H_∞ switched-gain based observer vs nonlinear transformation based observer for a vehicle tracking model

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$\mathcal{H}_\infty$ Switched-Gain Based Observer vs Nonlinear Transformation Based Observer for a Vehicle Tracking Model ⋆

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Abstract: The main objective of this paper is to provide a comparison between two nonlinear observer design methods for a vehicle tracking problem based on a kinematic model. This challenging tracking issue is difficult to solve because of the nature of the nonlinearities in the dynamics process, which are all non-monotonic. First, we will extend the switched-gain based approach, established recently in the literature, to systems with disturbances in both the state dynamics and in the output measurements, and investigate its $\mathcal{H}_\infty$ performance. Then we will introduce a new way to avoid non-monotonic nonlinearities by using a specific nonlinear transformation to bring the system into a suitable form for which available techniques in the literature can be applied. Finally, numerical comparisons between the two design methods are provided, where the $\mathcal{H}_\infty$ based switched gain observer has better performance with respect to measurement noise.

Keywords: $\mathcal{H}_\infty$ based Observers, Nonlinear systems, Hybrid systems, Linear matrix inequalities, Nonlinear Transformation

1. INTRODUCTION

The control of the autonomous vehicle is an emerging topic in the current era. There has been significant literature on control of autonomous vehicle but not sufficiently addressed. The problem of motion tracking arrives in the case of the collision avoidance problem and the adaptive cruise control (ACC) which are illustrated in Abou-Jaoude (2003), Mukhtar et al. (2015). In these two cases, authors had used radar or lessor sensors for estimating azimuth angle or distance to achieve vehicle motion tracking in autonomous driving. However, these sensors are not sufficient to fully estimate the trajectories of vehicles. In Kayacan et al. (2015), the problem of vehicle motion tracking is resolved with help of the nonlinear model predictive control approach while Linear parameter varying (LPV) based observer approach used in Wang et al. (2016) for estimating trajectories. These methods had their limitations like heavy computation cost, complex calculation. Authors of Kaempchen et al. (2004), Kang et al. (2012) had used the interacting multiple model (IMM) filter approach to tackle the problem of vehicle tracking, but these methods are restricted to particular models only, and can not used in different scenarios.

However, in Jeon et al. (2019), the problem of vehicle tracking is tackled using a single model to represent all the possible motions consisting of both lateral and longitudinal maneuvers which provide the stability of the observer used for the estimation of trajectories of a vehicle. Authors of Rajamani et al. (2020) had proposed a switched gain hybrid observer to overcome the drawback of infeasiblity of LMI's under presence of nonmonotonic nonlinearities. Though the observer approach proposed in Rajamani et al. (2020) is effective in the case of nonlinear and nonmonotonous systems as compared to the other observer methods, the considered system dynamics and outputs are noise-free. Thus proposed method is not useful in case of system containing noise.

One of the major contributions is to propose a solution for resolving the limitation of Rajamani et al. (2020). A nonlinear switched-Gain observer based on $\mathcal{H}_\infty$ criterion is developed in order to eliminate the noise from measurement and process dynamics of the system. Another contribution is development of an alternative approach for the switched gain observer. The existing nonlinear model is transformed using coordinate transformation in order to obtain feasible solution of LMIs. Then, an observer based on LMI approach is developed for the transformed model and the developed observer is called as a nonlinear transformation based observer. Further, these two methods are compared with the help of MATLAB/simulation. The objective of proposing a nonlinear transformation based observer is to highlight the effectiveness of switched gain observer.

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2. NONLINEAR VEHICLE MODEL

In the literature, there are various vehicle models described for vehicle tracking problems in autonomous vehicle application or safety driving application.

A bicycle model from Rajamani et al. (2020) is used for tracking of autonomous vehicle model and it is shown in Fig. 1.

