (P_{er} - P_{er}^*) + \frac{5D_r k_{\delta r}}{4\omega_0} \omega_r + \frac{2H_r}{\omega_0} \omega_r \\
&\quad - \frac{k_r}{4} (P_{er} - P_{er}^*) \left[ P_{er}^2 + I_{dr}^2 I_{qr}^2 + \frac{I_{dr}^2 P_{er}^2}{I_{qr}^2} \right. \\
&\quad \left. + \frac{P_{er}^4}{I_{qr}^4} (1 + \omega_r^2) + \omega_r^2 (I_{dr}^2 + I_{qr}^2)^2 \right] \\
&\quad - \frac{k_r}{4} \left( \frac{5}{4}k_{\omega r} + \frac{5}{4}k_{\delta r} + \frac{5}{4}k_{per} \right. \\
&\quad \left. + \frac{k_r}{4} + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2} k_{\omega pr} \right. \\
&\quad \left. + \frac{1}{k_{\omega pr}} \right)^2 (P_{er} - P_{er}^*) + \hat{\theta}_{1r} P_{er} + \hat{\theta}_{2r} I_{dr} I_{qr} \\
&\quad + \hat{\theta}_{3r} \frac{I_{dr} P_{er}}{I_{qr}} + \hat{\theta}_{4r} \frac{P_{er}^2}{I_{qr}^2} + \hat{B}_{rr} \frac{P_{er}^2}{I_{qr}^2} \omega_r \\
&\quad + \hat{x}'_{dr} (I_{dr}^2 + I_{qr}^2) \omega_r + Q_{er} \omega_r - \frac{5k_{\delta r}}{4\omega_0} \left[ D_r \omega_r \right. \\
&\quad \left. - \omega_0 (\hat{P}_{mr} - P_{er}) \right] \\
&\quad - \frac{2H_r}{\omega_0} \left( \frac{5}{4}k_{\omega r} + \frac{1}{k_{\omega pr}} \right) \left[ \left( \frac{5}{4}k_{\omega r} \right. \right. \\
&\quad \left. \left. + \frac{D_r}{2H_r} \right) (\omega_r - \omega_r^*) + (\delta_r - \delta_{rs}) + \frac{\omega_0}{2H_r} (P_{er} \right. \\
&\quad \left. - P_{er}^*) + \frac{1}{k_{\omega pr}} (\omega_r - \omega_r^*) \right] - \frac{k_r}{4I_{qr}^2} P_{er}^2 (P_{er} \\
&\quad - P_{er}^*) \left[ \bar{\varphi}_{\mu r}^2 \left( \max_{0 \leq \tau \leq t} \{[\delta_r(\tau) \right. \right. \\
&\quad \left. \left. - \delta_{rs}]\} \right) + \bar{\varphi}_{\nu r}^2 \left( \max_{0 \leq \tau \leq t} \{[\omega_r(\tau) \right. \right. \\
&\quad \left. \left. - \omega_r^*(\tau)]\} + \frac{5}{4}k_{\delta r} \max_{0 \leq \tau \leq t} \{[\delta_r(\tau) - \delta_{rs}]\} \right) \right]
\end{aligned}$$