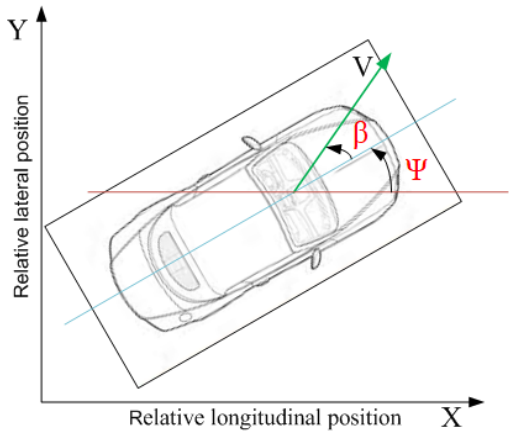

Fig. 1. Vehicle motion model

The state vector is assumed to be

$$x = [X \ Y \ \psi \ \delta_f]^\top, \tag{1}$$

where

- X is a relative longitudinal position of vehicle;
- Y is a relative lateral position of the vehicle;
- ψ is the yaw angle of the vehicle;
- δ_f is a steering angle of the front wheel.

Under the assumption that the derivative of the steering angle is zero, then the model equations are described as,

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\psi} \\ \dot{\delta}_f \end{bmatrix} = \begin{bmatrix} v\cos(\psi) \\ v\sin(\psi) \\ \dfrac{v}{l_f + l_r}\tan(\delta_f) \\ 0 \end{bmatrix}, \tag{2}$$

where

- v is the total velocity of the vehicle which is assumed to be constant;
- l_f and l_r are the distances to the front and rear tires from the center of gravity of the vehicle.

The location of the vehicle is measured with help of a radar or LIDAR sensor. Therefore, the output equations are written as,

$$y \triangleq \begin{bmatrix} X \\ Y \end{bmatrix}, \tag{3}$$

In section 6, the vehicle model described in (2) and (3) is used for validation of both proposed observer.

3. A GENERAL LMI-BASED OBSERVER DESIGN METHOD

Before tackling the estimation problem for the vehicle tracking model (2), we will provide a general observer design method based on feasibility of LMI conditions. We will present in this section, an extension of the LMI-based technique given in Rajamani et al. (2020) to systems in presence of $\mathcal{L}_2-$ bounded disturbances in both the dynamics process and the output measurements. Indeed, what we propose in this paper is twice: 1) First, we generalize the method established in Rajamani et al. (2020) to systems with disturbances and investigate the $\mathcal{H}_\infty-$ optimality criterion; and 2)We show that the switched gain based technique works better than the alternative method (which will be presented in the next section) based on nonlinear transformation of the model. To this end, we consider the family of nonlinear systems described in the following set of equations:

$$\begin{aligned} \dot{x} &= Ff(x) + K\omega \\ y &= Cx + J\omega, \end{aligned} \tag{4}$$

where $x \in R^n, y \in R^p$, and $w \in R^q$ are respectively the system state, the output measurement, and the disturbance vectors. The matrices F, K, C, and J are constants with appropriate dimensions. The nonlinear function $f(.)$ is assumed to under the form

$$f(\Lambda x) \triangleq \begin{bmatrix} f_1(\Lambda_1 x) \\ \vdots \\ f_i(\overbrace{\Lambda_i x}^{v_i}) \\ \vdots \\ f_s(\Lambda_s x) \end{bmatrix}, \qquad \Lambda_i \in R^{1\times n}, \ \Lambda = \begin{bmatrix} \Lambda_1 \\ \vdots \\ \Lambda_s \end{bmatrix}.$$

Also, assume that $f(.)$ satisfies the condition

$$-\infty < a_i \leq \frac{\partial f_i}{\partial v_i}(v_i) \leq b_i < +\infty. \tag{5}$$

As in almost all LMI-based techniques, we consider the following Luenberger state observer:

$$\dot{\hat{x}} = Ff(\hat{x}) + L(y - C\hat{x}). \tag{6}$$

It follows that the dynamics of the estimation error $\tilde{x} \triangleq x - \hat{x}$ is given by

$$\dot{\tilde{x}} = F\tilde{f}(x, \hat{x}) - LC\tilde{x} + (K - LJ)\omega, \tag{7}$$

where $\tilde{f}(x) = f(x) - f(\hat{x})$. The objective is to determine the gain matrix L such that the estimation error satisfies the $\mathcal{H}_\infty-$ optimality criterion.

$$\|\tilde{x}\|_{\mathcal{L}_2^n} \leq \sqrt{\gamma^2\|\omega\|^2_{\mathcal{L}_2^q} + \nu\|\tilde{x}_0\|^2}, \tag{8}$$

where $\gamma > 0$ is the disturbance attenuation level and $\nu > 0$ is to be determined later.