$$\begin{aligned}
&\quad + \bar{\varphi}_{\rho r}^2 \left( \max_{0 \leq \tau \leq t} \{[P_{er}(\tau) - P_{er}^*(\tau)]\} \right. \\
&\quad \left. + P_{mrM} - P_{mrm} + \frac{\dot{P}_{Mr}}{k_{per}} + \frac{2H_r}{\omega_0} \left( \frac{5}{4}k_{\omega r} \right. \right. \\
&\quad \left. \left. + \frac{1}{k_{\omega pr}} + \frac{5}{4}k_{\delta r} \right) \max_{0 \leq \tau \leq t} \{[\omega_r(\tau) - \omega_r^*(\tau)]\} \right. \\
&\quad \left. + \left[ \frac{2H_r}{\omega_0} \left( 1 + \frac{25}{16}k_{\delta r}^2 \right) + \frac{5D_r k_{\delta r}}{4\omega_0} \right] \cdot \right. \\
&\quad \left. \max_{0 \leq \tau \leq t} \{[\delta_r(\tau) - \delta_{rs}]\} \right) + 1 \Big] \\
\omega_r^* &= -\frac{5}{4}k_{\delta r}(\delta_r - \delta_{rs}) \\
P_{er}^* &= \frac{2H_r}{\omega_0} \left[ \frac{5}{4}k_{\omega r}(\omega_r - \omega_r^*) + \frac{5}{4}k_{\delta r}\omega_r \right. \\
&\quad \left. - \frac{D_r}{2H_r}\omega_r^* + (\delta_r - \delta_{rs}) \right. \\
&\quad \left. + \frac{1}{k_{\omega pr}}(\omega_r - \omega_r^*) \right] + \hat{P}_{mr} \\
\hat{P}_{mr} &= \phi_r + \frac{2H_r}{\omega_0} \left( \frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} \right. \\
&\quad \left. + \frac{\omega_0^2}{16H_r^2} k_{\omega pr} \right) \omega_r \\
\dot{\phi}_r &= \left( \frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2} k_{\omega pr} \right) \left[ -\phi_r \right. \\
&\quad \left. + \frac{D_r}{\omega_0} \omega_r + P_{er} - \frac{2H_r}{\omega_0} \left( \frac{5}{4}k_{per} + \frac{k_r}{4} \right. \right. \\
&\quad \left. \left. + \frac{1}{k_r} + \frac{\omega_0^2}{16H_r^2} k_{\omega pr} \right) \omega_r \right] \\
\phi_r(0) &= \hat{P}_{mr}(0) - \frac{2H_r}{\omega_0} \left( \frac{5}{4}k_{per} + \frac{k_r}{4} + \frac{1}{k_r} \right. \\
&\quad \left. + \frac{\omega_0^2}{16H_r^2} k_{\omega pr} \right) \omega_r(0), \\
P_{mrm} &\leq \hat{P}_{mr}(0) \leq P_{mrM} \\
\dot{\hat{\theta}}_{1r} &= \text{Proj} \left[ -\frac{P_{er}(P_{er} - P_{er}^*)}{\beta_{1r}}, \hat{\theta}_{1r}, \theta_{1rm}, \right. \\
&\quad \left. \theta_{1rM}, \varepsilon_{1r} \right], \theta_{1rm} \leq \hat{\theta}_{1r}(0) \leq \theta_{1rM} \\
\dot{\hat{\theta}}_{2r} &= \text{Proj} \left[ -\frac{I_{dr} I_{qr} (P_{er} - P_{er}^*)}{\beta_{2r}}, \hat{\theta}_{2r}, \theta_{2rm}, \right. \\
&\quad \left. \theta_{2rM}, \varepsilon_{2r} \right], \theta_{2rm} \leq \hat{\theta}_{2r}(0) \leq \theta_{2rM} \\
\dot{\hat{\theta}}_{3r} &= \text{Proj} \left[ -\frac{I_{dr} P_{er} (P_{er} - P_{er}^*)}{I_{qr} \beta_{3r}}, \hat{\theta}_{3r}, \theta_{3rm}, \right.
\end{aligned} \tag{3}$$