To investigate such optimality criterion, as commonly in LMI context, we use the standard quadratic Lyapunov function

$$V(\tilde{x}) \triangleq \tilde{x}^\top P\tilde{x},$$

where $P = P^\top > 0$.

By developing the derivative of $V(\tilde{x})$ along the trajectory of (7), we get

$$\dot{V}(\tilde{x}) = \zeta \begin{bmatrix} -C^\top L^\top P - PLC & PF & P(K - LJ) \\ F^\top P & 0 & 0 \\ (K - LJ)^\top P & 0 & 0 \end{bmatrix} \zeta^\top, \tag{11}$$

where $\zeta^\top = \left[\tilde{x}^\top \ \tilde{f}^\top \ \omega^\top\right]$.

$$\begin{bmatrix} -C^\top \mathcal{X}^\top - \mathcal{X}C + \mathbb{I}_n - \Lambda^\top \Big[\Gamma_a^\top \Gamma_b + \Gamma_b^\top \Gamma_a\Big]\Lambda & PF + \Lambda^\top(\Gamma_a + \Gamma_b)^\top & PK - \mathcal{X}J \\ F^\top P + (\Gamma_a + \Gamma_b)\Lambda & -2\mathbb{I}_s & 0 \\ \Big(PK - \mathcal{X}J\Big)^\top & 0 & -\mu\mathbb{I}_s \end{bmatrix} \leq 0. \tag{9}$$

$$\begin{bmatrix} -C^\top \mathcal{X}_i^\top - \mathcal{X}_iC + \mathbb{I}_n - \Lambda^\top \Big[\Gamma_a^\top \Gamma_b + \Gamma_b^\top \Gamma_a\Big]\Lambda & P_iF + \Lambda^\top(\Gamma_a + \Gamma_b)^\top & P_iK - \mathcal{X}_iJ \\ F^\top P_i + (\Gamma_a + \Gamma_b)\Lambda & -2\mathbb{I}_s & 0 \\ \Big(P_iK - \mathcal{X}_iJ\Big)^\top & 0 & -\mu\mathbb{I}_s \end{bmatrix} \leq 0 \tag{10}$$

On the other hand, the $\mathcal{H}_\infty$ criterion (8) is satisfied if the following inequality holds:

$$\vartheta(t) \triangleq \dot{V}(\tilde{x}) + \tilde{x}^\top \tilde{x} - \gamma^2 \omega^\top \omega \leq 0. \tag{12}$$

From (11), we get

$$\vartheta(t) = \zeta^\top \begin{bmatrix} -C^\top L^\top P - PLC + I & PF & P(K - LJ) \\ F^\top P & 0 & 0 \\ (K - LJ)^\top P & 0 & -\gamma^2 I \end{bmatrix} \zeta. \tag{13}$$

From the mean value theorem, there exists $\bar{v} \in R^s$ such that

$$\tilde{f}(x, \hat{x}) = \frac{\partial f}{\partial v}(\bar{v})\Lambda\tilde{x} \tag{14}$$

$$= \mathrm{diag}\Big(\frac{\partial f_i}{\partial v_i}(\bar{v}_i), i = 1, \dots, s\Big)\Lambda\tilde{x}. \tag{15}$$

By setting

$$\Gamma_a = \mathrm{diag}\Big(a_i, i = 1, \dots, s\Big),\ \Gamma_b = \mathrm{diag}\Big(b_i, i = 1, \dots, s\Big),$$

we deduce from (5) that we can write

$$\Big(\tilde{f} - \Gamma_a\Lambda\tilde{x}\Big)^\top \Big(\tilde{f} - \Gamma_b\Lambda\tilde{x}\Big) + \Big(\tilde{f} - \Gamma_b\Lambda\tilde{x}\Big)^\top \Big(\tilde{f} - \Gamma_a\Lambda\tilde{x}\Big) \leq 0. \tag{16}$$

Inequality (16) is written under the following matrix form:

$$\zeta^\top \underbrace{\begin{bmatrix} \Lambda^\top \Big[\Gamma_a^\top \Gamma_b + \Gamma_b^\top \Gamma_a\Big]\Lambda & -\Lambda^\top(\Gamma_a + \Gamma_b)^\top & 0 \\ -(\Gamma_a + \Gamma_b)\Lambda & 2\mathbb{I}_s & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathbb{M}} \zeta \leq 0 \tag{17}$$

Consequently, from (13) and (16), the criterion (12) is satisfied if

$$\vartheta(t) - \zeta^\top \mathbb{M}\zeta \leq 0. \tag{18}$$

Now we are ready to state the main theorem.

Theorem 1. Assume there exist a symmetric positive definite matrix $P \in R^{n\times n}$, and a matrix $\mathcal{X} \in R^{n\times p}$, such that the following convex optimization problem is solvable:

$$\min(\mu) \ \text{ subject to (9)}, \tag{19}$$

then for $L = P^{-1}\mathcal{X}$, the estimation error $\tilde{x}$ satisfies the $\mathcal{H}_\infty$ criterion (8) with $\gamma = \sqrt{\mu}$ and $\nu = \lambda_{\max}(P)$.

Proof. Now, as we have (18), the proof is straightforward. Indeed, by using the change of variables $PL = \mathcal{X}$ and $\mu = \gamma^2$, we deduce that (18) holds if the LMI given in (9) is feasible.

4. CASE OF SYSTEMS HAVING NONMONOTONIC NONLINEARITIES

The solution of LMI described in (9) is unfeasible when system dynamics contain nonmonotonous terms and it is the limitation of proposed observer and the solution for it was proposed as a switched gain or hybrid observer in Rajamani et al. (2020). The observer proposed in section 3 is need to be modified as a switched gain observer which will be discussed in first subsection of section 4. Further, an alternative approach for tackling the non monotonic nonlinearities is proposed.

4.1 Switched gain observer design

For the system having nonmonotonous terms, the nonlinear functions do not satisfy the condition defined in (5) which leads to the unfeasible solution of the proposed LMI equation (9). In order to obtain the feasible solution of (9), it is necessary to eliminate the nonmonotonous terms from nonlinear functions. The finite local extrema for a nonlinear function f on a compact set is always exists and it can be represented in piecewise monotonous functions f_i in $\mathcal{S}$ number of regions. Thus, we can write f as f_i where, $i = 1, 2, ..., \mathcal{S}$ in total $\mathcal{S}$ numbers such as each function is monotonous as well as piecewise continuous in its regions. Now, for each f_i, $\mathcal{S}$ numbers of constant gain observers can be developed with the help of (10), where, $i = 1, 2, ..., \mathcal{S}$. As these functions does not have any nonmonotonous terms, it has feasible solution in its region. The stability of this swicthed gain observer is guaranteed because, in each region, the LMI observer with a constant gain is designed with $\mathcal{H}_\infty$ criterion which provides convergence of error in its region.

Theorem 2. Let symmetric positive definite matrices $P_i \in R^{n\times n}$, and matrices $\mathcal{X}_i \in R^{n\times p}$ exit for each i^{th}(where $i = 1, 2, ..., \mathcal{S}$) region as such that the following convex optimization problem is solvable:

$$\min(\mu) \ \text{ subject to (10)}, \tag{20}$$

then for each i^{th} region, gain matrix $L_i = {P_i}^{-1}\mathcal{X}_i$, the estimation error $\tilde{x}$ satisfies the $\mathcal{H}_\infty$ criterion (8) with $\gamma = \sqrt{\mu}$ in each regions.