$$\begin{aligned}
& \left. \theta_{3rM}, \varepsilon_{3r} \right], \theta_{3rm} \leq \hat{\theta}_{3r}(0) \leq \theta_{3rM} \\
\dot{\hat{\theta}}_{4r} &= \text{Proj} \left[ -\frac{P_{er}^2(P_{er} - P_{er}^*)}{I_{qr}^2 \beta_{4r}}, \hat{\theta}_{4r}, \theta_{4rM}, \right. \\
& \left. \theta_{4rM}, \varepsilon_{4r} \right], \theta_{4rM} \leq \hat{\theta}_{4r}(0) \leq \theta_{4rM} \\
\dot{\hat{\theta}}_{5r} &= \text{Proj} \left[ \frac{I_{qr}^2 \nu_r (P_{er} - P_{er}^*)}{(\hat{\theta}_{5r} I_{qr}^2 + \hat{\theta}_{6r} P_{er}) \beta_{5r}}, \hat{\theta}_{5r}, \theta_{5rM}, \right. \\
& \left. \theta_{5rM}, \varepsilon_{5r} \right], \theta_{5rM} \leq \hat{\theta}_{5r}(0) \leq \theta_{5rM} \\
\dot{\hat{\theta}}_{6r} &= \text{Proj} \left[ \frac{P_{er} \nu_r (P_{er} - P_{er}^*)}{(\hat{\theta}_{5r} I_{qr}^2 + \hat{\theta}_{6r} P_{er}) \beta_{6r}}, \hat{\theta}_{6r}, \theta_{6rM}, \right. \\
& \left. \theta_{6rM}, \varepsilon_{6r} \right], \theta_{6rM} \leq \hat{\theta}_{6r}(0) \leq \theta_{6rM} \\
\dot{\hat{x}}'_{dr} &= \text{Proj} \left[ -\frac{(I_{dr}^2 + I_{qr}^2) \omega_r (P_{er} - P_{er}^*)}{\beta_{xr}}, \hat{x}'_{dr}, \right. \\
& \left. x'_{drm}, x'_{drM}, \varepsilon_{xr} \right], x'_{drm} \leq \hat{x}'_{dr}(0) \leq x'_{drM} \\
\dot{\hat{B}}_{rr} &= \text{Proj} \left[ -\frac{P_{er}^2 \omega_r (P_{er} - P_{er}^*)}{\beta_{Br} I_{qr}^2}, \hat{B}_{rr}, B_{rrM}, \right. \\
& \left. B_{rrM}, \varepsilon_{Br} \right], B_{rrM} \leq \hat{B}_{rr}(0) \leq B_{rrM} \\
\theta_{1rm} &= \frac{1}{T'_{d0rM}}, \quad \theta_{1rM} = \frac{1}{T'_{d0rM}} \\
\theta_{2rm} &= \frac{(x_{drm} - x'_{drM})}{T'_{d0rM}} \\
\theta_{2rM} &= \frac{(x_{drM} - x'_{drM})}{T'_{d0rM}} \\
\theta_{3rm} &= \frac{G_{rrM}(x_{drm} - x'_{drM})}{T'_{d0rM}} \\
\theta_{3rM} &= \frac{G_{rrM}(x_{drM} - x'_{drM})}{T'_{d0rM}} \\
\theta_{4rm} &= \frac{G_{rrM}}{T'_{d0rM}}, \quad \theta_{4rM} = \frac{G_{rrM}}{T'_{d0rM}} \\
\theta_{5rm} &= \frac{k_{crm}}{T'_{d0rM}}, \quad \theta_{5rM} = \frac{k_{crM}}{T'_{d0rM}} \\
\theta_{6rm} &= \frac{G_{rrM} k_{crm}}{T'_{d0rM}}, \quad \theta_{6rM} = \frac{G_{rrM} k_{crM}}{T'_{d0rM}}
\end{aligned}$$

$$\text{Proj}[\zeta, \hat{z}_r, z_{rm}, z_{rM}, \varepsilon_{zr}] = \begin{cases} \zeta \zeta_{p1} & \text{if } \mathcal{C}_a \\ \zeta \zeta_{p2} & \text{if } \mathcal{C}_b \\ \zeta & \text{otherwise} \end{cases}$$

$$\begin{aligned}
\zeta_{p1} &= \left[ 1 - \frac{z_{rm}^2 - \hat{z}_r^2}{z_{rm}^2 - (z_{rm} - \varepsilon_{zr})^2} \right] \\
\zeta_{p2} &= \left[ 1 - \frac{\hat{z}_r^2 - z_{rM}^2}{(z_{rM} + \varepsilon_{zr})^2 - z_{rM}^2} \right] \\
0 &\leq z_{rm}(z_{rm} - \varepsilon_{zr}), \quad 0 \leq z_{rM}(z_{rM} + \varepsilon_{zr}) \\
\mathcal{C}_a : \hat{z}_r &< z_{rm} \ \& \ \zeta < 0, \quad \mathcal{C}_b : \hat{z}_r > z_{rM} \ \& \ \zeta > 0
\end{aligned}$$