Proof.

Consider a constant gain observer with gain L_i is designed from the LMI equation (9) in each i^{th} region such that we have total $\mathcal{S}$ number of observers. This leads to LMI equation (10) where $i = 1, 2, ..., \mathcal{S}$.

Without loss of generality, consider the switching of the function f from region i to region j, where $(i,j) \in \{1,2,...,\mathcal{S}\} \times \{1,2,...,\mathcal{S}\}, i \neq j$ and it must take place at time t_s grater than minimum dwell time (T_d) i.e. $(t_s > T_d)$. Consider the standard quadratic Lyapunov functions V_i and V_j with positive definite symmetric matrices P_i andP_j in i^{th} and j^{th} region respectively, such as,

$V_i(t_s) = \tilde{x}^\top P_i \tilde{x}$ and $V_j(t_s) = \tilde{x}^\top P_j \tilde{x}$.

As the function f switches from i^{th} region to j^{th} region, the switching of observers from L_i to L_j will take place. The process of switching must occurred at the time (t_s) grater than the minimum dwell time T_d. According to Rajamani et al. (2020), this switching time condition provides the guarantee of asymptotic stability of observer during switching. Thus, in order to maintain stability of observer during switching, the time required for each switching must be greater than the minimum dwell time T_d. And after every switching, $\vartheta(t)$ goes on decreasing under the condition that each switching take place at time greater than minimum dwell time. It should be noted that inside each region, a single observer with gain L_i satisfies $\mathcal{H}_\infty$ criterion.

Thus, the proposed observer in (20) provides globally asymptotic stability.

The switching of the observer completely depends on f and its regions. Sometimes, we have irregular domain of function or shorter duration for switching of the observer. So, it is very difficult to construct the switched gain observer in these conditions. If the switching of functions from one region to another region takes place in time less than a minimum dwell time, the stability of the proposed observer during switching is not guaranteed. Thus, this proposed switched gain observer is quite difficult to develop. So, in such cases, there is need of an alternate approach.

4.2 Observer design based on transformation

In the previous sub-section, we have seen the development of $\mathcal{H}_\infty$ based switched gain observer. But the drawback of the switched gain observer is the unavailability of switching conditions or less switching time. In such situations, we might fail to develop this observer. Thus, an alternative approach for the switched gain observer is developed in this subsection. With the help of a nonlinear transformation, the existing system is transformed into a new form of system which does not contain any nonmonotonous term. After transformation, any type of observer like a high gain observer, sliding mode observer, or the LMI based observer can be implemented on the transformed model to estimate the state of systems. Here, in this manuscript, LMI based observer is used. For the simplicity, noiseless system dynamics and noiseless outputs are considered.

Consider a transformation $\chi : x \to z$ of class C^2 and a system (4) with $\omega = 0$. It is assumed that χ has invertible Jacobians. After applying the transformation χ on system, the transformed model is described as,

$$\begin{aligned} \dot{z} &= \mathcal{A}z + \eta(z) \\ \bar{y} &= \mathcal{C}z \end{aligned} \tag{21}$$

where, $z \in R^{n_1}$, $\bar{y} \in R^{p_1}$,$\mathcal{A} \in R^{n_1 \times n_1}$, $\mathcal{C} \in R^{p_1 \times n_1}$. Here, n_1 and p_1 be number of state variables and number of outputs in transformed system respectively. $\mathcal{A}$ is triangular matrix with η as nonlinear function. It is assumed that the transformed system does not contain any nonmonotonous term.

The observer for system defined in (21) is designed as,

$$\dot{\hat{z}} = \mathcal{A}\hat{z} + \eta(\hat{z}) + \mathcal{L}(\bar{y} - \mathcal{C}\hat{z}) \tag{22}$$

where, $\hat{z} \in R^{n_1}$ is estimated state of observer with $\mathcal{L} \in R^{n_1 \times p_1}$ as the gain of designed observer. As we already assumed that the transformed system (21) has all monotonous terms, a simple LMI based observer approach is easily used for the design of observer.