is bounded and guarantees the closed loop system (2)-(3) to satisfy, in terms of  $y_r(t) = [\delta_r(t) - \delta_{rs}, \omega_r(t), P_{er}(t) - P_{mr}(t)]^T$ ,  $x_r(t) = [y_r(t)^T, P_{mr}(t) - \hat{P}_{mr}(t)]^T$ ,  $w_{dr}(t) = [\dot{P}_{mr}(t), \theta_{1rM} - \theta_{1rm} + \varepsilon_{1r}, \theta_{2rM} - \theta_{2rm} + \varepsilon_{2r}, \theta_{3rM} - \theta_{3rm} + \varepsilon_{3r}, \theta_{4rM} - \theta_{4rm} + \varepsilon_{4r}, \max\{\theta_{5rM} - \theta_{5rm} + \varepsilon_{5r}, \theta_{6rM} - \theta_{6rm} + \varepsilon_{6r}\}, x'_{drM} - x'_{drm} + \varepsilon_{xr}, B_{rrM} - B_{rrm} + \varepsilon_{Br}]^T$ , the following properties:

(S1) exponential convergence and  $\mathcal{L}_\infty$  disturbance attenuation, i.e.

$$\|y_r(t)\|^2 \leq h_{1r} e^{-c_r t} + \frac{1}{k_r} \gamma_{1r} (\|w_{dr}(\cdot)\|_\infty)$$

holds for all  $t \geq 0$ , where  $h_{1r}(x_r(0)) \geq 0$ ,  $c_r > 0$  and  $\gamma_{1r}(r)$  is a class  $\mathcal{K}_\infty$  function;

(S2)  $\mathcal{L}_2$  disturbance attenuation, i.e.

$$\int_0^T \|y_r(\tau)\|^2 d\tau \leq h_{2r} + \frac{1}{k_r} \int_0^T \gamma_{2r} (\|w_{dr}(\tau)\|) d\tau$$

holds for any given  $T > 0$ , where  $h_{2r}(x_r(0)) \geq 0$  and  $\gamma_{2r}(r)$  is a class  $\mathcal{K}_\infty$  function.

If, in addition to previous assumptions, there exist a non-negative real  $t_0$  and positive reals  $M_{Rr}$ ,  $\gamma_{\delta r}$ ,  $\gamma_{\omega r}$ ,  $\gamma_{pr}$  such that

vii)  $P_{mr}(t)$ ,  $T'_{d0r}(t)$ ,  $x_{dr}(t)$ ,  $x'_{dr}(t)$ ,  $k_{cr}(t)$  are constant for all  $t \geq t_0$ ;

viii)  $\lim_{t \rightarrow \infty} \int_{t_0}^t R_r^2(\tau) d\tau \leq M_{Rr}$   
 $+\gamma_{\delta r} \lim_{t \rightarrow \infty} \int_{t_0}^t [\delta_r(\tau) - \delta_{rs}]^2 d\tau$   
 $+\gamma_{\omega r} \lim_{t \rightarrow \infty} \int_{t_0}^t [\omega_r(\tau)]^2 d\tau$   
 $+\gamma_{pr} \lim_{t \rightarrow \infty} \int_{t_0}^t [P_{er}(\tau) - P_{mr}(\tau)]^2 d\tau$ ;

ix)  $\min \left\{ \left( \frac{k_{\delta r}}{4} - \frac{\bar{\gamma}_{\delta r}}{k_{Rr}} \right), \left( \frac{k_{\omega r}}{4} - \frac{\bar{\gamma}_{\omega r}}{k_{Rr}} \right), \left( \frac{k_{pr}}{4} - \frac{4\gamma_{pr}}{k_{Rr}} \right), \left( \frac{k_{per}}{4} - \frac{4\gamma_{pr}}{k_{Rr}} \right) \right\} > 0$  with  $\bar{\gamma}_{\delta r} = \gamma_{\delta r} + \frac{25}{8} \gamma_{\omega r} k_{\delta r}^2 + 4 \left[ \frac{2H_r}{\omega_0} \left( \frac{25}{16} k_{\delta r}^2 + 1 \right) + \frac{5}{4} \frac{D_r k_{\delta r}}{\omega_0} \right]^2 \gamma_{pr}$  and  $\bar{\gamma}_{\omega r} = 2\gamma_{\omega r} + 4 \left[ \frac{2H_r}{\omega_0} \left( \frac{5}{4} k_{\omega r} + \frac{5}{4} k_{\delta r} + \frac{1}{k_{\omega pr}} \right) \right]^2 \gamma_{pr}$ ,

then controller (3) guarantees the additional property:

(S3) asymptotic convergence, i.e.

$$\lim_{t \rightarrow \infty} \left\| [\delta_r(t) - \delta_{rs}, \omega_r(t), P_{er}(t) - P_{mr}] \right\| = 0.$$

*Remark 1:* The proposed control algorithm (3) relies on the locally measured signals:  $\delta_r$ ,  $\omega_r$ ,  $P_{er}$ ,  $I_{dr}$ ,  $I_{qr}$ ,  $Q_{er}$  (the method for measuring the power angle  $\delta_r$  can be found in [3], [2] and [5]).

*Remark 2:* Let  $\Sigma_r(y_{r1}, y_{r2}, y_{r3}, R_r)$  be the system consisting of the  $j$ -th generators ( $j \neq r$ ) with inputs the  $y_r(t)$  vector components  $y_{r1}(t)$ ,  $y_{r2}(t)$ ,  $y_{r3}(t)$  and output the function  $R_r(t)$ . Less restrictive conditions than those required in the single machine-infinite bus approximation are to be satisfied by the remaining part of the network: assumption ii)<sub>a</sub>) is satisfied if system  $\Sigma_r(y_{r1}, y_{r2}, y_{r3}, R_r)$  is Input to Output Stable with gain functions  $\psi_{\mu r}(\cdot)$ ,  $\psi_{\nu r}(\cdot)$ ,  $\psi_{\rho r}(\cdot)$ ; assumption ii)<sub>b</sub>) requires the knowledge of suitable functions  $\bar{\varphi}_{\mu r}(\cdot)$ ,  $\bar{\varphi}_{\nu r}(\cdot)$ ,  $\bar{\varphi}_{\rho r}(\cdot)$  which majorize the gain functions  $\psi_{\mu r}(\cdot)$ ,  $\psi_{\nu r}(\cdot)$ ,  $\psi_{\rho r}(\cdot)$ ; assumption viii) is satisfied if system  $\Sigma_r(y_{r1}, y_{r2}, y_{r3}, R_r)$  has finite  $\mathcal{L}_2$  gains, bounded from above by the positive numbers  $\sqrt{\gamma_{\delta r}/3}$ ,  $\sqrt{\gamma_{\omega r}/3}$ ,  $\sqrt{\gamma_{pr}/3}$ .

### 3 Simulation Results

In this section we will test by simulation the control algorithm (3) with reference to each generator of the popular Western System Coordinating Council (WSCC) 3-machine, 9-bus system described in [10] and [1] [ $D_i = 0$ ,  $1 \leq i \leq 3$ ], with the aim of numerically checking the conservativeness of the conditions provided by the theorem, of testing the performance and of verifying the robustness of the proposed control law with respect to unmodelled dynamics: the simulation is carried out by using the two-axis model (see [10]) from which the model (1) has been derived. The initial conditions for the state variables are computed by systematically solving the load-flow equations of the network and by computing the values of the algebraic variables. For  $1 \leq i \leq 3$ , the functions  $\bar{\varphi}_{\mu i}(\cdot)$ ,  $\bar{\varphi}_{\nu i}(\cdot)$ ,  $\bar{\varphi}_{\rho i}(\cdot)$  are set equal to  $id(\cdot)|_{[0, \infty)}$  [ $id(\cdot)$  is the identity function], the control parameters are chosen as  $k_{\delta i} = k_{\omega i} = 0.01$ ,  $k_{p i} = 720$ ,  $k_{p e i} = k_{\omega p i} = 1$ ,  $k_i = 0.001$ ,  $k_{R i} = 0.1$ ,  $\beta_{1 i} = \beta_{2 i} = \beta_{3 i} = \beta_{4 i} = \beta_{5 i} = \beta_{6 i} = \beta_{x i} = \beta_{B i} = 48000$ ,  $\varepsilon_{1 i} = \varepsilon_{2 i} = \varepsilon_{3 i} = \varepsilon_{4 i} = \varepsilon_{5 i} = \varepsilon_{6 i} = \varepsilon_{x i} = \varepsilon_{B i} = 0.00001$ , while  $\dot{P}_{M i} = 1$  and the known upper and lower bounds on the model parameters values are  $k_{c1m} = 0.6$ ,  $k_{c1M} = 1.4$ ,  $x_{d1m} = 0.1$ ,  $x_{d1M} = 0.2$ ,  $x'_{d1m} = 0.05$  p.u.,  $x'_{d1M} = 0.07$  p.u.,  $T'_{d01m} = 6$  p.u.,