The estimated states of observer are in terms of transformed variable z. In order to measure output in its original terms, there is a need for an inverse transformation which is defined as $\chi' : z \to x$. As the transformation function $\mathcal{F}$ is an invertible function, $\mathcal{F}^{-1}$ exists.

Nonlinear transformation of the model: In this subsection, the transformation of the autonomous vehicle model in order to eliminate the nonmonotonous terms from system is illustrated. The vehicle model described in (2) is used here. It is assumed that dynamics and outputs of model are noise-free.

Consider the following nonlinear transformation on the vehicle model (2),

$$\begin{aligned} z_1 &= y_1 = X \\ z_2 &= y_2 = Y \\ z_3 &= \dot{z}_1 = \dot{X} = v\cos(\psi) \\ z_4 &= \dot{z}_2 = \dot{Y} = v\sin(\psi) \\ z_5 &= \dot{z}_3 = \ddot{X} = -v\sin(\psi)\dot{\psi} = -\alpha \times v\sin(\psi) \\ z_6 &= \dot{z}_4 = \ddot{Y} = v\cos(\psi)\dot{\psi} = \alpha \times v\cos(\psi) \end{aligned} \tag{23}$$

where $\alpha = \frac{v}{l_f + l_r} tan(\delta_f)$

With some mathematical calculation, the value of α is obtained as,

$$\alpha = \frac{1}{z_3^2 + z_4^2}(-z_4 z_5 + z_3 z_6)$$

Thus, the transformed system under the transformation χ is defined as:

$$\begin{aligned} \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} &= \begin{pmatrix} z_3 \\ z_4 \end{pmatrix} \\ \begin{pmatrix} \dot{z}_3 \\ \dot{z}_4 \end{pmatrix} &= \begin{pmatrix} z_5 \\ z_6 \end{pmatrix} \\ \begin{pmatrix} \dot{z}_5 \\ \dot{z}_6 \end{pmatrix} &= \left(\frac{-z_4 z_5 + z_3 z_6}{v^2} \right)^2 \begin{pmatrix} -z_3 \\ -z_4 \end{pmatrix} \\ y &= \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \end{aligned} \tag{24}$$

5. IMPLEMENTATION OF THE OBSERVERS

The observers proposed in previous section can be used for many industrial applications. However, in this manuscript, we are using the autonomous vehicle model for observer development. The designing of $\mathcal{H}_\infty$ based switched gain observer is illusrated in this section. Further, the design of an observer for a transformed nonlinear autonomous vehicle is explained.

5.1 Implementation of $\mathcal{H}_\infty$ switched gain observer

This subsection deals with the development of switched gain observer based on $\mathcal{H}_\infty$ criterion. Consider the vehicle model described by (2) and (3) in section 2, along with the noise w present in both system dynamics and measurements obtained from sensors.

The Jacobian matrix for nonlinear vehicular model is described as:

$$\frac{\partial f(x)}{\partial(\Lambda x)} = \begin{bmatrix} -v\sin(\psi) \\ v\cos(\psi) \\ \frac{v}{l_f + l_r} sec^2(\delta_f) \\ 0 \end{bmatrix}; \tag{25}$$

Functions $f_1 = \cos(\psi)$ and $f_2 = \sin(\psi)$ are non-monotonous which leads to the unfeasible solution of LMI equation (9). Hence, It is not possible to apply LMI equation (9) directly on this model. The infeasibility of LMI equations is resolved with the help of the $\mathcal{H}_\infty$ based switched gain observer.

The functions f_1 and f_2 , i.e., $\sin(\psi)$ and $\cos(\psi)$ are monotonic in the region of $0° < \psi < 90°$ and $90° < \psi < 180°$. Thus, in order to obtain the feasible LMI solution, system dynamics is operated in two different regions in which these functions are monotonous. Thus, the value of Γ_a and Γ_b in each region $0° < \psi < 80°$ and $60° < \psi < 140°$ is obtained from (25). In both regions, the LMI equation (10) with $i = 1, 2$ is applied in order to estimate the state of an autonomous vehicle in presence of noise.