$T'_{d01M} = 12$  p.u.,  $G_{11m} = 0.5$  p.u.,  $G_{11M} = 1.5$  p.u.,  $P_{m1m} = 0.3$  p.u.,  $P_{m1M} = 1.1$  p.u.,  $B_{11m} = -6$  p.u.,  $B_{11M} = -1$  p.u.,  $k_{c2m} = 0.6$ ,  $k_{c2M} = 1.4$ ,  $x_{d2m} = 0.7$ ,  $x_{d2M} = 1.1$ ,  $x'_{d2m} = 0.09$  p.u.,  $x'_{d2M} = 0.14$  p.u.,  $T'_{d02m} = 4$  p.u.,  $T'_{d02M} = 8$  p.u.,  $G_{22m} = 0.3$  p.u.,  $G_{22M} = 0.6$  p.u.,  $P_{m2m} = 1.2$  p.u.,  $P_{m2M} = 2$  p.u.,  $B_{22m} = -6$  p.u.,  $B_{22M} = -1$  p.u.,  $k_{c3m} = 0.6$ ,  $k_{c3M} = 1.4$ ,  $x_{d3m} = 1.2$ ,  $x_{d3M} = 1.5$ ,  $x'_{d3m} = 0.1$  p.u.,  $x'_{d3M} = 0.3$  p.u.,  $T'_{d03m} = 4$  p.u.,  $T'_{d03M} = 8$  p.u.,  $G_{33m} = 0.1$  p.u.,  $G_{33M} = 0.4$  p.u.,  $P_{m3m} = 0.4$  p.u.,  $P_{m3M} = 1.2$  p.u.,  $B_{33m} = -6$  p.u.,  $B_{33M} = -1$  p.u.. The goal of the simulation is to verify the effect of a three-phase fault occurring near bus 7 at the end of line 5-7 at  $t = 0.001$  s, which is cleared in five cycles (0.083 s) by opening line 5-7. The same simulation is performed by testing separately (with reference to each generator of WSCC) the proposed controller and both the IEEE-Type I and the fast excitors with PSS (the exciter data are taken from [10], while  $K_0 = \tau_0 = 24$ ,  $\tau_1 = \tau_3 = 0.5$ ,  $\tau_2 = \tau_4 = 0.05$  and the saturating limits are  $\pm 0.1$  in the PSS diagram in [1]). The results are reported in Figures 1-3: the proposed controller restores the synchronous speeds, while the linear controllers either cannot survive the severe fault or can only maintain the system stable.