5.2 Nonlinear transformation based observer design

The procedure of developing an observer for the transformed nonlinear autonomous vehicle model (24) is illustrated in this section.

Observer design: The gain of an observer defined in (22) is calculated using the LMI equation (26) which is explained in Rajamani et al. (2020),

$$\begin{bmatrix} \eta_1 & -\Lambda^\top(\Gamma_a + \Gamma_b)^\top \\ -(\Gamma_a + \Gamma_b)\Lambda & 2\mathbb{I}_s \end{bmatrix} \leq 0, \tag{26}$$

where $\eta_1 = -C^\top R^\top - RC + \Lambda^\top \left[\Gamma_a^\top \Gamma_b + \Gamma_b^\top \Gamma_a\right] \Lambda + \sigma P$

In (26), $P = P^\top$, $P > 0$ and σ indicates the exponential convergence rate of designed observer and $\mathcal{L} = P^{-1}R$. The solution of this LMI equation is feasible because of monotonous model (24). The terms Λ, Γ_a, and Γ_b are same as the terms defined in section 3 and calculated from (24) and its Jacobian matrix.

Inverse transformation: The proposed observer is used to estimate the transformed variable i.e. z vector, however, the main purpose is to compute the state vector i.e. X, Y, ψ and δ_f of the vehicle model. The vehicle trajectories are computed from the estimated transformed state vectors, with the help of the inverse transformation.

$$\begin{aligned} \hat{x}_1 &= z_1 \\ \hat{x}_2 &= z_2 \\ \hat{x}_3 &= tan^{-1}(\frac{z_4}{z_3}) \\ \hat{x}_4 &= \tan^{-1}\left(\frac{l_f + l_r}{v^3}\right)(z_3 z_6 - z_4 z_5) \end{aligned} \tag{27}$$

In this way, the observer based on nonlinear transformation for the nonlinear autonomous vehicle model is developed.

6. SIMULATION RESULTS

The nonlinear transformation based observer, as well as $\mathcal{H}_\infty$ based switched gain observer are developed for a nonlinear autonomous vehicle model in section 5. In order to illustrate the performances of the two proposed observers, both observers are implemented in MATLAB/simulink. The obtained results are summarized in the next subsections. For simulation of a vehicle model, its parameters are considered as $l_f = 1.35m$, $l_r = 1.45m$, $v = 10\ m/s^2$ and $\delta_f = 3.5rad$.

6.1 Results of $\mathcal{H}_\infty$ based switched gain observer

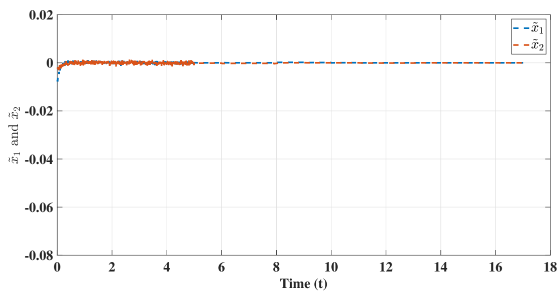

Fig. 2. Error in vehicle trajectory estimation

In the MATLAB/Simulink environment, the proposed $\mathcal{H}_\infty$ based switched gain observer for autonomous vehicle is implemented in MATLAB. In the Fig. 2, estimation errors $\tilde{x}_1$ and $\tilde{x}_2$ of observer is shown. In simulation, some noise in output is considered and from the figure, it is easily shown that the proposed $\mathcal{H}_\infty$ based switch gain observer estimate outputs x_1 and x_2 efficiently under noisy condition. Further, the estimation error of ψ is illustrated in the Fig. 3 where the term $\hat{\psi} - \psi$ indicates the error in yaw angle ψ.

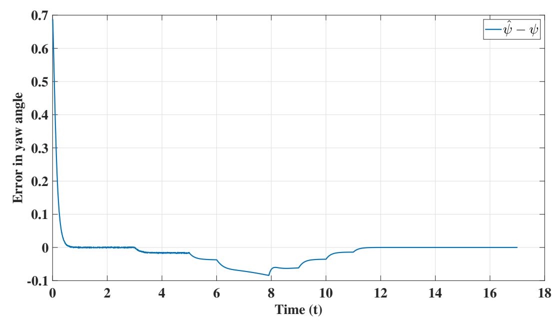

Fig. 3. Error in yaw angle estimation

The performance of $\mathcal{H}_\infty$ based switched gain observer for nonlinear autonomous vehicle model is effective under presence of noise in system dynamics which is validated with help of results obtained from MATLAB/simulink.

6.2 Results obtained from nonlinear transformation based observer

Similarly, the proposed nonlinear transformation based observer for autonomous vehicle is implemented in MATLAB and with the help of the inverse transformation (27), states of vehicle model are computed.

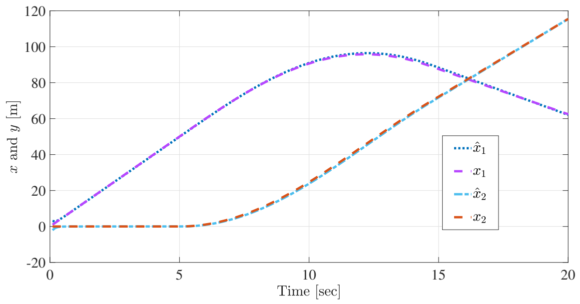

Fig. 4. Estimation state vectors x_1 and x_2

The Fig. 4 shows the estimation of state vectors x_1 and x_2 and it has high accuracy which helps to validate the transformation based observer method. However, in the case of estimation of ψ, the proposed nonlinear transformation based observer fails. The Fig. 5 is elaborating the estimation of ψ vector, which shows that the estimated $\hat{\psi}$ from inverse transformation is not the same as ψ of nonlinear autonomous vehicle model. Thus, the method of estimation using nonlinear transformation fails in case of the motion tracking of autonomous vehicle model.

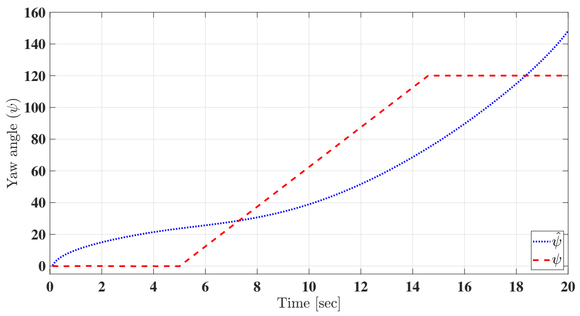

Fig. 5. Estimation state vector ψ

After comparing the performance of two proposed observer, the switched gain observer shows accurate and efficient estimation over the nonlinear transformation based observer. One of the state variable i.e. ψ of nonlinear autonomous vehicle model is not estimated accurately in nonlinear transformation based observer. However, with the help of switched gain observer, all state are estimated with very minute error. If the system dynamics contain noise, observer implementation is failed, while in case of switched gain observer, the problem of noise in system dynamics or in output is resolved with the $\mathcal{H}_\infty$ based switched gain observer and it is another drawback of nonlinear transformation based observer.

7. CONCLUSION

This paper addressed the problem of nonlinear observer design for a vehicle tracking problem where the nonlinearities of the kinematic model are all non-monotonic in which two design methods has been proposed. The first one consists of the extended result of Rajamani et al. (2020) to systems with disturbances in both the system process and in the output measurements. The $\mathcal{H}_\infty-$ optimality criterion has then been combined with the switched gain based observer. To overcome the nonmonotonic nonlinearities of the model, a second and alternative method has been proposed which is based on the use of a specific nonlinear transformation to put the system with new coordinates in a particular structure allowing the design of a nonlinear observer by using any known design method available in the literature, like high-gain methodology or LMI-based approach. Although mathematically the nonlinear transformation based technique is more systematic than the switched gain based approach, it is unfortunately very sensitive to measurement noises. This has been shown through numerical simulations in the section 6.

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