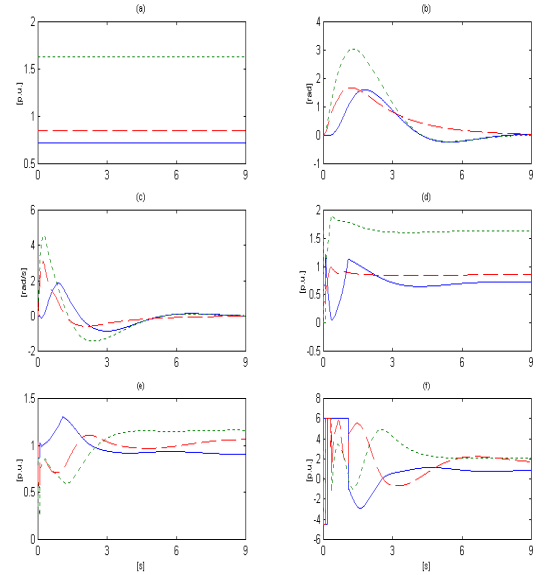


Figure 1: Proposed control [Generator 1 (solid), Generator 2 (dot), Generator 3 (dashed)]; a)  $P_{mi}$  b)  $\delta_i - \delta_{is}$  c)  $\omega_i$  d)  $P_{ei}$  e)  $V_{ti}$  f)  $u_{fi}$

### 4 Conclusions

A robust adaptive nonlinear feedback control (3) has been designed on the basis of a third order model

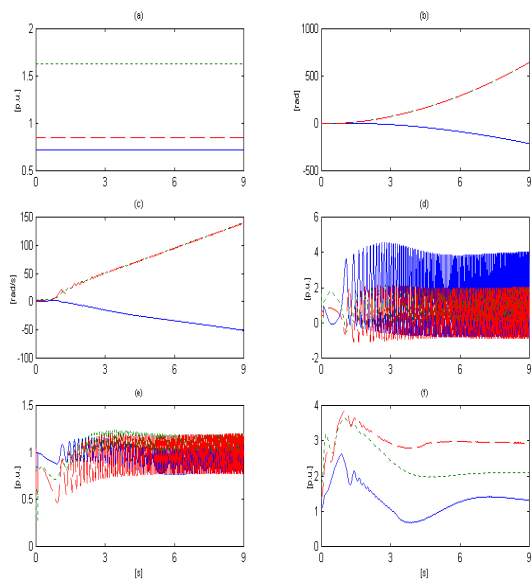


Figure 2: IEEE-Type I exciter with PSS [Generator 1 (solid), Generator 2 (dot), Generator 3 (dashed)]; a)  $P_{mi}$  b)  $\delta_i - \delta_{is}$  c)  $\omega_i$  d)  $P_{ei}$  e)  $V_{ti}$  f)  $u_{fi}$

(1) of a synchronous generator in a multimachine power network with nontrivial conductances. Only three model parameters and upper and lower bounds on the uncertain ones are required to be known.  $\mathcal{L}_2$  and  $\mathcal{L}_\infty$  disturbance attenuation and asymptotic regulation properties (S1)–(S3) are guaranteed under suitable conditions to be satisfied by the network dynamics which generalize those required by the single machine-infinite bus approximation.

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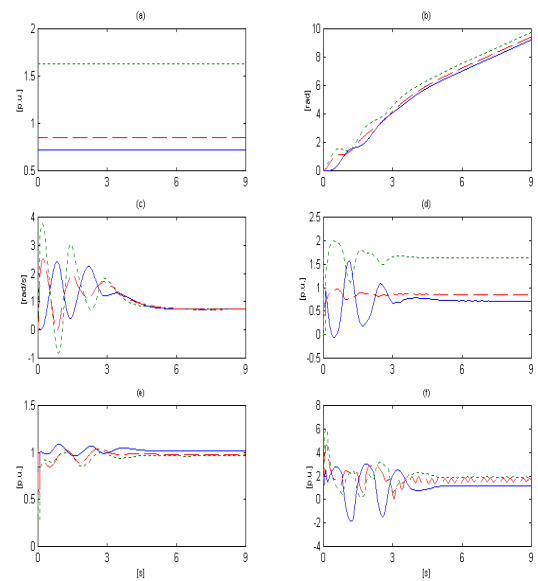


Figure 3: Fast exciter with PSS [Generator 1 (solid), Generator 2 (dot), Generator 3 (dashed)]; a)  $P_{mi}$  b)  $\delta_i - \delta_{is}$  c)  $\omega_i$  d)  $P_{ei}$  e)  $V_{ti}$  f)  $u_{fi}$

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